A New Method for Estimating Spectral Performance of ADC from INL

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Abstract

Linearity test and spectral test are two main contributors of ADC test cost which includes data acquisition time and accurate instrumentation. This paper presents a new method for estimating an ADC's spectral performance from its tested INL data. The method does not require additional dedicated test circuitry or data acquisition. The results from INL test are used to compute harmonic distortions and other spectral specifications of the ADC. Memory and computation requirements are very small comparing to those in traditional spectral testing. When combined with a BIST approach for INL testing, the proposed method offers a very low cost BIST solution to ADC spectral testing. Both simulation and experimental results show that the proposed method can estimate THD and SFDR values accurately.

1. Introduction

As more mixed-signal functions are deeply embedded in System on Chip (SoC) applications and as customers demand higher performance, accurate and cost-effective testing of ADCs becomes significantly more challenging. In production test, ADC static linearity and spectral performance are the two categories of specifications that are most time consuming and impose most stringent hardware and software test requirements. Static linearity, including INL and DNL, is conventionally tested using the histogram method with either a sine wave or triangular wave input. Spectral performance, including SNR, THD, and SFDR, is tested using the FFT method with a sine wave input having very high spectral purity [1].

To reduce test time, methods have been introduced for estimating ADC static linearity from spectral testing results [2, 3]. However, due to the loss of "high-frequency" details of ADC's transition levels, these methods are unacceptable in real applications in which transition levels matter. For example, in measurement instrumentation, automotive control, and high resolution imaging, INL performance is critical and must be measured accurately. This paper takes another direction to achieve test time reduction by trying to estimate spectral performance based on INL test results. One might argue that saving spectral test time is not as big a saving as saving linearity test time. But when accurate linearity test is mandated by the application, saving spectral test time is the best one can hope for. When accurate spectral testing results are needed, both data acquisition time and computation time in traditional methods are significant. Furthermore, the stringent spectral purity requirement on the input sine wave generator is a major challenge, especially for on-chip built-in self test. Removing this challenge is a giant step toward enabling built-in self test of deeply embedded ADCs.

The idea of estimating the spectral performance using linearity test data becomes more valuable in applications of ADC built-in self-test (BIST), where testing circuitry's area is more concerned than test time. Significant research results of ADC BIST have been published in literature over the last two decades. BIST schemes of SNR and other frequency specifications testing were presented in [4, 5]. Low cost BIST schemes of testing static performances have been presented in [6, 7]. Recently, research results have been published on reducing the accuracy requirement on linearity testing signal and simplifying its generation circuitry, which makes it possible to realize ADC linearity test on chip [8]-[10]. Using the method developed in this paper, it takes very little additional resources to obtain the spectral performance of an ADC based on BIST results of its linearity. This method eliminates the need of accurate sine wave generation on chip for spectral testing, making ADC BIST one step easier to implement.

In this paper, a method of estimating THD and SFDR based on INL of an ADC is introduced. The method computes THD and SFDR without requiring any additional hardware or data acquisition. Only a small amount of computation is required to estimate THD and SFDR accurately. The rest of this paper is organized as following. In Section 2, the traditional method of testing THD and SFDR and its challenges are reviewed. In Section 3, the new method is described in detail. First a model of ADC test is presented. Then how distortion is extracted from INL data is presented. A way of efficiently compute harmonic power is present at the end. Error analysis is given in Section 4. Section 5 and Section 6 gives simulation results and measurement results respectively.

2. Existing Challenges

In traditional ADC spectral performance testing, a pure, sine wave with large amplitude is used as input signal of the

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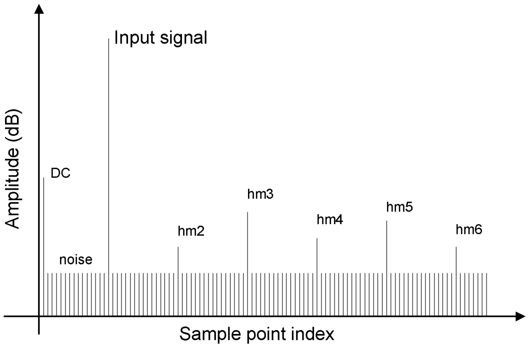


Fig.1. Spectrum of output signal

ADC under test [11, 12]. The frequency is set to satisfy coherent sampling condition, which usually leads to odd number of signal periods. M sampling points from the input signal are converted into digital binary codes by the ADC. DFT of these digital codes are computed, the magnitude spectrum of which looks like what is shown in Fig.1. From the magnitude spectrum, THD and SFDR can be computed as following.

$$THD = 10 \cdot \log_{10}\left(\sum_{i=2}^{H} hm_i^2 \Big/ A_{rms}^2 \right) \tag{1}$$

$$SFDR = 10 \cdot \log_{10}\left(A_{rms}^2 \Big/ \max_{i=2\sim H}\left(hm_i^2\right) \right) \tag{2}$$

In above equations, H is the number of harmonics to be computed, hm_i is the magnitude of the component at the i^{th} harmonic of DFT, A_{rms} is the RMS value of input sine wave amplitude.

Two conditions must be satisfied in traditional spectral testing to achieve valid testing. The first condition is that the sine wave must be pure enough so that its distortion is much lower than ADC resolution under test. The second condition is input signal frequency must be well controlled to achieve coherent sampling. To make the sine wave pure enough, a low pass or band pass filter is usually put after the sine wave generator. Harmonics of the input sine wave must be attenuated to be much lower than ADC resolution. At board level testing, a passive LC filter can be used to perform this function [13]. LC filter shall not be implemented on chip since it consumes large area. Even building active filter for this purpose needs large area which can be shown by the following example. Assume the input frequency is f_0, the harmonic at frequency f_h needs to be attenuated by R dB. The order of the Butterworth lowpass filter is given by the following expression.

$$r = \frac{R}{20 \cdot \log_{10}\left(f_h / f_0 \right)} \tag{3}$$

Assume the 2nd order harmonic of generated sine wave is 50dB lower than fundamental. If we want to attenuate it by 40dB so that the 2nd order harmonic is 90dB lower than fundamental, a 7th order lowpass filter will be needed. Building 7 poles at low frequency on chip consumes large area and hence is not practical.

To meet the second condition, the number of input signal periods must be a coprime number of the total number of sampling points. The relation between input sine wave frequency and ADC sampling frequency is given by (4).

$$f_0 = \frac{P}{M} f_s \tag{4}$$

P is the number of input signal periods, M is the total number of points will be sampled, and f_s is the sampling frequency of ADC. The value of M is usually a power of 2. P is usually chosen to be an odd integer to guarantee integer number of periods is sampled and different phase of each period is sampled. The coprime relation makes f_0 be a fractional frequency of f_s. For example, when f_s is 10M Hz, M is 8192, and P is 799, the input signal frequency is 975.342K Hz. A frequency synthesizer is used in traditional testing to generate the fractional frequency [11]. Frequency synthesizer design itself is a challenging task in current SOC design. This block also consumes large area thus is unaffordable in ADC BIST. The high precision frequency requirement may be avoid by using window in DFT. But the windowing will increase the computation complexity.

As described above, generating a pure sine wave on chip at proper frequency with low cost is very challenging and is unpractical for ADC BIST. A new method proposed in this paper avoids above challenges by estimating spectral performance from INL data which has been acquired in linearity test. INL can be tested on chip with low overhead by adopting SEIR method which only needs nonlinear triangular stimulus [8]. Estimating spectral performance from INL data does not need additional data acquisition and only needs very small amount computation.

3. New Method of Estimating THD and SFDR

In this section, a new method of estimating harmonic distortion power and then THD and SFDR values is presented. THD and SFDR are computed from existing INL data without additional data acquisition. Digital circuitry needed for this computation is available in SoC. In other word, THD and SFDR can be estimated with almost no extra overhead. Another advantage of this approach is that noise in INL is much lower than normal ADC output codes because of average effect of histogram testing.

The new method estimates THD and SFDR from tested INL data. Because of the static characteristic of INL, estimated THD and SFDR are pseudo static. This method cannot capture spectral performance at high frequency or spurious not at harmonic frequencies.

A new way of modeling ADC testing process is given at first. Secondly, how the harmonic distortion power is extracted from INL data and calculated is discussed. At last, efficient computation is presented.

3.1. Model of ADC testing

When a sine wave is converted into digital codes by an ADC, transfer characteristic of the ADC can be represented by equation (5).

$$V_{in}(t_k)+n(t_k)=T_{C(t_k)}+E_{os}\cdot LSB+\frac{C(t_k)}{2^n}E_g\cdot LSB+Q(t_k) \quad (5)$$

In this equation, t_k is the testing time index, $V_{in}(t_k)$ is voltage of input sine wave at time t_k, $n(t_k)$ is the input referred noise including ADC noise and signal source noise, $C(t_k)$ is the output code at time t_k, $T_{C(tk)}$ is the transition voltage corresponding to output $C(t_k)$, $Q(t_k)$ is the quantization error at time t_k, E_{os} is the offset, and E_g is the gain error of the ADC. Continuous input sine wave is represented by discrete transition voltages of the ADC plus error and noise. The transition voltage corresponding to output code $C(t_k)$ can be expressed by equation (6)

$$T_{C(t_k)}=C(t_k)\cdot LSB+INL_{C(t_k)}\cdot LSB \quad (6)$$

in which, $INL_{C(tk)}$ is the INL error of transition level $T_{C(tk)}$. Equation (7) can be obtained by substituting (6) into (5) and switching sides.

$$\begin{aligned}&C(t_k)\cdot LSB+\frac{E_g}{2^n}C(t_k)\cdot LSB\\&\quad=V_{in}(t_k)+n(t_k)-Q(t_k)-INL_{C(t_k)}\cdot LSB-E_{os}\cdot LSB\end{aligned} \quad (7)$$

$C(t_k)\cdot LSB$ is the output data of ADC. All values of $C(t_k)\cdot LSB$ over the testing time $0\le t_k\le 1$ represents the input signal which is a single tone sine wave.

After Fourier transform, equation (7) becomes to

$$\begin{aligned}&FT\left(C(t_k)\cdot LSB\right)\\&\quad=\left(\frac{1}{1+E_g/2^n}\right)\cdot\left\{FT\left(V_{in}(t_k)\right)+FT\left(n(t_k)-Q(t_k)\right)\right.\\&\qquad\left.-FT\left(INL_{C(t_k)}\cdot LSB\right)-FT\left(E_{os}\cdot LSB\right)\right\}\end{aligned} \quad (8)$$

In this equation, $FT(C(t_k)\cdot LSB)$ is the Fourier transform of ADC output codes which is the key data in traditional FFT testing. $FT(C(t_k)\cdot LSB)$ consists of several components that are shown at the right side of (8). $FT\left(V_{in}(t_k)\right)$ is the Fourier transform of input sine wave, $FT(n(t_k)\text{-}Q(t_k))$ is the noise floor, $FT\left(INL_{C(t_k)}\cdot LSB\right)$ is the harmonic distortion caused by nonideality of ADC, and $FT\left(E_{os}\cdot LSB\right)$ is the part DC component from ADC offset. Fig.1 shows a typical spectrum of a digitized sine wave contains all components in equation (8). Signal power, harmonic distortion power, and noise power can be computed from the spectrum and eventually SNR, THD, and SFDR can be computed.

It can be observed from equation (8) that all harmonic distortion power is carried by $FT\left(INL_{C(t_k)}\right)$ term which also contains a small amount of noise and a small part of input signal. Spectrum of INL data contains the same harmonic distortion power as the spectrum of digital output data shown in Fig.1. To achieve the purpose of computing THD and SFDR value, we only need to do Fourier transform of INL instead of output codes. All harmonics distortion power can be calculated from spectrum of INL.

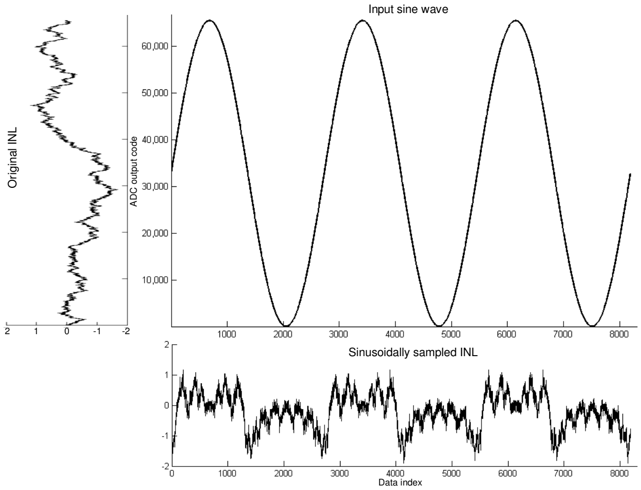


Fig.2. Sinusoidal sampling of INL

3.2. Extracting distortion power from INL

In traditional spectral testing, a pure sine wave is applied to ADC. Distortion information carried by output code is the distortion experienced by the sine wave, which means that the distortion term $FT\left(INL_{C(t_k)}\right)$ in equation (8) is the INL experienced by the sine wave. To obtain INL corresponding to the input sine wave, we can sinusoidally sample INL by the ADC output codes of the sine wave. Regard the original INL data of the ADC as a series INL_{orig} which has 2^n-2 elements as shown in (9).

$$INL_{orig}(i) \qquad i=1,2,3,\cdots 2^n-2 \quad (9)$$

in which n is the resolution of ADC. Sampling process constructs a new series based on output codes and INL_{orig}

$$INL_{\sin}=INL_{orig}\left(C(t_k)\right) \qquad k=1,2,3,\cdots M \quad (10)$$

In (10), series $INL_{\sin}$ is the distortion experienced by sine wave, $C(t_k)$ is the ADC output code of sine wave, M is the total number of points in sine wave test. The value of M can be either larger or smaller than 2^n-2. Fig.2 shows the process of constructing a new data sequence by sinusoidally sampling INL according to output codes of ADC. The curve at the left side is the original INL, which is sampled by ADC output codes of sine wave. The new INL sequence experienced by the sine wave is shown at the bottom of Fig.2. The pattern of the new sequence consists of 6 repeats of original INL. But the total number of points of the new sequence can much smaller than the number of points of original INL. From spectrum of this new INL sequence, we can calculate the spectral performance as following.

$$THD = \frac{\sum_{i=2}^{H} P_h\left(i\right)}{P_0 + P_h\left(1\right)} \tag{11}$$

$$SFDR = \frac{P_0 + P_h\left(1\right)}{\max\limits_{i=2\sim 20} P_h\left(i\right)} \tag{12}$$

In above two equations, $P_h(i)$ is the i^{th} harmonic power in the power spectrum of the new INL sequence INL_{sin}. The power spectrum corresponds to the term $FT\left(INL_{C(t_k)}\cdot LSB\right)$ in (8). This term also contains a part of input signal power $P_h(1)$. P_0 is the signal power corresponding to $FT\left(V_{in}\left(t_k\right)\right)$ term in (8). The numerator of (11) is the total power of first H order harmonics, and the denominator is the signal power.

The difficulty of sinusoidal sampling is that ADC's output code of sine wave is not available because only code density is recorded in histogram testing. To overcome this, we acquire the sinusoidal digital codes by virtually testing a sine wave. Assume a sine wave X_{in} has frequency of f_0 and amplitude of 1. An ideal ADC with the same full scale range converts this sine wave into digital codes which can be simply calculated by

$$C(k) = \left\lfloor \frac{N}{2}\cdot\left(1+\sin\left(2\pi f_0\cdot k\right)\right)\right\rfloor \qquad k = 1,2\cdots M \tag{13}$$

in which, N is the number of transition level of ADC, $C(k)$ is the output code, and M is the total number of samples that will be used for spectral performance estimation. Now the value of $C(k)$ can be used as the index to read the value of INL from the original INL data and construct a new data set INL_{vsin}. Constructing a new data set from INL according to sine wave does not change distortion power. Frequency of the sine wave in (13) can be selected to be any value that makes computation convenient. Assume H is the number of harmonics will be calculated. In order to let the first H harmonics distribute within half sampling frequency, the sine wave frequency can be set as

$$f_0 = \frac{1}{2H} \tag{14}$$

The value of f_0 should be slightly adjusted to achieve coherent sampling. From these ideal digital codes for sine wave, another new sequence of INL is constructed as

$$INL_{\text{vsin}} = INL_{orig}\left(C\left(k\right)\right) \qquad k = 1,2,3,\cdots M \tag{15}$$

From the spectrum of INL_{vsin}, we can calculate THD and SFDR as following.

$$THD = \frac{\sum_{i=2}^{H} P_h\left(i\right)}{A^2/8} \tag{16}$$

$$SFDR = \frac{A^2/8}{\max\limits_{i=2\sim 20} P_h\left(i\right)} \tag{17}$$

where A is the full scale range of ADC. Equation (16) and (17) are computations carried out in the new method. Because there is no input signal component in spectrum of INL data, the signal power is theoretical full scale sine wave power. Every harmonic power can be calculated from the spectrum of INL data.

3.3. Reducing computation requirement

Though the Fourier transform of INL can be easily computed by on chip processer, the computation can be further simplified. To calculate THD and SFDR, we only need distortion power when full scale input signal is applied. Instead of implementing FFT algorithm on chip, discrete-time Fourier series (DFS) of INL is computed as (18).

$$X\left(k\right) = \frac{1}{M}\sum_{n=0}^{M-1} x\left(n\right)\cdot e^{-j\cdot\frac{2\pi}{M}\cdot n\cdot k} \qquad k = 0,1,2,...,M-1 \tag{18}$$

In which, $x(n)$ is the value of $L(n)$, $X(k)$ is the k^{th} coefficient of the Fourier series, M is the total number of points used in THD and SFDR estimation. The coefficient of the fundamental component is given by (19)

$$X\left(k_1\right) = \frac{1}{M}\sum_{n=0}^{M-1} x\left(n\right)\cdot e^{-j\cdot\frac{2\pi}{M}\cdot n\cdot k_1} \tag{19}$$

The relation between input signal frequency and sampling frequency is set beforehand, thus value of k_1 is known. Coefficient of i^{th} order harmonic can be calculated by (20)

$$X\left(i\cdot k_1\right) = \frac{1}{M}\sum_{n=0}^{M-1} x\left(n\right)\cdot e^{-j\cdot\frac{2\pi}{M}\cdot n\cdot i\cdot k_1} \qquad i = 2,3,4...H \tag{20}$$

There is no need to calculate fundamental component since it is not the power of input signal or part of distortion power. Only 19 coefficients need to be calculated for good estimation of THD and SFDR value. The frequency of input sine wave is selected by tester so that value of k_1 and total number of points M are always known. Rewrite (20) in to (21)

$$X\left(i\cdot k_1\right) = \frac{1}{M}\sum_{n=0}^{M-1} x\left(n\right)\cdot\left(E_i\right)^n \tag{21}$$

In which,

$$E_i = e^{-j\cdot\frac{2\pi}{M}\cdot i\cdot k_1} \tag{22}$$

Instead of creating a look up table for exponential term, only the exponential value E_i needs to be stored on chip and used for DFS coefficient computation. In (21), there are n times of multiplication. When n is large, it can be expressed in binary form to reduce computation further.

$$n = b_0 + b_1 2 + b_3 4 + \cdots + b_{12} 2^{12} \tag{23}$$

The exponential term in (21) can be rewritten as

$$\left(E_i\right)^n = \left(E_i\right)^{b_0} \cdot \left(E_i^2\right)^{b_1} \cdot \left(E_i^4\right)^{b_2} \cdots \left(E_i^{4096}\right)^{b_{12}} \tag{24}$$

Values of different power E_i can be stored in memory. The number of multiplications in exponential value computation is reduced to 13.

4. Error analysis

4.1. Approximations in equation derivation

Comparing (16) and (17) with equation (11) and (12), we can see 2 approximations may cause estimation error in THD and SFDR value. The first approximation is using ideal sine wave to sample INL instead of real output codes, so that $FT(INL_{\text{vsin}})$ will be slightly different from $FT(INL_{\text{sin}})$. It can be seen from equation (13) that $C(k)$ is different from the actual output code of ADC $C(t_k)$. For reasonably good ADC, $C(k)$ is only several codes away from $C(t_k)$ and the value of INL changes very slowly. The difference between INL_{sin} and INL_{sin} will never be larger than the peak-to-peak value of INL which we denote as ΔINL. INL consists of three parts including a part of input signal, distortion, and noise. So we can express INL as following.

$$INL_{\text{sin}}\left(k\right) = h_1 e^{j\cdot\frac{2\pi}{M}\cdot k\cdot p} + h_2 e^{j\cdot\frac{2\pi}{M}\cdot k\cdot 2p} + h_3 e^{j\cdot\frac{2\pi}{M}\cdot k\cdot 3p} + \ldots$$
$$+\, higher \quad order \quad terms + noise \tag{25}$$

in which h_1 is the coefficient of fundamental component, h_2 and h_3 are coefficients of 2nd and 3rd harmonic components, and p is the number of periods of sine wave. From (11) and (12) to (16) and (17), INL_{sin} is replaced by INL_{vsin}, so the coefficient of each harmonic will change.

$$INL_{\text{vsin}}\left(k\right) = \tilde{h}_1 e^{j\cdot\frac{2\pi}{M}\cdot k\cdot p} + \tilde{h}_2 e^{j\cdot\frac{2\pi}{M}\cdot k\cdot 2p} + \tilde{h}_3 e^{j\cdot\frac{2\pi}{M}\cdot k\cdot p} + \ldots$$
$$+\, higher \quad order \quad term + noise \tag{26}$$

in which h_1 is the coefficient of fundamental component, h_2 and h_3 are coefficients of 2nd and 3rd harmonic components. Due to the difference between $C(t_k)$ and $C(k)$, the value of $INL_{\text{vsin}}(k)$ is actually equal to the value of $INL_{\text{sin}}(k')$, in which k' is very close to k. Equation (26) can be rewritten as

$$INL_{\text{vsin}}\left(k\right) = INL_{\text{sin}}\left(k'\right)$$
$$= h_1 e^{j\cdot\frac{2\pi}{M}\cdot k'\cdot p} + h_2 e^{j\cdot\frac{2\pi}{M}\cdot k'\cdot 2p} + h_3 e^{j\cdot\frac{2\pi}{M}\cdot k'\cdot 3p} + \ldots$$
$$+\, higher \quad order \quad term + noise \tag{27}$$

Because k and k' are very close to each other and frequency of harmonic is low, we can expand each harmonic term.

$$\tilde{h}_i e^{j\frac{2\pi}{M}\cdot i\cdot k} = \tilde{h}_i e^{j\frac{2\pi}{M}\cdot i\cdot k'} + \tilde{h}_i \cdot \left(j\frac{2\pi}{M}\cdot i\right)\cdot e^{j\frac{2\pi}{M}\cdot i\cdot k'}\left(k - k'\right)$$
$$= \tilde{h}_i\left(1 + j\frac{2\pi}{M}\cdot i\cdot\left(k - k'\right)\right)e^{j\frac{2\pi}{M}\cdot i\cdot k'} \tag{28}$$

Comparing (27) and (28), we have

$$h_i = \tilde{h}_i\left(1 + j\frac{2\pi}{M}\cdot i\cdot\left(k - k'\right)\right) \tag{29}$$

The i^{th} harmonic power is calculated from the Fourier series coefficient at $i\cdot p$. The estimation of the i^{th} order harmonic distortion power is

$$e_i = 10\log\left(1 + \left(\frac{2\pi}{M}\cdot i\cdot\left(k - k'\right)\right)^2\right) \tag{30}$$

Assume difference between k and k' is 5 LSB and M is 8192, the estimation error of THD and SFDR values will be smaller than 0.025dB which is negligible. From above analysis, it causes negligible error by using ideal quantized sine wave to sample INL instead of real ADC output codes.

The second approximation is using theoretical power of full scale sine wave as fundamental component power, which does not cause error either. In traditional spectral testing, in order to avoid clipping, the amplitude of sine wave is set to be smaller than ADC full scale range. THD and SFDR can still be computed correctly from the spectrum, but nonlinearity at two ends is not excited in this case. In this INL based new method, amplitude of the ideal sine wave used in INL sampling can be very close to full scale range so that $A^2/8$ is the fundamental component power. In addition, all codes are hit and ADC nonlinearity is fully excited.

4.2. Noise effect

Input noise is another error source of spectral testing which exists in both traditional method and the new method. Assume the equivalent input noise is P_n, the noise floor in M points FFT spectrum is

$$P_{nfloor} = \frac{P_n}{M} \tag{31}$$

This noise causes error in every harmonic component. The actual k^{th} harmonic component is

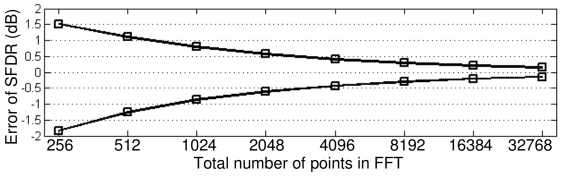


Fig.3. Bound of SFDR error caused by noise

$$X'_k = X_k + 3\delta_{X_k} \tag{32}$$

in which X_k is the true value of k^{th} harmonic components, δ_{X_k} is the error of k^{th} harmonic component whose rms value is at noise floor level. And the power of this harmonic component is calculated as

$$X_k'^2 = X_k^2 \left(1 + \frac{3\delta_{X_k}}{X_k}\right)^2 \tag{33}$$

The noise power floor also causes error in fundamental component, which is negligible since the ratio of signal power and noise power is very large. Error term in (33) causes error in SFDR value.

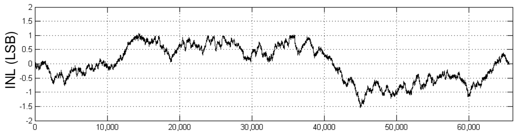
$$SFDR = SFDR_{true} + 20\log_{10}\left(1 + \frac{3\delta_{X_k}}{X_k}\right) \tag{34}$$

Since the error is analyzed base on ratio, error in THD is similar to the error term in (34). But H harmonic components will be added together, error of THD is smaller than error of SFDR. For a 16 bits ADC, assume the input equivalent noise is 0.5LSB which includes thermal and quantization noise. In M points FFT test, the SNR is about 93dB and the noise floor is about -147dB for M equal to 2^{15}. Assume the SFDR is at 102dB level, noise may cause error within ±0.15dB. Fig.3 shows the boundary of error caused by noise when M changes from 2^8 to 2^{15}. It can be seen that when M decreases, noise causes more error in SFDR.

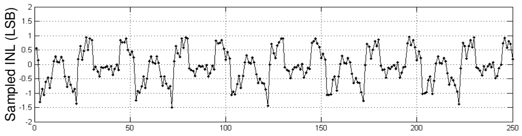
5. Simulation Results

The method of estimating THD and SFDR from INL data has been investigated and validated by simulations. Both traditional and the new method are implemented in Matlab. Comparing with results of traditional method, the new method provides comparable testing accuracy as traditional method. With no additional data acquisition and small amount of computation, the new method is more attractive in ADC BIST.

A 16 bits ADC is modeled as a set of transition levels that are randomly generated in MATLAB. Fig.4.a shows the true INL of the ADC which is +1.1/-1.5LSB. Based on the INL data, the new method estimates THD and SFDR of the ADC. First, an ideal sine wave is quantized into a set of digital codes by an ideal 16-bit ADC as shown in equation (13). The total number of samples is 2^{15} and the frequency is set to be a value so that the first 20 harmonic components are distributed within half sampling frequency. Secondly, the 2^{15} digital codes are used as index to sample the INL to construct a new data set as shown in (15). The first 5 periods of the sinusoidally sampled INL is shown in Fig.4.b.



(a) True INL of ADC



(b) First 5 periods of sinusoidally sampled INL

Fig.4. INL of ADC

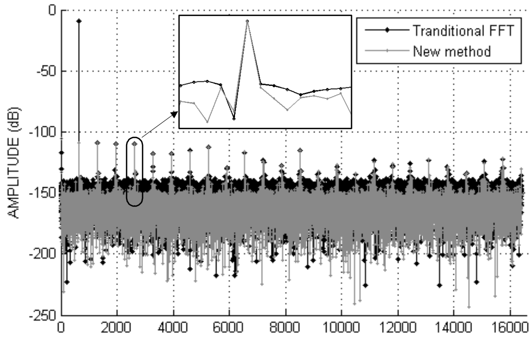


Fig.5. Spectrum of output code and sampled INL

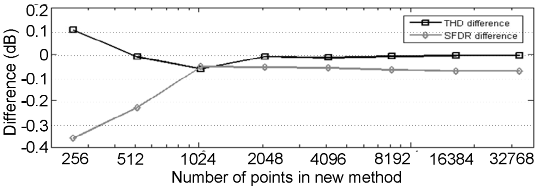
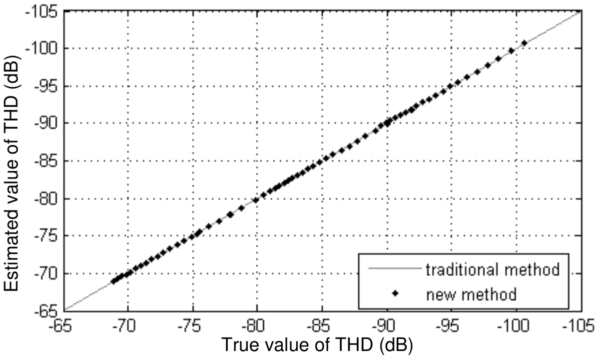
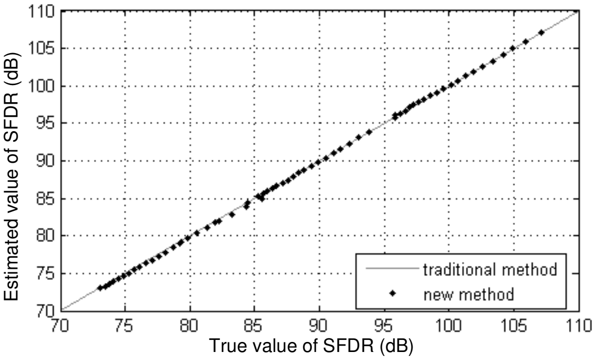


Fig.6. THD and SFDR difference between new method and traditional 32768 points FFT test

Fig.5 compares the spectrum of ADC output codes in traditional spectral testing and the spectrum of sinusoidally sampled INL data. The black curve is the spectrum of 2^{15} digital output codes of the ADC when the input is a sine wave. For good investigation of harmonic distortion, input noise in traditional method is set to be zero so the SNR of this spectrum is 97.2dB which is very close to the theoretical SNR value of a 16 bits ADC. The gray curve is the spectrum of sinusoidally sampled INL data shown in Fig.4.b. Fundamental frequencies in two cases are set to be equal to each other for convenient comparison. From the gray curve, we can see three parts including small component at fundamental frequency, components at harmonic frequencies, and noise floor. It can be observed from this plot that spectrum of sinusoidally sampled INL has the same harmonic distortion power as the spectrum of digital output codes. The zoomed in plot around the 4th



(a) THD estimation results



(b) SFDR estimation

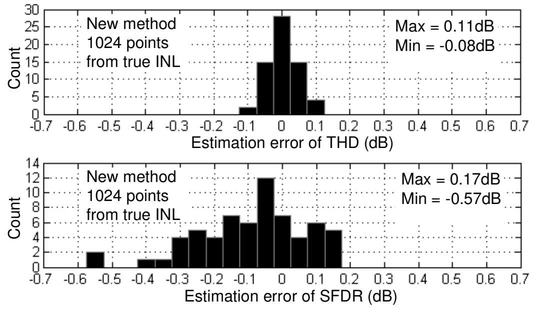
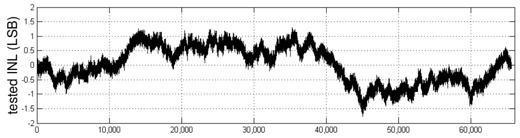


Fig.7. (c) THD and SFDR estimation errors



(a) INL tested by histogram method

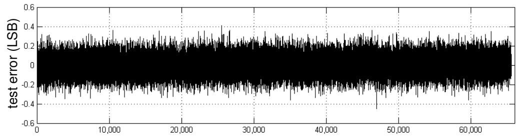
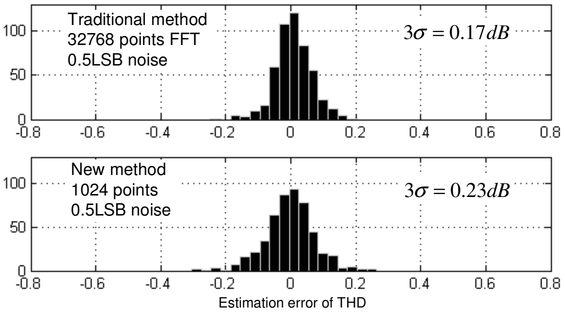
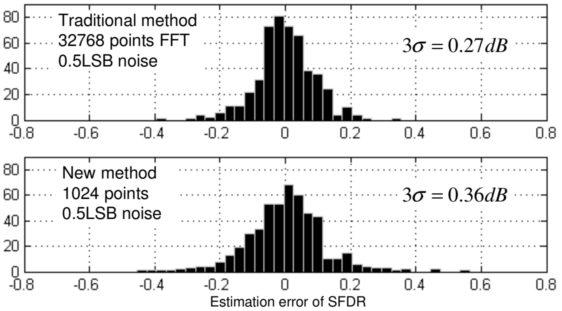


Fig.8. (b) Test error of INL



(a) Estimation error distribution of THD



(b) Estimation error distribution of SFDR

Fig.9. Comparison of tradition and new method

harmonic frequency shows more details of this. From all these harmonic bins, total harmonic distortion power and maximum harmonic distortion power can be calculated. There is no real fundamental bin in the gray curve. When the INL is sinusoidally sampled, the sine wave amplitude is set to be full scale. Therefore, the signal power is simply $A^2/8$ or -9dB. THD and SFDR values are computed based on total harmonic distortion power and maximum harmonic power. Another observation is the noise floor of INL spectrum is much lower than that of digital output code spectrum, which is an advantage of INL based estimation.

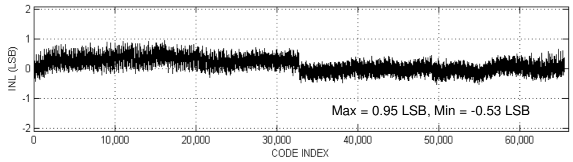
In Fig.5, the number of samples of new method is chosen to be the same as traditional FFT method which is 32768. Smaller number of points is enough in the new method to obtain good enough estimation results. THD and SFDR values tested from traditional 32768 points FFT method are -93.7dB and 100.2dB and considered as the reference values. Estimation results of the new method will be compared with the reference and differences are regarded as estimation error. Fig.6 shows THD and SFDR estimation error of new method versus different number of points are used. The number of points in the new method changes from 2^8 to 2^{15}. From this figure, it can be seen that larger number of points gives more accurate THD and SFDR estimation which agrees with Fig.3. When the number of points is larger than 1024, estimation errors of THD and SFDR are smaller than 0.02dB and 0.1dB respectively.

To verify the new method for different ADCs, 64 ADCs with resolution 12 bits, 14 bits, and 16 bits and THD values from -69dB to -100.6dB are created and tested by both traditional 32768 points FFT method and 1024 points new method. Results of traditional method are considered as references as gray curves showing in Fig.7. From the plot, estimated THD and SFDR values track true values at all linearity level. From Fig.7.c, estimation error of THD is always smaller than 0.15dB, and most estimation errors of

SFDR are smaller than 0.3dB. Estimation error of SFDR is larger than that of THD. Overall, estimation errors of THD and SFDR are small.

Above estimations are based on true INL of ADCs and have shown good accuracy. In real world, only tested INL can be used in the new method. Due to limited testing resources such as stimulus generator and number of hits per code, tested INL may have large amount of error. Fig.8.a shows the INL of the 16-bit ADC tested by using 32 hits per code histogram method. The linear input ramp used for testing contains 0.5LSB additive noise so that the tested INL is noisy comparing with the true INL curve shown in Fig.4.a. The test error can be as large as 0.4 LSB as shown in Fig.8.b even when the ramp is ideal. Due to testing limitations, INL is always different from test to test.

To investigate how the INL test error affect the new method, 500 different ADCs are randomly generated and tested. THD and SFDR values measured by traditional 32768 points FFT method with noise free pure sine wave are considered as true THD and SFDR. INL of these ADCs are tested by 32 hits per code histogram method with linear ramp. 0.5 LSB additive noise is added to the ramp so that tested INLs have similar error shown in Fig.8.b. Tested INL is sinusoidally sampled according to 1024 sine wave codes. These 1024 points are used in new method to estimate THD and SFDR of each ADC. Estimation error is the subtraction of new method results and true values. Fig.9 shows the distribution of 500 estimation errors. The traditional 32768 points FFT method uses pure sine wave plus 0.5 LSB additive noise as input. Due to noise effect, result the traditional method differs from test to test. The distributions of traditional test results are shown below the new method. Mean of all distributions are zero, and 3σ values are very small. The THD and SFDR estimation error 3σ of the new method are 0.06 dB and 0.09dB larger than traditional method, which is insignificant. The number of points used in the new method is only 1024 which is much smaller than the number of points used in traditional method which is 2^{15}.



(a) INL of ADC1

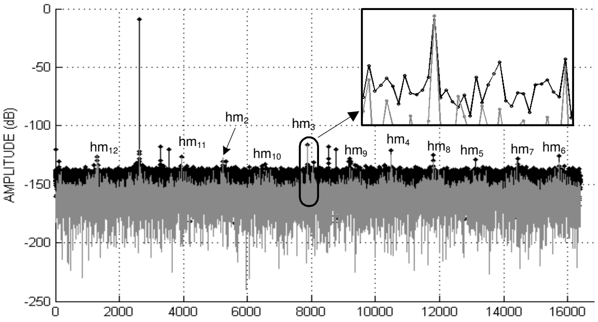
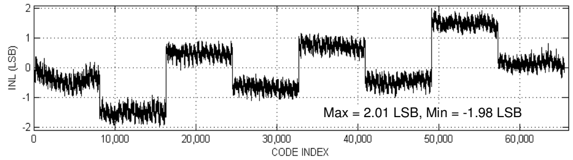


Fig.10. (b) Spectrum of ADC1 output code and INL



(a) INL of ADC4

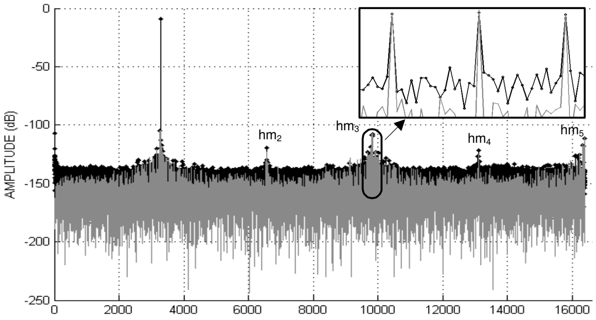


Fig.11. (b) Spectrum of ADC4 output code and INL

6. Measurement Results

The new method of estimating THD and SFDR from sinusoidally sampled INL has also been validated by measurement results. Based on tested INL data, the new method estimates THD and SFDR values with only simple computations. Accuracy of estimation results is comparable with that of traditional method which needs high quality sine wave as input and 32768 points FFT computation.

Four different 16 bits SAR ADCs (ADC161S626) from National Semiconductor product line are tested by both traditional FFT method and the new method. The INLs are tested by 128 hits per code histogram method with sin wave input. THD and SFDR values of all ADCs are tested by 2^{15} points traditional FFT method in which sampling frequency is 250 KHz and input signal frequency is about 20KHz. Spectral performances measured by traditional method are considered as the reference. Fig.10 to Fig.11 show test results of two ADCs including linearity and spectral testing. In the spectrum plots, black curves are obtained from traditional 2^{15} points FFT method and the gray curves are obtained from FFT of sinusoidally sampled INL data. Spectrum plot of sinusoidally sampled INLs is obtained for comparison only. Only Fourier series coefficients of the first 20 harmonic components are computed in the new method to estimate THD and SFDR values. Zoomed in plots of region around maximum harmonic frequency show that spectrum of sinusoidally sampled INL has the same harmonic bins as the spectrum of output codes. Performances of these four ADCs are listed in Table.1, in which SNR value varies from 91.36 dB to 92.23 dB, and INL varies from 0.9 LSB to 2 LSB. ADCs with various performances can provide better validation.

Table.1. Performance of four ADCs

	ADC1	ADC2	ADC3	ADC4
INL(LSB)	+0.95/-0.54	+1/-1.27	+1.83/-1.83	+2.01/-1.98
SNR(dB)	92.23	91.99	91.83	91.36

Table.2. Estimation results of 1024 points new method (Unit: dB)

	ADC1	ADC2	ADC3	ADC4
THD_FFT	-103.85	-97.60	-90.78	-91.29
THD_new	-104.8	-97.83	-90.85	-91.11
Δ(THD)	-0.95	-0.23	-0.07	0.18
Δ(P_d)	-119.06	-118.82	-124.95	-114.3
SFDR_FFT	106.83	102.81	95.14	95.47
SFDR_new	106.92	102.48	95.66	95.49
Δ(SFDR)	0.09	-0.33	0.52	0.02
Δ(Ph_max)	-144.69	-121.72	-114.67	-121.49

Table.2 shows estimation results of the new method with 1024 points sinusoidally sampled INL data. The 2nd row and 6th row lists THD and SFDR values of these ADCs tested by traditional 2^{15} points FFT method. The 3rd row and 7th row lists THD and SFDR values of these ADCs tested by the new method with 2^{10} points sinusoidally sampled INL. The 4th row is the direct subtraction of THD values tested by the new method and THD values tested by traditional FFT method. The 8th row is the direct subtraction of SFDR values tested by the new method and THD values tested by traditional FFT method. All differences are very small, which means very good estimation accuracy can be achieved by the new method with a number samples that is much smaller than traditional FFT method.

It can be noticed that THD value difference of ADC1 is larger than others in row 4 which is misleading. Direct subtraction of THD or SFDR value gives ratio of estimation error to distortion power. Absolute distortion power also affects the subtraction result. For example, when the true value and estimated value of a quantity Q1 are -103 dB and -102 dB, the direct subtraction value ΔQ1 is 1dB. The estimation error e_1 is 1.3×10^{-11} or -108.9dB. Another similar case is the true value and estimated value of a quantity Q2 are -97dB and -96dB, the direct subtraction value ΔQ2 is also 1dB. But the estimation error e_2 is 5.2×10^{-11} or -102.9 dB which is 4 times larger than e_1.

Therefore, comparing estimated distortion power which is shown in the 5th row provides more reasonable way of judging estimation accuracy. Estimation errors of distortion power of all ADCs are at the same level which is very small and comparable with noise effect. The noise floor of ADC1 is -143.51dB, and the effect of total distortion power from noise is -125.95dB. Besides, noise floor in INL spectrum will also have effect on distortion power estimation, which will increase the effect up to about -123dB. Similar calculations of other ADCs give results at the same level. On the whole, Table.2 shows that the new method gives good enough estimation of THD and SFDR.

Table.3. Bound of SFDR error due to noise in 32768 points FFT (Unit: dB)

	ADC1	ADC2	ADC3	ADC4
Ph_max	-115.96	-111.95	-104.28	-104.61
Noise floor	-143.51	-143.27	-143.11	-142.64
Error bound	+1/-1.2	+0.68/-0.74	+0.3/-0.3	+0.32/-0.33

Similar to Section 3.3, the error bounds of SFDR caused by noise are calculated for these four ADCs. The results are shown in Table.3. Error bounds are calculated base on the number of samples in FFT, the noise floor, and the maximum harmonic components that are obtained from FFT spectrum. Comparing the last row of Table.3 with the estimation errors in Table.2, it can be seen that estimation errors of all ADCs are well within the boundaries. It is necessary to notice that this method can only provides accurate estimation of spectral performance at low frequency due to the pseudo static feature of INL.

7. Conclusion

A new method has been introduced to estimate ADC THD and SFDR values from INL test results. In the circumstance of INL has been tested, this method requires only simple computations to estimate THD and SFDR. The new method avoids another round of data acquisition for spectral testing and additional dedicated circuitry. Simulation and experimental results show accuracy of the new method is comparable with traditional FFT method which needs high quality signal generator and FFT computation with large number of points. This method can only estimate spectral performance at low frequency and cannot identify SFDR when spur frequency is not harmonic frequency.

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