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A warranty forecasting model based on piecewise statistical distributions and stochastic simulation

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Abstract

This paper presents a warranty forecasting method based on stochastic simulation of expected product warranty returns. This methodology is presented in the context of a high-volume product industry and has a specific application to automotive electronics. The warranty prediction model is based on a piecewise application of Weibull and exponential distributions, having three parameters, which are the characteristic life and shape parameter of the Weibull distribution and the time coordinate of the junction point of the two distributions. This time coordinate is the point at which the reliability 'bathtub' curve exhibits a transition between early life and constant hazard rate behavior. The values of the parameters are obtained from the optimum fitting of data on past warranty claims for similar products. Based on the analysis of past warranty returns it is established that even though the warranty distribution parameters vary visibly between product lines they stay approximately consistent within the same product family, which increases the overall accuracy of the simulation-based warranty forecasting technique. The method is demonstrated using a case study of automotive electronics warranty returns. The approach developed and demonstrated in this paper represents a balance between correctly modeling the failure rate trend changes and practicality for use by reliability analysis organizations.

Keywords: Stochastic simulation; Warranty analysis; Piecewise distribution; Monte Carlo; Automotive warranty forecasting; Weibull analysis

1. Introduction

Market conditions have traditionally been the main factor that determines the terms of automotive warranties. While expected reliability and quality of the product is considered an important supporting factor, in reality, the actual warranty terms are most often determined by marketing pressures. Currently the terms of the standard automotive warranty, often referred to as the manufacturer's basic warranty are 36 months or 36,000 miles (whichever comes first) on the majority of vehicle parts (see, for example, [1]) with additional extended warranties on selected subsystems. Longer warranty periods are often used as an enhanced marketing tool; warranty history and warranty expectations greatly affect the market value of new and used cars sold, and lease residual values. Because of these and other financial and marketing considerations, a multitude of business decisions are being made based on the forecasted number of warranty returns for the overall warranty period and subsets thereof. All the aforementioned makes the process of improving warranty claims forecasting even more important, further increasing the need for models that provide an acceptable accuracy for business decision making. In addition, a parallel need for warranty forecasting in industry also arises when the first few months of warranty claims are being analyzed for the purpose of forward extrapolation and development of appropriate corrective actions.

In many industries quality and reliability engineers who are involved in the warranty forecasting process use empirical models based on past warranty claims of products with similar design and complexity adjusted by certain,

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experience-based correction factors accounting for the design and technology changes in the product. A reasonably accurate, scientific, and user-friendly model could help to accomplish warranty prediction tasks with better precision and improve the overall quality of business decisions requiring estimates of future warranty claims.

2. Warranty forecasting model

In this paper, we will cover the two most common types of warranty forecasting activities. The first type includes future product estimates, which are usually conducted in a product planning stage in order to anticipate the costs associated with future warranty returns. This type of analysis is based on the product complexity, technology, and other design aspects known in advance about the product. The second type is the ongoing forecasting for current products, where the warranty returns are known for the first few months of service and the objective is to anticipate the final numbers of warranty returns at the end of the warranty cycle and beyond.

Warranty data usually contains information on all incidents reported during the warranty period. It is conventionally accepted that product failure behavior can be modeled by a 'bathtub curve' that is widely used in reliability literature [2]. There exist a variety of mathematical models that adequately represent the reliability bathtub curve [3–8]. For our purposes we are interested in a model's ability to fit the data presented in the automotive warranty reporting formats described in Section 3. Many bathtub-curve models are mathematically expressed in terms of hazard rates (or failure rates), while reliability engineers are usually more accustomed to working with reliabilities and percentages of failures. Also since reliability forecasting is usually the ultimate goal of this kind of analysis, a model expressed in terms or reliability would typically be easier to apply directly in engineering calculations.

Based on the fact that a typical automotive part is designed for a mission life of 10–15 years, it is very unlikely that it would be subjected to wear-out failures during either warranty or extended warranty periods of 3–7 years.

Fig. 1 provides an illustration of an automotive electronics product family failure rates recorded in terms of incidents per thousand vehicles (IPTV) see Eq. (1) for seven different model years[1] of the same product family (model years A–G in Fig. 1). The data shows no wear-out mode for at least the first 4 years of service

$$\mathrm{IPTV}(t) = \frac{\mathrm{Claims}(t)}{N(t)} 1000 \quad (1)$$

where

Claims(t) = number of claims reported in the period t.
$N(t)$ = number of vehicles in the field in the period t.

For any time interval T the relationship between IPTV and conventional failure rate would be:

$$\lambda(T) = \frac{\mathrm{IPTV}(T)}{1000T} \quad (2)$$

where $\lambda(T)$ = failure rate for the time interval T.

Fig. 1 suggests that in the majority of the cases the warranty failure model is sufficiently represented by the infant mortality and useful life phases of bathtub curve.

A detailed study of the existing warranty of various product lines of automotive parts performed at Delphi Corporation showed a clear trend of diminishing failure rate for the first 8–18 months (see Fig. 2) followed by a flattening of the failure rate curve for the remainder of the time period where warranty and extended warranty data were available.

To combine the first two sections of the bathtub curve and to provide a best fit for the warranty data in Fig. 1 or Fig. 2 we suggest using a conditional reliability equation

$$R(t) = R(t_s)R(t_s \rightarrow t) \quad (t > t_s) \quad (3)$$

where

$R(t)$ = reliability at the time interval t.
t_S = predetermined time coordinate.
$R(t_S)$ = reliability at the time t_S.
$R(t_S \rightarrow t)$ = probability of reaching the time point t, under the condition that time t_S has already been reached.

As mentioned earlier, most of reliability and quality engineers are more accustomed to working with reliabilities expressed in terms of commonly used distributions: Weibull, Exponential, Normal, and Lognormal. Analysis of the existing data (Fig. 2) shows that t_S, can be determined as the time coordinate where hazard rate stabilizes, and the failure data with decreasing failure rate in the range $[0; t_S]$ could be fitted by Weibull distribution. Similarly the failure data in the range $[t_S; t]$ could be fit with Exponential distribution, since the failure rate would remain relatively constant in this range. This approach is similar to the method discussed in [9], where the combination of Weibull and Exponential distributions were used to calculate the expected MTBF of a hard disk drive. Under the piecewise scheme described above, (3) becomes

$$R(t) = \mathrm{e}^{-(t_s/\eta)^{\beta}}\, \mathrm{e}^{-\lambda(t-t_s)}, \quad t \geq t_s \quad (4)$$

where

β = Weibull slope often referred as shape parameter.

[1] Model year is a manufacturer's annual production period. In the automotive industry new model year production may start as early as July of the previous calendar year.

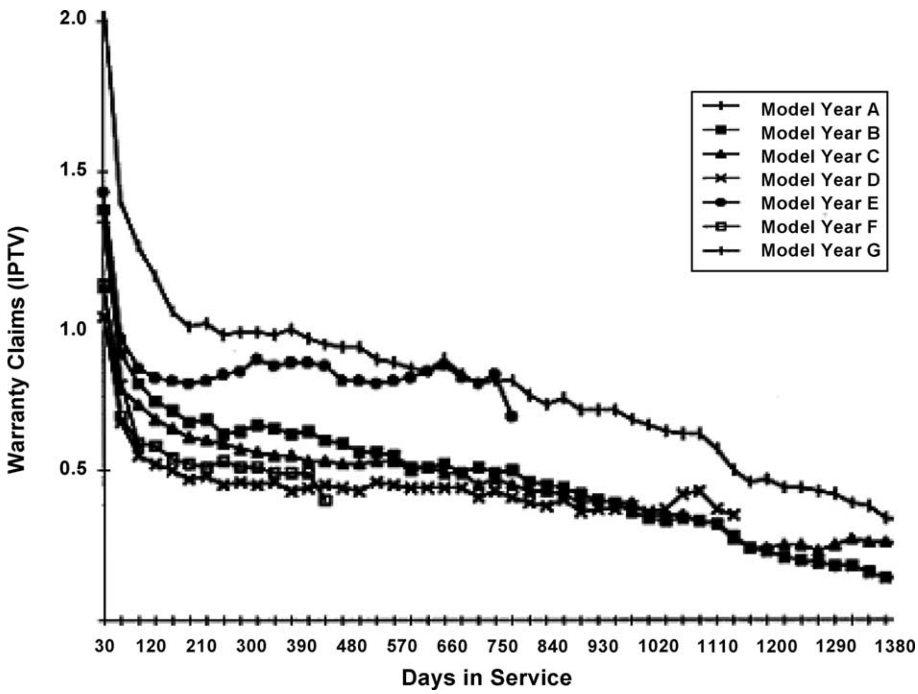

Fig. 1. Extended warranty charts compiled from Delphi Corporation warranty data for the several model years of the same electronic product mounted in the engine compartment of an automobile.

η = Weibull scale parameter, often referred as characteristic life.
λ = constant failure rate after t_S.

The time t_S can be referred as a *change point*, the coordinate where the pattern of data changes requiring a different data-fitting model [10]. The continuity at the junction point t_S can by achieved by equating the hazard rates at the point t_S. The hazard rate for Weibull distribution h_{Weibull} at t_S is:

$$h_{\text{Weibull}}(t_s) = \frac{\beta}{t_s}\left(\frac{t_s}{\eta}\right)^{\beta} \tag{5}$$

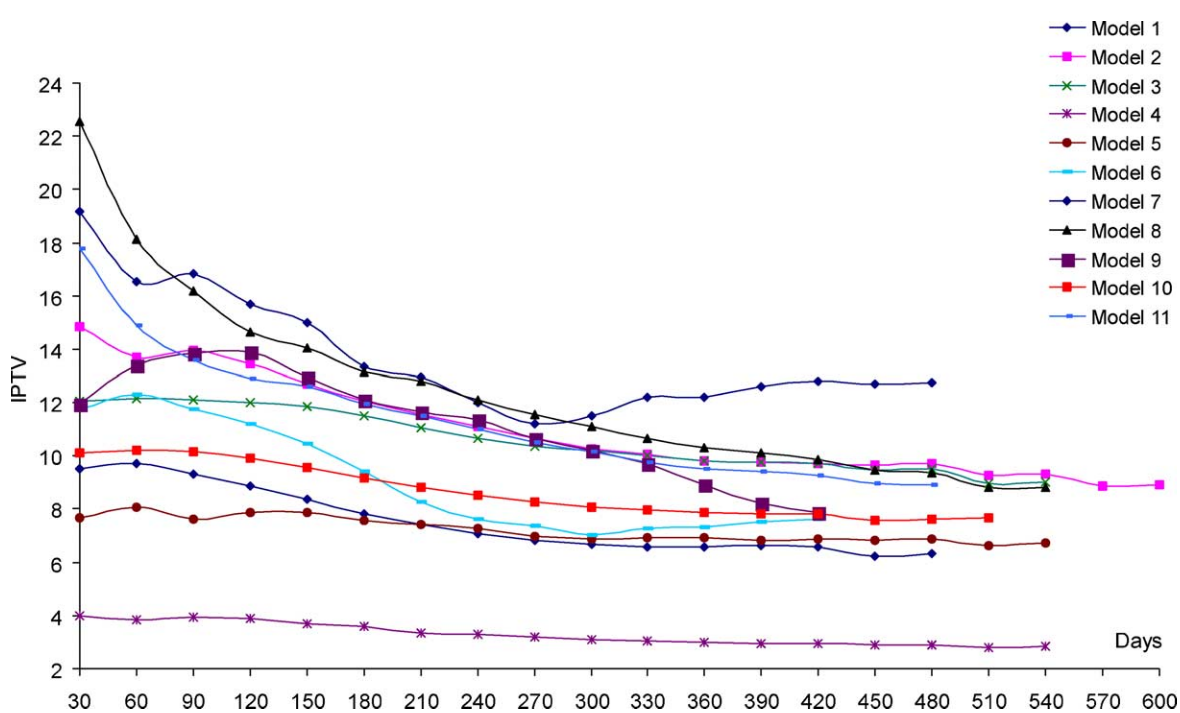

Fig. 2. Failure rates expressed in incidents per thousand vehicles (IPTV) for selected passenger compartment mounted electronic products recorded by Delphi Corporation. The actual IPTV values have been modified to protect the proprietary nature of the data.

Thus equating h_{Weibull} with the constant failure rate λ past the point t_S would produce:

$$\lambda = \frac{\beta}{t_s}\left(\frac{t_s}{\eta}\right)^{\beta} \tag{6}$$

The overall reliability expressed in (4) has four parameters: β, η, t_S, and λ, using (6) to eliminate λ, (4) can be transformed into:

$$R(t) = \exp\left[-\left(1 + \frac{\beta(t - t_s)}{t_s}\right)\left(\frac{t_s}{\eta}\right)^{\beta}\right], \quad t \geq t_s \tag{7}$$

Eq. (7) is in a suitable format for a stochastic simulation such as Monte Carlo analysis, which has been successfully applied in a variety of parametric studies of reliability [11]. Each of the parameters, β, η, t_S, is a random variable and could be represented by a statistical distribution. The best way of obtaining those distributions is by observing the past history of the product. The authors studied warranty returns for several automotive electronics product families and identified some common trends in the data. While the variation of statistical parameters between those groups was significant, parameter variation within the same product group was far less apparent. An important factor governing variation within a product family was found to be the number of years in production with a tendency for the first year to have the highest number of warranty claims.

Besides forecasting the expected warranty returns for future products, this model can also be used for ongoing forecasting of current products, where the final warranty prediction is based on the number of claims reported after the product's first few months in the field and is subject to continuous updates. This type of forecasting is often used to compile monthly reports to the management as well as to detect potentially serious field reliability problems.

3. Determining the distribution parameters

In this paper, we are going to consider two data formats commonly used for automotive warranty data reporting. In the first format, the data is presented in '30-day buckets' where the failure data is divided into 30-day service time intervals counted from the date of vehicle sale, where all the failures occurring within each 30-day time interval are reported in failed quantities or IPTV. The IPTV format (see example in Table 1) is an easier, faster, and more common form of data reporting and is usually sufficient for the first-level approach to data analysis. The raw warranty data typically contains additional information including vehicle identification number (VIN), vehicle mileage, geographical information, cumulative costs, cumulative IPTV, and many other parameters.

Table 1
Example '30-day Bucket' data

Days in service	Vehicles in the field during the time period	Reported number of failures	IPTV (incidents per thousand vehicles)
1–30	10,000	8	0.80
31–60	9000	2	0.22
61–90	7000	9	1.29

Table 2
Example 'Layer cake' data

Month	New vehicles sold	Number of failures by month			
		Month 1	Month 2	Month 3	Month 4
1	15,980	5	3	12	1
2	23,340		5	7	12
3	26,541			6	1
4	18,510				2

If the failed units can be traced to a specific production lot, this data can be converted into a more comprehensive format sometimes referred by quality professionals as 'layer cake', which usually combines all sold and failed units on a monthly basis, as presented in the Table 2. This format provides information, which allows the user to trace each failure to a particular production group and can be used to conduct more sophisticated statistical analyses.

The actual data in Tables 1 and 2 was made up for example purposes and is not linked to any real product or to each other. The format in Table 2 is easier to understand and data in this form can be easily processed with commercially available software like Weibull++ from ReliaSoft Corporation [12] and be converted into interval-based life data.

Both formats discussed above are acceptable for obtaining the distribution parameters β, η, t_S, however, the 30-day bucket data can be analyzed only on a percentage-failed basis and is thus unusable for the calculation of confidence bounds. In contrast, the layer cake data provides more options for determining a best-fit distribution including the estimation of confidence bounds. However, it is important to mention that the 30-day bucket format can be considered as a cost/time saving version of layer cake since it involves fewer data processing steps.

The procedure for determining distribution parameters starts with obtaining the change point estimation t_S. Since any real data would demonstrate some form of variation between consecutive 30-day intervals, we suggest using the Bayesian smoothed hazard function described in [13]. It would modify the stepwise pattern of the interval-based hazard function and would provide a continuous transition between adjacent 30-day intervals using Bayesian estimation of hazard rates. For simplicity purposes the average hazard rate $h_{\text{avg}}(t)$ given by Eq. (8) can also be used for this type of analysis:

$$h_{\text{avg}}(t) = \frac{\text{Number of accumulated failures } (t)}{\text{Total accumulated time in service } (t)} \tag{8}$$

Graphic analysis of the average hazard rate shows the general trend of saturation starting at t_S. One of the criteria

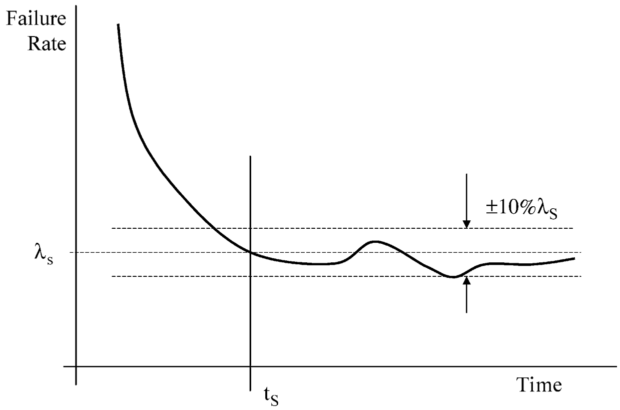

Fig. 3. Change point estimation for t_S, λ_S is the failure rate at t_S.

used for determining the exact change point t_S could be the flattening of the curve to fit within $\pm 10\%$ of the boundaries of the saturated hazard rate value as shown in Fig. 3 (other criteria may be practical depending on the specific nature of the data and the shape of the curve).

If the characteristics of the data are different from that presented in Fig. 3 and do not have a pattern of diminishing failure rate followed by stabilization, then the parameter t_S can be estimated from visual observation of the plotted data whenever possible. The majority of the datasets initially studied by the authors (42 out of 45) did exhibit the stabilization trend. The remaining three cases did not display the similar pattern being special cases with assignable causes of failure. For that reason they were considered outliers and were not included in the pool of datasets used for evaluation of distribution parameters of t_S.

Procedurally for each set of data the failure numbers should be split into pre-t_S and post-t_S intervals. Then each of the two data sets should be Weibull-fit as a separate group for determining Weibull parameters β and η. Analysis of the product groups mentioned previously, demonstrated stable trends, showing that the pre-t_S Weibull slope β (we will refer to it as β_1) typically stays in the range of 0.65–0.85. The statistical analysis of more than 40 different data sets with the @Risk software package [14] demonstrated that a two-parameter Weibull distribution was indeed the best-fit distribution for the pre-t_S data in almost half of the cases. For the remainder of the datasets Weibull was in the top five out of 28 different distribution options thus supporting the choice of Weibull distribution for this procedure. The similar analysis of post-t_S data showed that Weibull slopes β_2 in all analyzed cases were within $\pm 10\%$ of $\beta_2 = 1.0$, thus confirming the constant failure rate assumption for the post-infant-mortality stage.

4. Forecasting procedure

Two separate procedures are suggested for the two data formats mentioned in Section 3. Both can be performed using commercially available reliability analysis software.[2]

The data presented in layer cake format allows more sophisticated data processing, since the user is able to obtain exact failure time intervals and the number of suspended items. This more detailed information allows the implementation of Maximum Likelihood Estimator (MLE) Weibull analysis or other distribution best-fit approaches and provides the confidence intervals on the results of the best-fit approximation. It is also important to address the effect of the production year. For example, it has been observed that quality usually improves with the number of years in production due to continuous improvement of manufacturing procedures. Thus the obtained β and η coefficients can be categorized not only by product features, like model numbers, functionalities, car platforms, etc. but also by the number of years in production for the same product group.

It is also important to mention the multi-dimensional aspect of warranty specifications. Since automotive warranties are usually expressed in both time and mileage terms, i.e. 36 months or 36,000 miles whichever comes first [1], it is a two-dimensional warranty [15], which can be accounted for by using the methodologies presented, for example, in Refs. [16,17]. For automotive electronic parts it is more appropriate to use time as the primary usage variable since there are no moving parts involved in the process of wear-out, though the mileage variable is also important in estimating the expected warranties. Any of the methodologies described in the referenced literature can be applied to the proposed model in order to add an additional dimension of warranty. For example, if warranty is expressed in terms of $\{T_0, M_0\}$ with T_0 being specified maximum time period and M_0 specified maximum mileage, and if we can obtain the probability distribution function of reaching mileage M_0 at time t, $f(t|M_0)$, then

$$F(T)_{\text{Warranty}} = \int_0^T [1 - R(t)] f(t|M_0)\,\mathrm{d}t \tag{9}$$

where $F(T)_{\text{Warranty}}$ = fraction of accumulated failures covered by warranty for the time period T.

Or after substituting (7) into (9):

$$F(T)_{\text{Warranty}} = \int_0^{t_s} [1 - \mathrm{e}^{-(t/\eta)^\beta}] f(t|M_0)\,\mathrm{d}t + \int_{t_s}^{T} \left[1 - \exp\left[-\left(1 + \frac{\beta(t - t_s)}{t_s}\right)\left(\frac{t_s}{\eta}\right)^\beta\right]\right] \times f(t|M_0)\,\mathrm{d}t, \quad T \geq t_s \tag{10}$$

[2] When using ReliaSoft Weibull++ with the '30-day bucket' format it is best to use a 'free form data' format, which is made up of X time to failure data and Y position data in % in which ranks are not assigned to the times. The solution method is least squares (rank regression in X or Y). The time interval (30-day 60-days, etc.) would represent the X-axis and $0.1 \times$ IPTV (cumulative percent failed) would be plotted on the Y-axis.

Procedurally, each trial in the Monte Carlo simulation would include solving the integral (10) for every generated set of random variables β, η and t_S.

In addition, (7) can be used for ongoing warranty forecasting for current products. Direct application of Eq. (7) in conjunction with (10) allows using pre-t_S data (data from several months of warranty return) to predict the post-t_S data expanding to the normal warranty period, extended warranty period, and beyond.

5. Automotive electronics application example

For simplicity we will consider only the data presented in 30-day bucket format. Let us assume that we must forecast the 5-year/50,000 miles extended warranty of the new passenger compartment mounted product and let us also consider the effect of production start (usually the first year production) on the rate of returns for this part. The warranty data is available for four different models with similar features and complexities. Due to limited space we will present the initial data for only one model called Product 1, 1st year production lot (Table 3), and show the rest of the data in a statistical distribution format. As before, the presented warranty numbers will be altered due to proprietary nature of the data.

Since the data comes in 30-day bucket format it is best to apply the free form data format (percentages failed) to pre-t_S (β_1) and post-t_S (β_2) separately.

Typically the type of information presented in Table 4 would contain a much larger amount of data with more automotive product categories due to the large number of parts and applications. For instance, the same models can be subdivided by vehicle platforms, where the same type of products would be considered as a different group if they were installed on light trucks as opposed to mid-size cars. The larger the number of similar product lines, the better the confidence intervals for the results obtained with Monte Carlo simulation.

There are several possible ways of processing the data presented in the Table 4. All the data can be analyzed together by finding the best distributions for each of the three parameters β_1, η, t_S, and based on the obtained distributions, model those values for Monte Carlo simulation with (7) or (10). However, if we are, for example, interested in the warranty of the product manufactured within the first year after the start of production, only the data pertinent to the first year of production will be analyzed (see the four bold rows in Table 4). Based on these four data groups the following distributions were obtained:

t_S Lognormal distribution: $\mu=5.71$, $\sigma=0.186$

β_1 2-parameter Weibull distribution: $\beta=6.62$, $\eta=0.815$ (please note that those are the distribution parameters of β_1, and not that of the original failure data)

η_1 Normal distribution: $\mu=247{,}830$, $\sigma=30{,}069$.

Table 3
Product 1 (1st year production lot)

Days in service	IPTV	Total % failed
0–30	3.03	0.30
31–60	1.50	0.45
61–90	1.41	0.59
91–120	1.39	0.73
121–150	1.32	0.87
151–180	1.31	1.00
181–210	1.37	1.13
211–240	0.49	1.18
241–270	0.36	1.22
271–300	1.70	1.39
301–330	0.45	1.43
331–350	1.70	1.60
361–390	1.76	1.78
391–420	1.74	1.95
421–450	0.65	2.02
451–480	2.90	2.31
481–510	0.92	2.40
511–540	2.80	2.68
541–570	0.30	2.71
571–600	1.20	2.83
601–630	0.20	2.85
631–660	0.15	2.87
661–690	0.55	2.92
691–720	2.40	3.16
721–750	0.60	3.22
751–780	2.00	3.42
781–810	2.50	3.67
811–840	0.90	3.76
841–870	5.00	4.26
871–900	3.27	4.59
901–930	1.12	4.70
931–960	0.21	4.72

'30-Day bucket' warranty data for 960 days of service.

In order to account for the effect of two-dimensional characteristics of warranty we need to estimate the probability distribution function $f(t|50{,}000$ miles) of mileage reaching 50,000 miles at time t. First, using the dealership data containing the analysis of dates and mileages associate with each warranty claim we can construct a probability distribution function of daily mileage $f_{\mathrm{Daily}}(m)$. Our data was based on the sample of 1000 warranty claims containing both time and mileage information that was best-fit with two-parameter Weibull distribution with shape parameter $\beta=1.55$ and scale parameter $\eta=41.1$ miles (66.1 km). Using those parameters we obtained a conditional probability distribution $f(t|50{,}000$ miles), which is based on daily distribution mileage above and is best represented by Lognormal distribution with parameters: $\mu=7.53$ and $\sigma=0.904$.

A 10,000 sample Monte Carlo simulation of expected warranty returns at the 5-year mark (1825 days) produced the following results according to Eq. (10).

Mean value for cumulative return of claims covered by warranty was $F(5\ \mathrm{yr})=2.2\%$ (50% confidence). With upper 80% confidence this value reaching $F_{80\%}(5\ \mathrm{yr})=3.1\%$.

Table 4
Results of Weibull analysis of each data set for four product groups

Product	t_S (days)	β_1 (pre-t_S)	η_1 (days)	β_2 (post-t_S)
Product 1. 1st year production lot	**390**	**0.668**	**205,781**	**1.21**
Product 1. 2nd year production lot	270	0.761	378,248	0.961
Product 1. 3rd year production lot	420	0.872	501,320	1.03
Product 2. 1st year production lot	**330**	**0.890**	**290,258**	**0.920**
Product 2. 2nd year production lot	420	0.793	483,692	0.986
Product 3. 1st year production lot	**240**	**0.731**	**242,725**	**1.06**
Product 3. 2nd year production lot	180	0.903	618,440	1.02
Product 4. 1st year production lot	**270**	**0.912**	**252,551**	**0.946**

This example demonstrates the use of (10) with real data to perform a reliability/warranty prediction. A common simplistic method to treat the data associated with this example would have been a Weibull analysis of early failures for existing parts with similar design features. In our case, a simple Weibull analysis of early failure data that accounted for the 2-D aspect of the warranty would produce $F(5\text{ yr}) = 0.74\%$, which is significantly lower than the result obtained from Monte Carlo simulation using (10). Weibull analysis of the early failures often presents an over-simplification of the science that does not capture the trend change in the failure rate. Alternatively, detailed statistical approaches [3–8] adequately represent the bathtub curve, but are not formulated for forecasting and are generally not practical for use with real data and its associated uncertainties.

6. Conclusions

The model presented here offers a straightforward solution to a complex two-dimensional warranty prediction problem. The solution is easy to implement within Monte Carlo or other types of stochastic simulations because it is represented by a single closed-form equation. The procedure allows the user to practically accomplish two major reliability prediction tasks: (1) the forecasting of future product warranty at a product planning stage, and (2) the ongoing forecasting for current products, where the warranty returns are known for the first several months of production. This method can be used to predict the number of failed parts, which would not be reflected by warranty claims due to mileage exceeding warranty limit. In addition, the methodology also enables the accurate calculation of various life cycle cost components.

The approach developed and demonstrated in this paper represents a balance between correctly modeling the failure rate trend changes and practicality for use by real reliability analysis organizations. This approach can be generalized to work with a mixture of other applicable statistical distributions and should be suitable for implementation using other non- Monte Carlo stochastic methods. The method has been demonstrated on an automotive electronics example and shown to predict the expected number of claims for any specified period accounting for the effect of two-dimensional versus single-variable warranty. The demonstration clearly showed that simplistic data fitting approaches do not adequately model the real application data.

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