A Non-Convex Robust Simulation Optimization Model for Inventory Management Problem by System Dynamics

Abdollah Sharifi[1], Abdollah Aghaie[2*]
[1]Industrial Engineering Doctoral Student, K. N. Toosi University of Technology, Tehran, Iran
[2]Industrial Engineering Professor, K. N. Toosi University of Technology, Tehran, Iran

Abstract
Perishable product inventory management is a challenging issue because of its direct effect on companies' profits. The dependence of a product order cost on the order quantity is one of the practical but less examined assumptions in this problem literature. Hence, this paper considers the dependency between the order cost and order quantity as well as between the holding cost and the inventory level. This problem will have a non-convex object, and is not solvable through the usual mathematical methods. Thus, simulation-optimization approach is used to determine the perishable product inventory management policy with stochastic demand. The system dynamics approaches have been used to simulate the problem by minimizing the cost function. The casual diagram, inputs, output, and relation of the system are determined. A numerical example of a hypermarket is presented, and the optimal amount of the objective function is determined with optimization of the input variables via the experimental design's method. Then, to rule out the effects of different errors, a robust optimization of the model is presented. The results show that the proposed replenishment policy could benefit the necessary decisions regarding inventory management and control of the perishable products which count in different errors.
Keywords: Inventory Management Simulation, Perishable, System Dynamics, Non-convex, Robust Optimization, Meta-model.

1. Introduction

In today's global economy, products are perishable for various reasons such as changing customer expectations, high levels of competition, product lifecycle, technological advancement, and so on. Products that loses value over time are called perishable. Products such as vegetables, fruits, cooked products, bread, milk, meat, blood, radioactive materials, chemicals, medicines, seasonal clothes, fashion clothes and advanced technology products (such as computers and mobile phones) are perishable. Proper managing of perishable products inventory can lead to a noticeable profit for the system.

Many researchers have been trying to model and solve inventory management problems throughout history. The economic order quantity (EOQ), first introduced by Harris [1], was one of the earliest attempts to use mathematical modelling to assist managers in their daily decision making. The purpose of this model is to determine the optimum order quantity of products to minimize the total costs of system. The basic assumptions of this model are as follows: 1) The annual rate of demand is constant, and total demand is known in advance. 2)The ordering cost per item is known in advance and constant during planning. 3) Lead time is constant and known in advance. 4) The products ordering cost is constant and known in advance. 5) The annual holding cost of inventory is constant and known in advance. 6) There are no limits on storage, ordering or financials. 7) The products shortage is not allowed.

Many scholars have tried to apply new assumptions and adapt them to the current business world. Thus, many assumptions have changed and different models were created with assumptions such as offering discounts on purchase prices, allowed shortage, lost orders, uncertain demand, and so on.

One of the basic assumptions that have been less addressed in the research literature is the constant and independent order cost of the products from the order quantity. In practice, this assumption does not always true. The order cost is dependent on the cost of shipment, the cost of issuing a permit for the preparation of products and the cost of preparing machinery for the production of products by the supplier in some cases. In an unpublished research, Gupta and Gupta, for the first time addressed the order cost of a product as a continuous incremental function which depends on the order quantity time [2]. Other scholars like Hariri and Abuo-El-Ata [3], Abuo-El-Ata et al. [2], Kotb and Fergany [4], El-Wakeel [5], El-Wakeel and Al-yazidi [6], Banu and Mondal [7], Zhou et al. [8] and Kumar [9], also assume the dependency of the order cost on the order quantity.

Between the years of 2001 and 2018, 317 papers with the focus on perishable products inventory models were reviewed by Perez and Torres [10] who were concerned that although there has been increased attempts to represent reality in this field, a noticeable gap still exists between theory and practice. Hence, they stressed that more empirical evidence and practical implications need to be urgently considered.

Given that the assumption of the dependency of the order cost on the order quantity is not addressed in the literature as it should be, the current paper first examines the modelling of the inventory management problem in the state of varying ordering cost and its dependence on order quantity. Despite this assumption, the mathematical models of inventory management problems are far from commercial problems in this field. In practice, there are so many hypotheses and constraints that the uncertain nature and dynamics of many of them make mathematical modelling not suitable for meeting the business world's needs. One method that can take such complexities into account is the system dynamics that used in the current paper. With the help of modelling by system dynamics, we can consider stochastic data, the dynamic nature of different parts of the model and their effect on each other [11]; however, there is a

[*]Corresponding author.
E-mail addresses: Aaghaie@kntu.ac.ir (A. Aghaie), a.sharifi@email.kntu.ac.ir (A. Sharifi).
Address: No. 17, Pardis Avenue, Mollasadra Street, Vanak Square, Department of Industrial Engineering, K.N.Toosi University of Technology, Tehran, Iran. P.O.Box: 19395-1999, Tel.: +982184063363 Mobile:+989121487957

fundamental limitation to such simulations. They work based on input data provided by the decision-maker, which produces unreliable responses if the data is not accurate or uncertain. To solve this problem, we can use the design of experiments. As it is used in this paper, the design of experiments can be implemented to obtain the optimal input points of the simulation, for which the problem's objective function can be optimized. One of the optimization methods in the design of experiments literature is meta-model based optimization and a full factorial design. This scheme has the possibility of examining all interactions, conducting experiments in parallel and the independence of test results [12].

After obtaining the optimal response of the model, its validity should be checked. The reason for this is the presence of various noises and errors in the problem. Three types of conceptual errors can occur in meta-model based optimizations. A) Simulation error that occurs due to differences between reality and the computer's model. B) Meta-model error occurs due to differences between the computer's model and meta-model, and C) implementation error that occurs in practice when implementing optimal solutions of the problem [13]. The current paper applies the robust optimization method to take into account all these errors, which according to Yanıkoğlu et al. [14], can help in the robustness of the obtained optimal response against the mentioned errors. Fig. (1) depicts the schematic view of the proposed framework stages in the current paper.

In the following, section (2) provides the literature of the relevant research and the mathematical modelling of the problem is presented in the section (3). In section (4), other assumptions have been added to the model to get the problem assumptions closer to the reality and then system dynamics modelling has been used to simulate it. Lastly, section (5) provides the robust optimization of the problem and the final section discusses the conclusion and future researches.

2. Literature Review

The current paper addresses the inventory management problem as one of the most widely used of industrial engineering fields in practice. In the following, we survey the literature on varying ordering costs, non-convex objective function in inventory management problems for perishable products, the application of system dynamics simulation in inventory management problems, simulation optimization methods, and robust simulation optimization papers.

2.1 Varying Order Cost

Hariri and Abuo-El-Ata [3] research on the dependency of the order cost to the order quantity is the first published research that expanded the Gupta and Gupta model to determine the policy of a multi-product inventory management system whose ordering cost is a continuous function of the order quantity. They considered the total cost of the inventory system for all products to be limited and noted that their proposed cost function in real-world conditions could be effective and applicable. Their proposed function for the ordering cost is given in Eq. (1).

$$C_O(Q) = C_0 Q^b \qquad C_0 \geq 0 \ , 0 \leq b < 1 \tag{1}$$

In this formula, the ordering cost $(C_O(Q))$ is dependent on the order quantity (Q), and the b and C_0 values can be estimated using the two points of the ordering cost in practice. They also noted that if $b=0$, then $C_O(Q) = C_0$, which is the constant ordering cost (the basic assumption of the EOQ model). Fig. (2) depicts the ordering cost for $0<b<1$ [3].

In another study, Abuo-El-Ata et al. [2] investigated a probabilistic inventory management problem with two constraints: (a) limited and constant maximum inventory levels; and (b) the ordering cost dependency on the order quantity. They have used the geometric programming approach to get the optimal answer, and they have not considered any shortage of products in their model.

In two other researches, Fergany and El-Wakeel [15-16] have studied the problem of lost sales in the probabilistic inventory management model with normal and continuous distribution function. In both studies, the ordering cost is also variable. El-Wakeel [5] studied the inventory management problem with a limited delayed order and varying ordering cost. She considered the demand and the lead time period as uniform distributions with the attempt to minimize the total annual expected cost under the holding cost limitation.

The interesting point about all the previous studies is that all of them use the ordering function given in Eq. (1); but this function has a fundamental constraint. The ordering cost will increase significantly with the increase in the order quantity. However in practice, there is a definite upper limit for the ordering cost, and this cost can not be infinite. This limitation is explained in section (3.3) and is addressed in this paper.

Mendoza and Ventura [17] addressed a supply chain inventory system aimed at selecting the supplier and assigning the order quantity to them. In their inventory model, instead of ordering cost, they used a setup cost which is common in production models, and was dependent on the order quantity. Similarly, Matsuyama [18] has also changed the size of the economic order quantity to take the presence of discount on the purchase price and the increased cost of adjusting the device (setup cost) into account. The setup cost function considered in his research is presented in Fig. (3).

In Choi's [19], the order cost is considered general to reflect the economic scale and its impact on the cost, and its impact on a variety of models of economic order quantity. He examined this function in concave or convex terms and cited more than 20 related researches that use these types of ordering functions. It is also noted that the ordering cost and inventory cost functions in general can cause a non-convex problem.

El-Wakeel and Al-yazidi [6] considered a continuous review model (r, Q) with constrained varying order cost, and modelled their probabilistic inventory model with trapezoidal fuzzy numbers. To survey their proposed model, they used two crisp and fuzzy numerical examples and minimized the expected total cost of the model. Banu and Mondal [7] considered the EOQ model for deteriorating products with the link between credit and demand of the supplier to a retailer. They considered the ordering cost to be dependent on the number of replenishment cycles. A numerical example is presented to explain the model.

Additionally, Zhou et al. [8] modelled a constrained multi-item lot-sizing problem with a mixed integer linear programming. They considered a time-varying production setup cost in their model. Kumar [9] examined a varying order cost per cycle in his EOQ model for deteriorating products which includes a time-dependent exponential demand rate and penalty cost.

2.2 Perishable Inventory Management

Some papers are addressing the inventory management problems of perishable products, as well. An inventory management model for perishable products was first developed by Abramovitz [20] in the fashion and apparel industry, where the level of products was taken into account at the end of the sales period. He was followed by Ghare [21] on the consumption of perishable products with the aid of a negative time exponentiation. This research trend is ongoing. Recent researches have combined the effect of price, product innovation, and inventory levels to build the demand function and maximize the profit function [22].

Sarker et al. [23] developed a model that provides optimal ordering policies for perishable products. They have modelled their problem by considering the inflation rate, delays in paying the purchased product, and the allowed shortage. They concluded that the inventory time cycle and the products order quantity have a non-convex pattern associated with the inflation rate.

Chen and Chen [24] examined the pricing and order quantity for perishable products using the inventory management model, while considering shortage. They considered a perishable item with a multivariate demand function that depends on price and time and solved it using dynamic programming techniques.

Van Donselaar et al. [25] have examined the perishable products inventory management problem in supermarkets. They used the statistical data of two supermarket chains in Netherlands, and concluded that there is a significant statistical difference between the perishable products (with a shelf life of less than 30 days) and non-perishable products. These characteristics created the need to use different inventory management policies for each of these types of products. On average, perishable products experience higher sales, fewer items per packet, less dependency on weekdays, possible lower lead times, and lower average inventories.

Sana [26] research addressed mathematical modelling of inventory management of perishable products problem in the supermarket. He assumed that reducing sales prices could stimulate product demand. He solved his problem algebraically and used a numerical example and sensitivity analysis to test his model.

Shen et al. [27] provided a mathematical model for analyzing the supply of inventory for perishable agricultural products by taking the cooperation between the buyer and the seller into account. They have used a simple two-tier supply chain and analyzed their proposed inventory management policy performance using numerical analysis.

Duan and Liao [28] developed a policy based on shelf life for an inventory management problem in a supply chain with high levels of perishable products. They considered the blood platelet (with a life span of less than 5 days) as a perishable item, aimed at minimizing the rate of perishable products in the system, and also assumed that the maximum level of authorized shortage for the product was known in advance. They used an inventive heuristic method to solve the problem.

Avinadav et al. [29] have modelled the optimal price, order quantity, and replenishment interval for perishable products, in which the demand is dependent on price and time, and concluded that three goals could be reduced to a single-variable problem, duration of the replenishment interval. Their cost function is non-convex, and the demand is considered deterministic. Three numerical examples are also presented to provide a sensitivity analysis of the problem.

Chen et al. [30] combined the inventory management and pricing problems and modelled it for perishable products with fixed shelf life and price-dependent demand. Their goal was to maximize profits by determining the optimal price and product consumption policy.

Kouki et al. [31] provided a mathematical model for a lost sales (r, Q) inventory model for perishable products with fixed lead time. They have developed the Chiu [32] research that is based on a continuous review model (r, Q), as well as Chiu [33] research that has been used as a periodic review inventory model (R, T).

Chen et al. [34] have considered the food supply chain in terms of accidental demand to minimize the cost of the entire supply chain at the time span. They applied system dynamics simulation model in their inventory management problem due to the nonlinear relations existing in the problem and the dynamic forces in the model to determine the optimum lot size and replenishment interval. They have solved their problem with two models: the economic order quantity (EOQ) and the economic production quantity (EPQ).

Akbari Kaasgari et al. [35] provided an inventory management to a vendor in the supply chain of perishable products, with the consideration of price discounts. They considered a two-tier supply chain with a supplier and several retailers and supposed discounts on product prices after a specific time (if not sold by a specific time) to stimulate demand in their model. They used a nonlinear programming model to minimize the cost of the entire supply chain objective function and used the combination of genetic algorithm and particle optimization to solve it.

Dobson et al. [36] provided a model of the economic order quantity for perishable products with age-dependent demand and in deterministic setting. They solved their mathematical model with a numerical example.

In their research, Feng et al. [22] have examined the relationship between price, product freshness rate and displayed volume of product on the shelf with the product's demand. Their objective function is to minimize the total cost, which is non-convex by considering the decision variables as product price, cycle time, and final level of inventory. Chaudhary et al. [11] reviewed the literature on perishable products inventory management modelling. They examined 418 related researches between 1990 and 2016.

2.3 Non-Convex Objective Function

There are also some researches on the non-convex objective function in inventory management problems. An example would be Ting et al. [37] research in which he considered an (r, Q) inventory system at a fixed shortage cost and proved that the cost function of the problem is non-convex in general. They also provide a numerical example to show the non-convex objective function.

Chung et al. [38] presented a complete mathematical procedure to find the optimum values of fixed demand inventory management models that their order quantity is dependent on the buyer's reputation. Also they prove the non-convexity of their model's cost function. Lagodimos et al. [39] have also considered the EOQ model in the case of discrete time, and concluded that the obtained two-dimensional integer number optimization problem is non-convex type.

Other researches have taken different types of non-convex cost functions in different inventory models and pricing problems into account, for more information see: [40-45].

2.4 System Dynamics Simulation

There are many ways to model and solve inventory management problems. A particular category of these problems is modelled and solved by simulation, because of the complexity and dynamic nature of the problem. Thus, the related research literature has been reviewed, and number of them are given below.

Minegishi and Thiel [46] modelled and simulated the supply chain (chicken and cock) by system dynamics. They have examined the impact of a particular disease on the supply chain. Kumar and Nigmatullin [47] analyzed the food supply chain with the help of system dynamics. They researched unsustainable products and measured the effects of demand variability and lead time on supply chain performance.

Lee and Chung [48] used systems dynamics to explore the dynamics of existing inventory management models. They have examined an inventory management model for perishable products in a supply chain. In his research, Poles [49] worked on modelling systems with system dynamics to produce and control the inventory of production, to improve the system evaluation strategies.

Abbasi et al. [50] have modelled the hybrid intelligent control in fluidized bed reactors by dynamic process modelling. Their work presents an excellent application of mathematical dynamic modelling which can be presented by simulation as well.

Piewthongngam et al. [51] have modelled the supply chain of pig breeding in an uncertain environment using the system dynamics. Because of unexpected events that may lead to disruptions in supply chain order, such as contagious diseases occurrence, they have used system dynamics tools to manage the optimal supply chain so that they can examine different scenarios.

Chen et al. [34] have examined two different policies to fill the stock of randomly applied agricultural products with system dynamics. The weakness of their research is the lack of consideration of the lead time of the product (taken as zero) in the model. In their research, Langroodi and Amiri [52] studied the modelling of a multi-level, multi-product, multi-region supply chain, and under the conditions of uncertainty in demand, by the system dynamics approach. They used a numerical example to illustrate the output of their model.

Tordecilla et al. [53] utilized a systematic literature review approach to survey papers in the field of simulation-optimization methods which were used for designing and assessing resilient supply chain networks. They considered papers that were published during the period of 2000-2020 and found that the combination of dynamic conditions, uncertainty and simulation are research opportunities.

2.5 Simulation Optimization

Ayanso et al. [54] also used a full factorial design of 2^5 to analyze the sensitivity and control of inventory in a supply chain. Finally they obtained a model that provided the objective function of the problem in question. Barton and Meckesheimer [55] in their book chapter entitled "Metamodel-Based Simulation Optimization" have examined computationally costly discrete and random simulations. They used meta-models for simulation modelling, which simplifies simulation optimization, because firstly, the meta-model response (output) is a definite (not random) value. Secondly, the time to solve the meta-models and obtain their response is much less than the original model simulation.

Pourdaryaei et al. [56] proposed a hybrid forecasting method for a real-world electricity demand market and they optimized the price of the power market by simulation results. Pirhooshyaran and Snyder [57] focused on a complex and multi-echelon inventory optimization problem. They optimized the interaction between different agents with the environment simultaneously and provided a better performing method.

Jalali and Nieuwenhuyse [58] stated that meta-model based optimization is "considerably common", with Response Surface Methodology (RSM) as the most popular method of all meta-model based optimization methods. Kumar et al.

[12] used simulation to investigate the effects of inventory management in a global organization. To examine their simulation model, they used a full factorial experimental design with five factors at two different levels and calculated the main and interaction effects of the factors with the help of this method.

2.6 Robust Simulation Optimization

The uncertainty of the input data in the simulation model plays a decisive role. The three main components in a quality control process for a simulation process or model include design variables (input or controllable), uncertainties or noise factors (uncontrollable), and response variable(s) (output). Accounting for uncertainty or noise parameters in the model leads us to robust optimization methods [59]. Stinstra and Den Hertog [13] noted that the source of uncertainty can be the execution error that occurs in practice, when implementing the recommended values for decision variables. For example, implementing continuous variables in practice is difficult because the execution accuracy is limited. In addition to execution error, validation error can also occur for a simulation model (compared to the real world) and for a meta-model (compared to the simulation model).

Bertsimas et al. [60] proposed a robust optimization method for unconstrained problems whose cost function is non-convex and calculated based on simulation. Their proposed method can be based on Response Surface Methodology (RSM) and Kriging-based meta-models because they did not take specific assumptions for the problem structure into account. Their method can also take implementation errors and uncertainties in parameters in non-convex optimization problems into consideration. Yanıkoğlu et al. [14] stated in their research that the robust optimization method proposed by Bertsimas et al. [60] is suitable for the most commonly used meta-models in implementation.

Complicated simulations which utilize data for optimization purposes are predominantly favored by decision makers because they can facilitate the comprehension of non-linear processes. Additionally, more robust solutions can be created when uncertainty is included in the simulation-based optimization, resulting in a better protection of decision makers from impracticable solutions [61].

2.7 Paper Stand Point According to Literature Review

Finally, the literature review shows that managing the inventory of perishable products, in variable ordering cost (dependence on order quantity), stochastic product’s demand and non-convex objective function addressed by the current paper is rarely modelled and simulated. This paper is an integrated framework for considering all the noise and errors in the problem, which ultimately provides a robust and optimal response to the problem. As far as the authors know, this problem has not been resolved yet. Therefore, the current paper takes the combination of these assumptions as a gap in the literature and a research innovation. Fig. (4) depicts the innovation combination in each stages of the current paper based on the proposed framework (Fig. 1).

3. Mathematical EOQ Model with Varying Order Cost

3.1 Mathematical Problem Statement

The current paper applies the EOQ model provided by Harris [1], which is one of the main components of inventory control theory. This basic model considers a product’s demand as constant and known in advance, in an unlimited time span. The purpose of this model is to determine the order quantity in such a way that minimizes the average long-term cost of the system. These costs include the ordering, holding and purchasing costs. Given the constant purchase cost, the main tradeoff is between the order cost and holding cost. Large order quantity increases the cost of holding per item in a unit of time, while the low order quantity also increases the number of ordering the products and consequently increases the order cost per item in a unit of time. In this paper, the basic assumptions of the Harris [1] model are considered, with the difference that the ordering cost per each item of products is determined by the order quantity rather than being fixed, which are referred to in section (3.3) and Eq. (3), as well.

3.2 Notation

The parameters used in the mathematical model are given in Table (1).

3.3 Problem Modelling

The total annual cost of the inventory system includes the cost of purchase, the cost of holding and the ordering cost annually (Eq. 2).

Total annual cost = Annual order cost + Annual holding cost + Annual purchase cost (2)

As stated, in this paper the ordering cost of products depends on the order quantity. Given the limitation for Eq. (1) (ordering costs approaches infinity with an increase in order quantity), by combining the costing function provided by Hariri and Abuo-El-Ata [3] (Eq. 1) and the cost function provided by Matsuyama [18] (Fig. 3), the cost of each ordering time is incremental as follows (Eq. 3).

$$C_O(Q) = \begin{cases} C_0 + (\alpha q_1 - \beta)^{\gamma} & 0 < Q \le q_1 \\ C_0 + (\alpha Q - \beta)^{\gamma} & q_1 < Q \le q_2 \\ C_0 + (\alpha q_2 - \beta)^{\gamma} & q_2 < Q \end{cases} \tag{3}$$

Where, q_1 and q_2 are constant values $(0 \le q_1 < q_2)$, which determine the intersect points of the ordering cost function. C_0 also represents the fixed ordering cost $(C_0 > 0)$. α, β and γ are constant coefficients and $\alpha > 0$ (if $\alpha < 0$, the ordering cost function is descending). Also, for the ordering cost function to be ascending, $\gamma \ge 1$ and is an odd number. Fig. (5) shows the ordering cost function (Eq. 3) for a numerical example with the following values: $q_1 = 100$, $q_2 = 200$, $C_0 = 300$, $\alpha = 0.05$, $\beta = 7.5$ and $\gamma = 5$ (E.g. 1).

According to Fig. (5), for the left-hand side of the figure to be positive (for $0 < Q \le q_1$), the constant value of $C_0 + (\alpha q_1 - \beta)^\gamma > 0$ must be obtained, which implies:

$$\beta < \alpha q_1 + C_0^{\frac{1}{\gamma}} \tag{4}$$

Also, for the right-hand side of the Fig. (5) to be positive (for $q_2 < Q$), the constant value of $C_0 + (\alpha q_2 - \beta)^\gamma > 0$ must be obtained, which implies: $\alpha > \left(\left(-C_0^{\frac{1}{\gamma}} + \beta \right) / q_2 \right)$. Also the assumption of $\alpha > 0$, implies:

$$\alpha > \max \left\{ 0, \frac{-C_0^{\frac{1}{\gamma}} + \beta}{q_2} \right\} \tag{5}$$

And for the middle side of the Fig. (5) to be positive (for $q_1 < Q \le q_2$), the value of $C_0 + (\alpha Q - \beta)^\gamma > 0$ must be obtained, which implies: $Q > \left(\left(-C_0^{\frac{1}{\gamma}} + \beta \right) / \alpha \right)$. If we input the maximum value of β (Eq. 4), we get $Q > q_1$, which always holds. On the other hand, for the function to be ascending, the derivative of the function implies: $\gamma\alpha(\alpha Q - \beta)^{\gamma-1} \ge 0$. Since $\gamma \ge 1$ is an odd number and $\alpha > 0$, the result of $\gamma - 1$ is an even number, and this relation $\left(\gamma\alpha(\alpha Q - \beta)^{\gamma-1} \ge 0\right)$ always holds true.

The annual ordering cost is $C_O(Q) D/Q$, where D/Q represent the number of times an order is made per year. The cost of holding and purchasing is also calculated based on the basic assumptions of the EOQ model, which is equal to [1]:

$$\text{Annual holding cost} = \frac{1}{2} hQ \tag{6}$$

$$\text{Annual purchase cost} = PD \tag{7}$$

Finally, given formulas 2, 3, 6 and 7, we have:

$$TC = C_O(Q)\frac{D}{Q} + \frac{1}{2} hQ + PD \tag{8}$$

Fig. (6) shows the function of the total annual cost (Eq. 8) for a numerical example with the following values: $q_1 = 100$, $q_2 = 200$, $C_0 = 300$, $\alpha = 0.05$, $\beta = 7.5$, $\gamma = 5$, $D = 1000$, $h = 10$ and $P = 15$ (E.g. 2).

Given Fig. (6), we visually realize that the total annual cost function (Eq. 8) for the E.g. (2) data is a non-convex function. Since the values of α and β are dependent on each other, it is impossible to obtain a formula for the order quantity, for which the total annual cost function is always non-convex. Hence, similar to Liu and Yang [62], Kalpakam and Shanthi [63], Olsson and Tydesjö [64] and Kouki et al. [31]'s researches, numerical examples can be used to prove the non-convexity of the annual cost function. We have tested 36 different numerical examples and all of them showed the non-convexity of the annual cost function, but for the sake of keeping the paper on point, they are not presented in this paper. However, in section (4.4) we present a mathematical proof for the non-convexity of the annual cost function.

The model presented in this section is based on the basic assumptions of the EOQ model, but in practice, inventory management models are very complex and require much more assumptions to have high performance in the business world. Thus, a functional model of the mathematical model is presented in the next section.

4. Simulation by System Dynamics

4.1 Problem Statement

Controlling the inventory of perishable products is essential because it has a direct impact on sales, prices, inventory levels, costs (waste, deterioration, transportation, holding, etc.), and product availability, all of which affect on the amount of the system profit. Therefore, many managers and researchers have tried to understand and model the inventory management problems of perishable products with various factors such as product characteristics, competition level, internal and external constraints, price impact's on demand, product availability, and nature of demand in different environments [11].

Chaudhary et al. [11] concluded that the use of dynamic methods to address various purposes (including product availability) is appropriate in modelling the inventory management of perishable products in their literature review paper. They also mentioned that some of the real factors and events that occur are rarely considered in the models. For example, only a few researchers have considered discount on prices assumption that is found to be abundant in real-world conditions.

This paper addressed the inventory management problem of perishable products. This is a single-product type, and the product is perishable according to a known and fixed deterioration rate, which is a fraction of its inventory, and has a limited life span. Other main assumptions used in modelling this problem are as follows:

- The demand for the product is stochastic and dependent on price and time.
- The inventory policy is a continuous review (r, Q) type, and whenever the inventory level reaches the reorder point (r), the order quantity (Q) of the product is ordered.
- The lead time is stochastic and follows a normal distribution.
- Product shortage is allowed and is of lost order type.
- The purchase price of the product is dependent on the order quantity. As the purchase quantity increases, the discount is applied to the purchase price.
- The product sales price is dynamically determined by the model.
- The inventory management problem is modelled for a hypermarket, which has no limit on the order quantity from the product supplier and on the product's holding capacity.
- The ordering cost of the product varies depending on the order quantity.
- The cost of product holding is also variable and depends on the level of inventory.
- The cost of waste and product purchase is also considered.

For the aforementioned problem, the following goal is considered:

- ❖ Minimizing the average net costs.

Next section of the paper (section 4.2) is an extension of Sharifi et al. [65] research which implemented an inventory management problem for perishable products and tried to find an optimal (but not a robust) solution with the help of system dynamics. Additionally, their research considered the purchase price, ordering cost and holding cost as fixed values. However, in this paper these are counted to be dependent on other parameters (as stated in the previous paragraph). While Sharifi et al. [65] used different numerical examples for sensitivity analysis of the problem, this paper has utilized a systematic sensitivity analysis that is provided by Vensim DSS which is a more precise procedure.

4.2 Simulation by Vensim DSS

The system dynamics was used in the simulation to face the problems in modelling. The system dynamics is suitable for modelling and simulating systems with nonlinear relationships and dynamic effects. This type of model can change itself by input and output flows from the system. System dynamics models include stock (level), flow (rate) variables and constant values. This structure of stock and flow systems and the relationships between these variables can be considered and form a simulation model of the system dynamics by stock-flow diagrams [34]. The causal loop diagram of the problem is presented in Fig. (7). Positive marks at the end of the arrows indicate that the two variables on both sides of the arrow are co-directional. Negative marks represent the inverse behavior of the variables on both sides of the arrows. The two parallel lines were drawn in the middle of some of the arrows also indicate a time delay for applying the first variable's effect on the second variable.

Fig. (8) depicts the stock-flow diagram of the model plotted in Vensim DSS software. In this figure, the ReOrder and Order Quantity variables are exogenous variables. The optimal values of these two input variables need to be found to achieve the goal of minimizing the Gross Cost Average. Other exogenous variables, such as the holding cost of each unit per time, the order cost of each unit, deterioration cost of each unit and the buy price of a product, are also depend on the problem's environment and conditions.

The mathematical equations in this study were based entirely on the relationships existing in Vensim DSS. In these relationships, constant parameters represent the values of the exogenous inputs and the initial values of the stock variables. Relationships are written in such a way that they can accurately draw real-world concepts. Table (2) illustrates the formulas of all the variables of the problem. Also fixed values are considered as inputs of the problem.

4.3 Numerical Example and finding the optimal values of inputs

Table (3) shows the fixed and definite values required to solve the numerical example for the model presented in Section (4.1) (E.g. 3).

The stochastic value of the demand (which depends on the time and price of the sale of the product) is considered according to the criteria for demand in researches by the Poles [49] and Abad and Jaggi [66] as follows (Eq. 9):

$$\text{Demand} = \left[\left(27 + SIN\left(Time\right) + 10 * SIN\left(\frac{3.14}{18} * Time\right) + RANDOM\ UNIFORM\left(10,20,15\right) \right) * \frac{Base\ Sale\ Price}{Sale\ Price} \right] \quad (9)$$

To find the optimal input values, we used the optimization toolbox in Vensim DSS and the results show the optimal values (ReOrder Point=97, Order Quantity=51). The demand values figure during the simulation period (365 days) for the numerical example (E.g. 3) and the optimal input values are shown in Fig. (9).

Fig. (10) shows the optimal inventory level for optimal input values (ReOrder Point=97, Order Quantity=51). The sinusodal behavior of the optimal inventory level is caused by the fluctuation in demand and is occurred by the nature of the stochastic values of demand. Fig. (11) depicts the objective function (Gross Cost Average) for the optimal input values during the simulation period according to E.g. (3) data. At the beginning of the simulation, the amount of volatility of the variable of the objective function (Gross Cost Average) is very high and the fluctuations have decreased over time. The reason for this lies in the nature of the simulation of system dynamics problems. One of the main capabilities of system dynamics is to modify the model over time, which does this by changing the variable weight of the lost sales (Total Lost Profit Weight). This variable changes by changing the discount rate (Discount Percent) and, consequently, the change in the demand for the product, the amount of sales and the lost sales, and with the help of them, the weight of the lost sales variable varies. These changes, in aggregate, lead to the balance of the model, reduce the volatility of the objective function variable and, in the long run, help stabilize the model.

4.4 Calculating the objective function's meta-model

To calculating the objective function's meta-model and surveying its concavity, the full multi-level factorial design by Minitab software have been used. In Minitab software, with the help of the "Create Factorial Design" command, we generate a complete full multi-level factorial design with two factors of ReOrder Point and Order Quantity. Given the E.g. (3) data, the factor values of (80, 90, 100, 110) and (40, 50, 60, 70, 80, 90, 100, 110) have been selected for levels of ReOrder Point and Order Quantity respectively. Also, the experimental design was performed in 3 iterations (for Seed= 10, 15, 20). Seed used in the demand formula (Eq. 9), and present the initial value of the product's demand. Vensim DSS software has been used to simulate the inventory management problem of perishable products for the input values obtained by the full multi-level factorial design and the objective function values (Gross Cost Average) have been calculated. These values have been added to the Minitab software and the test design has been analyzed using the "Fit Regression Model" command. Finally, the meta-model obtained for the simulation problem is as follows (Eq. 10):

$$\begin{aligned} \text{Gross Cost Average} = {} & 23714978 - 944700\ \text{ReOrder Point} + 441657\ \text{Order Quantity} \\ & + 11923\ \text{ReOrder Point*ReOrder Point} + 893\ \text{Order Quantity*Order Quantity} \\ & - 9365\ \text{ReOrder Point*Order Quantity} \\ & - 44.1\ \text{ReOrder Point*ReOrder Point*ReOrder Point} \\ & - 11.04\ \text{Order Quantity*Order Quantity*Order Quantity} \\ & + 31.2\ \text{ReOrder Point*ReOrder Point*Order Quantity} \\ & + 16.53\ \text{ReOrder Point*Order Quantity*Order Quantity} \end{aligned} \quad (10)$$

To investigate the concavity of this meta-model, the Hessian matrix and Hessian matrix determinant have been calculated as follows (Eq. 11-12):

$$H = \begin{pmatrix} 23846 + 62.4 OrderQuantity - 264.6 ReOrderPoint & -9365 + 62.4 ReOrderPoint + 33.06 OrderQuantity \\ -9365 + 62.4 ReOrderPoint + 33.06 OrderQuantity & 1786 - 66.24 OrderQuantity + 33.06 ReOrderPoint \end{pmatrix} \quad (11)$$

$$detH = -12641.4 ReOrderPoint^2 + 15464.2 ReOrderPoint * OrderQuantity + 1484530 ReOrderPoint - 5226.34 OrderQuantity^2 - 848899 OrderQuantity - 45114269 \quad (12)$$

Eq. (12) shows that the Hessian matrix determinant is neither positive nor negative and the matrix is indeterminate. The Hessian matrix determinant is negative per specific inputs values and resulting in a non-convex meta-model. Fig. (12) shows a three-dimensional diagram of the objective function's meta-model (Eq. 10) . It shows the inputs variable's behavior (ReOrder point and Order Quantity) on the objective function (Gross Cost Average). The areas shown in red indicate the range of the minimum values of the objective function.

5. Robust Optimization of Objective Function based on its Meta-model

5.1 Robust optimization algorithm considering implementation errors

The solution obtained from the full multi-level factorial design (Section 4.4) can deal with some uncertainties and noise factors; however, given the non-convexity of the obtained meta-model and the implementation errors that may occur when applying the optimal response in practice, it is necessary to use robust optimization approaches to achieve a stable optimal solution that takes all types of errors (simulation, meta-model, and implementation errors) into account. So, in the following, the method presented by Bertsimas et al. [60] is adjusted to our problem to achieve the robust optimal solution under these conditions **.**

The "robust local search" method is an iterative algorithm with two parts per iteration. In the first part, we examine the current iteration neighborhood to estimate the worst-case cost and collect high-cost neighbors. Then, using the knowledge gained from this neighborhood a robust local movement is created, i.e. a step towards the descent of the robust problem. These two parts are repeated to meet the completion conditions, which is when another appropriate descending direction cannot be found. The steps of the algorithm used in the current paper are as follows **.**

Step 0. Initialization: x^1 is the arbitrary decision vector that is arbitrarily selected. Set k = 1 **.**

Step 1. Neighborhood exploration:

Find M^k. the set contains the worst implementation errors Δx^* that occur the highest cost in the neighborhood x^k.

Step 2. Local robust move **:**

(1) Solve the following problem (Eq. 13). If this is not possible, quit **.**

$$\begin{aligned} &min_{d,\beta}\ \beta \\ &s.t\ \|d\|_2 \le 1, \\ &d'\ \Delta x^* \le \beta \qquad \forall \Delta x^* \in M^k \\ &\beta \le -\epsilon \end{aligned} \tag{13}$$

Where, ϵ is a positive value. When this problem has a practical solution, its optimal solution, d^*, Forms the maximum possible angle θ_{max} with all Δx^***.** This angle is always greater than 90 due to the limit $\beta \le -\epsilon < 0$. $\beta \le 0$ is not used instead of $\beta \le -\epsilon$, because we want to remove this obvious solution $\left(d^*, \beta^*\right) = (0,0)$. When ϵ is small enough, this problem is impossible. Note that if the problem is possible, the limit $\| d^* \|_2 = 1$ is satisfied automatically.

(2) Set $x^{k+1} = x^k + t^k d^*$ where d^* is the optimal solution for Eq. (16).

(3) Set k=k+1. Go to step 1 **.**

Steps 1 and 2 details are discuss in the following.

- **Neighborhood exploration**

In this section, we describe a neighborhood exploration algorithm that uses $n+1$ ascending gradients from different starting points in the neighborhood. When exploring the $\hat{x}$ neighborhood, we are basically trying to solve the problem of internal maximization. First, apply an ascending gradient of descending step size. The size of the initial step used is $\frac{\Gamma}{5}$, which decreases by 0.99 after each step. The ascending gradient ends after the time limit. Then use the last point inside the neighborhood as the initial solution to solve the following sequence (Eq. 14):

$$max_{\Delta x}\, f\left(\hat{x} + \Delta x\right) + \epsilon_r ln\left\{\Gamma - \| \Delta x \|_2\right\} \tag{14}$$

Note that if we use $u = \left\{\Delta x \in R^n \mid \| \Delta x \|_2 \le \Gamma\right\}$, We must use $\| \Delta x \|_p$ in Eq. (14). The positive value ϵ_r is chosen so that the additional expression $\epsilon_r ln\left\{\Gamma - \| \Delta x \|_2\right\}$ maps the gradient step into the neighborhood's interior to ensure that the ascent remains firmly inside. In this way a good estimation of the local maximum can be found quickly. A general neighborhood exploration algorithm is:

For a n-dimensional problem, we use the n + 1 gradient ascent starting at $\Delta x = 0$ and $\Delta x = sign\left(\vartheta f\left(x = \hat{x}\right) / \vartheta x_i\right)\left(\Gamma / 3\right) e_i$ for $i = 1, \ldots n$ where e_i is a unit vector along the coordinates i. The results of all function evaluations performed during multiple gradient ascents are recorded in a set of H^k with all previous histories along with neighborhood exploration in k iteration. This history set is then used to estimate the cost of the worst-case $x^k, \hat{g}\left(x^k\right)$.

- **Local robust move**

In the second part of the robust local search algorithm, we update the current iteration with a local scheme (which is more robust) based on our knowledge of the N^k neighborhood. A new iteration is found by finding a direction and distancing oneself from it, so that all high-cost neighbors are removed from the new neighborhood. Here's how direction and distance can be found effectively **.**

- **Finding the direction**

To find the direction in x^k that improves $\tilde{g}\left(x^k\right)$, we include all known high-cost neighbors of H^k in our set (Eq. 15) **.**

$$M^k = \left\{ x \mid x \in H^k, x \in N^k, f\left(x\right) \geq \tilde{g}\left(x^k\right) - \sigma^k \right\} \tag{15}$$

The cost factor σ^k controls the size of the set and may be changed in one iteration to ensure transfer conditions. In the first iteration σ^1 is set to $0.2 \times \left(\tilde{g}\left(x^1\right) - f\left(x^1\right)\right)$. In subsequent iterations, σ^k is set using the final value σ^{k-1}. The probem of finding a good direction d, away from bad neighbors, can then be formulated as following (Eq. 16):

$$\begin{aligned}
& min_{d,\beta} \beta \\
& s.t \ \|d\|_2 \leq 1, \\
& d' \left(\frac{x_i - x^k}{\| x_i - x^k \|_2} \right) \leq \beta \qquad \forall x_i \in M^k \\
& \beta \leq -\epsilon
\end{aligned} \tag{16}$$

Where ϵ there is a positive scalar value. When Eq. (13) is infeasible, x^k is surrounded by unsuitable neighbors. However, since we may have trouble classifying bad neighbors, we reduce σ^k, rebuild M^k, and solve the updated Eq. (16). When σ^k decreases, we divide it by a factor of 1.05. The termination condition is available when Eq. (16) is impossible and σ^k is below a threshold. If x^k is surrounded by "bad" neighbors and σ^k is small, we assume that we have achieved at least one robust local minimum.

- **Finding the distance**

After finding the path d^*, we want to select the smallest step size ρ^* so that each element in the set of bad neighbors M^k is at least at the new iteration neighborhood border $x^{k+1} = x^k + \rho^* d^*$. To ensure that we made significant progress in each iteration, we set the minimum step size $\Gamma / 100$ in the first iteration and reduced it by 0.99, respectively.

- **Checking the direction**

Knowing the d^* update direction and the step size ρ^*, we update the set of bad neighbors as follows (Eq. 17).

$$M^k_{updated} = \left\{ x \mid x \in H^k, \| x - x^k \|_2 \leq \Gamma + \rho^*, f\left(x\right) \geq \tilde{g}\left(x^k\right) - \sigma^k \right\} \tag{17}$$

This set includes all known neighbors that are further away from the neighborhood and will cost more than $\tilde{g}\left(x^k\right) - \sigma^k$. We examine whether the desired direction d^* is still a descending direction that moves away from all members in the set $M^k_{updated}$. If so, we accept the upgrade step $\left(d^*, \rho^*\right)$ and continue the next iteration. If d^* is not the new set's descending direction, we repeat the local robust transmission by solving Eq. (13), but with $M^k_{updated}$ instead of the original M^k. Again, the value of σ^k may be reduced to find a possible direction. As a result, during a repetition, the local robust transmission may occur several times.

5.2 Computation Results

Finally, we considered the obtained objective function's meta-model (Eq. 10), which is a non-convex function and coded the problem with a robust optimization approach in MATLAB software. The solution was solved by the proposed robust optimization algorithm (section 5.1). The robust optimal solution obtained by the robust optimization approach is $\left(ReOrder\, Point = 87, Order\, Quantity = 43\right)$ where $Gross\, Cost\, Average = 216280$. Table (4) presents a summary of all values that are calculated in section (4.3, 4.4, and 5.2). Similar to Fig. (10), in Fig. (13) three optimal input sources that were presented in Table (4) are used and the simulated optimal inventory level of them is presented. It shows that the inventory level on average is lower in the robust optimization input data.

Hughes [67] proposed various metaheuristic methods for the black-box optimization problems. He stated that when the best solution could not be implemented in practice (implementation error), we should attempt to find a robust solution, which should be within the uncertainty space and yet performs well. Based on Table (4) data, the result obtained by the robust optimization approach, compared to other methods provides the worst solution $\left(Gross\, Cost\, Average = 216290\right)$. It is not favorable for a minimizing goal problem; however as stated in Bertsimas et al. [60], by taking different errors (simulation, meta-model, and implementation errors) into account, the

$\left(ReOrder\ Point = 87, Order\ Quantity = 43\right)$ are considered as the robust optimal values for the problem's inputs.

6. Conclusion and Future Research

In this work, the objective function of the inventory management model of the EOQ concerning the ordering cost dependent on order quantity is defined as a multi-criteria and non-convex function. Additional assumptions are added to make the model from the basic state to an operational and practical model in today's business world. Solving non-convex models has a particular complexity due to several local minimum points in their objective function. Nevertheless, in this paper, the inventory management model of EOQ is more applicable.. Fig. (1) presented a schematic view of this paper's proposed framework and Fig. (2) presented the combination of innovation in this paper.

Accordingly, the mathematical representation of the proposed ordering cost formula is presented in Section (3). The inventory management problem is modelled in Section (4), where the problem is for perishable products (with a deterioration rate) and the product's demand is stochastic (depending on time and price of product sales). Because of the computational complexity and dynamic nature of the model, the modelling by the usual mathematical methods is complicated and, in some instances, impossible. Therefore in this paper, a dynamic and flexible simulation method is presented to solve the inventory management problems of perishable products, regardless of certain assumptions (such as demand function following a particular distribution function). Simulation of the proposed model is done by system dynamics in Vensim DSS. The system dynamics model and relation between variables is determined by studying a real-world problem and then the problem is surveyed via a numerical example for a perishable product in a hypermarket. The simulation model presented in this paper allows for the modelling of a variety of perishable products with stochastic demand. Besides, the order cost of the product is dependent on the order quantity, which is one of the prevailing assumptions in the real world. The cost of product holding dependents on the level of inventory, which is believed to be right in the real world, and a discount on the purchase and sale price of the product is also considered.

To find the optimal input values of the simulated model, a full factorial design method is used, and a meta-model approximation of the simulation's objective function is calculated. Also by taking three different noises, which are simulation, meta-model, and implementation errors into account, a robust optimization algorithm that is adjusted from Bertsimas et al. [60] work is presented. Finally, the robust optimal value of the objective function (Gross Cost Average) is obtained by optimizing the input variables (ReOrder Point and Order Quantity). Table (4) summarized the results and Fig. (13) presents a visual representation of the simulated optimal inventory level. The results indicate that the policy for filling inventory can be useful in making decisions to manage and control the inventory of perishable products.

For future research, it is proposed to examine the model in a production state or within the supply chain concept. Other simulation base methods can be used for solving the problem and comparing the obtained results with the result of this paper. Considering further assumptions including order quantity, shipping or holding limitations, descending demand, multi products and suchlike can also help to make the model more operational and applicable.

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Figure Caption

Fig. 1: Schematic view of the proposed framework (in stages)
Fig. 2: The ordering cost function $(0 < b < 1)$ [3]
Fig. 3: The incremental setup cost function [18]
Fig. 4: Innovation combination in each stages of the proposed framework
Fig. 5: Ordering cost function (E.g. 1)
Fig. 6: Total annual cost function (E.g. 2)
Fig. 7: The causal loop diagram for the inventory management of perishable products
Fig. 8: The stock-flow diagram for the inventory management of perishable products in Vensim DSS
Fig. 9: Demand values during the simulation period
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Fig. 11: Gross Cost Average during the simulation period
Fig. 12: 3-D figure of Gross Cost Average
Fig. 13: Optimal inventory level for Table (4) input sources

Table Caption

Table 1: Mathematical model parameters
Table 2: Variables used in the problem modelling
Table 3: Fixed values (E.g. 3)
Table 4: Summary of optimal inputs, calculated by different methods

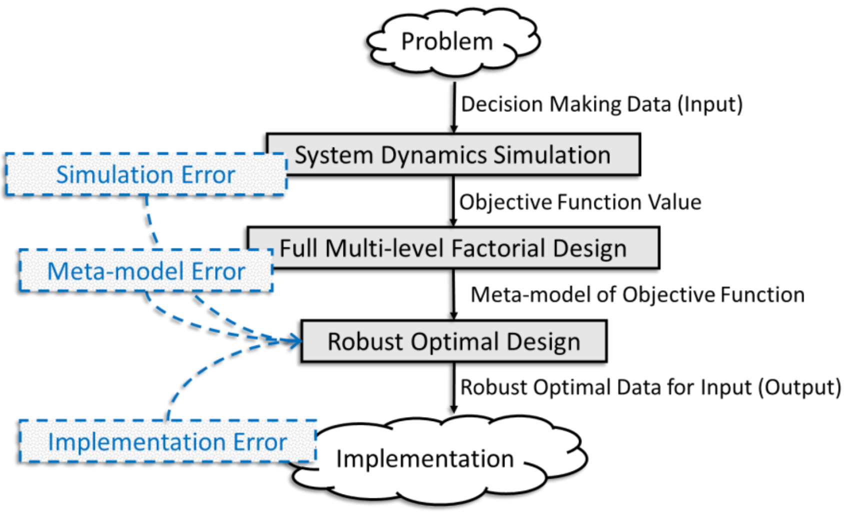

Fig. 1: Schematic view of the proposed framework (in stages)

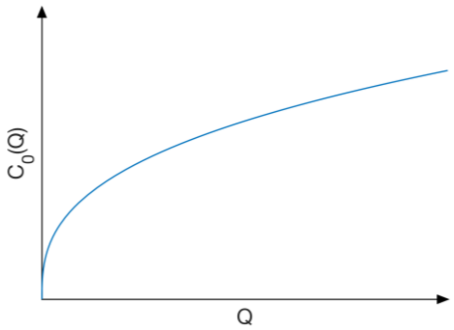

Fig. 2: The ordering cost function $(0 < b < 1)$ [3]

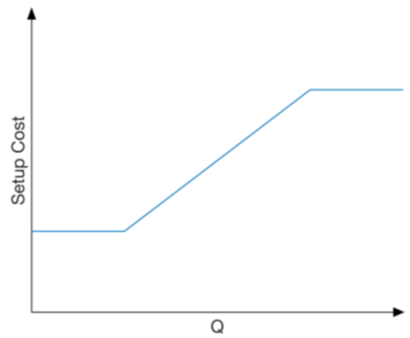

Fig. 3: The incremental setup cost function [18]

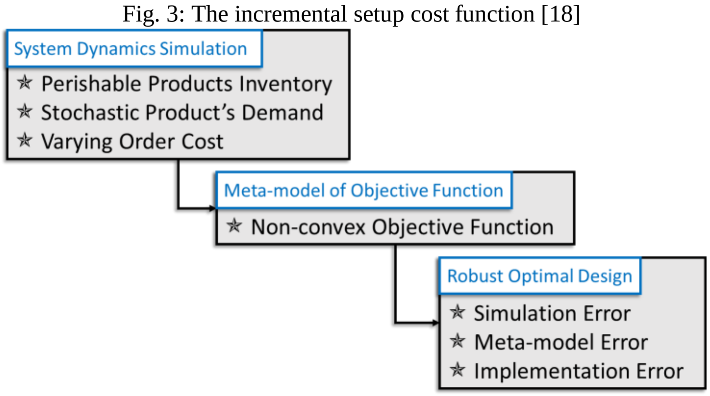

Fig. 4: Innovation combination in each stages of the proposed framework

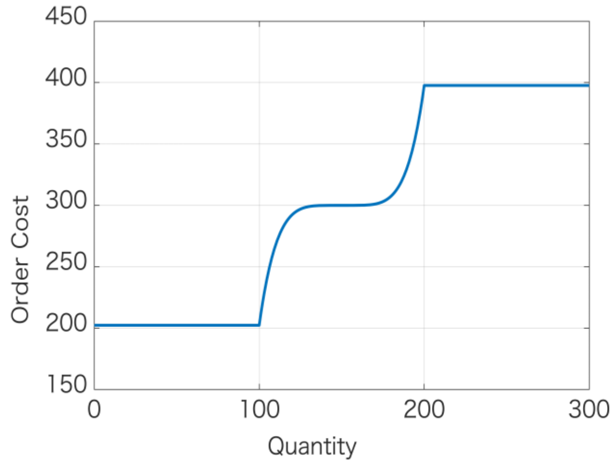

Fig. 5: Ordering cost function (E.g. 1)

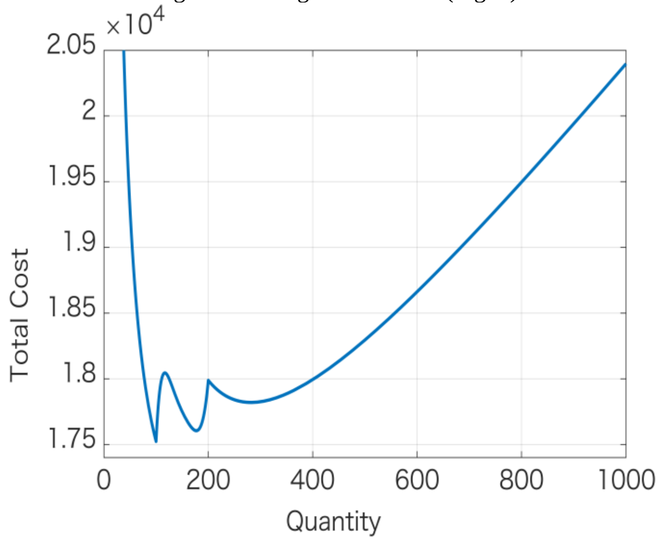

Fig. 6: Total annual cost function (E.g. 2)

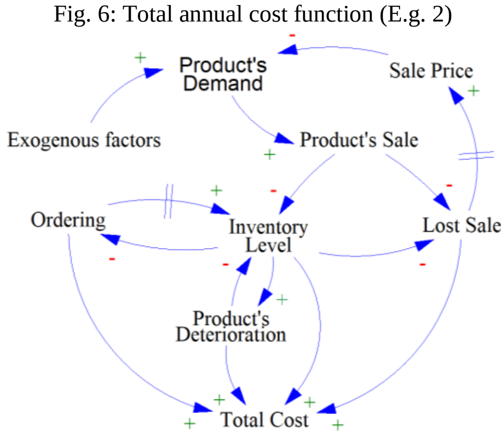

Fig. 7: The causal loop diagram for the inventory management of perishable products

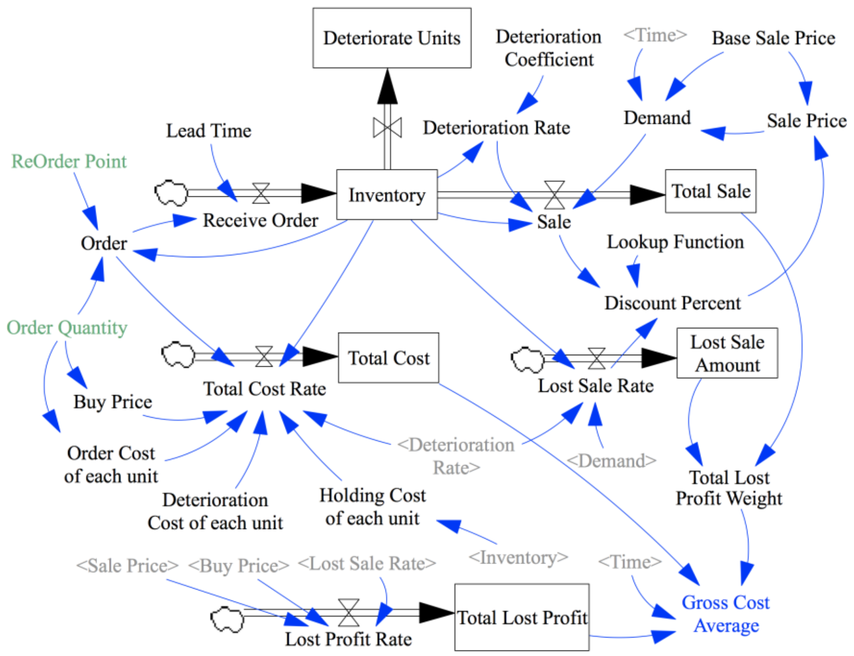

Fig. 8: The stock-flow diagram for the inventory management of perishable products in Vensim DSS

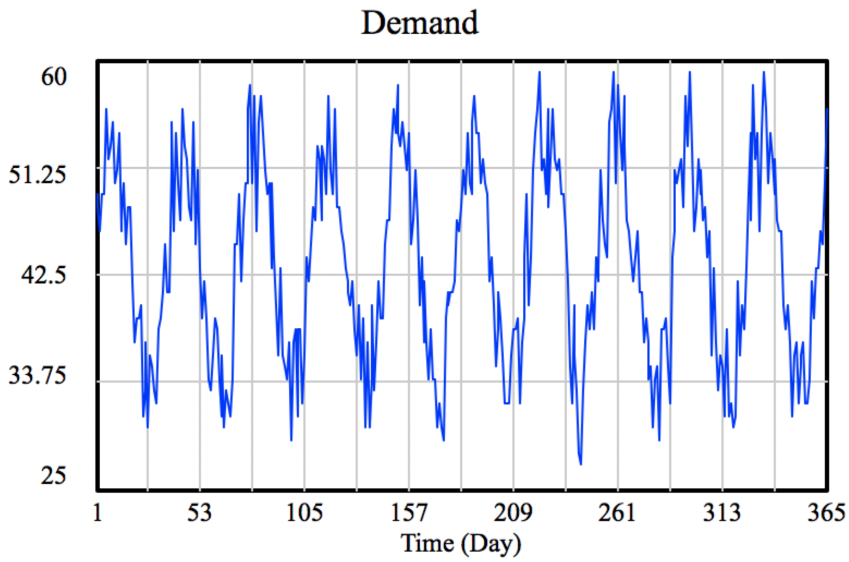

Fig. 9: Demand values during the simulation period

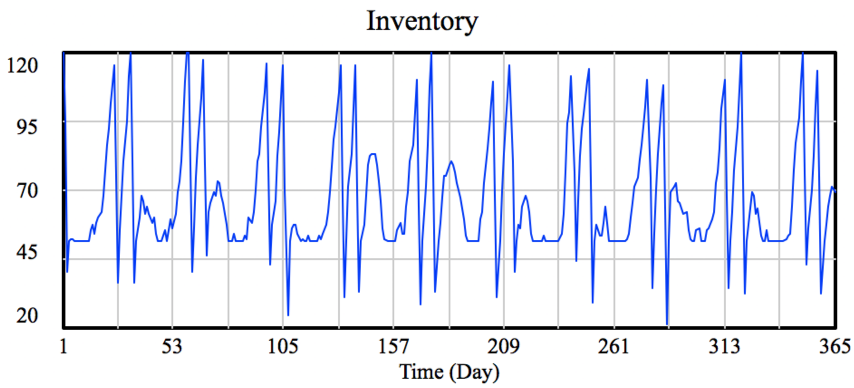

Fig. 10: Optimal inventory level

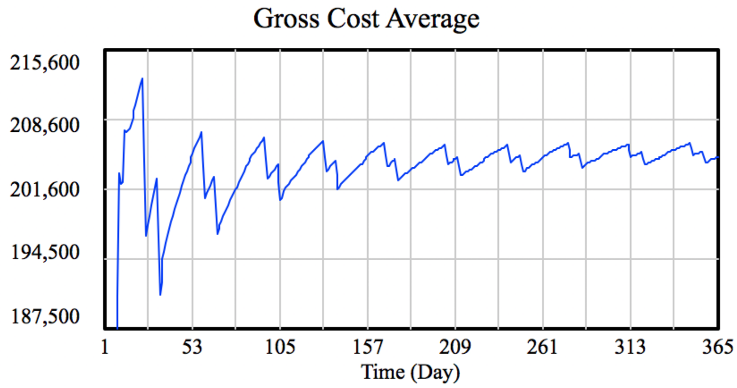

Fig. 11: Gross Cost Average during the simulation period

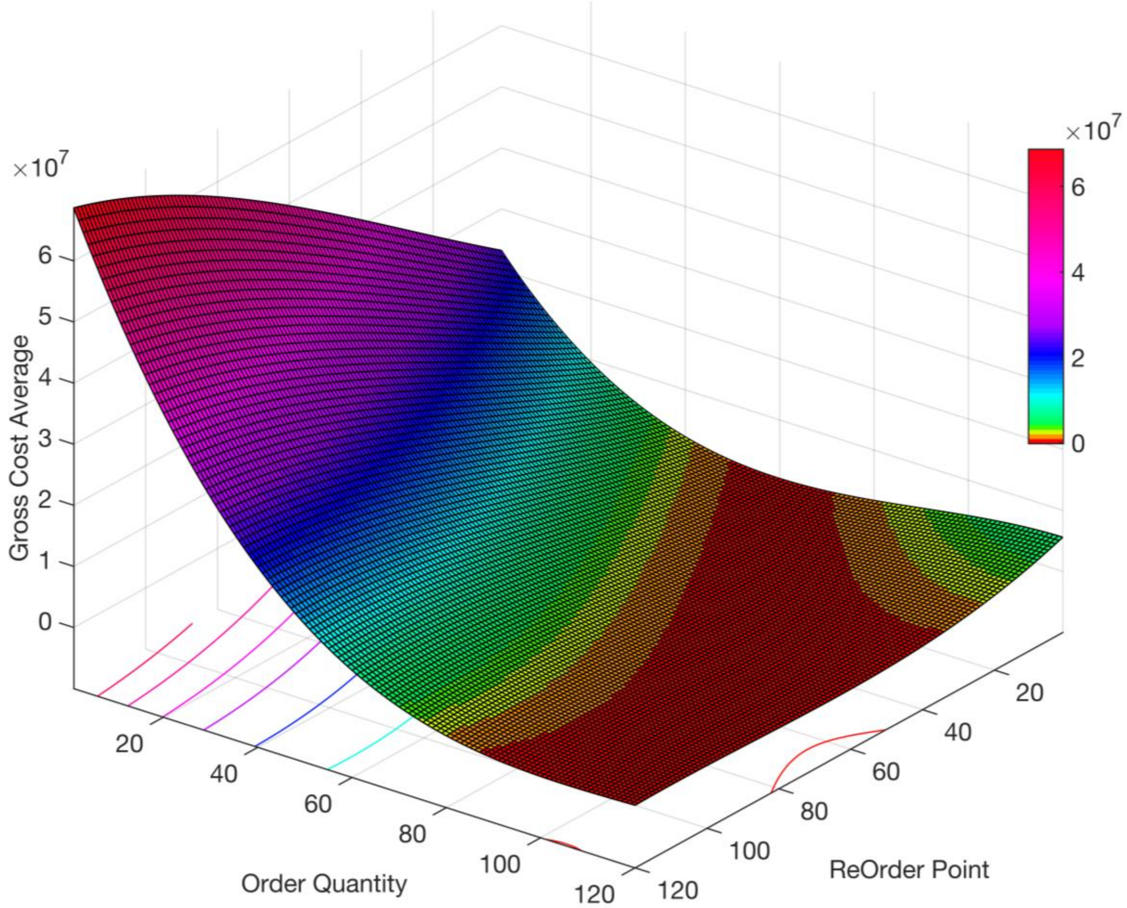

Fig. 12: 3-D figure of Gross Cost Average

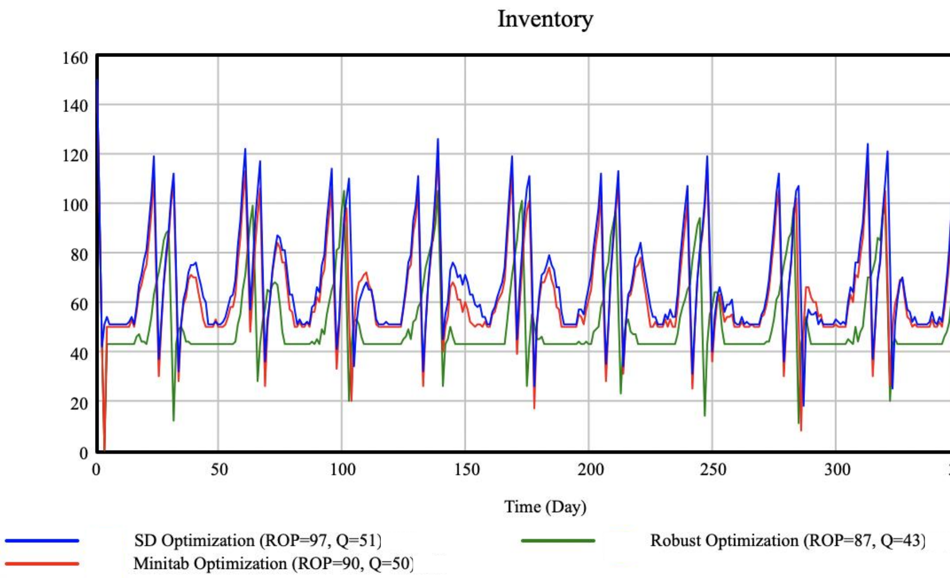

Fig. 13: Optimal inventory level for Table (4) input sources

Table 1: Mathematical model parameters

Parameter	Description
D	Annual demand rate (fix, certain and known)
h	Holding cost for each product per unit of time (fix, certain and known)
P	Buy (purchasing) price for each product (fix, certain and known)
Q	Order quantity (decision making variable)
$C_O(Q)$	Order cost function (dependent to order quantity (Q))
TC	Total annual cost

Table 2: Variables used in the problem modelling

Parameter	Formula
$Inventory(t)$	$=\int_{t_0}^{t}\left(Receive\,Order(t)-Sale(t)-Deterioration\,Rate(t)+Inventory(t_0)\right)dt$
$Deterioration\ Rate(t)$	$=Inventory(t)*Deterioration\,Coefficient$
$Sale(t)$	$=IF\ THEN\ ELSE\left(Inventory(t)\geq Demand(t)+Deterioration\,Rate(t),\ Demand(t),Inventory(t)-Deterioration\,Rate(t)\right)$
$Receive\ Order(t)$	$=DELAY\ FIXED\left(Order(t),Lead\,Time,0\right)$
$Order(t)$	$=IF\ THEN\ ELSE\left(Inventory(t)\leq ReOrder\ Point,Order\ Quantity,0\right)$
$Deteriorate\ Units(t)$	$=\int_{t_0}^{t}\left(Deterioration\,Rate(t)+Deterioration\,Rate(t_0)\right)dt$
$Total\ Sale(t)$	$=\int_{t_0}^{t}\left(Sale(t)+Sale(t_0)\right)dt$
$Lost\ Sale\ Rate(t)$	$=IF\ THEN\ ELSE\left(Inventory(t)\geq Demand(t),0,Demand(t)-Inventory(t)\right)$
$Lost\ Sale\ Amount(t)$	$=\int_{t_0}^{t}\left(Lost\,Sale\,Rate(t)+Lost\,Sale\,Amount(t_0)\right)dt$
$Total\ Lost\ Profit\ Weight(t)$	$=\left(XIDZ\left(Lost\,Sale\,Amount(t),Total\,Sale(t),\frac{Lost\,Sale\,Amount(t)}{100}\right)*100\right)^2$
$Discount\ Percent(t)$	$=Lookup\,Function\left(XIDZ\left(Lost\,Sale\,Rate(t),Sale(t),0.2\right)\right)$
$Sale\ Price(t)$	$=Base\,Sale\,Price*\left(1-Discount\ Percent(t)\right)$
$Total\ Cost\ Rate(t)$	$=\left(Order(t)*Buy\,Price(t)\right)+\left(Deterioration\,Rate(t)*Deterioration\,Cost\,of\ each\,unit\right)+\left(Inventory(t)*Holding\,Cost\,of\ each\,unit(t)\right)+\left(Order(t)*Order\,Cost\,of\ each\,unit\right)$
$Total\ Cost(t)$	$=\int_{t_0}^{t}\left(Total\,Cost\,Rate(t)+Total\,Cost\,Rate(t_0)\right)dt$
$Lost\ Profit\ Rate(t)$	$=Lost\,Sale\,Rate(t)*\left(Sale\,Price(t)-Buy\,Price\right)$
$Total\ Lost\ Profit(t)$	$=\int_{t_0}^{t}\left(Lost\,Profit\,Rate(t)+Lost\,Profit\,Rate(t_0)\right)dt$
$Gross\ Cost\ Average(t)$	$=\frac{Total\,Cost(t)+Total\,Lost\,Profit\,Weight(t)*Total\,Lost\,Profit(t)}{Time}$
$Demand(t)$	Stochastic values of demand depend on the time and price of the sale of the product (Eq. (9))

Table 2: Variables used in the problem modelling (Continued)

Parameter	Formula
Deterioration Coefficient	Stochastic values that follow normal distribution
Lead Time	Stochastic values that follow normal distribution
ReOrder Point	Fixed value (model input)
Order Quantity	Fixed value (model input)
Buy Price	Dependent on Order Quantity
Deterioration Cost of each unit	Fixed value
Holding Cost of each unit	Dependent on inventory level
Order Cost of each unit	Dependent on Order Quantity
Base Sale Price	Fixed value

Table 3: Fixed values (E.g. 3)

Parameter	Value
Deterioration Coefficient	$= RANDOM\ NORMAL(0.01, 0.09, 0.05, 0.03, 0.01)$
Buy Price	$= \begin{cases} 4400 & Order\ Quantity \le 40 \\ 4380 & 40 < Order\ Quantity \le 50 \\ 4350 & 50 < Order\ Quantity \end{cases}$
Holding Cost of each unit	$= \begin{cases} 30 & Inventory \le 80 \\ 60 & 80 < Inventory \end{cases}$
Order Cost of each unit	$= \begin{cases} 100 & \text{Order Quantity} \le 70 \\ 125 & 70 < \text{Order Quantity} \le 90 \\ 150 & 90 < \text{Order Quantity} \le 100 \\ 200 & 100 < \text{Order Quantity} \end{cases}$
Lead Time	$= RANDOM\ NORMAL(1, 4, 1, 0.5, 1)$
Deterioration Cost of each unit	$= 100$
Base Sale Price	$= 5225$
Inventory (t=1)	$= 150$

Table 4: Summary of optimal inputs, calculated by different methods

Optimal Input Sources	Input		Output
	ReOrder Point	Order Quantity	Gross Cost Average
System dynamics (Vensim DSS optimization toolbox)	97	51	204188
Meta-model (Minitab response optimizer toolbox)	90	50	206470
Robust optimization (MATLAB)	87	43	216280

Abdollah Sharifi is currently an industrial engineering doctoral student at K. N. Toosi University of Technology, Tehran, Iran. He received his BSc from Shahid Bahonar University in Kerman, Iran, MSc from University of Sistan and Baluchestan, Zahedan, Iran. His main area of interest is simulation and inventory management. Also, He has translated and published two books in Farsi and has been a technical editor of more than five books.
Abdollah Aghaie is a professor of Industrial Engineering at K .N. Toosi University of Technology in Tehran, Iran. He received his BSc from Sharif University of Technology in Tehran, Iran, MSc from New South Wales University in Sydney, Australia and PhD from Loughborough University in the U.K. His main research interests are in Modeling and Simulation, Queuing System, Quality Management and Control, Supply Chain and Data Science.