A Technical Development in Realizing Hybrid Quantum Memory

A thesis presented

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A Technical Development in Realizing Hybrid Quantum Memory

Advised by Jonathan Simon

Abstract
In the following undergraduate thesis, I fully explore the theory behind how lasers can be used as both measuring tools and preparation devices for studying atomic physics. This theory naturally leads to a full understanding of light-matter interactions and the ground-breaking physics that can be studied by looking at this regime. An experiment is then proposed that could realize hybrid quantum memory through precise magnetic control over the atomic transitions in a Rydberg atom. The work done in this thesis explores the first step in succeeding at this endeavor. By using a hybrid optical and condensed matter table-top experiment, the hyperfine ground state transition of ^{87}Rb was properly coupled to resonant cavity. Further steps are then explored in an effort to bring this success to the high photon-photon interaction regime present in Rydberg physics.

Contents

Introduction

The story begins with Charles H. Townes and his 1953 discovery of the ammonia maser. The maser, an acronym for **M**icrowave **A**mplification by the **S**timulated **E**mission of **R**adiation, was the immediate precursor to the modern laser, a similar acronym but for a device that works at higher frequencies in the rang of visible light. This Nobel-winning research revolutionized atomic physics for two big reasons. Firstly, the laser gave scientists a precise method for accessing atoms– or, in other words, a measuring tool. Secondly, the laser acts as a preparatory step towards putting atoms into a state that we want to study.

A working knowledge of the laser is only the first baby-step into foraging towards Atomic, Molecular, and Optical (AMO) physics. In recent years, AMO physicists have sought to play in the realm of light and matter, be it atoms or ions, in some sort of meaningful way. Particularly, research has been centering around the ways photons, which normally do not notice one another[1], can be forced to say 'Hi'. To see this happen, we study photons through their interactions with matter.

This regime is referred to as 'quantum non-linear optics', and it is a regime in which an atomic system behaves differently for two photons than for one [12]. The system behaves differently because an atom can only absorb one photon, so its response to the second will be different. Luckily enough, this difference can be measured. Research topics in this area are already quite plentiful with experiments ranging from building optical transistors and switches[23, 3] to studying many-body phenomena like massively entangled atomic systems[7].

Not only does this type of physics give rise to electromagnetically induced transparency (EIT) which can be used to study many interesting aspects of atoms such as Rydberg polaritons[6], but it also points toward a great system for probing quantum information[25]. By integrating the best methods of achieving light-matter interactions, through 'hybrid' systems, simple tabletop experiments arise that exhibit groundbreaking physics at the forefront of AMO. Thus, one such system for realizing hybrid quantum memory through controlling Rydberg atoms with microwave fields is proposed. This is an ambitious project, but we have found success in the first step: using microwaves to drive atomic transitions in atoms. To understand how it all works, one must start from the beginning and understand the laser.

1. Theory

[1] Although this is true, the natural absence of interactions between photons is what makes them ideal carriers of information in the first place. Because photons do not interact with one another, their information will not mix and subsequently distort.

1.1 The Innerworkings and Uses of Lasers

1.1.1 Diode Lasers

An easy way to understand how lasers work is through the lens of a quantum mechanical system of three atomic levels. Each level is an energy eigenstate of the system. Typically, they are referred to as the ground (1) and excited (2) states: $|0\rangle$, $|1\rangle$, and $|2\rangle$, respectively. A pumping mechanism then causes a $|0\rangle \rightarrow |2\rangle$ transition. The result of this pump, or spark, would be for every sparked particle to eventually decay to $|1\rangle$. This 'metastable' energy eigenstate also eventually decays back down to the ground. Individually, each spontaneous decay results in a photon emission. Eventually, an exponential number of photons at the same frequency and phase will build up within the system since every spark can generate two spontaneously emitted photons. If this rate surpasses the rate of photons absorbed and emitted at the $|0\rangle \rightarrow |1\rangle$ transition, then an outpouring of coherent photons will result[16]. A visual depiction of such a quantunm mechanical system is seen in Figure 1.

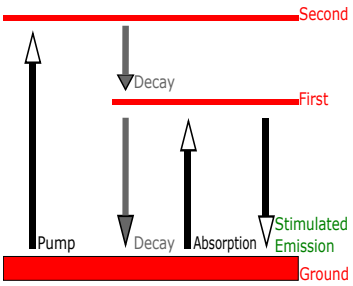

Figure 1. A general schematic for a laser with one ground and two excited states, in which stimulated emission is achieved through a gradual population inversion centered around a series of pumps to the excited states and subsequent decays back down to the ground state.

To study 'Ultracold Quantum Materials', laser diodes are great to use largely due to how inexpensively they can be made in the laboratory. Diode lasers work based off a system of PIN junctions, analogous to the lasing principles just discussed. A PIN junction is a structure made up of a p-type[2] semiconductor that is separated from an n-type[3] by some sort of intrinsic semiconductor[4].

The three types of semiconductors present in a PIN junction are the energy eigenstates of a quantum mechanical system. The pumping mechanism is a current which drives carriers into the intrinsic region where they annihilate with each other. This annihilation gives rise to photons through a process dubbed 'recombination radiation'. Recombination radiation is the basic method through which laser light is produced– sparing more in-depth technicalities that are present elsewhere[14].

[2]Doped to contain excess electron holes.
[3]Doped to contain excess free electrons.
[4]No dopant.

1.1.2 Proper Linewidths

A basic rule atomic physicists work under is the Electric-Dipole-Approximation which states that the wavelength of laser light must be much larger than the size of the atom it is probing. Relative sizes must therefore be understood. For the experiment to be laid out in this thesis, rubidium was used. Rubidium's atomic radius is .235nm, while visible light traverses a spectrum of 380nm→ 750nm. With a wavelength of 780nm, the Electron-Dipole-Approximation is well-satisfied.

Space is great but time is a size dimension that is arguably more important! The frequency domain of a laser must be small enough to resolve the transition frequencies in an atom. Call this size in frequency the linewidth, and denote it Γ, where

$$\Gamma = \frac{1}{4\pi\varepsilon_0}\frac{4\omega^3\mathscr{D}^2}{3\hbar c^3} \tag{1}$$

and $\mathscr{D}$ is the non-zero dipole matrix element of the electric dipole operator $\hat{d} = q\vec{r}$.

If the linewidth is larger than the feature it is trying to measure, then the feature will be obscured in broadening. Some experiments in the realm of cavity QED [24] and Rydberg EIT [9] actually require lasers with linewidths much smaller than the intrinsic linewidth of a diode laser. There are various methods of creating narrow linewidth lasers. Such methods include the use of a fast servo-loop [1] and the addition of diffraction gratings [11]. Another simple method, use in Jonathan Simon's Ultracold lab, is the integration of a saturated Doppler-free absorption signal, narrow cavity, and PI lockbox whose dither amplitude can be narrowed to lock to small linewidths [21].

1.1.3 Saturated Doppler-Free Absorption Spectroscopy

Recall that a laser also effectively prepares atoms into a state that can be experimented on. First take note that a gas of atoms will inevitably be moving with some speed at room temperature. High School physics dictates that the thermal motion in an atom will therefore cause the atom to see Doppler shifted light. To get the clearest results, a system of measurement that does not get broadened by random atomic thermal motion is used.

Saturated Doppler-free absorption spectroscopy is the method through which very small atomic features can be resolved by aligning simple optics. The picture is simplified by imagining a cloud of atoms confined to $1D$. These atoms could either be moving to the left at various speeds, the right at various speeds, or sitting still. Seeing as the frequency source is a non-moving laser beam, there are three possible Doppler scenarios– given by Figure 2.

In the first case, where an atom is moving towards the laser source, the speed of the atom is defined by v_r while the speed of the laser source is defined by v_s. Given the Doppler effect equation,

$$f = \left(\frac{c+v_r}{c+v_s}\right) f_0 \tag{2}$$

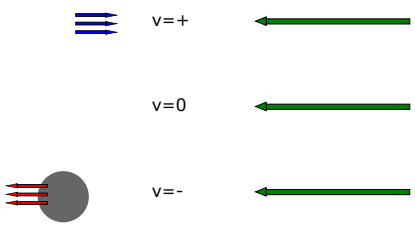

Figure 2. The Doppler effect for atoms confined to one dimension experiencing a laser-induced frequency source, where the non-moving atom sees the light at the same frequency, the atom moving to the right ($v = +$) sees blue-shifted frequencies, and the atom moving to the left ($v = -$) sees red-shifted frequencies.

in which f is the frequency seen by the atom, f_0 is the emitted frequency, and c is the speed of light, we can rewrite Equation 2 as

$$f = \left(1 + \frac{v_r}{c}\right) f_0 \tag{3}$$

since $v_s = 0$[5].

The importance of this argument is that if a laser whose frequency can cause an atomic excitation is shined on an atom, only some atoms in the gas will actually see the correct frequency. The atoms which are moving to the left or right will see a different frequency of light and therefore not be resonant. To account for the thermal motion of atoms, the dithering system referred to in Section 1.1.2 can be employed. This dithering system lets the laser scan back and forth across a broad range of frequencies. A broader range of frequencies being scanned will guarantee that moving and non-moving atoms will see some light they are resonant with.

The result of this scan is a broadening of the spectrum. However, if two counter-propagating beams that are precisely aligned with each other as depicted in Figure 3 are used, then greater precision can be achieved. These counter-propagating beams will select for $v = 0$ atoms to be the ones being measured in the following manner. If the laser is detuned about the resonant frequency of the atom, then there will be a saturation of moving atoms in either the positive or negative direction. However, if the laser is tuned to the resonant frequency of the atom, then the difference between the first pass and second pass of the beam gives rise to a measurable intensity change.

1.1.4 Laser Cooling

When atoms are excited, they accept a photon. Since a photon has been removed from the beam path, a change in light intensity as measured by a photodiode is observed. A secondary effect of this collision involves the conservation of momentum.

Given that momentum is directly proportional to speed, lasers can be used to excite atoms as well as precisely control

[5] v_r is positive when the particle is moving towards the source, and negative when it is moving away.

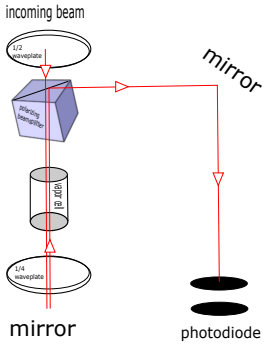

Figure 3. An optical schematic for exhibiting saturated Doppler-free absorption spectroscopy, in which the small energy splittings in atoms can be resolved from a more broadened spectrum that results from random atomic thermal motion.

their motion since the momentum of a photon is added to the atom when the photon is absorbed. Thus, a precise amount of control over speed has a direct effect on the amount of precise control scientists can have over temperature[8]. It follows that if the velocity of atoms in a gas are confined to near zero, then laser cooling will be achieved. This level of cooling is actually required in many experiments to keep the system from decohering[18], effectively prepare matter into exact quantum states across all degrees of freedom [28], and, simply enough, confine the atoms spatially so they are easily found. Although this project did not make use of laser cooling, the concept is ubiquitous in atomic physics to such a degree that it should be heavily looked in to by any aspiring atomic physicist. There are countless sources on the matter[26, 5, 10] as well as on Magneto-Optical-Trapping which takes the argument a bit further[13]

The equations resulting from formally working through the theory of trapping and cooling have been reproduced below to be thorough,

$$S = \frac{I}{I_s} = \frac{I}{1.1\,\frac{\text{mw}}{\text{cm}^2}} \tag{4}$$

$$R_{sc} = \frac{\frac{1}{2} S\Gamma}{1+S+\left(\frac{2\delta}{\Gamma}\right)^2} \tag{5}$$

$$F_{sc} = pR_{sc} = \frac{\frac{1}{2} S\Gamma\hbar k}{1+S+\left(\frac{2\delta}{\Gamma}\right)^2} \tag{6}$$

$$F_{\text{motion}} = -\beta v \tag{7}$$

$$\beta = -\frac{\hbar k^2 8 S \frac{\delta}{\Gamma}}{\left(1+S\left(\frac{2\delta}{\Gamma}\right)^2\right)^2} \tag{8}$$

where R_{sc} is the scattering rate, I_s is the saturation intensity of the laser, I is the intensity of the laser, δ is the detuning, and β is the damping coefficient.

Cooling to absolute zero is not physical possibility, and the furthest limit attainable without making use of other techniques can easily be derived. Again, to be concise, this argument has been left out for its been covered by many scientists over the decades[8]. The relevant parameters have been repro-

duced below to be thorough,

$$E_r = \frac{\hbar^2 k^2}{2m} = \hbar\omega_r \quad (9)$$

$$\omega_{absorb} = \omega_a + \omega_r \quad (10)$$

$$\omega_{emit} = \omega_a - \omega_r \quad (11)$$

$$\hbar(\omega_{absorb} - \omega_{emit}) = 2\hbar\omega_r \quad (12)$$

$$\langle E_k \rangle = \tfrac{1}{2} k_B T \quad (13)$$

$$E_{\text{stead-state}} = \frac{\hbar\Gamma}{8} \frac{1+\left(\frac{2\delta}{\Gamma}\right)}{\frac{2\delta}{\Gamma}} \quad (14)$$

$$T_D = \frac{\hbar\Gamma}{2k_B} \quad (15)$$

where ω_r is the recoil frequency, E_r is the recoil energy, E_k is the average kinetic energy, k_B is the Boltzmann constant, T is temperature, $E_{ss} = \frac{\hbar\Gamma}{2}$ is the maximum value of the steady state energy when the detuning is maxed out at $\delta = \frac{\Gamma}{2}$, and T_D is the final Doppler limit we can cool atoms to without using extra techniques. It has been experimentally determined for $^{87}{}_{\text{Rb}}$ to be $T_D = 145.57\mu_{\text{K}}$[20].

1.2 Talking to Matter with Light

Light-matter interactions are derivable under both semi-classical and quantum mechanical formalism. Semi-classical formalism sees atoms act as quantum objects while laser beams are purely classical fields. Meanwhile, quantum mechanical formalism sees atoms and lasers as quantum objects, since laser beams are made up of coherent photons that each carry quantized energy $\hbar\omega$. The ability to describe atomic systems with both formalisms gives scientists an incredible amount of flexibility and convenience when trying to understand physics in the best way possible.

1.2.1 Coupling and Interactions

The easiest type of atom to look at quantum mechanically is the hydrogenic atom. Hydrogenic atoms are defined as such because they only have one electron in the outer shell. Therefore, all interactions that take place are with respect to this single electron, making it much easier to not only trap and cool but to manipulate the states thereof. When $|g\rangle$ is time evolved, the physical representation is an accumulation of phase. Phase is not energy and therefore it does not make sense for the ground state to excite to $|e\rangle$ without accepting a photon carrying an energy equal to the splitting between these two levels, as per discussed in Section 1.1.3.

The story just told is not the only method for exciting an atom. In fact, some much more interesting physics lies in a much different method of excitation. This method of excitation is brought about by coupling the ground and excited states. Coupling of the ground and excited states is achieved through the use of external fields, namely electromagnetic fields. This electromagnetic field source mixes together $|g\rangle$ and $|e\rangle$ such that time evolution will eventually bring an atom lying in the ground state to the excited state. However, this excited state is not the end of the road for the atom. Because an electromagnetic field is time-dependent, the atom will oscillate back and forth between $|g\rangle$ and $|e\rangle$ if the coupling is strong enough. So, just how strong is a coupling?

Surprisingly or not, this question is easily answered. After correctly writing out the Hamiltonian of the atom and field, the coupling strength can be found in one of the Hamiltonian's matrix elements. This matrix element has a derivable formula. To derive it, think of an atom as a dipole with the following energy

$$\hat{d} = q\vec{r} \quad (16)$$

where q is the charge of the dipole and $\vec{r}$ is the position vector. Given that $\vec{E}(\vec{r},t)$ is a time varying electric field, we can write the Hamiltonian using Equation 16 as

$$\hat{H} = \hat{d} \cdot \vec{E}(\vec{r},t) \quad (17)$$

It follows from Equation 17 and an understanding of quantum mechanics that the matrix element that will give us the coupling strength of states $|g\rangle$ and $|e\rangle$ is therefore

$$\langle g | \hat{H} | e \rangle \quad (18)$$

In other words, this matrix element gives the strength of the dipole transition from the ground to the excited state. The question now arises, is the strength of coupling completely dependent on the electric field and two states being coupled? Anyone's first guess would be 'yes' because that is what the equations say. However, this dipole element becomes strengthened if the atoms and field are actually placed within a cavity. Thus, cavity quantum electrodynamics (cQED) comes into play in this intricate story of how light and matter interact.

1.2.2 Interactions in a Cavity

The coupling involved with a cavity not only increases the ability for an atom to hold a photon, but for a photon to be absorbed by an atom in the first place. So, it is clear that cavities are extremely useful tools in the atomic physicists utility belt that should be commanded with ease.

There are two major types of cavities used in atomic physics experiments. The first is an optical cavity; built using mirrors reflecting a laser beam into a waist shape that holds atoms. The second is a microwave cavity; built using antennas and metal boxes that allow for electromagnetic wavemodes to develop within. For a simple depiction of the these two cavity types, see Figure 4. In this experiment, the latter was solely used.

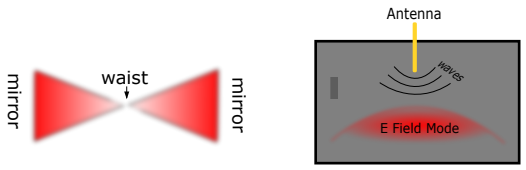

Figure 4. Two basic physical representations of a cavity are depicted; an optical concentric-spherical cavity on the left and a rectangular microwave cavity on the right.

Although semi-classical formalism was used in Section 1.2.1 to explicitly define what is meant by coupling and how to find the strength of coupling, recall that a quantum mechanical picture tells the same story. After adding a cavity, the story should still not change. For example, when talking about the behavior of photons, note that the spatial wavefunctions of photons from quantum mechanics have a direct analog to linear combinations of classical cavity modes from electrodynamics. Therefore, the spatial occupation of a quantum mechanical field is directly analogous to Laguerre-Gaussian modes from classical optics. In such comparisons, the limit of high-field intensity will bring the quantum mechanical picture back to the semi-classical due to a high-field intensity's signal-to-noise ratio becoming large enough to blur out individual photon characteristics.

Although the atom experiences coupling to photons within a cavity, the atom is not *only* coupled to the photons and cavity. In fact, as previously hinted in Section 1.1.4, the concept of decoherence implies that atoms begin coupling to the surrounding environment. Coupling to the environment introduces enough noise to the system that information can be completely lost track of. Although this is obviously a bad effect, the ability for an atom to couple to multiple sources is actually requisite in doing atomic physics experiments. As an atom couples to waves in a cavity, it stays coupled to free space modes. Due to this free space mode coupling, the atoms can eject photons out of the cavity so they can be picked up by a photodiode, for example.

Thankfully, a formula exists that describes how much a cavity and its coupling can be relied on. In other words, a formula exists that determines "how good" a laser and cavity are at making sure atoms absorb and hold on to atoms. A physical cavity is never going to be perfect, so light will always be leaking out in some manner. An atom will also eventually exhibit spontaneous emission and eject a photon, no matter how strong the energy eigenstate coupling is. These types of losses are non-conservative which means they cannot be described as a Hamiltonian matrix and rigorously deal with. However, it can be shown that a parameter mathematically relates the coupling between atoms and a light field. This parameter is called the Cooperativity parameter, and its individual values will show how much coupling exists between an atom and a light field.

1.2.3 Cooperativity Parameter

Cooperativity is not a measure of the probability that an atom will scatter photons into a cavity mode or free space. It can, however, be used to determine this, which sorta showcases its strength! Depending on a simple ratio, exhibited in Equation 19, an atom will be expected to either lose information into the path leading to a photodiode so it can be measured, or eject it back into a cavity with the probability of being reabsorbed at a later time. Although there is a free space mode leading towards a photodiode, there are also countless modes that simply go nowhere. This means that the information can be lost forever. Thus, it is important to have a strong formula defining how well we know the information is still in our system, or most likely lost to free space. It is easy to see this as a method of determining the efficiency of a quantum system. The probability is marked as

$$P^{\text{probability}}_{\text{cavity mode}} = \frac{\eta}{\eta+1} \tag{19}$$

where η is the Cooperativity parameter.

Th paramater is calculated in a similar way as to what was done in Section 1.2.2 by using the Optical Bloch Equations. This exact derivation will not be shown, but note that such a method determines the semi-classical version of the Cooperativity parameter. From a geometric perspective, it is easy to see that the Cooperativity parameter is

$$\eta = \text{proportion of } 4\pi \text{ steradians the cavity mode covers} * \text{finesse}$$

in which the finesse is defined as the number of round trips a photon takes in a cavity before randomly leaking out. Semi-classically, this is equal to

$$\eta_{\text{classic}} = \frac{24\mathscr{F}}{\pi(k\omega_o)^2} \tag{20}$$

when an atom is sitting in the anti-node of a cavity mode. The anti-node of the cavity mode depicted on the right-hand side of Figure 4 would be at the center of the arc where the $\vec{E}$-field intensity is greatest. In Equation 20, $\mathscr{F}$ is the finesse of the cavity while ω_o is the size of the waist determined by the geometry of our mirrors or the cavity walls. The quantum mechanical form of this is equation can be found to be

$$\eta_{\text{quantum}} = \frac{4g^2}{\kappa\Gamma} \tag{21}$$

in which Γ is the natural linewidth of the atom, κ is the free spectral range of the finesse, and $g = \mu\sqrt{\frac{\omega_o}{2\varepsilon_o\hbar V}}$ is the dipole coupling in a volume V[22]. Equation 20 and Equation 21 can then be shown to be equal to one another if we merely rewrite g, Γ, and κ in terms of the dipole matrix element of an atom, d, and do some algebra[17].

1.2.4 Two-Level Atomic Systems

In the following discussion, imagine a two-level atomic system consisting of just a single atom, with a ground and excited state. This system is depicted in Figure 5. The goal is to determine a clearer way of describing which state the atom is occupying as well as solving a Hamiltonian describing the entire system such that energies and frequencies of oscillations can be determined. To the first point, matrix math is used to solve the Schrödinger Equation of a two level-system. For the second point, one must understand how to build and extract eigenvalues from the Jaynes-Cummings Hamiltonian.

The time-dependent Schrödinger equation is written as

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi \tag{22}$$

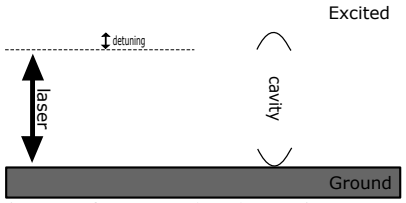

Figure 5. Diagram for a two-level atomic system in the presence of a cavity. The laser populates atoms from the ground to the excited state with the cavity tuned to this transition.

in which $|\psi\rangle$ is a spinor made up of the probabilities an atom is located at state $|g\rangle$ or $|e\rangle$. Equation 22 can then be rewritten as

$$i\hbar\frac{d}{dt}\begin{pmatrix} C_e \\ C_g \end{pmatrix} = \begin{pmatrix} 0 & \Omega e^{i\delta t} \\ \Omega e^{-i\delta t} & 0 \end{pmatrix}\begin{pmatrix} C_e \\ C_g \end{pmatrix} \tag{23}$$

where δ is the detuning, $\Omega = |\mathscr{D}\cdot E|$, and $\mathscr{D}$ is equal to Equation 18: the non-zero matrix element of the dipole operator $\hat{d} = q\vec{r}$. The solution to this formula explains a lot of interesting atomic physics such as Rabi flopping, AC stark shift, and even laser cooling!

Now the Jaynes-Cummings Hamiltonian, which is the simple Hamiltonian for describing almost all atomic systems, can be modeled. It has the basic form of

$$\hat{H} = \hat{H}_{\text{field}} + \hat{H}_{\text{atom}} + \hat{H}_{\text{interaction}}$$

although it can be written much more exactly. In the two-level picture, the levels are split by the energy of a photon, $E = \hbar\omega$. The field used to create interactions has a single mode of frequency ν. If the rotating wave approximation[6] is satisfied, the Jaynes-Cummings Hamiltonian will have the form of

$$\hat{H}_{\text{JC}} = \hbar\nu a^\dagger a + \tfrac{1}{2}\hbar\omega\sigma_z + \hbar g\left(a\sigma_+ + a^\dagger\sigma_-\right) \tag{24}$$

in which a and $a^\dagger$ are the annihilation and creation operators, respectively.

Finding a way to simplify Equation 24 to a matrix form would would make solving for its eigenvalues much easier. So, the eigenstates of this system after adding a cavity can be written as

$$|e,n\rangle \tag{25}$$
$$|g,n+1\rangle \tag{26}$$

in which n refers to the number of photons in the cavity and $|e,n\rangle$ is written as such because a photon has been absorbed by the atom such that it populates a new state and a photon is removed from the cavity in doing so. The individual energy levels of these two states are

$$n\hbar\nu + \tfrac{1}{2}\hbar\omega \tag{27}$$
$$(n+1)\hbar\nu - \tfrac{1}{2}\hbar\omega \tag{28}$$

therefore giving an energy separation of $\hbar(\omega-\nu) = \hbar\delta$. In the uncoupled basis, Equation 24 is rewritten as

$$\hat{H}_{\text{JC}} = \begin{pmatrix} n\hbar\nu + \frac{1}{2}\hbar\omega & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1)\hbar\nu - \frac{1}{2}\hbar\omega \end{pmatrix} \tag{29}$$

The eigenvalues of the Jaynes-Cummings Hamiltonian are then solved to be

$$\alpha = (n+\tfrac{1}{2})\hbar\nu \pm \tfrac{1}{2}\hbar\Omega_R \tag{30}$$

where $\Omega_R = \sqrt{\delta^2 + 4g^2(n+1)}$ is the Rabi frequency. The Rabi frequency is the rate of oscillation between two uncoupled states. In other words, the uncoupled eigenstates ended up with are merely linear combinations of the original uncoupled eigenstates but now with raised and lowered energies. This energy shift is given by

$$\Delta E = \hbar\Omega_R \tag{31}$$

1.3 Three-Level Atomic Systems

By adding a third energy level to the discussion, an example of which is depicted in Figure 6, a theoretical basis is allowed for in which to study some more interesting physics like spontaneous emission and the photon leaking from one of the states to somewhere else. Because of the high level of control scientists have over the movement of atoms and the coupling of states with lasers and cavities, the three-level atomic system gives a massive amount of leeway with what atoms and photons can be told to do. One of the clearest avenues to take this idea is that of a hybrid quantum memory scheme in which information is carried by photons.

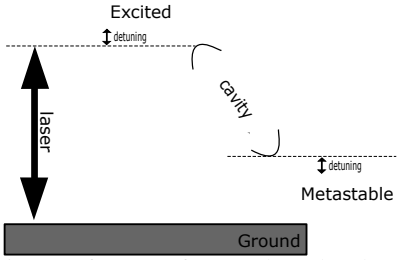

Figure 6. Diagram for one of many three-level atomic system types in the presence of a cavity. This figure specifically features a Λ system in which a laser populates an atom to the excited state and cavity is detuned at the excited-metastable transition.

The first step in understanding the three-level system is to imagine states similar to Equation 25 without any photons. The degenerate states of $|g,0\rangle$ and $|e,0\rangle$ can instead be written as a linear combination of each other like $|e,0\rangle \pm |g,0\rangle$.

[6] The rotating wave approximation is a regime in which the field being applied is near resonance to the atom's energy transition under low intensity. It lets us remove all energy non-conserving terms from the Hamiltonian because the atom is perceived as rotating with the system in such a way that it's evolution and the evolution of the system are equal and cancel if they are oppositely rotating.

Depending on the linear combination, a metastable state is created at some energy splitting. Depending on the splitting, a ladder (Ξ) system, or one of two different lambda (Λ and V) systems is created. To stay grounded in reality, the physical representation of a Λ system is two hyperfine ground states in which one has the photon in the cavity, and then an excited state that has absorbed the photon. This is what is depicted in Figure 6.

Coupling two levels of the three-level atomic system will result in an atom oscillating in time between the two. By the addition of a third level, we can even guarantee that the entire atomic population will either be in one of the ground states or the excited state. Given that each state accrues phase with time, the interference of these two phase profiles will change the various probabilities an atom will be in one state or another. A case of perfect destructive interference can even occur at the site of one of the energy levels, therefore making it a 'dark state' since atoms will be entirely eliminated from here. Thus, photons will not exist at this transition. In other words, a dark state is an eigenstate of the Jaynes-Cummings Hamiltonian that is completely separated from the original eigenstates before time evolution or any sort of coupling has occurred.

2. Results

The groundwork for which to begin building a system for studying quantum memory has been laid. I have touched on all the relevant theory needed to couple photons and atomic states in a way that could prove useful for a hybrid system of optics and condensed matter. This hybrid system would make use of a microwave cavity that at a specific frequency will support an electromagnetic wavemode. The shape and strength of the mode is such that an intense magnetic field acts as the coupling mechanism between two atomic levels of ^{87}Rb. To be specific, the two levels are the hyperfine ground states, $F = 1$ and $F = 2$. Using Saturated Doppler-Free absorption spectroscopy as a method for measuring this coupling, I tested a number of microwave cavities in order to determine that such a method can even give distinguishable results. This is only the first step of a much more ambitious attempt to realize a hybrid quantum memory, whose further steps will be laid out in Section 3.

2.1 Atomic Blueprint of ^{87}Rb

Rubidium is a hydrogenic atom whose atomic blueprint has been fully documented, and we now know the exact energy splittings of its various states. Since ^{87}Rb has a $j = \frac{1}{2}$ and $j = \frac{3}{2}$ fine structure splitting, two blueprints exist. One for a 794nm transition and another for a 780nm transition from the ground state. These two transitions correspond to what are called the D_2 and D_1 lines, respectively[20]. In my experiment, we use a 780nm laser, and thus only focus on the D_2 transition line corresponding to the $j = \frac{3}{2}$ splitting. I have reproduced a sketch of this blueprint for quick reference in Figure 7.

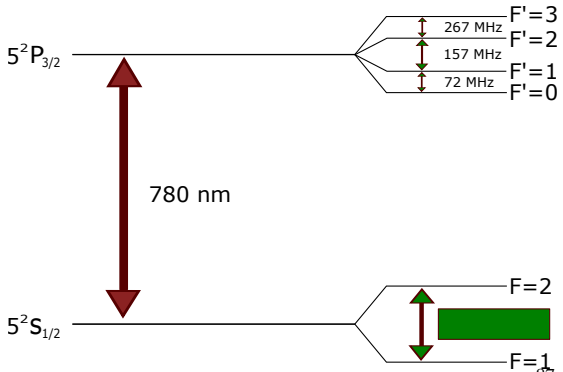

Figure 7. Diagram for the D_2 atomic energy levels of ^{87}Rb. Energy splittings have been determined by experiment decades ago, while quantum mechanical addition of angular momentum explains how L, J, and F values. In performing the addition of angular momentum calculations, we note that spin $s = \frac{1}{2}$ and nuclear spin $i = \frac{1}{2}$ are such due to protons, electrons, and the nuclei of a rubidium atoms being fermionic. Important values we will actually use are boxed.

A major issue in solely relying on this blueprint is that rubidium naturally exists in two isotopes, ^{87}Rb as well as ^{85}Rb. Quartz vapor cells can be ordered that are enriched to overwhelmingly carry the first isotope. These vapor cells are much more expensive, and thus when measuring energy levels of rubidium, I must take note that ^{85}Rb still has a large presence. The atomic blueprint of this isotope of rubidium has been equally researched so have a clear understanding of its energy splittings[19]. Because of its presence in my experiment, I have reproduced a sketch of its D_2 line in Figure 8 considering it also occurs with a 780nm laser on resonance.

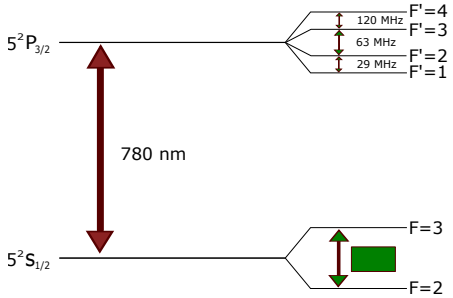

Figure 8. Diagram for the D_2 atomic energy levels of ^{85}Rb. Energy splittings and quantum numbers have been determined by researchers and through calculations similarly to those in Figure 7. Important values we will actually use are boxed.

Although I am not experimenting on ^{85}Rb, taking note of its hyperfine ground state transition frequency is important because of its use a sanity check to be described in Section 2.4. To be clear, the choice of using ^{87}Rb over ^{85}Rb is not merely due to the population density of the two isotopes within a vapor cell. In fact, naturally occurring rubidium is typically in the form of the latter isotope rather than the former. We force

the population of rubidium within a vapor cell to favor ^{87}Rb because it is more radioactive. Therefore, it is much more likely to undergo gamma decay which produces photons. A full Saturated Doppler-free Absorption spectrum is provided in Figure 9 to see the relative intensities as well as energy splittings of the various states in rubidium.

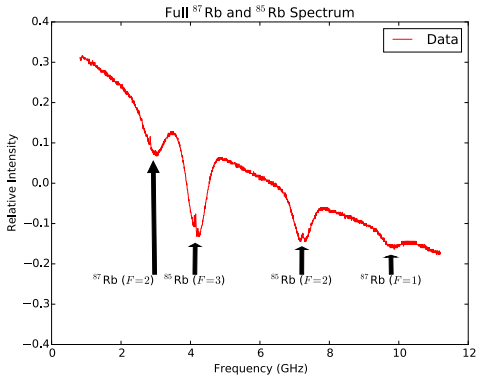

Figure 9. A spectrum exhibiting the hyperfine splitting of naturally occurring rubdium in a quartz vapor cell. The measurement was done using Saturated Doppler-free Absorption Spectroscopy on the D_2 line with a dithering 780nm laser made in house.

2.2 Experimental Design and Simulations

In a solely optical atomic system, a laser beam is used as the electromagnetic field source that couples atomic states. This field is typically called the "control" field and is seen as entirely classical. Instead of taking this approach, we create a rectangular cavity whose dimensions support a strong magnetic field mode that passes through a sample of rubidium. These dimensions also give arise to the mode occurring at a very specific frequency. If this frequency is equal to the hyperfine ground state transition frequency of ^{87}Rb, then the theory laid out in Section 1.2 dictates the two levels will become coupled. As discussed, this coupling will cause the atomic sample of rubidium to oscillate back and forth between the two energy levels with frequency Ω_R. This system in which microwaves are used to drive atomic transitions is not only quite similar to that used in atomic clock experiments[2] but has recently been shown to be robust enough to control all of the rotational and hyperfine states in entire molecules[27]. Thus, I developed a microwave cavity that would exhibit such results.

Basic electrodynamics is used to derive the formula dictating the resonant frequency of a rectangular cavity. By properly applying boundary conditions to Maxwell's Equations, it can be shown that this rectangular cavity resonant frequency will occur at

$$f = \frac{c}{2\pi\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m\pi}{x}\right)^2+\left(\frac{n\pi}{y}\right)^2+\left(\frac{l\pi}{z}\right)^2} \tag{32}$$

where $\mu_r = 1$ and $\varepsilon_r = 1$ because the relativity permeability and permittivity of air are roughly 1 and air is the cavity filling. In Equation 32, x, y, and z refer to the length, width, and height of the cavity walls while m, n, and l are the mode numbers. In an attempt to keep the cavity as small as possible such that the magnetic field is intense across the entirety of the atomic sample, we say $l = 0$. This approximation will simplify Equation 32 to

$$f = \frac{c}{2\pi\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m\pi}{x}\right)^2+\left(\frac{n\pi}{y}\right)^2} \tag{33}$$

therefore giving us complete freedom over how tall the cavity is. The choice was made to make it just large enough that a small rubidium vapor cell could fit inside without breaking. Taking into account the design parameters in Table 1.

	Cavity (m)	**Vapor Cell** (m)	**Cell Tip** (m)
length	.0510	.0142	—
width	.0232	—	—
height	.0110	—	.0044
diameter	—	.0050	.0018

Table 1. A table featuring the final design parameters used to machine a microwave cavity with $f_{\text{res}} \approx 6.834$ GHz. Note that the vapor cell is a cylinder with flat windows and the tip-ff is conical, with the widest value that is recorded being located at the base. The cavity was machined by myself under the supervision of Luigi Mazzenga, while the vapor cell was blown by the engineers at Thorlabs, Inc.

the resonant frequency of the microwave cavity should be $f_{\text{res}} = 7.10306$ GHz. Do not worry, this expected resonance is purposefully ≈ 300 MHz away from the actual hyperfine ground state splitting of ^{87}Rb for reasons to be discussed in Subsection 2.3.

Although the mathematical formulas hinting toward a machinable apparatus have hereby been explored, more tools are at our disposal to check the theoretical likelihood that a 6.834682 GHz microwave resonator can be achieved. The Schuster Lab has graciously supported my use of ANSYS High-Frequency Structure Simulator (HFSS) to check the design parameters I concluded were best in Table 1. Not only can this program simulate the frequency at which microwave cavities will resonate, but it will estimate Quality factors, Q[7], for a number of other potential electromagnetic modes, as well as dynamically plot the resultant $\vec{E}$ and $\vec{B}$ fields.

Simulating a cavity that takes into account aluminum walls and the quartz vapor cell yields a 110 electromagnetic wavemode that is properly shaped to extract the interesting physics we are actively looking for, as seen in Figure **??**. In particular, we hope to use magnetic field intensities to drive the hyperfine ground state of ^{87}Rb. The reason we are looking for a magnetic field based coupling system of an electric field based one is that achieving positive results would help us move towards

[7] See Section 2.3

realizing a system of hybrid quantum memory. The level of control we have over magnetic fields is quite large such that through the integration of YIG resonators, which couple to magnetic fields, and rydberg atoms, which aid in photon-photon interactions, a memory system based off magnetically coupled atomic states would be quite powerful.

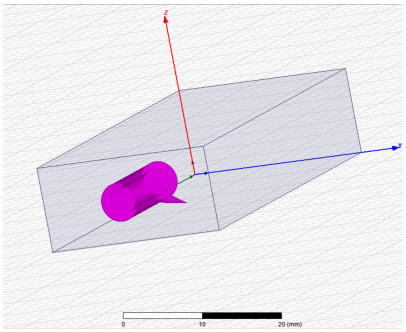

Figure 10. A sketch of a rectangular microwave cavity machined from aluminum with a small quartz vapor cell containing rubidium placed inside. This sketch was simulated to determine $\vec{B}$-field strength, location, and wavemode resonant frequency.

The parameters calculated by HFSS for this system are laid out in Table 2.

Freq. (GHz)	**Quality Fac.**
6.846	4200

Table 2. A table featuring simulated resonant frequency and the pertaining Quality factor as calculated by ANSYS HFSS, graciously on loan from the Schuster Lab.

2.3 Fine Tuning the Cavity Onto Resonance

The parameters given in Table 1 were deliberately chosen to support a 110-mode that was just off-resonance from the hyperfine ground state transition of ^{87}Rb. The reason I "incorrectly" machined the cavity was because the quartz vapor cell would placed inside of it. Quartz has a non-zero dielectric constant, which disturbs the mode supported by the cavity dimensions. Increasing the cavity dimensions will lower the resonant frequency of the cavity. Decreasing the dimensions will raise the resonant frequency. Although adding a quartz cell technically lowers the free space volume within the cavity, the fact that quartz is a dielectric makes the supported mode see larger dimensions. Thus, the vapor cell brings down the resonant frequency of the microwave cavity.

The issue with this occurrence is that it makes the calculation done in Equation 33 no longer correct. Instead, an incredibly non-trivial calculation must be done. This requires much more work than is needed since we can add a tuning screw to the cavity to account for discrepancies. Just as a dielectric makes the cavity larger, adding a metal screw will make the cavity smaller. By taking into account these two foreign objects placed within the cavity, the resonant frequency of the 110-mode could be exactly tuned from $f_{\text{res}} = 7.10306 \rightarrow 6.834682$ GHz.

This process was frustrating due to the number of ball-park guesses and trial-and-error machining attempts that had to be made. Simulating the effects of foreign objects in a cavity were not terribly helpful due to the imprecise methods I had to measure dimensions. Very slight perturbations can cause extremely drastic changes to the resonant frequency of a microwave cavity. Given that the vapor cell is not only filled with macroscopic Rubidium crystals but also quite asymmetric, the HFSS simulations done in Section 2.2 were frequently anomalous to the experimental values. Henceforth, the addition of a fine-tuning screw became an absolute must.

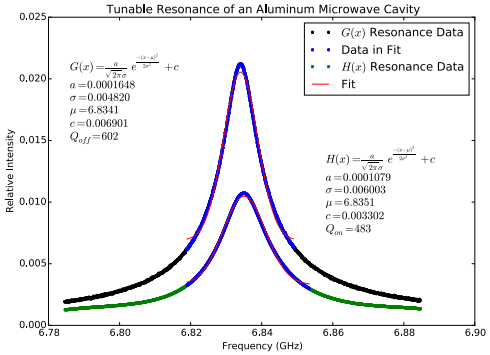

Figure 11. $S_{1,2}$ reflection measurements performed by a Network Analyzer owned by the Schuster Lab on my Microwave cavity. This data was taken after screwing a metal screw into the lid to show that although the fine-tuning range covers 6.834682 GHz, small perturbations in dimensions drastically affect the wavemode. The quality factor of this cavity is given by the ratio of the resonance peak over the bandwidth, $Q = f_{\text{res}}/FWHM = \mu/(\sigma 2\sqrt{2}\ln 2)$

The quality factor, Q, of a resonant cavity is characterizes the bandwidth of the resonator with respect to the resonant frequency. The higher the number this is, the slower the oscillations will die out. Having a high quality factor means that the 110-mode will support an oscillating microwave photon for a longer amount of time. The longer it oscillates, the stronger the entire control electromagnetic field will become that is being used to couple atomic states. This dimensionless parameter therefore seems very similar to the finesse, $\mathscr{F}$ from Equation 20. In fact, the two are related by

$$Q = \frac{\mathscr{F} f_{\text{res}}}{2\lambda} \tag{34}$$

in which 2λ is what is called the Free Spectral Range. A high Q can be achieved in a number of ways. First, one should make sure the inside of a cavity is as clean as possible such that the supported wavemode is not disturbed. Sadly, because of existence of large vapor cell and tuning screw on the inside in our experiment, we suffer major dimension dependent losses even when the perturbations are small as

can be seen in Figure 11. Secondly, all cracks and holes should be as unobtrusive as possible since the Q will suffer substantial losses if microwave photons are able to leak out of the cavity. Due to the necessity of antennae that support $S_{1,2}$ reflection measurements of the resonant frequency as well as their use as a method for sending microwaves into the cavity in the first place, there must be at least two holes. Two more holes were added to give the lasers optical access to the rubidium atoms. One last hole was put into the apparatus to support the fine-tuning screw. All of these holes could allow for light to escape. In order to combat this loss, I made the walls of the cavity very thick such that the holes act as a long waveguide. Before the light is given the chance to leak out of the cavity, they exponentially decay within the waveguide. Lastly, we do our best fill the large crack existing as a result of the cavity needing a lid. The crack is filled with indium wire; a practice used by many scientists to create seals for vacuums and cryogenics.

2.4 Driving the Hyperfine Ground State Transitions of ^{87}Rb

It is now clear how the apparatus was built to exhibit coupling between the hyperfine ground state transition of ^{87}Rb. After taking much care to machine the pieces as exactly as possible and fine-tune the cavity to resonance, the cavity was attached to an optical breadboard housing an aligned Saturated Doppler-free Absorption Spectroscopy setup. A frequency synthesizer that was programmed to generate microwave photons at $f = 6.834682$ GHz with 18 dBm of power passed through a microwave amplifier whose specs indicated 35 dBm of amplification. This circuit would heat up very quickly, so it was attached to a water cooling system. In order to couple the hyperfine ground state of ^{87}Rb, a switch just needed to be flipped on the frequency synthesizer to begin driving microwave photons into the resonant cavity. The results across the spectrum of rubidium can be seen in Figure 12.

The data shows a positive result depicting ^{87}Rb atoms being driving back and forth between their hyperfine ground state levels. Because these two atomic states are coupled by the electromagnetic field supported within the cavity, rubidium atoms will oscillate back and forth at the Rabi frequency. The cavity keeps this effect going for some amount of time as governed by the Cooperativity parameter. Therefore, we have effectively enhanced the ^{87}Rb atoms probability of being excited by the laser to their excited state at the expense of ^{85}Rb atoms. It should now be experimentally apparent as to why we took note of the ^{85}Rb hyperfine ground state transition frequency in Section 2.1. Although we do not care about this isotope of rubidium, the data shows that as ^{87}Rb exhibits ground state coupling, ^{85}Rb is adversely affected. This was a helpful sanity check in determining whether I was on the right track.

Step one towards realizing a hybrid quantum memory therefore was successful. I adequately have shown that using a hybrid system of optics and condensed matter physics,

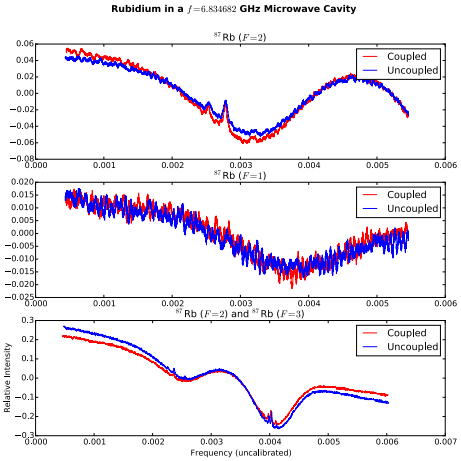

Figure 12. The effects of amplifying a $f = 6.834682$ microwave signal in a cavity designed to resonate a 110 mode at this frequency. An atomic vapor cell is located in the maximum of the magnetic field. The ^{87}Rb hyperfine ground states are clearly showing coupling through an increase in intensity, while the ^{85}Rb hyperfine ground state exhibits a roughly equal decrease.

magnetic fields can be used to couple atomic states thereby favoring certain atomic transitions within a sample. This is an extremely important building block and it paves the way for further research in this lab.

3. Outlook

Having driven the hyperfine ground state transition in rubidium, the undergraduate experiment I embarked upon was succesful. This success is not the end of the project, and hopefully future undergraduates working in the Ultracold Quantum Material lab will take it to the next step.

This next step is to use YIG resonators that couple to the cavity and therefore to the rubidium atoms in the cavity as well. The successful additional coupling of YIG modes will cause cavity and atomic resonances to split apart. The trouble with this next step is that atomic resonances get smeared out in the presence of strong magnetic fields. Strong magnetic fields are applied to YIGs in order to get them to work. Thus, a major next step will be to find a way to shield the atoms from this magnetic field while still remaining strongly coupled to the cavity and the YIG. It is proposed that creating a magnetic quadrupole whose field falls of like $1/r^4$ could do the trick if placed far enough away from an atomic vapor cell.

The end goal is to add a control field to the atoms that excite them to Rydberg states. Rydberg states are special states of atoms where photons have been shown to strongly interact through what's known as the Rydberg Blockade. If we can successfully control Rydberg transitions with a strong magnetic field, then a very interesting quantum memory arises. I did not get that far in my own studies, but there is a clear path to be taken next.

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