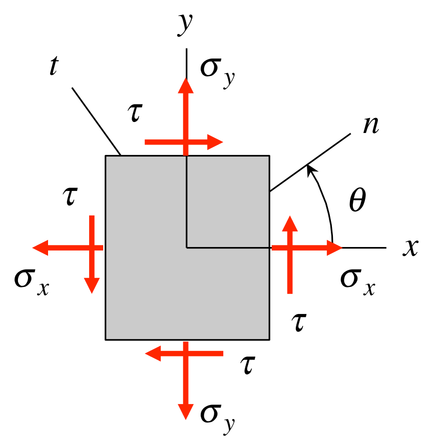
### *Example 13.10*

A state of plane stress at point A on a body is known to be $\left(\sigma_x,\sigma_y,\tau\right)=\left(-10,2,8\right)\,ksi$ .

a) Determine the n-t components of stress corresponding to $\theta=36.87°$ .

b) Draw the Mohr's circle for this state of stress. On your Mohr's circle, clearly indicate: the location of the circle's center, the radius of the circle and the location of the x-axis.

c) Determine the angles $\theta$ at which the shear component of stress is zero. What are the corresponding values of normal stress at these rotations?



### ***Example 13.11***

Consider the three states of stress given below in a), b) and c). For each state of stress, do the following:

- Draw the corresponding *Mohr's circles* (both in-plane and out-of-plane).
- Locate the *x-axis* on the in-plane Mohr's circle.
- Determine the maximum *in-plane* shear stress
- Determine the *absolute* maximum shear stress

a) $\left(\sigma_x, \sigma_y, \tau\right) = \left(-28, -52, -16\right)\ MPa$

b) $\left(\sigma_x, \sigma_y, \tau\right) = \left(60, -60, 80\right)\ MPa$

c) $\left(\sigma_x, \sigma_y, \tau\right) = \left(22, 10, 8\right)\ MPa$

### *Example 13.12*

The Mohr's circle for a state of stress is presented above.

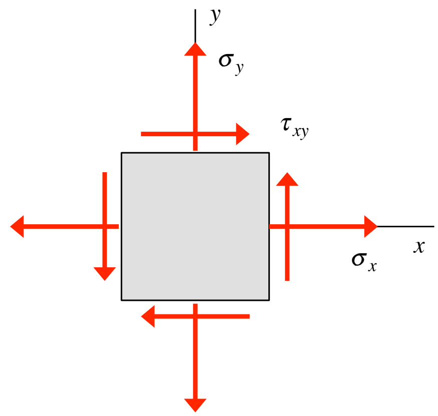
a) Determine the values for $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ for this stress state.

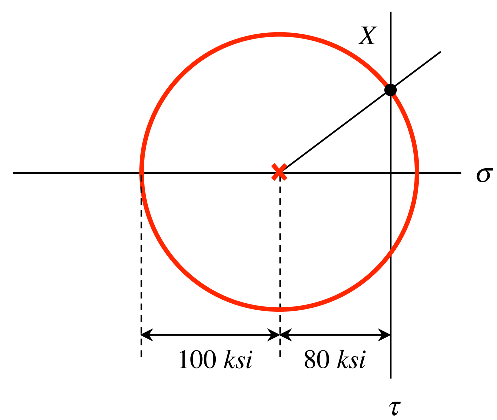
b) What counterclockwise in-plane rotation of the stress element produces the principal stress $\sigma_{P1}$?

c) What counterclockwise in-plane rotation of the stress element produces the principal stress $\sigma_{P2}$?

d) What is the smallest counterclockwise in-plane rotation of the stress element produces $\left|\tau\right|_{max,in-plane}$?

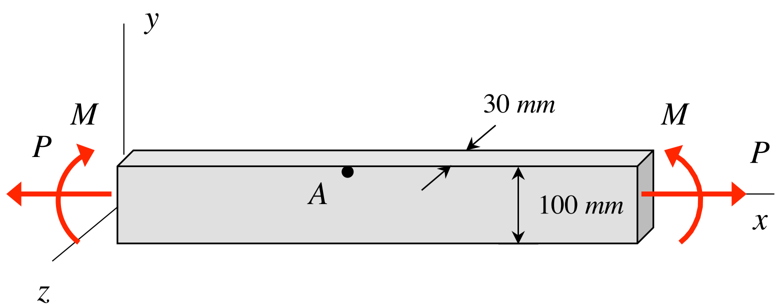
e) Determine the *absolute* maximum shear stress

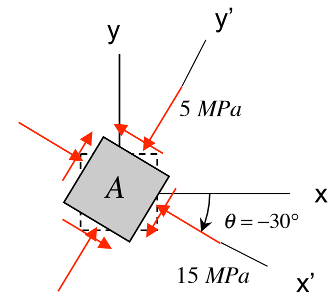


### *Example 14.1*

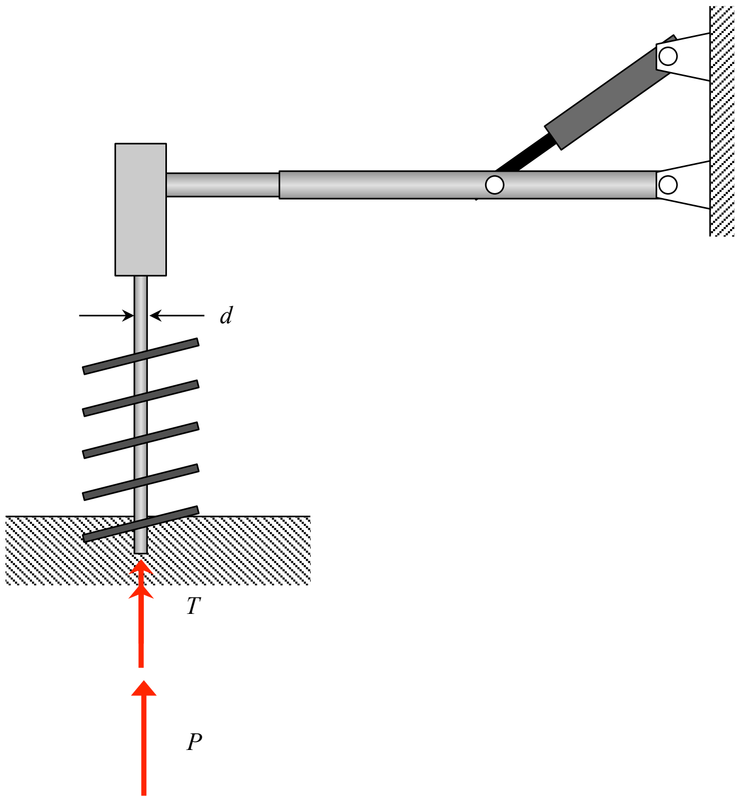
The rectangular bar shown is subjected to bending in the xy plane and, simultaneously, to an axial force P. The state of plane stress at point A at the top edge of the bar is shown below. If the bending moment acting on the bar is known to be $M = 2kN \cdot m$, what is the magnitude of the axial load P?



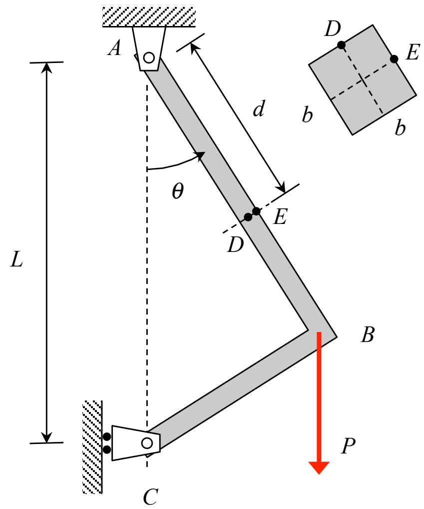
### *Example 14.3*

The auger of a post-hole digger experiences a torque T and an axial load P during its operation. Consider a point on the circular rod section (of diameter d) of the auger. Determine the principal stresses and the maximum shear stress at that point on the outer surface of the rod.



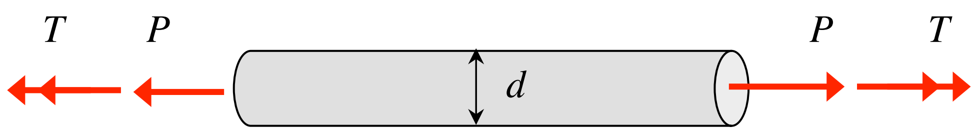
### *Example 14.7*

The L-shaped frame ABC has a square cross section of side dimension b. This frame is supported by a fixed pin at A and by a roller at C. Determine the maximum principal stresses and the maximum in-plane shear stress at points D and E in the cross section at the location at a distance of d from A.



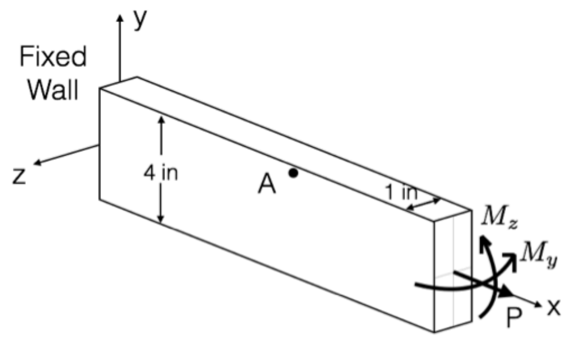
## *Example 14.8*

A solid shaft of diameter $d = 2\ in$ experiences a simultaneous application of an axial load $P = 10\ kips$ and torque $T = 2.5\ kip \cdot in$ Determine the two principal stresses and the maximum in-plane shear stress at any point on the outer surface of the shaft.



## *Example 14.10*

A rectangular bar is fixed on a wall and subjected to the loads shown in the following figure with values P = 5 kips, $M_z$ = 2 kips*in and $M_x$ = 3 kips*in. Plot the stress at A in a properly oriented stress element and find the principal stresses, and maximum shear stress using a Mohr's circle.



***Example 14.13***

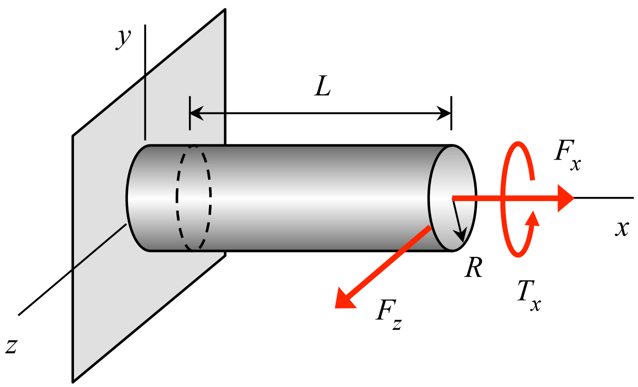
A rod having a radius of $R$ is attached to a fixed wall on its left end, as shown in the figure on the next page. The rod is acted upon by a pair of forces $F_x$ and $F_y$, and a torque $T_x$ at its free end. It is desired to understand the states of stress at points "a", "b" and "c" at a location at a distance $L$ from the free end.

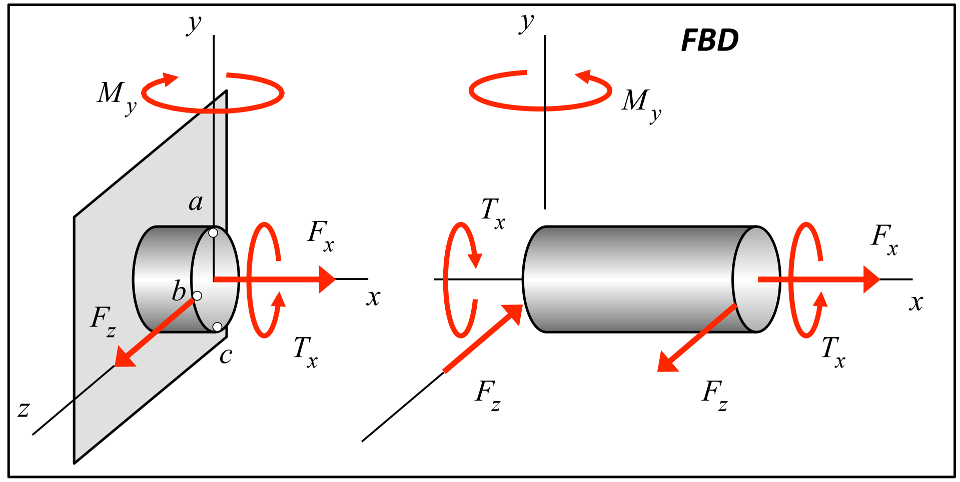
a) Determine the cross-sectional area A, the polar area moment $I_P$ and the second-order area moment $I$ for the shaft.

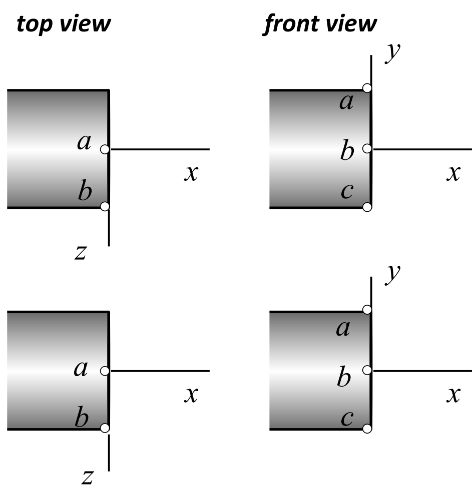
b) Determine the bending moment $M_y$.

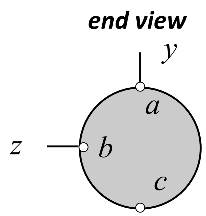
c) Determine the states of stress at locations "a", "b" and "c" on the cross section of the cut. Use the figures provided for drawing the distribution of stresses and the table provided for identifying the components of stress for each of the internal loads at the cross section.

d) Draw the Mohr's circle for the state of stress at point "c". Determine the principal components of stress, the maximum in-plane shear stress and the absolute maximum shear stress for this state of stress. At this step, use the following parameters in your analysis: $F_x = 10kN$, $F_y = 20\ kN$, $T_x = 50kN \cdot m$, $R = 0.10\ m$ and $L = 2m$.

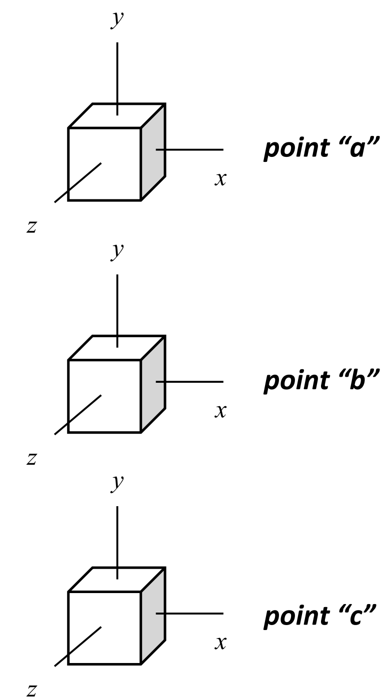



| loading | stress comp. @ "a" | stress comp. @ "b" | stress comp. @ "c" |
|---|---|---|---|



### *Example 14.14*

Consider the structure shown on the next page with loading applied at end H.

f) Determine the internal resultants at location B on section OC.

g) Show the stress distribution due to each internal resultant on the supplied figures.

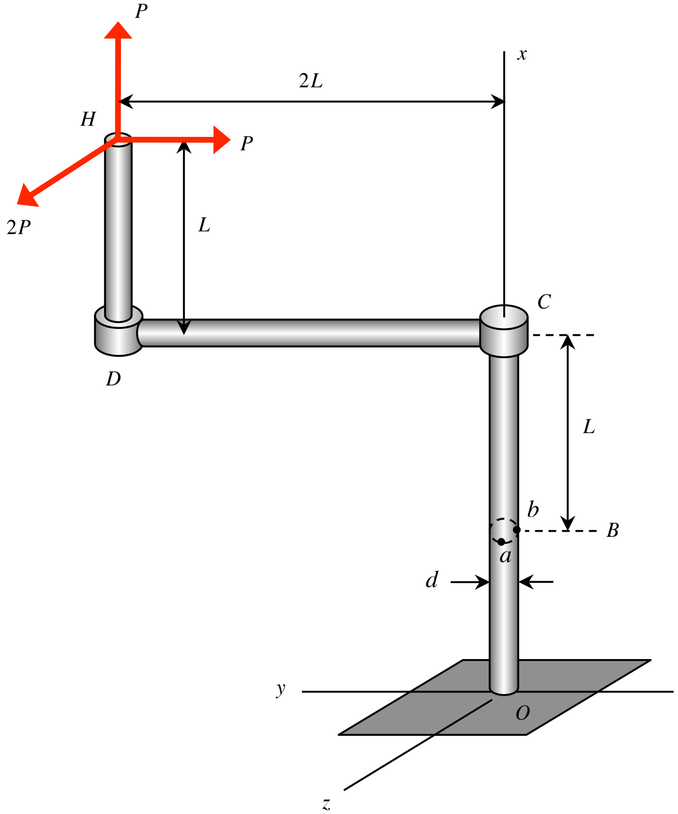
h) Complete the table quantifying the stresses at points "a" and "b".

i) Show the stress components on the stress elements at "a" and "b".

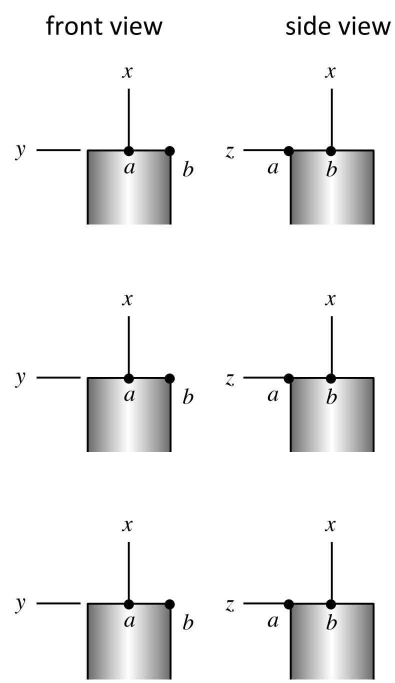
j) Determine the principal components of stress and maximum absolute shear stress at point "b".

k) *BONUS POINTS* (2 points): Explain in words or equations why the maximum in-plane shear stress is equal to the maximum absolute shear stress at point "b", regardless of the loading on the structure at end H.

*Addi.*

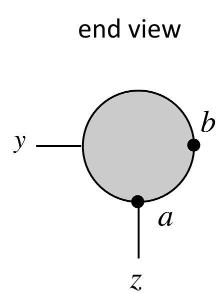


P
2L
x
H
P
2P
L
C
D
L
b
B
a
d
y
O
z

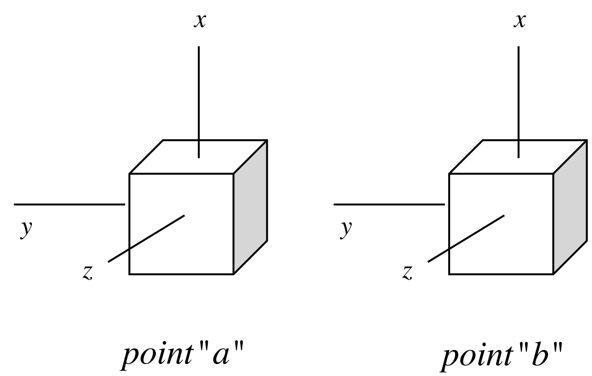



front view
side view
x
y
a
b
z


| load | "a" | "b" |
|---|---|---|



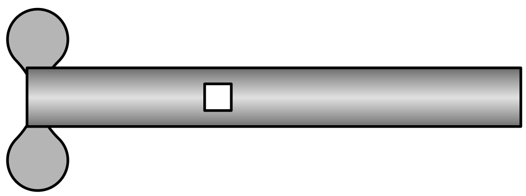
end view
y
b
a
z

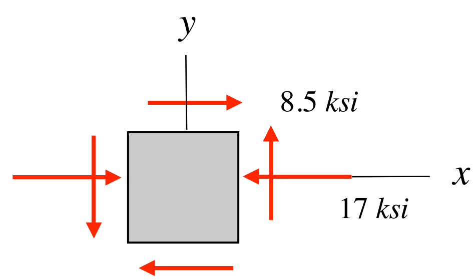



x
y
z
point "a"
point "b"

### *Example 15.5*

The components of stress on a propeller shaft made up of a material having a yield strength of $\sigma_Y = 36ksi$ is as shown below. What is the factor of safety as predicted by the maximum shear stress theory for the material? What is the factor of safety as predicted by the maximum distortional energy for the material?



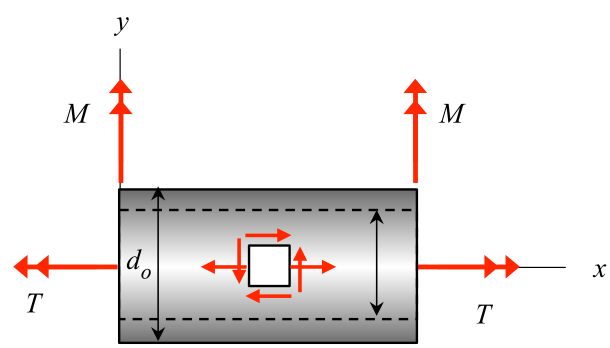


***Example 15.3***

A section of pipe is loaded as shown below with bending couple $M = 35\,kip \cdot in$ and axial torque $T = 175\,kip \cdot in$. The yield strength of the pipe's ductile material is known to be $\sigma_Y = 100\,ksi$, with the inner and outer diameters of the pipe given as $d_i = 3\,in$ and $d_o = 3.5\,in$, respectively.

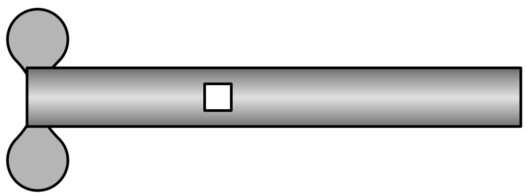
a) What is the factor of safety $FS_S$ predicted by the *maximum shear stress* theory of failure for this loading on the pipe section?

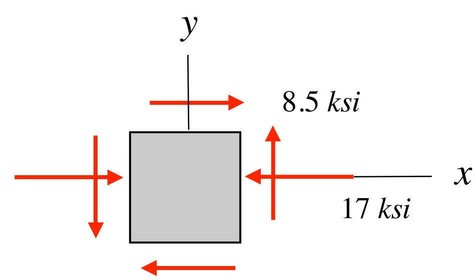
b) What is the factor of safety $FS_D$ predicted by the *maximum distortional energy* theory of failure for this loading on the pipe section?



***Example 15.5***

The components of stress on a propeller shaft made up of a material having a yield strength of $\sigma_Y = 36ksi$ is as shown below. What is the factor of safety as predicted by the maximum shear stress theory for the material? What is the factor of safety as predicted by the maximum distortional energy for the material?



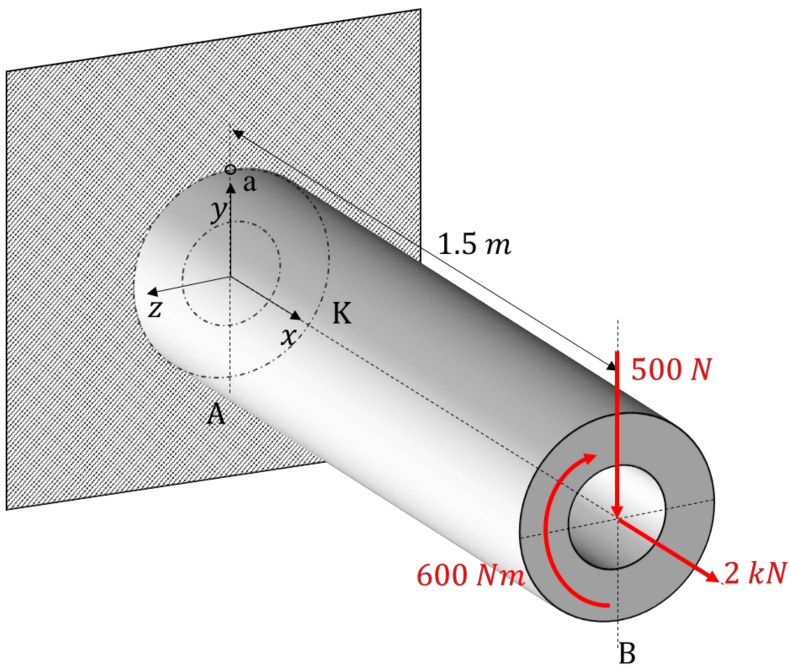


### *Example 15.8*

A tube AB of length 1.5 $m$ is acted upon by an axial load 2 $kN$, a downward shear 500 $N$, and a clockwise torque 600 $Nm$, at B as shown below. The outer diameter of the tube is 40 $mm$, and its thickness is 5 $mm$. The tube is composed of material whose yield stress is specified as $\sigma_Y = 220\ MPa$. Calculate the yielding factor of safety point a (0, 20, 0) $mm$ based on:

a) maximum shear stress (MSS) theory

b) maximum distortional energy (MDE) theory



***Example 16.10***

An elastic rod BC of uniform cross section is bent into the form of a three quarter (270°) circle such that its mean radius is $R$. The rod is fixed to a wall at B and is pinned to a roller at C. The rod is composed of a material of Young's modulus $E$, and the second area moment of the cross section is $I$. A downward load $P$ is applied at end C of the rod as shown in Fig 9.3. Assuming that elastic strain energies due to shear and axial loads are negligible as compared to bending strain energy:

a) Set up the integral to calculate the total bending strain energy in the rod BC as a function of load $P$, the unknown reaction $F_C$ at C, and the angle $\theta$.

b) Use Castigliano's theorem to determine the reaction force $F_C$.

c) Use Castigliano's theorem to determine the downward deflection of end C.

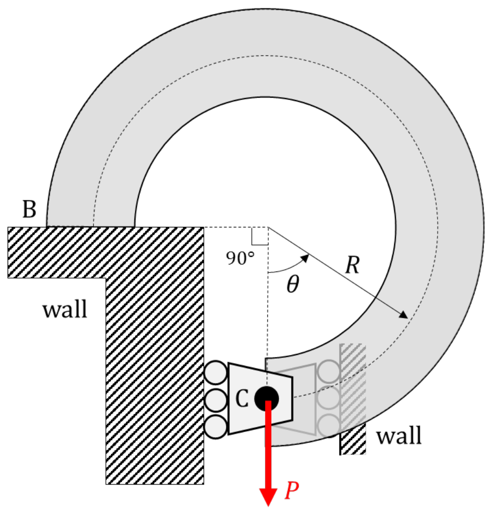
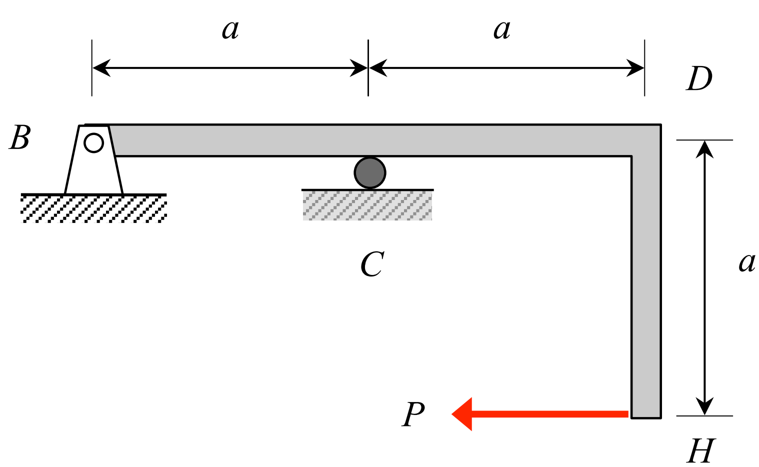



Fig 9.4

***Example 16.11***

An L-shaped structural member is supported by a pin at B and a roller support at D. The cross section of the beam is square having cross-sectional dimensions of $(b \times b)$, and is made of a material with a Young's modulus of $E$.

Use Castigliano's theorem to determine the horizontal and vertical deflection of end H of the member. You may ignore the influence of shear in the strain energy expression for the member.



### *Example 16.12*

Consider the beam shown below that is supported by rollers at C and D, and by a fixed wall at end H. A end load $P$ acts at B. The cross section has a second area moment of $I$ and is made up of a material having a Young's modulus of $E$. It is desired to determine the reactions at supports C and D using Castigliano's method. To this end:

a) Draw a free body diagram of the entire beam and write down the equilibrium equations. Show that the problem is statically indeterminate.
b) Choose an appropriate set of redundant constraint forces from your FBD above.
c) Write down the strain energy expression for the beam. You may neglect the contributions to the strain energy from shear.
d) Using Castigliano's method to determine the reactions at C and D.