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A Stochastic Cash-Flow Model of a General Insurance Company

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Summary

A cash flow model is proposed as a way of analysing uncertainty in the future development of a general insurance company. A computer model is presented for use in practical applications by actuaries advising the management of general insurance companies. The company is modelled alongside the market in aggregate so that the impact of changes in premium rates relative to the market can be assessed. Simulation methods are used to explore the consequences of uncertainty, particularly in regard to inflation and investments.

Résumé

Modèle de Cash-Flow Stochastique d'une Compagnie d'Assurance IARD

Un modèle de "cash-flow" est proposé comme une façon d'analyser l'incertitude dans le développement futur d'une compagnie d'assurance toute branche. Un modèle informatisé est présenté pour être utilisé dans des applications pratiques par des actuaires conseillant la direction de compagnies d'assurance toute branche. La société est transformée en modèle parallèlement au marché en aggrégat de façon à ce que l'impact des changements dans les taux de primes relatifs au marché puisse être évalué. Des méthodes de simulation sont utilisées pour explorer les conséquences de l'incertitude, en particulier en ce qui concerne l'inflation et les investissements.

1. INTRODUCTION

1.1 Traditional accounting models are of limited value in assessing the financial strength of a general insurance company, partly because of difficulties of measurement and valuation and partly because they do not help the user to understand the inherent uncertainty.

1.2 The accounting model is even less promising as a vehicle for providing management with the information necessary to make proper decisions about future business strategy. Valuation problems are compounded, the impact of writing further business, investment returns and movements in asset values are difficult to handle satisfactorily, and there is no ready mechanism for exploring the consequences of uncertainty in a structured framework.

1.3 The accounts are concerned with measuring what has happened in the past, to give an account of stewardship to shareholders and to provide a modest level of disclosure of the company's financial status to present and potential shareholders and policyholders, to intermediaries and to supervisors. Income and expenditure are recognized on an accruals basis as they are incurred, rather than reflecting the actual movement of cash. Thus premiums received to cover risks in a future accounting period are not treated as income until that later period. Appropriate provisions have to be set up in respect of future settlement of claims arising from cover that has already been granted.

1.4 The magnitude of the provisions fundamentally affects the picture shown by the accounts. There is considerable scope for debate about the size of the provision for outstanding claims, including questions of whether the liabilities should be conservatively estimated and whether they should be discounted. Many different approaches are adopted in practice and this presents a major difficulty for the interpretation of companies' financial results and apparent financial strength.

1.5 Problems also arise on the assets side of the balance sheet. Showing assets at historic cost discloses nothing about true financial strength and often produces substantial hidden investment reserves. Market values present different problems, because the values can fluctuate from day to day as the terms on which the market is willing to buy and sell change. Such fluctuations may or may not reflect any real change in the anticipated stream of income to be generated by the assets. It is on

this income flow that the company depends, rather than on the current market value of the assets. A forced realization of assets is unlikely to be necessary for an insurance company operating as a going concern.

1.6 There is scope for improving the comparability of reporting in insurance accounts. However, in our view, there are inherent difficulties and shortcomings in the accounting perspective of an insurer's business which constrain the usefulness of even the most informative accounts from the point of view of an outside observer, and mean that management will be ill-advised to focus attention only on traditional accounting measures.

1.7 It is important for management to view the company as a dynamic entity. The impact of the business which is being written and which will be written in future is fundamental to the financial health of the company. Management should be interested in the cash flow impact of writing particular types of business, the effect on the balance sheet, and on the emergence of profit, of reserving requirements, the capital that is implicitly tied up in writing the business and the rate of return that can be expected on that capital. Then there are questions of the trade-off of risk and reward in different investment strategies, the possibility of developing an investment portfolio which pays proper attention to the nature of the corresponding liabilities, the potential variability in projected financial measures such as solvency margin and profit, and strategies for maximizing the value of the company as a going concern, subject to an acceptable level of risk.

2. MODELLING FUTURE CASH FLOWS

2.1 To answer key management questions of this sort, a model is required which takes into account future cash flows and brings together the asset and liability components of the strategy. Since the accounting implications cannot be ignored, the model needs to operate on two levels: the level of actual cash transactions and the level at which the transactions will be reported using accounting conventions. It may be necessary to consider yet a further set of calculations to determine the tax payable.

2.2 The essential basis is the emerging costs paradigm. This approach differs in principle from the accounting model in focusing on the future cash flows. Important features of the business can be taken into account in as realistic a way as possible, including tax and dividend payments, as well as premium income, expenses, claim outgo,

investment income, asset values and investment strategy. At a more sophisticated level appropriate allowance can be made for feedback mechanisms and interactions between the variables. Uncertainty can be modelled directly using simulation.

2.3 Such a model is presented here. Regard is had to the operation of the insurer in a competitive market. The market and the insurer in question are modelled in parallel, so that rating decisions relative to the general market level of premiums can be taken into account. Fuller details are given in Daykin and Hey (1990) and in the users' manual which accompanies the diskette (Hey and Daykin, 1990). Somewhat similar modelling techniques are used by Pentikäinen et al (1989).

3. ASSET/LIABILITY MANAGEMENT

3.1 Central to the management model is the view that assets and liabilities must be considered together. The traditional balance sheet approach does not encourage this. Assets and liabilities are often seen as the responsibilities of quite different groups of people. If the assets are taken at market value, what meaning can be ascribed to the comparison between the assets and the liabilities? If asset values fall as a result of a change in the market, does the change in surplus give a fair indication of a change in underlying financial strength of not?

3.2 Fundamentally the need is to compare cash flows into and out of the insurance company. For some purposes it is necessary to convert the cash flows into present value figures but, if this is to be done in a consistent way, both income and outgo should be discounted at the same valuation rate of interest.

3.3 In general insurance the liabilities may not be discounted at all, or if they are, the rate at which they are discounted is unlikely to correspond to that implied by the valuation of assets. Indeed, the issues are rarely considered from this perspective. If the market value of the assets changes, the value of the liabilities may well be left unchanged. This can create spurious fluctuations in the apparent margin of assets over liabilities. The process of valuation itself thus introduces additional uncertainty and may fail to capture all the information that is available about the emerging cash flows.

3.4 The emerging costs paradigm focuses attention on the actual assets held and the way in which they might behave. It enables specific assumptions to be made about how cash

surpluses are invested and what rules should be followed for selling assets to meet a shortfall. Simulation enables the full variability of the assets to be taken into account by means of stochastic asset models.

4. THE MANAGEMENT MODEL

4.1 The management model is designed to simulate a realistic market situation, by modelling the effect on the volume of business written of increases or decreases in the premium rates charged relative to the average market level. The model involves projecting the behaviour of the market as a whole alongside the behaviour of the particular insurer under investigation and allows the user to test the behaviour of the insurer against that of the market.

4.2 The model allows for six types of liabilities to be written. These need not correspond on a one-to-one basis with particular classes of business or lines. They are characterized by a series of parameters, including target profitability, claim settlement pattern, initial premium volume and rate of real growth in volume. The target profitability is the percentage margin for profit in the gross premiums, allowing for expenses and for the expected run-off of claims in accordance with the claim settlement pattern, discounted at a realistic real rate of interest.

4.3 The company under investigation is assumed to have a specified initial premium volume. The market volume is set arbitrarily at a level much higher than the company. The precise market volume is not important. Growth rates in the market volume are specified both for the past and for the future, with flexibility to allow for different growth rates in two separate past periods and in two separate future periods. The volume of business written by the company is assumed to increase or decrease according to whether premiums are lower than or higher than the market rates, with appropriate gearing factors to reflect the elasticity of demand.

4.4 Market premiums are assumed to be set in order to achieve the specified target profit for each liability type. This can be on the basis of an assumed perfect knowledge of inflation throughout the underwriting year or can be on the basis of a formula estimate of the current rate of inflation, derived from the increases in the RPI over the past four years. As the simulation moves forward, the starting point for the calculation of projected inflation includes the actual rates of inflation for previous years according to the particular simulation.

4.5 Premium rates in the market may be assumed to vary

cyclically according to a cycle of specified length and shape. The amplitude and phase of the cycle may be specified separately for each liability type. Furthermore, the market may be assumed to set premiums higher than would be implied by the profit target, in order to attempt to recoup past shortfalls relative to target profits, or to set premiums lower in order to pass on to policyholders the benefit of past surpluses relative to target profits.

4.6 The company under observation is assumed to operate within the framework of the market which we have described, starting with a specified premium volume. It is then assumed to set premiums higher or lower than the market as a result of one or more of the following considerations:

(a) with a view to increasing profits or in order to grow or contract, it may aim to set its general premium levels at a stated percentage below or above the market level;

(b) it may estimate the current rate of inflation on the basis of a different formula from that implicit in the behaviour of the market as a whole:

(c) it may attempt to recoup past shortfalls relative to target profits by charging higher premiums, or pass on to policyholders the benefits of past surpluses relative to target profits through lower premiums, to a different extent than the market;

(d) it may charge a supplement for a superior product or for some element of cover which is being offered over and above that offered by the market generally.

4.7 In addition to the actual market premium, a notional 'claims' premium is calculated as part of the process of deriving the related claim expenditure. This is similar to the actual market premium, but allows for the actual outcome of inflation even when a formula estimate is used in setting the actual premium. It ignores any premium cycle and any adjustments in respect of past shortfalls or excesses of profits relative to target. Corresponding to the notional 'claims' premium for the market, a notional company 'claims' premium is also calculated. This allows for the actual volume of business written, but for a premium level which assumes perfect knowledge of inflation, no premium cycle and no adjustments to increase or decrease profitability. It does, however, include the cost of any supplementary cover (see paragraph 4.6(d)).

4.8 The extent to which the company sets its premiums above the market level can be made to vary directly

according to the level of the asset margin at the end of the previous year relative to a specific level. This is intended to provide a feedback mechanism so that a worsening solvency position is responded to automatically by increasing premium rates. The user can specify whether or not this is to apply, the asset margin level below which it applies and the factor of proportionality.

4.9 Facility is also given for the introduction of intervention strategies. One such can be triggered by the asset margin falling below the required level. This corresponds to the system of supervision in the E.C., whereby the supervisor can require the company to produce a plan for the restoration of a sound financial position once it has failed to maintain the statutorily required solvency margin. The facility can be used with a higher trigger point than the statutorily required solvency margin, on the assumption that management would introduce similar measures to those which might be required by the supervisor, and that they would do so before the point was reached where the supervisor could intervene. The intervention strategy permits the company's premium levels to be reset at a different percentage above the market level. Further provision is made for intervention in the area of asset distributions (see paragraph 7.2).

4.10 The intervention can be constrained so that it does not occur if the whole market is below a specified asset margin level. This is to cater for situations where a temporary severe drop in asset values could cause many companies to be technically insolvent, but where the supervisor might be reluctant to close them all to new business.

4.11 An alternative strategy can also be specified to be introduced after a fixed period of years (which can differ for each liability type).

4.12 If the company charges higher premiums than the market, it may expect to lose business volume. Similarly, if it charges lower premiums than the market, it will expect to gain business volume. The relationship between premium differentials and volume response, or the elasticity of demand, is controlled by gearing factors which are specified by the user, separately for each liability type and for premiums above and below the market level. The company is assumed to be small enough for its behaviour not to have an effect on the market as a whole.

4.13 Up to 20 years' future business can be incorporated. There would be no problem in principle in extending this, but it is difficult to obtain stable results over long

periods without incorporating additional feedback mechanisms, so it is doubtful whether useful results would be obtained on a longer timescale. Experience shows that for most purposes no more than 10 years should be considered.

4.14 Unearned premium reserves are set up at the end of the year as specified percentages of gross premiums written during the year. These percentages may differ between liability types and between the company and the market.

5. THE MANAGEMENT MODEL - EXPENSES

5.1 Expenses other than claim settlement expenses are taken into account explicitly. Claim settlement expenses are assumed to be included in the cost of settling claims. Separate provision is made for fixed expenses, which are assumed to increase in future years in line with inflation or at a fixed percentage rate faster or slower than inflation. In addition, some expenses can be assumed to go up in steps, triggered by a specified percentage increase in the volume of business. This could be to reflect the need to increase staff numbers and office space, once business has increased by a certain amount. The percentage rise in expenses may be less than the percentage rise in volume to represent an improvement in efficiency.

5.2 The variable component of expenses includes brokerage or commission and expenses related to the volume of business. These expenses are taken to be proportional to gross written premiums, but may also increase at a fixed percentage rate faster or slower than this to allow for improvements in efficiency or expected changes in commission levels.

6. THE MANAGEMENT MODEL - CLAIM OUTGO

6.1 In order to estimate the claims which are expected to be incurred in respect of the business written, it is assumed that the notional claims premium rate calculated for the market, or for the company (see paragraph 4.7), gives a mean estimate of the basic level of claims likely to be incurred, subject to any addition to company claim costs in respect of any supplementary cover (see paragraph 4.6(d)). The profit rate as a fraction of gross premiums assumes that a real rate of return will be achieved on the investment of the provisions at the same rate as the rate at which they are discounted.

6.2 Provision is made for the claim ratios to vary on a cyclical basis about the basic level. The length and shape of the cycles may be specified and the amplitude and phase

may be specified separately for each liability type. Allowance is also made for random variation in the claim ratios. The variability is assumed to be normal, with standard deviations specified by the user separately for each liability type for the market as a whole and for the company.

6.3 The resulting claims are subdivided into claim amounts in successive years in accordance with the specified settlement patterns for each liability type, and are added to expected claim payments in those years of development from earlier underwriting years. In this process, allowance is made for the claim payments, in each successive year, to be inflated in accordance with the retail price index, and by an additional rate of increase of claims where specified by the user. The total claim outgo in any particular future year is assumed to be subject to random variation, in accordance with a normal distribution with standard deviation $aX+b\sqrt{X}$, where a and b are suitable chosen constants, which may be specified by the user, and X is the mean level of claims.

6.4 For the purposes of establishing balance sheet provisions in respect of outstanding claims, future projected claim payments are taken at their nominal amount (before allowance for inflation) and discounted at a specified real rate of return, which may be different for the company and the market.

6.5 The above procedures enable premiums, expenses and claims to be projected year by year in terms of cash flow, and appropriate balance sheet provisions to be set up in respect of unearned premium reserves, outstanding claim reserves, amounts due from agents and outstanding tax liabilities. In each simulation a set of random numbers is chosen, which gives rise to a particular realization, in which the program proceeds forward one year at a time, evaluating the various items in the revenue account and the balance sheet. The receipt of premiums and investment income and the payment of claims, taxes and dividends give rise either to a surplus available for investment or to a shortfall requiring assets to be sold.

7. THE MANAGEMENT MODEL - ASSETS

7.1 The model allows for seven types of investment (cash, bonds of three different lengths to be specified by the user, index-linked government securities, equities and property) and provides that, at the end of every year, purchases and/or sales take place in order that the liabilities of each type, and the asset margin, are represented by the specified fraction of some or all of the

asset types, ready for the start of the next year of the projection. Any assets that need to be sold in order to achieve this are assumed to be disposed of at the current market value, followed by the purchase of assets of the appropriate type if necessary. No explicit allowance is made for dealing expenses.

7.2 An alternative set of proportions can be specified as applying on intervention by a supervisor (or by the management) if the asset margin falls below a given level or after a specified number of years (as in paragraphs 4.9 to 4.11). Investment proportions for the market as a whole are specified separately.

7.3 The asset movements (and inflation) are modelled using the autoregressive stochastic processes which have been described by Wilkie (1986). They are further discussed in Daykin and Hey (1990).

7.4 A proportionate holding of cash may be assumed for any of the liability types or for the asset margin. Cash is assumed to be interest-bearing, with the rate of interest assumed to be one percentage point lower than the yield on irredeemable gilts as defined by the Wilkie model. Non-interest bearing cash is assumed to be held in respect of the balance due from agents. These are taken as a proportion of gross written premiums, as specified by the user. Other current assets and liabilities are ignored.

7.5 Wilkie's model provides only for irredeemable bonds and, in order to introduce dated bonds, we have assumed a simple yield curve relationship, whereby 20-year bonds are assumed to have the same gross redemption yield as irredeemable bonds and the yield is assumed to fall linearly to one percentage point below the yield on irredeemables as the term to maturity falls to zero.

7.6 Redeemable bonds are assumed to be purchased at par at the current yield and sold one year later at the ruling price for stocks with one year less to redemption, to be replaced by gilts of the original term, once more purchased at par, but at the then current yield for such stock.

7.7 Wilkie's model also generates future values for the Retail Prices Index (RPI) and these have been used whenever an estimate of future inflation is needed, either in respect of premiums or claims. The inflation model also provides a basis for simulating the behaviour of index-linked government securities. The yield on index-linked government securities is taken to be a constant below the yield on equities. The constant is chosen by the user. Their value is assumed to rise in line with the RPI. This

is something of a simplification since yields may vary relative to yields on equities and profits or losses relative to inflationary growth may occur if index-linked government securities are not held to redemption.

7.8 Equities are provided for by using the standard Wilkie model, with the facility for the user to specify the parameters. For property Wilkie (1988) has provided a model similar to, but simpler than, the model for equities, and parameters to reflect the less volatile nature of the market.

8. DIVIDENDS TO SHAREHOLDERS

8.1 Dividends are assumed to be paid annually, at the end of the year following that in which they are assumed to be earned. The initial rate of dividend is specified by the user as a percentage of the initial written premiums and the amount of dividends payable each year, in money terms, is increased annually at the rate of increase of the RPI. A further percentage increase may be specified by the user if it is desired to give shareholders a better return than merely following the RPI. This clearly ignores the actual profit or loss achieved in the past year, but seems more realistic than relating dividends directly to actual current profits. Dividend payments may be assumed to be deferred or paid at a reduced level for a few years.

9. PROFIT AND LOSS ACCOUNT

9.1 Only realized gains and losses on investments are taken into account in determining the balance subject to tax. However, unrealized gains can be quite important in relation to measuring profit, particularly where there is a substantial level of investment in equities, property and index-linked gilts. A company will invest in variable income securities at a running yield lower than that available on gilts only in the expectation that capital growth will exceed the difference in running yields by at least sufficient to provide a fair reward for the uncertainty. Incorporating in full the actual changes in asset values resulting from the volatility of the market might introduce a confusing level of additional variability into the already quite variable profit results. To help in dealing with this problem the model allows the user to specify expected rates of growth for equity, property and index-liked gilt prices and growth at this rate is brought into the final tabulated profit for each year.

9.2 The model distinguishes four different definitions of 'profit' for different purposes. These may be characterized as follows:

(a) The profit for Corporation Tax purposes. This is defined separately for each of the liability types and for the business as a whole. The total figure is adjusted in respect of any past losses carried forward and, if negative, may be subject to adjustment as a result of carry-back.

(b) The profit shown in the output. This is defined separately for each of the liability types and for the business as a whole. It includes underwriting profit and investment income, and a notional allowance for capital gains (see paragraph 9.5). It is not adjusted for past losses carried forward, nor is it affected by carry-back. This profit is also used, after discounting, as the basis for the calculation of company worth. Inflation is taken into account in fixing the appropriate risk rate of discount.

(c) The profit used for calculating return on capital. This is the same as in (b) but deflated by the RPI.

(d) The 'excess' profit; namely the amount by which the profit in (b) exceeds the target profit used for setting premium levels. This is used for adjusting premiums in the following year where this option is required.

9.3 The resulting figures enable a profit and loss account to be drawn up each year, in order to establish taxable profits and show the results in a form familiar to management. The profit and loss account is built up as follows:

Written premiums
+Unearned premium reserve brought forward from previous year
+Outstanding claim reserve brought forward from previous year
-Claim payments
-Expenses
-Unearned premium reserve carried forward at the end of the year
-Outstanding claim reserve carried forward at the end of the year
+Investment income (gross)
=Balance subject to corporation tax on trading profits
-Taxation for year (provision set up - see paragraph 9.4)
-Dividends to shareholders
+Capital gains/losses (see paragraph 9.1)
=Profit (loss) retained.

9.4 Taxation for the year includes tax on chargeable gains or losses as well as tax on trading profits. The amount required to meet the tax liability is set aside as a provision in the balance sheet and settlement of the liability is assumed to take place 12 months later, just before the next balance sheet is drawn up. The tax provision is assumed to be invested in the same way as the asset margin.

9.5 If there are taxable losses, these are carried forward to the following year, unless they can be carried back to be offset against profits in previous years. The number of years for which carry-back is permitted can be specified by the user, as circumstances may differ in different territories. Taxable losses from previous years are brought forward to offset against any taxable profits.

9.6 As far as the profit and loss account is concerned, the program output displays the means and standard deviations of the distribution of values of the profit (loss) for each future year, and the full distribution in a selection of future years. The program tracks the progress both of the company under investigation and the market as a whole and shows variation of profit for each. The profits (losses) in future years are expressed per thousand units of written premium and therefore take into account changes in the volume of business and the RPI.

9.7 As far as cash flow in the year is concerned, we have the following:

Net cash flow
=Written premiums
+Portion of written premiums in the previous year that was held by agents at the beginning of the year
-Portion of written premiums in the year that is still held by agents at the end of the year
+Investment income
-Claim payments
-Expenses
-Payment of tax at end of year
-Dividend payment at end of year
=Cash available for investment.

9.8 The net cash flow gives rise to a net amount of surplus cash available for investment or a net cash requirement for which assets need to be sold.

10. BALANCE SHEET

10.1 The balance sheet may be drawn up as follows:

Funds
=Unearned premium reserves
+Outstanding claim reserves
+Provision for tax liabilities
+Asset margin

Assets
=Investments (7 types)
+Agents' balances.

10.2 The unearned premium reserve is defined as a percentage of written premiums to be specified by the user. No separate provision is made for any additional amount for unexpired risks. Given the philosophy of the model, that reserves should not contain margins but give a realistic picture, the unexpired risk reserve should be calculated on a discounted basis and hence will not normally require a significant addition to the unearned premium reserve.

10.3 The funds are made up by a mixture of assets, including, in the case of the asset margin, balances held by the agents. Once the outstanding claim reserves and unearned premium reserves have been calculated, the requirements for each of the types of assets to match the six liability types and the asset margin plus tax reserve can be established according to the specified investment proportions, and assets bought or sold accordingly. Any capital gains tax liability that is created as a result of these transactions will result in a payment of tax a year later but does not require any further rearrangement of the assets.

10.4 The program output displays the means and standard deviations of the distribution of values of the asset margin, i.e. the excess of total assets over total liabilities, at the end of each year, and the full distribution at the end of a selection of future years. This is shown both for the company under investigation and for the market as a whole, expressed as a fraction of gross written premium for the year and then multiplied by 1000. It therefore takes into account changes in the RPI and in the volume of business to produce figures more comparable with the initial asset margin.

10.5 The program also calculates and displays the distribution of average return on capital for the company and company net worth, but care needs to be taken in interpreting these quantities. For return on capital the

profits displayed in the output (as described in paragraph 9.6(b)) are deflated by the RPI, averaged over the number of future years used, and then expressed as a percentage of the initial asset margin, which is taken as a proxy for capital employed.

10.6 The definition of capital employed raises a number of problems, since a high proportion of the funds of an insurer is provided by policyholders rather than by shareholders. In some cases there are no shareholders. It is common for there to be a substantial margin in the technical provisions, leading to a corresponding understatement in the real level of the asset margin. Figures from Daykin et al (1987) suggest that the hidden margin could well exceed the disclosed margin in many cases. Unless some way can be found to standardize for this, comparisons with other companies may not be very meaningful.

10.7 Using company net worth instead of return on capital avoids the problem of defining capital employed. We take net worth in the conventional sense of the present value of the future streams of profits or losses. A real risk rate of return can be specified by the user for this purpose and applied to amounts which have been deflated by the RPI. However, a disadvantage is that the projected stream of profits is available only for a limited number of years, whereas in principle the calculation involves discounting profits in perpetuity. An estimated approximation is adopted whereby the discounted sum in perpetuity is estimated from the profit stream during the projection period.

11. USING THE MANAGEMENT MODEL

11.1 The program has not been designed with a view to producing a single set of results on the basis of a hypothetical company, but as a tool which actuaries will be able to use in the context of different companies to explore the sensitivity of future prospects for the company to changes in key parameters. The program is available in compiled program form on a diskette.

11.2 The description in this paper has concentrated on the main features of the model, in order to focus attention on the most important aspects. Parameters need to be chosen carefully in combination in order to produce realistic scenarios and, to test for stability, the program should be run first without any stochastic variation. This shows whether there is an inbuilt bias towards abnormal growth or disaster. Different sets of parameter values can be explored to find a stable scenario and to discover what

features are giving rise to any strong trends in either direction.

11.3 When a sensible combination of parameters has been identified, the program can be run in stochastic mode. The same sets of random numbers should be used each time the program is run with different values for the parameters, so that comparison between the results using different parameter values is not affected simply by the choice of random numbers. For initial exploration of the effect of introducing stochasticity, some 50 to 100 simulations should be adequate. In order to reduce the dependence of the results on the particular sets of random numbers chosen, and hence to produce more stable distributions for the purposes of making comparisons between the results with different parameter values, a larger number of simulations can be used. For most practical purposes, runs with 1,000 simulations should provide the right balance between economizing on running time and providing stable estimates of the underlying distributions for such comparisons.

11.4 The authors believe that the model provides an invaluable tool which should enable management to explore the consequences of uncertainty in the operations of a general insurance company. Few models designed to assist in the forward planning process give adequate recognition to the uncertainty inherent not only in the claim outgo but also in future inflation, asset movements and yields. This model enables all those aspects to be taken into account in a integrated fashion.

11.5 Use of the model within a company will focus attention on key parameters for determining the future course of the business and managing the inherent uncertainties. The focus on emerging cash flows enables the assets and liabilities to be taken into consideration together in a coherent way. Management (and actuaries) will need to be educated to an appreciation of the results of projections presented in the form of distributions to show the impact of variability and to appreciate how these can be used to understand the trade-off between expected outcome and uncertainty.

11.6 The model may also be used independently of the market by specifying volumes and growth rates and setting the gearing factors to zero. This enables the user to specify both premium rates and volumes for the company. From this the user can deduce the capital required, or, by adding a further liability type, the additional capital required to write a further line of business, or the capital that would ultimately be released by discontinuing a line. This approach would also enable the capital

requirements of a new insurance operation to be explored, past business being set to zero.

11.7 Appendix 1 illustrates a few results from the model on a hypothetical set of assumptions, and shows the effect of varying some of the parameters.

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APPENDIX 1

ILLUSTRATIVE RESULTS FROM THE MANAGEMENT MODEL

1. In view of the very large number of parameters employed by the model, it is not possible to give more than a brief illustration of the type of information available from the model and the variability which is exhibited.

2. Table 1 sets out a selection of the key parameters which we have used in the standard basis for these illustrations. Figure 1 shows the development over 10 years of the company's asset margin from an initial level of 50% of gross written premiums. The graph shows the mean of the distribution of asset margins in each successive year and the mean plus or minus one standard deviation to illustrate the widening funnel of doubt. Figure 2 shows the distribution of the asset margin at the end of years 2, 4, 6 and 10 on the standard basis. Figure 3 shows the corresponding distributions for the profit.

3. In the standard basis the experience of the company does not diverge significantly from that of the market, so the development of the market would be very similar. The standard basis shown here allows for stochastic variation in the assets as well as in the claims. The asset margin in the corresponding non-stochastic case would follow quite closely the mean in Figure 1, although always slightly below. This is because the log-normal characteristics of the investment model produce mean values in the stochastic case which are greater than the equivalent non-stochastic values.

4. Table 2 gives a summary of key results for year 5 from a variety of alternative scenarios. Figure 4 shows the development of the mean asset margin over 10 years on the standard basis and 4 other scenarios. Figure 5 shows the corresponding development of mean overall profit. The high company premium scenario assumes that the company charges 3% more than the market, whilst the low company premium scenario assumes 3% less. High gearing implies gearing factors of 2.5 for premiums above the market level and 2.0 for premiums below, compared to 1.5 and 1.0 respectively in the standard case. The gearing factor represents the ratio of the proportionate change in volume to the proportionate deviation of the company premium from the market level. They are of opposite signs, i.e. the volume falls more if the premium charged is higher.

Table 1

Principal parameters for standard case

Asset margin	50% of gross written premium (gwp)
No. of liability types	6
Target profitability	5% of gwp (company and market)
Rate of dividends	5% of gwp
Real rate of discount	2½%
Rate of discount for net worth	15%
Unearned premium reserve	40% of gwp
Cash retained by agents	25% of gwp
Real growth of premium volume	None
Company premiums over market	No
Claim cost supplement	Nil
Gearing factors	1.5 above; 1.0 below
Fixed expenses	10% of gwp (rising in line with RPI)
Variable expenses	20% of gwp
Step increase expenses	Every 2% of real premium growth
Efficiency gains	Nil
Standard deviation of claim ratios	10% for short-tailed, rising to 30% for long-tailed
Overall claim variability	$aX+b\sqrt{X}$ where a=0.07 and b = 70 for short-tailed, rising to 100 for long-tailed
Premium cycles	None
Claim cycles	None
Required solvency margin	16% of gwp
Tax rates	35% on profits; 35% on capital gains
Notional capital gains profit	7% a year
No of simulations	100

Table 2

Summary of key statistics for company

Parameters	Asset margin end of in year 10 (per mille of gross written premium)		Overall profit in year 10 (per mille of gross written premium)		Company worth (per mille of asset margin)		
	Mean	S.D.	Mean	S.D.	Mean	S.D.	
Standard	444	370	71	102	1310	894	
Premiums 3% higher[1]:							
Low gearing[2]	671	452	110	110	1515	846	Variant 1
High gearing[3]	630	527	90	119	1351	830	Variant 2
Premiums 3% lower[1]:							
Low gearing[2]	241	331	32	97	981	933	Varaint 3
High gearing[3]	270	290	44	92	1108	954	Variant 4
No investment variability	504	55	90	20	1205	133	Variant 5
No claim variability	452	361	74	97	1320	880	Variant 6

Notes

1. Premiums 3% higher (lower) implies that the company sets its premium rates 3% higher than the market level for identified risks.

2. Low gearing implies a gearing factor of 1.5 when premiums are above the market level, i.e. when the company's premium is x% above the market premium level (for all liability types) its market share will fall by 1.5x%, and a gearing factor of 1.0 when premiums are below the market leve l.

3. High gearing implies a gearing factor of 2.5 when premiums are above the market level, i.e. when the company's premium is x% above the market level (for all liability types) its market share will fall by 2.5x%, and a gearing factor of 2.0 when premiums are below the market level.

Figure 1

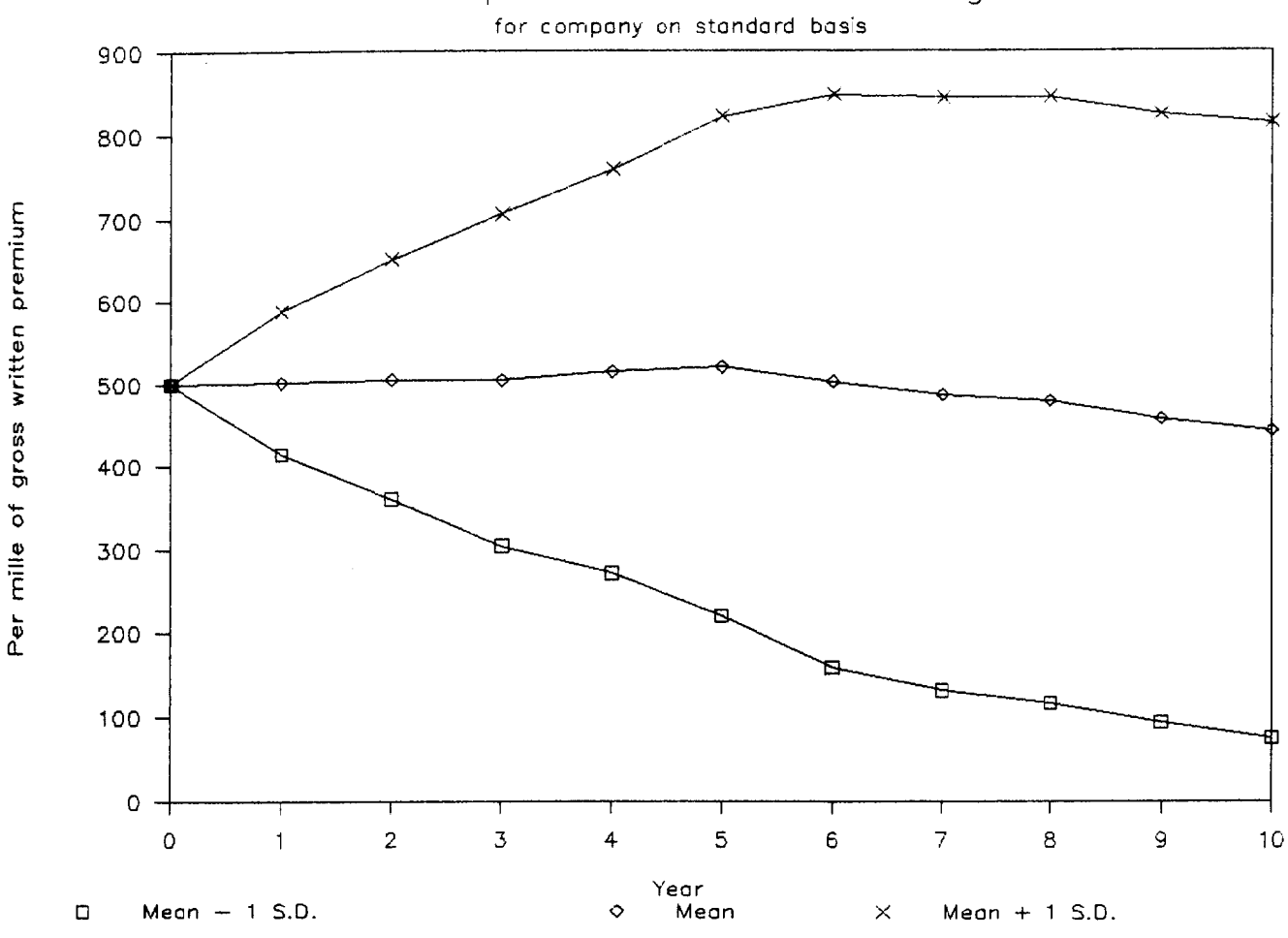
Development of asset margin
for company on standard basis
Per mille of gross written premium
900
800
700
600
500
400
300
200
100
0
0
1
2
3
4
5
6
7
8
9
10
Year
Mean − 1 S.D.
Mean
Mean + 1 S.D.

Figure 2 Distribution of company asset margin

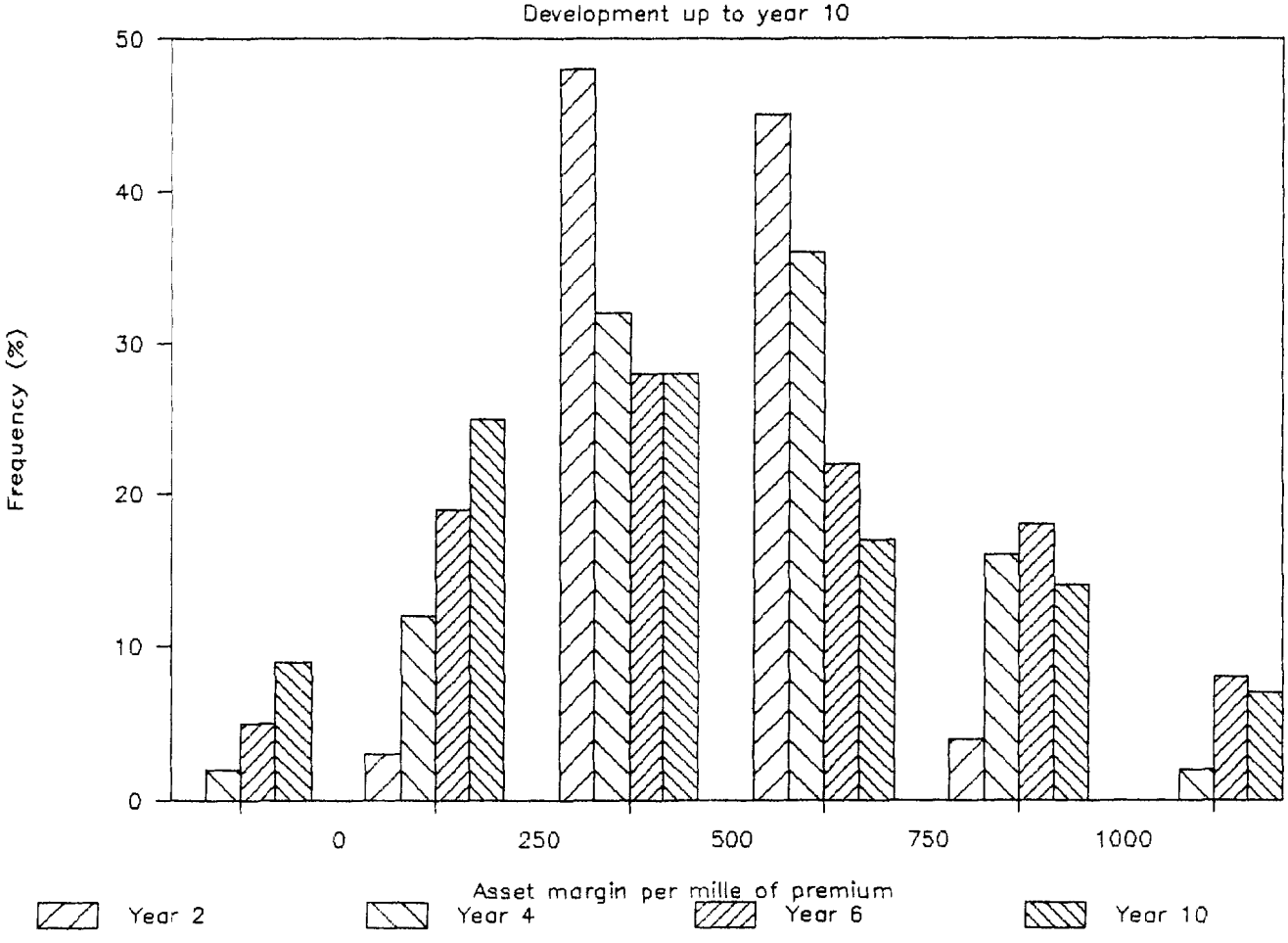

Figure 3 Distribution of company overall profit

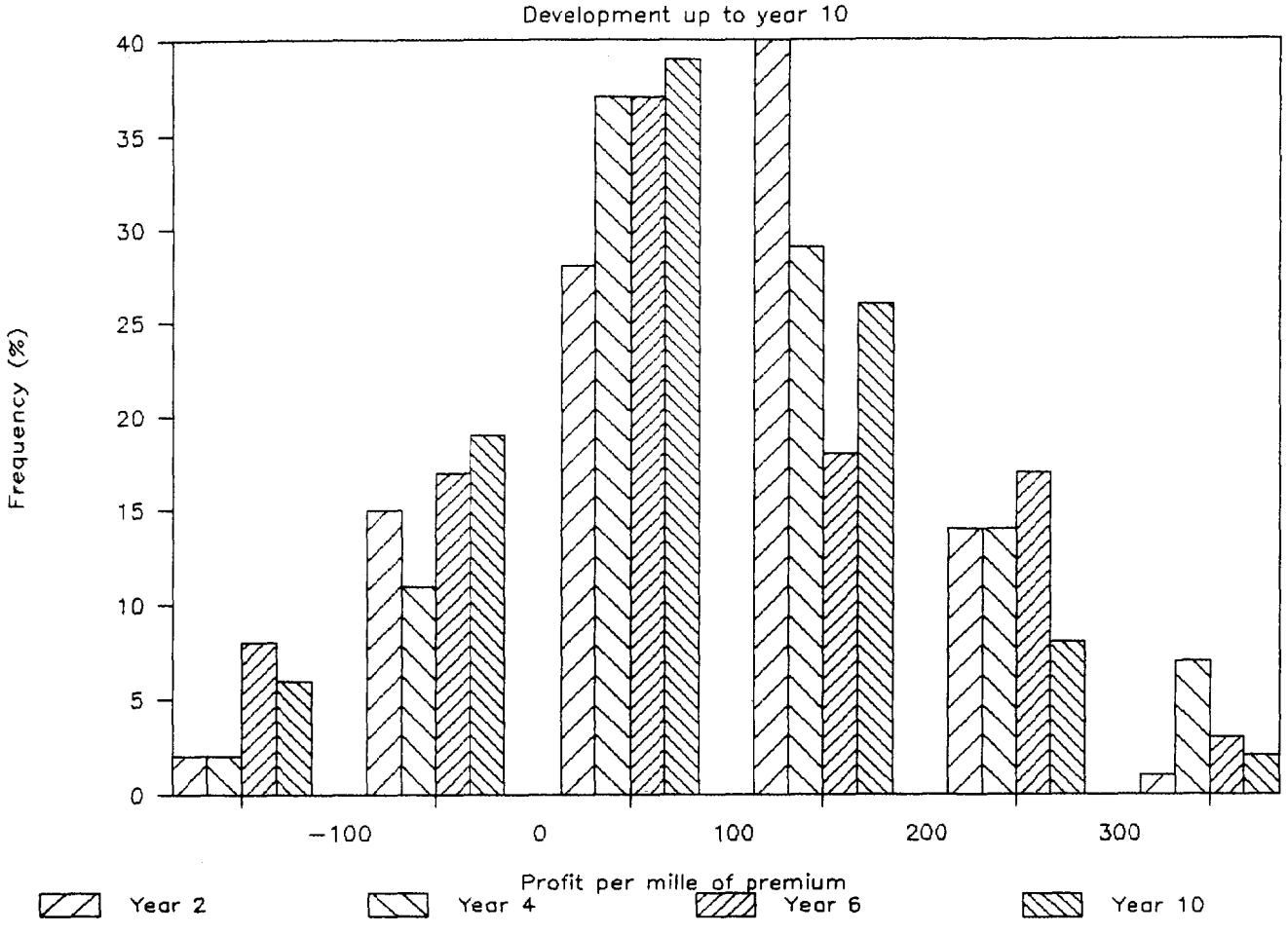

Figure 4 Development of mean asset margin

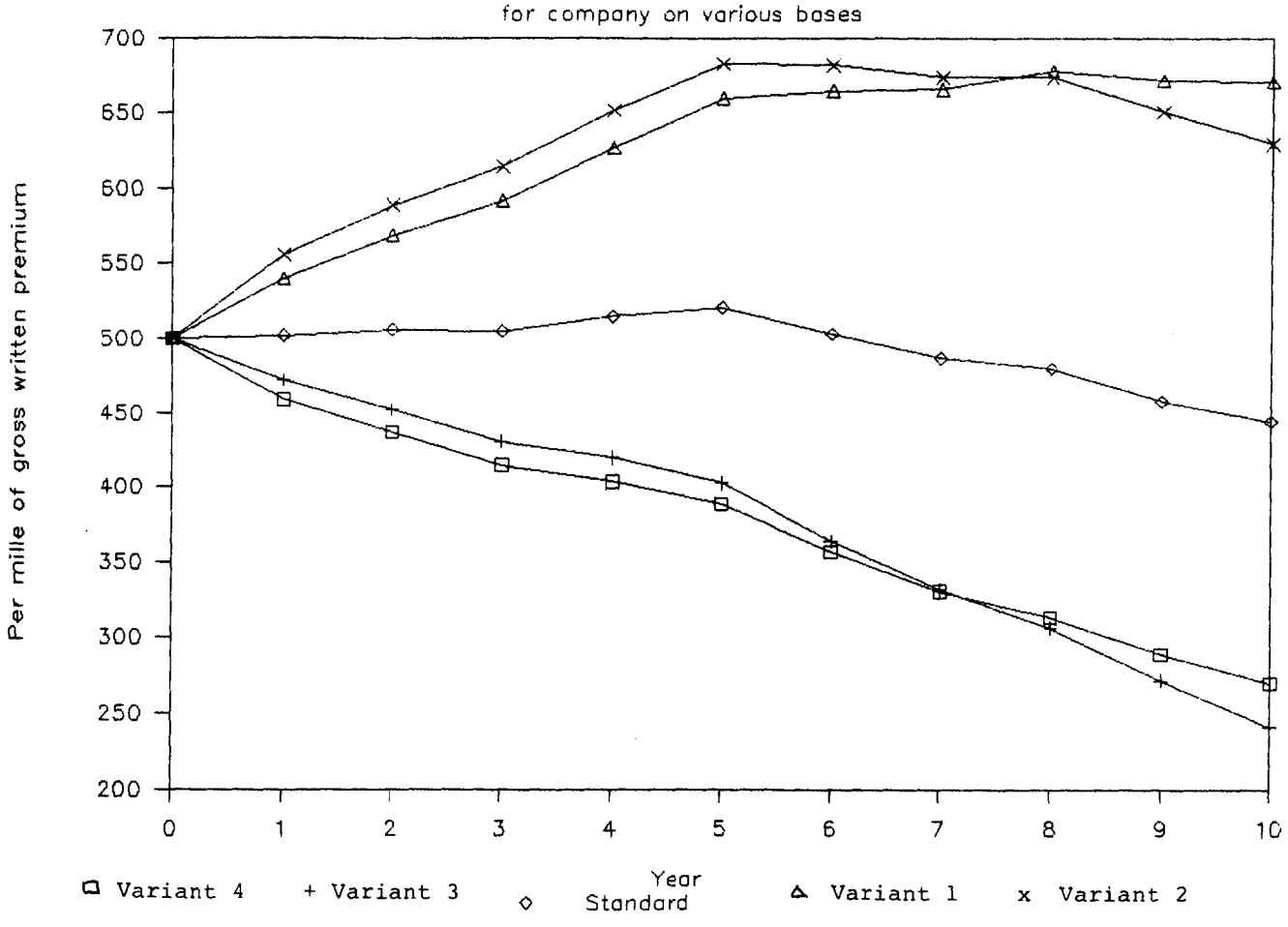

Figure 5

Development of mean overall profit

for company on various bases