A Model for Designing Functionally Gradient Material Joints

An analytical model allows estimation of thermally induced stresses in discrete-layer FGM joints, facilitating design for processing

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ABSTRACT. An analytical, thin-plate layer model was developed to assist research and development engineers in the design of functionally gradient material (FGM) joints consisting of discrete steps between end elements of dissimilar materials. Such joints have long been produced by diffusion bonding using intermediates or multiple interlayers; welding, brazing or soldering using multiple transition pieces; and glass-to-glass or glass-to-metal bonding using multiple layers to produce matched seals. More recently, FGM joints produced by self-propagating high-temperature synthesis (SHS) are attracting the attention of researchers. The model calculates temperature distributions and associated thermally induced stresses, assuming elastic behavior, for any number of layers of any thickness or composition, accounting for critically important thermophysical properties in each layer as functions of temperature. It is useful for assuring that cured-in fabrication stresses from thermal expasion mismatches will not prevent quality joint production. The model's utility is demonstrated with general design cases.

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Introduction

Joining of dissimilar materials into hybrid structures to meet severe design requirements is becoming more necessary and common (Ref. 1). Joints between heat-resistant or refractory metals and refractory or corrosion-resistant ceramics and intermetallics are especially in demand for advanced aerospace engines and airframes, advanced energy generation and conversion system components, chemical processing plant components, and advanced automobile engines (Refs. 2, 3). The drivers for such hybrid structures are multifold and include: 1) optimization of mechanical properties (e.g., strength, modulus, toughness); 2) optimization of environmental utility and durability (e.g., wear,-corrosion-, and oxidation-resistance); 3) attainment of special properties or combinations of properties (e.g., special electrical, optical, magnetic or thermal properties with strength); 4) minimization of weight; 5) minimization of raw material costs and conservation of limited material resources; 6) ease of fabrication; 7) minimization of fabrication and life-cycle costs; and, increasingly, 8) consideration of environmental compatibility (Refs. 3, 4).

KEY WORDS

Functionally Gradient Materials
Diffusion
Brazing/Welding
FGM Joints
Brazing and Soldering
Modeling
SHS Joining
Temperature Distributions
Stress Calculations
Joint Design
Dissimilar Materials

Joining dissimilar combinations of oxide and nonoxide ceramics, intermetallics, glasses, and heat-resistant or refractory metals and alloys, whether in monolithic or reinforced forms, poses particular challenges (Refs. 2, 3, 5). Drastic differences in atomic structure, chemical composition, and physical or mechanical properties between different material types cause serious problems of incompatibility that prevent direct joining by fusion welding and, often, by nonfusion welding relying on plastic deformation, and pose challenges even to indirect methods employing heterogeneous filler materials (e.g., brazing or soldering) (Ref. 3).

Joining techniques using functionally gradient materials or FGM joints bridge incompatibilities in chemistry and properties by changing composition from one joint element to the other. Composition changes can be accomplished in steps using discrete layers or continuously by blending materials from a composition that is compatible with (even if not identical to) one end element to a composition that is compatible with (even if not identical to) the other end element. In either case, property differences or mis-

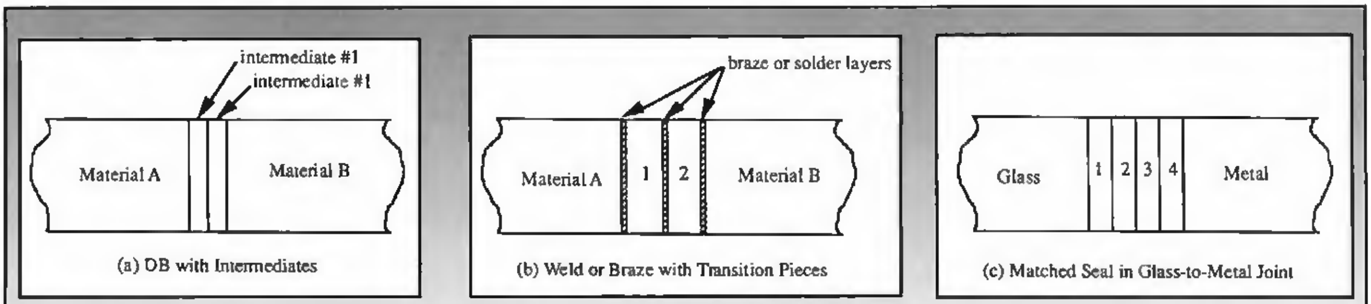

Fig. 1 — Schematic of stepped functionally gradient material (FGM) joints in: A — diffusion brazing/welding using single or multiple intermediates or interlayers; B — welded, brazed or soldered joints using multiple layers or transition pieces; C — multiple transition pieces used in producing "matched seals" between glasses or between a glass and a metal.

matches that would otherwise preclude atomic bonding or lead to bond failure through mismatch stresses are avoided or, at least, are lessened.

The use of discrete layers for joining is not new. Single or multiple interlayers are commonly used in solid-state diffusion welding (Refs. 7, 8), multiple layers or transition pieces are commonly used in "sandwich" brazing and in soldering of dissimilar metals or alloys (Refs. 9–11), of metals to ceramics or glasses (Ref. 2), or even of different glasses in so-called "matched seals" (Ref. 12). While such intermediates can perform many functions, the most important is almost always to gradually change the coefficient of thermal expansion across the joint to preclude development of destructively high stresses. Examples of the use of such stepped FGMs are shown in Fig. 1.

The use of continuous functionally gradient materials for joining is rather new, as, in the past, they could not be easily accomplished due to lack of suitable processing options (*e.g.*, plasma spraying, chemical or physical vapor deposition).

Self-propagating high-temperature synthesis (SHS) offers particular promise as a joining process for many difficult-to-join ceramic or intermetallic materials or dissimilar material combinations (Refs. 13, 14). Briefly, the process involves the reaction of powdered elemental solids, solids and gases, or elements and compounds to produce a new compound or compound and element with the release of substantial exothermic heat of formation. The reaction begins when the mixture reaches a triggering or initiation temperature (Ti), raises the temperature locally at a reaction front through adiabatic heating to a combustion temperature (Tc), and the reaction progresses spontaneously using the heat of formation to drive the reaction front.

The process is attractive for joining for several reasons, including: 1) capability for generating high temperatures to enable atomic bonding through localized melting and wetting of substrates or through solid-state reactions or interdiffusion with substrates; 2) minimal heat effect in substrates due to the speed of the reaction and the highly localized nature of heating caused by the reaction; 3) high-energy efficiency (with the majority of heat needed for joining being generated internal to the process); 4) facility for production of functionally gradient material (FGM) joints; 5) ability to incorporate reinforcing phases in the filler material (for joining composites or producing composite filler materials *in situ*); and 6), and not incidentally, essentially the same process for joining as was used or could have been used to produce the joint end elements in the first place (Refs. 15, 16). Beyond these advantages, SHS has potential for joining as either a primary or secondary process, where in primary joining end elements are joined at the time they are, themselves, being synthesized and where in secondary joining preexisting joint components are joined as in classic welding and brazing.

As promising as SHS is for producing FGM joints between dissimilar materials, processing engineers engaged in research and development, and often in production, need to design these joints to overcome process-induced stresses from mismatch of coefficients of thermal expansion (CTE) between end-elements and/or intermediates. Likewise for the fabrication of discrete FGM joints by other processes. Such "cured-in" stresses arise during joint fabrication due to the wide excursion in temperature between the temperature at which the joint is produced and room temperature. If too high, they can cause adhesive failure at interfaces or cohesive failure immediately adjacent to interfaces in weak-boundary layers. Thermally induced stresses can also arise in service from excursions of temperature and/or gradients in temperature associated with operation, but these are routinely dealt with rigorously by structural designers.

While other critical design requirements of FGM joints are that they bridge chemical incompatibilities between end elements, provide required mechanical properties in the joint (*e.g.*, strength, modulus, or toughness, as required), and exhibit microstructural stability under service temperatures, coefficient of thermal expansion (CTE) mismatch is most important during joint formation.

Objective

The objective of the work presented here was to develop a simple model for assisting with the design of functionally gradient material joints consisting of discrete steps or layers between dissimilar end elements. The model was to be capable of predicting the temperature distribution and associated thermally induced stresses in such joints for a wide variety of materials, and to facilitate design to reduce cured-in stresses associated with joint fabrication, regardless of the process employed.

Description of Model

Heat Transfer Analysis and Temperature Distributions in FGM Joints

The basic differential equation of heat conduction for a three-dimensional Cartesian coordinate system is:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_v\frac{\partial T}{\partial t} \quad (1)$$

where x, y and z are the three Cartesian directions, k is the thermal conductivity, ρ is the density, C_v is the volume specific heat, and q is the value of any internally generated heat. To determine the heat distribution in a typical FGM joint during cooling following processing (or under the influence of an imposed operating temperature gradient), one can consider the joint to be a simple planar slab with constant surface area, ends at prescribed boundary temperatures of T_1 and T_2, and with no internal heat generation, so the problem reduces to a case of one-di-

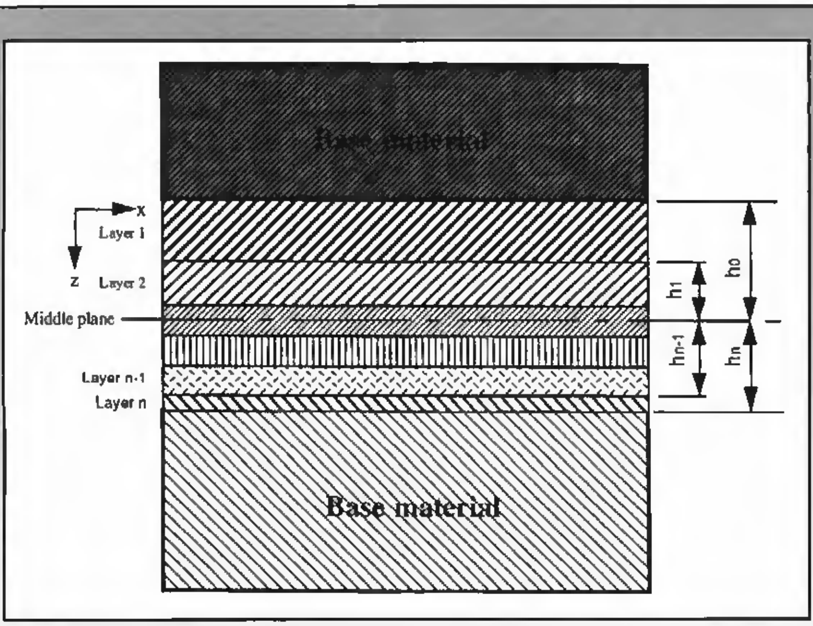

Fig. 2 — Schematic of a typical multilayered FGM joint.

mensional, steady-state heat conduction. Thus, Equation 1 reduces to

$$\frac{d}{dz}\left(k\frac{dT}{dz}\right)=0 \quad \text{or}$$
$$\frac{d^2T}{dz^2}+A(T)\frac{dT}{dz}=0 \qquad (2)$$

$$\text{where } A(T)=\frac{\partial lnk}{\partial z}.$$

The temperature distribution in this slab can be obtained by integrating Equation 2 twice, using the conditions T_1 (for $z = z_1$), T_2 (for $z = z_2$), giving

$$T(z)=C_0'+C_0\int_{T1}^{T2} exp^{-\int_{T1}^{T2}A(T)dz}dz \qquad (3a)$$

where C'_0 and C_0 are constants.

The temperature profile T(z) resulting from Equation 3A, above, depends solely but strongly on thermal conductivity k. If k is a constant, T(z) is linear, and Equation 3A becomes

$$\frac{T-T_1}{T_2-T_1}=\frac{z-z_1}{z_2-z_1} \qquad (3b)$$

But, if, as is usually the case in real materials, k is a function of temperature (Ref. 17), T(z) is nonlinear.

In FGM joints, material in the joint experiences wide excursions of temperature during fabrication and both wide excursion and, often, steep gradients in service. Therefore, thermal conductivity must be considered as a function of temperature, so the temperature distribution T(z) in FGM joints must be expressed as

$$T_i(z)=C_i'+C_i\int_{T_1}^{T_2} exp^{-\int_{T_1}^{T_2}\frac{\partial lnk}{\partial z}dz}dz \qquad (3c)$$

where i represents a particular layer in a multilayered joint as shown in Fig. 2.

Thermally Induced Stresses in FGM Joints

As the temperature of a material changes, its length, width and thickness change in response. The incremental change is called the linear coefficient of thermal expansion (CTE). For most materials, linear dimensions increase with increasing temperature, and they increase at different rates in different crystallographic directions. The change in dimensions arising from temperature changes or differences leads to thermally induced stresses when the material is not free to expand or contract as required. Such thermally induced stresses can arise during fabrication of joints or in joints in service, and can, in either case, lead to joint failure if stresses rise high enough. The challenge, then, is to calculate the stresses that are induced in a material joint due to the prevailing temperature distribution calculated from the general heat flow equation presented earlier.

The ratio of thickness to length or width for an individual layer in a FGM joint is almost always very small. Thus, a thin plate assumption can be used to derive thermal stresses. The normal strain, ε^T_z, is neglected under this assumption, reducing the thermal strains in a Cartesian system to ε^t_x, ε^t_y and ε^T_{xy}. As stated above, thermal strains develop in every layer of material as a result of temperature changes or differences from excursions or gradients. The thermal strain, $\{\varepsilon^T\}$, is equal to the product of the coefficient of thermal expansion, α, of the material and the change in temperature, ΔT, given by $\{\varepsilon^T\} = \alpha\Delta T$. The coefficient of thermal expansion $\{\alpha\}$, however, varies with temperature (Ref. 17). Again, since FGM joints are subjected to large temperature changes either during fabrication or in service or both, the coefficient of thermal expansion $\{\alpha\}$ must be considered as a function of temperature.

Thermal strains do not produce stresses when the material is completely free to expand, bend and twist. However, an individual layer in a multilayered material is not completely free to so deform. The deformation of each and every layer is influenced by adjoining layers, and thermal stresses are induced by the constraints placed on deformation. The strains that cause stresses are given by:

$$\begin{Bmatrix}\varepsilon_x^s\\ \varepsilon_y^s\\ \varepsilon_{xy}^s\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0+z\gamma_x\\ \varepsilon_y^0+z\gamma_y\\ \gamma_{xy}^0+z\gamma_{xy}\end{Bmatrix}-\alpha\Delta T \qquad (4)$$

where $\{\varepsilon^0\}$ and $\{\gamma\}$ are midplane strains and midplane curvatures. Therefore, the induced thermal stresses can be obtained by

$$\begin{Bmatrix}\sigma_x^T\\ \sigma_y^T\\ \sigma_{xy}^T\end{Bmatrix}=[Q]\begin{Bmatrix}\varepsilon_x^0+z\gamma_x\\ \varepsilon_y^0+z\gamma_y\\ \gamma_{xy}^0+z\gamma_{xy}\end{Bmatrix}-\alpha\Delta T \qquad (5)$$

where [Q], a matrix, represents material stiffness within a layer in each and every direction. For an isotropic material, the [Q] matrix is

$$[Q]=\begin{bmatrix}\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0\\ \frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0\\ 0 & 0 & G\end{bmatrix} \qquad (6)$$

This [Q] matrix affects thermal stresses, and its components contain the material properties of elasticity modulus (E), shear modulus (G), and Poisson's ratio (v). Once again, these material properties are affected by temperature changes (Ref. 17) and must be considered as functions of temperature in FGM joints.

In Equation 5, the midplane strains, $\{\varepsilon^0\}$, and the curvatures, $\{\gamma\}$, are unknown, but can be evaluated from the equivalent forces $\{N^T\}$ and moments $\{M^T\}$ in

$$\begin{Bmatrix}\varepsilon^0\\ \gamma\end{Bmatrix}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{bmatrix}N^T\\ M^T\end{bmatrix} \qquad (7)$$

where [A], [B] and [D] are the extensional stiffness matrix, the coupling matrix, and the bending stiffness matrix for multilayered materials, respectively. They are, in turn, defined as

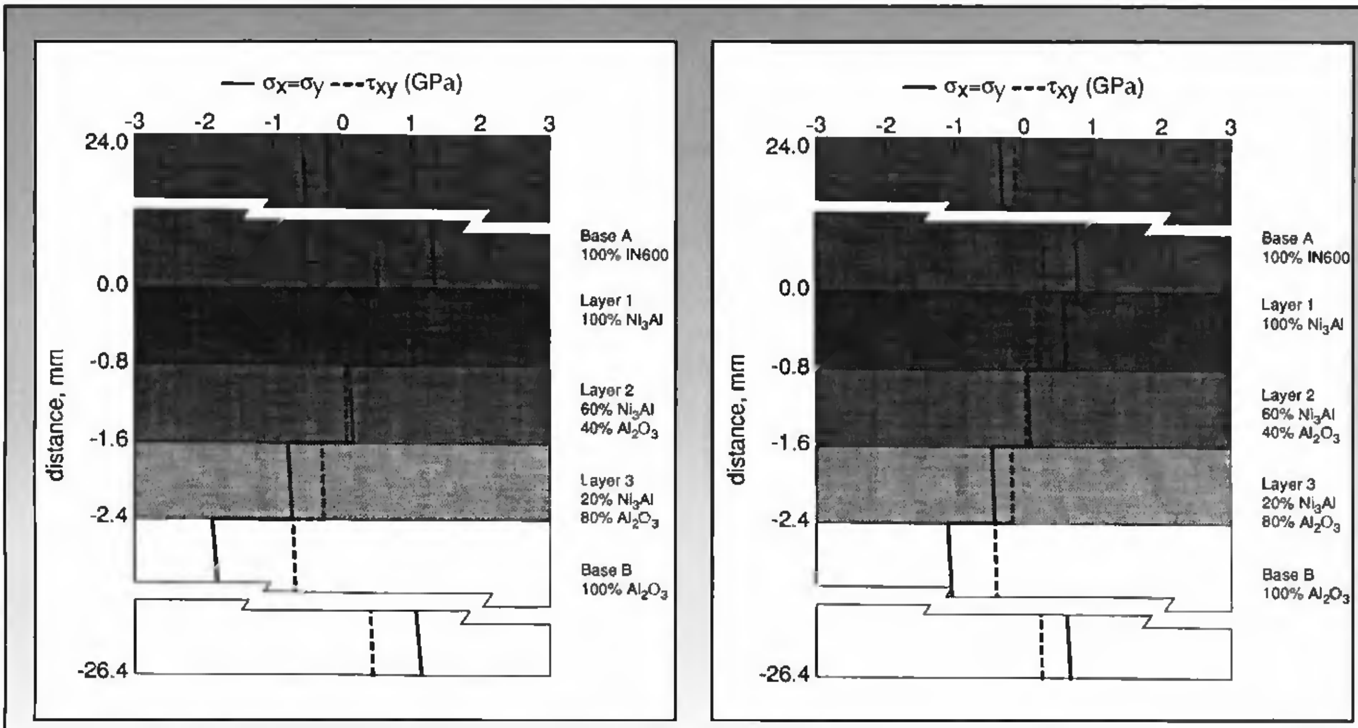

Fig. 3 — Predicted cured-in stresses for two different simulated processing cycles. A — 1500 to 300 K; B — 1000 to 300 K.

$$[A_i]=\sum_{i=1}^{n}[Q_i](h_i-h_{i-1}) \tag{8a}$$

$$[B_i]=\frac{1}{2}\sum_{i=1}^{n}[Q_i]\left(h_i^2-h_{i-1}^2\right) \tag{8b}$$

$$[D_i]=\frac{1}{3}\sum_{i=1}^{n}[Q_i]\left(h_i^3-h_{i-1}^3\right) \tag{8c}$$

The thermal forces in a layer are produced by the constraints placed on its free deformation by adjacent layers. Because the coefficient of thermal expansion of an individual layer and its adjacent layers are different, a temperature change (or difference) will result in unequal thermal strains so thermal moments are induced. The thermal forces and moments can be obtained by

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \Delta T \times \sum_{i=1}^{n}[Q_i]\alpha_i\left(h_i-h_{i-1}\right) \tag{9}$$

and

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \frac{1}{2}\Delta T \times \sum_{i=1}^{n}[Q_i]\alpha_i\left(h_i-h_{i-1}\right) \tag{10}$$

where h_i and h_{i-1} define the position of a layer in a multilayered material — Fig. 2.

From the above analysis, it is clear that the magnitude of induced thermal stresses depends on the thickness of individual layers, the number of layers, the temperature change or prevailing gradients, and material properties (α, k, E, G and ν) as functions of temperature. In order to reduce cured-in stresses from fabrication processes and residual stresses arising during service, a computer program has been developed to analyze the relationship between thermal stresses and joint design parameters. The source code for computing temperature distribution and thermal stresses was developed using FORTRAN computer language on an IBM ES/9000 Model 580 large-scale computer using the MTS (Michigan Terminal System) operating system. This source code can be transferred to any computer system (*e.g.*, Macintosh, IBM PC, etc.) provided a FORTRAN compiler is available.

Results Obtained for Typical Cases

The true test of the utility of a model is its ability to predict outcomes for meaningful situations or cases. Ultimately, predictions should be verified by experimental measurements (*e.g.*, here, residual stresses using neutron diffraction). Initially, however, a test of the validity and utility of a model's predictive capability can be seen from whether it allows cured-in stresses to be reduced to permit successful joint fabrication. Also, predictions of the model can be used to identify general effects of joint design changes on process-induced stresses.

To assess and demonstrate the utility of the analytical thin-plate layer model described here, the results obtained for several cases representative of the variety of situations and conditions that must be dealt with during the production of FGM joints by SHS or other joining processes are presented below. The relationship of each case to general joining is emphasized, where appropriate.

For illustration here, in each case the joint consists of a ductile, nickel-based alloy (Alloy 600) end element (with a nominal CTE of 14 x 10^{-6} K^{-1}) joined to a brittle, alumina ceramic (Al_2O_3) end element (with nominal CTE of 6 x 10^{-6} K^{-1}) using multiple steps or layers of gamma (γ) nickel aluminide (Ni_3Al) (with nominal CTE of 12 x 10^{-6} K^{-1}) mixed with various volume fractions of alumina (Al_2O_3) particles. Overall joint processing temperatures and individual end-element operating temperatures were varied to produce different cured-in or service-induced residual stresses. Joint end elements were taken to be either circular or square in cross-section to make the normal stresses σx and σy the same (*i.e.*, σx = σy). The assembled joint was free to expand or contract in its axial (or longitudinal) direction, making the normal stress σ_z zero. The particular joint material combination is not important to this presentation; what is important is the effect of various design changes on the magnitude of thermally induced normal (σ_x and σ_y) and shear (σ_{xy} or τ_{xy}) stresses.

Before employing the model for more complicated discrete FGM cases, its gen-

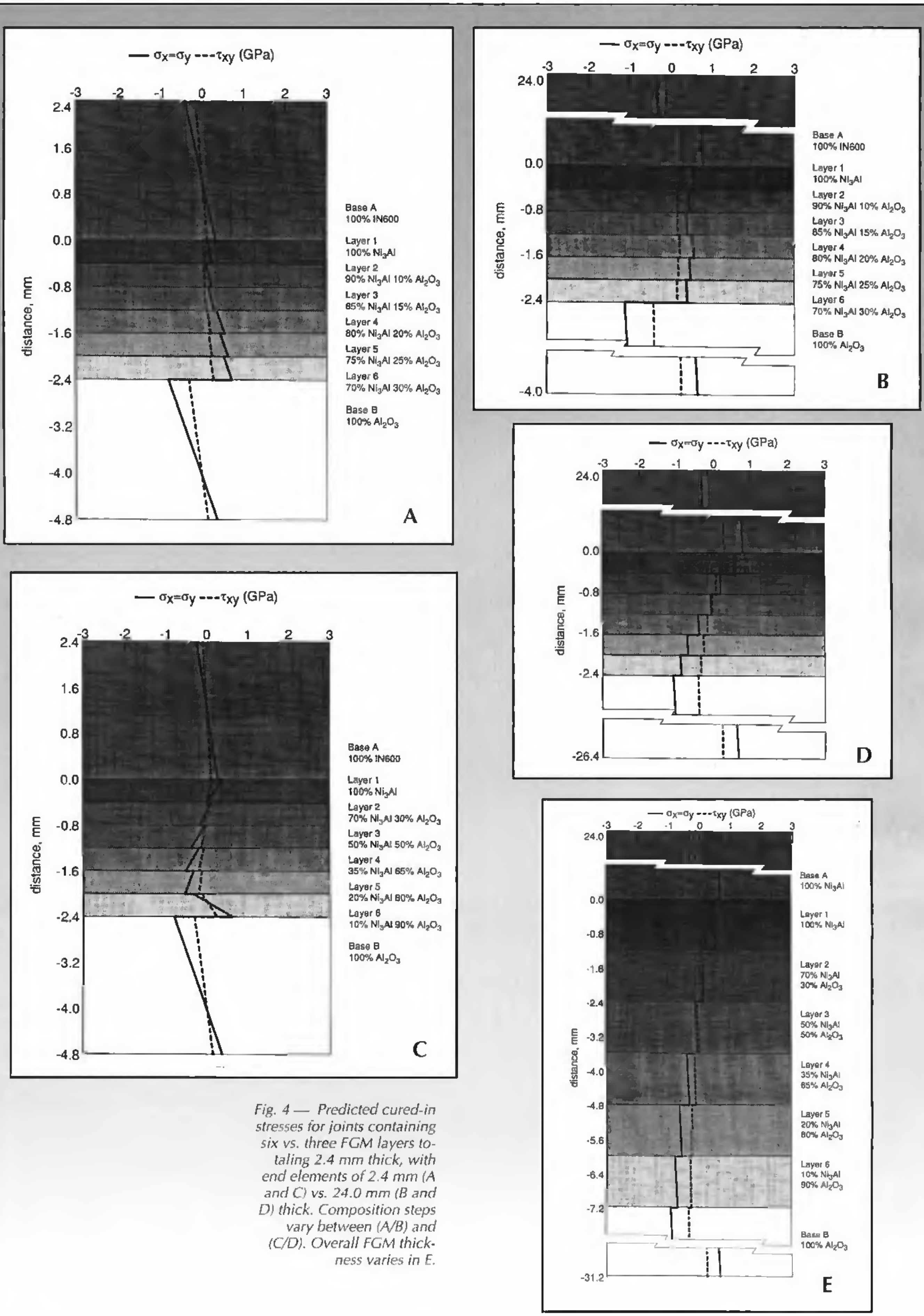

Fig. 4 — Predicted cured-in stresses for joints containing six vs. three FGM layers totaling 2.4 mm thick, with end elements of 2.4 mm (A and C) vs. 24.0 mm (B and D) thick. Composition steps vary between (A/B) and (C/D). Overall FGM thickness varies in E.

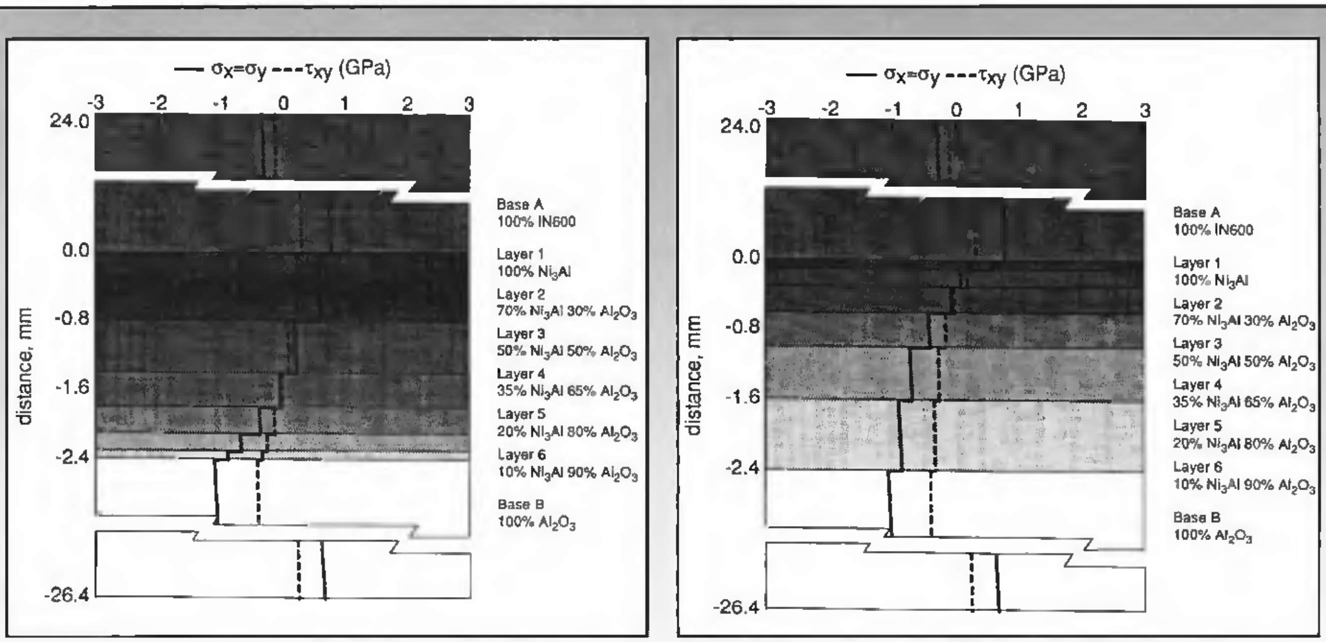

Fig. 5 — Predicted cured-in stresses for a 2.4-mm-thick joint consisting of six steps of equal, incremental composition change but unequal thickness and arrangement between (A) and (B).

eral validity was tested for the trivial case of several thin layers of Alloy 600 sandwiched between thick Alloy 600 end elements. As it should, where there is no mismatch of CTE between joint elements or interlayers, the model predicted no residual stresses were cured-in following an excursion from 1300 K during reaction to 300 K following cooling to room temperature. For all cases, equilibrium (*i.e.*, balance) between tensile and compressive stresses was tested and found to exist.

Variable Temperature Excursions

The production of FGM joints by SHS requires either of two conditions: 1) that the end elements of the joint are kept at a reasonably moderate temperature while the combustion reaction adiabatically raises the temperature of the joint filler material and immediately surrounding material to much higher temperatures, thereby producing a substantial temperature gradient through the joint for a short time; or 2) that the joint end elements are heated to near the reaction-initiation temperature, the reaction is triggered and the atomic bond is produced, and, then, the assembly is cooled back to room temperature, in which case a wide excursion in temperature takes place. The first condition represents the propagation mode of SHS, while the second represents the simultaneous combustion mode (Ref. 13). The second condition is far more representative of what occurs with most other joining options, including diffusion welding, brazing, soldering and glass-bonding, *i.e.*, the temperature of substrate is raised to accelerate diffusion or promote melting, wetting and spreading of filler material (and, possibly, substrate). Nonfusion welding by friction would be representative of the first condition. Both conditions, regardless of specific process, lead to thermally induced stresses.

Model predictions for two different temperature excursions for two different, simulated processing cycles (*e.g.*, reaction and bonding at 1500 and 1000 K) are shown in Fig. 3A and B, respectively, for a functionally gradient material joint consisting of three equally thick (0.8-mm), equally incremental composition layers between much thicker (24.0 mm) end elements[1]. As would be expected, the greater temperature excursion (a difference of 1200 vs. 700 K) produced greater normal and shear stresses. The general distribution of stresses (between tensile and compressive components) was quite similar. The magnitude of the cured-in residual stresses developed were of the same order as those predicted by other investigators using a simple finite element model in FGMs between NiAl to Al_2O_3, *i.e.*, 1–2 GPa, with high compressive stresses occurring near and in the alumina end element (Ref 18).

Variable Overall Joint Thickness

Relatively little can be done to change the magnitude of temperature excursions and only slightly more can be done to reduce prevailing gradients imposed during joining, so cured-in stresses must be dealt with in other ways. One apparent approach is to vary the overall thickness of the joint using some fixed number of intermediate, graded composition steps, intermediates or transition pieces. Such a case was run for a joint like that run above (in Fig. 3A) with 24.0-mm-thick end elements, but with the grading of composition occurring in three steps of much greater thickness (*i.e.*, 2.4 mm vs. 0.8 mm each, 7.2 vs. 2.4 mm overall). Increasing the overall graded joint thickness lowered the normal and shear stresses slightly, but had little effect on the direction of the stresses. It was clear that much greater effects could be obtained by varying the number of layers and/or the incremental change in composition from layer to layer.

Decreasing the overall thickness of the FGM joint layers (toward zero thickness as a limit), increased the level of stresses. For many other joining options, the use of actual transition pieces, as opposed to simple composition layers, greatly increases the thickness of the joint, so mismatch stresses can be much more effectively lowered by spreading them out. Examples prevail in diffusion brazing between metals and ceramics and in glass-to-metal joining. In such

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[1] *Because the very thick end elements would have forced the plot of stresses to be severely compressed if shown to scale, end elements were broken, showing the value of residual stress at the interface to the FGM and at the end of the end element itself. Within the end elements, stress varied linearly from the value at the interface back to the end of the element.*

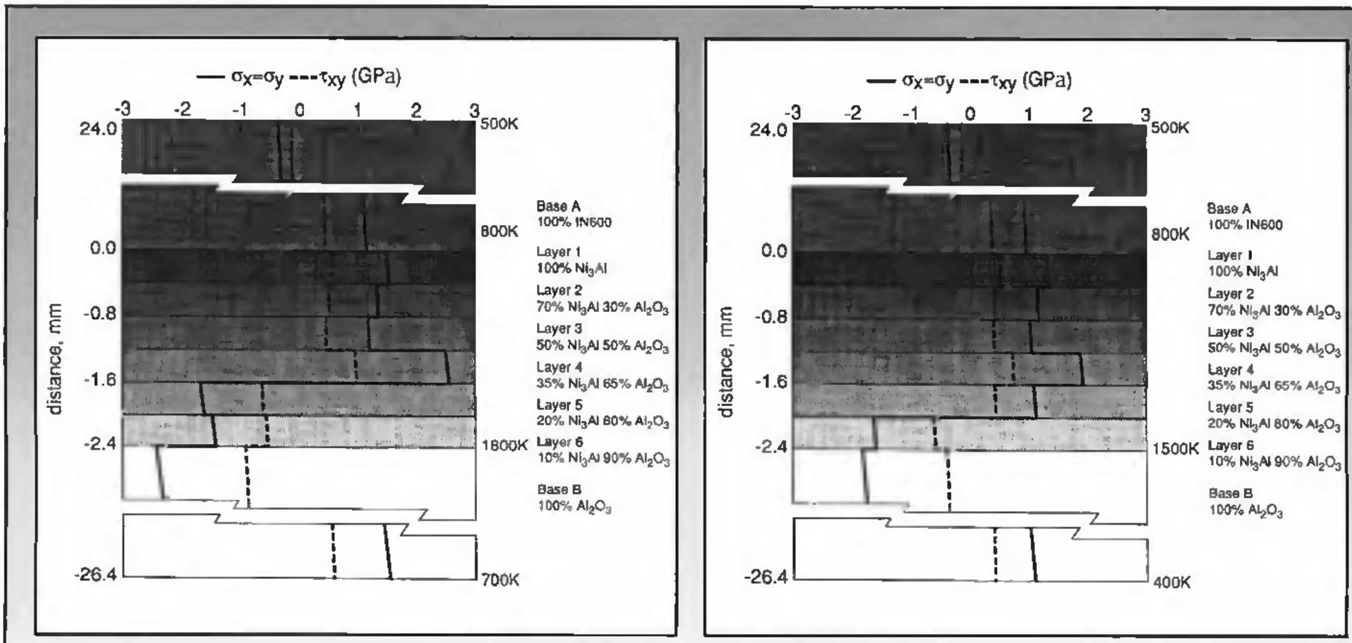

Fig. 6 — Predicted stresses for a six-layer FGM under two different prevailing temperature gradients. A — A gradient of 1000 K; B — A gradient of 700 K.

joints, however, stresses are often not purely elastic, as yielding occurs in ductile metals used as interlayers.

Variable Number of Steps

Another approach for reducing cured-in stresses during joining is to change the number of incremental composition steps or interlayers across the joint, provided the greater number of steps reduces the incremental mismatch in CTE between adjacent layers. Figure 4A and B shows the result of changing the number of steps between end elements from three (in Fig. 3A and B) to six. In each case, the overall joint thickness is the same (*i.e.*, 2.4 mm), although the thickness of end elements differs between Fig. 4A (2.4 mm thick) and 4B (24.0 mm). In each case, the change in composition across the joint is made in equal increments of equal thickness. The excursion from processing at 1000 K to room temperature (300 K) is also held constant. The effect of increasing the number of steps, reducing the mismatch from step to step, was to lower the normal and shear stresses (compare Figs. 3B and 4A). Increasing the number of steps to reduce cured-in stresses is most common in the production of matched seals between glasses of significantly different CTE or between glasses and metals, although there are other examples in diffusion welding/brazing (e.g., of gray cast iron to ceramics) and brazing using transition pieces between metals and ceramics.

In the limit, increasing the number of steps to infinity would very drastically, but not totally, reduce the cured-in stresses. This is one of the advantages of employing continuous-gradient FGMs.

Changing the Thickness of Joint End Elements

Changing the thickness of the joint's end elements (from 24.0 to 2.4 mm) relative to the graded joint thickness (2.4 mm) had some effect on the distributions for both normal and shear stresses, but did not change the magnitude of these stresses very much. In general, thicker end elements tended to increase cured-in stresses somewhat, due to their greater restraining effect. These effects can be seen by comparing Fig. 4A and 4B to one another and to Fig. 3B.

Changing Composition Increments between Steps

Clearly, the most significant effect one can have on cured-in stresses is to minimize the mismatch between the CTE of the end elements by appropriate selection of composition in the graded joint interlayers, minimizing the difference in CTE between adjacent layers[2]. This effect is shown in Fig. 4C and D for end element thicknesses of 2.4 and 24.0 mm, respectively. The number of steps is six, just as in Fig. 4A and B, but the compositions of steps has been changed to drastically reduce the severity of the mismatch from the last interlayer to the alumina end element. The result is a slight lowering of both normal and shear stresses, and a change in the distribution of stresses later-to-layer (comparing 4C to 4A and 4D to 4B). This approach is widely employed in assembling brittle joints, as between a glass and a metal. As a rule of thumb, the increment of CTE between adjoining layers should not exceed about 10%.

Figure 4E shows a joint comprised of the same number and composition of steps as in Fig. 4D, only the thickness of individual steps, as well as the overall joint, are increased. The effect is to lower normal and shear stresses slightly.

Steps of Different Thicknesses

In many dissimilar material joints, the material in one end element is better able to accommodate strain than that in the other, although this cannot show up in purely thermoelastic analysis if yielding occurs. In general, however, cured-in stresses can be reduced by using steps of different (*i.e.*, unequal) thickness, with thinner steps of greater mismatch near the higher ductility end of the joint and thicker steps of smaller mismatch near the brittle end of the joint. Figure 5A and B compare the results for joints consisting of six steps of different thicknesses vs. steps of equal thickness, but similar composition increments, as shown in Fig. 4D. In Fig. 5A, the thickest step abuts the more ductile end element, while in Fig. 5B the thicker step abuts the more brittle end element. As shown in Fig. 5B, much

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[2] *Without question, there are interaction effects among layers not immediately adjacent to one another. That is, a restraint due to a CTE mismatch can be felt across several interlayers.*

more of the joint can be placed in compression by increasing layer thicknesses near the brittle end element, even though the magnitude of the stresses is not altered much.

Obviously, both the thickness of individual steps and the incremental change in composition between adjoining steps can be varied simultaneously to possible advantage, and this is frequently done, as in glass-to-metal sealing.

Different Steady-State Temperature Gradients

A temperature gradient can exist across an FGM joint due either to the effect of localized heating during joint formation using the propagation mode or in service due to different exposure temperatures imposed at each of the end elements. Figure 6A and B show the states of stress for two different prevailing gradients in a six-layer FGM of the same compositions as the joint in Figure 4D: for a gradient of 1000 K in 6A and for a gradient of 700 K in 6B. As expected, the stresses are higher for the more severe gradient.

Different Constitutive Laws

Whenever different materials are mixed to create a composite, it is obvious the various properties of the composite are a function of its constituents' properties. What is not obvious is what constitutive law governs the composite's properties. A widely used constitutive law is $X_c^n = v_1X_1^n + v_hX_h^n$, where v_l and v_h are the volume fractions of the lower and higher modulus constituent, respectively, X is an appropriate property, such as modulus of elasticity (E), coefficient of thermal expansion (α), thermal conductivity (k), mass diffusivity (D), or electrical conductivity (σ), and where n has a value of 1, 1/2, 0 (actually, approaching 0, say ± 0.01), –1/2, or –1, depending on the relative values of the modulus of the reinforcement vs. the matrix, starting with a low modulus reinforcement at n = 1 and moving to an extremely high modulus reinforcement at n = –1. The constitutive law used in all cases (shown in Figs. 3–6) is that above, with n = 1. Other constitutive laws have been tested, but results are not shown. The importance of constitutive law is rather uniquely important to SHS, although it could be important in joining of composite or monolithic materials using composite filler materials employing other processes (*e.g.*, diffusion welding).

Summary

An analytical model based on a thin-plate assumption has been developed for determining the stresses induced in functionally gradient material joints consisting of discrete composition steps by the joint production process. The layer-model incorporates important material properties (*e.g.*, coefficient of thermal expansion, k, thermal conductivity, α, elastic modulus, E, and Poisson's ratio, ν) as functions of temperature. Such inclusion is critical to meaningful predictions, and is rather unique in current analyses. The model has been shown to be user-friendly and quite versatile, enabling joint design optimization through variation of the overall joint thickness, the thickness of end-elements, the number of steps comprising the overall joint, the relative thicknesses and arrangement of the various steps employed, the composition of individual steps, and the composite properties for various composition using different constitutive laws. Predicted results give good insight to the state of stress developed during processing, and, so, are useful for aiding in joint design by process engineers by comparing to end-element failure stress. A more elaborate model based on finite element/finite difference analysis methods is needed to analyze FGM joints with continuous gradients, such as that developed by others (Ref. 19).

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