A Novel Technique of Noise Cancellation based on Stationary Bionic Wavelet Transform and WATV: Application for ECG Denoising

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Abstract: *In this paper, is proposed a novel technique of Electrocardiogram (ECG) denoising. It is based on the application of Wavelet/Total-Variation (WATV) denoising approach in the domain of the Stationary Bionic Wavelet Transform (SBWT). It consists firstly in applying the SBWT to the noisy ECG signal for obtaining two noisy coefficients named wtb1 and wtb2 which are respectively details and approximation coefficients. For estimating the level of noise altering the signal, named σ, we use wtb1. This noise is an additive Gaussian white noise. The thresholding of wtb1 is secondly performed employing the soft thresholding and a denoised coefficient wtd1 is obtained. This thresholding requires the use of a certain threshold, thr which is computed using σ. Thedenoising of wtb2 is performed using WATV denoising method and we obtain a denoised coefficient, wtd2. This WATV denoising method also uses σ. The denoised ECG signal is finally obtained by applying the inverse of SBWT ($SBWT^{-1}$) to wtd1 and wtd2. The proposed technique performance is justified by the results obtained from the computations of Signal to Noise Ratio (SNR), Minimum Square Error (MSE), Mean Absolute Error (MAE), Peak-SNR (PSNR) and Cross-Correlation (CC).*

Keywords: *Convex optimization, denoising, ECG, WATV, stationary bionic wavelet transform.*

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1. Introduction

The signal of Electrocardiogram (ECG) is a combination of P wave-QRS complex-T wave. The employment of ECG is performed for the detection of heart related diseases where the P wave, QRS complex and T wave, are diverse functions [5]. The ECG is the cardiac activity recording and is largely used for heart diseases diagnosis. Furthermore, it is an indispensable tool that permits monitoring the patient at home, thus advancing tele-medical applications. Inside the clinical environment under ECG signals observation, those signals have diverse sorts of artifacts or noises and this is caused by the poor channel conditions [5]. In ECG denoising, the purpose is to perform separation between the valid ECG and the undesired artifacts in order to have a signal permitting easy visual analysis. The diverse sorts of noise degrading the ECG signal are as follow [16]: Power line interference, Electrode contact noise, Motion artifact, Muscle contractions, Baseline wander. Many approaches have been introduced in the literature in order to address ECG denoising. They include the Wiener filtering and Kalman filtering based approaches which are the pioneering works that aim to eliminate the additive noises [11, 13]. Though, due to the fact that the ECG signal is non-stationary, frequency domain filters can introduce degradations in a transient interval of the signal and the loose of important clinical information [11, 13]. In [9], was proposed a denoising technique based on stationary wavelet transform. According to the results obtained from the computations of Signal-to-Noise Ratio (SNR), percentage root-mean-square difference, and Root Mean Square Error, this technique [9] out performs many denoising approaches like low-pass filtering, high-pass filtering, Empirical Mode Decomposition (EMD), Fourier decomposition technique, Discrete Wavelet Transform (DWT). This stationary wavelet transform based ECG denoising approach [9] permits to conserve more ECG signal components than the other denoising techniques. Karimipour and Homaeinezhad [8] employed wavelets for denoising and detecting QRS complex [1] using curves interpretation. Wang *et al*. [24] proposed an approach of modified wavelet design. It was applied for ECG denoising [24]. The optimized filter coefficients are obtained by the approximation of the amplitude frequency response of the ideal filter, and the wavelet is obtained with the optimized filter coefficients [24]. Ling *et al*. [10] have proposed a fuzzy rule based multi wavelet ECG denoising. Sharma *et al*. [15] introduced an ECG denoising technique employing higher order statistics in wavelet sub-bands. Üstündağ *et al*. [23] have developed a weak ECG signal denoising technique based on fuzzy thresholding and wavelet packet analysis. At the first step, the weak ECG signal is decomposed into different levels by wavelet packet transform. After that, the threshold value is gotten employing the fuzzy s-function. The denoised ECG signal is finally reconstructed from the retained coefficients by using the inverse of the wavelet packet transform. Rakshit and Das [12] developed an effective ECG denoising method using EMD and Adaptive Switching Mean Filter (ASMF). The advantages of EMD and ASMF are employed for reducing the noises corrupting ECG signals with minimum degradation. Unlike traditional EMD based techniques, which eliminate the initial Intrinsic Mode Functions (IMFs) or utilize a window based technique for reducing high frequency noises, in [12], a wavelet based soft thresholding scheme is adopted for reducing high-frequency noises and preserving QRS complexes. Chunqiang *et al*. [2] have employed Local Means (LM) for ECG denoising. They presented a simple technique for obtaining standard deviation of an additive gaussian white noise degrading the ECG signal. After that, is proposed a fast ECG signal denoising method, Local Means approach, which is a “local” version of the NLM method. The LM technique has about second order of magnitude lower computational cost than the NLM approach due to the “local search’’. In a low SNR level condition, the SNR amelioration increases via the LM technique, by 21% in comparison with to the NLM approach. Many techniques have devoted to fractional calculus in signal processing [6]. According to [6], few researchers have explored ECG signal processing using fractional wavelets. A technique based on this type of wavelets, was proposed in [6], for cancelling Gaussian White Noise and power-line interference. Unlike classical wavelet, the principal advantage of employing fractional wavelet is its flexibility in term of modifying parameters in order to have diverse bandwidths. In this context, the technique proposed in [6] relies on the application of fractional wavelets for obtaining a better-quality signal employing threshold approaches. Fractional wavelets were compared by means of hard and soft threshold methods with other traditional wavelets, to prove their efficiency. In this paper is developed a novel ECG denoising technique. It is based on applying the Wavelet/Total-Variation (WATV) denoising method [4] in the domain of the Stationary Bionic Wavelet Transform (SBWT) [20]. In the rest of this paper, the section 2 is devoted with the SBWT. In section 3, is presented the WATV based denoising method. In section 4, is detailed our proposed ECG denoising approach. In section 5, is presented the results and discussion and we will conclude in section 6.

2. The Stationary Bionic Wavelet Transform (SBWT)

The SBWT was proposed in our previous research work developed in [20], for solving the perfect reconstruction problem existing with the Bionic Wavelet Transform (BWT) [7, 20, 25, 26]. In [20], the SBWT application is for enhancing speech signals and in [21], it was also applied for ECG denoising. The applications of SBWT and its inverse, $SBWT^{-1}$, are summarized by the block diagram illustrated in Figure 1.

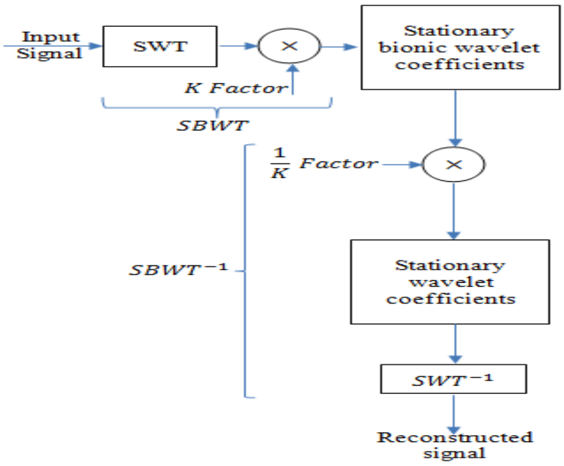


Figure 1. The applications of (SBWT) and its inverse, $SBWT^{-1}$.

According to Figure 1, the first step of SBWT application is to apply the Stationary Wavelet Transform (SWT) to the input signal for obtaining stationary wavelet coefficients. Those coefficients are then multiplied by a K factor for obtaining finally the stationary bionic wavelet coefficients. Also, according to Figure 1, the first step of the application of $SBWT^{-1}$ is to multiply the stationary bionic wavelet coefficients by the factor 1/K for obtaining stationary wavelet coefficients. To those coefficients is applied the inverse of SWT, SWT^{-1}, for obtaining finally the reconstructed signal. The K factor is a function of the adaptation factor, T and this is detailed in [7, 20, 25, 26].

3. The WATV based Denoising Method

Ding and Selesnick [4] have proposed a unified WATV technique which is estimating the wavelet coefficients (both in significant and significant) at the same time through the minimization of a single objective function. In order to induce wavelet-domain sparsity, Ding and Selesnick [4] have used non-convex penalties and this due to their strong sparsity inducing properties [4]. Generally, when employing non-convex penalties, the convexity of the objective function is commonly sacrificed. Though, Ding and Selesnick [4], have restricted the non-convex penalty in order to guarantee the strict convexity of the objective function; then, the minimizer is unique and can be reliably gotten via convex optimization. The WATV denoising technique proposed in [4], is quite resistant to pseudo-Gibbs oscillations and spurious noise spikes. Ding and Selesnick [4] have derived a computationally effective algorithm for the objective function proposed in [4].

3.1. Problem Formulation

Let $x \in \mathrm{IR}^N$, is a signal corrupted by an additive white Gaussian noise, so we have:

$$y_n = x_n + v_n, n = 0, 1, \dots, N1 \qquad (1)$$

Let W denotes the wavelet transform and therefore the wavelet coefficients obtained from x as follow:

$$w = Wx \qquad (2)$$

Let $W_{j,k}$ denote the indexes of those wavelet coefficients with k and j are respectively the time and scale indices. Ding and Selesnick [4] employed the translational-invariant (i.e., undecimated) Wavelet Transform (WT), satisfying the Parseval frame condition expressed as follow:

$$W^T W = I \qquad (3)$$

The WATV denoising algorithm proposed in [4] can be employed with any Wavelet Transform, W which satisfies (3). The Total Variation (TV) of a signal $x \in \mathrm{IR}^N$ is as follow:

$$\mathrm{TV}(x) := \|Dx\|_1 \qquad (4)$$

With D is the first-order difference matrix and $\|\cdot\|_1$ is the $\ell_1 -$ norm of x, ie., $\|x\|_1 = \sum_n |x_n|$. Also, we have $\|x\|_2^2 = \sum_n |x_n|^2$ and for a set of doubly-indexed wavelet coefficients, we have $\|w\|_2^2 := \sum_{j,k} |w_{j,k}|^2$. The matrix D is expressed as follow:

$$D = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix} \qquad (5)$$

The WATV denoising technique proposed in [4], permits to find the coefficients $w$and this by solving the following optimization problem:

$$\hat{w} = \underset{w}{\operatorname{argmin}} \left\{ F(w) = \frac{1}{2}\|Wy - w\|_2^2 + \sum_{j,k} \lambda_j \phi(w_{j,k}; a_j) + \beta \|DW^T w\|_1 \right\} \qquad (6)$$

The estimate of x which is $\hat{x}$, is consequently obtained from the application of the inverse of the wavelet transform to $\hat{w}$ so we have:

$$\hat{x} = W^T \hat{w} \qquad (7)$$

The penalty term, $\|DW^T w\|_1$ is the TV of $\hat{x}$ [4]. The regularization parameters are $\beta > 0$ and $\lambda_j > 0$. Ding and Selesnick [4] allowed the wavelet regularization and the penalty parameters (a_j and λ_j) varying with j.

4. The proposed ECG Denoising Technique

In this work, is proposed a novel ECG denoising technique. It is based on the application of WATV denoising method [4] in the domain of the SBWT [20]. This proposed ECG denoising technique can be summarized by the block diagram illustrated in Figure 2.

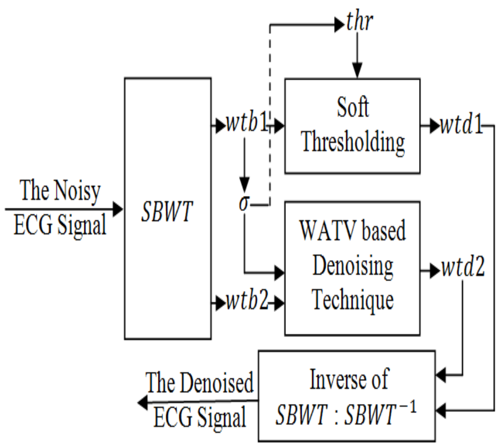


Figure 2. The flowchart of the proposed ECG denoising approach.

As illustrated in Figure 2, the SBWT [20] is firstly applied to the noisy ECG signal in order to have two noisy stationary bionic wavelet coefficients, *wtb1* and *wtb2*. Then, is estimated the noise level, σ from the detail coefficient *wtb1* and the threshold *thr* is computed as follow:

$$\mathrm{thr} = \sigma \cdot \sqrt{2 \times \log(N)} \qquad (8)$$

Where N is the samples number in *wtb1* and σ is expressed as follow:

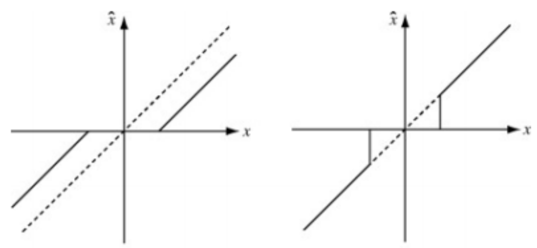
$$\sigma = MAD(|wtb1|)/0.6745 \qquad (9)$$

After that, *wtb1* is thresholded and we obtain a denoised coefficient, *wtd*1. This thresholding is the soft thresholding function, S_{thr} which requires the use of a certain threshold, *thr*. The latter is computed using the formula in (Equation (8)). The Soft thresholding function is expressed as follow:

$$\hat{x} = S_{thr}(x) = \begin{cases} sign(x)(|x| - thr) & if \ \ |x| > thr \\ 0 & if \ \ |x| \le thr \end{cases} \qquad (10)$$

Apart from Soft thresholding function, there are other functions such as hard thresholding which is expressed as follow:

$$\hat{x} = H_{thr}(x) = \begin{cases} x & \text{if } |x| > thr \\ 0 & \text{if } |x| \le thr \end{cases} \qquad (11)$$



a) Soft thresholding. b) Hard thresholding.

Figure 3. Illustration of two Types of thresholding function.

Hard thresholding (Figure 3-b) permits to maintain the scale of the signal and is introducing ringing and artifacts after reconstruction and this due to a discontinuity in the wavelet coefficients. Although, Soft thresholding (Figure 3-a) permits to eliminate this discontinuity and this results in a smoother signal. However, Soft thresholding is slightly decreasing the magnitude of the reconstructed signal [18].

Also, according to Figure 2, the denoising technique based on WATV [4] is applied to *wtb2* for obtaining a denoised coefficient, *wtd2*. This denoising technique [4] requires the use of the noise level, σ (Equation (9) and Figure 2). The inverse of SBWT, SBWT^{-1} is applied to *wtd1* and *wtd2* for obtaining finally the denoised ECG signal (Figure 2).

5. Results and Discussion

In this part, we make a comparison between the proposed ECG denoising approach and five other denoising ones which are as follows: our ECG denoising method introduced in [19] and based on the BWT [7, 20, 25, 26] and Translation Invariant Forward Wavelet Transform (FWT_T1). The second one is the 1-D double-density complex DWT denoising approach [14]. The third one is the ECG denoising approach based on non local means [5, 22]. The fourth one is the denoising approach based on wavelets and hidden Markov models [3]. The fifth one is the WATV based denoising technique [4]. All these approaches including our technique proposed in this work, are applied to 28 noisy ECG signals. Those signals are obtained in the following manner: each clean ECG signal chosen from a set of seven clean ECG signals belonging to MIT-BIH database, is corrupted by an additive white Gaussian noise with four different values of SNRi (SNR before denoising). These values are -5dB, 0dB, 5dB and 10dB. Consequently, for each clean ECG signal, we obtain five different noisy ECG signals. The seven clean ECG signals are 100.dat, 101.dat, 102.dat, 103.dat, 104.dat, 105.dat and 106.dat.

For this comparative study, we have used as evaluation criterions, the Signal to Noise Ratio (SNRf), the Peak-SNR (PSNR), the Mean Square Error (MSE), the Mean Absolute Error (MAE) and the Cross-Correlation (CC). Those criterions are expressed as follow [6, 17, 27]:

$$MSE = E\left(\left(x(n) - \hat{x}(n)\right)^2\right) = \frac{1}{N}\sum_{n=1}^{N-1}\left(x(n) - \hat{x}(n)\right)^2 \quad (12)$$

$$SNR_{dB} = 10 \cdot log_{10}\left[\frac{\sum_{n=1}^{N} x^2(n)}{\sum_{n=1}^{N}\left(x(n)-\hat{x}(n)\right)^2}\right] \quad (13)$$

$$PSNR = 20 \cdot log_{10}\left[\frac{\max(x(n))}{\sqrt{MSE}}\right] \quad (14)$$

$$MAE = \frac{1}{N}\sum_{n=1}^{N-1}|x(n) - \hat{x}(n)| \quad (15)$$

$$CC = \frac{\sum_{n}^{N}[x(n)-\bar{x}]\cdot[\hat{x}(n)-\bar{\hat{x}}]}{\sqrt{\sum_{n=1}^{N}[x(n)-\bar{x}]^2}\sqrt{\sum_{n=1}^{N}[\hat{x}(n)-\bar{\hat{x}}]^2}} \quad (16)$$

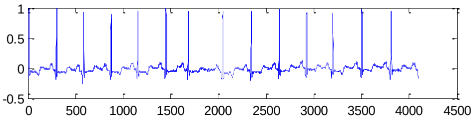
Where $x(n)$ and $\hat{x}(n)$ are respectively the clean and the denoised signals and N is the samples number in $x(n)$. The quantities $\bar{x}$ and $\bar{\hat{x}}$ are respectively the mean value of $x(n)$ and the mean value of $\hat{x}(n)$.

Table 1. Comparative study in terms of CC, MAE, MSE, SNR and PSNR: results obtained from the computations of the mean of seven values of MAE, of the mean of seven values of PSNR, of the mean of seven values of CC, the mean of seven values of SNR and the mean of seven values of MSE. Each mean is computed for seven clean ECG signals 100 to 106.dat corrupted by Gaussian white noise with different values of *SNRi* before denoising (varying from $-5dB$ to $10dB$ with step of $5dB$).

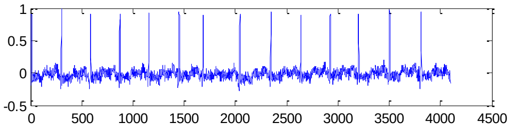
Denoising Technique	*SNRi*= –5*dB*	*SNRi*= 0*dB*	*SNRi*= 5*dB*	*SNRi*= 10*dB*
The proposed ECG Denoising Technique	MAE:0.0458 MSE:0.0043 SNRf:7.5563dB PSNR:23.9852dB CC:0.9174	MAE:0.0283 MSE:0.0016 SNRf:11.6926dB PSNR:28.1215dB CC:0.9670	MAE:0.0186 MSE:6.8571e-04 SNRf:15.4771dB PSNR:31.9060dB CC:0.9864	MAE:0.0139 MSE:3.1429e-04 SNRf:18.7926dB PSNR:35.2214dB CC:0.9934
WATV Denoising Technique [4]	MAE:0.0473 MSE:0.0044 SNRf:7.2861dB PSNR:23.9303dB CC:0.9136	MAE:0.0295 MSE:0.0018 SNRf:11.2690dB PSNR:27.6979dB CC:0.9648	MAE:0.0193 MSE:7.5714e-04 SNRf:14.9927dB PSNR:31.4215dB CC:0.9852	MAE:**0.0135** MSE:3.7143e-04 SNRf:18.0846dB PSNR:34.5135dB CC:0.9931
The ECG Denoising Technique proposed in [19]	MAE:0.0755 MSE:0.0121 SNRf:3.2999dB PSNR:19.7287dB CC:0.7830	MAE:0.0391 MSE:0.0031 SNRf:8.9095dB PSNR:25.3384dB CC:0.9377	MAE:0.0233 MSE:0.0011 SNRf:13.4812dB PSNR:29.9101dB CC:0.9788	MAE:0.0147 MSE:4.1429e-04 SNRf:17.6433dB PSNR:34.0722dB CC:0.9920
The 1-D double- density complex DWT denoising method [14]	MAE :0.0711 MSE :0.0103 SNRf :3.6629dB PSNR: 20.0918dB CC: 0.8005	MAE:0.0414 MSE:0.0034 SNRf : 8.4147dB PSNR: 24.8436dB CC: 0.9292	MAE:0.0247 MSE:0.0012 SNRf:12.9861dB PSNR: 29.4150dB CC: 0.9754	MAE:0.0140 MSE:5.4000e-04 SNRf:17.1191dB PSNR: 33.5480dB CC: 0.9908
The ECG Denoising Technique based on Non Local Means [2, 5]	MAE :0.0678 MSE :0.0096 SNRf :3.9834dB PSNR :20.8533dB CC :0.8165	MAE :0.0431 MSE :0.0035 SNRf :8.5354dB PSNR :24.9957dB CC:0.9323	MAE :0.0282 MSE :0.0014 SNRf :12.3277dB PSNR :28.7880dB CC:0.9730	MAE :0.0170 MSE :5.0000e-04 SNRf :16.7405dB PSNR :33.2008 dB CC : 0.9905
The technique based on wavelets and hidden Markov models [3]	MAE: 0.0668 MSE:0.0087 SNRf:4.3536dB PSNR: 20.4772 dB CC: 0.8294	MAE: 0.0477 MSE:0.0035 SNRf:8.9189dB PSNR: 23.6498 dB CC: 0.9168	MAE: 0.0258 MSE:0.0012 SNRf:13.1256dB PSNR: 27.9770 dB CC: 0.9685	MAE: 0.0148 MSE:4.0000e-04 SNRf:17.6421dB PSNR: 33.8433 dB CC: 0.9914

In Table 1, are listed the different results obtained from the applications of the proposed technique and the other previously mentioned ones, used for our comparative study. Those results are in terms of SNR after denoising SNRF, the PSNR, the MAE, the MSE and the CC. According to the Table 1, the results in blue color, are the best ones. Those best results are obtained from the application of the ECG denoising technique proposed in this work. In fact, this technique permits to have the highest values of SNRF, PSNR and CC and the lowest values of MAE and MSE. Consequently, this proposed approach outperforms all the other previously mentioned techniques, used for our comparative study.

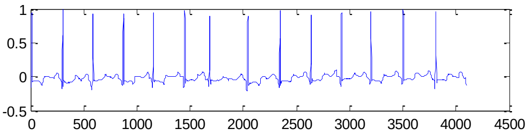
Figures 4 to 7 show some examples of ECG denoising using thedenoising technique proposed in this work.



a) The clean ECG signal (100. dat).

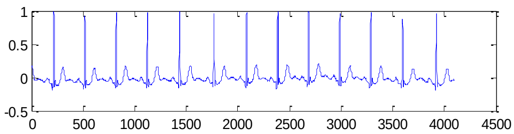


b) The corresponding noisy ECG signal with $SNRi = 10\ dB$

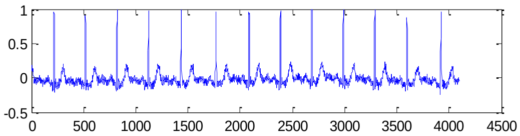


c) The denoised ECG signal ($SNRf = 18.714938dB$, $PSNR = 36.418703\ dB$, $CC = 0.993442$, $MAE = 0.011001$, $MSE = 0.000228$

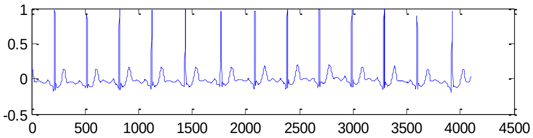
Figure 4. First example of ECG denoising by the proposed approach.



a) The clean ECG signal (103. dat).

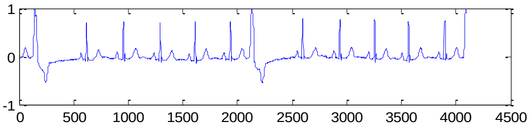


b) The corresponding noisy ECG signal with SNRi = 15 dB.

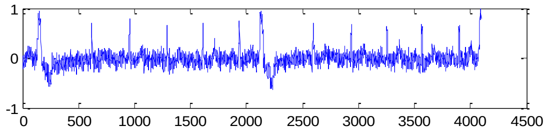


c) The denoised ECG signal (SNRf = 22.581725dB, PSNR = 39.240232 dB , CC = 0.997524, MAE = 0.007968, MSE = 0.000228).

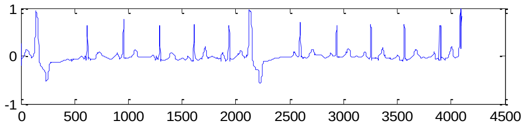
Figure 5. Second example of ECG denoising by the proposed approach.



a) The clean ECG signal (119. dat).

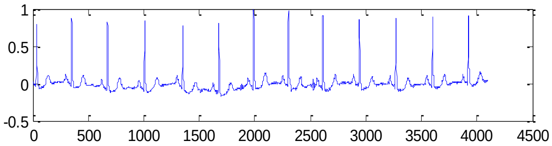


b) The corresponding noisy ECG signal with $SNRi = 5\ dB$

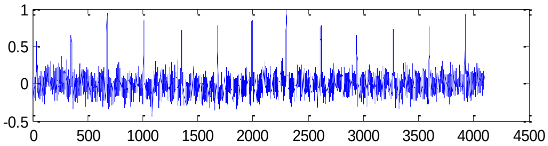


c) The denoised ECG signal (SNRf = 14.758179dB, PSNR = 29.715380dB, CC = 0.983566, MAE = 0.019725, MSE = 0.001068).

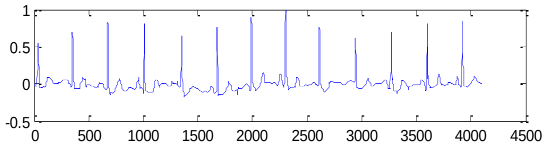
Figure 6. Third example of ECG denoising by the proposed denoising approach.



a) The clean ECG signal (101. dat).



b) The corresponding noisy ECG signal with SNRi = 0 dB.



c) The denoised ECG signal (SNRf = 12.276751dB, PSNR = 30.590276dB , CC = 0.970692, MAE = 0.021001, MSE = 0.000873).

Figure 7. Fourth example of ECG Denoising by the proposed denoising approach.

Figures 4 to 7 show the performance of the ECG denoising technique proposed in this work. In fact, the noise is significantly reduced and the diverse P waves-QRS complexes-T waves of the original signal are practically preserved.

6. Conclusions

In this paper, is proposed a novel ECG denoising approach based on the application of the WATV denoising method in the domain of the SBWT. For its performance evaluation, it is compared to five other denoising methods. These methods are the 1-D double-density complex DWT denoising one, the denoising technique based on Wavelets and Hidden Markov Models, the approach based on Non Local Means, the proposed technique based on theBWT and the FWT_TI with hard thresholding and also the WATV based denoising approach. The results obtained from the computations of SNR, MSE, MAE, PSNR and CC, show that the proposed ECG denoising technique outperforms the other ones used for this evaluation. In fact, our proposed approach permits to have the highest values of SNR, PSNR and CC and the lowest values of MAE and MSE. Moreover, it considerably reduces the noise and the diverse waves P-QRS-T of the original signal are practically preserved.

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