A Word by the Author

Russian mathematicians have suggested that ***mathematics is the gymnasium of the brains***. Likewise, in China, Mathematics Olympiad is ferociously practised by the large student population, all for a coveted place in one of the elite high schools.

Children who exhibit certain traits and penchant for numbers at the age of 5 or 6 years old, or even earlier, have great potential to be the mathematical olympians among their peers – provided they are groomed via a systematic, rigorous and routinized training.

Singapore was ranked 3rd in Mathematics in a recent TIMSS survey, after Hong Kong and Taiwan. Notably, China was not among the list of countries surveyed.

The most prestigious competition locally is **RIPMWC** (Raffles Institution Primary Mathematics World Competition). Meant for primary 6 students or younger, the top 50 to 60 or so participants are selected from Round 1 to compete in Round 2. Thereafter, 6 top participants emerge to take part in the world competition for primary school mathematics in Hong Kong. Another popular competition, also meant for primary 6 students, is **APMOPS** (Asia Pacific Mathematical Olympiad for Primary Schools), which is organized by Hwa Chong Institution since 1989. The following awards are being given at the end of two rounds of competition: ***Platinum, Gold, Silver and Bronze*****.**

At primary 5 level, the yearly **NMOS** (National Mathematical Olympiad of Singapore) competition has also captured the attention of parents since 2006, who eye NUS High as their most preferred high school.

The first series of books ***Maths Olympiad: Unleash the Maths Olympian in You!*** published in 2007 and 2008, has served as an ideal companion to students looking to establish a strong foundation in Mathematics – be it for PSLE preparation or in hope that they might one day take part in the various local and international competitions. The books are, therefore, also first-choice materials for parents of primary 3 students looking for quality content in gifted programme training.

In this new edition you will find the following additions:
- Write Equations
- Geometric Problem
- Ratio
- Complex Fraction
- Speed 2
- Approximation

Area and Perimeter of Circles has also been largely enhanced.

The objective is to cater to increasingly smarter children who have been exposed to a wide variety of topics. Some of these topics, which overlap the local mathematics syllabus, have also been adopted by schools here for students to practice on.

I feel extremely privileged and honoured to be able to continue serving students in this field. My latest series ***Wicked Mathematics!*** is currently out on shelves.

For related courses and workshops, please visit **www.terrychew.org**.

Terry Chew
(2015)

Foreword

Occasionally, in some difficult musical compositions there are beautiful,
but easy parts - parts so simple a beginner could play them.
So it is with mathematics as well.
- Professor Sherman K. Stein -

Mathematical Olympiad has been widely practised in some countries due to the following characteristics:

- the wide range of topics that link mathematics to most everyday events,
- the witty and tricky nature of the problems that bring out the best in the students' thinking skills and creative imagination,
- encourages the students to use more than one method to solve the problems, thus stimulating them to think outside the box,
- the students will be equipped with abundant resources to devise their own methods in problem-solving due to the extensive training and exposure.

This book consolidates the materials that I have used to teach my students over the years. Although the problems are of Mathematical Olympiad type, I realised that all students can benefit by working on them. Built on and beyond the school syllabus, the importance of attitude and enthusiasm surpasses that of capability in learning Mathematical Olympiad.

Many children whom I have guided and their parents alike are mesmerized by the materials presented in this book.

I hope you and your child will be too!

Terry Chew
(2008)

CONTENTS

1 Whole Numbers

The most important and sophisticated technique that we will learn from this chapter is to be able to express, say, *abcd*, which denotes a 4-digit number, in the form of

$$1000a + 100b + 10c + d.$$

The usefulness of this simple expression is to help us solve and appreciate a unique set of mathematical problems such as Example 3, Example 4 and Question 10 in this chapter.

The other category of mathematical problems, namely multiplication of two extremely long string of numbers, uses a simple concept (10 – 1 = 9) for problem-solving. This concept is demonstrated in Example 2.

Last but not least, we will learn to simplify the computation of the sum or difference of two sets of products through skilful factorisation. Example 1 and Question 7 illustrate this technique.

EXAMPLES

1 999 999 × 222 222 + 333 333 × 333 334 = ?

***Solution*:** 999 999 × 222 222 + 333 333 × 333 334
= 333 333 × 3 × 222 222 + 333 333 × 333 334
= 333 333 × 666 666 + 333 333 × 333 334
= 333 333 × (666 666 + 333 334)
= 333 333 × 1 000 000
= **333 333 000 000**

2 Find the sum of all the digits of $\underbrace{333\ldots333}_{2008\text{ 3s}} \times \underbrace{666\ldots666}_{2008\text{ 3s}}$.

***Analysis*:** It is not possible to multiply two numbers of such magnitude. The whole trick to this question lies in a simple relationship:
$10 - 1 = 9$.

***Solution*:**

$\underbrace{333\ldots333}_{2008\text{ 3s}} \times \underbrace{666\ldots666}_{2008\text{ 3s}}$

$= \underbrace{333\ldots333}_{2008\text{ 3s}} \times 3 \times \underbrace{222\ldots222}_{2008\text{ 2s}}$

$= \underbrace{999\ldots999}_{2008\text{ 9s}} \times \underbrace{222\ldots222}_{2008\text{ 2s}}$

$= (1\underbrace{000\ldots000}_{2008\text{ 0s}} - 1) \times \underbrace{222\ldots222}_{2008\text{ 2s}}$

$= \underbrace{222\ldots222}_{2008\text{ 2s}}\,\underbrace{000\ldots000}_{2008\text{ 0s}} - \underbrace{222\ldots222}_{2008\text{ 2s}}$

$= \underbrace{222\ldots222}_{2007\text{ 2s}}\ 1\ \underbrace{777\ldots778}_{2007\text{ 7s}}$

$2 + 7 = 9$ (there are 2007 pairs of 9)
$1 + 8 = 9$ (there is one more pair of 9)
$2008 \times 9 = 18\ 072$
The sum of all the digits of $\underbrace{333\ldots333}_{2008\text{ 3s}} \times \underbrace{666\ldots666}_{2008\text{ 6s}}$ is **18 072**.

3 The sum of all the digits of a three-digit number is 21. The digit in the ones place is greater than the digit in the tens place. A new number, which is 198 more than the original one, is formed by interchanging the digit in the ones place with the digit in the hundreds place. What is the original number?

***Analysis*:** It will not take us long to figure out that $876 - 678 = 198$. Hence, the answer is 678. The question lies with whether this is the only answer.

***Solution*:**

$100a + 10b + c = abc$ —— (1)
Interchange the digits in the ones and hundreds places, it will become
$100c + 10b + a$ —— (2)
(2) – (1)

$$100c + 10b + a - abc = 198$$
$$100c + 10b + a - 100a - 10b - c = 198$$
$$99c - 99a = 198$$
$$99(c - a) = 198$$
$$c - a = 198 \div 99 = 2$$

Since the digit in the ones place is greater than the digit in the hundreds place by 2, we try 597. However, 9 is greater than 7.

Next, we try 759.

957 – 759 = 198

Hence, the original number can be **678** or **957**.

4 Miss Cussler was born on the 1st of January many years ago. In 2002, her age was the sum of all the four digits of the year that she was born in. How old was Miss Cussler in 2002?

***Analysis*:** Again, we express the year that she was born in as $1000 + 100a + 10b + c$, if we assume it is $1abc$.

***Solution*:**

$$2002 - (1000 + 100a + 10b + c) = 1 + a + b + c$$
$$1002 - 100a - 10b - c = 1 + a + b + c$$
$$1001 = 101a + 11b + 2c$$

Take $a = 9$, so that it becomes $19bc$.

$$1001 = 101 \times 9 + 11b + 2c$$
$$1001 - 909 = 11b + 2c$$
$$92 = 11b + 2c$$
$$b = \frac{92 - 2c}{11}$$

When $c = 2$,

$$b = \frac{92 - 2 \times 2}{11} = \frac{88}{11} = 8$$

Miss Cussler was born in 1982.
1 + 9 + 8 + 2 = 20
as 1982 + 20 = 2002.
Miss Cussler was **20** years old in 2002.

PRACTICE

1 For each question below, do only the first three multiplication problems. Write out the next three products based on your conjecture.

(a) $3 \times 4 =$
$33 \times 34 =$
$333 \times 334 =$
$3333 \times 3334 =$
$33\,333 \times 33\,334 =$
$333\,333 \times 333\,334 =$

(b) $6 \times 7 =$
$66 \times 67 =$
$666 \times 667 =$
$6666 \times 6667 =$
$66\,666 \times 66\,667 =$
$666\,666 \times 666\,667 =$

(c) $5 \times 9 =$
$55 \times 99 =$
$555 \times 999 =$
$5555 \times 9999 =$
$55\,555 \times 99\,999 =$
$555\,555 \times 999\,999 =$

(d) $8 \times 9 =$
$88 \times 99 =$
$888 \times 999 =$
$8888 \times 9999 =$
$88\,888 \times 99\,999 =$
$888\,888 \times 999\,999 =$

2 Find the value of $1\,111\,111\,122\,222\,222 \div 33\,333\,334$.