Aalto University
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A Review of the Valuation Methods for Real Options

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Abstract

The traditional valuation methods, such as net present value, are incapable of capturing the value of managerial flexibility under uncertainty in real investments. Real options analysis offers a solution by incorporating option-pricing theory into the evaluation of real investments. However, the approach has not gained much traction among practitioners despite ample recognition in the literature. The availability of multiple valuation methods for real options analysis and the difficulty of choosing the optimal method are some of the reasons for the lack of application in the corporate world.

This review of real options and valuation literature examines the concept of real options analysis, the attributes of different real options and the available valuation methods. The information is used to analyse the advantages and shortcomings of the methods in various situations.

The aim of the study is to determine the optimal valuation method for real options analysis of different real investments. The study also provides the reader with a basic understanding of the real options approach to valuation, and the capability to identify the various real options embedded in strategic investments.

The results show that there is no universally optimal method, as real investments tend to contain varying complicating features that benefit from different approaches to valuation. However, for individual valuation problems, the ideal method can often be determined, and it is mostly dependent on the complexity of the problem.

Keywords real options, valuation methods, investments

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1. Introduction

In order to create shareholder value, companies must be able to identify sufficiently profitable investment opportunities, which in turn requires choosing the correct valuation method. Traditional discounted cash flows based analysis has been, and still is the method preferred by practitioners (Graham & Harvey, 2001; Ryan & Ryan, 2002; Teach, 2003; Alkaraan & Northcott, 2006; Siziba & Hall, 2021), even when it is well known that it cannot accurately capture the value of managerial flexibility and strategic opportunities embedded in real investments, possibly leading to incorrect investment decisions (Trigeorgis, 1996; Amram & Kulatilaka, 1999; Copeland & Antikarov, 2001; Brealey et al. 2008).

Real options, a term coined by Myers (1977), provide a way of quantifying these opportunities and managerial flexibility. Regardless of ample recognition in literature, real options analysis is yet to reach widespread popularity among practitioners (Ryan & Ryan, 2002; Teach, 2003; Block, 2007). One reason behind this is the difficulty of choosing the optimal valuation method for real options analysis, and their relative complexity compared to traditional methods (Block, 2007; Mathews et al. 2007).

This review of real options and valuation literature focuses on different real options and the available valuation methods, their features, accuracy and suitability for various situations. The aim of the study is to determine the optimal valuation method for real options analysis of different real investments.

The study provides its reader with a basic understanding of the real options approach, along with the capabilities to identify real options embedded in investments and to select the correct valuation method for their needs. To allow for meaningful comparison of the methods, the study assumes shareholder value maximization as the primary objective of a company.

The rest of the study is organized as follows; section two introduces the basic concepts related to financial options, real options and real options analysis. Section three focuses on identifying the different real option types and examining their convertibility to financial options. Section four assesses the core valuation models used for real options analysis by giving an overview of each valuation process, their advantages and disadvantages. Section five combines the findings of the previous sections, evaluating the suitability of each valuation method in different real option settings. Finally, section six summarises the observations made during the study and discusses possible future subjects on studying real options valuation.

2. Basic Concepts & Terminology

2.1 Options

Options are contracts that give their holder the right, but not an obligation, to buy or sell an underlying asset or instrument at a specified price, the exercise price. There are two types of options: call options, which give the right to buy an asset, and put options, which give the right to sell an asset. Options have an expiration date, and *European options* can be exercised only at that specific date, while *American options* can be exercised at any time up to the expiration date. (Hull, 2017)

For the purpose of this study, it is necessary to understand the drivers of option value. The value of any option increases with the volatility of the underlying asset and the time until expiration. For call (put) options, the option value also increases (decreases) when the underlying asset's value increases, the exercise price decreases or the risk-free interest rate increases (Brealey et al. 2020). The term intrinsic value will also be used, and refers to the option value without risk or time value, i.e. the value at expiration date, calculated as follows:

$$C = \max(S - X; 0) \tag{1}$$

$$P = \max(X - S; 0) \tag{2}$$

where C and P are the intrinsic values of a call and a put option, S is the market price of the underlying asset at expiration date, and X is the exercise price of the option.

There are also more complex instruments called exotic options, such as barrier options (additional restrictions to option payoff), compound options (another option as the underlying asset), and rainbow options (two or more sources of uncertainty) (Damodaran, 2005). An exhaustive description of the exotic options is outside of the scope of this study, but it is worth noting that real option counterparts of them are also frequently found in strategic investments.

2.2 Real Options

There are multiple definitions for real options, such as "opportunities to acquire real assets" (Dixit & Pindyck, 1994), "managerial operating flexibility and strategic interactions" (Trigeorgis, 1996) and "options to modify projects" (Brealey et al. 2020). Regardless of phrasing, real options can be understood as option-like opportunities related to real, or nonfinancial assets. The underlying asset could be a project or some other strategic investment.

The same factors affect the value of real options as of their financial counterparts, and they share the key attribute of providing a right, but not an obligation. These similarities indicate that like financial options, real options have a measurable monetary value, and we should be able to utilize option-pricing models to value real options as well.

However, there are significant differences that make the valuation process rather difficult at times. Unlike financial options, real options aren't publicly traded, and therefore finding a comparable investment or historical data to evaluate risk profiles is difficult or even impossible (Copeland & Antikarov, 2001). Additionally, the expiration date and exercise price of a real option may not be constant over time, or even known at all (Bowman & Moskowitz, 2001). Furthermore, while financial options are easy to observe, real options must be explicitly identified and specified. Almost every real investment contains some degree of flexibility, but classifying this flexibility into individual real options can be challenging (Amram & Kulatilaka, 1999; Block, 2007). Lastly, Yeo and Qiu (2003) note that contrary to financial options, real options are also affected by competitive interactions.

The aforementioned features make real options more complicated than financial assets, and consequently their valuation significantly less straightforward. Bowman and Moskowitz (2001) suggested that there is a need for customized option valuation models to account for the specialties of real options, and novel approaches have indeed followed (see for example Datar & Mathews, 2004; Collan et al. 2009).

2.3 Real Options Analysis

Real options analysis, or real options valuation, refers to the valuation of real investments as collections of real options. It can also be considered as the valuation of managerial flexibility, or an active management's added value. Amram and Kulatilaka (1999) describe real options analysis as "an extension of the financial option theory to options on real assets" and a way of thinking, rather than a strict methodology. This is true, as there is no clear guideline on how to value real options, rather the important part is to acknowledge and identify the existence of real option value and adjust the investment process accordingly.

A distinctive characteristic of real options analysis compared to other techniques is how it views uncertainty. Uncertainty, or risk, generally decreases the value of an asset. With options however, uncertainty is the primary driver that increases the value. Capturing this upside of uncertainty is at the heart of real options analysis, and intuitively, utilizing it is most beneficial when surrounding uncertainty is high. Correspondingly real options analysis is not useful without uncertainty – if we know the future state of nature, we know the optimal investment strategy now. Thus, active management deviating from this optimal strategy cannot create added value under certainty.

The value of managerial flexibility can be viewed more technically as an alteration of the probability distribution of investment outcomes. Instead of a symmetric distribution, active management's opportunities to limit downside losses and extend the upside potential result in the probability distribution being skewed towards the right, i.e. the positive side of investment outcomes, as demonstrated in Figure 1. (Yeo & Qiu, 2003)

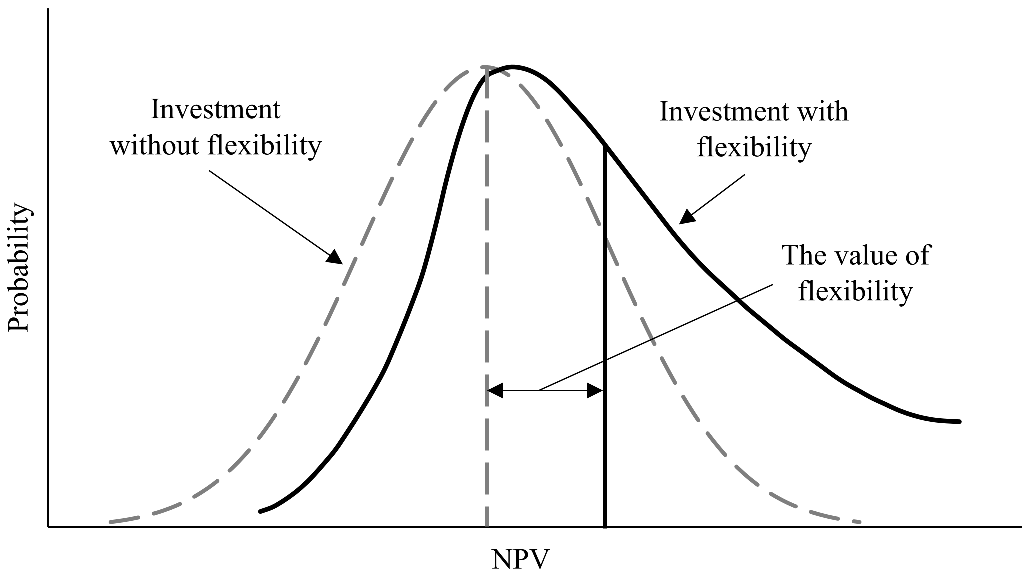

Figure 1. The Value of Managerial Flexibility

The vertical lines represent the means of the distributions, i.e. the expected NPVs of the investment. The added value of managerial flexibility is the difference between the two expected NPVs. (adapted from Yeo & Qiu, 2003)

3. Common Real Options

The real options analysis process begins with the identification of individual real options that the investment consists of, and perhaps those that could be acquired. While there isn't a clear answer for which valuation method is optimal for each real option type, knowing the degree of complexity and the comparison to financial options is essential for determining the suitable methods. Due to the diverse nature of business, there are countless different variations and combinations of real options. The following list contains the primary real option types recognized in the literature so far.

3.1 Option to Abandon

After initiating a project, management has the option to permanently abandon it and realize the resale value of committed assets through secondhand markets or alternative use within the company. Abandonment might become beneficial in case of adverse market developments, or if the initially forecasted cash flows of the project turn out to be overly optimistic.

From a valuation perspective, the option to abandon can be viewed as an American put option on the project's current value, where the potential salvage or best-alternative-use value is the exercise price (Trigeorgis, 1996). Accordingly, it should be exercised if the salvage value exceeds the present value of cash flows obtained by continuing (Brealey et al. 2008).

The abandonment option is particularly valuable in capital-intensive industries, financial services and new product introductions in uncertain markets. The option value of abandonment is also affected by the quality of committed assets, as more general-purpose assets tend to be easier to resell than special-purpose assets, increasing the salvage value. (Trigeorgis, 1996)

3.2 Option to Delay

If the management has control over the start date of a project, they possess the option to delay the investment. Reasons to wait include, for example, anticipated favourable price changes in inputs or outputs of the project. The option to delay an investment is particularly valuable in all natural resource extraction industries, real-estate development, farming and paper products. (Trigeorgis, 1996)

According to McDonald and Siegel (1986), correctly timing an investment is a rather significant decision; they argue that suboptimal timing can often lead to a 10-20 percent loss of project value when the option value is taken into account.

The option to delay is comparable to an American call option on the present value of the project's expected cash flows, with an exercise price equal to the initial investment cost. Initiating the project before necessary essentially means giving up the option value of waiting, and thus should be considered an opportunity cost of early investment. This implies that to justify an early investment, a substantial cash flow premium should be obtained to compensate for the loss of option value. (Trigeorgis, 1996)

When valuing an option to delay, there are some strategic considerations. Delaying an investment may allow competitors to undertake the investment and capture a significant part of the market, potentially decreasing the expected profits of the investment. Or if the investment is exclusive to the company, the source of exclusivity may expire, a patent for example. These would imply the existence of a cost to delaying the investment, which must be taken into account in the valuation. The barriers of entry must be very restrictive for the option to delay to be valued as a simple call option without considering the cost. (Damodaran, 2005)

3.3 Option to Alter Operating Scale

Management's flexibility to alter operating scale according to market conditions can be divided into three distinct real options: option to expand, to contract and to shut down and restart operations.

If market conditions turn out to be more favourable than expected, management may have the option to expand the scale of operations. Option to expand is analogous to a call option on the added cash flows resulting from the increased operating scale. Upscaling operations usually implies incurring an additional cost (increased variable costs of operations), which acts as the exercise price of the option (Trigeorgis, 1996). In addition to variable costs, being able to upscale production often implies initial overcapacity. While this is in retrospect a sunk cost, it should be considered in the development phase by comparing the benefit of flexibility and the cost of building in overcapacity.

Similarly, adverse market conditions can make operating below normal capacity beneficial to mitigate losses. The option to contract is analogous to a put option on the reduced part of the base-scale project, where the cost savings form the exercise price. In an extreme case it might be beneficial to temporarily halt the project and restart it at a later date. If such option is available, operation in a given time period can be viewed as a call option on the period's cash flows by incurring the exercise price, i.e. the variable costs of operating. (Trigeorgis, 1996)

3.4 Option to Switch

The management might have the option to switch between inputs (process flexibility) and/or outputs (product flexibility) of the project (Trigeorgis, 1996). These options can often be acquired via decisions in procurement, sourcing and product development – an indication that it could be beneficial for companies to adopt the real options way of thinking in other areas as well, not just capital budgeting.

Switching options are slightly problematic to value – the option is often reversible and repeatable, there can be varying costs associated with switching, and there may be no time limitations to exercise these options. Their financial option counterparts are exchange options, the options to switch between two volatile assets (Adkins & Paxson, 2011).

The option to switch outputs is important in industries where products are sought in small batches, or the demand is particularly volatile due to factors such as seasonality. Such industries include e.g. consumer electronics, toys and cars. The option to switch inputs is especially relevant in feedstock-dependent industries such as oil, electric power and chemicals. (Trigeorgis, 1996)

3.5 Staged Investment Option

The possibility to stage an investment instead of immediate full capital commitment retains the option to abandon the project at a later stage if it is beneficial according to new information, thus limiting the potential downside in case of project failure. Examples of users of staged investment include R&D intensive industries such as pharmaceuticals and the multi-staged development of oil and gas exploration. Staged development can also provide a learning and uncertainty reduction effect. (Yeo & Qiu, 2003)

In a staged investment, each stage can be viewed as a call option on the value of subsequent stages, by incurring the cost of proceeding to the next stage. Therefore it can be valued as a compound option. (Trigeorgis, 1996)

Similarly to the option to delay, Damodaran (2005) notes that there is a cost associated with this real option – staging an investment may lead to higher costs due to the firm not fully utilizing the economies of scale or allowing competitors to capture the market by entering immediately at full scale. This cost must be weighed off against the gain of limiting downside.

3.6 Growth Option

An initial investment may be a prerequisite for future growth opportunities. For example, an R&D investment's directly measurable cash flows' present value may seem very unattractive, but it could open up profitable growth opportunities in the future and therefore act as a call option on another investment. If the option value of these future opportunities offsets the loss of the initial R&D investment, it may still be worth undertaking. (Kester, 1984)

Amram and Kulatilaka (1999) argue that support function investments such as advertising or improved customer service can also be considered as growth options. Damodaran (2005) notes that growth options are often the drivers behind acquisitions, for example to gain entry into a growing market, technological expertise or a valuable brand name. Since a growth option creates *external* opportunities, but no obligations, in the future, it can be viewed as an *inter-project* compound option (Trigeorgis, 1996).

3.7 Multiple Interacting Options

Real investments often involve several different real options on the same underlying asset, the project's value. While they can be valued individually, it is important to bear in mind that they may interact, and thus the combined value of an investment's real options may not be equal to the sum of the individual real option values (Trigeorgis, 1996). Therefore a straightforward decomposition of an investment into individual options and separate valuation afterwards can lead to an incorrect solution, and more sophisticated models should be applied (Gamba, 2002).

The interaction often appears as prior options affecting the value of the underlying asset. For example, consider the option to expand; by altering the scale of the project, the value of the underlying asset (project's cash flows) is changed, and thus the values of all subsequent real options change as well. Another self-evident interaction is that by exercising a prior option, subsequent options may be lost. Exercising the option to abandon implies forfeiting all future real options on that investment, and this loss of option value should be considered in the valuation of the abandonment option. (Trigeorgis, 1993)

4. Valuation Methods

There are several methods for valuing investments under uncertainty. For real options, the most relevant are the traditional discounted cash flows (DCF) based methods, mainly net present value (NPV), and various option-based methods. In real options valuation, they aren't mutually exclusive, but rather complement each other. NPV captures the value of directly measurable expected cash flows, and the suitable real options valuation model accounts for the option value created by managerial flexibility. (Trigeorgis, 1996; van Putten & MacMillan, 2004)

The following comparison will help in choosing the correct valuation method by evaluating their advantages and shortcomings. The list is not exhaustive, but covers the core models used for real options valuation, as other methods are more or less extensions of them.

4.1 Traditional DCF Methods

Out of the traditional valuation methods, only net present value is consistent with the objective of shareholder value maximization under certainty. Other methods such as payback period, return on investment, profitability index, book rate of return and internal rate of return (IRR) either fail to consider the time value of money, cost of capital, or have other shortcomings, which is why they are generally considered inferior to NPV. (Trigeorgis, 1996; Brealey et al. 2020)

The discounted cash flows methods, NPV and IRR, are the most used capital budgeting techniques, especially among large companies (Graham & Harvey, 2001; Ryan & Ryan, 2002; Alkaraan & Northcott, 2006; Siziba & Hall, 2021), and can thus be considered the industry standard.

The NPV formula is as follows: (Brealey et al. 2020)

$$\text{NPV} = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} \tag{3}$$

where C_t is the net cash flow at time period t, and r is the discount rate.

The method suggests that if the net sum of all future cash flows discounted at the required rate of return (the opportunity cost of capital) is larger than the initial investment, i.e. the project's NPV is positive, one should undertake the project.

Uncertainty can be accounted for with two different approaches in NPV. The *certainty-equivalent approach* uses expected cash flows adjusted with a certainty-equivalent coefficient, which are then discounted at the risk-free rate. The adjusted cash flows contain the business and financial risks of the investment, while the discount rate accounts only for the time value of money. The second, more widely used method is the *risk-adjusted discount rate approach*, where the expected cash flows are left as is, and the discount rate is modified to reflect the risk associated with uncertain cash flows in addition to the time value of money. Regardless of the approach, determining the correct risk profile for real investments can be challenging as information on comparable investments may be limited or non-existent. (Trigeorgis, 1996)

While NPV can account for uncertainty, it does implicitly make an assumption of management acting passively according to the expected cash flow scenario. Therefore the added value of managerial flexibility is completely omitted in NPV valuations (Trigeorgis, 1996; Brandão et al. 2005). As volatility is a key driver of option value, this flexibility is most valuable under high uncertainty and thus the probability of NPV leading to an incorrect valuation is accordingly at its highest (van Putten & MacMillan, 2004).

Additionally, van Putten and MacMillan (2004) suggest that as DCF models require cash flows to be discounted at a high rate to reflect the uncertainty around project returns, the valuation captures all of the risks of uncertainty while neglecting the potential rewards. This is intuitive, as uncertainty naturally decreases asset value, but increases option value, and DCF models disregard the option value of management's decisions (Trigeorgis, 1996; Brandão et al. 2005).

Another issue is timing. A positive NPV implies that the investment should be undertaken, but the method does not comment on when to do it. Just because an investment has a positive NPV, it might not be optimal to commit to it today (Brealey et al. 2008).

4.2 Decision-Tree Analysis

Decision-Tree Analysis (DTA) is a DCF-based approach that incorporates managerial flexibility as decisions at distinct, discrete time points. The possible decisions are mapped in a tree structure, and their consequences (cash flows) are dependent on some uncertain future events, which are described by probabilities (see Figure 2 for example). Management is expected to make decisions based on the expected risk-adjusted NPV of the following states of nature. The optimality of decisions must therefore be evaluated by working backwards from the final outcomes, determining the expected NPVs of prior decisions. (Trigeorgis, 1996)

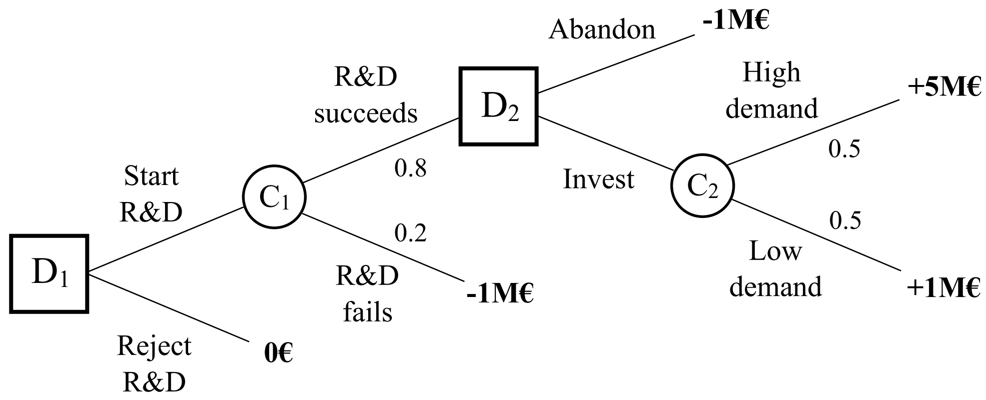

Figure 2. A Simple Decision Tree

D_n represents a decision, and C_n represents a consequent state of nature.

DTA is not originally an option-based method, but it can be applied to real options valuation, given risk neutral probabilities are used. The inclusion of management's ability to optimize their decision-making changes the risk characteristics of the investment, and therefore the discount rate used without real options in the investment is not viable (Brandão & Dyer, 2005). Smith and Nau (1995) proposed that this problem can be solved by discounting the cash flows at the risk-free rate, and adjusting for risk in the probabilities of individual outcomes.

While DTA forces management to acknowledge the existence and consequences of their future decisions, it still has several issues. Real investments are rarely as simple as the example above, and mapping out more complex investments may be very cumbersome. The model also assumes that decisions and events occur at discrete time points, when in reality the resolution of uncertainty and availability of decisions are likely to be continuous. Similarly to NPV, choosing the appropriate discount rates is also very challenging, as they should reflect the gradually resolving uncertainty and therefore vary at different time points. (Trigeorgis, 1996)

DTA requires perhaps too many subjective inputs (events, probabilities, cash flows, discount rates) prone to forecasting errors to be viable for accurate valuation of real investments. Therefore it might be more useful for providing management with a clear and transparent strategic investment plan, as long as the investment is simple enough to be structured as a tree. This helps with identifying the individual real options, after which more accurate methods can be applied.

4.3 The Black-Scholes Model

The Black-Scholes or Black-Scholes-Merton model is a closed-form mathematical option pricing model originally derived by Black and Scholes (1973), and later extended by Merton (1973). Various modifications have been created to allow for broader applications and more relaxed assumptions. The Black-Scholes formula can be written as:

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

where

C = price of the call option
t = time in years
T = time of option expiration
S_t = price of the underlying asset at time t
$N(\ )$ = cumulative standard normal distribution
K = the exercise price of the call option
r = the annualized risk-free rate
σ = standard deviation of the underlying asset's returns

Option valuation in a single equation as a direct function of relatively few inputs, such as Black-Scholes, is the easiest and fastest method of obtaining the option value. However, real options tend to have features that make them too complex for such analytical solutions. (Amram & Kulatilaka, 1999; Block, 2007)

For real option applications, Black-Scholes makes many assumptions that must be considered. For example, it is intended for European (or perpetual American) options and assumes a known and constant interest rate, volatility and exercise price. Unfortunately, as most real options are American options, have unknown or variable inputs and the returns might not follow a lognormal distribution, these assumptions are often too restrictive for accurate valuation. (Copeland & Antikarov, 2001)

Regardless, due to their relative straightforwardness, Black-Scholes and its extensions can act as useful limiting cases and valuation boundaries for other methods (Damodaran, 2005; Sick & Gamba, 2010). A fringe case of an analytical solution's effectiveness in a more complex real option is the option to switch. Margrabe's formula, derived for pricing exchange options, can be applied to input and output-switching options (Margrabe, 1978). However, it is also intended only for European options (and by later extensions perpetual American options).

4.4 Binomial Option Pricing Model

Binomial option pricing models or binomial lattices, first proposed for financial options by Cox, Ross and Rubinstein (1979), present uncertainty as the evolution of the value of the underlying asset at discrete time points. There are two possible values the underlying asset can take at each time point, represented by up and down in the binomial tree. Boyle (1986) later extended the model to a trinomial version, where each node has three possible future paths, but the concept is otherwise similar.

The most widely used method is the multiplicative binomial model of uncertainty, where the asset has an initial value, *A*, and at the first time point the asset value moves either up, to *Au*, or down, to *Ad*. At the next time point, the possible outcomes are Au^2, *Aud,* and Ad^2. Following all possible branches of the binomial tree results in a distribution of underlying asset value outcomes, as demonstrated in Figure 3. (Amram & Kulatilaka, 1999)

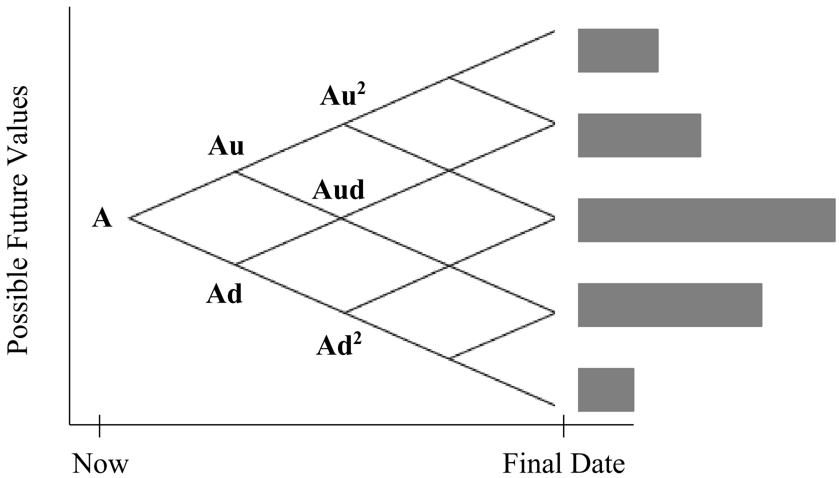

Figure 3. The Binomial Tree and its Outcome Distribution

The binomial tree results in a probability distribution of possible asset values at the final date. (adapted from Amram & Kulatilaka, 1999)

After the binomial tree is constructed, the option valuation starts from each of the final nodes, where the value is simply the intrinsic value at the given node. The binomial values of the earlier nodes are then calculated using the risk neutrality assumption on its following nodes, until we arrive at the very first node, where the option value is found.

A key advantage of the binomial models over Black-Scholes is the ability to value American options, or allowing for early exercise of the option (Brandão et al. 2005). This is performed by taking the greater of binomial and intrinsic value at the node(s) where early exercise is permitted, or all of them in the case of American options. Amram and Kulatilaka (1999) emphasize the transparency provided by a tree structure, which is useful for observing the sources of option value. Brealey et al. (2008) add that binomial models are tailor-made for most abandonment options.

There are trade-offs however, as binomial models require a large number of inputs in the form of the expected underlying asset values at each node (Damodaran, 2005). This implies that they are less suitable for projects where forecasting the cash flows and their probabilities is particularly time-consuming. According to Trigeorgis (1996), binomial models are useful in valuing complex projects with multiple, possibly interacting real options, dividend-like effects and staged investments. This is true when compared to Black-Scholes, but Gamba (2002) notes that the model quickly becomes computationally intractable when complexity increases.

4.5 Monte Carlo Simulation

Traditional Monte Carlo simulation, first applied to financial options by Boyle (1977), draws many random scenarios for the underlying asset's price evolution. The optimal investment strategy for each scenario is determined and the ending pay-off (intrinsic value) calculated accordingly. The option value is then found by averaging the pay-offs and discounting that average to its present value. (Amram & Kulatilaka, 1999)

The Monte Carlo simulation method is more flexible than closed-form models such as Black-Scholes, as it can use distributions as its input parameters (e.g., for returns or volatility) rather than requiring known constants (Boyle, 1977). This is especially helpful with real investments, where reliably determining constant input parameters is often challenging due to the lack of comparable assets.

In the past simulation models were mainly used to analyse European options, and weren't considered suitable for American options or sequences of options (Amram & Kulatilaka, 1999). However, Carriere (1996) showed that options with early exercise features can also be valued using Monte Carlo simulation, and today we know that simulation models have a clear advantage over the previous methods in their ability to handle increasing complexity and multiple sources of uncertainty (Sick & Gamba, 2010).

Computational power is not as significant of an issue as it was when the real options approach was starting to gain traction, but simulation still requires technical expertise and external tools. Thus it might be worthwhile to consider the traditional methods first, and resort to simulation if they are deemed unsuitable.

4.6 Datar-Mathews Method

The Datar-Mathews method, or DM method, proposed by Datar and Mathews (2004) is specifically tailored for real options. According to Mathews et al. (2007), their goal was to "create a real options approach that uses the language and frameworks of standard DCF analysis" because the existing methods weren't particularly appealing to practitioners due to their complexity or inclusion of lesser known concepts. The method is algebraically equivalent to Black-Scholes, but uses the same inputs as a traditional DCF valuation (Mathews et al. 2007). Instead of valuing individual real options, this practitioner-oriented approach views the whole investment as a real option, when the investment can be terminated if a loss is forecasted (Kozlova et al. 2015).

The method is based on management's cash flow scenarios (often triangular distributions), from which hundreds or thousands of possible NPV outcomes are drawn using Monte Carlo simulation. The resulting histogram is the NPV pay-off distribution of the project. Negative NPV outcomes are considered as zero, as they would be terminated by a rational manager. The real option value is then calculated as the mean NPV of the resulting pay-off distribution multiplied with the risk-adjusted success probability, see Figure 4. (Mathews, 2009; Kozlova et al. 2016)

In cases of limited information, such as very new business ideas, a three-point estimation of the NPV outcomes (triangular distribution) can also be used. By using a triangular distribution of the operating profits and the mean of exercise price estimates, one can arrive at a conservative estimate, called the range option value, without the burden of simulation. (Mathews, 2009)

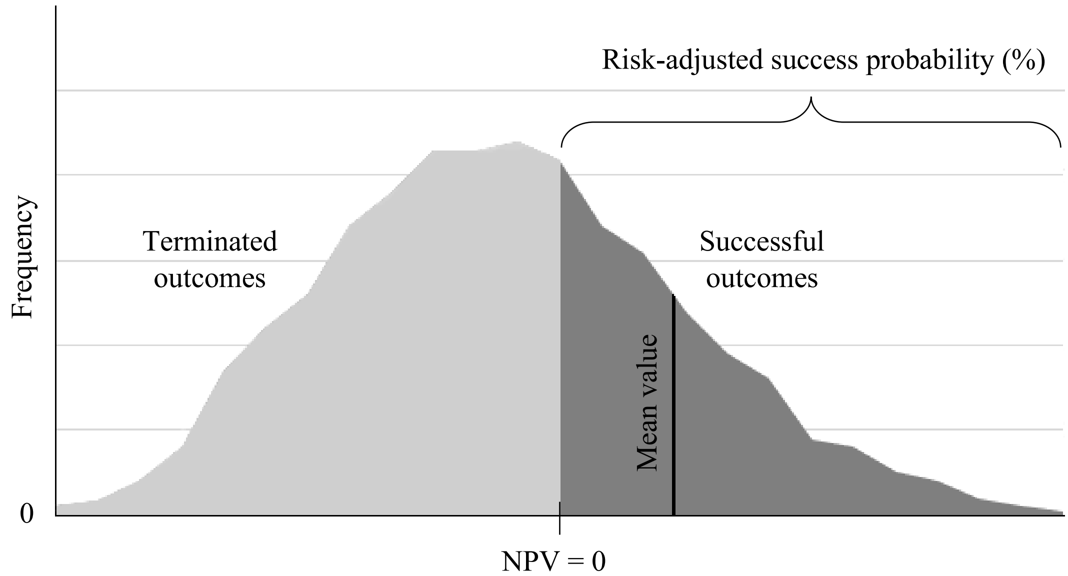

Figure 4. Datar-Mathews Pay-Off Distribution

(adapted from Mathews et al. 2007)

The formula for the DM method is as follows: (Datar & Mathews, 2004)

$$C_0 = E_0\left[\max\left(\tilde{S}_T e^{-\mu T} - \tilde{X}_T e^{-rT}\right), 0\right] \tag{5}$$

where

C_0 = option value of the project
$\tilde{S}_T$ = random variable representing operating profits at time T
$\tilde{X}_T$ = random variable representing exercise price
r = risk-free rate
μ = market risk rate / hurdle rate

As seen in the formula, DM uses two separate discount rates. The project's operating profits, subject to market risk, are discounted at the regular hurdle rate while more secure cash flows, which the company has fairly extensive control over, such as the exercise price (the launch cost), are discounted at the risk-free rate (Mathews, 2009). This proper discounting of cash flows with differing risk profiles is something that the traditional models often lack.

The method's main advantages in the real options setting are that it doesn't require volatility or the accurate current value of the project as an input, both of which can be difficult to estimate for real assets, and that as a simulation-based model various types of distributions may be used. While simple to use for practitioners, Barton and Lawryshyn (2010) note that the DM method does not provide a transparent strategic investment plan, like lattice methods do. Kozlova et al. (2016) add that a significant amount of information is required to build a credible model for the Monte Carlo simulation.

The original DM method was created to value European call options; however it has been further extended to value put options, compound options and American options. (Mathews et al. 2007)

4.7 Fuzzy Pay-Off Method

The fuzzy pay-off method, also specifically created for real options, was proposed by Collan, Fullér and Mezei in 2009. The logic is otherwise similar to the DM method, but it accounts for uncertainty by using fuzzy numbers instead of probabilities. Fuzzy numbers are, instead of single real values, sets of possible values that each have a weight between 0 and 1. The weight represents the degree of membership in the given set, and fuzzy numbers can thus be viewed as possibility distributions. (Collan et al. 2009)

The valuation process starts with managers providing three or four scenarios, usually minimum, maximum and best-estimate, from which cash flows and then NPVs are estimated. The scenarios form the fuzzy NPV (a triangular pay-off distribution, see Figure 5), where the minimum and maximum scenarios establish the lower and upper limits (having degrees of membership of 0), and the best-estimate acts as the highest probability pay-off (having a degree of membership of 1). The real option value (ROV) is then calculated as the mean of the positive side of the distribution, multiplied with the “success ratio”, i.e. the area of the positive side divided by the total area of the distribution, see formula below: (Kozlova et al. 2016)

$$\text{ROV} = \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} * E(A_+) \tag{6}$$

where A is the fuzzy NPV, $\int_{-\infty}^\infty A(x)dx$ computes the total area of the distribution, $\int_0^\infty A(x)dx$ computes the positive area of the distribution and $E(A_+)$ is the fuzzy mean value of the positive side of the distribution. (Collan et al. 2009)

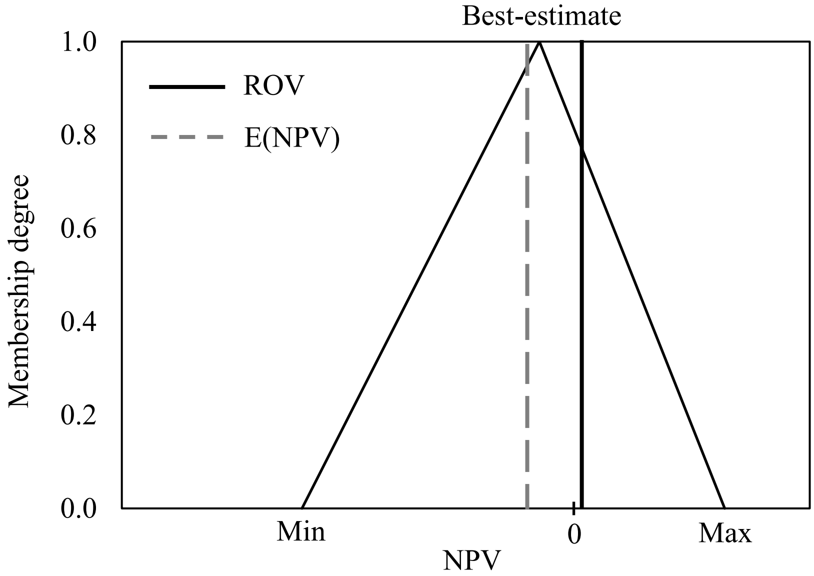

Figure 5. Triangular Fuzzy NPV

(adapted from Kozlova et al. 2016)

Collan et al. (2009) argue that fuzzy numbers allow for more realistic models, as they represent the inaccuracies of human decision-making and forecasting, instead of simplifying uncertainty to a single-point estimate. Perhaps the greatest advantage of the fuzzy pay-off method lies in the usage of management's scenarios as inputs with no simulation necessary, thus no computational power or additional tools are needed. It is worth noting that the simplified DM range option value works similarly, albeit it provides only a rough estimate instead of an accurate valuation. The fuzzy pay-off method also allows for costs and revenues to be discounted at different rates (Kozlova et al. 2016).

Kozlova, Collan and Luukka compared the DM and fuzzy pay-off methods with case examples. They found that the extremities of the distributions and final valuations tend to be nearly identical for the two methods. However, more complex problems with conditioning variables resulted in the Monte Carlo simulated distributions to be atypical with multiple summits – capturing these local maxima can be valuable information, and the fuzzy method with its triangular distribution could be too rigid for increasingly complex valuations. They note that the fuzzy pay-off method is significantly easier and faster to implement, while still offering sufficient precision for less complex valuations. (Kozlova et al. 2016)

5. Analysis

The suitability of a given valuation method is dependent on several factors, such as the complexity of the real option, availability of information and the desired degree of accuracy. Additionally, interaction between individual real options, strategic considerations and the existence of competition can affect the usability of a method. This section focuses on analysing how well the valuation methods handle the aforementioned aspects, and therefore how suitable they are for various situations.

5.1 Complexity

The complexity of the individual real option is perhaps the most influential factor for valuation method selection. A simple valuation method may be unable to value a complex real option, and likewise an unnecessarily complicated method implies using excessive time and resources for the valuation. Table 1 classifies the real option types based on their complexity, and lists the individual sources of complexity.

Table 1. Complexity of the Real Option Types

Real Option	Complexity	Complicating Features
Abandon	Simple	-
Delay	Simple	Competitive interactions
Switch	Moderate	Reversibility, repeatability, multiple underlying assets
Alter Operating Scale	Moderate	Reversibility, repeatability
Staged Investment	Complex	Compound option, competitive interactions, possible learning and uncertainty reduction effects
Growth	Complex	Interproject compound option; involves external investments

For the simplest real options, simulation is often unnecessary, as an accurate valuation can be achieved with faster methods. While Black-Scholes is the easiest and fastest of the methods presented, its assumptions based on financial options are often too restrictive. Binomial models offer a balance of flexibility and simplicity – they are able to include most of the complicating real asset features, but do not require external tools or intensive calculation. As an option-based method, while logically similar, the binomial model is better suited for real options than DTA, and thus the optimal choice for the valuation of simple, individual real options.

When complexity increases, the tree structure of binomial models starts to limit its suitability as it grows to impractical sizes, becoming slow and intractable. With today's advancements in computational power, simulation can handle almost any level of complexity as long as the user is able to model it. Because of this, and the ability to use distributions as inputs, simulation models are the best option for the most difficult valuations, such as the real options comparable to exotic options.

Because DM and the fuzzy pay-off method recognize the complete investment as a real option, a similar comparison can be drawn between them if we evaluate the complexity of the complete investment. As noted in Section 4.7, the methods tend to arrive at the same values, but the fuzzy pay-off method omits potentially valuable information if the investment is very complex. DM on the other hand requires using simulation, making it more time-consuming and difficult. Accordingly, the fuzzy pay-off method should be used for simple to moderately complex investments, and DM reserved for the most complex problems or when comprehensive information about the outcome distribution is desired.

5.2 Real Option Interaction

As mentioned in Section 3.7, it is often the case that real investments contain multiple, possibly interacting real options that affect the values of each other. This issue can apply to any of the real option types. Out of the valuation methods presented, only Black-Scholes is generally unusable when significant interaction between real options is applied, as it is a closed-form solution intended to value single options.

DTA and binomial models allow for option interaction, as different options and their joint outcomes can be mapped to the tree structures. Unfortunately, the aforementioned issue of tree structures becoming intractable with complexity is further amplified with option interaction.

Regardless, for investments with only few interacting real options, the tree-structure models are likely very practical choices.

Simulation, offering the greatest degree of flexibility, is probably the most accurate method for the valuation of interacting real options. As long as the user has the knowledge required to specify and quantify the dependencies between the individual real options, the outcomes can be simulated and the investment valued. That being said, this process of configuring the simulation can be significantly more challenging than simply mapping the real options into a tree structure, and therefore simulation should be reserved for the more complex interactions.

Due to their approach of valuing the complete investment as a real option instead of valuation through individual options, DM and fuzzy pay-off mostly avoid the issue of interaction between real options. However, this does not imply superiority; unless the management's scenarios used as inputs capture the effects of option interaction, the information is largely omitted from the valuation.

5.3 Strategic Considerations

While competitive interaction and other strategic considerations can affect the value of any real option type, they are the most relevant for the options to delay and to stage an investment. When exercised, both of them leave the company vulnerable to competitors taking action and capturing market share, which likely affects the value of the investment and option value accordingly.

Most of the methods are to some degree capable of including the effects of competitive interaction, as naïvely they can be thought of as simply changes to the underlying asset's value as a result of exercising an option and the consequent reaction from competitors. However, assuming that the management is able to accurately predict the actions of competitors and their impact on the investment's value might not be very realistic. To account for the difficulty in prediction, using distributions and/or multiple scenarios as the inputs rather than single-point estimates is necessary. Consequently, simulation models, DM and the fuzzy pay-off are the methods capable of realistically including the strategic aspects of real investments into the valuation.

6. Summary and Conclusions

This study started with a brief overview of the concept of real options analysis and relevant terminology, then followed by identifying the different real option variations and analysed their comparability to financial options in order to provide a basis for their valuation. The study then proceeded to explore the available valuation methods and their features with a focus on their applicability to a real options setting. Finally, all of this information was used to analyse the valuation methods from the most relevant aspects.

The study found that the traditional DCF methods have crucial shortcomings that require an options-based method to be applied along with them to efficiently value managerial flexibility under uncertainty. Decision-tree analysis was found to quickly become too cumbersome when complexity increases, and to require a lot of subjective forecasting about future states of nature. While suboptimal for real options valuation, it can be very useful by providing management with a transparent investment plan and by assisting in identifying the investment's embedded real options.

Derived for financial options, Black-Scholes and its extensions make too restrictive assumptions to be used in most real option problems. It also stumbles when interaction between multiple real options is added. However, it may be used as a boundary to guide other valuation methods. The binomial models allowed for increasing complexity and more relaxed assumptions than Black-Scholes, but this resulted in a complex tree structure, slower valuation and a rapidly increasing number of inputs. The binomial model was found to be a great model for simple, single real option valuation.

Monte Carlo simulation is a very flexible method, today capable of solving the most complex problems as long as the user has the required technical expertise and tools. It has to be noted that simulation usually makes the valuation process significantly more time-consuming, and thus it should not be immediately applied to every problem, but rather used when the simple methods aren't sufficient.

The two practitioner-oriented methods provided a different approach by valuing the complete investment as a real option. DM method, based on Monte Carlo simulation, was found to be better suited for complex problems out of the two, with the trade-off being slower computation times and technical requirements that come with simulation. The fuzzy pay-off model is significantly easier to use, and sufficiently precise if the problem structure isn't overly complicated. The utilization of management's scenarios as a fuzzy number makes it very user-friendly, and captures humane inaccuracy better than traditional probability theory.

It is apparent that there is no clear answer for which valuation method is always optimal, as it is highly dependent on the attributes of the problem and the desired outcome. As Brealey et al. (2020) said, sometimes an approximate answer now is more useful than a perfect answer later. Regardless, the results of the study should be useful for determining the best available method after considering the attributes affecting the valuation problem in question. Ultimately, by enhancing the understanding of real options valuation, I hope the approach can be more widely applied to improve investment processes in the corporate world.

There are some aspects of real options valuation that would benefit from further study. The effects of competitive interaction on real option value are recognized in the literature (see for example Smit & Trigeorgis, 2004), but aren't well represented in the currently available valuation methods. Most of the methods implicitly make an assumption of the company operating in a vacuum, not considering how the real options affect the competitive environment and vice versa, leaving the user responsible for modifying the methods to include the strategic aspects.

This study was limited to the theoretical side of real options valuation, but an incorporation of the practical side through numerical real-world case examples could be greatly beneficial for a more thorough comparison. While there are plenty of individual studies on applying real options valuation to real-world problems, most apply at most two different methods. Comparing all of the methods, especially the newer approaches, in various different case-example settings could prove beneficial in identifying the best practices and features for real-world problems, and thus for developing even more efficient methods.

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