A Thermodynamic Entropy Based Approach for Prognosis and Health Management

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ABSTRACT

Data-driven stochastic and probabilistic methods that underlie reliability prediction and structural integrity assessment remain unchanged for decades. This paper provides a method to explain the Prognostics and Health Management (PHM) in terms of fundamental concepts of science within the irreversible thermodynamic framework. The common definition of damage, which is widely used to measure the reduction of reliability over time, is based on observable markers of damage at different geometric scales. Observable markers are typically based on evidences of any change in the physical or spatial properties or the materials, and exclude unobservable and highly localized damages. Thermodynamically, all forms of damage share a common characteristic: "energy dissipation". Energy dissipation is a fundamental measure of irreversibility that within the context of non-equilibrium thermodynamics is quantified by "entropy generation". The definition of damage in the context of thermodynamics allows for incorporation of all underlying dissipative processes including unobservable markers of damage. Using a theorem relating entropy generation to energy dissipation associated with damage producing failure mechanisms, this paper presents an approach that formally describes and measures the resulting damage.

Having developed the approach to derive the damage over time, one could assess the health of structures and components subject to known degradation processes. This paper presents a prognostic approach on the basis of thermodynamically derived cumulative damage, whereby the thermodynamic entropy, as a broad measure of damage, is assessed.

1. INTRODUCTION

The definition of damage due to the physical mechanisms varies at different geometric and scales. For example, the definition of fatigue damage can vary from nano-scale through the macro-scale. At the atomic level the grain boundary is a likely location where atoms are more loosely packed. At the micro-scale damage is the accumulation of micro-stresses in the neighborhood of cracks. At the meso-scale level, damage might be defined as growth and coalescence of micro-cracks to meso-cracks. However, measuring damage is subject to the physically measurable variables (i.e., observable marker) when dealing with specific failure mechanisms. For example, in the fatigue mechanism material density, change of hardness, module of elasticity, accumulated number of cycles-to-failure, and crack length may be used as "observable markers" that measure the damage. Therefore, defining a consistent and broad definition of damage is necessary and plausible. To reach this goal, we elaborate on the concept of material damage within the thermodynamic framework.

Thermodynamically, all forms of damage share a common characteristic, which is the dissipation of energy. In thermodynamics, dissipation of energy is the basic measure of irreversibility, which is the main feature of the degradation processes in materials (Tang & Basaran, 2003). Chemical reactions, release of heat, diffusion of materials, plastic deformation, and other means of energy production involve dissipative processes. In turn, dissipation of energy can be quantified by the *entropy generation* within the context of irreversible thermodynamics. Therefore, dissipation (or equivalently entropy generation) can be considered as a substitute for characterization of damage. We consider this characterization of damage highly general, consistent and scalable.

The common practice in damage analysis and prediction of structural life and integrity is based on the traditional generic handbook-based reliability prediction methods, data driven prognostics approaches and Physics-of-Failure (PoF) methods. The traditional generic handbook-based reliability prediction methods such as those advocated in MIL-HDBK-217F (U. S. Department of Defence, 1965), Telcordia SR-332 (Telcordia Technologies, 2001), and FIDES (FIDES Guidance Issue, 2004) rely on the analysis of field data (with incoherent operating and environmental conditions), with the assumption that the failure rates are constant. Numerous studies have shown that these methods cause misleading and inaccurate results and can lead to poor design and incorrect reliability prediction and operating decisions (IEEE Standard 1413, 1998; IEEE Standard 1413.1, 2002). The PoF models (Manson, 1996; Norris & Landzberg, 1969; Bayerer, Hermann, T. Licht, Lutz, & Feller, 2008; Shi & Mahadevan, 2001; Harlow & Wei, 1998) are more rigorous in terms of employing the specific knowledge of products, such as failure mechanism, material properties, loading profile and geometry. However, such empirical methods are limited to simple failure mechanisms and are hard to model when multiple competing and common cause failure mechanisms are involved. Finally, the data driven methods such as neural networks (Byington, Watson, & Edwards, 2004), decision tree classifiers (Schwabacher & Goebel, 2007) and Bayesian techniques (Bhangu, Bentley, Stone, & Bingham, 2005) do not capture the difference between failure modes and mechanisms, although they can obtain the complex relationship and degradation trend in the data without the need for the particular product characteristics such as degradation mechanism or material properties. Moreover, these methods require rich historical knowledge of materials and structural degradation behavior that may not always be available.

In this paper, we introduce an entropy-based prognostic approach to predict the Remaining Useful Life (RUL) of components and structures. This approach is based on the second law of thermodynamics and defines entropy as a more consistent measure of damage. As compared to other existing PoF or fusion prognostics methods (Held, Jacob, Nicoletti, Scacco, & Poech, 1999; Ciappa, 2002; Cheng & Pecht, 2009), this approach captures the effect of multiple failure mechanisms[1], more effectively. Moreover, the results of entropy approach are favorably used in fracture mechanics, fatigue damage analysis (Bryant, Khonsari, & Ling, 2008; Tang & Basaran, 2003) and tribological processes such as friction and wear (Amiri & Khonsari, 2010; Nosonovsky & Bhushan, 2009). Furthermore, it is a powerful technique to study the synergistic effects arising from interaction of multiple processes (Amiri & Khonsari, 2010).

The remainder of this paper is organized as follows. Section 2 describes our construction of the entropy model. Section 3 describes an entropic based framework for prognosis. Section 4 provides a case study which explores the application of the proposed prognostics framework, and section 5 offers concluding remarks.

2. TOTAL ENTROPY PRODUCED IN A SYSTEM

Consistent with the second law of thermodynamics, entropy does not obey a conservation law. Therefore, it is essential to relate the entropy not only to the entropy crossing the boundary between the system and its surroundings, but also to the entropy produced by the processes taking place inside the system. Processes occurring inside the system may be reversible or irreversible. Reversible processes inside a system may lead to the transfer of the entropy from one part of the system to other parts of the interior, but do not generate entropy. Irreversible processes inside a system, however, result in generation of the entropy, and hence in computing the entropy they must be taken into account.

Using the second law of thermodynamic, it is possible to express the variation of total entropy flow per unit volume, dS, in the form of

$$dS = d^r S + d^d S \qquad (1)$$

where, S is defined for a domain g by means of specific entropy, s, per unit mass as $S = \int_g \rho s dV$, and the super scribes r and d represent the reversible and irreversible part of the entropy, respectively. The term $d^r S$ is the entropy supplied to the system by its surroundings through transfer of mass and heat (e.g., in an open system where wear and corrosion mechanisms occur). The rate of exchanged entropy is obtained as

$$\frac{d^r S}{dt} = -\int^{\Omega} J_s . n_s dA \qquad (2)$$

where, J_s is a vector of the total entropy flow per unit area, crossing the boundary between the system and its surroundings, and n_s is a normal vector. Similarly, $d^d S$ is the entropy produced inside of the system, which can be obtained from Eq. 3

$$\frac{d^d S}{dt} = \int^{V} \sigma dV \qquad (3)$$

where, σ is the entropy generation per unit volume per unit time. The second law of thermodynamics states that $d^d S$ must be zero for reversible transformations and positive ($d^d S > 0$) for irreversible transformations of the system.

The balance equation for entropy shown in Eq. 4 can be derived using the conservation of energy and balance equation for the mass.

$$\frac{ds}{dt} + \nabla J_s = \sigma \qquad (4)$$

[1] Particularly, in contrast with the empirically-based PoF approach which considers only the most predominant failure mechanisms, the definition of damage in the context of the entropic approach allows for the incorporation of all underlying dissipative processes. For example, in the case of corrosion-fatigue, both stress and electrochemical affinity of the oxidation-reduction electrode reaction ($Me \Leftrightarrow Me^{z+} + ze$) of a metal are considered.

This gives us an explicit expression for total entropy in terms of reversible and irreversible processes as (De Groot & Mazor, 1962; Kondepudi & Prigogine, 1998)

$$\frac{ds}{dt} = -\nabla.\left(\frac{J_q - \sum_{k=1}^{n}(c_m\psi + \mu_k)J_k}{T}\right) + \frac{1}{T^2}J_q.\nabla T - \Sigma_{k=1}^{n}J_k\left(\nabla\frac{\mu_k}{T}\right) + \frac{1}{T}\tau:\dot{\epsilon_p} + \frac{1}{T}\Sigma_{j=1}^{r}v_j\, A_j + \frac{1}{T}\Sigma_{m=1}^{h}c_m J_m(-\nabla\psi) \quad (5)$$

where, T is the temperature, μ_k the chemical potential, J_q the heat flux, J_k the diffusion flow, J_m any fluxes resulting from external fields (magnetic and electrical) such as electrical current, v_i the chemical reaction rate, τ the stress tensor, $\dot{\epsilon_p}$ the plastic strain rate tensor, $A_j = -\Sigma_{i=1}^{u}\mu_i v_{ji}$ the chemical affinity or chemical reaction potential difference, ψ the potential of the external field such as electrical potential difference, and c_m the coupling constant. External forces may be resulted from different factors including electrical field, magnetic field, gravity field, etc., where the corresponding fluxes are electrical current, magnetic current and velocity. For example, in the case of an electric field, $E = -\nabla\psi$ is the electric potential, $I = \Sigma_{m=1}^{h}c_m J_m$, the current density and $c_m = Fz_m$, where F is the Faraday constant and z_m is the number of ions. Each term in Eq. 5 is derived from the various mechanisms involved, which define the macroscopic state of the complete system.

By comparing Eq. 5 with Eq. 4 we can make the identifications as

$$J_s = \frac{J_q - \sum_{k=1}^{n}(c_m\psi + \mu_k)J_k}{T} \quad (6)$$

$$\sigma = \frac{1}{T^2}J_q.\nabla T - \Sigma_{k=1}^{n}J_k\left(\nabla\frac{\mu_k}{T}\right) + \frac{1}{T}\tau:\dot{\epsilon_p} + \frac{1}{T}\Sigma_{j=1}^{r}v_j\, A_j + \frac{1}{T}\Sigma_{m=1}^{h}c_m J_m(-\nabla\psi) \quad (7)$$

where, Eq. 6 shows the entropy flux resulted from heat and material exchange. Eq. 7 represents the total energy dissipation terms from the system that from left to the right include heat conduction energy, diffusion energy, mechanical energy, chemical energy, and external force energy. Eq. 7 is fundamental to non-equilibrium thermodynamics, and represents the entropy generation σ as the bilinear form of forces and fluxes as

$$\sigma = \Sigma_{i,j}X_i J_i(X_j);\ (i, j=1, \ldots, n) \quad (8)$$

It is through this form that the contribution from the applicable thermodynamic forces and fluxes are expressed. When multiple failure mechanisms are involved in a degradation process such as corrosion fatigue, summing the contributions of the mechanical and electrochemical processes, one can write the total entropy generation for combined effect of plastic deformation and anodic and catholic dissolution as:

$$T\sigma = \tau:\dot{\epsilon_p} + \tilde{A}i_{corr} \quad (9)$$

where $\tilde{A}$ is the electrochemical potential losses (over-potential) (Imanian & Modarres, 2014). Additionally, using forces and fluxes enables one to take into account complex loading scenarios and operating conditions in computing entropy produced in degradation processes.

3. RUL PREDICTION USING ENTROPY AS AN INDEX OF DAMAGE

It was stated earlier that damage caused through a degradation process could be viewed as the consequence of dissipation of energies that can be measured and expressed by entropy such that:

Damage ≡ Entropy

In the earlier discussion in this paper it was shown (Eq. 5) that one could express the total entropy per unit time per unit volume for individual dissipation processes resulting from the corresponding failure mechanisms. Therefore, the evolution trend of the damage, D, is obtained from

$$D|t \sim \int_0^t [\sigma|X_i(u), J_i(u)]du \quad (10)$$

where, $D|t$ is the monotonically increasing cumulative damage starting at time t from a theoretically zero value or practically some initial damage value. In this study, the evaluation of damage is performed relative to the initial damage value. The initial damage can be calculated using the correlation between the rate of damage and damage at different stage of degradation (Liakat & Khonsari, 2014).

When D reaches a predefined (often subjective) level of endurance, it may be assumed that beyond that point the component or structure will fail. It is worth to note that failure in this context is the point when an item becomes effectively nonfunctional (but possibly still operational) – i.e., failure happens when the item is no longer meeting its functionality requirements (e.g., acceptable performance level or endurance limit such as a given level of thermodynamic efficiency). The rate of entropy or damage can vary according to the type of degradation. However, damage in the system mounts up over time. For example, in the case of fatigue crack closure, while the crack as an observable marker of damage disappears, causing damage rate decrease, the damage accumulation keeps rising as unobservable markers of damage such as loading asymmetry, hardening properties, residual stresses and loading ratio increase (Romaniv, Nikiforchin, & Andrusiv, 1983).

Material, environmental, operational and other types of variability in degradation forces impose uncertainties on the cumulative damage, D. Existence of any uncertainties about the parameters and independent variables in this thermodynamic-based damage model leads to a time-to-failure distribution. Imanian et al. showed how such a distribution and corresponding reliability function can be derived from the thermodynamic laws rather than estimated from the observed time to failure histories (Imanian & Modarres, 2014).

Currently, most of the health management of components and structures is based on reliability analysis and maintenance scheduling. However, in many cases this is neither sufficient nor efficient because each of these components can undergo different life cycles and hence different aging. Therefore, if maintenance or replacement is done solely based on reliability analysis, in most circumstances the components will either be abandoned before they have reached their end of life, or worse, they will fail before their scheduled replacement.

Prognostics and health management modeling approaches are used to reduce the costs of the physics based propagation damage. The techniques included in the PHM provide warnings before failures happen; they also optimize the maintenance schedule, reduce life cycle cost of inspection, and improve qualification tests assisted in design and manufacturing. Prognostics and health management modeling methods are implemented through three stages of diagnostics, prognostics, and health management. Diagnostics techniques identify the operational states of a working component or a structure. These techniques use statistics features such as mean, standard deviation, Mahalanobis distance and Euclidean distance of a component's degradation operating data (e.g. temperature, current, voltage, acoustic signals) to find out if the component is in a healthy condition or not regarding the feature's level degradation (Schwabacher & Goebel, 2007; Bock, Brotherton, Grabill, Gass, & Keller, 2006; Fraser, Hengartner, Vixie, & Wohlberg, 2003).

Prognostics methods provide information about the performance and RUL of components by modeling degradation propagation. These methods rely on the condition of the data which can roughly be divided into data driven based models and PoF based models. PoF based prognostics methods employ knowledge of products life cycle loading profile, failure mechanisms, geometry, and material properties. However, using PoF models is challenging because these methods are based on the interactions among multiple failure mechanisms which are not easy to analyze. Data driven based models are able to obtain the complex relationship and degradation trend in the data without the need for the particular product characteristics such as degradation mechanism or material properties (Amin, Byington, & Watson, 2005; Byington, Watson, & Edwards, 2004; Roemer, Ge, Liberson, Tandon, & Kim, 2005; Goebel, Saha, & Saxena, 2008). However, they cannot capture the difference between failure modes and mechanisms.

Since entropy function includes all of the failure mechanisms' dissipative energies when multiple competing and common cause failure mechanisms are involved, using it as a damage parameter for diagnosis and prognostics is more favorable in comparison with the PoF models and data driven models which merely rely on the most predominant failure mechanisms and the statistical analysis, respectively. What follows presents an entropy based prognostics method for RUL prediction. The proposed prognostics framework is depicted in Figure 1.

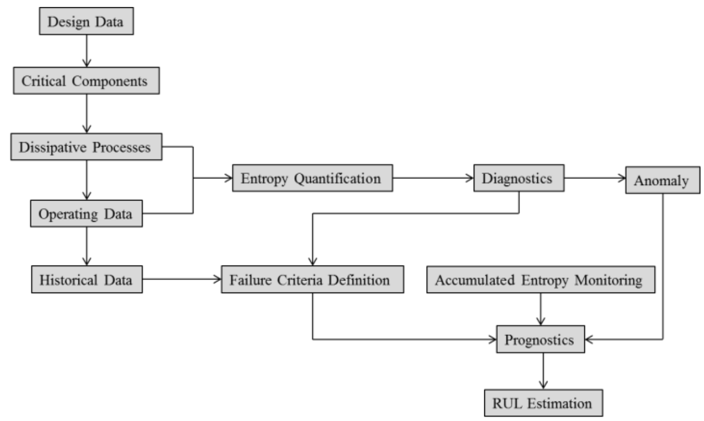

Figure 1. RUL prediction by entropy based prognostic method.

According to this framework the entropic base prognostics method can be implemented in four steps. First, the dissipative processes and associated data in the critical components under aging are determined. The identification of these processes and relevant parameters can be aided by failure modes, mechanisms, and effects analysis (FMMEA) which identifies the potential failure mechanisms for products, under certain environmental and operating conditions. The entropy as a parameter of damage which includes all the interactive failure mechanisms is quantified then.

The second step is to extract the features of the monitored entropy data and compare them with the healthy baseline data features to detect anomalies. The traditional diagnostic approaches are mainly designed for stationary and known operating conditions. The problem of a fault diagnosis under fluctuating load and operating conditions has been successfully addressed by methods such as order tracking method (Stander & Heyns, 2005), instantaneous power spectrum statistical analysis (Bartelmus & Zimroz, 2009), and diagnosis algorithms such as clustering algorithms (Schwabacher & Goebel, 2007; Vapnik, 1995; He & Wang, 2007).

Because entropy as a parameter of degradation includes all observable damage markers (cracks, wear debris and pit

densities) and unobservable damages such as subsurface dislocations, slip and micro-cavities, definition of a single failure threshold might not be possible due to long stretch of damage measurement from nano-scale to macroscopic scale. In this case, the cumulative damage and alternatively entropy endurance level can be estimated through the measurement of certain observable damage markers. The correlation between the observable damage markers and entropy, justified by several studies (Naderi, Amiri, & Khonsari, 2010; Bryant, Khonsari, & Ling, 2008), enables the definition of failure threshold on the basis of observable markers. In the other word, the damages grow, coalesce and eventually the weakest link among all coalesces damages manifests itself as an observable damage which causes failure.
Additionally, records of the entropy data from historical data can be used to obtain the entropy to failure values. Entropy, as a thermodynamic state function is independent of the path to failure (loading values, frequency and geometry) and provides an overall constant failure criterion (Kondepudi & Prigogine, 1998; Bryant, Khonsari, & Ling, 2008).

The third step is to use an appropriate prognostics approach using entropy as an index of damage. Some of the conventional methods used for prognostics are artificial neural network (Byington, Watson, & Edwards, 2004; Amin, Byington, & Watson, 2005), fuzzy logic (Amiri & Khonsari, 2010), wavelet theory (Roemer, Ge, Liberson, Tandon, & Kim, 2005), support vector machine (Vapnik, 1995), relevance vector machine (Tipping, 2000), Bayesian methods (like Kalman filter and Particle filter (Arulampalam, Maskell, Gordon, & Clapp, 2002)), time series analysis (Kumar & Pecht, 2007) and PoF based prognostics models. The application of these methods depends to the complexity of accumulated entropy signal from two extremes of periodic and purely random signal.

The fourth and final step is RUL prediction. Remaining useful life is defined as the time when the entropy meets the failure criteria. There are different techniques for RUL estimation using data driven methods. For example one approach uses a pattern matching technique on data to estimate the RUL. Another strategy estimates the RUL indirectly by estimating damage trend, performing an appropriate extrapolation to the damage trend, and the calculation of RUL from the intersection of the extrapolated damage and the failure criteria (Schwabacher & Goebel, 2007). In comparison with the end of life prediction from entropy trend, the conventional RUL prediction methods are based on a damage mechanism with different failure mechanisms. These various failure mechanisms with different failure criteria and parameters' trends have various RULs which needs them to be prioritized accordingly (Cheng & Pecht, 2009).

Generally speaking, using entropy as a damage parameter has various advantages. The entropy based prognostics method is capable of shortening the prognostics procedure by isolating the damage parameter to entropy which includes multiple degradation mechanisms. It offers a science based foundation for prognostic methods which could combine with the conventional data driven techniques, as compared to the methods suggested by previous studies such as fusion prognostic approach suggested by Cheng et al (Cheng & Pecht, 2009). Furthermore, it uses a constant failure threshold and suggests a straightforward process to predict RUL (Amiri & Khonsari, 2010).

4. CASE STUDY

The entropy based prognostics approach was employed to obtain the remaining useful life of the AL7075-T651 coupons subjected to fatigue loading, using an MTS servo-hydraulic uni-axial load frame, from Ontiveros et al. experimental results (Ontiveros, 2013). Geometries of the coupons used are shown in Figure 2. All tests were performed at peak stress of 248 MPa with load ratio of 0.1 and frequency of 2Hz. Since the focus of Ontiveros et al. study was crack initiation, so most of experiments were stopped when, a crack was detected at the notch by visual inspection.

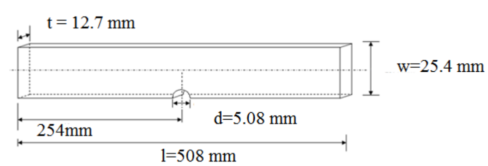

Figure 2. Al7075-651 edge notch specimen.

The formulation for entropy generation using Eq. 7 can be derived as

$$\sigma = \frac{\tau : \dot{\varepsilon}_p}{T} + \frac{1}{T} Z\dot{D} + \frac{1}{T^2} J_q \cdot \nabla T \qquad (11)$$

where, Z is the elastic energy release rate and $\dot{D}$ is the damage rate variable.
In Eq. 11, the first two terms can be captured directly from the hysteresis loop as depicted in Figure 3. In Figure 3, the largest area represents the energy dissipated due to plastic deformation. The remaining portion represents the energy dissipation as a result of elastic damage which can be observed as degradation of the Young's modulus (Lemaitre & Chaboche, 1990).

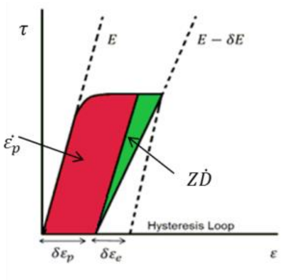

Figure 3. Hysteresis Energy (Reproduced from (Ontiveros, 2013)).

Results of Ontiveros et al. analysis showed that when compared to the plastic and elastic energy dissipations the fraction of the entropy generation due to heat conduction is considered to be negligible. Therefore, the third term does not take into account in the entropy calculation.

The prognostic framework implemented in this study involves the measurement of parameters included in the Eq. 11 and using the entropy as a parameter to be monitored. Specific Mahalanobis Distance (MD) is used as a diagnostic threshold which triggers the prediction. Once an anomaly is detected, the Particle Filter (PF) procedure is initiated for time to failure prognostic. The failure threshold in this approach is the mean of the failure threshold of the 3 samples considered as training samples.

4.1. Anomaly Detection

To obtain the anomaly threshold for every entropy data point, the MD values are calculated based on the distance between healthy and anomalous data. Then, the calculated MD values are transformed into a normal distribution using the Box-Cox transformation method (Box & Cox, 1964). After that, a detection threshold is quantified upon the mean and standard deviation of the transformed healthy MD data. The calculations are repeated for every test data, and anomaly is marked for every test point which goes beyond the detection threshold.

To implement the MD, entropy data are divided into two categories: (i) healthy data and (ii) test data. The observations between 4000 and 5500 cycles were classified as healthy data and the whole set of observations was considered as test data. The number of observations recorded for entropy parameter is denoted by k, where $k = 1, 2, \dots n$. S_k is the values of entropy at cycle k. Each individual observation of entropy data vector was normalized using the mean, $\overline{S_h}$, and standard deviation, D_h, from the healthy entropy data using Eq. 12.

$$Y_k = \frac{\mathrm{S} - \overline{S_{h_k}}}{D_{h_k}} \tag{12}$$

The MD values were computed by using Eq. 13.

$$MD_k = Y_k^T C^{-1} Y_k \tag{13}$$

Where C is the correlation matrix which can be obtained by

$$C = \frac{1}{n-1} \sum_{k=1}^{n} Y_k Y_k^T \tag{14}$$

Since the healthy MD values were found to not follow a normal distribution, the Box-Cox power transformation was employed to convert the healthy MD values into a normal distribution. This transformation allows for the use of statistical mean to determine the healthy or unhealthy conditions of the data. The Box-Cox transformation is defined by Eq. 15, where $MD(\lambda)$ is the transformed vector, MD is the original vector, and λ the transformation parameter.

$$\begin{aligned} MD(\lambda) &= \frac{MD^{\lambda} - 1}{\lambda} \qquad \lambda \neq 0 \\ MD(\lambda) &= \ln(MD) \qquad \lambda = 0 \end{aligned} \tag{15}$$

The mean and standard deviation of the transformed healthy values were used to define the threshold for anomaly detection as $\overline{S_{h_k}} + 3D_h$. When a transformed test $MD(\lambda)$ values (based on the Box-Cox transformation using parameter λ learned from the healthy data) crosses this threshold, an anomaly was considered to have occurred.

4.2. Particle Filter Prediction

By choosing the entropy data as a feature of damage, Bayesian method can be used to update the parameters of the model and the age predictions. Bayesian approaches provide a general rigorous method for dynamic state estimation problems. The idea is to build a Probability Density Function (PDF) of the system states based on all available information. Particle Filter (PF) is a method for implementing a recursive Bayesian filter using Monte Carlo simulations. Particle Filter (PF) approximates the model parameters' PDF by a set of particles sampled from the distribution and a set of associated weights denoting probability masses (Arulampalam, Maskell, Gordon, & Clapp, 2002).

In particle filter method, the particles are generated and recursively updated by process model shown in Eq. 16, a measurement model depicted in Eq. 17 and an *a priori* estimate of the state PDF.

$$\vec{x}_k^i = f_k\left(\vec{x}_{k-1}^i, \Omega_{k-1}\right) \tag{16}$$

$$\vec{y}_k = H_k(\vec{x}_k, \Psi_k) \tag{17}$$

where, Ω_k and Ψ_k are the system and measurement noises, respectively. Defining the model parameter vector at cycle k as $\vec{x}_k = [a_1,\ a_2, \dots,\ a_n]$ and damage level measurements as $\vec{y}_k = [S_0,\ S_1, \dots,\ S_m]$, the particle filter is implemented by initiating the state of the system by a set of particles $\vec{x}_0^i$, where $i = 1, 2, \dots, N_s$.

If $\{(w_k^i, \vec{x}_k^i)\}$ denotes a random measure that characterizes the posterior PDF, $p(x_{0:k}|y_{1:k})$ (where $\{x_{0:k}^i, i = 0,, \dots N_s\}$, is a set of support points with associated weights $\{w_k^i, i = 0,, \dots N_s\}$, normalized such that $\sum_{i=1}^{N_s} w_k^i = 1$) the posterior density at cycle k can be approximated as

$$p(\vec{x}_{0:k}|\vec{y}_{1:k}) = \Sigma_{i=1}^{N_s} w_k^i \delta(\vec{x}_{0:k} - \vec{x}_{0:k}^i) \tag{18}$$

where, $\vec{x}_{0:k}$ and $\vec{y}_{1:k}$ are the set of all states and measurements up to cycle k. *Sampling importance resampling* is a commonly used algorithm to attribute importance weight, w_k^i, to each particle, i,

$$w_k^i = \frac{p(\vec{y}_{1:k}|\vec{x}_k^i)p(\vec{x}_k^i)}{\pi(\vec{x}_k^i|\vec{y}_{1:k})} \tag{19}$$

The posterior PDF is then calculated by

$$w_k^i = w_{k-1}^i \frac{p(\vec{y}_k|\vec{x}_k^i)p(\vec{x}_k^i|\vec{x}_{k-1}^i)}{\pi(\vec{x}_k^i|\vec{x}_k^i, \vec{y}_{1:k})} \tag{20}$$

where the importance distribution $\pi(\vec{x}_k^i|\vec{x}_k^i, \vec{y}_{1:k})$ is approximated by $p(\vec{x}_k^i|\vec{x}_{k-1}^i)$ (Arulampalam, Maskell, Gordon, & Clapp, 2002).

4.3. Remaining Useful Life Prediction

To tie in the aforementioned technique, namely PF approach, with the entropic based prognosis, the system model can be represented by a regression model, based on accumulated entropy values, S', from experimental data analysis

$$S'_k = a_{1_k} k + a_{0_k} \tag{21}$$

which delivers a good fit for the entropy increment of Al specimens subjected to fatigue mechanism. Here, k is the cycle number, and a_1 and a_0 are the model parameters subjected to a Gaussian error as

$$\begin{aligned} &a_{0_k} = a_{0_{k-1}} + \omega_{a_0} \\ &\text{where: } \omega_{a_0} \sim N(0, std_{a_0}) \\ &a_{1_k} = a_{1_{k-1}} + \omega_{a_1} \\ &\text{where: } \omega_{a_1} \sim N(0, std_{a_1}) \end{aligned} \tag{22}$$

Given a series of measured entropy values, S', subjected to a Gaussian noise, $N(0, std)$ with zero mean and standard deviation std, as

$$\begin{aligned} &S'_k = a_{1_k} k + a_{0_k} + \psi \\ &\text{where: } \psi \sim N(0, std) \end{aligned} \tag{23}$$

the PF technique enables the estimation of the model parameters (a_1 and a_0) where in the updating process, N_s samples are used to approximate the posterior PDF. Each sample denotes a candidate for the model parameter vector $\vec{x}_k^i = [a_{0_k}, a_{1_k}]$, $i = 1, 2, \dots, N_s$, so the prediction of S' would have N_s possible trajectories with the corresponding importance weight w_k^i. The h^{th} steps ahead prediction of each trajectory at cycle k is calculated by

$$S'^i_{k+h} = a_1{}_k^i (k + h) + a_0{}_k^i \tag{24}$$

The estimated PDF of the entropy prediction can be obtained by

$$p(S'_{k+h}|S'_{0:k}) = \sum_{i=1}^{N_s} \omega_k^i \delta(S'_{k+h} - S'^i_{k+h}) \tag{25}$$

Since the failure threshold is defined as the mean of entropy to failure of training entropy data taken from 3 samples, S'_f, the remaining useful life probability estimation, R_k^i, of the i^{th} trajectory at cycle k can be obtained by solving the following equation

$$S'_f = a_1{}_k^i (k + R_k^i) + a_0{}_k^i \tag{26}$$

The PDF of the RULs at cycle k can be approximated by

$$p(R_k|S'_{0:k}) \approx \sum_{i=1}^{N_s} \omega_k^i \delta(R_k - R_k^i) \tag{27}$$

4.4. Prognostics Results

Using the MD approach, anomalies were identified when the transformed MD threshold of the test entropy data crosses the anomaly detection threshold. Once the anomaly was detected, the PF algorithm was initiated to predict RUL. The system model used for particle filter prediction follows Eq. 23. The initial values of the model parameters were obtained from the least square regression for each specimen, using the healthy interval of the data. Figure 4 shows prediction results for specimen number 6. The yellow zone shows the shape of RUL probability density function estimation after anomaly criteria detected.

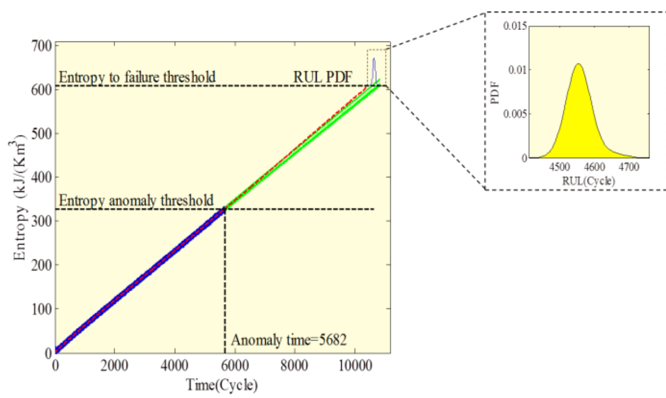

Figure 4. Predicted failure distribution at the time of anomaly detection for specimen number 6.

The same procedure applied to the 6 remaining specimens. The values for the mean of the predicted RULs and actual RULs are shown in Table 1. The error between mean of estimated RULs and actual RULs falls in the reasonable range of 4% to 18%.

Table 1. Comparisons of the actual and estimated RULs

Sample no.	RUL_{actual}(Cyc)	Mean($RUL_{estimated}$(Cyc))	Error
1	2829	2635.5	7%
2	3827	3563	7%
3	11165	10696.5	4%
4	1987	1621	18%
5	1018	835.5	17%
6	4792	4596	4%
7	3604	3444	4%

5. CONCLUSION

This paper presents an effort to use a thermodynamic framework, using entropy generation as a measure of damage, to assess RUL of a component or structure. It introduces a unified measure of damage in terms of energy dissipations for multiple irreversible processes with reference to physically measurable quantities. As compared to other existing PoF, data driven, or fusion prognostics methods, entropic-damage models capture the effect of multiple competing and common-cause failure mechanisms. The RUL predicted by this method includes the effect of all failure mechanisms and unlike conventional RUL prediction methods, where various RULs correspond to different failure mechanisms, it provides a unified RUL.

This paper also demonstrates a case study for implementation of an entropy-based prognostics method. Particle filter is applied to update the states of the model, reduce uncertainties and predict the RUL probability distribution function. The proposed method provides satisfactory RUL predictions.

While the entropy method proves to be theoretically more relevant for reliability analysis, its advantages remain to be explored practically. One practice in this regard is the authors' current project on introducing the entropy growth rate as a degradation parameter to the corrosion-fatigue mechanisms in materials.

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