A Unified Approach for Serviceability Design of Prestressed and Nonprestressed Reinforced Concrete Structures

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Prestressing is now widely used in reinforced concrete structures to improve their performance during service conditions without necessarily eliminating cracking. The total reinforcement, prestressed and nonprestressed, must be sufficient to satisfy requirements for ultimate strength. This aspect of the design is relatively simple and is not treated in this paper.

The checks for serviceability are more difficult because they involve a determination of stress and strain distributions in a cross section at various loading stages. The analysis must account for the time-dependent effects of creep and shrinkage of concrete and relaxation of prestressed steel which include the effects of cracking when the tensile strength of concrete is exceeded.

In this paper one set of equations is presented to determine the changes in stress and strain in concrete sections reinforced with prestressed and nonprestressed steels when subjected to normal forces and bending moments. The equations are applicable to statically determinate or indeterminate structures with or without cracking.

A part of the tension applied at the jacking end of a post-tensioned tendon is lost by friction and anchor set. It is assumed that the initial tension excluding the loss due to these causes has been determined prior to starting the analysis presented herein. Other prestress losses common to post-tensioning and pretensioning are discussed below.

The forces in the prestressed and nonprestressed steels and in the concrete in a cross section are subject to instantaneous changes whenever a prestressing force or other loads are introduced. Creep, shrinkage and relaxation produce other gradual changes. The equations presented below, based on equilibrium and compatibility, account for the instantaneous and time-dependent changes in the forces in the three components and give the strain and stress distributions, without the need of a separate computation of prestress loss.

The main reason that designers calculate the loss in tension in a prestressed tendon is to determine the effect of this loss on the strain and stress distributions. These variables are determined in the analysis presented below and the stress in the prestressed reinforcement is obtained as a part of the results of the analysis.

The present practice of estimating the loss of tension in prestressed tendons due to creep, shrinkage and relaxation while considering the effect of the force thus calculated on a plain concrete section ignores the change in force in the nonprestressed steel (which can be of the same importance[1]). Such a procedure disregards the fact that the change in force in any of the three components affects the strain in all three. The equations presented below ensure compatibility of strain in concrete and steels at all reinforcement layers. Thus, they give a more accurate prediction of strain and displacements calculated therefrom.

A comprehensive review of partial prestressing and suggested methods of design have been covered in Refs. 1-7 and therefore will not be addressed here. The intent of this paper is to present one general procedure for checking the design for serviceability requirements for reinforced concrete sections with any degree of prestressing involving minimum assumptions and avoiding the use of approximations or empirical equations.

The equations presented apply to any reinforced concrete cross section of a beam or a frame, with or without prestressing subjected to loads producing a normal force (tensile or compressive) combined with a bending moment. Thus, the cross section can be that of a column, a tie or a flexural member.

The prestressing or the loading may be introduced in single or in multistages and the same equations employed to determine the instantaneous or time-dependent changes in stress and strain between any two consecutive stages.

Synopsis

Distributions of strain and stress in a reinforced concrete section subjected to a normal force combined with bending due to prestressing and other loads introduced in stages are derived. The analysis does not need to be preceded by an estimate of the prestress loss due to creep and shrinkage and relaxation of prestressed steel.

The derived equations apply to cross sections made up of one type of concrete or to composite sections made of two concrete types or of concrete and structural steel. The approach presented is applicable to statically determinate or indeterminate structures with any amount of prestressing varying from zero allowing cracking to full prestressing eliminating cracks.

The stress and strain and the corresponding elongations and deflections are calculated using three numerical examples of members with and without prestressing subjected to normal force and bending to demonstrate the proposed method.

The calculations are simple enough to be performed by hand-held calculators, as will be demonstrated below by three numerical examples. However, for expediency, an available computer program[8] may be employed when the calculations are desired for many sections or the prestressing and the loads are applied in stages.

Cross sections made of more than one type of concrete or of concrete and structural steel can be analyzed by the same equations; see the numerical examples in Refs. 9 and 10.

STRESS-STRAIN RELATION OF CONCRETE

The following linear relations are adopted. A stress increment $\Delta\sigma_c(t_o)$ introduced at time t_o and sustained without change in magnitude to time t produces instantaneous strain plus creep of total value:

$$\Delta\epsilon_c(t) = \frac{\Delta\sigma_c(t_o)}{E_c(t_o)}[1 + \phi(t, t_o)] \quad (1)$$

where $E_c(t_o)$ is the secant modulus of elasticity of concrete at t_o and $\phi(t, t_o)$ is the creep coefficient, which is equal to the ratio of creep during the period t_o to t to the instantaneous strain at t_o. The value of ϕ depends, in addition to t and t_o on the humidity of the atmosphere and the cross section dimensions (see graphs in Refs. 10 and 12).

The change in forces in the reinforcements in a section develop gradually and thus produce time-dependent stress. A stress increment $\Delta\sigma_c(t, t_o)$ introduced gradually on the concrete from zero at t_o to full value at time t produces instantaneous strain and creep of total magnitude at time t given by:

$$\Delta\epsilon_c(t, t_o) = \frac{\Delta\sigma_c(t, t_o)}{E_c(t_o)}[1 + \chi\phi(t, t_o)] \quad (2)$$

where χ is the aging coefficient, a function of t_o and t. The value χ, usually between 0.6 and 0.9 can be read from graphs or tables.[10-12] For many practical calculations it may be sufficient to assume $\chi = 0.8$.

Eq. (2) may be rewritten:

$$\Delta\epsilon_c(t, t_o) = \frac{\Delta\sigma_c(t, t_o)}{\bar{E}_c(t, t_o)} \quad (3)$$

where $\bar{E}_c(t, t_o)$ is an age-adjusted modulus of elasticity of concrete:

$$\bar{E}_c(t, t_o) = \frac{E_c(t_o)}{1 + \chi\phi(t, t_o)} \quad (4)$$

RELAXATION OF PRESTRESSED STEEL

A tendon stretched between two fixed points loses gradually a part of its initial tension due to creep of steel. The change in stress $\Delta\sigma_{pr}$, referred to here as the intrinsic relaxation depends upon the quality of steel and on the magnitude of the initial stress. Very little relaxation occurs when the initial stress is smaller than $0.5 f_{pu}$ and the magnitude of the intrinsic relaxation increases rapidly as the initial stress approaches f_{pu}, where f_{pu} is the specified tensile strength of prestressing tendons.

For the same initial stress, the relaxation of a prestressed tendon in a concrete member is smaller than the intrinsic relaxation. A reduced relaxation value must be used in the design of concrete structures given by:

$$\Delta\bar{\sigma}_{pr} = \chi_r \Delta\sigma_{pr} \quad (5)$$

in which χ_r is a reduction factor:[10,14]

$$\chi_r = e^{(-6.7 + 5.3\lambda)\Omega} \quad (6)$$

where λ is the ratio of the initial stress in the tendon σ_{po} to its specified tensile strength f_{pu}:

$$\Omega = -\frac{\Delta\sigma_{ps} - \Delta\sigma_{pr}}{\sigma_{po}} \quad (7)$$

where $\Delta\,\sigma_{ps}$ is the change in stress in prestressed steel in a given period of time due to the combined effect of creep, shrinkage and relaxation. This value is not known a priori and thus has to be assumed and later corrected if necessary.

In the majority of cases Ω is positive and Eq. (6) gives a value of χ_r smaller than 1.0. Note that an exception will be discussed in a subsequent section.

SIGN CONVENTION

Tensile stress, tensile force and elongation are assumed positive. A positive bending moment produces tension at the bottom fiber; the corresponding curvature and slope of the stress diagram are also positive.

The y coordinate of any fiber is measured downwards from an arbitrarily chosen reference point O.

The symbol Δ represents a change in value; a positive Δ represents an increase. Thus, the symbols $\Delta\,\epsilon_{cs}$ and $\Delta\,\sigma_{pr}$ used for shrinkage of concrete and relaxation of prestressed steel always represent negative quantities.

INSTANTANEOUS STRESS AND STRAIN

The cross section shown in Fig. 1a is subjected at any instant to changes ΔN and ΔM in normal force and in bending moment. The axial force ΔN is at an arbitrary reference point O. The section is reinforced with prestressed and/or nonprestressed steel.

Note that forces ΔN and ΔM are the statical equivalents of the internal forces introduced on the section at the instant considered; e.g., the dead load plus the effect of the initial prestressing (including the statically indeterminate effects of these forces if any).

Assuming that plane cross sections remain plane, the changes in strain and in stress will be linear. At any fiber with coordinate y their values will be:

$$\Delta\,\epsilon = \Delta\,\epsilon_o + \Delta\,\psi\, y \qquad (8)$$

$$\Delta\,\sigma = \Delta\,\sigma_o + \Delta\,\gamma\, y \qquad (9)$$

where the subscript o refers to the reference point and $\Delta\,\psi$ and $\Delta\,\gamma$ are changes in slopes of strain and stress diagrams; the first of these two represents also a change in curvature.

Stress and strain changes at any concrete fiber or at a steel layer are related:

$$\Delta\,\sigma = E\,\Delta\,\epsilon \qquad (10)$$

where E is modulus of elasticity. Each of the symbols, σ, ϵ and E in this equation may have subscripts c, ns or ps to refer to concrete, nonprestressed or prestressed steel, respectively. When concrete is considered, the value E_c represents the modulus of elasticity of concrete at the instant considered.

The stress resultants ΔN and ΔM may be expressed:

$$\Delta N = \int \Delta\,\sigma\, dA \qquad (11)$$

and

$$\Delta M = \int \Delta\,\sigma\, y\, dA \qquad (12)$$

Substitution of Eq. (9) into Eqs. (11) and (12) gives:

$$\Delta N = A\,(\Delta\,\sigma_o) + B\,(\Delta\,\gamma) \qquad (13a)$$

$$\Delta M = B\,(\Delta\,\sigma_o) + I\,(\Delta\,\gamma) \qquad (13b)$$

where A, B and I are the area and its first and second moment about an axis through the reference point O of a transformed section composed of the area of concrete plus the areas of the two steels each multiplied by its modulus of elasticity and divided by E_c.

Substituting $\Delta\,\sigma_o = E_c\,(\Delta\,\epsilon_o)$ and $\Delta\,\gamma = E_c\,(\Delta\,\psi)$ and solving Eqs. (13a) and (13b) gives the instantaneous changes in axial strain and curvature:

$$\Delta\,\epsilon_o = \frac{1}{E_c\,(A\,I - B^2)}[I\,(\Delta N) - B\,(\Delta M)] \qquad (14a)$$

$$\Delta \psi_o = \frac{1}{E_c (A I - B^2)} [-B(\Delta N) + A(\Delta M)] \tag{14b}$$

When the reference point O is chosen at the centroid of the transformed section, $B = 0$ and Eqs. (14a) and (14b) take the more familiar forms:

$$\Delta \epsilon_o = \frac{\Delta N}{E_c A} \tag{15a}$$

$$\Delta \psi = \frac{\Delta M}{E_c I} \tag{15b}$$

Because E_c changes with time, the centroid of the transformed section changes position. In this paper, Eqs. (14a) and (14b) will be preferred because they refer to a reference point O which will be kept unchanged through the analysis of the instantaneous and the time-dependent changes in stress and strain.

The equations presented above give the instantaneous changes in strain and stress in any reinforced concrete section when the section is uncracked or when the section is fully cracked (Figs. 1a and 1b, respectively). In the latter case, the concrete in tension is ignored and use of the equations must be preceded by calculation of the depth c of the compressed zone; it is only the concrete in this zone that may be included in calculating the transformed section properties A, B and I.

When the equations of the present section are used to analyze the effect of prestressing a post-tensioned tendon, its cross section area should be excluded from A_{ps} and its duct area excluded from A_c in calculating the properties A, B and I of the transformed section. Note that with multistage prestressing the area of tendons prestressed and anchored at earlier stages of loading or ducts previously grouted should not be excluded.

The equations presented are also applicable to a composite section made of a reinforced concrete part and a structural steel section. The latter may be treated in the same way as a steel layer, but the second moment of area of the structural steel section about its own centroid must not be ignored in calculating I as commonly done with a reinforcing steel layer. The same should be noted in calculating the time-dependent changes in strain and in stress in a composite section using the equation presented in a subsequent section.

DEPTH OF COMPRESSION ZONE IN A FULLY-CRACKED SECTION

Fig. 1b represents the strain and stress distributions in a cross section reinforced with several layers of steel due to forces ΔN and ΔM which produce cracking at the bottom fiber. It is assumed that the stress in concrete is zero prior to the instant t when ΔN and ΔM are applied. Concrete in tension is ignored and the plane cross section is assumed to remain plane; thus, Eqs. (8) and (10) apply with $E_c = 0$ for the concrete below the compressed zone c.

The stress in concrete or in steel at any fiber:

$$\sigma = E \left(1 - \frac{y}{y_n}\right) \epsilon_o \tag{16}$$

The symbols σ and E in this equation may have subscripts, c, ps or ns to refer to concrete or the two types of steel. When reference is made to concrete, E_c = modulus of elasticity of concrete at the instant considered and $E_c = 0$ when $y > y_n$; where $y_n = -\epsilon_o/\psi = y$ coordinate of the neutral axis.

Substitution of Eq. (16) into Eqs. (11) and (12) gives:

$$\int_{y_t}^{y_n} (y_n - y) dA + \sum \left[\frac{E_s}{E_c} A_s (y_n - y_s) \right] = 0$$

$$(\text{when } \Delta N = 0) \tag{17}$$

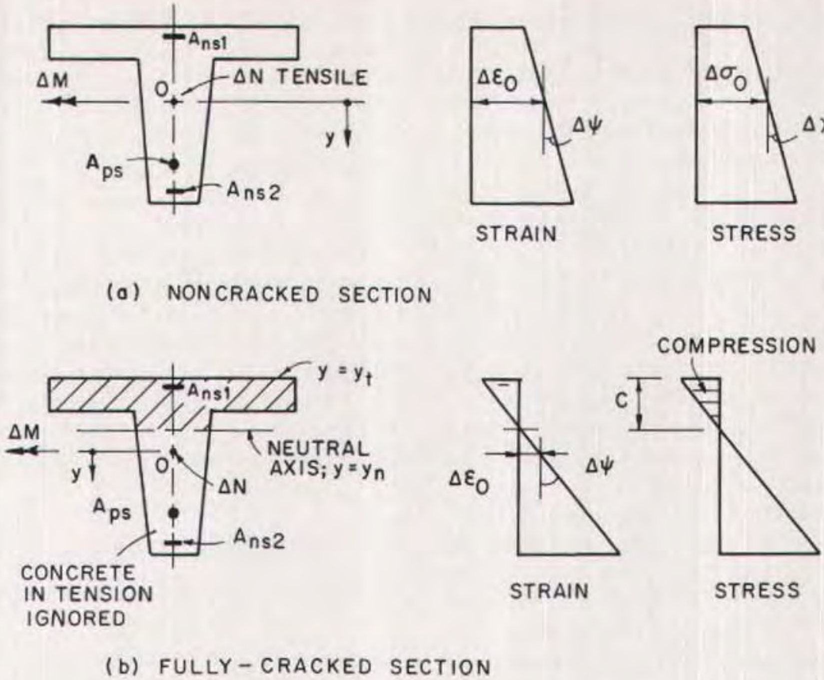


Fig. 1. Definition of symbols used in Eqs. (8) to (18).

$$\frac{\int_{y_t}^{y_n} y\,(y_n - y)\,dA + \sum\left[\frac{E_s}{E_c} A_s\, y_s\,(y_n - y_s)\right]}{\int_{y_t}^{y_n} (y_n - y)\,dA + \sum\left[\frac{E_s}{E_c}\ A_s\,(y_n - y_s)\ \right]} - \frac{\Delta M}{\Delta N} = 0 \text{ (when } \Delta N \neq 0) \qquad (18)$$

where y_t is the y coordinate at the extreme compression fiber (the top of the section); the summation is for all steel layers. Subscripts of p or n may be added to each of the symbols A_s, y_s and E_s to refer to prestressed or nonprestressed steel.

Eq. (17) indicates that when $\Delta N = 0$, the neutral axis is at the centroid of the transformed section. When $\Delta N \neq 0$, Eq. (18) shows that y_n depends upon the eccentricity ($\Delta M/\Delta N$), not on the separate magnitudes of ΔM and ΔN.

For a T section, Eq. (17) or (18) yields respectively a quadratic or a cubic equation in c, where $c = y_n - y_t$ = depth of the compression zone. Solution of the quadratic equation when $\Delta N = 0$ or the cubic equation when $\Delta N \neq 0$ gives the value of c. Tables and graphs are available[10] for determining c and properties of the transformed fully-cracked section.

Eqs. (17) and (18) apply only when the stress at top fiber is compressive and when the stress changes sign within the section depth as illustrated in Fig. 1b. This occurs when the resultant of ΔN and ΔM is a compressive or a tensile force situated respectively above or below the core of the noncracked transformed section. In other cases, the section has a stress of one sign, either compressive (noncracked) or tensile (fully cracked, all concrete ignored).

DECOMPRESSION FORCES

Partially prestressed sections are often designed to have no cracking under sustained dead load, but cracking is allowed due to live load. Assume, for a noncracked section, that at a specified instant t the stress distribution is known and that this is defined by the values of stress σ_o at a reference point and the slope γ of the stress diagram. It is required to determine the changes in strain and stress due to additional loading introduced at this instant producing internal forces $\{\Delta N, \Delta M\}$, which are of magnitudes sufficient to produce cracking.

Cracking occurs only after introduction of a part of the added load. Thus the pair of forces $\{\Delta N, \Delta M\}$ must be partitioned into a decompression part to be applied on a noncracked section and a remaining part on the fully-cracked section.

The decompression forces are the values of the normal force at O and the bending moment which reduce the stress in concrete to zero; from Eqs. (13a) and (13b):

$$\Delta N_{decompression} = A\,(-\sigma_o) + B\,(-\gamma) \quad (19a)$$

$$\Delta M_{decompression} = B\,(-\sigma_o) + I\,(-\gamma) \quad (19b)$$

Note that A, B and I are the properties of the transformed noncracked section at time t.

The decompression forces are to be applied on a noncracked section and the corresponding changes in axial strain and curvature are $[-\sigma_o/E_c(t)]$ and $[-\gamma/E_c(t)]$.

The forces to be applied on a fully-cracked section are:

$$\Delta N_{fully\ cracked} = \Delta N - \Delta N_{decompression} \quad (20a)$$

$$\Delta M_{fully\ cracked} = \Delta M - \Delta M_{decompression} \quad (20b)$$

The depth c of the compression zone based on the eccentricity $(\Delta M/\Delta N)_{fully\ cracked}$ must be determined as discussed in the preceding section. Eqs. (14a), (14b), and (10) may be used to calculate the changes in strain and in stress distributions in the cracking stage using for A, B and I properties of the fully-cracked section.

It is important to note that the width of cracks in reinforced concrete members depends mainly upon the increment of steel stress which occurs at the cracking stage, not on the steel stress after cracking. In fact, it is possible for cracking to occur while the steel is in compression as will be demonstrated in Example 1.

TIME-DEPENDENT CHANGES IN STRAIN AND IN STRESS

The values $\epsilon_o\,(t_o)$ and $\psi\,(t_o)$ defining the strain distribution at time t_o are assumed known for a cross section reinforced with prestressed and nonprestressed steels (Fig. 1a). It is required to find the changes in strain and in stress due to creep and shrinkage of concrete and relaxation of prestressed steel.

The following values which depend upon t_o and t and material properties are assumed to be known: $\Delta\bar{\sigma}_{pr}$, ϕ, χ and ϵ_{cs}. Guidance on the values of the four parameters for use in design may be found in Refs. 10, 12 and 15. The symbol ϵ_{cs} represents the shrinkage which occurs when the shortening of a member is not restrained by the reinforcement or by end forces.

Assume that creep and shrinkage are artificially restrained by a gradually introduced stress increment which attains at time t the value:

$$\Delta\,\sigma_{restraint} = -\,\bar{E}_c\,[\phi\,\epsilon_c\,(t_o) + \epsilon_{cs}] \quad (21)$$

where $\bar{E}_c$ is the age-adjusted elasticity modulus [Eq. (4)].

The restraining stress has the following resultants:

$$\Delta N_{creep} = -\bar{E}_c \phi [A_c \epsilon_o (t_o) + B_c \psi (t_o)] \quad (22a)$$

$$\Delta M_{creep} = -\bar{E}_c \phi [B_c \epsilon_o (t_o) + I_c \psi (t_o)] \quad (22b)$$

$$\Delta N_{shrinkage} = -\bar{E}_c \epsilon_{cs} A_c \quad (23a)$$

$$\Delta M_{shrinkage} = -\bar{E}_c \epsilon_{cs} B_c \quad (23b)$$

where A_c, B_c and I_c are the properties of the concrete section without any reinforcements.

Eqs. (22) and (23) are derived from Eqs. (10) and (13), assuming that the free shrinkage ϵ_{cs} is uniform. The area of post-tensioned ducts (not grouted before the instant t_o) may be excluded from A_c, B_c and I_c in Eq. (22).

The strain due to relaxation of prestressed steel can be artificially prevented by the forces:

$$\Delta N_{relaxation} = \Sigma (A_{ps} \Delta \bar{\sigma}_{pr}) \quad (24a)$$

$$\Delta M_{relaxation} = \Sigma (A_{ps} y_{ps} \Delta \bar{\sigma}_{pr}) \quad (24a)$$

The summation is for prestressed steel layers. All strains can be prevented by the restraining forces:

$$\Delta N = \Delta N_{creep} + \Delta N_{shrinkage} + \Delta N_{relaxation} \quad (25a)$$

$$\Delta M = \Delta M_{creep} + \Delta M_{shrinkage} + \Delta M_{relaxation} \quad (25b)$$

The artificial restraint can now be eliminated by the application of $\{-\Delta N, -\Delta M\}$ on an age-adjusted transformed section composed of the area of concrete plus the areas of the reinforcements, each multiplied by its modulus of elasticity and divided by the age-adjusted modulus of elasticity of concrete. The strain thus resulting is the change due to creep shrinkage and relaxation during the period t_o to t and may be determined by Eqs. (14a) and (14b).

$$\Delta \epsilon_o (t, t_o) = \frac{1}{\bar{E}_c (\bar{A} \bar{I} - \bar{B}^2)} [\bar{I} (-\Delta N) - \bar{B} (-\Delta M)] \quad (26a)$$

$$\Delta \psi_o (t, t_o) = \frac{1}{\bar{E}_c (\bar{A} \bar{I} - \bar{B}^2)} [-\bar{B} (-\Delta N) + \bar{A} (-\Delta M)] \quad (26b)$$

Note that $\bar{A}$, $\bar{B}$ and $\bar{I}$ are properties of the age-adjusted transformed section. The change in strain $\Delta \epsilon (t, t_o)$ at any concrete fiber or in any layer of reinforcement can now be calculated by Eq. (8). The corresponding changes in stress in concrete, prestressed and nonprestressed steel are:

$$\Delta \sigma_c (t, t_o) = \Delta \sigma_{restraint} + \bar{E}_c \Delta \epsilon_c (t, t_o) \quad (27)$$

$$\Delta \sigma_{ps} (t, t_o) = \Delta \bar{\sigma}_{pr} + E_{ps} \Delta \epsilon_{ps} (t, t_o) \quad (28)$$

$$\Delta \sigma_{ns} (t, t_o) = E_{ns} \Delta \epsilon_{ns} (t, t_o) \quad (29)$$

For a statically indeterminate structure, the time-dependent effects change the reactions and produce statically indeterminate (secondary) increments of internal forces at all sections. These increments must be estimated (e.g., as a ratio of the initial secondary forces) and added to $(-\Delta N)$ and $(-\Delta M)$, and the sum used to calculate the time-dependent strain changes by Eqs. (26a) and (26b).

The equations of this section may be applied for a reinforced concrete section without prestressing simply by setting $A_{ps} = 0$.

If a section is cracked before time t_o and the stress distribution at time t_o is known for the fully-cracked section, the same equations presented above may be used if the depth c of the compression zone is assumed constant during the period t_o to t. Thus, A_c, B_c and I_c used in Eqs. (22) and (23) or $\bar{A}$, $\bar{B}$ and $\bar{I}$ in Eqs. (26a) and (26b) include only the concrete in the compression zone at time t_o. This assumption greatly simplifies the analysis of time-dependent changes in strain and stress and does not result in a significant error.

Creep and shrinkage of concrete generally result in an increase in the depth c of the compression zone. Thus, with the assumption that c does not change with time, an area adjacent to the neutral axis is ignored although it is subjected to compressive stress. Because the ignored area is close to the neutral axis, the error involved is small. It should be noted that the assumption does not result in violation of the requirements of equilibrium of forces in the concrete and the reinforcements and of compatibility of strain in concrete and steel at all the reinforcement layers.

At the bottom fiber of a cracked flexural member the time-dependent change in strain is usually positive. Thus, a prestressed steel layer located near the soffit of a member exhibits a time-dependent elongation. In this case, the parameter Ω, defined by Eq. (7), will be negative and the coefficient χ_r greater than 1.0 [Eq. (6)], indicating that the relaxation in the prestressed member is greater than the intrinsic relaxation.

TENSION STIFFENING

Concrete in the tension zone contributes to some extent to the stiffness of a cracked member. Thus, when the properties of the fully-cracked transformed section are used in Eqs. (14a) and (14b), an overestimation of the absolute values of the axial strain $\Delta\,\epsilon_o$, the curvature $\Delta\,\psi$ and the displacements determined therefrom will result.

To account for the stiffening effect of the concrete in tension, two pairs of values $\{\Delta\,\epsilon_o, \Delta\,\psi\}$ must be calculated using the properties of a noncracked and of a fully-cracked section and interpolation is performed to obtain mean values from which the displacements are determined. The following empirical interpolation equation[12,13] is adopted here:

$$\Delta\,\epsilon_{o\ mean} = (1-\zeta)\,\Delta\,\epsilon_{o\ noncracked} + \zeta\,\Delta\,\epsilon_{o\ fully\ cracked} \qquad (30a)$$

$$\Delta\,\psi_{o\ mean} = (1-\zeta)\,\Delta\,\psi_{o\ noncracked} + \zeta\,\Delta\,\psi_{o\ noncracked} \qquad (30b)$$

The interpolation coefficient ζ is given by:

$$\zeta = 1 - \beta_1\,\beta_2 \left(\frac{f_{ct}}{\Delta\,\sigma_{max}}\right)^2 \qquad (31)$$

where $\Delta\,\sigma_{max}$ is the change in stress at the extreme tension fiber due to $\{\Delta\,N, \Delta\,M\}_{fully\ cracked}$ applied on a noncracked section; f_{ct} is the concrete strength in tension; $\beta_1 = 1$ or 0.5 for high bond or plain reinforcing bars, respectively; $\beta_2 =$ 1 or 0.5; the value 1 is to be used for first loading and 0.5 for the case when the load is applied in a sustained manner or in a large number of cycles.

The mean value of crack width at the level of a steel layer:

$$w = \zeta\,s\,\Delta\,\epsilon_{s\ fully\ cracked} \qquad (32)$$

where $\Delta\,\epsilon_{s\ fully\ cracked}$ is the strain increment at the level of a reinforcement layer calculated for a fully-cracked section; s is the spacing between cracks. An empirical equation is available[12,13] for calculating s as a function of parameters including concrete cover, bar spacing, bar diameter, bond quality and reinforcement ratio.

Eqs. (30a), (30b) and (32) give mean values of strain increment and crack width for a cross section for which the stress in concrete is zero prior to application of the load considered. For partial prestressing, when cracking is produced by live load, Eqs. (30a), (30b) and (32) apply for the change in strain due to the increments $\{\Delta\,N, \Delta\,M\}_{fully\ cracked}$, introduced after the decompression stage [see Eqs. (20a) and (20b)].

STEPS OF ANALYSIS

The following are the steps of analysis of strain and stress at time t_o and t in a reinforced section with or without prestress; t_o is the time at which the dead load and prestress (if any) are introduced and sustained to time t at which additional live load is applied:

1. Determine the instantaneous increments of stress and strain at t_o using Eqs. (14a), (14b) and (10) with the properties of a transformed noncracked section (or fully cracked if cracking occurs at this stage).

2. Calculate the time-dependent increments of strain and stress during the period t_o to t, using Eqs. (26) to (29). When no additional load is applied at time t, the calculations may be concluded here by summing up the increments in Steps 1 and 2 (see Example 2).

3. Add the stress increments in Steps 1 and 2 to obtain the stress distribution before application of the live load and use this stress to determine the decompression forces by Eqs. (19a) and (19b). The change in strain in the decompression stage is simply equal to minus the stress divided by $E_c(t)$.

4. Use Eqs. (20a) and (20b) to determine the part of internal forces due to live load which should be applied to a fully-cracked section. Determine the eccentricity and the corresponding depth of the compression zone using Eq. (18) or available[10] graphs or tables. Apply Eqs. (14a), (14b) and (10) with the properties of a fully-cracked section to determine the changes in strain and in stress in the cracking stage. The stress thus calculated is also the final stress in concrete because the stress is zero after decompression.

The final stress in the reinforcements is determined by summing up the initial stress to the changes calculated in Steps 1 to 4. The final axial strain ϵ_o and the curvature ψ are the sum of increments determined in the same steps. The values ϵ_o and ψ calculated do not account for the tension stiffening. If the displacements determined from these values are more than allowed in design, the increments of strain determined in Step 4 may be adjusted to account for tension stiffening by Eqs. (30a) and (30b). In most cases the adjustment results in a small reduction of displacements.

A computer program which performs the above steps of analysis is available.[7]

EQUILIBRIUM CHECKS

The calculation for the time-dependent stress increments (Step 3) may be checked by verifying that the stress changes in the concrete and the reinforcements are self equilibrating when the analysis is for a cross section of a statically determinate structure.

Note that when the structure is statically indeterminate the time-dependent stress increment has a resultant of known or assumed magnitude, discussed in an earlier section, immediately below Eq. (29).

The final stress determined in Step 4 may also be checked by verifying that the resultant of stresses introduced in concrete and steel is statically equivalent to the resultant of the initial prestress and the normal force and bending moment due to dead load and live load.

MULTISTAGE PRESTRESSING AND LOADING

Structures are often prestressed in a number of stages during construction. This procedure is often used in bridge construction where ducts are left in the concrete for the prestressing tendons to be inserted and prestressed in stages to suit the development of forces due to the structure self weight as the construction progresses.

For each set of forces introduced simultaneously (e.g., prestressing plus dead load), the calculations in Step 1 are performed as outlined above and the increments of stress and strain added to the values existing before this stage. If the increment of internal forces in this stage produces cracking, Step 1 must be replaced by Steps 3 and 4 and the stress and strain updated after each step.

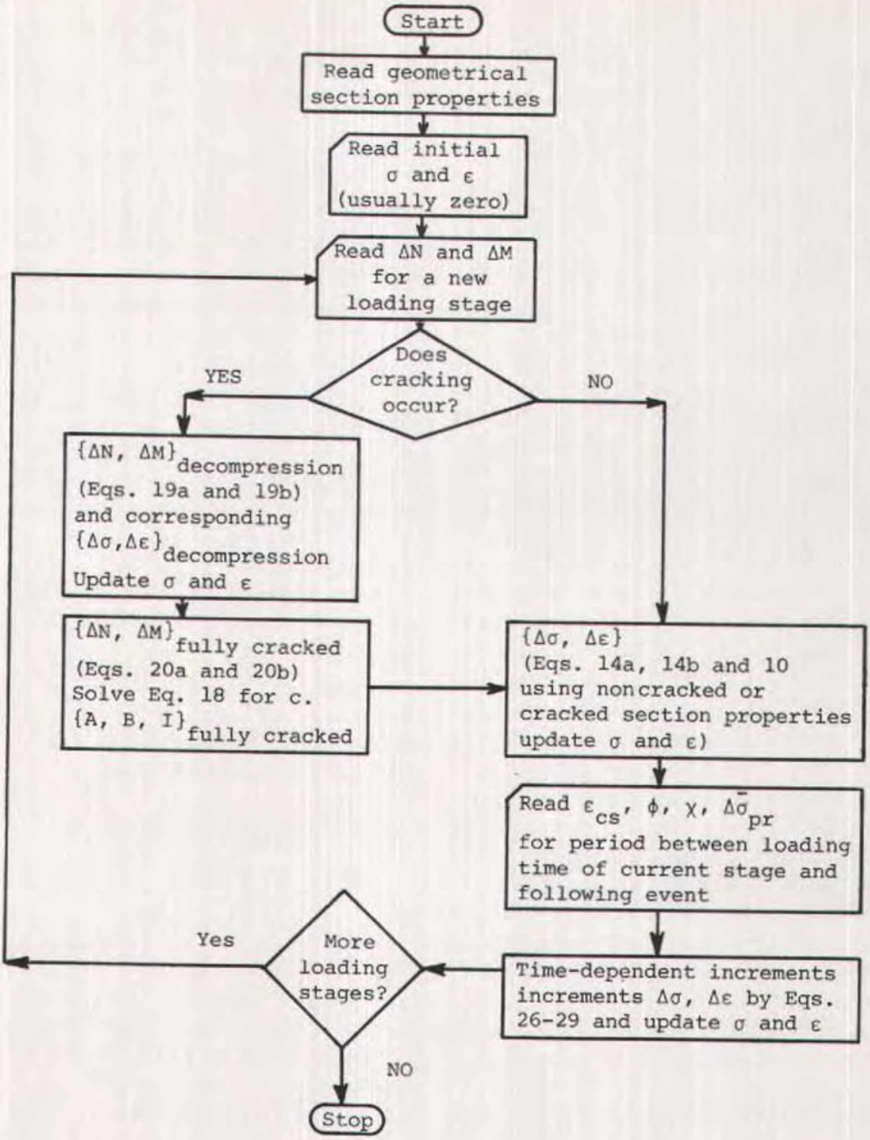


Fig. 2. Flow chart for analysis of stress and strain in a concrete section prestressed and/or loaded in a multiple of stages.

Performing the calculations in Step 2, and updating σ and ϵ, will complete a load cycle which is repeated for each loading stage. For a quick visualization of this process, see the flow chart above in Fig. 2.

The computer program CRACK,[8] referred to earlier, can be used with multistage loading for sections composed of one or more types of concrete or of concrete and steel with or without prestressing. The shape of the cross sections must be idealized as a set of rectangles.

CALCULATION OF DISPLACEMENTS

The displacement at any location may be determined from the axial strain $\Delta \epsilon_o$ and the curvatures $\Delta \psi$ by virtual work or by other methods of structural analysis. The following two equations may be conveniently used to find the change in length Δl and in transverse deflection, $\Delta \delta$ at the center with respect to the ends of a straight member:

$$\Delta l = \frac{l}{6}(\Delta \epsilon_{01} + 4\,\Delta \epsilon_{02} + \Delta \epsilon_{03}) \qquad (33)$$

and

$$\Delta \delta = \frac{l^2}{96}(\Delta \psi_1 + 10\,\Delta \psi_2 + \Delta \psi_3) \qquad (34)$$

The subscripts 1, 2 and 3 refer to sections at the left end, middle and right end, respectively. These equations are derived assuming a parabolic variation of the axial strain and the curvature. The equations are applicable for an isolated member or for a member in a continuous structure.

In a preliminary calculation of $\Delta \delta$, it may suffice to calculate the curvature at the middle section of the member and ignore or estimate the other two curvatures in Eq. (34). Other expressions similar to Eqs. (33) and (34) using five sections instead of three can be found in Ref. 10.

NUMERICAL EXAMPLES

The following three numerical examples demonstrate the applicability of the proposed design method.

Example 1

The cross section of a tie in Fig. 3a is post-tensioned at time t_o with a prestressing force equal to 225 kips and no dead load is simultaneously applied with the prestress.

Find the total change in length of the tie and the stresses in the reinforcement at time t before and after application of a live load producing a tensile force equal to 270 kips at the center of the section. What is the mean width of the cracks?

Given data: $E_c(t_o) = 3600$ ksi; $E_c(t) = 4500$ ksi; $E_{ps} = 27000$ ksi; $E_{ns} = 29000$ ksi; $\phi(t, t_o) = 2.5$; $\chi = 0.75$; $\Delta\bar{\sigma}_{pr} = -13$ ksi; $\epsilon_{cs} -250 \times 10^{-6}$; $\beta_1 = 1.0$; $\beta_2 = 0.5$; $f_{ct} = 0.4$ ksi. Length of tie = 100 ft; crack spacing = 8 in.

(a) Instantaneous increments at t_o

The area of the transformed section (excluding A_{ps} and area of duct) = 162 in.² Substitution in Eqs. (15a) and (10) with $\Delta N = -225$ kips, gives $\Delta \epsilon_0(t_o) = -385 \times 10^{-6}$; $\Delta \sigma_c(t_o) = -1.39$ ksi; $\Delta \sigma_{ns} = -11.2$ ksi.

(b) Changes between t_o and t

The age-adjusted modulus of elasticity of concrete [Eq. (4)], $\bar{E}_c = 1252$ ksi. Forces necessary to restrain creep and shrinkage and relaxation are [Eqs. (22) to (25)]:

$\Delta N_{creep} = 166$; $\Delta N_{shrinkage} = 44$

$\Delta N_{relaxation} = -16$; ΔN (total) = 194 kips

The age-adjusted transformed area (including the two steels and assuming the duct filled) = 235 in.² Substitution in Eq. (26a) (with $B = 0$) gives:

$\Delta \epsilon_0(t, t_o) = -194/(1252 \times 235) = -661 \times 10^{-6}$

From Eq. (21):

$$\Delta \sigma_{restraint} = -1252\,[2.5\,(-385 \times 10^{-6}) + (-250 \times 10^{-6})] = 1.52 \text{ ksi}$$

Changes in stresses during the period t_o to t [Eqs. (27) to (29)]:

$$\Delta \sigma(t, t_o) = 1.52 + 1252\,(-661 \times 10^{-6}) = 0.69 \text{ ksi}$$

$$\Delta \sigma_{ns}(t, t_o) = 29000\,(-661 \times 10^{-6}) = -19.1 \text{ ksi}$$

$$\Delta \sigma_{ps}(t, t_o) = -13 + 27000\,(-661 \times 10^{-6}) = -30.8 \text{ ksi}$$

(c) The decompression stage

Stress in concrete at time t, before live

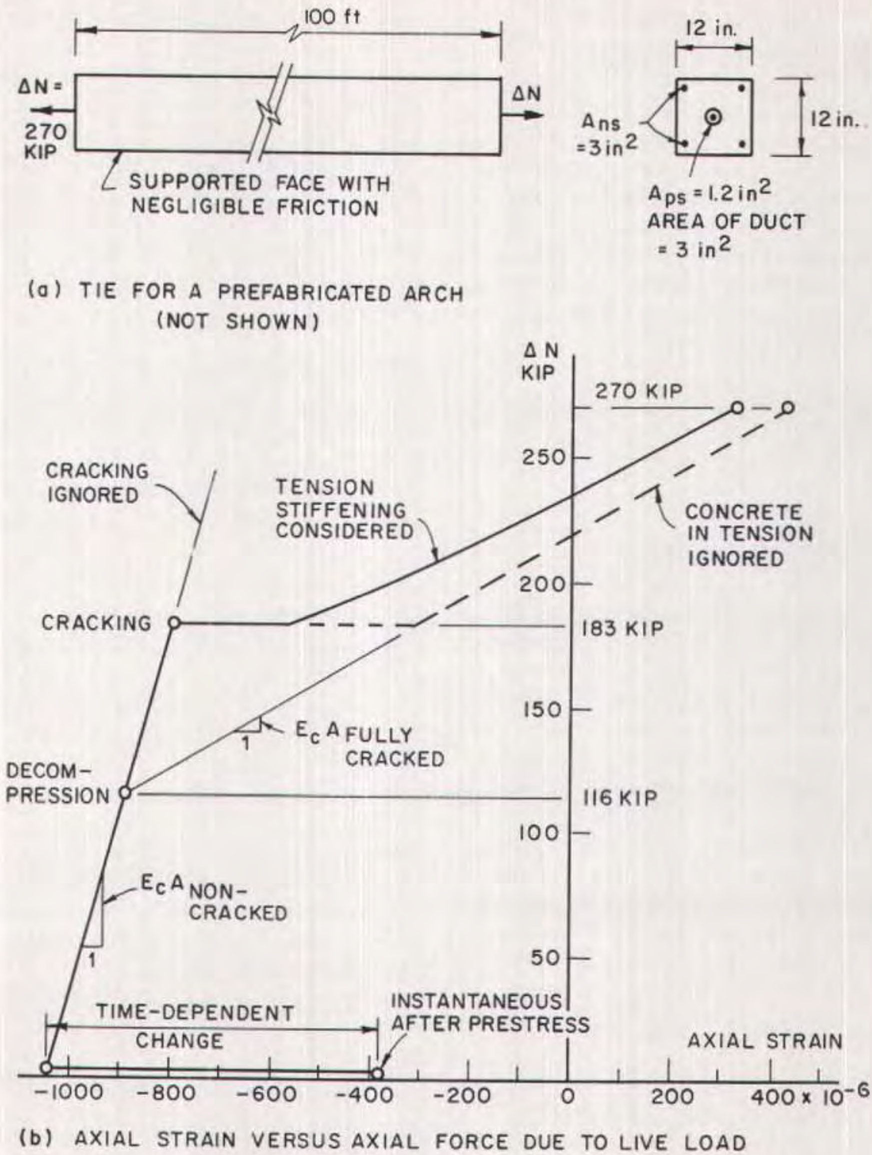


Fig. 3. Analysis of strain and stress in a partially prestressed tie (Example 1).

load = −1.39 + 0.69 = −0.70 ksi.
Area of transformed section at time t = 166 in.²
Decompression force [Eq. (19)]:
$\Delta N_{decompression} = -166\,(-0.70) = 116$ kips
Changes in strain and in stress by decompression [Eqs. (15a) and (10)]:

$$\Delta \epsilon_{0\ decompression} = \frac{116}{4500\,(166)} = 155 \times 10^{-6}$$

$$\Delta \sigma_{ns} = 29000\,(155 \times 10^{-6}) = 4.5 \text{ ksi}$$

$$\Delta \sigma_{ps} = 27000\,(155 \times 10^{-6}) = 4.2 \text{ ksi}$$

(d) The cracking stage

Using Eq. (20):

$\Delta N_{fully\ cracked} = 270 - 116 = 154$ kips

Area of transformed fully-cracked section = 26 in.^{2}

Changes in strain and in stress at cracking [Eqs. (15a) and (10)]:

$$\Delta \epsilon_{o\ fully\ cracked} = \frac{154}{4500\,(26)} = 1292 \times 10^{-6}$$

$\Delta \sigma_{ns} = 29000\,(1292 \times 10^{-6}) = 37.5$ ksi

$\Delta \sigma_{ps} = 27000\,(1292 \times 10^{-6}) = 34.9$ ksi

Concrete stress cracking ignored:

$$\Delta \sigma_{max} = \frac{154}{166} = 0.93 \text{ ksi}$$

$$\Delta \epsilon_{o\ noncracked} = \frac{154}{4500\,(166)} = 206 \times 10^{-6}$$

Interpolation coefficient [Eq. (31)]:

$$\zeta = 1 - 0.5\left(\frac{0.40}{0.93}\right)^2 = 0.91$$

Mean strain [Eq. (30a)]:

$$\Delta \epsilon_{o\ mean} = [(1 - 0.91)\,206 + 0.91 \times 1292]\,10^{-6} = 1191 \times 10^{-6}$$

Change in length of tie just before live load = $(100 \times 12)\,(-385 - 661)\,10^{-6} = -1.26$ in.

Sum of axial strain and strain changes in the above four steps:

$\Delta \epsilon_o = 10^{-6}\,(-385 - 661 + 155 + 1191) = 300 \times 10^{-6}$

$\Delta\sigma_{ns} = -11.2 - 19.1 + 4.5 + 37.5 = 11.7$ ksi

$\Delta \sigma_{ps} = 0 - 30.8 + 4.2 + 34.9 = 8.3$ ksi

Final change in length of tie:

$(100 \times 12)\,(300 \times 10^{-6}) = 0.36$ in.

Crack width [Eq. (32)]:

$w = 0.91 \times 8\,(1292 \times 10^{-6}) = 0.009$ in.

A graph of the axial strain versus the axial force due to live load is shown in Fig. 3b. The dotted line is calculated ignoring the tension stiffening.

It may be noticed that the crack width depends upon the change in strain in the cracking stage, which is the strain occurring after decompression. The final stress in the nonprestressed steel after cracking should not be used to determine the crack width. In fact, cracking can occur while the nonprestressed steel is still in compression.

In this example, after decompression of concrete the nonprestressed steel has a compression of:

$(-11.2 - 19.1 + 4.5) = -25.8$ ksi

At the first load cycle, cracking occurs when $\Delta N = 183$ kips, the corresponding stress in the nonprestressed steel equals -23.2 ksi.

The crack width may be reduced by an increase of A_{ps} and the prestressing force and a reduction of A_{ns}, without a change in the ultimate strength.

Example 2

Find the instantaneous deflection and crack width at the center of an 80 ft simple span T beam, reinforced without prestressing (Fig. 4a) and subjected to a uniform load of 2.1 kips per ft applied at time t_o and sustained to time t.

What is the time-dependent change in deflection due to creep and shrinkage between time t_o and t?

Given data: $E_c\,(t_o) = 3600$ ksi; $E_{ns} = \epsilon_{cs}$ 29000 ksi; $f_{ct} = 0.5$ ksi; $\beta_1 = 1.0$; $\beta_2 = 0.5$; $s = 8$ in.; $\phi = 3$, $\chi = 0.8$; $\epsilon_{cs} = -300 \times 10^{-6}$.

Consider a section at midspan and use a reference point O at the top fiber. The analysis is done in two steps:

(a) Instantaneous changes at t_o

The bending moment at midspan, 20160 kip-in., is sufficient to produce cracking. The depth of the compression zone $c = 12.2$ in. is found from Eq. (17) or Ref. 9.

The properties of the transformed fully-cracked section at time t_o are:

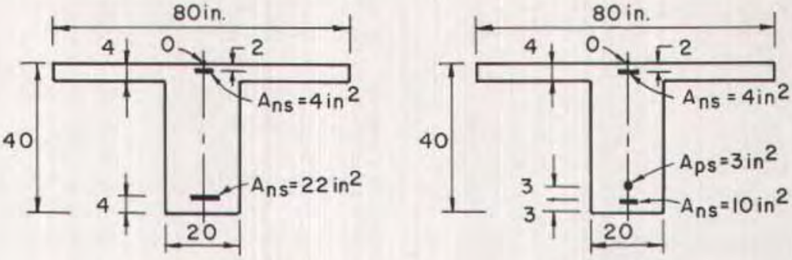
$A = 689$ in.^{2}; $B = 8403$ in.^{3}; $I = 243150$ in.^{4}

Substitution of these values in Eqs. (14a) and (14b) with $\Delta N = 0$, $\Delta M = 20160$ and $E_c = 3600$ gives:

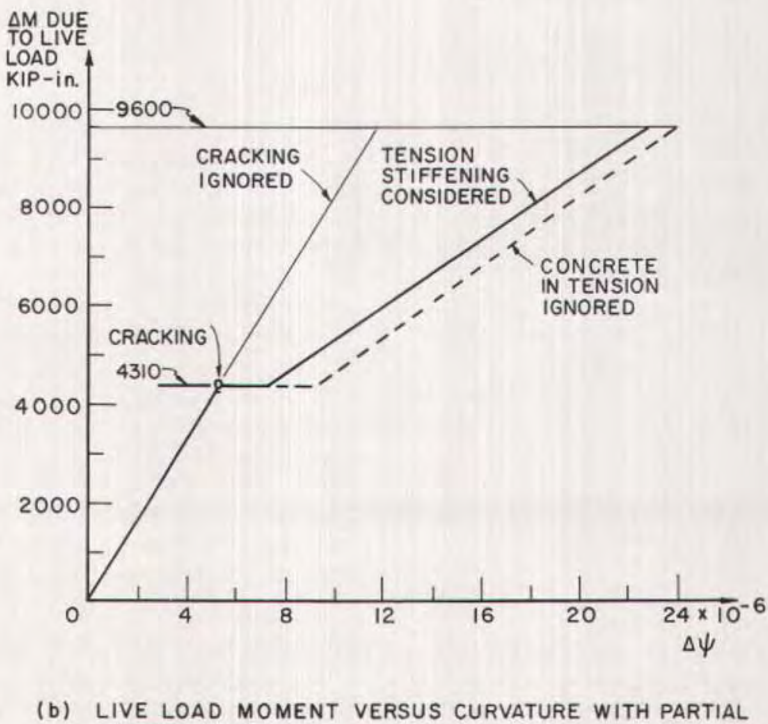
$\Delta \epsilon_{o\ fully\ cracked} = -485 \times 10^{-6}$

$\Delta \psi_{fully\ cracked} = 39.8 \times 10^{-6}$ in.$^{-1}$

The strain at the level of the bottom



(a) MIDSPAN SECTIONS FOR BEAMS OF EXAMPLES 2 AND 3



(b) LIVE LOAD MOMENT VERSUS CURVATURE WITH PARTIAL PRESTRESSING.

Fig. 4. Analysis of stress and strain for beams with and without prestressing (Examples 2 and 3).

steel [Eq. (8)], $(\Delta \epsilon_s)_{fully\ cracked}$ is 948 x 10^{-6}.

To account for tension stiffening, the calculation is repeated using the properties of a noncracked section, giving:

$\Delta \epsilon_{o\ noncracked} = -442 \times 10^{-6}$

$\Delta \psi_{noncracked} = 24.4 \times 10^{-6} \text{in.}^{-1}$

From Eqs. (10) and (8), the maximum tension at the bottom if the section were noncracked is:

$$\sigma_{max} = \sigma_{bot} = 3600\,[-442 + 24.4\,(40)]10^{-6} = 1.93 \text{ ksi}$$

Interpolation coefficient [Eq. (31)]:

$$\zeta = 1 - 0.5 \left(\frac{0.5}{1.93}\right)^2 = 0.97$$

Crack width at the level of the bottom steel [Eq. (32)]:

$$w = 0.97\,(8)\,(948 \times 10^{-6}) = 0.007 \text{ in.}$$

Mean strains [Eqs. (30a) and (30b)]:

$$\Delta\epsilon_{o\,mean} = (1 - 0.97)(-442 \times 10^{-6}) + 0.97\,(-485 \times 10^{-6}) = -484 \times 10^{-6}$$

$$\Delta\psi_{mean} = (1 - 0.97)(24.4 \times 10^{-6}) + 0.97\,(39.8 \times 10^{-6} = 39.3 \times 10^{-6} \text{in.}^{-1}$$

Instantaneous deflection at t_o [Eq. (34)]:

$$\Delta\delta = \frac{(80 \times 12)}{96}[0 + 10\,(39.8 \times 10^{-6}) + 0] = 3.82 \text{ in.}$$

(b) Changes between time t_o and t

Age-adjusted elasticity modulus [Eq. (4)]:

$\bar{E}_c = 1059$ ksi

The properties of area of concrete in the compression zone are:

$A_c = 480$ in.^{2}; $B_c = 1958$ in.^{3}; $I_c = 13340$ in.^{4}

The forces needed to prevent deformations due to creep and shrinkage of concrete are found from Eqs. (22), (23), and (25):

$$\Delta N_{creep} = -1059\,(3)\,[480\,(-485 \times 10^{-6}) + 1958\,(39.8 \times 10^{-6})] = 492 \text{ kips}$$

$$\Delta M_{creep} = -1059\,(3)\,]1958\,(-485 \times 10^{-6}) + 13340\,(39.8 \times 10^{-6})] = 1330 \text{ kip-in.}$$

$$\Delta N_{shrinkage} = -1059(-300 \times 10^{-6})\,(480) = 153 \text{ kips}$$

$$\Delta M_{shrinkage} = -1059(-300 \times 10^{-6})\,(1958) = 622 \text{ kip-in.}$$

Summing the above forces:

$\Delta N = 492 + 153 = 645$ kips

$\Delta M = 1330 + 622 = 1952$ kip-in.

The age-adjusted transformed fully-cracked section properties are:

$\bar{A} = 1192$ in.^{2}; $\bar{B} = 23870$ in.^{3}; $\bar{I} = 794700$ in.^{4}

Substitution in Eqs. (26a) and (26b) gives:

$\Delta\epsilon_o\,(t, t_o) = -1164 \times 10^{-6}$

$\Delta\psi\,(t, t_o) = 32.7 \times 10^{-6}$ in.$^{-1}$

Change in deflection due to creep and shrinkage [Eq. (34)]:

$$\Delta\delta\,(t, t_o) = \frac{(80 \times 12)^2}{96}[0 + 10\,(32.7 \times 10^{-6}) + 0] = 3.14 \text{ in.}$$

The total deflection (3.82 + 3.14 = 6.96 in.) may be too large to be accepted for design. In the following example partial prestressing will be used to reduce the deflection.

Example 3

The bottom 22 in.^{2} of nonprestressed steel in Example 2 is here replaced by 3 in.^{2} of prestressed and 10 in.^{2} of nonprestressed steel as shown in Fig. 4a (without appreciable change in the ultimate strength). At time t_o a prestressing force 600 kips and a dead load 1.1 kips per ft are introduced.

Find the deflection at the center of the span at time t immediately after application of a live load of 1 kip per ft which is sufficient to produce cracking.

Given data: $E_c\,(t_o) = 3600$ ksi; $E_c\,(t) = 4000$ ksi; $E_{ps} = 27000$ ksi; $E_{ns} = 29000$ ksi; $\phi\,(t, t_o) = 3$; $\chi = 0.8$; $\Delta\bar{\sigma}_{pr}\,(t, t_o) = -13$ ksi; $\beta_1 = 1.0$; $\beta_2 = 0.5$; $f_{ct} = 0.5$ ksi; $\epsilon_{cs} = -300 \times 10^{-6}$.

Consider a cross section at midspan and a reference point O at its top. The bending moments at this section due to dead load and due to live load are 10560 and 9600 kip-in., respectively. Four steps of analysis are performed:

(a) Instantaneous changes at t_o

The prestressing and dead load moment introduced at t_o are equivalent to a force, $\Delta N = -600$ kip at O (at top fiber) combined with a moment, $\Delta M = -9840$ kip-in.

The properties of the noncracked transformed section are:

$A = 1158$ in.^{2}; $B = 19810$ in.^{3}; $I = 547200$ in.^{4}

Substitution in Eqs. (14a), (14b) and (10) gives:

$\Delta \epsilon_o (t_o) = -154 \times 10^{-6}$
$\Delta \psi (t_o) = 0.57 \times 10^{-6}$ in.$^{-1}$
$\{\Delta \sigma_c (t_o)\}_{top,\ bottom} = \{-0.554, -0.472\}$ ksi

Therefore, no cracking occurs at this stage.

(b) Changes between time t_o and t

$\bar{E}_c = 1059$ ksi; $A_c = 1023$ in.^{2};
$B_c = 16000$ in.^{3}; $I_c = 410800$ in.^{4}
Eqs. (22) to (25) give:
$\Delta N_{creep} = 470$; $\Delta N_{shrinkage} = 325$; $\Delta N_{relaxation} = -39$
$\Delta M_{creep} = 7068$; $\Delta M_{shrinkage} = 5083$; $\Delta M_{relaxation} = -1326$
Summing:
$\Delta N = 470 + 325 - 39 = 756$ kips
$\Delta M = 7068 + 5083 - 1326 = 10825$ kip-in.

The properties of the age-adjusted transformed section are:
$\bar{A} = 1483$ in.^{2}; $\bar{B} = 28950$ in.^{3}; $\bar{I} = 874500$ in.^{4}

Substitution in Eqs. (26a) and (26b) gives:
$\Delta \epsilon_o (t, t_o) = -717 \times 10^{-6}$
$\Delta \psi (t, t_o) = 12.03 \times 10^{-6}$ in.$^{-1}$
Stress changes at top and bottom fibers [Eq. (27)]:
$\{\Delta \sigma_c (t, t_o)\}_{top,\ bottom} = \{0.047, 0.485\}$ ksi

(c) The decompression stage

The stress in the concrete at time t, just before live load is:

$$[\sigma_c (t)]_{top} = -0.554 + 0.047 = -0.507 \text{ ksi}$$
$$[\sigma_c (t)]_{bottom} = -0.472 + 0.485 = 0.013 \text{ ksi}$$

Thus, $\sigma_o = -0.507$ ksi and
$\gamma = [0.013 - (-0.507)]/40 = 0.013$ ksi/in.

The properties of the transformed section at time t are:
$A = 1145$ in.^{2}; $B = 19430$ in.^{3}; $I = 533600$ in.^{4}

Substitution in Eqs. (19a) and (19b) gives:

$$\Delta N_{decompression} = 1145(0.507) + 19430(-0.013) = 327 \text{ ksi}$$

$$\Delta M_{decompression} = 19430(0.507) + 533600(-0.013) = 2908 \text{ kip-in.}$$

The changes in strain by decompression [Eqs. (14a) and (14b) or from $-\sigma_o/E_c (t)$ and $-\gamma/E_c (t)$] are:
$\Delta \epsilon_{o\ decompression} = 127 \times 10^{-6}$
$\Delta \psi_{decompression} = -3.24 \times 10^{-6}$ in.$^{-1}$

(d) The cracking stage

Internal forces due to live load:
$\Delta N = 0$; $\Delta M = 9600$ kip-in.
Forces to be applied on a fully-cracked section [Eqs. (20a) and (20b)]:
$\Delta N_{fully\ cracked} = 0 - 327 = -327$ kips
$\Delta M_{fully\ cracked} = 9600 - 2908 = 6692$ kip-in.

This pair of forces is equivalent to a compressive force of -327 kips at 20.5 in. above the top edge.

The depth of the compression zone is $c = 13.9$ in. [by solving Eq. (18) or using Ref. 9].

The properties of the fully-cracked section are:
$A = 635$ in.^{2}; $B = 5824$ in.^{3}; $I = 141800$ in.^{4}

Substitution in Eqs. (14a) and (14b) gives:
$\Delta \epsilon_{o\ fully\ cracked} = -380 \times 10^{-6}$
$\Delta \psi_{fully\ cracked} = 27.41$ in.$^{-1}$

If cracking is ignored, the same forces will produce the following changes in strain [note that Eqs. (14a) and (14b) can be used again with section properties of noncracked section at time t, see Step 3]:
$\Delta \epsilon_{o\ noncracked} = -326 \times 10^{-6}$
$\Delta \psi_{noncracked} = 15.02 \times 10^{-6}$ in.$^{-1}$

From Eq. (10) the maximum tension (at bottom fiber) will be:

$$\sigma_{max} = 4000 \times 10^{-6} (-326 + 15.04 \times 40) = 1.10 \text{ ksi}$$

Interpolation coefficient [Eq. (31)]:

$$\zeta = 1 - 0.5 \left(\frac{0.5}{1.10}\right)^2 = 0.90$$

Mean values of axial strain and curvature increments [Eqs. (30a) and (30b)]:

$$\Delta \epsilon_{o\ mean} = (1 - 0.90)(-326 \times 10^{-6}) + 0.9(-380 \times 10^{-6}) = -375 \times 10^{-6}$$

$$\Delta \psi_{mean} = (1 - 0.90)(15.02 \times 10^{-6}) + 0.9(27.41 \times 10^{-6}) = 26.12 \times 10^{-6} \text{ in.}^{-1}$$

The final curvature after application of the live load is the sum of the increments calculated in Steps 1 to 4.

$$\Delta \psi_{final} = 10^{-6}(0.57 + 12.03 - 3.24 + 26.12) = 35.48\ 10^{-6}$$

The corresponding deflection, using Eq. (34) and ignoring curvatures at the ends is:

$$\Delta \delta = \frac{(80 \times 12)^2}{96}[0 + 10(35.48\ 10^{-6}) + 0] = 3.41 \text{ in.}$$

The graph in Fig. 4b represents the bending moment increment due to live load versus the corresponding instantaneous increment in curvature with and without accounting for tension stiffening.

The deflection in the prestressed beam in this example is obviously smaller than the beam without prestressing in Example 2. If further reduction of deflection is desired, the area of prestressed steel can be further increased, e.g., to $A_{ps} = 4$ in.^{2} and the bottom nonprestressed steel reduced to $A_{ns} = 6$ in.^{2} This would reduce the final deflection to 1.78 in. without appreciable change in ultimate strength.

CONCLUSION

A set of equations is presented for the analysis of strain and stress in noncracked or cracked reinforced concrete sections subjected to normal force and moments due to loads or prestressing introduced in stages. The cross section may have prestressing of any amount varying from zero allowing cracking to full prestressing eliminating cracking.

The analysis is based on requirements of equilibrium and compatibility of strain in concrete and all layers of reinforcements. In this way, the time-dependent changes in forces in the prestressed and nonprestressed steels and in the concrete are analyzed and their effects on strains accounted for without the need to precede the analysis by an estimate of the prestress loss by empirical or code-dependent equations.

The effect of tension stiffening when cracking occurs is determined by interpolation between lower and upper limits of strains calculated for a noncracked section and for a fully-cracked section ignoring concrete in tension. Once the forces on a section exceed the limit which produces cracking, the strains quickly approach the values of a fully-cracked section; thus, the upper limit calculated ignoring concrete in tension is not in many cases much larger than the improved values accounting for tension stiffening.

An empirical equation for the interpolation is adopted here from the 1978 CEB-FIP Code,[13] but other interpolation procedures should not appreciably change the results. Thus, the unified approach presented above for prestressed and nonprestressed members can be used in design to check serviceability requirements for deflection and crack width specified in any code.

Although the equations are fairly simple, designers may dislike the repetitious calculations of transformed section properties involved in the steps of the analysis. To save time, programmable hand-held calculators can perform this task. Reference is made to an available relatively simple computer program[8] which performs the entire analysis. It should be possible to use, or adapt, the program for most micro-computers.*

*A version of the program on disc for IBM personal computers is also available.

* * *

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Metric (SI) Conversion Factors

1 ft = 3.28 m	1 ksi = 6.90 MPa
1 in. = 25.4 mm	1 kip/ft = 14.6 kN/m
1 kip = 4.45 kN	1 kip-in. = 0.113 kN·m

APPENDIX — NOTATION

$A, \bar{A}$ = area of transformed and of age-adjusted transformed sections
$B, \bar{B}$ = first moment of area of transformed section and of age-adjusted transformed section
c = depth of compression zone in a fully-cracked section
$E, \bar{E}$ = modulus of elasticity and age-adjusted elasticity modulus
f_{ct} = strength of concrete in tension
$I, \bar{I}$ = moment of inertia of transformed section and of age-adjusted transformed section
M = bending moment
N = normal force
s = crack spacing
t = time
y = coordinate of any fiber, measured downwards from a reference point O
β_1, β_2 = coefficients 0.5 or 1 as specified below Eq. (31)
γ = slope of stress diagram
δ = deflection
Δ = increment or decrement
ϵ = normal strain
$\Delta \sigma_{pr}, \Delta \bar{\sigma}_{pr}$ = intrinsic and reduced relaxation of prestressed steel
σ = stress
ϕ = creep coefficient
χ = aging coefficient
χ_r = relaxation reduction factor
ψ = curvature (slope of strain diagram)

Subscripts

c, ps, ns = concrete, prestressed and nonprestressed steel
cs = shrinkage of concrete
o = initial time
O = reference point

* * *

NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by November 1, 1986.