

A Study of System Splitting Strategies for Island Operation of Power System: A Two-Phase Method Based on OBDDs

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Abstract—System splitting problem, also known as controlled system separation problem, is to determine the proper splitting points for splitting the entire power network into islands when island operation of system is unavoidable. By “proper” we mean that the splitting strategies should guarantee both the power balance and satisfaction to capacity constraints of transmission lines and other facilities in each island. The system splitting problem is very hard because the strategy space is huge for even middle-scale power networks. This paper proposes a two-phase method to search for proper splitting strategies in real-time. The method narrows down the strategy space using highly efficient OBDD-based algorithm in the first phase, then finds proper splitting strategies using power-flow analysis in the reduced strategy space in the second phase. Simulation with symbolic model checking tool SMV indicates that this method is very promising.

Index Terms—Graph theory, island operation, OBDD, system splitting, splitting strategy.

I. INTRODUCTION

SYSTEM splitting, also known as controlled system separation, is to split an interconnected transmission network into islands of load with matched generation at proper splitting points by opening selected transmission lines. Applied together with load shedding and perhaps generator dropping, each island of load and generation would theoretically remain in balance, thus avoiding cascading instability or even blackout of the entire system. The studies of historic blackouts or outages, such as the November 9, 1965, Northeast USA power interruption [1], the blackout of New York’s power in July 1977[2], the July 2, 1996, cascading outage of Western USA power system [3], the Brazilian blackout on March 11, 1999[4], etc., show that if proper system splitting combined with load shedding and generator dropping had been performed in time, some blackouts should have been avoided and much of the losses should have been reduced. Active and viable system splitting can efficiently avoid blackout of the entire power system and is typically better than passive system islanding. One reason is that after split-

ting, “although the power system is operating in an abnormal degraded state, customers are continuing to be served” [19].

Since power system emergencies may strike suddenly and cripple the system within a minute or even seconds, the system splitting should be completed within such a short period. It is necessary to guarantee both speediness and correctness in determining the splitting strategy to avoid more serious and catastrophic faults in time. To solve a real splitting problem in real-time, at least two challenges must be faced. First, we have to find a splitting strategy corresponding to an acceptable steady-state operating point after splitting (satisfying short or long term “emergency” limits), such that the system can avoid collapse. This is a strategy search procedure. Second, we have to make sure that the system can securely reach that steady-state operating point. This is a control or simulation procedure. Both procedures are quite challenging. To our best knowledge, almost all existing literatures focus on the control or simulation procedure. Among them, detecting islanding and determining asynchronous groups of generators [16], [25], [26], dynamic analysis and stabilization control of subsystem in each island [16], [17], [19], [23] were studied. There is little effort on addressing the strategy search procedure due to its combinatorial explosion.

In this paper, we focus on finding only the splitting strategies satisfying steady-state constraints because: 1) the steady-state constraints are necessary and the most important conditions and, in fact, all strategies satisfying steady-state constraints give acceptable steady-state operating points; 2) after system splitting according to a splitting strategy satisfying steady-state constraints, available stabilization measures (e.g., island AGC function [19]), etc., can make each island system reach a acceptable steady-state operating point. Therefore transient dynamics of the system after splitting is not considered in this paper.

Thus, in this paper, the so-called system splitting problem refers to the problem only to determine proper splitting points (i.e., a splitting strategy), when island operation of the system is unavoidable. By “proper” we mean that the splitting strategies should satisfy both the power balance and capacity constraints of transmission lines and other facilities in each island.

Thus, in this paper, we propose a two-phase method to find all proper splitting strategies in real-time. In the first phase, strategies satisfying power balance condition for islands are found by employing highly efficient (ordered binary decision diagram [5]) OBDD-based algorithm on a node-weighted graph model. The graph model omits all irrelevant information of the power system except the topology of the network and power generation

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or load. In the second phase, power-flow analyses are used to exclude the strategies violating transmission capacity constraints. The output of our two-phase method will be the strategies corresponding to acceptable steady-state operating points. It should be clear that a proper splitting strategy perhaps does not automatically make the system reach an acceptable steady-state operating point because transient period is not considered in our method. Hence, other control measures often have to be applied. In practice, when system splitting is unavoidable, one can use our method to obtain the set of all proper splitting strategies, which provide the decision maker or system dispatchers the freedom to select a final good strategy according to additional criteria such as easy recovery to normal operation state and economical considerations. Further detail simulation on transient dynamics corresponding to the chosen splitting strategy and furthermore control measures may also be needed when other major practical constraints such as the bus voltage constraints are crucial.

Simulation results employing the popular symbolic model checker symbolic model verifier (SMV) [6], [7] and power-flow analyses show that our method can quickly find all proper splitting strategies which can guarantee both the power balance in each island and satisfaction to the capacity constraints of transmission lines and other facilities.

The rest of the paper is arranged as follows. Section II defines the system splitting problem and introduces a node-weighted graph model to express a power network. We also present the framework of our two-phase method for the system splitting problem in this section. An implementation of the OBDD-based method with SMV and power-flow analysis is given in Section III. In Section IV, the method is applied to the IEEE 30-bus test network. Section V provides some concluding remarks and discussions on further work.

II. SYSTEM SPLITTING PROBLEM AND A FRAMEWORK OF THE TWO-PHASE METHOD

In this section, we will formulate the system splitting problem. A framework of our two-phase method to solve the system splitting problem will also be presented.

A. System Splitting Problem

In the state transition diagram of power system depicted by Elgerd [9], normally, a power system works in its normal state at a certain security level. In this state, the frequency and bus voltages are kept at prescribed values and all generators run synchronously. Both equality and inequality constraints must be satisfied. “Equality,” also known as power balance, means that the total system generation equals total system load. “Inequality” means that generator and transformer loads must not exceed the rated values and transmission lines cannot be loaded above their thermal or static stability limits. After some contingencies, the security level may fall or, even worse, one or several components of the system may be overloaded. Thus, some inequality constraints are violated and the system enters the emergency state. If emergency control actions (see [20]–[22], [24]), that is, disconnection of faulted section or rerouting of power or load shedding, should fail, the system will go into the ex-

tremitis state. In this state, one or several generators may become asynchronous with the others and no protection measures can keep the integrity of power system. One of the most efficient measures, known as controlled system separation, is to properly split the transmission network into two or several isolated subsystems like “islands” by the tripping relay and protective devices on the system to avoid passive collapse or blackout of the entire power system. Before island operation, the system splitting problem, which asks to find a proper splitting strategy, must be solved. Here, by “proper” we mean that the “equality” and “inequality” constraints are satisfied: in each island, power should be balanced and currents and voltages should be kept within rated limits; in other words, power system in each island can work in an acceptable state after splitting. After faults are cleared, the whole system will return to normal state by restoration operation such as resynchronization of separated islands and some other actions. It is helpful for making the decision of system splitting to install power swing detectors together with the associated out of step protection on crucial lines and the lines with higher probability of tripping such as exposed lines [18], by which the instability and asynchronization of generators are observed much easier.

Let us consider a five-bus power system shown in Fig. 1 to illustrate what is meant above. The IEEE 30-bus system is used in Section IV to illustrate the capacity of our method. Assume that the complex generations and demands of all buses satisfy

$$S_G^1 + S_G^2 - S_L^1 - S_L^4 \approx 0, \quad S_G^3 - S_L^3 - S_L^5 \approx 0. \quad (1)$$

Suppose a fault occurred on the line l_{23} at $t = 0$ second, generator 3 may become asynchronous with generators 1 and 2. Fig. 2 gives the angular rotor swings of three generators.

When emergency control actions failed and the system went into the extremis state, one of the most efficient measures is to open the circuit breakers on transmission lines l_{23} and l_{45} to split the system into two islands. Thus, balanced subsystems are created due to (1). Then generators in each island can work synchronously at a frequency near the original system frequency. If the powers in transmission lines l_{12} , l_{14} , l_{24} , and l_{35} are below their thermal or static stability limits, both of the two islands will work in an acceptable state. They may be reconnected and resynchronized by restoration operation after the fault is removed from the system.

From the above discussion, it is clear that the following constraints must be satisfied after system splitting.

- 1) Asynchronous groups of generators must be separated into different islands and generators in each island must be synchronous. (We call this condition *separation and synchronization constraint* or SSC.)
- 2) In each island, power is balanced. This means that power generation roughly equals power load. [This is just the equality constraint and we call it *power balance constraint* (PBC)].
- 3) Transmission lines and other transmission services must not be loaded above their transmission capacity limits (e.g., thermal capacity limits and steady-state stability limits). (They are inequality constraints and we call them *rated value & limit constraints* or RLC).

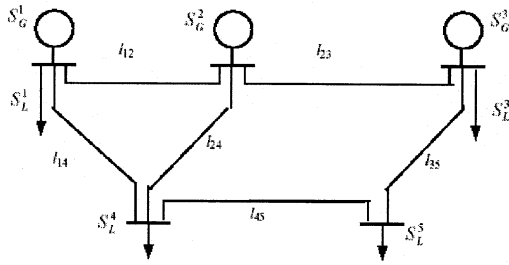


Fig. 1. Five-bus power system.

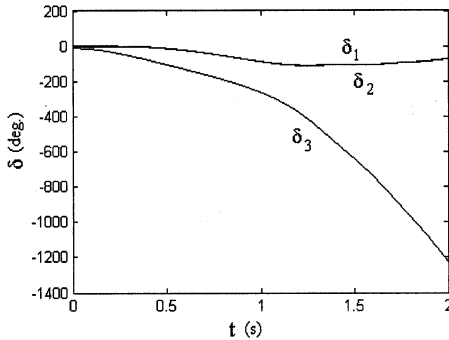


Fig. 2. Angular rotor swings of three generators.

With these constraints, the system splitting problem can be formulated as follows.

1) *System Splitting Problem*: When system separation is unavoidable, is there a strategy for determining splitting points to meet SSC, PBC, and RLC after system splitting?

Since the constraints SSC, PBC, and RLC concern only steady-state stability of islands after splitting, they are necessary conditions for successful system separation. We will use the term *proper splitting strategies* in this paper to refer to the strategies satisfying all these three kinds of constraints. Decision maker can locate correct splitting points by evaluating only proper splitting strategies instead of all strategies through sophisticated and time consuming procedure such as simulation. Because power system emergencies may strike suddenly and cripple the system within a minute or even seconds, the system splitting should be completed within such a short period. It is necessary to guarantee both speediness and correctness in solving the system splitting problem because a tardy or wrong strategy may cause more catastrophic faults and even blackout. In the above five-bus power system example, if only the transmission lines l_{23} and l_{35} are opened before any other actions, and generator dropping and load shedding are not performed immediately, it will lead to two unbalanced subsystems and even lead to a blackout of the whole system. The Brazilian blackout on March 11, 1999 [4] was a real story for incorrect operations of relays leading to unbalanced islands and even outage of system.

B. Node-Weighted Graph Model for System Splitting Problem

To solve the system splitting problem, we draw an undirected, connected, and node-weighted graph $G(V, E, W)$ for a networked power system. It is the model encoding the static system information for SSC and PBC. $V = \{v_1, \dots, v_n\}$ is

a set of nodes where v_i stands for a node called a bus in the power network. E is a set of unordered node pairs, called edges. Its element e_{ij} ($i < j$) represents a transmission line connecting bus i and bus j . Here we assume that there is only one transmission line connecting every pair of buses. $W = \{w_1, \dots, w_n\}$ is a set of weights of all nodes. The weight w_i of Node v_i stands for the sum of the injected power from the devices (generators or loads) on the bus i into network. The weight w_i can be determined as

$$w_i = \sum_k S_{G,k}^i - \sum_l S_{L,l}^i \quad (2)$$

where $S_{G,k}^i$ and $S_{L,l}^i$ are the complex powers of generator k and load l on bus i . Let $S_G^i = \sum_k S_{G,k}^i$ and $S_L^i = \sum_l S_{L,l}^i$. If the power in the network is balanced and losses of power network are omitted, ideally w_i should satisfy

$$\sum_i w_i = 0. \quad (3)$$

The node-weighted graph model of the five-bus power system above is shown in Fig. 3, where black dots denote the nodes connected with generators (perhaps also loads) and white dots denote the nodes only connected with loads, where

$$w_1 = S_G^1 - S_L^1, w_2 = S_G^2, w_3 = S_G^3 - S_L^3, \\ w_4 = -S_L^4 \quad \text{and} \quad w_5 = -S_L^5.$$

The node-weighted graph model of the well-known IEEE 30-bus system is shown in Fig. 4, where white dots and black dots are as just described and the gray dots denote, for example transformer substations, the nodes not connected with generators and loads.

Let V_G be a set of all the nodes connected with generators and V_L be a set of the other nodes. So we have

$$V = V_G \cup V_L. \quad (4)$$

In Fig. 4, $V_G = \{v_1, v_2, v_3\}$ and $V_L = \{v_4, v_5\}$.

Assume that when some faults occur, generators are found to be divided into several asynchronous groups. Let V_G^u , $u = 1, \dots, U$ denote the node sets that correspond to these groups of generators. It holds that

$$V_G = \bigcup_u V_G^u. \quad (5)$$

If we separate the groups of generators with matched loads from the rest of the system in a proper way, generators in each island will be synchronous and customers are continuing to be served, which achieve exactly the aim of system splitting.

We introduce the following graph partition problem focusing only on the constraints SSC and PBC for system splitting.

1) *BALANCED PARTITION Problem (BP Problem)*: Given an undirected, connected, and node-weighted graph $G(V, E, W)$ and U subsets V_G^u ($u = 1, \dots, U$), of V , is there a cut set $E_C \subset E$ to split the graph G into U subgraphs

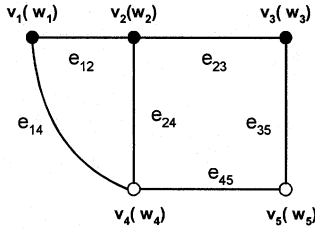


Fig. 3. Graph model of the five-bus power system.

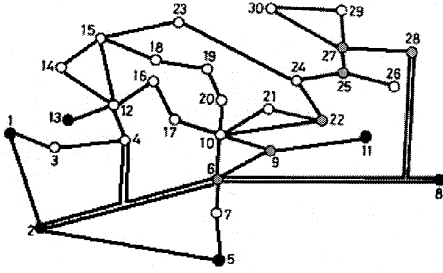


Fig. 4. Graph model of the IEEE 30-bus network (The weight of each node is not shown.)

$G^u(V^u, E^u, W^u)$ ($u = 1, \dots, U$) (SSC) such that $V_G^u \subset V^u$, and the following constraint (PBC) are satisfied

$$\left| \sum_{v_i \in V^u} w_i \right| < d, \quad u = 1, \dots, U, \quad (6)$$

where $d > 0$ is a small positive constant mainly reflecting the tolerance on power unbalance in each island. In order to guarantee that the frequency offset in each island after system splitting is not too large (normally, 1–2 Hz), d must be small enough, the value of which can be estimated from speed-power curve easily.

Remark: In the BP problem formulation, all edges are considered for possible cutoff. In practice, additional information on possible splitting points is helpful to restrict the search scope of cut sets. The algorithm that follows can be extended directly to handle such restrictions.

Obviously, the BP problem is a relaxed version of the system splitting problem since a solution to the BP problem corresponds to a strategy of system splitting satisfying SSC and PBC but perhaps not satisfying the RLC. The system splitting problem will be shown hard by the following complexity result for the BP problem.

Theorem 1: BP problem is NP-complete.

Proof: See Appendix.

Noting that when there is enough capacity in transmission lines and other facilities (RLC is automatically satisfied), (i.e., SSC and PBC are the only conditions to be satisfied by proper splitting strategies), we have the following corollary directly from theorem 1. \square

Corollary 1: The system splitting problem is NP-hard.

Remark: NP-completeness and NP-hardness¹ of a problem are concepts used by computer scientists to quantify the difficulty of problems. The purpose here is to justify theoretically the fact that a large number of possible ways to select splitting strategies contributes essentially to the computational challenge in solving a real splitting problem. In other words, no method can be successful if the huge search space problem is not suitably addressed. The formal definitions of these terms can be found in the standard [12] and will not be presented here because they need too many background terminologies of computational complexity theory.

III. A TWO-PHASE METHOD FOR SYSTEM SPLITTING PROBLEM

As shown in the previous section, for large-scale power systems, their splitting problem is very hard because of combinatorial explosion of the search space. This is justified by looking at the strategy space of the IEEE 30-bus network with 41 lines. There are $2^{41} \approx 2.2 \times 10^{12}$ possible choices for system splitting. To speed up the solving of the system splitting problem, we propose the following two-phase method.

- 1) Identify a new strategy satisfying SSC and PBC using node-weighted graph model by OBDD-based satisfiability checking algorithm. In this phase, the BP problem will be solved. If no further strategy is found, the whole search procedure stops.
- 2) Check RLC for the strategy given by phase 1) using power-flow analysis. If the strategy satisfies RLC, output it as a proper splitting strategy. If not, go to phase 1).

Remark: The set of all proper splitting strategies can be obtained in phase 2) when the two steps are applied repeatedly. As we have pointed out in the introduction that these strategies provide the decision maker or system dispatchers the freedom to select a final good strategy according to additional criteria such as easy recovery to normal operation state and economical considerations. Further detail simulation on transient dynamics corresponding to the chosen splitting strategy and more control measures may also be needed when other major practical constraints are considered.

The speediness potential of our method lies in the fact that the search space can be efficiently reduced. The flowchart of our method and the reduction process of the search space are shown in Fig. 5, where the black ellipse denotes the set of proper splitting strategies. The two phases of our method are realized individually in two modules: satisfiability checking module and power-flow analysis module.

A. Satisfiability Checking Module

Since the BP problem is NP-complete, heuristic algorithms based on the specific information of the problem have to be used to speed up the solution algorithm. Since a major part of the information of the BP problem is encoded in the relationship of a large number of decision variables, it is reasonable to believe that a good representation of this relationship can improve

¹Informally, an NP problem is a problem such that each of its candidate solution can be tested to be true or false within polynomial time. NP-complete problems are most difficult NP problems. NP-hard problems are general problems that are at least as difficult as the NP-complete problems.

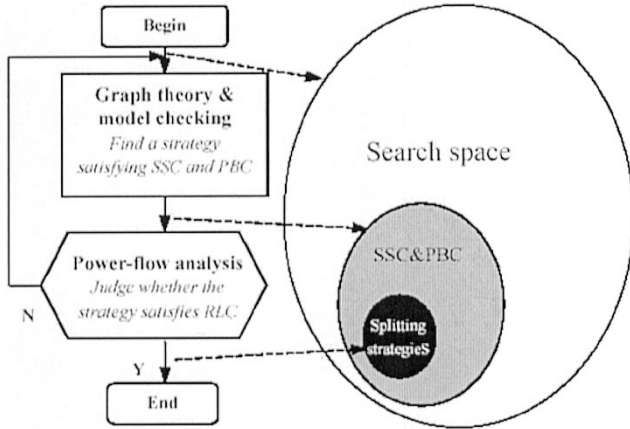


Fig. 5. Flowchart of the method and the reduction of search space.

the solving efficiency by orders of magnitude. The OBDD technique is a representation whose power has been demonstrated through large industry examples [8]. It enables us to solve very large BP problems in real time. We use problem specific information through efficient transformation of the BP problem into a satisfiability checking problem which can be solved by OBDD-based model checking tools such as symbolic model verifier (SMV). The main advantages of using OBDD-based methods for the BP problem are

- 1) OBDD representation provides a systematic way of exploring structural information in the problem. Very complex truth table of Boolean functions may have very concise OBDD representation.
- 2) The BP problem is a typical problem of satisfaction checking.
- 3) They check the whole strategy space. So if there is a solution, they will never miss it.

Remark: The feature 3) is different from many stochastic search methods which can draw no conclusion when the tools fail to find a solution. There always might be a solution that the stochastic search tools are not lucky enough to hit.

In applying model checking tools, we have to transform the BP problem into a satisfiability checking problem, namely we verify the BP problem against some specifications. Counterexamples should correspond to the feasible splitting strategies that are solutions to the BP problem. To do so, we choose SMV as our model checker and define semicertain Boolean variable matrix and semicertain adjacency matrix.

Definition 1 (SBVM, Semicertain Boolean Variable Matrix): A $n \times n$ matrix is called semicertain Boolean variable matrix if its elements are 0 or a Boolean variable (only taking value 0 or 1).

Denote the set of all $n \times n$ SBVM's by $SBVM(n)$.

Definition 2 (SAM, Semicertain Adjacency Matrix): Given an n -node, undirected, and connected graph $G(V, E)$, its semicertain adjacency matrix A_G is a symmetric SBVM $(a_{ij})_{n \times n}$ such that a_{ij} is an undetermined Boolean variable b_{ij} if there is an edge from v_i to v_j and $a_{ij} = 0$ otherwise. And $\forall i \neq j, b_{ij} = b_{ji}$. Dually, a graph G determines and is determined by its semicertain adjacency matrix A_G .

Obviously, there is one-to-one correspondence between all b_{ij} and e_{ij} . Every b_{ij} denotes a decision variable². To decide its value 1 or 0 is to decide “on” or “off” of e_{ij} or the transmission line connected node i with j after splitting. So to split a power network or a graph G is to decide the values of all $b_{ij}(i, j \in \{1, \dots, n\})$. For example, SAM of the graph in Fig. 3 is

$$A_G = \begin{bmatrix} 0 & b_{12} & 0 & b_{14} & 0 \\ b_{12} & 0 & b_{23} & b_{24} & 0 \\ 0 & b_{23} & 0 & 0 & b_{35} \\ b_{14} & b_{24} & 0 & 0 & b_{45} \\ 0 & 0 & b_{35} & b_{45} & 0 \end{bmatrix}. \quad (7)$$

We use a Boolean Matrix $\hat{M} \in \{0, 1\}^{n \times n}$ to denote a decision of a SBVM M . Consider the SAM A_G in (7). If we want to separate node 3 and 5 from the rest of system, $\{e_{23}, e_{45}\}$ will be the cut set E_C . So a possible and the only decision of A_G is

$$\hat{A}_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \quad (8)$$

In the following text, we give what kinds of decisions of all b_{ij} or what kinds of \hat{A}_G s are the possible strategies for system splitting. We use symbol \oplus for the “OR” operation and symbol \otimes for the “AND” operation. Then, for all $P, Q \in SBVM(n)$, we have

$$(P \otimes Q)_{ij} \stackrel{\text{def}}{=} \oplus_k [(P)_{ik} \otimes (Q)_{kj}] \quad (9)$$

$$P^2 = P \otimes P, \dots, P^{k+1} = P^k \otimes P. \quad (10)$$

From Boolean algebra and Boolean matrix theory [13], it is clear that for A_G of graph G , the entry $(A_G^k)_{ij}$ is a polynomial of operations “ \otimes ” and “ \oplus ” representing all paths of length (number of edges) no more than k from node v_i to node v_j ; especially when there is no such path, $(A_G^k)_{ij}$ will be 0. To separate nodes v_i and v_j , the decision variables have to be chosen such that the expression for all paths are zero simultaneously.

Let us consider the graph of five-bus power network shown in Fig. 3. From v_1 to v_4 , we have three paths of length ≤ 4 , namely e_{14} , $e_{12}e_{24}$, and $e_{12}e_{23}e_{35}e_{45}$. This is encoded as

$$(A_G^4)_{14} = b_{14} \oplus (b_{12}b_{24}) \oplus (b_{12}b_{23}b_{35}b_{45}), \quad (11)$$

where “ \otimes ” is omitted. It is clear that a way to separate v_1 and v_4 , is to set b_{14} and b_{12} as 0, in other words, to cut e_{14} and e_{12} .

It can be seen that if the longest path in G is of length L and the longest path from v_i to v_j is of length k , we have

$$(A_G^L)_{ij} = (A_G^k)_{ij}, \quad (\forall l > k) \quad (12)$$

$$A_G^L = A_G^L, \quad (\forall l > L). \quad (13)$$

²When information on possible splitting points is available, some b_{ij} may no longer be decision variables. Their value will be stuck to 1 or stuck to 0 according to this information with 1 as “must (or prefer) not cut” and 0 as “must cut.” The decision maker’s preference on priorities for cutting different edges can also be modeled by forcing low priority edges stuck to 1 during the early search period.

We define

$$A_G^* \stackrel{\text{def}}{=} I \oplus A_G \oplus A_G^2 \oplus \cdots \oplus A_G^L, \quad (14)$$

where I refers to the identity matrix; 1 on the diagonal and 0 elsewhere. If there is a Hamiltonian path in G , $L = n - 1$, otherwise $L < n - 1$. For the convenience in expression, we define U sets

$$I_u = \{i \mid v_i \in V_G^u\}, \quad u = 1, \dots, U. \quad (15)$$

Then the constraints SSC and PBC can be expressed as

$$\begin{aligned} SSC = & [(\forall u \in \{1, \dots, U\})(\exists g \in I_u)(\forall h \in I_u)((A_G^*)_{hg} = 1)] \\ & \otimes \left[\forall k \in \{1, \dots, n\} \setminus (I_1 \cup \cdots \cup I_U)(\exists g_u \in I_u, \right. \\ & \left. u = 1, \dots, U) \left(\sum_{u=1, \dots, U} (A_G^*)_{g_u k} = 1 \right) \right] \end{aligned} \quad (16)$$

where " $A \setminus B$ " means the set of elements in A but not in B . In (16), $(\exists g \in I_u)(\forall h \in I_u)((A_G^*)_{hg} = 1)$ says that there is a node g in group I_u s.t. all nodes in I_u are connected to it; $[\forall k \in \{1, \dots, n\} \setminus (I_1 \cup \cdots \cup I_U)(\exists g_u \in I_u, u = 1, \dots, U)(\sum_{u=1, \dots, U} (A_G^*)_{g_u k} = 1)]$ says that every node not in $I_1 \cup \cdots \cup I_U$ connects and only connects to one of $\{g_u, u = 1, \dots, U\}$.

When SSC is true, we have for PBC

$$PBC = (\forall u \in \{1, \dots, U\})(\exists g \in I_u)(|(A_G^*)_{g*} \cdot W| < d) \quad (17)$$

where $(A_G^*)_{k*}$ is the k -th row of A_G^* and W is the weight vector of nodes. The inner product $(A_G^*)_{g*} \cdot W \equiv \sum_i (A_G^*)_{gi} w_i$ is the algebraic sum of the weights (in words, sum of the power) of all nodes connected to node g . So it is clear that (17) is modeling the request (6) of power balance. Combing (16) and (17) together, we have

$$\begin{aligned} SSC \otimes PBC = & [(\forall u \in \{1, \dots, U\})(\exists g \in I_u)(\forall h \in I_u)((A_G^*)_{hg} = 1) \\ & \otimes (|(A_G^*)_{g*} \cdot W| < d)] \otimes [\forall k \in \{1, \dots, n\} \setminus (I_1 \cup \cdots \cup I_U) \\ & \otimes (\exists g_u \in I_u, u = 1, \dots, U) \left(\sum_{u=1, \dots, U} (A_G^*)_{g_u k} = 1 \right)]. \end{aligned} \quad (18)$$

Remark: In (18), we only need calculate U rows the g_1 -th, \dots , g_U -th rows of elements in A_G^* .

In satisfiability checking for (18), we will find out all decisions \hat{A}_G of A_G satisfy the specification $SSC \otimes PBC$ or obtain a counterexample violating the specification $SSC \otimes PBC$. Our specification of (18) will be in form of the temporal logic computation tree logic (CTL) [6]. We choose the popular tool SMV system as our model checking tool. SMV employs the highly structured symbolic representation of Boolean functions—OBDD instead of explicit storage of individual strategies. The check of specifications is directly accomplished through manipulations on the OBDDs [5]. Cadence SMV [7] is

used as our programming language. The program for describing the BP problem has the following structure.

- 1) initialize A_G whose undetermined Boolean variable elements are as many as the edges in graph G ;
- 2) calculate A_G^* and $SSC \otimes PBC$;
- 3) use the following syntax to verify the expression:

$$\neg(SSC \otimes PBC) : \text{assert } G\neg(SSC \otimes PBC);$$

where " G " means "globally," " \neg " means "NOT." The program running SMV tries all possible values of A_G to verify the assertion. If there is a counterexample \hat{A}_G , SMV will stop and give \hat{A}_G satisfying constraints SSC and PBC . From \hat{A}_G , we can obtain a cut set which is a solution to the BP problem. Normally, load shedding and generator tripping are allowed at some nodes when the system is pushed into the emergency state. To model these different choices, we assume each weight of the corresponding nodes may take value only from a finite set. In other words, each weight w_i of the nodes is treated as a variable taking value from a finite set in the SMV program. The model checker will select proper weight for each node when finding solutions of the BP problem.

B. Power-Flow Analysis Module

In this module, for a solution of the BP problem given by the satisfiability checking module, the power in each transmission line is obtained by power-flow calculation, and then whether RLC is satisfied and whether the solution is a proper splitting strategy will be known.

When system splitting must be performed immediately to avoid outage or blackout, speediness is more important than accuracy for power-flow calculation. So high-accuracy of the calculation is not necessary in this module. Thus, we prefer to use some approximate calculations to speed up the power-flow calculation process. But in order to ensure the safety of the splitting strategies given by the module, the errors of approximate calculations must be considered. One of the efficient measures is to strengthen the security limits of transmission lines and other facilities to ensure certain margin of safety.

In real power networks, local reactive power compensators can compensate reactive power for unbalance. And the balance of the real power in each island is more important after system splitting. Thus, *dc power-flow method*, a fast approximate method of power-flow calculation, can be used in this module. As a result, w_i is redefined as a real number or a set of real numbers (for the nodes allowing load shedding and generator dropping). Equation (2) becomes

$$w_i = \sum_k P_{G,k}^i - \sum_l P_{L,l}^i \equiv P_i. \quad (19)$$

Normally, when an n_I -bus island reached a acceptable steady-state point after system splitting, its bus voltage $V_i = 0.95\text{--}1.05V_0$ ($i = 1, \dots, n_I$) where V_0 is rated voltage of system, so we assume voltages of all nodes are equal approximately, that is $|V_i| = V_0$ ($i = 1, \dots, n_I$), and the difference between receiving-end power angle δ_i and sending-end power angle δ_j of any line l_{ij} is very small ($|\delta_i - \delta_j| < 20^\circ$) and

for the line impedance z_{ij} , the resistance r_{ij} is far less than the reactance x_{ij} ($x_{ij} > 2r_{ij}$). Then, the transmission complex power S_{ij} is

$$\begin{aligned}
 S_{ij} &= V_i I_i^* \\
 &= V_i \left(\frac{V_i - V_j}{z_{ij}} \right)^* \\
 &= \frac{|V_i|^2}{z_{ij}^*} - \frac{V_i V_j^*}{z_{ij}^*} \\
 &= \frac{V_0^2}{z_{ij}^*} (1 - \cos \delta_{ij} - j \sin \delta_{ij}) \\
 &= \frac{V_0^2}{|z_{ij}|} (1 - \cos \delta_{ij} - j \sin \delta_{ij}) (\cos \theta_{z_{ij}} + j \sin \theta_{z_{ij}}).
 \end{aligned} \tag{20}$$

The transmission real power P_{ij} is

$$\begin{aligned}
 P_{ij} &= \frac{V_0^2}{|z_{ij}|} [\cos \theta_{z_{ij}} (1 - \cos \delta_{ij}) + \sin \theta_{z_{ij}} \sin \delta_{ij}] \\
 &\doteq \frac{V_0^2}{|z_{ij}|} \sin \theta_{z_{ij}} \cdot \delta_{ij} \\
 &= \frac{V_0^2}{|z_{ij}|} \sin \theta_{z_{ij}} (\delta_i - \delta_j) = -P_{ji}
 \end{aligned} \tag{21}$$

where $\theta_{z_{ij}}$ is the impedance angle or z_{ij} . From *Kirchhoff's current law*, we have

$$\begin{aligned}
 P_i &= \sum_{j, s.t. e_{ij}=1} P_{ij} \\
 &= \sum_{j, s.t. e_{ij}=1} \frac{\delta_i - \delta_j}{|z_{ij}|} \sin \theta_{z_{ij}} V_0^2, \quad (\forall i = 1, \dots, n_I)
 \end{aligned} \tag{22}$$

$$\sum_{i=1}^{n_I} P_i = 0. \tag{23}$$

Given real power of any $n_I - 1$ nodes and a reference power angle δ_0 , we can solve all δ_i by (22) and (23), and then we can calculate P_{ij} by (21). Because transmission power in all lines can be obtained by solving a group of linear equations including (22) and (23), the speed of calculation can be very fast. Normally, the error of the results are in 3%–10%.

Given the real power security limits of transmission lines $PSL_{ij}(i, j = 1, \dots, n_I)$ and considering the error of calculation above, the steps in the power-flow analysis module are

- 1) obtain a strategy S form model checking module;
- 2) calculate P_{ij} by (21)–(23) for S ;
- 3) if $P_{ij} < \alpha \cdot PSL_{ij}, (\forall e_{ij} \in E), S$ is a proper splitting strategy; otherwise, turn back to satisfiability checking module.

Here, α is a safety coefficient for 0.7–0.9.

IV. SIMULATION RESULTS

An adaptation of the IEEE 30-bus standard test network (shown in Fig. 4) is used to demonstrate the performance of the method. We use SMV (version 2.5.4) system on PC

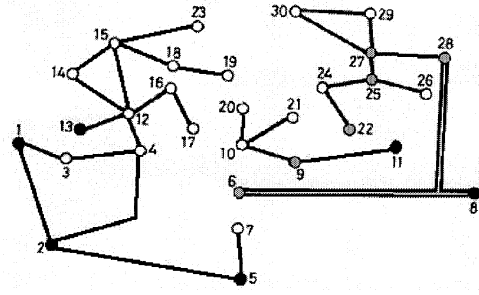


Fig. 6. Splitting strategy (no. 2: Cut 2–6, 4–6, 6–7, 6–9, 6–10, 10–17, 10–22, 19–20, 21–22, 23–24).

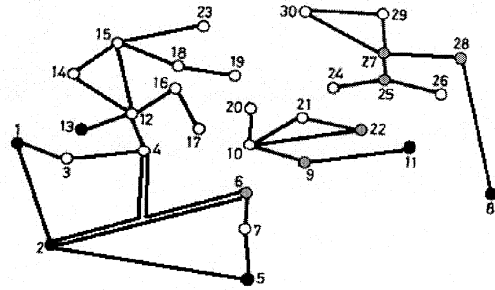


Fig. 7. Splitting strategy (no. 6: 6–8, 6–9, 6–10, 6–28, 10–17, 19–20, 22–24, 23–24).

with Pentium III-1 GHz CPU. The generation P_G of each generator is shown in Table I and the real power security limit PSL of each transmission line is shown in Table II. We select $V_G^1 = \{v_1, v_2, v_5, v_{13}\}, V_G^2 = \{v_8\}, V_G^3 = \{v_{11}\}$, and $d = 10$ MW. To investigate the performance of our method, we ask all proper splitting strategies. In the first phase, satisfiability checking module found 48 solutions of the BP problem. The benefit of our two-phase method can be seen clearly at this point. In order to find a proper splitting strategy, we need only consider 48 candidates instead of all 2^{41} strategies. This is a very big saving in time. In the second phase, if the safety coefficient $\alpha = 0.9$ in power-flow analysis module, there will be six solutions satisfying $RLC(\forall i, j, P_{ij} < \alpha PSL_{ij})$ which are shown in Table III. We use Δ to denote the degree of unbalance in each island

$$\Delta = \max_k \left| \sum_{i \in V_k} P_i \right|. \tag{24}$$

Two of the feasible splitting strategies are shown in Figs. 6 and 7, respectively.

Computing time for each step in simulation is listed in Table IV. Note that the OBDD generation and optimization for a given graph $G(V, E, W)$ are performed independently of the checking of satisfiability. Fortunately, OBDD presentation has a remarkable advantage: the OBDD of a problem can be built by stages, in other words, the final OBDD can be obtained by combining the OBDDs of all subproblems. That implies that, if there is no change in some graph data of V, E and W , the corresponding some OBDDs can be built offline and then used to form the final OBDD online. Thus, only the calculations dynamically depending on what and where the faults are must be executed online. Thus, in fact, the total online computing

TABLE I
GENERATOR DATA

Bus No.	1	2	5	8	11	13
P_G (MW)	94	52	48	55	18	17

TABLE II
BRANCH DATA

Bus No's	PSL (MW)	Bus No's	PSL (MW)	Bus No's	PSL (MW)
1-2	130	4-12	65	21-22	32
1-3	130	12-13	65	15-23	65
2-4	65	12-14	32	22-24	32
3-4	130	12-15	65	23-24	65
2-5	130	12-16	32	24-25	32
2-6	65	14-15	16	25-26	16
4-6	65	16-17	16	25-27	16
5-7	65	15-18	32	28-27	65
6-7	130	18-19	16	27-29	16
6-8	32	19-20	32	27-30	16
6-9	65	10-20	32	29-30	16
6-10	32	10-17	32	8-28	32
9-11	65	10-21	32	6-28	32
9-10	65	10-22	32		

TABLE III
PROPER SPLITTING STRATEGIES

No.	Proper Splitting Strategies (Cut sets)	Δ (MW)
1	2-6, 4-6, 6-7, 6-8, 6-28, 10-17, 10-22, 19-20, 21-22, 23-24	8.3
2	2-6, 4-6, 6-7, 6-9, 6-10, 10-17, 10-22, 19-20, 21-22, 23-24	8.3
3	6-8, 6-9, 6-10, 6-28, 10-17, 10-22, 19-20, 21-22, 23-24	8.3
4	2-6, 4-6, 6-7, 6-8, 6-28, 10-17, 19-20, 22-24, 23-24	8.3
5	2-6, 4-6, 6-7, 6-9, 6-10, 10-17, 19-20, 22-24, 23-24	8.3
6	6-8, 6-9, 6-10, 6-28, 10-17, 19-20, 22-24, 23-24	8.3

TABLE IV
SIMULATION TIME

Steps	Computing time (s)
BDD generation and optimization	<2.47
Finding one <i>SSC&PBC</i> strategy	<0.03
Perform power-flow calculation one time	<0.01

time for finding out a proper splitting strategy for our example can be less than 2 s.

V. CONCLUSIONS AND FURTHER WORK

The method proposed here is just the beginning instead of a finishing point of solving the extremely important and challenging power system splitting problem. The contribution of our OBDD-based two-phase method lies in reducing the volume

of computation in a real time assessment of a proper splitting strategy. At this stage, it can be used as a tool to study the nature of proper splitting strategies. Further improvement and study of the method are possible.

It is well known that there are symbolic model checking methods without OBDD. Many of these methods are based on SAT (satisfiability of Boolean function). We did a simulation study for the IEEE30-bus example using bounded model checking (BMC) [27]. The result is rather discouraging. To use BMC, to our best knowledge, a (conjunctive normal form) CNF format version of the problem has to be generated. For the IEEE 30-bus example, although our SMV program is a short one, it takes enormous time to transform it into CNF format. The in-efficiency of an SAT-based model checker to our example might be due to the extra large size of the CNF description. In regarding the worst-case complexity nature of the system splitting problem, to apply our method successfully to large-scale system splitting, we summarize some of our experiences below with our future research topics.

1) Increase the offline work:

The No Free Lunch theorem [15] says: "Without specific structural analysis of the problem you studying, no algorithm can do any better than a blind search on the average." For a given power network, we need to analyze and make the most of its structural properties sufficiently to decrease the search space before beginning searching splitting strategies. In other words, we need to increase the offline work and decrease the online work. The following measures can narrow the search space efficiently:

- a) For the BP problem, if an edge e_{ij} has no effect on separating asynchronous groups of generators, e_{ij} should be directly set to 1 against an undetermined Boolean variable. Thus, the number of undetermined Boolean variables will be cut down. For IEEE 30-bus system shown in Fig. 4, obviously e_{12} and e_{25} have no effect on separating V_G^1, V_G^2 , and V_G^3 then e_{12} and e_{25} are set constantly to 1 all through the process of satisfiability checking.
- b) In $G(V, E, W)$, we use $d(i)$ to denote the degree of node i . If $d(i) = 1$ and node i is incident with the node j and w_i cannot take the value 0 (normally there is not a node in actual power networks whose degree is 1 and weight is always 0), we should cut off node i and let $w_j = w_j + w_i$. The "+" is addition of sets. For IEEE 30-bus system shown in Fig. 4, node 13 can be cut off and let $w_{12} = w_{12} + w_{13}$.
- c) More efforts should be given to gain a better understanding of the splitting problem so that OBDD-based method can be further acculturated with the help of other existing or yet to be invented satisfiability checking techniques.

2) Parallel search:

If the technology of parallel search is adopted, the process of searching will be completed much more quickly. This is one of our future research topics.

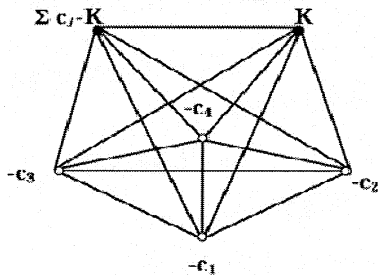


Fig. 8. BP problem constructed to solve 0-1 KNAPSACK problem ($n = 4$).

APPENDIX

To establish the complexity result, we need to cite the following well-known NP-complete problem.

Problem-3 the 0-1 KNAPSACK Problem: Given integers $c_j, j = 1, \dots, n$, and K , is there a subset S of $\{1, \dots, n\}$ such that $\sum_{j \in S} c_j = K$?

Remark: Our version of 0-1 KNAPSACK problem is slightly different from the one given in [11]. We require $c_j \geq 0$. However, this does not change the NP-completeness nature of the problem as stated in the following lemma.

Lemma 1: [11] The 0-1 KNAPSACK problem is NP-complete (Fig. 8).

Proof: The proof given in Theorem 15.8 of [11] is still valid for the case of $c_j \geq 0, j = 1 \dots n$.

Proof of Theorem 1: Obviously, the BP problem is in NP; we shall polynomially transform 0-1 KNAPSACK problem to the BP problem with $U = 2$. Given any instance c_1, \dots, c_n, K of 0-1 KNAPSACK, we construct the following instance of the BP problem. The undirected and node-weighted graph $G(V, E, W)$ with $n + 2$ nodes is fully connected (i.e., it is a complete graph) with node weights set as $w_i = c_i, i = 1, \dots, n, w_{n+1} = K - \sum_{j=1}^n c_j, w_{n+2} = -K$. Let $V_G^1 = \{v_{n+1}\}, V_G^2 = \{v_{n+2}\}$ and $d = 1$. An instance of $n = 4$ is shown in Fig. 5. We claim that there exists a subset S of $\{1, 2, \dots, n\}$ s.t. $\sum_{j \in S} c_j = K$ if and only if there exists a cut set $E_C \subset E$ to split G into two subgraphs $G^1(V^1, E^1, W^1)$ and $G^2(V^2, E^2, W^2)$ s.t. $v_{n+1} \in V^1, v_{n+2} \in V^2$ and

$$\left| \sum_{v_i \in V^1} w_i \right| < 1 \quad \text{and} \quad \left| \sum_{v_j \in V^2} w_j \right| < 1.$$

If: Because $w_i, i = 1, \dots, n + 2$ are nonnegative integers, we have the equivalence

$$\left| \sum_{v_j \in V^2} w_j \right| = 0 \Leftrightarrow \sum_{v_j \in V^2 \setminus \{v_{n+2}\}} c_j = K.$$

It follows directly that $S = V^2 \setminus \{v_{n+2}\}$.

Only if: Suppose that $\sum_{j \in S} c_j = K$ for some $S \subseteq \{1, 2, \dots, n\}$. Let $V^1 = \{v_i | i \in S\} \cup \{v_{n+2}\}$ and $V^2 = V \setminus V^1$. (Obviously $v_{n+2} \in V^2$) Then immediately

$$\sum_{v_i \in V^1} w_i = \sum_{v_j \in V^2} w_j = 0.$$

So the cut set splitting G into $G^1(V^1, E^1, W^1)$ and $G^2(V^2, E^2, W^2)$ is just the E_C to be found. This concludes the proof. ■

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