A105 assessment Vectors SOLUTIONS

Do the questions as a test – circle questions you cannot answer

Red

1)		Given that the point A has position vector 6**i** – 7**j** and the point B has position vector -2**i** + 3**j**,	
	a)	find the vector $\overrightarrow{AB}$	[2]
	b)	Find $\lvert\overrightarrow{AB}\rvert$ Give your answer as a simplified surd.	[2]
	a)	$\overrightarrow{AB} = \begin{pmatrix}-2\\3\end{pmatrix} - \begin{pmatrix}6\\-7\end{pmatrix} = \begin{pmatrix}-8\\10\end{pmatrix}$	M1A1
	b)	Magnitude $\lvert\overrightarrow{AB}\rvert$ $\sqrt{(-8)^2+(10)^2} = 2\sqrt{41}$	M1A1

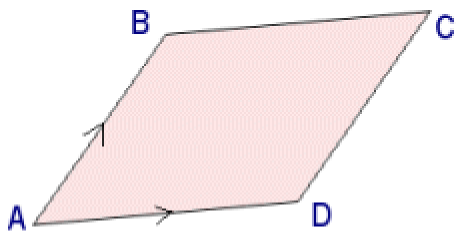
2)		Find in exact form the unit vector in the same direction as **a** = 7**i** – 14**j**	[3]
		Makes an attempt to use Pythagoras' theorem to find $\lvert\mathbf{a}\rvert$ $\sqrt{7^2+(-14)^2} = 7\sqrt{5}$ So unit vector is $\frac{1}{7\sqrt{5}}\begin{pmatrix}7\\-14\end{pmatrix} = \frac{1}{\sqrt{5}}(i-2j) = \frac{\sqrt{5}}{5}(i-2j)$	M1A1 A1

3)		Given that the point P has position vector -4**i** – 5**j** and the point Q has position vector 5**i** – 8**j**,	
	a)	find the vector $\overrightarrow{PQ}$	[2]
	b)	Find a vector in the same direction as that has magnitude of $\sqrt{10}$	[2]
	a)	$\overrightarrow{PQ} = \begin{pmatrix}-4\\-5\end{pmatrix} - \begin{pmatrix}5\\-8\end{pmatrix} = \begin{pmatrix}-9\\3\end{pmatrix} = -9i+3j$	M1A1
	b)	Magnitude is $\sqrt{(-9)^2+(3)^2} = 3\sqrt{10}$ Vector in same direction is $\frac{1}{3}\begin{pmatrix}-9\\3\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix} = -3i+j$	M1A1

4)		Consider the points $A(7, 6)$, $B(5, 5)$ and $C(12, -4)$	
	a)	Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$	[3]
	b)	Find the sine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$	[3]
	a)	$\overrightarrow{AB} = \begin{pmatrix}-2\\-1\end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix}5\\-10\end{pmatrix}$	M1A2
	b)	$\theta = 90°$, so $\sin\theta = 1$	M2A1

Amber

5)		Given that point A has the position vector $7\mathbf{i} - 14\mathbf{j}$ and point B has the position vector $11\mathbf{i} + q\mathbf{j}$, where q is a constant, find	
	a)	the vector $\overrightarrow{AB}$ in terms of q	[2]
	b)	Given further that $\lvert\overrightarrow{AB}\rvert = 4\sqrt{17}$, find the two possible values of q showing detailed reasoning in your working	[5]
	a)	Makes an attempt to find the vector $\overrightarrow{AB}$ For example, writing $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or $\overrightarrow{AB} = \binom{11}{q} - \binom{7}{-14} = \binom{4}{q+14} = 4i + (q+14)j$	M1 A1
	b)	Interprets correctly giving $4^2 + (q+14)^2 = \left(4\sqrt{17}\right)^2 = 252$ Method to solve resulting quadratic $q^2 + 28q - 60 = 0$ Factorising $(q+30)(q-2) = 0$ or formula Solves to $q = -30$ *or* $q = 2$	M1 M2 A2

6)			
		The vectors $\overrightarrow{AB} = 7\boldsymbol{i} + 9\boldsymbol{j}$ and $\overrightarrow{AD} = 9\mathbf{i} + 2\mathbf{j}$ represent two sides of the parallelogram *ABCD* shown	
	a)	Find vectors which represent each of the diagonals of *ABCD*.	[4]
	b)	Find the length of *AC*.	[2]
	a)	$\overrightarrow{AC} = 16\mathbf{i} + 11\mathbf{j}$ $\quad\overrightarrow{BD} = 2\mathbf{i} - 7\mathbf{j}$	M2A2
	b)	$\lvert\overrightarrow{AC}\rvert = \sqrt{377}$	M1A1

7)		Points A $(7, 1)$, B $(10, 5)$ and C $(14, 2)$ lie on a plane surface Show that triangle ABC is right angled and find its area	[3]
		Distance AB $\sqrt{(10-7)^2 + (5-1)^2} = 5$ Distance AC $\sqrt{(14-7)^2 + (2-1)^2} = \sqrt{50} = 5\sqrt{2}$ Distance BC $\sqrt{(14-10)^2 + (2-5)^2} = 5$ So $\sqrt{AB^2 + BC^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = AC$ hence ABC is a RAT	M1 any M1A1

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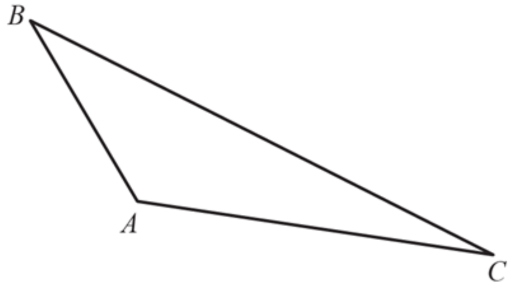
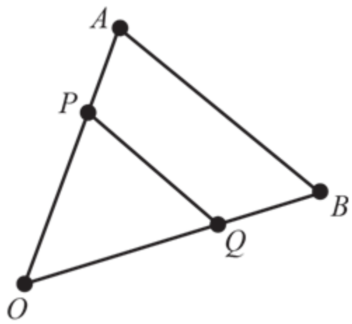
8)		In $\Delta\, ABC$, $\overrightarrow{AB} = -3i + 6j$ and $\overrightarrow{AC} = 10i - 2j$	
			
	a)	Find the size of $\angle BAC$, in degrees , to 1 decimal place	[5]
	b)	Find the exact value of the area of $\Delta\, ABC$	[3]
	a)	States or implies that $\overrightarrow{BC} = \begin{pmatrix}10\\-2\end{pmatrix} - \begin{pmatrix}-3\\6\end{pmatrix} = \begin{pmatrix}13\\-8\end{pmatrix} = 13i - 8j$ and recognises that cosine rule needed $\overrightarrow{BC}^2 = \overrightarrow{AB}^2 + \overrightarrow{AC}^2 - 2\overrightarrow{AB} \times \overrightarrow{AC} \cos A$ $\left(\sqrt{233}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{104}\right)^2 - 2\sqrt{45}\sqrt{104}\cos A$ oe $\cos A = \frac{-7}{\sqrt{130}}$ gives $A = 127.9°$ A is obtuse	M2 M2 A1
	b)	Area $= \frac{1}{2} \times \sqrt{45} \times \sqrt{104} \times \sin 127.9 = 27$ uses correct formula; correct values Note – exact answer if not rounded angle when using calculator	M2 A1

Green

9)			
		Given that the resultant of the vectors $\mathbf{a} = 2p\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = 6\mathbf{i} - 3p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 4\mathbf{i} - 5\mathbf{j}$, find	
	a)	the value of p	[4]
	b)	the resultant of the vectors **a** and **b**	[2]
	a)	Equates the **i** components for the equation $\mathbf{a} + \mathbf{b} = m\mathbf{c}$ oe $2p + 6 = 4m$ Equates the **j** components for the their equation $\mathbf{a} + \mathbf{b} = m\mathbf{c}$ $-5 - 3p = -5m$ Makes an attempt to find p by eliminating m in some way. For example, $\begin{matrix}10p + 30 = 20m\\20 + 12p = 20m\end{matrix}$ o.e. or $\frac{2p+6}{-5-3p} = -\frac{4}{5}$ o.e. $p = 5$	B1 B1 M1A1
	b)	Using their value for p from above, makes a substitution into the vectors to form $\mathbf{a} + \mathbf{b}$ $10\mathbf{i} - 5\mathbf{j} + 6\mathbf{i} - 15\mathbf{j}$ Correctly simplifies $16\mathbf{i} - 20\mathbf{j}$	M1 ft A1 ft

10)		A particle P of mass 6 kg moves under the action of two forces, F_1 and F_2, where $F_1 = (8\mathbf{i} - 10\mathbf{j})$ N and $F_2 = (p\mathbf{i} + q\mathbf{j})$ N, p and q are constants. The acceleration of P is $\mathbf{a} = (3\mathbf{i} - 2\mathbf{j})$ ms^{-2}	
	a)	Find, to 1 decimal place, the angle between the acceleration and $\mathbf{i}$	[2]
	b)	Find the values of p and q	[3]
	c)	Find the magnitude of the resultant force R of the two forces F_1 and F_2. Simplify your answer fully	[3]
	a)	States that $\tan\theta = \pm\frac{2}{3}$ or $\theta = \tan^{-1}\pm\frac{2}{3}$ (if θ shown on diagram sign must be consistent with this). Finds −33.7° (must be negative)	M1 A1
	b)	Makes an attempt to use the formula **F** = m**a** Finds p = 10 Note: $8 + p = 6\times 3 \Rightarrow p = 10$ Finds q = −2 Note: $-10 + q = 6\times -2 \Rightarrow q = -2$	M1 A2
	c)	Attempt to find **R** (either $6(3\mathbf{i} - 2\mathbf{j})$ or $8\mathbf{i} - 10\mathbf{j} + '10'\mathbf{i} + '-2'\mathbf{j}$) Makes an attempt to find the magnitude of their resultant force. For example, $\lvert R\rvert = \sqrt{'18'^2 + '12'^2}\ \left(= \sqrt{468}\right)$ Presents a fully simplified exact final answer. $\lvert R\rvert = 6\sqrt{13}$	M1 M1A1

11)		In ΔOAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P divides OA in the ratio 3 : 2 and Q divides OB in the ratio 3 : 2	
	a)	Show that PQ is parallel to AB	[4]
	b)	Given that AB is 10 cm in length find the length of PQ	[1]
	a)	States that $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ States $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ or $\overrightarrow{PQ} = -\frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ States $\overrightarrow{PQ} = \frac{3}{5}(-\mathbf{a} + \mathbf{b})$ or $\overrightarrow{PQ} = \frac{3}{5}\overrightarrow{AB}$ Draws the conclusion that as $\overrightarrow{PQ}$ is a multiple of $\overrightarrow{AB}$ the two lines PQ and AB must be parallel	M1 M1 A1 A1
	b)	$PQ = \frac{3}{5}\times 10$ cm = 6 cm cao	B1

TOTAL 60

A	B	C	D	E
80%	70%	60%	50%	40%

EBI: (What you are going to do)