A.P. Calculus –Summer Packet

Going into AP calculus, there are certain skills that have been taught to you over the previous years that we assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

We assume that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example should now be a part of you. If not, you would not be going onto AP calculus. So, only the topics I see that students consistently do not have down in their basic skill set are included here. These are skills that are used continually in A.P. Calculus.

On the following 15 pages, you have 9 to 12 problems per page. Each problem should be done in the space provided. Rather than give you a textbook to remind you of the techniques necessary to solve the problem, I have given you several websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, examine these websites. Don't fake your way through these problems. As stated, students are notoriously weak in them, even students who have achieved well prior to AP Calculus. Use the websites.

Realize also that certain concepts are interrelated. Domain, for example, may require you to be expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations.

This packet is due the first day back in school in the fall. It will be graded. You need to get off to a good start so spend some quality time on this packet this summer. Tear off these first two sheets and return the 15 sheets stapled together. Be sure your name appears on the first sheet. Work needs to be shown when needed. Also do not rely on the calculator. Half of your AP exam next year is taken without the calculator. So paper and pencil techniques only.

It is a mistake to decide to do this now. Let it go until mid-summer. I want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time. If you do two concepts a day, the whole packet will take you about a week to complete.

If you have questions about any of these problems or techniques used in solving them, contact me at the school website/email address. Have a good summer and see you in the fall.

The topics listed are below. You can certainly do Google searches for any of these topics. But I have given you several sites that will cover pretty much all of these topics.

Here is a good site for most algebra topics:

http://www.purplemath.com/modules/index.htm

Beginning algebra topics

Exponents

1. Negative and fractional exponents

Intermediate algebra topics

2. Domain
3. Solving inequalities: absolute value
4. Solving inequalities: quadratic
5. Special Factoring formulas
6. Function transformation
7. Factor theorem (*p* over *q* method)
8. Even and odd functions
9. Solving quadratic equations and quadratic formula

Advanced algebra topics

10. Asymptotes
11. Complex fractions
12. Composition of functions
13. Solving Rational (fractional) equations

Trig Information

http://www.mathematicshelpcentral.com/index.html

Once in the site, go to lecture notes.

14. Basic right angle trig
15. Trig equations

Name_____

Topic 1: Fractional & Negative Exponents

Simplify using only positive exponents

1. $-3x^{-3}$

2. $-5\left(\frac{3}{2}\right)(4-9x)^{\frac{-1}{2}}(-9)$

3. $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]$

4. $\left(16x^2y\right)^{\frac{3}{4}}$

5. $-\frac{x^{\frac{-1}{2}}}{2}\sin\sqrt{x}$

6. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

7. $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

8. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

9. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{\frac{-1}{2}}$

Topic 2: Domain

Find the domain of the following functions:

1. $y = \dfrac{3x-2}{4x+1}$

2. $y = \dfrac{x^2-4}{2x+4}$

3. $y = \dfrac{x^2-5x-6}{x^2-3x-18}$

4. $y = \dfrac{2^{2-x}}{x}$

5. $y = \sqrt{x-3} - \sqrt{x+3}$

6. $y = \dfrac{\sqrt{2x-9}}{2x+9}$

7. $y = \dfrac{x^2+8x+12}{\sqrt[4]{x+5}}$

8. $y = \sqrt{x^2-5x-14}$

9. $y = \dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

10. $y = \log(2x-12)$

11. $y = \sqrt{\tan x}$

12. $y = \dfrac{x}{\cos x}$

Topic 3: Solving inequalities (absolute value)

Write the following absolute value expressions as piecewise expressions

1. $y = |2x - 4|$

2. $y = |6 + 2x| + 1$

3. $y = |4x + 1| + 2x - 3$

Solve the following absolute value inequalities

4. $|x - 3| > 12$

5. $|x - 3| \leq 4$

6. $|10x + 8| > 2$

7. $|3x - 4| > -2$

8. $|x - 6| > -8$

9. $|x + 1| \leq |x - 3|$

Topic 4: Solving inequalities (quadratic)

Write the following absolute value expressions as piecewise expressions

1. $|x^2 - 1|$

2. $|x^2 + x - 12|$

3. $|x^2 + 4x + 4|$

Solve the following by factoring and making appropriate sign charts.

4. $x^2 - 16 > 0$

5. $x^2 + 6x - 16 > 0$

6. $x^2 - 3x \geq 10$

7. $2x^2 + 4x \leq 3$

8. $x^3 + 4x^2 - x \geq 4$

9. $2\sin^2 x \geq \sin x \qquad 0 \leq x < 2\pi$

Topic 5: Special factorization

Factor completely

1. $x^3 + 8$

2. $x^3 - 8$

3. $27x^3 - 125y^3$

4. $x^4 + 11x^2 - 80$

5. $ac + cd - ab - bd$

6. $2x^2 + 50y^2 - 20xy$

7. $x^2 + 12x + 36 - 9y^2$

8. $x^3 - xy^2 + x^2y - y^3$

9. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

Topic 6: Function transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1. $f(x) - 4$

2. $f(x-4)$

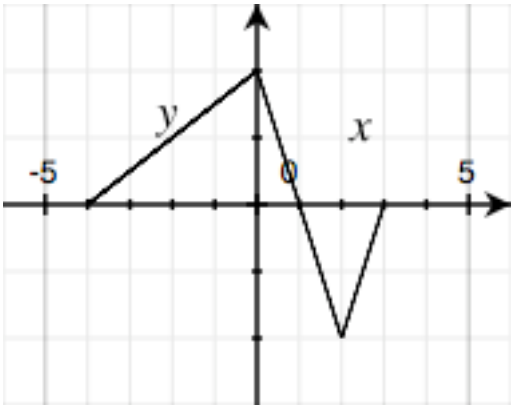
3. $-f(x+2)$

4. $5f(x) + 3$

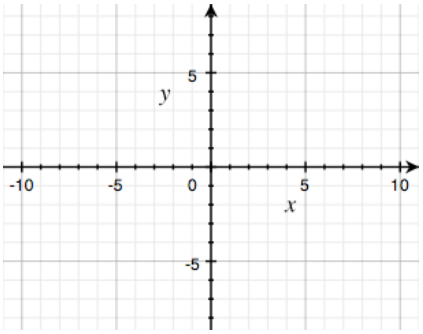
5. $f(2x)$

6. $|f(x)|$

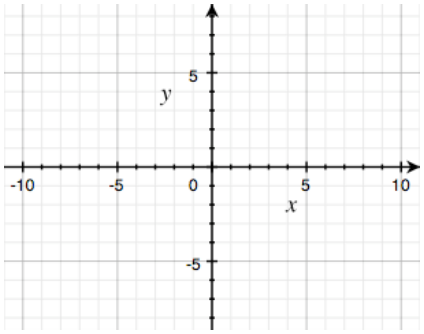
Here is a graph of $y = f(x)$. Sketch the following graphs



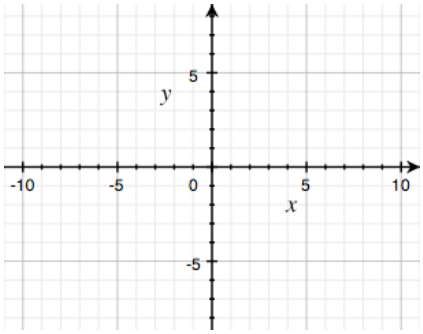
7. $y = 2f(x)$



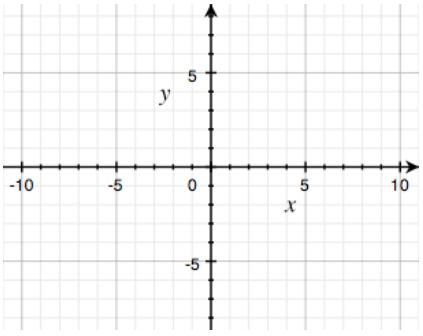
8. $y = -f(x)$



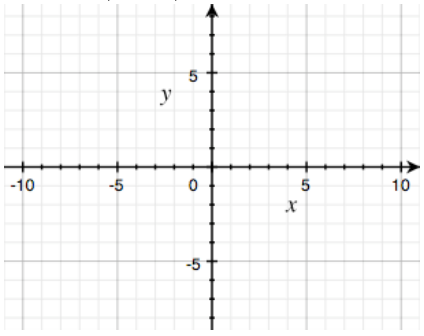
9. $y = f(x-1)$



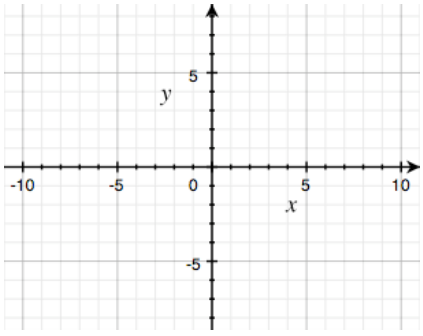
10. $y = f(x+2)$



11. $y = |f(x)|$



12. $y = f|x|$



Topic 7: Factor theorem (*p* over *q* method/synthetic division)

Use the *p* over *q* method and synthetic division to factor the polynomial $P(x)$. Then solve $P(x) = 0$.

1. $P(x) = x^3 + 4x^2 + x - 6$

2. $P(x) = x^3 + 5x^2 - 2x - 24$

3. $P(x) = x^3 - 6x^2 + 3x - 10$

4. $P(x) = x^3 + 2x^2 - 19x - 20$

5. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

6. $P(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$

Topic 8: Even and odd functions

Show work to determine if the relation is even, odd, or neither

1. $f(x) = 2x^2 - 7$

2. $f(x) = -4x^3 - 2x$

3. $f(x) = 4x^2 - 4x + 4$

4. $f(x) = x - \frac{1}{x}$

5. $f|x| = |x| - x^2 + 1$

6. $5x^2 - 6y = 1$

7. $y = e^x - \frac{1}{e^x}$

8. $3y^3 = 4x^3 + 1$

9. $3x = |y|$

Topic 9: Solving quadratic equations and quadratic formula

Solve each equation

1. $7x^2 - 3x = 0$

2. $4x(x-2) - 5x(x-1) = 2$

3. $x^2 + 6x + 4 = 0$

4. $2x^2 - 3x + 3 = 0$

5. $2x^2 - (x+2)(x-3) = 12$

6. $x + \frac{1}{x} = \frac{13}{6}$

7. $x^4 - 9x^2 + 8 = 0$

8. $x - 10\sqrt{x} + 9 = 0$

9. $\frac{1}{x^2} - \frac{1}{x} = 6$

Topic 10: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptotes (if they exist)

1. $y = \dfrac{x}{x-3}$

2. $y = \dfrac{x+4}{x^2-1}$

3. $y = \dfrac{x+4}{x^2+1}$

4. $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

5. $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

6. $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

7. $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

8. $y = \dfrac{2x^3}{x^3-1}$

9. $y = \dfrac{\sqrt{x}}{2x^2-10}$

Topic 11: Complex fractions

Simplify the following

1. $\dfrac{x}{x-\frac{1}{2}}$

2. $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

3. $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

4. $\dfrac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

5. $\dfrac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

6. $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

7. $\dfrac{x^{-3}-x}{x^{-2}-1}$

8. $\dfrac{\frac{x}{1-x}+\frac{1+x}{x}}{\frac{1-x}{x}+\frac{x}{1+x}}$

9. $\dfrac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}+3}$

Topic 12: Composition of functions

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find the following

1. $f(g(2))$

2.. $f(g(2))$

3. $f(h(-1))$

4. $h(f(-1))$

5. $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

6. $f(g(x))$

7. $g(f(x))$

8. $g(g(x))$

9. $f(h(x))$

Topic 13: Solving Rational (fractional) equations

Solve each equation for x

1. $\frac{2}{3}-\frac{5}{6}=\frac{1}{x}$

2. $x+\frac{6}{x}=5$

3. $\frac{x+1}{3}-\frac{x-1}{2}=1$

4. $\frac{x-5}{x+1}=\frac{3}{5}$

5. $\frac{60}{x}-\frac{60}{x-5}=\frac{2}{x}$

6. $\frac{2}{x+5}+\frac{1}{x-5}=\frac{16}{x^2-25}$

7. $\frac{x}{x-2}+\frac{2x}{4-x^2}=\frac{5}{x+2}$

8. $\frac{x}{2x-6}-\frac{3}{x^2-6x+9}=\frac{x-2}{3x-9}$

9. $\frac{2x+3}{x-1}=\frac{10}{x^2-1}+\frac{2x-3}{x+1}$

Topic 14: Solving Rational (fractional) equations

Solve the following problems.

If point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture.

1. $P(-2,4)$

2. $P(\sqrt{5},-2)$

3. If $\cos\theta = \frac{5}{13}, \theta$ in quadrant II, find $\sin\theta$ and $\tan\theta$

4. If $\cot\theta = 3, \theta$ in quadrant III, find $\sin\theta$ and $\cos\theta$

Find the exact value of the following without calculators:

5. $\sin^2 225° - \cos^2 300°$

6. $\left(6\sec 180° - 4\cot 90°\right)^2$

7. $\left(4\cos 30° - 6\sin 120°\right)^{-2}$

Solve the following triangles (3 decimal place accuracy)

8. $A =$, $a = 21.7$, $B = 16°$, $b =$, $C = 90°$, $c =$

9. $A =$, $a = 6$ feet, $B =$, $b =$, $C = 90°$, $c = 95$ inches

Topic 15: Solving Trigonometric equations

Solve each equation on the interval $[0, 2\pi)$

1. $\sin x = \frac{1}{2}$

2. $\cos^2 x = \cos x$

3. $2\cos x + \sqrt{3} = 0$

4. $4\sin^2 x = 1$

5. $2\sin^2 x + \sin x = 1$

6. $\cos^2 x + 2\cos x = 3$

7. $2\sin x \cos x + \sin x = 0$

8. $8\cos^2 x - 2\cos x = 1$

9. $\sin^2 x - \cos^2 x = 0$