

# **A Simple 2-sector CGE Model for Cambodia with Excel Implementation**

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## Introduction

This brief note describes a simple two-sector single country CGE model of Cambodia and its implementation in Excel. The base SAM is derived from Version 5.0 of the GTAP data set. The first section describes the specification for the single country model. It contains many of the structural features of the GTAP data set, though with a simplified trade structure.

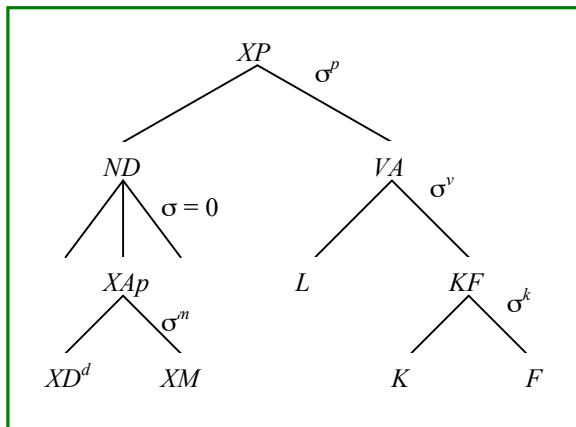
The second section describes the implementation this model in an Excel worksheet. The Excel worksheet contains data for the 57-sector version of the GTAP SAM for Cambodia. An aggregation feature is contained in the worksheet which enables the user to aggregate to any 2-sector SAM.

## Model Specification

### Production

Production,  $XP$ , is modeled as a series of nested CES functions which determines the substitution and complementarity relations across the different inputs into production (figure 1).

Figure 1: CES Production Nest



- $XP$ : Output
- $ND$ : Aggregate intermediate demand
- $VA$ : Value added bundle
- $L$ : Demand for labor
- $KF$ : Capital/sector specific capital bundle
- $K$ : Demand for capital
- $F$ : Demand for sector specific factor
- $XAp$ : Input/output matrix (at the Armington level)
- $XD^d$ : Domestic demand for domestic goods
- $XM$ : Demand for imports
- $\sigma^p$ : Top level substitution elasticity ( $ND$  and  $VA$ )
- $\sigma^v$ : Substitution elasticity between  $L$  and  $KF$
- $\sigma^k$ : Substitution elasticity between  $K$  and  $F$
- $\sigma^m$ : Armington elasticity

The top nest determines demand for an aggregate bundle of intermediate goods,  $ND$ , and the value added bundle,  $VA$ . The relevant prices of these two bundles are  $PND$  and  $PVA$ , respectively. At this level the CES cost function will determine the final unit cost of production,  $PX$ . Equations (1) and (2) reflect the reduced form CES demand functions, and equation (3) determines the unit cost of production.<sup>1</sup>

$$(1) \quad ND_i = \alpha_i^{nd} \left( \frac{PX_i}{PND_i} \right)^{\sigma_i^p} XP_i$$

<sup>1</sup> The model specification is written in its most general form. The indices  $i$  and  $j$  refer to sectors. Summation signs only refer to the sectoral index and it is implicitly assumed that the summation is from 1 to  $N$ , where  $N$  is the number of sectors. A subscript of 0 refers to an initial value.

$$(2) \quad VA_i = \alpha_i^{va} \left( \frac{PX_i}{PVA_i} \right)^{\sigma_i^p} XP_i$$

$$(3) \quad PX_i = \left[ \alpha_i^{nd} PND_i^{1-\sigma_i^p} + \alpha_i^{va} PVA_i^{1-\sigma_i^p} \right]^{1/(1-\sigma_i^p)}$$

where the substitution elasticity (the *ND-VA* substitution) is given by  $\sigma^p$ .

Assuming perfectly competitive markets, the output price,  $PP$ , is equal to the unit cost of production multiplied by an *ad valorem* production tax.

$$(4) \quad PP_i = (1 + \tau_i^p) PVA_i$$

The value added bundle is composed of three factors: labor ( $L$ ), capital ( $K$ ), and a sector-specific factor ( $F$ ). It is also decomposed using nested CES functions. At the top level, labor is combined with a capital-fixed factor bundle ( $KF$ ). And at the next level, the  $KF$  bundle is decomposed into capital on the one hand and the fixed factor on the other hand. Equations (5) and (6) determine demand for labor and the  $KF$  bundle, respectively, where the relevant prices are  $W$ , the wage rate, and  $PKF$ , the price of the  $KF$  bundle. The substitution elasticity is given by  $\sigma^v$ . Equation (7) determines the price of the value added bundle using the CES cost function. Note that the labor demand function is expressed in efficiency units, where the parameter  $\lambda^l$  incorporates (potentially sector-specific) changes to labor productivity. Note that the wage rate is not sector specific. The model explicitly assumes that labor is fully mobile across sectors and hence there is a uniform economy-wide wage rate.

$$(5) \quad L_i^d = \alpha_i^l (\lambda_i^l)^{\sigma_i^v - 1} \left( \frac{PVA_i}{W} \right)^{\sigma_i^v} VA_i$$

$$(6) \quad KF_i = \alpha_i^{kf} \left( \frac{PX_i}{PKF_i} \right)^{\sigma_i^v} VA_i$$

$$(7) \quad PVA_i = \left[ \alpha_i^l \left( \frac{W}{\lambda_i^l} \right)^{1-\sigma_i^v} + \alpha_i^{kf} PKF_i^{1-\sigma_i^v} \right]^{1/(1-\sigma_i^v)}$$

Equations (8) and (9) express the decomposition of the  $KF$  bundle, where the parameters  $\lambda^k$  and  $\lambda^f$  incorporate productivity changes for capital and the sector specific factor respectively, and the substitution elasticity is given by  $\sigma^k$ . Equation (10) determines the price of the  $KF$  bundle.

$$(8) \quad K_i^d = \alpha_i^k (\lambda_i^k)^{\sigma_i^k - 1} \left( \frac{PKF_i}{R_i} \right)^{\sigma_i^k} KF_i$$

$$(9) \quad F_i^d = \alpha_i^f (\lambda_i^f)^{\sigma_i^k - 1} \left( \frac{PKF_i}{PF_i} \right)^{\sigma_i^k} KF_i$$

$$(10) \quad PKF_i = \left[ \alpha_i^k \left( \frac{R_i}{\lambda_i^k} \right)^{1-\sigma_i^k} + \alpha_i^f \left( \frac{PF_i}{\lambda_i^f} \right)^{1-\sigma_i^k} \right]^{1/(1-\sigma_i^k)}$$

The left-hand most branch of the nest, aggregate intermediate demand ( $ND$ ), is decomposed into the input-output matrix of the production side. A simple Leontief structure is assumed, therefore there is no substitution across intermediate inputs. Equation (11) determines intermediate demand for goods and services,  $XAp$ .<sup>2</sup> Finally, equation (12) determines the price of aggregate intermediate demand. Given the assumption of the Leontief technology, it is equal to the weighted sum of the tax inclusive Armington prices, where the weights are given by the Leontief share coefficients.

$$(11) \quad XAp_{ij} = a_{ij}ND_j$$

$$(12) \quad PND_j = \sum_i a_{ij} (1 + \tau_{ij}^{ip}) PA_i$$

### **Household Income and Final Demand**

#### *Consumption*

Household income,  $YH$ , is derived from factor income augmented by transfers from the government.<sup>3</sup> Disposable income,  $YD$ , is equal to after-tax household income adjusted by depreciation.

$$(13) \quad YH = \sum_i [WL_i^d + R_iK_i^d + PF_iF_i^d] + P.TR_h^g$$

$$(14) \quad YD = (1 - \kappa) YH - DeprY$$

Consumer demand is modeled using the extended linear expenditure system (ELES) which is similar to the (LES), but incorporates household saving into the consumer's objective function. Equation (15) specifies consumer demand for the Armington good,  $XAc$ . It is the sum of two components. The first component,  $\theta$ , is the so-called *subsistence minima*. The second is a share,  $\mu$ , of *supernumerary income*,  $Y^*$ , which is residual income after aggregate expenditures on the subsistence minima. Supernumerary income is defined in equation (16). Equation (17) defines household saving by residual. Equation (18) defines the depreciation allowance.

$$(15) \quad XAc_i = \theta_i + \frac{\mu_i Y^*}{(1 + \tau_i^{ic}) PA_i}$$

$$(16) \quad Y^* = YD - \sum_j (1 + \tau_j^{ic}) PA_j \theta_j$$

$$(17) \quad S^h = YD - \sum_i (1 + \tau_i^{ic}) PA_i XAc_i$$

$$(18) \quad DeprY = P.DeprY_0$$

<sup>2</sup> All domestic demand components are expressed as Armington goods. The suffix  $p$  is used to refer to production demand,  $c$  for private consumption,  $g$  for public consumption, and  $i$  for investment demand.

<sup>3</sup> Exogenous transfers are multiplied by a price index to insure model price homogeneity. The variable  $P$  is the GDP price deflator (at factor cost).

### Government

The volume of aggregate government expenditures,  $XG$ , is fixed. The government is assumed to have a CES expenditure function (potentially with a zero elasticity). Equation (19) determines the volume of aggregate government expenditures. Equation (20) specifies sectoral government demand,  $XAg_i$ . And equation (21) determines the government expenditure price,  $PG$ .

$$(19) \quad XG = XG_0$$

$$(20) \quad XAg_i = \alpha_i^g \left( \frac{PG}{(1 + \tau_i^{ig})PA_i} \right)^{\sigma^g} XG$$

$$(21) \quad PG = \left[ \sum_i \alpha_i^g \left[ (1 + \tau_i^{ig})PA_i \right]^{1-\sigma^g} \right]^{1/(1-\sigma^g)}$$

### Investment

Investment is savings determined. The value of domestic investment is identically equal to the value of domestic savings augmented (or diminished) by the level of foreign savings. The volume of aggregate investment is given by  $XI$ , and the investment price deflator is given by  $PI$ . Equation (22) represents the investment-savings closure rule, with public savings given by  $S^g$ , foreign savings given by  $S^f$ , and the  $ER$  representing the exchange rate which will be discussed below. Similar to government expenditures, a CES expenditure function is assumed to allocate aggregate investment into sectoral demand,  $XAi$ .

$$(22) \quad PI.XI = S^h + S^g + ER.S^f + DeprY$$

$$(23) \quad XAi_i = \alpha_i^i \left( \frac{PI}{(1 + \tau_i^{ii})PA_i} \right)^{\sigma^i} XI$$

$$(24) \quad PI = \left[ \sum_i \alpha_i^i \left[ (1 + \tau_i^{ii})PA_i \right]^{1-\sigma^i} \right]^{1/(1-\sigma^i)}$$

### Trade Volumes

Through now, domestic demand has been determined at the so-called Armington level. This model assumes that there is a single Armington agent who allocates aggregate demand into two components: demand for goods produced domestically and imports.<sup>4</sup> Given the uniformity in preference, Armington demand is aggregated across all domestic agents into a single variable,  $XA$ , which is allocated to domestic goods,  $XD^d$ , and to imports,  $XM$ . Equation (25) determines aggregate Armington demand. Equation (26) is the reduced form demand for domestic goods using a CES preference function with a substitution elasticity of  $\sigma^m$ . Equation (26) determines the demand for imports. Finally, equation (27) expresses the aggregate Armington price,  $PA$ , which is the CES aggregation of the domestic price,  $PD$ , and the import price,  $PM$  (which is tariff inclusive).

$$(25) \quad XA_i = \sum_j XAp_{ij} + XAc_i + XAg_i + XAi_i$$

<sup>4</sup> The GTAP data set allows implementation of agent-specific Armington preferences. This specification, though richer, would significantly increase the dimensions of the model.

$$(26) \quad XD_i^d = \alpha_i^d \left( \frac{PA_i}{PD_i} \right)^{\sigma_i^m} XA_i$$

$$(27) \quad XM_i = \alpha_i^m \left( \frac{PA_i}{PM_i} \right)^{\sigma_i^m} XA_i$$

$$(28) \quad PA_i = \left[ \alpha_i^d PD_i^{1-\sigma_i^m} + \alpha_i^m PM_i^{1-\sigma_i^m} \right]^{1/(1-\sigma_i^m)}$$

It is typical to treat the export supply decision in a symmetric fashion using a CET transformation function. Thus a producer has the capacity to supply domestic and export markets, but the supply decision is constrained by a transformation frontier, where the transformation elasticity determines the degree to which suppliers can switch from one market to the other as a function of relative prices. At one extreme, the transformation elasticity is zero and the markets will be supplied in constant proportions of output. At the other extreme, the transformation elasticity is infinite, and suppliers can seamlessly switch from one market to the other. In the case of the latter, goods to each market are uniform and the law of one price holds. The equations below are formulated for all possible cases.

With  $XP$  representing aggregate output, the component supplied to the domestic market is  $XD^s$ , and the component allocated to foreign markets is  $ES$ . Equations (29) and (30) specify the allocation decision. When the transformation elasticity is finite, equations (29) and (30) reflect the reduced form CET supply functions, where the transformation elasticity is given by  $\sigma^x$ . If the transformation elasticity is infinite, the supply functions are replaced with the law-of-one-price conditions. Equation (31) in fact represents an equilibrium condition in both cases. In the first case, with a finite transformation elasticity, aggregate supply equals the aggregation of supply across both markets, using the CET aggregation function. Since it is equivalent to the CET revenue function (the CET dual), equation (31) uses the dual formulation (which tends to have better numerical properties).<sup>5</sup> With an infinite elasticity, aggregate supply is identically equal to the sum of supply to the individual markets.

$$(29) \quad \begin{cases} XD_i^s = \gamma_i^d \left( \frac{PD_i}{PP_i} \right)^{\sigma_i^x} XP_i & \text{if } \sigma_i^x \neq \infty \\ PD_i = PP_i & \text{if } \sigma_i^x = \infty \end{cases}$$

$$(30) \quad \begin{cases} ES_i = \gamma_i^e \left( \frac{PE_i}{PP_i} \right)^{\sigma_i^x} XP_i & \text{if } \sigma_i^x \neq \infty \\ PE_i = PP_i & \text{if } \sigma_i^x = \infty \end{cases}$$

<sup>5</sup> The CET primal expression is given by the following formula:

$$XP_i = \left[ g_i^d (XD_i^s)^{\nu_i} + g_i^e (ES_i)^{\nu_i} \right]^{1/\nu_i}$$

where the following relations hold:

$$\nu_i = (\sigma_i^x + 1) / \sigma_i^x \quad \gamma_i^d = (g_i^d)^{-\sigma_i^x} \quad \gamma_i^e = (g_i^e)^{-\sigma_i^x}$$

$$(31) \quad \begin{cases} PP_i = \left[ \gamma_i^d PD_i^{1+\sigma_i^x} + \gamma_i^e PE_i^{1+\sigma_i^x} \right]^{1/(1+\sigma_i^x)} & \text{if } \sigma_i^x \neq \infty \\ XP_i = XD_i^s + ES_i & \text{if } \sigma_i^x = \infty \end{cases}$$

Export demand is allowed to respond to price signals, i.e. the small country assumption does not necessarily hold for export markets. Equation (32) determines export demand,  $ED$ , using a constant elasticity demand function (with a demand elasticity of  $\varepsilon$ ), where  $WPE^*$  represents a world price index which is exogenous, and  $WPE$  is the world export price of domestic exports, i.e. the FOB price (in international currency units). If the small country assumption holds, the world export price is constant, and the world is assumed to be able to absorb any quantity of exports at the given price.

$$(32) \quad \begin{cases} ED_i = \alpha_i^e \left( \frac{WPE_i^*}{WPE_i} \right)^{\varepsilon_i} & \text{if } \varepsilon_i \neq \infty \\ WPE_i = WPE_i^* & \text{if } \varepsilon_i = \infty \end{cases}$$

### **Trade Prices**

World import prices,  $WPM$ , are given and are converted to domestic import prices,  $PM$ , using the exchange rate,  $ER$ , and applying the tariff rates,  $\tau^m$ . World export prices will be determined by an equilibrium equation (see below) in the case of a finite export demand elasticity, or are given otherwise. They are converted to domestic export prices using the exchange rate and adjusted for export taxes/subsidies,  $\tau^e$  (which are applied to the producer price, not the world price).

$$(33) \quad PM_i = ER(1 + \tau_i^m)WPM_i$$

$$(34) \quad PE_i = ER.WPE_i / (1 + \tau_i^e)$$

### **Goods equilibrium**

In this model, there are two goods market<sup>6</sup>, the domestic market for domestic production, and the export market. The domestic price of domestic goods,  $PD$ , is determined by the equilibrium expressed in equation (35).<sup>7</sup> The world price of domestic exports is determined by the equilibrium expressed in equation (36).<sup>8</sup>

$$(35) \quad XD_i^d = XD_i^s$$

$$(36) \quad ED_i = ES_i$$

<sup>6</sup> The small country assumption is assumed for imports, and thus there is no equilibrating price mechanism on this market.

<sup>7</sup> If the transformation elasticity is infinite, these equations trivially set supply equal to demand, and the price is determined via the law-of-one-price. If further, the small country assumption holds, all producer prices will equal the prevailing world export price (adjusted by the export tax/subsidy).

<sup>8</sup> In the case of an infinite export demand elasticity, equation (36) trivially sets world export demand to domestic export supply.



### Factor market equilibrium

There are three factor markets which will be dealt with separately. The labor market is assumed to clear at the national level, with labor perfectly mobile across sectors. There is a uniform wage rate which equilibrates supply and demand. Supply is allowed to be a function of the real wage. The labor supply function is given in equation (37), with a supply elasticity of  $\omega^l$ . Equation (38) determines the equilibrating wage rate.

$$(37) \quad L^s = \chi^l \left( \frac{W}{P} \right)^{\omega^l}$$

$$(38) \quad L^s = \sum_i L_i^d$$

The capital market is modeled using a CET supply allocation function. Aggregate capital is allocated across sectors according to sector-specific rates of return. Unless the transformation elasticity is infinite, the allocation is imperfect and sectoral rates of return will not be uniform. In the extreme, with a zero transformation elasticity, capital would be completely sector specific. Equation (39) sets the aggregate capital stock,  $TK^s$ . Equation (40) determines its sectoral allocation,  $K_i^s$ , using the reduced form CET supply functions. (With an infinite elasticity, the law of one price holds.) Equation (41) determines the average rate of return,  $TR$ , using the CET dual price aggregator. (With an infinite elasticity, the aggregate rate of return is determined by an equilibrium condition.) Equation (42) determines the equilibrium rate of return specific to each sector. The transformation elasticity is given by  $\omega^k$ .

$$(39) \quad TK^s = TK_0^s$$

$$(40) \quad \begin{cases} K_i^s = \chi_i^k \left( \frac{R_i}{TR} \right)^{\omega^k} TK^s & \text{if } \omega^k \neq \infty \\ R_i = TR & \text{if } \omega^k = \infty \end{cases}$$

$$(41) \quad \begin{cases} TR = \left[ \sum_i \chi_i^k (R_i)^{1+\omega^k} \right]^{1/(1+\omega^k)} & \text{if } \omega^k \neq \infty \\ TK^s = \sum_i K_i^d & \text{if } \omega^k = \infty \end{cases}$$

$$(42) \quad K_i^d = K_i^s$$

The sector specific factor is modeled using a constant elasticity supply function (for each sector). Equation (43) reflects sectoral supply,  $F_i^s$ . Equation (44) is the equilibrium condition determining the equilibrating factor price,  $PF$ .

$$(43) \quad F_i^s = \chi_i^f \left( \frac{PF_i}{P} \right)^{\omega^f}$$

$$(44) \quad F_i^d = F_i^s$$

### Closure

Investment-savings closure has been discussed above. Fiscal closure usually has a fixed government fiscal balance with the direct tax rate,  $\kappa$ , adjusting to achieve the fiscal target. The components of government revenue are broken out into three equations. The first, (45), determines revenues generated by indirect taxes. The second, (46), determines revenues generated by the trade distortions. The third determines aggregate government revenues. Equation (47) is the fiscal balance equation determining the value of government savings. Equation (48) determines real government savings. Finally, equation (49) reflects the fiscal closure rule.

$$(45) \quad ITaxY = \sum_i PA_i \left( \sum_j \tau_{ij}^{ip} XAp_{ij} + \tau_i^{ic} XAc_i + \tau_i^{ig} XAg_i + \tau_i^{ii} XAi_i \right)$$

$$(46) \quad TradeY = ER \sum_i \tau_i^m WPM_i XM_i + \sum_i \tau_i^e PE_i ES_i$$

$$(47) \quad GRev = ITaxY + TradeY + \sum_i \tau_i^p PX_i XP_i + \kappa.YH$$

$$(48) \quad S^g = GRev - PG.XG - P.TR_g^h$$

$$(49) \quad RS^g = S^g / P$$

$$(50) \quad RS^g = RS_0^g$$

External closure assumes that the trade balance (equal to the current account balance in this model) is fixed. Or equivalently, that foreign capital flows are exogenous. The real exchange rate is the equilibrating mechanism. For example, removal of tariffs, which would tend to increase import demand, needs to be met by increasing exports. A real exchange rate depreciation (as measured by the GDP deflator for example), would tend to increase exports. Equation (51) reflects the external closure rule (note that it is expressed in terms of the foreign currency).

$$(51) \quad \sum_i WPE_i E_i^d + S^f = \sum_i WPM_i XM_i$$

Any price can be chosen as the model numéraire. It is perhaps natural to use the exchange rate,  $ER$ . The exchange rate is not the normal nominal exchange rate of macroeconomic models. Instead, it reflects a price index which evaluates a foreign bundle, for example foreign savings or imports, in terms of a domestic bundle.

The remaining equations are used to calculate the domestic price index. Equation (52) defines nominal GDP at factor cost,  $GDPFC$ . Equation (53) defines real GDP at factor cost,  $RGDP$ . Finally, equation (54) determines the domestic GDP price deflator.

$$(52) \quad GDPFC = \sum_i WL_i^d + R_i K_i^d + PF_i F_i^d$$

$$(53) \quad RGDP = \sum_i W_0 \lambda_i^l L_i^d + R_{i,0} \lambda_i^k K_i^d + PF_{i,0} \lambda_i^f F_i^d$$

$$(54) \quad P = GDPFC / RGDP$$

The model specified in this section has  $N(30+N)+23$  equations and  $N(30+N)+22$  endogenous variables. Invoking Walras' Law, one equation is redundant. It can be proven that the balance of payments equation can be derived as a linear combination of other equations in the model, and it will be dropped from the model specification. It can be calculated at each equilibrium to verify that the model is consistent. A two-sector model has 86 equations. The next section describes an implementation of the two-sector version of this model in Excel.

## Excel Implementation for Cambodia

### Introduction

The single country model described above has been implemented in Excel<sup>®</sup> using the Thai SAM available in GTAP (release 5.0). The implemented model has two sectors which are aggregated up from the 57 sector GTAP database. The two-sector version of the SAM is built up from the fully articulated 57-sector SAM available in GTAP. The next sections describe each of the components of the Excel worksheet and how to simulate the single country CGE model. Note that many cells in the Excel worksheet contain a small red triangle in the upper right hand corner. The triangle indicates that the cell contains a comment. Passing the cursor over the cell and holding it there will open a small text box containing the cell comment.

### The Base Social Accounting Matrix (SAM)

The base 57-sector SAM is stored in the worksheet BaseSAM. It would normally not be modified unless it was overwritten with a new SAM. In such a case, the new SAM should normally conform to the same structure as the original SAM or some work would be involved to make the rest of the worksheets consistent with the initial conception.

The user defines a two-sector aggregation using the *aggregation* matrix in the worksheet AggMat. An aggregation matrix is a matrix of 0's and 1's. The user should only modify the 0's and 1's in the first two columns of the aggregation matrix and the first 57 rows. This is the part of the aggregation matrix which maps the 57 sectors to the 2 sectors needed for the model. This 57 x 2 matrix must contain only 0's and 1's. Each of the 57 sectors must be assigned to one and only one of the 2 sectors, i.e. the sum across the two columns must be 0, and each column must have at least one cell with a 1 in it, i.e. the sum down the column must be at least 1. The original file comes with a two sector aggregation composed of agriculture and other goods and services. The user should also change the two labels at the top of the aggregation matrix to match the intended 2-sector aggregation.

The worksheet SAM contains two SAMs. The first is the initial base SAM which is the aggregated SAM. It is created by the following formula:

```
=mmult (mmult (transpose (aggmat) , BaseSAM) , aggmat)
```

which allows for the aggregation of the base 57-sector SAM into the base 2-sector SAM. The second SAM is the one resulting from a model simulation. This will be described further below.

The base SAMs are evaluated in millions of 1997 U.S. dollars. This may be inappropriate from a numerical point of view in terms of model convergence. A scaling factor has been introduced in the *SAM* worksheet which will convert the base units into more appropriate units. A good rule of thumb is that GDP should be of the order of magnitude of 10,000 or less. The scale variable is located in cell A47.

The base SAM, combined with some key parameter values, are the basis of calibrating the other parameters of the CGE model, defined below.

### ***The Model***

The model is implemented in the *Model* worksheet. The worksheet is divided into three main blocks, plus some auxiliary blocks. The three main blocks are: a) the block of endogenous variables; b) the equations block; and c) the block of exogenous variables. The two blocks of variables each contain four columns. The first column contains the name of the variables. The second column contains the initial (or base) values for the variables. The third column contains the simulation values of the variables. In the case of the exogenous variables, these are user determined. In the case of the endogenous variables, the Excel *Solver* will solve the model equations and replace the values of the endogenous variables with the model solution. The fourth column determines the percentage change from the base levels of the variables.

The equations block has two columns. The first column contains the relevant equation name. The second column contains a single equation of the model. All equations are written such that when an equilibrium has been solved the value of the equation is 0, i.e.  $y = f(x)$  is written as  $y - f(x) = 0$ . Any deviations from 0 indicate one of the following conditions: a) the equation has been mis-specified; b) a parameter has been mis-calibrated; c) an initial variable has been miscalculated; or d) the model solution is not an equilibrium solution.

The endogenous variables and equations have been written such that each variable is lined up with its “own” equation. Of course this is a heuristic device since all variables depend on all equations, but it is useful to make sure the model is square, i.e. that the number of endogenous variables matches the number of independent equations.

Another feature of the model is that all variables, parameters, and equations have a name, i.e. formulas never refer to individual cells, but always to named cells.<sup>9</sup> This reduces the number of errors and makes auditing of formulas more straightforward. It also facilitates reading and comprehension of the model. Each variable name corresponds to a specific mnemonic related to the variable as described in the model specification above. Since the model is multi-sectoral (and Excel has no indexing mechanism), each sectoral variable has a suffix. These suffixes are *\_a* and *\_o* respectively, for *agriculture* and the *other* sector. The input-output matrix (and its related variables) are doubly indexed using *\_aa*, *\_ao*, *\_oa*, and *\_aa*. All variables have two values (and names). The first has a suffix of zero indicating it contains the variable’s initial value. The second is without the zero suffix. The initial value will never change unless a new aggregation is specified or the user changes an initial price index (see below). Variable names can be seen by selecting a cell and looking at the name of the cell in the range name window (below the font window).

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<sup>9</sup> There is an exception to this rule. When initializing variables and parameters based on values from the base SAMs, the formulas make specific calls to cell references in the SAM.

## ***Initialization of variables***

Variables can be divided into three categories. First, there are the price indices. In most CGE models, prices are typically represented as price indices, not as monetary values. They can therefore be initialized to any level, with 1 representing the most natural initialization value.<sup>10</sup> Only base prices are initialized, such as factor prices, prices of composite goods, and producer prices. Other prices are normally equal to base prices adjusted by some price wedge such as a tax/subsidy or a trade margin, or by model specification (for example, domestic export prices are equal world export prices times the exchange rate). In this model the base prices include factor prices, producer prices (aggregate, domestic, and export), production cost prices, the Armington price, government and investment price deflators, the world price of imports, the price of world exports, and the exchange rate. These basic prices are visible in the green-shaded cells and can be modified by the user. The model equations should not be affected by changes in the base price indices. (A good test of the model calibration is to change the base price indices.)

A second set of variables are derived from the base SAM. Since the base SAM is expressed in value terms, and most variables being initialized are volumes, the SAM values will normally be divided by an appropriate price variable. For example, to set the volume of labor, labor remuneration will be divided by the base wage level. The final set of initialized values may be derived from other initial variables using accounting formulas or equations from the model specification. Except for the basic price indices, users should not modify any of the other cells in the columns representing the initial values.

## ***Key parameters and model calibration***

The model has four sets of key parameters: production elasticities, final demand elasticities, trade elasticities, and supply elasticities. Key parameters are user-determined. All other model parameters are calibrated such that the model represents an equilibrium when expressed in base levels of the variables. Table 1 lists the key parameters.

The key parameters are defined in the *Parameters* worksheet. They are in the pink-shaded cells and can be modified by the user. (A value of 1 should be avoided for the CES elasticities since the CES dual price functions are not defined for substitution elasticities of 1. To emulate a Cobb-Douglas function, use the value 0.99 or 1.01). The other parameters in the *Parameters* worksheet are calibrated. They are CES share parameters, shift parameters (for the supply and demand constant elasticity functions), and the consumer demand parameters. The calibration formulas use the model equations to calculate these parameters, using the initial values of the variables and the key elasticities as inputs. For example, the labor demand function is specified as:

$$L^d = \alpha^l \left( \frac{PVA}{W} \right)^\sigma XP$$

The share parameter will be calculated by solving the above equation for the  $\alpha^l$  parameter and inserting the base values for the variables:

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<sup>10</sup> There can be various reasons for initializing prices to actual base year nominal levels. For example, in the case of energy/climate change models, the absolute price of energy can be relevant when introducing new alternative energies (which are typically identified by their cost of introduction). In such cases, it is possible to initialize the price of oil and other sources of conventional energy to their actual nominal values. In this case, volumes will present actual physical units, for example barrels of oil, or tons of coal.

$$\alpha^l = \left( \frac{L_0^d}{XP_0} \right) \left( \frac{W_0}{PVA_0} \right)^\sigma$$

(Note that in formulating the calibration formulas, it is easy to use the value of the variables rather than their initial values (for example,  $Ld\_a$ , instead of  $Ld\_a0$ ). This can lead to problems when solving the model since the calibrated parameter depends on an endogenous variable and thus becomes endogenous.)

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**Table 1: Key Parameters**

*Production*

$\sigma^p$	<b>sigmap</b>	Substitution elasticity between total intermediate demand, $ND$ , and value added, $VA$ .
$\sigma^v$	<b>sigmav</b>	Substitution elasticity between labor, and the capital-sector specific factor bundle, $KF$ .
$\sigma^k$	<b>sigmak</b>	Substitution elasticity between capital and the sector specific factor.

*Final demand*

$\eta$	<b>eta</b>	Income elasticity. Note that in using the ELES, the income elasticity is only used in calibration of the other parameters of the model. The income elasticity is in fact an endogenous outcome of a model. Also note that there are consistency requirements on the income elasticities. The way calibration of the consumer demand system is constructed, the income elasticity of savings is determined residually to achieve the consistency requirements. Users should choose income elasticities of the two goods which lead to a plausible value of the savings income elasticity.
$\sigma^g$	<b>sigmag</b>	Government expenditure substitution elasticity (across goods).
$\sigma^i$	<b>sigmai</b>	Investment expenditure substitution elasticity (across goods).

*Trade elasticities*

$\sigma^m$	<b>sigmam</b>	Armington elasticity.
$\sigma^x$	<b>sigmax</b>	CET transformation elasticity (between domestic and export supply). Note that an infinite elasticity cannot be specified. Use a high value to approximate the law-of-one-price.
$\varepsilon$	<b>epse</b>	Export demand elasticity. Note that an infinite elasticity cannot be specified. Use a high value to approximate the small-country assumption.

*Supply elasticities*

$\omega^l$	<b>omegal</b>	(Aggregate) labor supply elasticity.
$\omega^k$	<b>omegak</b>	Capital mobility elasticity. 0 emulates sector-specific capital. An infinite value cannot be specified. Use a high value to approximate perfectly mobile capital.
$\omega^f$	<b>sigmak</b>	Sector-specific supply elasticities.

---

**Equations Block**

The equations block is relatively straightforward. Each model equation is formulated with all variables on the left hand side so that at equilibrium the value of the constraint is 0. Equations are written using variable and parameter names, and thus make no references to Excel cells. The region's redundant equation has been chosen to be the balance of payments equation. This expression of Walras' Law is defined at the bottom of the list of endogenous variables. After calibration and a model solution, this formula should be verified to make sure that the model solution is consistent.

## ***Model re-initialization***

After switching to a new region or changing a key model parameter the model solution may no longer be in equilibrium. For convenience, a copy of the model initialization formulas have been saved in a separate area in the *Model* worksheet. A copy of the endogenous variables is bounded by the area O4:O89. The initial exogenous variables are bounded by the area P4:P32. Re-initialization involves copying each one of these areas from this auxiliary storage in the worksheet and pasting into the relevant areas defining the model and its variables. Since this is a repetitive task, a macro has been created which simplifies the initialization procedure. The macro can either be run from the **Tools\Macro\Macro..** options of the menu bar. Choose the **InitMod** macro and click on the **Run** button. Alternatively, simply click on a floating button in the worksheet labeled **Initialize Model**. This button is located around cell H93 (or just below the equations block).

## ***Model simulation***

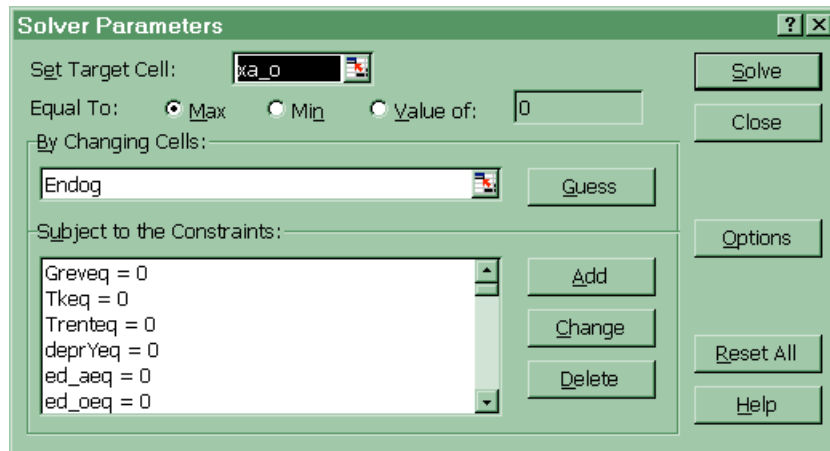
The following describes the various steps to simulate the model.

1. *Region selection.* Select a region from one of the 65 countries/regions in the pull-down menu at the top left of the SAM pivot table in the worksheet *SAM*.
2. *Data consistency check.* Verify that the base SAM is consistent in the *SAM* worksheet.
3. *Parameter choice.* Choose and enter the key parameters in the *Parameters* worksheet.
4. *Model initialization.* Re-initialize model variables. Click on the **Initialize Model** button located near cell H93 in the *Model* worksheet.
5. *Initialization consistency.* Verify that all model equations are consistent, i.e. that they evaluate to zero, or a small number. Verify that all percentage differences in the variables section (Endogenous and Exogenous) evaluate to 0. Verify that the expression of Walras' Law, at the bottom of the Endogenous variables section evaluates to 0 (or a small number).
6. *Model consistency.* Solve the model with no shock. This is simply to test that the model is able to reproduce the base solution. A mis-specified equation will normally lead to non-convergence, or an inconsistent solution. Excel contains a powerful equation solving package known as the *Solver*.<sup>11,12</sup> *Solver* is located under the **Tools\Solver..** option of the menu bar. The following dialog box should appear:

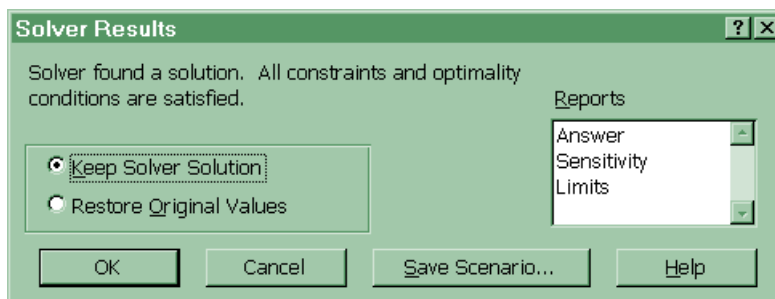
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<sup>11</sup> *Solver* is in fact more than just an equation solver, it is a non-linear optimizing algorithm. It will solve for a maximum (or minimum) of a given objective function, subject to non-linear constraints which can be either equalities or inequalities. The regional CGE model is specified as a square systems of equations. Assuming there is a single solution, the choice of objective function should not affect the final solution (though presumably it could impact on the convergence properties).

<sup>12</sup> The *Solver* algorithm is not necessarily loaded with Excel upon installation. If *Solver* is not installed, restart the Excel setup program, and click on the *Solver* option to have it installed.



All the options have already been entered and the *Solver* is ready to start. There are three main options. The first is the objective function. Since it is basically redundant for this model, it is simply a single endogenous variable, by default, Armington absorption of the *other* good (*xa\_o*). It will be maximized. The second option is the list of endogenous variables. A range name has been designated for the endogenous variables and it is called *Endog*. It corresponds to the area C4:C89 (i.e. it is a column vector with 86 cells). After initializing the model, the range name *Endog*, contains the initialization formulas. After running *Solver* it will contain actual numerical values, i.e. the solution of the model. The third option is the list of constraints. This list contains all 86 equation names which are all constrained to evaluate to zero. The user should not change these options. However, if convergence appears to be a problem, the user may want to modify some of the algorithm's convergence parameters which can be modified in a different dialog box by clicking on the **Options** button of the *Solver* screen. The *Solver* solution algorithm is invoked by clicking on the **Solve** button. The status bar at the bottom of the Excel screen displays (minimal) information on each iteration, including iteration count and value of the objective function. If successful, the solver will display the following dialog box:



To have the *Solver* overwrite the values of the endogenous variables, simply click on the **OK** button. Users can experiment with the other options.

If solution convergence was achieved, the model should have re-produced the base data set (within the limits of the convergence tolerance). All equations should evaluate to 0. The expression of Walras' Law should evaluate to 0. All deviations from initial values should evaluate to 0. A final test is to check the consistency of the resulting SAM.

The *SAM* spreadsheet, contains the solution SAM. The solution SAM is expressed in terms of the model solution. For example, the labor remuneration cell (in agriculture) contains the formula<sup>13</sup>:

<sup>13</sup> The formula is adjusted by the scale variable to make the solution SAM comparable with the initial SAM.



=wage\*ld\_a/scale

If the SAM is not consistent, either the solution is inconsistent, the model has been mis-specified, or the formulas in the SAM have been mis-specified.

7. *Homogeneity consistency.* If this is a new model, it is recommended to check model homogeneity. This involves a perturbation of the model numéraire. If the model is homogeneous in prices, perturbation of the model numéraire should leave all volumes constant, and adjust all prices and value variables by the same percentage amount as the percentage change in the numéraire (i.e. all relative prices remain constant). To check homogeneity, multiply the initial value of the numéraire by some constant (e.g. 1.1 would increase the model numéraire by 10 percent). The expression is contained in cell L23 (also labeled as ER). Enter the following formula, for example:

=er0\*1.1

Initially, the only equations which will be affected by this change are the domestic investment equation, the domestic trade prices, and the tariff revenue equation because these are the only equations where the numéraire (the exchange rate) appears. Invoke *Solver* to find a new solution to the model. If the homogeneity test fails (other than due to the lack of convergence), at least one of the equations has been mis-specified, or there could be a built-in nominal rigidity, such as a fixed nominal wage.<sup>14</sup> If both tests succeed, the model should be re-initialized, and the next step is to run one or more shocks to the model.

8. *Simulation of exogenous shocks.* Assessing the impacts of shocks to the exogenous variables is relatively straightforward. Table 2 lists the exogenous variables which can be modified.

Table 2: List of Exogenous Variables

<i>Variable Name</i>	<i>Description</i>
itp_aa, itp_ao, itp_oa, itp_oo	Indirect tax on intermediate demand
tp_a, tp_o	Output tax
lambdal_a, lambdal_o	Labor productivity
lambdak_a, lambdak_o	Capital productivity
lambdaf_a, lambdaf_o	Sector specific factor productivity
itc_a, itc_o	Indirect tax on private consumption
itg_a, itg_o	Indirect tax on public consumption
iti_a, iti_o	Indirect tax on investment expenditures
er	Exchange rate (model numéraire)
wpebar_a, wpebar_o	Price of world exports
wpm_a, wpm_o	World price of imports (in foreign currency)
tm_a, tm_o	Tariff rates
te_a, te_o	Export taxes/subsidies
savf	Foreign saving (in foreign currency).

<sup>14</sup> Convergence is sometimes achieved, and all consistency tests are valid, and yet the homogeneity test fails. This can sometimes occur because multiple solutions cannot be ruled out. Trying a different percent change to the numéraire may yield the desired result, or else re-scaling the model.

Implementing a shock requires modifying the formula in column L.<sup>15</sup> By default, the no-shock value is written as:

```
=Lambda1_a0*1
```

In other words, the default shocked value is expressed as the initial value (with the 0 suffix) times 1. A shock of x% would be written as:

```
=Lambda1_a0*(1+x/100)
```

For example, a 3% increase in labor productivity in the agriculture sector would be specified as:

```
=lambda1_a0*(1.03)
```

After introducing the modifications to the exogenous variables, *Solver* needs to be invoked to find the new equilibrium solution. If *Solver* converges successfully, the new equilibrium will replace the variables in the range named *Endog*. After each successful simulation run, the standard consistency checks should be verified: all equations evaluate to 0, Walras' expression evaluates to 0, and the solution SAM is consistent.

Two macros have been provided to perform trade reforms scenarios. The first sets import tariffs to 0. It can be invoked by clicking on the **Zero Tariff** button located near cell G98. (It is a good idea to re-initialize the model before each separate shock simulation). The second trade reform simulation sets export taxes/subsidies to 0, in addition to nullifying tariff rates. It can be invoked by clicking on the **Full Trade Reform** button located near cell G103.

9. *Post-simulation assessment.* Sample post-simulation results are expressed at the bottom of the *Model* worksheet. Users are of course free to modify and expand these. The *Model* worksheet also expresses deviations from baseline values of all endogenous and exogenous variables.

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<sup>15</sup> Other shocks can be implemented by directly modifying a model equation. For example, a labor supply shock can be introduced by adjusting the labor supply shift parameter (*als*) in the labor supply equation.