A Principles of Synthetic Aperture Radar

A.1 Introduction

Synthetic aperture radar (SAR) satellites collect swaths of side-looking echoes at a sufficiently high range resolution and along-track sampling rate to form high resolution imagery (see Figure A1). As discussed in this appendix, the range resolution of the raw radar data is determined by the pulse length (or 1/*bandwidth*) and the *incidence angle*. For real aperture radar, the along-track or *azimuth* resolution of the outgoing microwave pulse is diffraction limited to an angle corresponding to the wavelength of the radar (e.g. 0.05 m) divided by the length of the aperture (e.g. 10 m). When this beam pattern is projected onto the surface of the earth at a range of say 850 km, it illuminates 4250 m in the along-track dimension so the raw radar data are horribly out of focus in azimuth. Using the synthetic aperture method, the image can be focused on a point reflector on the ground by coherently summing thousands of consecutive echoes thus creating a synthetic aperture perhaps 4250 m long. Proper focus is achieved by summing the complex numbers along a constant range. The focused image contains both amplitude (backscatter) and phase (range) information for each pixel.

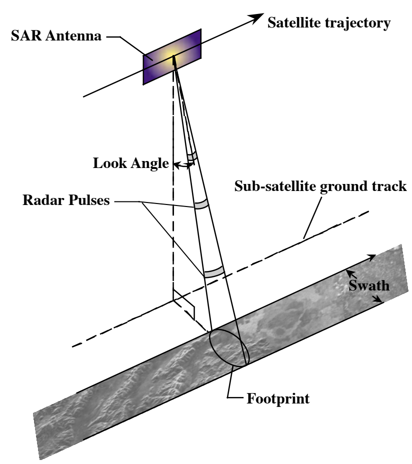


Figure A1: Schematic diagram of a SAR satellite in orbit. The SAR antenna has its long axis in the flight direction also called the *azimuth* direction and the short axis in the *range* direction. The radar sends pulses to one side of the *ground track* that illuminate the earth over a large elliptical footprint. The reflected energy returns to the radar where it is recorded as a function of *fast time* in the range direction and *slow time*, or echo number, in the azimuth direction.

A.2 Fraunhoffer diffraction

To understand why a synthetic aperture is needed for microwave remote sensing from orbital altitude one must understand the concepts of diffraction and resolution. Consider the projection pattern of coherent radiation after it passes through an aperture (see Figure A2). First we'll consider a 1-D aperture and then go on to a 2-D rectangular aperture to simulate a rectangular SAR antenna. The 2-D case provides the shape and dimension of the footprint of the radar. Although we will develop the resolution characteristics of apertures as transmitters of radiation, the resolution characteristics are exactly the same when the aperture is used to receive radiation. These notes were developed from Rees 2001 and Bracewell 1978.

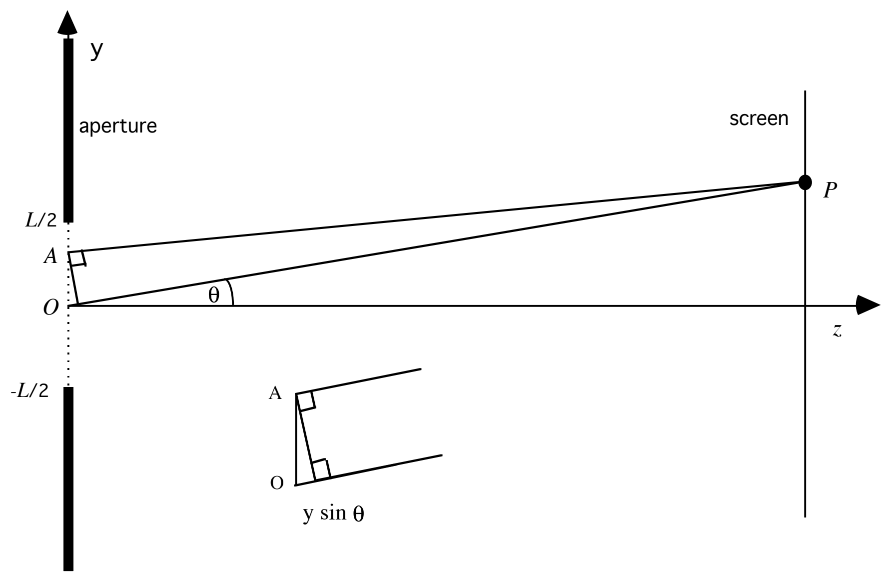


Figure A2: Diagram for the projection of coherent microwaves on a screen that is far from the aperture of length L.

We simulate coherent radiation by point sources of radiation distributed along the aperture between $-L/2$ and $L/2$. For simplicity we'll assume all the sources have the same amplitude, wavelength λ, and phase. Given these sources of radiation, we solve for the illumination pattern on the screen as a function of θ. We'll assume that the screen is far enough from the aperture so that rays AP and OP are parallel. Later we'll determine how far away the screen needs to be in order for this approximation to hold. Under these conditions, the ray AP is slightly shorter than the ray OP by an amount $-y \sin\theta$. This corresponds to a phase shift of $\frac{-2\pi}{\lambda} y \sin\theta$. The amplitude of the illumination at point P is the integral over all of the sources along the aperture

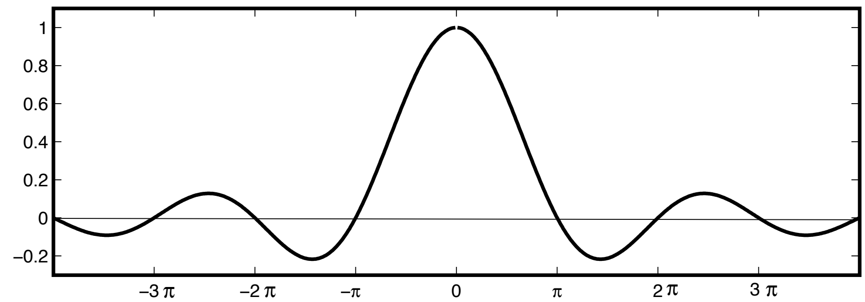


Figure A3: Sinc function illumination pattern for the aperture shown in Figure A2.

multiplied by their complex phase value

$$P(\theta) = \int_{-L/2}^{L/2} A(y) e^{-i2\pi yk\sin\theta}\,\mathrm{d}y \tag{A1}$$

where $k = 1/\lambda$. This is called the Fraunhoffer diffraction integral. The illumination across the aperture is uniform in both amplitude and phase so we set $A(y) = 1$. Now let $s = 2\pi k \sin\theta$ so the fourier integral is easy to evaluate.

$$P(s) = \int_{-L/2}^{L/2} e^{-isy}\,\mathrm{d}y = \frac{e^{-isL/2} - e^{isL/2}}{-is} = \frac{2}{s}\sin(sL/2) = L\,\mathrm{sinc}\,(sL/2) \tag{A2}$$

Replacing s with $2\pi\sin\theta/\lambda$ we arrive at the final result.

$$P(\theta) = L\,\mathrm{sinc}\left(\frac{L\pi\sin\theta}{\lambda}\right) \tag{A3}$$

The illumination pattern on the screen is shown in Figure A3.

The first zero crossing, or angular resolution θ_r of the sinc function occurs when the argument is π so $\sin\theta_r = \frac{\lambda}{L}$ and for small angles $\theta_r \cong \frac{\lambda}{L}$ and $\tan\theta_r \cong \sin\theta_r$. Note that one could modify the screen illumination by changing the strength of the illumination across the aperture. For example, a Gaussian aperture would produce a Gaussian illumination function on the screen. This would eliminate the sidelobes associated with the sinc function but it would also broaden the projection pattern. In addition, one could vary the phase along the aperture to shift the point of maximum illumination away from $\theta = 0$. Such a phased array aperture is used in some radar systems to continuously illuminate a feature as the satellite passed over it. This is called spotlight mode SAR and it is a favorite technique for military reconnaissance.

One can perform the same type of analysis with any 2-D aperture; many analytic examples are given in Figure 12.4 from Bracewell 1978. Below we'll be using a rectangular aperture when we discuss synthetic aperture radars. For now, consider a uniform circular aperture has an angular resolution given by $\sin\theta_r = 1.22\frac{\lambda}{L}$.

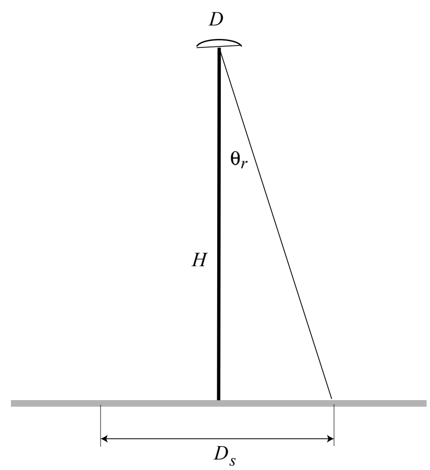


Figure A4: Beam-limited footprint D_s of a circular aperture (radar altimeter or optical telescope) at an altitude of H.

The Geosat radar altimeter orbits the Earth at an altitude of 800 km and illuminates the ocean surface with a 1-m parabolic dish antenna operating at Ku band (13.5 GHz, $\theta = 0.022$m) (Figure A4). The diameter of the beam-limited footprint on the ocean surface is $D_s = 2H \tan\theta_r \cong 2.44H\frac{\lambda}{D}$ or in this case 43 km.

An optical system with the same 1 m diameter aperture, but operating at a wavelength of $\theta = 5 \times 10^{-7}$ m, has a footprint diameter of 0.97 m. The Hubble space telescope reports an angular resolution of 0.1 arcsecond, which corresponds to an effective aperture of 1.27 m. Thus a 1-m ground resolution is possible for optical systems while the same size aperture operating in the microwave part of the spectrum has a 44,000 times worse resolution. Achieving high angular resolution for a microwave system will require a major increase in the length of the aperture.

Before moving on to the 2-D case, we should check the assumption used in developing the Fraunhoffer diffraction integral that the rays AP and OP are parallel. Suppose we examine the case when $\theta = 0$; the ray path AP is slightly longer than OP (Figure A5). This parallel-ray assumption breaks down when the phase of ray path AP is more than $\pi/2$ radians longer than OP which corresponds to a distance of $\frac{\lambda}{4}$. Let's determine the conditions when this happens.

The condition that the path length difference is smaller than 1/4 wavelength is

$$\left[\frac{L^2}{4} + z^2\right]^{\frac{1}{2}} - z < \frac{\lambda}{4} \qquad \text{(A4)}$$

or can be rewritten as

$$\left[\left(\frac{L}{2z}\right)^2 + 1\right]^{\frac{1}{2}} - 1 < \frac{\lambda}{4z} \qquad \text{(A5)}$$

Now assume $L \ll z$ so we can expand the term in bracket in a binomial series.

$$1 + \frac{L^2}{8z^2} - 1 < \frac{\lambda}{4z} \qquad \text{(A6)}$$

and we find $z_f > \frac{L^2}{2\lambda}$ where z_f is the Fresnel distance. So when $z < z_f$ we are in the near field and we need to use a more rigorous diffraction theory. However when $z \gg z_f$ we are safe to use the parallel-ray approximation and the Fraunhoffer diffraction integral is appropriate. Next consider some examples:

$$\textit{Geosat:} \qquad L = 1 \text{ m}, \ \lambda = 0.022 \text{ m} \qquad z_f = 23 \text{ m}$$

At an orbital altitude of 800 km the parallel-ray approximation is valid.

$$\textit{Optical telescope:} \qquad L = 1 \text{ m}, \ \lambda = 5 \times 10^{-7} \text{ m} \qquad z_f = 4000 \text{ km}.$$

So we see that an optical system with an orbital altitude of 800 km will require near-field optics.

What about a synthetic aperture radar (discussed below) such as ERS with a 4000 m long synthetic aperture?

$$\textit{ERS SAR:} \qquad L = 4000 \text{ m}, \lambda = 0.058 \text{ m} \qquad z_f = 140,000 \text{ km}.$$

Thus near-field optics are also required to achieve full SAR resolution for ERS. This near-field correction is done in the SAR processor by performing a step called range migration and it is a large factor in making SAR processing so CPU-intensive.

A.3 2-D Aperture

A 2-D rectangular aperture is a good approximation to a typical spaceborne strip-mode SAR. The aperture is longer in the flight direction (length L) than in the flight perpendicular direction (width W) as shown in Figure A6. As in the 1-D case, one uses a 2-D Fraunhoffer diffraction integral to calculate the projection pattern of the antenna. The integral is

$$P\left(\theta_x, \theta_y\right) = \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} A(x, y) \exp\left[i\frac{2\pi}{\lambda}\left(x \sin\theta_x + y \sin\theta_y\right)\right] \mathrm{d}x \ \mathrm{d}y \qquad \text{(A7)}$$

where λ is the wavelength of the radar. As in the 1-D case we'll assume the aperture $A(x, y)$ has uniform amplitude and phase. In this case the projection pattern can be

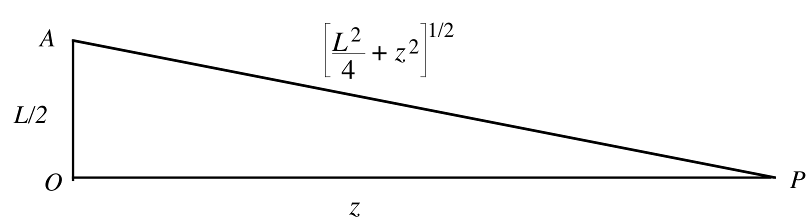


Figure A5: Diagram showing the increases length of path AP with respect to OP due to an offset of $L/2$.

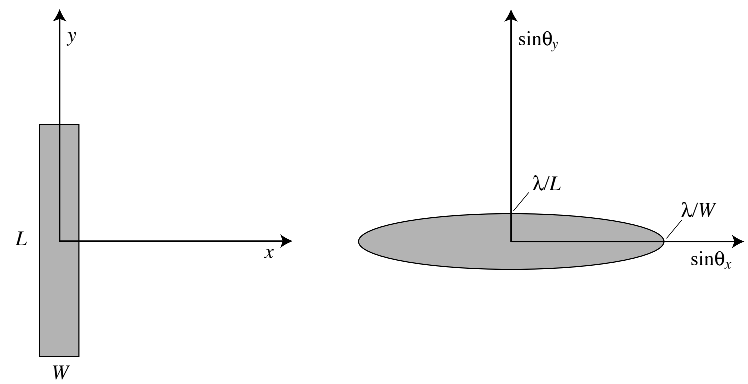


Figure A6: Diagram showing the projection pattern (right) for a rectangular SAR antenna (left).

integrated analytically and is

$$P\left(\theta_x, \theta_y\right) = LW \operatorname{sinc}\left(\frac{\pi W \sin\theta_x}{\lambda}\right) \operatorname{sinc}\left(\frac{\pi L \sin\theta_y}{\lambda}\right). \tag{A8}$$

The first zero crossing of this 2-D sinc function is illustrated in Figure A6 (right).

The ERS radar has a wavelength of 0.05 m, an antenna length L of 10 m, and an antenna width W of 1 m. For a nominal look angle of 23° the slant range R is 850 km. The footprint of the radar has an along-track dimension $D_a = 2R\lambda/L$, which is 8.5 km. As discussed below this is approximately the length of the synthetic aperture used in the SAR processor. The footprint in the range direction is 10 times larger or about 85 km.

A.4 Range resolution (end view)

The radar emits a short pulse that reflects off the surface of the earth and returns to the antenna. The amplitude versus time of the return pulse is a recording of the reflectivity of the surface. If adjacent reflectors appear as two distinct peaks in the return waveform then they are resolved in range. The nominal *slant range resolution* is $\Delta r = C\tau/2$ where τ is the pulse length, C is the speed of light and θ is the look angle. The factor of 2 accounts for the 2-way travel time of the pulse. Figure A7 shows how the *ground range resolution* is geometrically related to the slant range resolution $R_r = \frac{C\tau}{2\sin\theta}$.

Note the ground range resolution is infinite for vertical look angle and improves as look angle is increased. Also note that the range resolution is *independent of the height* of the spacecraft H. The range resolution can be improved by increasing the bandwidth of the radar. Usually the radar bandwidth is a small fraction of the carrier frequency so shorter wavelength radar does not necessarily enable higher range resolution. In

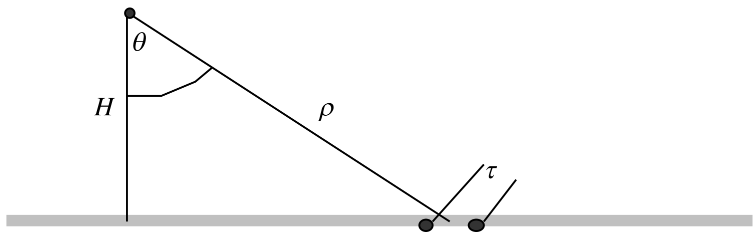


Figure A7: Diagram of radar flying into the page emitting a pulse of length ρ. That reflects from two points on the surface of the earth.

many cases the bandwidth of the radar is limited by the speed at which the data can be transmitted from the satellite to a ground station.

A.5 Azimuth resolution (top view)

To understand the azimuth resolution, consider a single point reflector on the ground that is illuminated as the radar passes overhead (Figure A8). From the Fraunhoffer diffraction analysis we know the length of the illumination (twice the angular resolution) is related to the wavelength of the radar divided by the length of the antenna. As discussed above the along-track dimension of the ERS footprint is 8.5 km so the nominal resolution R_a is 4.25 km, which is very poor. This is the azimuth resolution of the *real-aperture radar*

$$R_a = \rho \tan\theta_r \cong \frac{\rho\lambda}{L} = \frac{\lambda H}{L\cos\theta} \tag{A9}$$

where L is the length of radar antenna, ρ is the slant range, and λ is the wavelength of the radar.

If the scatterer on the ground remains stationary as the satellite passes overhead, then one can assemble a synthetic aperture with a length equal to the along-track beamwidth of length which is $2R_a$. This much longer aperture of 8.5 km results in a dramatic improvement in azimuth resolution given by

$$R_a^{'} = \frac{\lambda\rho}{2R_a} = \frac{L}{2} \tag{A10}$$

which is the theoretical resolution of a *strip-mode* SAR. Note the synthetic aperture azimuth resolution is *independent of spacecraft height* and improves as the antenna length is *reduced*. One could form a longer synthetic aperture by steering the transmitted radar beam so it follows the target as the spacecraft (aircraft) flies by. This is called *spotlight-mode* SAR.

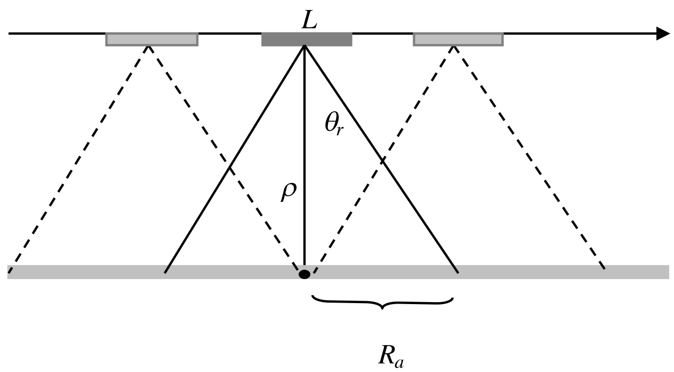


Figure A8: Top view of SAR antenna imaging a point reflector. The reflector remains within the illumination pattern over the real aperture length of $2R_a$.

A.6 Range and Azimuth Resolution of ERS SAR

The ERS SAR has a bandwidth of 15.6 MHz, an antenna length of 10 m and a look angle of 23°. Accordingly the ground range resolution is about 25 m and the maximum azimuth resolution is 5 m. In practice, one averages several "looks" together to improve the quality of the amplitude (backscatter) image. In the case of ERS, one could average 5 looks for a resolution cell of 25 m by 25 m.

A.7 Pulse repetition frequency

The discussion above suggests that we should make the antenna length L as short as possible to improve azimuth resolution. However, to form a complete aperture without aliasing longer wavelengths back into shorter wavelengths we must pulse the radar at an along-track distance of $L/2$ or shorter. To understand why such a rapid pulse rate is necessary, consider the maximum Doppler shift from a point that is illuminated at a maximum distance ahead of the radar (Figure A9) where ν_o is the carrier frequency $= C/\lambda$ and V is the velocity of spacecraft relative to the ground.

The maximum Doppler shift occurs at a maximum angle of

$$\begin{aligned} \sin\theta_a &= \frac{\lambda}{L} \\ \Delta\nu &= 2\nu_o \frac{V}{C} \sin\theta_a = \frac{2C}{\lambda}\frac{V}{C}\frac{\lambda}{L} = \frac{2V}{L} \end{aligned} \tag{A11}$$

This corresponds to a maximum along-track distance between samples of $L/2$. For ERS this corresponds to a minimum pulse repetition frequency (PRF) of 1400 Hz. The actual PRF of ERS is 1680 Hz.

A.8 Other constraints on the PRF

The PRF cannot be too large or the return pulses from the near range and far range will overlap in time as shown in Figure A10.

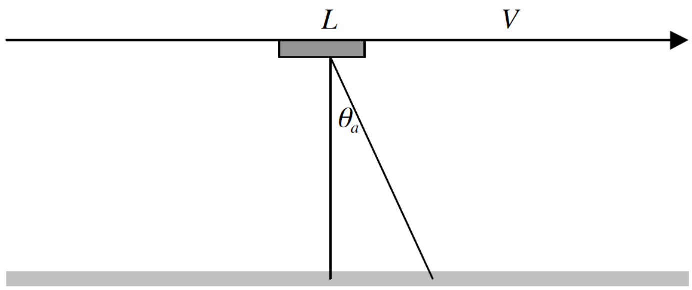


Figure A9: Diagram showing the angle between broadside to the antenna and the point where a reflector first enters the illumination of the antenna.

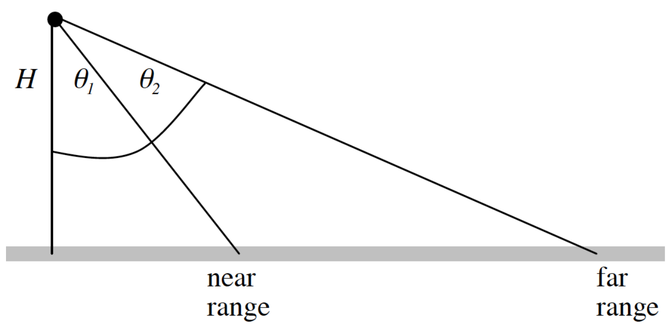


Figure A10: End view of the distance to the near range and far range of the radar illumination pattern.

$$
\begin{aligned}
PRF &< \frac{1}{t_2 - t_1} \\
t_1 &= \frac{2H}{C\cos\theta_1} \qquad t_2 = \frac{2H}{C\cos\theta_2} \\
\Delta t &= \frac{2H}{C}(\sec\theta_2 - \sec\theta_1) \\
\frac{2V}{L} &< PRF < \frac{C}{2H}(\sec\theta_2 - \sec\theta_1)^{-1}
\end{aligned}
\tag{A12}
$$

For ERS the look angles to the rear range and far range are 18° and 24°, respectively. Thus the maximum PRF is 4777 Hz. The actual PRF of 1680 is safely below this value and the real limitation is imposed by the speed of the data link from the spacecraft to the ground. A wider swath or a higher PRF would require a faster data link than is possible using a normal X-band communication link of 105 Mbps.

References

Bracewell, R.N. (1978). *The Fourier Transform and Its Applications*. Second Edition. New York City: McGraw-Hill Book Co.

Curlander, John C. and Robert N. McDonough (1991). *Synthetic Aperture Radar: Systems and Signal Processing*. Chapter 4. John Wiley & Sons.

Rees, W.G. (2001). *Physical Principles of Remote Sensing*. Cambridge, UK: Cambridge University Press.

A.9 Problems

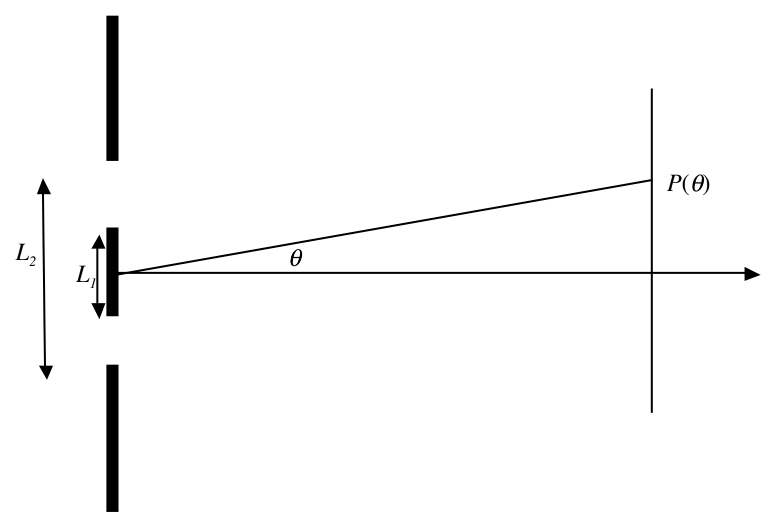
1. What is the illumination pattern for an aperture with a sign reversal at its center? What is $P(0)$? Is the function real or imaginary? Is the function symmetric or asymmetric?

   The aperture is

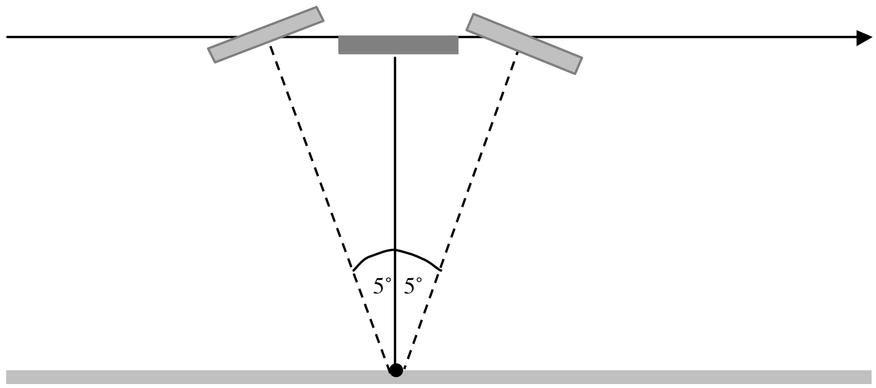
$$A(y) = \begin{cases} 0 & |y| > \frac{L}{2} \\ 1 & 0 < y \leq \frac{L}{2} \\ -1 & -\frac{L}{2} \leq y < 0 \end{cases}$$

2. (a) What is the illumination pattern for the aperture below?

   (b) What is the illumination pattern in the limit as L_1 approaches L_2?



3. What is the theoretical azimuth resolution of a spotlight-mode SAR that can illuminate the target over a 10° angle as shown in the diagram below?



4. Derive the projection pattern for a 2-D rectangular aperture of length L and width W given in equation (A8).

5. What is the ground-range resolution of side-looking radar with a pulse length of 6×10^{-8}s and a look angle of 45°?

6. (a) What is the period for a satellite in a circular orbit about the moon where the radius of the orbit is 1.9×10^6 m? The mass of the moon is 7.34×10^{22} kg.

   (b) You are developing a SAR mission for the moon. The length of your SAR antenna is 10 m. What minimum pulse repetition frequency is needed to form a complete aperture? The circumference of the moon is 1.1×10^7 m. You will need the orbital period from problem (a).

7. Derive equation (A10).

8. A SAR with a 10 m long antenna is orbiting the earth with a ground speed of 7000 ms^{-1}. What is the maximum possible swath width. Use a look angle to the near range of 45° and assume the earth is flat.

9. CryoSat is a nadir-looking altimeter that uses a burst-mode synthetic aperture to reduce the length of the radar footprint in the along-track direction. The radar has a pulse repetition frequency of 18 kHz and uses 64 echoes in one burst to form the synthetic aperture. What is the along-track resolution of the footprint? What is the Fresnel distance for this configuration (ground track velocity 7000 ms^{-1}, 700 km altitude)?