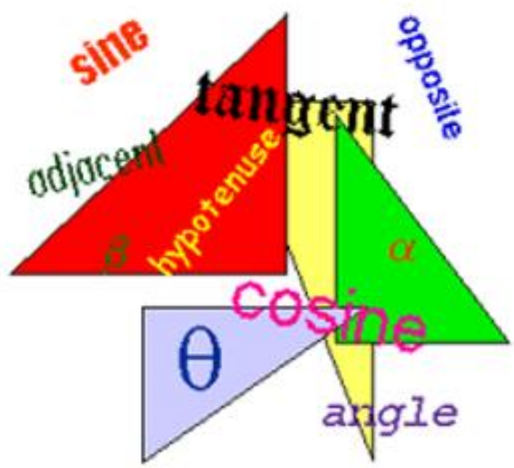
A2TH Trig Packet – Unit 1

In this unit, students will be able to:

- **Use the Pythagorean theorem to determine missing sides of right triangles**
- **Learn the definitions of the sine, cosine, and tangent ratios of a right triangle**
- **Set up proportions using sin, cos, tan to determine missing sides of right triangles**
- **Use inverse trig functions to determine missing angles of a right triangle**
- **Solve word problems involving right triangles**
- **Identify and name angles as rotations on the coordinate plane**
- **Determine the sign (+/-) of trig functions on the coordinate plane**
- **Determine sin, cos, and tangent of "special angles" (exact trig values)**
- **Determine reference angles for angles on the coordinate plane**
- **Determine the sine, cosine, and tangent of angles on the coordinate plane**
- **Do all of the above, using the reciprocal trig functions**



Name:________________________________

Teacher:______________________________

Pd: _______

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Day 1 – Reciprocal Trig Functions

Warm Up

Determine the *trigonometric ratios* for the following triangle:

(a) Sin A =

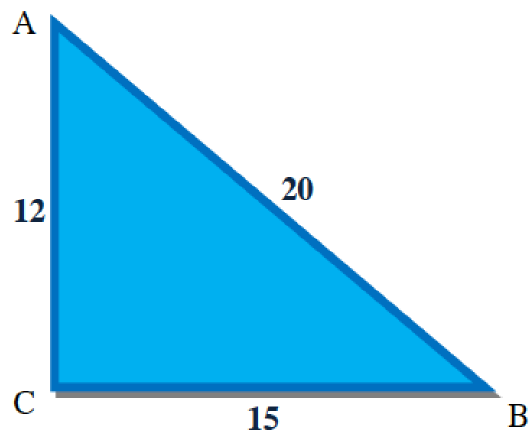
(b) Cos A =

(c) Tan A =

(d) Sin B =

(e) Cos B =

(f) Tan B =



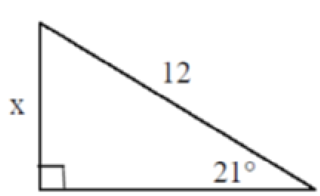
- ***What are the 3 trigonometry ratios?***
- ***What are the purposes of these ratios?***

$S\frac{O}{H}$	$C\frac{A}{H}$	$T\frac{O}{A}$
• ***What does θ represent?***		

Algebra REVIEW

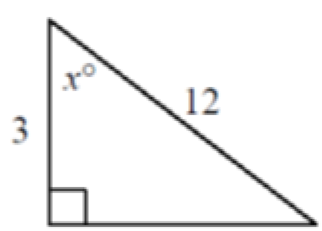
Problem 1: Using Trig to find a missing side

Find x.



Problem 2: Using Trig to find a missing angle

Find x.

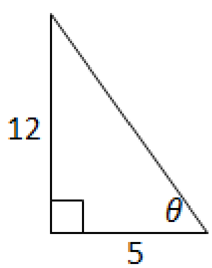


LEARNING GOAL: How Do We Use the Reciprocal Trig Functions?

- ***SECANT*** is the *reciprocal trigonometry* function of ____________
- ***COSECANT*** is the *reciprocal trigonometry* function of ____________
- ***COTANGENT*** is the *reciprocal trigonometry* function of ____________

Secant	***Cosecant***	***Cotangent***
sec θ =	csc θ =	cot θ =
sec θ =	csc θ =	cot θ =

Problem 3: Find the 3rd side first, then find all six trigonometric ratios.



$\sin\theta =$	$\csc\theta =$
$\cos\theta =$	$\sec\theta =$
$\tan\theta =$	$\cot\theta =$

Problem 4: $If\ \sin\theta = \frac{6}{7},\ find\ the\ other\ 5\ trigonometric\ ratios.$

Regents Question **Show work here!**

1. If $\csc\theta = -2$, what is the value of $\sin\theta$?

1) -2
2) 2
3) $-\frac{1}{2}$
4) $\frac{1}{2}$

2. The expression $\cot\theta \cdot \sec\theta$ is equivalent to

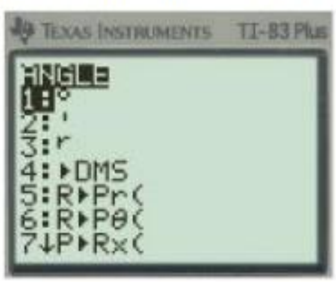
1) $\frac{\cos\theta}{\sin^2\theta}$
2) $\frac{\sin\theta}{\cos^2\theta}$
3) $\csc\theta$
4) $\sin\theta$

LEARNING GOAL: *Converting Angles into Degrees, Minutes, and Seconds*

➢ Angles are measured in degrees, minutes, and seconds.

14.88264119 14°52'57.508"

- ***Where do you find all the degree, minute, and second buttons in the calculator?***



a. Round to the nearest thousandth: $\sin 30°45'$	**b.** Round to the nearest tenth: $\tan 60°23'37''$	**c.** Round to the nearest hundredth: $\cos 210°15'37.025''$

d. Round to the nearest thousandth: $\sec 62°25'$	**e.** Round to the nearest hundredth: $\cot 125°5'48''$	**f.** Round to the nearest tenth. $\csc 280°31'20.125''$

ROUNDING WITH MINUTES, SECONDS

14.88264119 14°52'57.508"

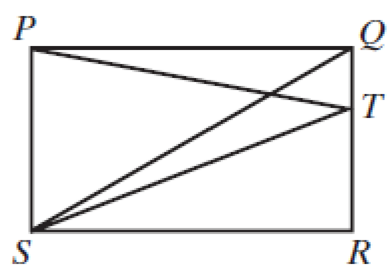
Rounded to the nearest minute: 14°52'57.508"	***Rounded to the nearest second:*** 14°52'57.508"	***Rounded to the nearest ten minutes:*** 14°52'57.508"
Rounded to the nearest minute: $\sin\theta = \frac{5}{23}$	***Rounded to the nearest second:*** $\tan\theta = \frac{5}{3}$	***Rounded to the nearest ten minutes:*** $\cos\theta = 0.7125689$
Rounded to the nearest minute: cot θ = .4663	***Rounded to the nearest second:*** csc θ = 7.1853	***Rounded to the nearest ten minutes:*** sec θ = 1.2521

Challenge

In rectangle $PQRS$, if $\tan\angle QPT = \frac{1}{5}$ and $\tan\angle TSR = \frac{1}{2}$, then $\tan\angle PQS =$

(A) $\frac{9}{10}$ (B) $\frac{4}{5}$ (C) $\frac{7}{10}$

(D) $\frac{1}{2}$ (E) $\frac{2}{5}$



Summary

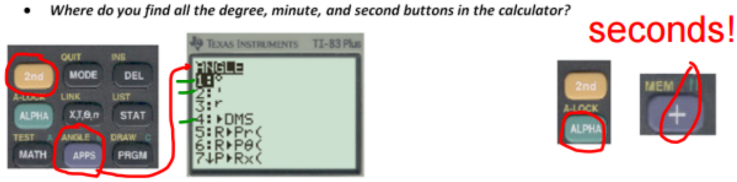
The Basic Trig Definitions

the sine function : $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	the cosecant function : $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$
the cosine function : $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	the secant function : $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$
the tangent function : $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	the cotangent function : $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

- Angles are measured in degrees, minutes, and seconds.

14.88264119 ——— 14°52'57.508"

- *Where do you find all the degree, minute, and second buttons in the calculator?*



a. Round to the nearest thousandth: $\sin 30°45'$	b. Round to the nearest tenth: $\tan 60°23'37''$	c. Round to the nearest hundredth: $\cos 210°15'37.025''$
sin(30°45') .5112930861	tan(60°23'37") 1.759861399	cos(210°15'37.025") -.8637450627

EXAMPLE 6 Convert 72.18° to D°M′S″ notation.

Solution On a calculator, we enter 72.18. The result is

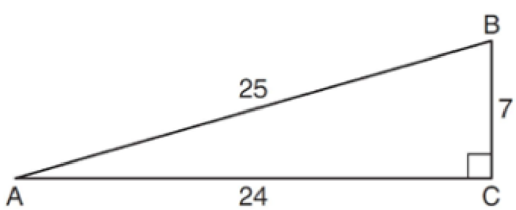
$$72.18° = 72°10'48''.$$

Without a calculator, we can convert as follows:

$$\begin{aligned}72.18° &= 72° + 0.18 \times 1° \\ &= 72° + 0.18 \times 60' \qquad 1° = 60' \\ &= 72° + 10.8' \\ &= 72° + 10' + 0.8 \times 1' \\ &= 72° + 10' + 0.8 \times 60'' \qquad 1' = 60'' \\ &= 72° + 10' + 48'' \\ &= 72°10'48''.\end{aligned}$$

Exit Ticket

Which ratio represents $\csc A$ in the diagram below?



1) $\dfrac{25}{24}$

2) $\dfrac{25}{7}$

3) $\dfrac{24}{7}$

4) $\dfrac{7}{24}$

Homework – Day 1

1. The angle of elevation from a point 25 feet from the base of a tree on level ground to the top of the tree is 30°. Which equation can be used to find the height of the tree?

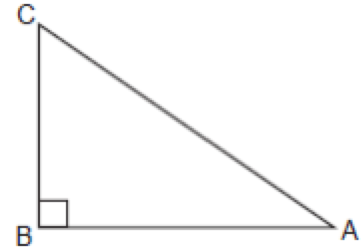
1) $\tan 30° = \frac{x}{25}$

2) $\sin 30° = \frac{x}{25}$

3) $\cos 30° = \frac{x}{25}$

4) $30^2 + 25^2 = x^2$

2. Cassandra is calculating the measure of angle *A* in right triangle *ABC*, as shown in the accompanying diagram. She knows the lengths of $\overline{BC}$ and $\overline{AC}$.



If she finds the measure of angle *A* by solving only one equation, which concept will be used in her calculations?

1) Pythagorean theorem

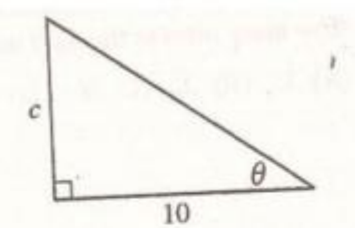
2) $\sin A$

3) $\cos A$

4) $\tan A$

3. In figure 7, if $\theta = 44°$, what is the value of *c*?

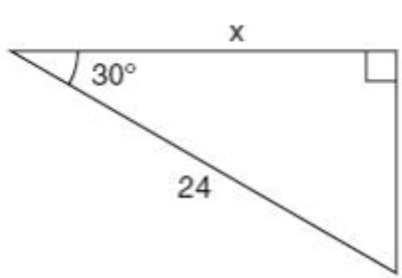
(A) 6.94
(B) 7.19
(C) 9.66
(D) 10.36
(E) 13.90



Note: Figure not drawn to scale.

Figure 7

4. In the right triangle shown in the diagram below, what is the value of *x* to the *nearest whole number*?



5. Convert to DMS form. Show work.

a. 37.285°

b. 314.42°

6. Convert to degree form. Show work.

a. $82^\circ 42'$

b. $213^\circ 15' 56''$

7. Use a calculator to determine the value of each trigonometric ratio: Round answers to the nearest ten-thousandths.

a) $\sin 52°47'$	**b)** $\cos 79°15'45''$	**c)** $\cot 36°$

8. If $\sin\theta = \frac{2}{5}$, find $\csc\theta$.

If $\sec\theta = 1.5$, find $\cos\theta$.

9. A person measures the angle of depression from the top of a wall to a point on the ground. The point is located on level ground 62 feet from the base of the wall and angle of depression is $52^\circ 27'$. How high is the wall, to the nearest tenth of a foot?

10. In a right triangle, θ is an acute angle and csc = $\frac{19}{18}$. Evaluate the other five trigonometric functions of θ.

a. $\sin\theta = \frac{18}{19}$

$\cos\theta = \frac{\sqrt{37}}{19}$ $\sec\theta = \frac{19\sqrt{37}}{37}$

$\tan\theta = \frac{18\sqrt{37}}{37}$ $\cot\theta = \frac{\sqrt{37}}{18}$

b. $\sin\theta = \frac{18}{19}$

$\cos\theta = \frac{19\sqrt{37}}{37}$ $\sec\theta = \frac{\sqrt{37}}{19}$

$\tan\theta = \frac{\sqrt{37}}{18}$ $\cot\theta = \frac{18\sqrt{37}}{37}$

c. $\sec\theta = \frac{19}{18}$

$\cos\theta = \frac{19\sqrt{37}}{37}$ $\sin\theta = \frac{\sqrt{37}}{19}$

$\tan\theta = \frac{\sqrt{37}}{18}$ $\cot\theta = \frac{18\sqrt{37}}{37}$

d. $\sec\theta = \frac{18}{19}$

$\cos\theta = \frac{\sqrt{37}}{19}$ $\sin\theta = \frac{19\sqrt{37}}{37}$

$\tan\theta = \frac{18\sqrt{37}}{37}$ $\cot\theta = \frac{\sqrt{37}}{18}$

Day 2 – Arcs and Angles as Rotations

Warm – Up

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

1. 28.955°

2. -57.3278°

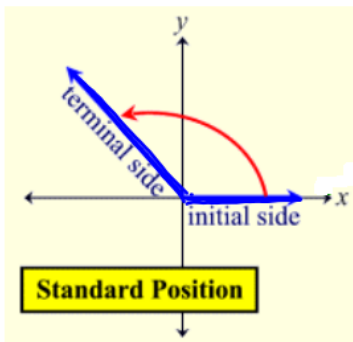
3. $32^\circ\ 28'\ 10''$

4. $-73^\circ\ 14'\ 35''$

AIM: ANGLES OF ROTATION

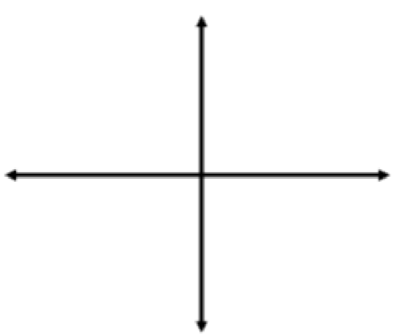
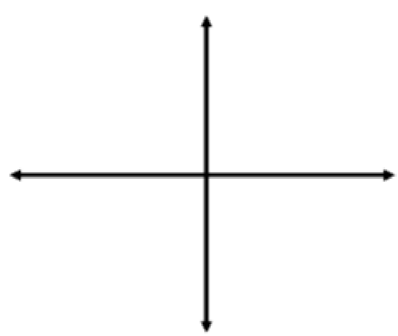
PART I: Initial vs. Terminal Side

- **Initial Side -** the ray (side) at which an angle of rotation begins
- **Terminal Side -** the ray (side) at which an angle of rotation ends

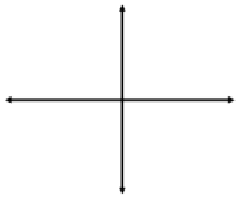
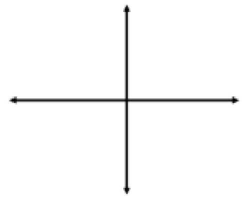


- **Standard Position -** an angle is in standard position if its vertex is located at the origin and one ray is on the positive x-axis

- **Clockwise vs. Counter Clockwise**

Counter Clockwise (positive angles)	*Clockwise (negative angles)*
	

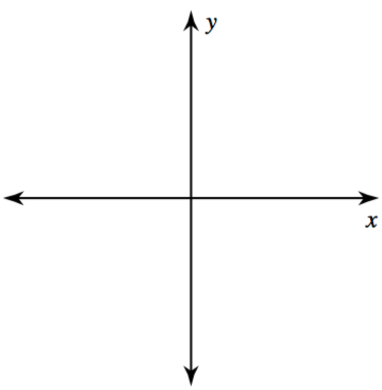
- **Quadrantal Angles –**

Counter Clockwise	***Clockwise***
	

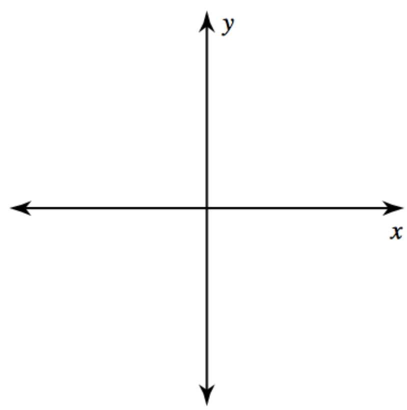
<u>You Try It!</u>

Draw an angle with the given measure in standard position and determine the quadrant in which the angle lies.

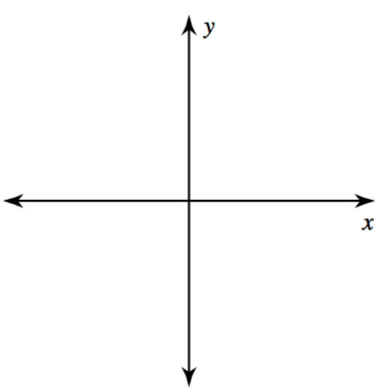
1. 60°



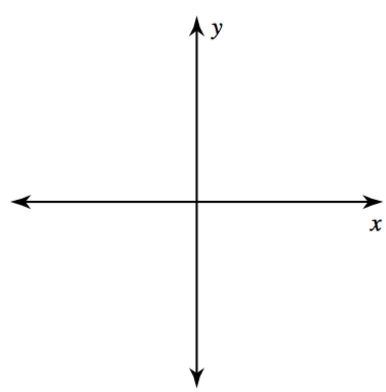
2. 210°



3. 450°

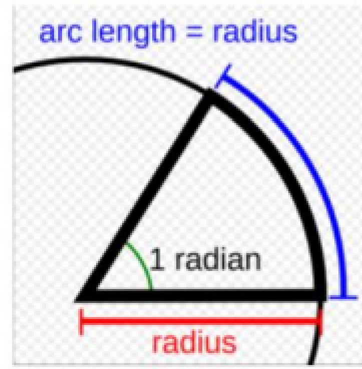


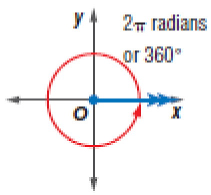
4. -40°



LEARNING GOAL: How Do We Convert Between Radian and Degree Measure?

- **What is a radian? –** a radian is the measure of an angle that, when drawn as a central angle of a circle, intercepts an arch whose length is equal to the length of a radius of the circle.





The circumference of any circle is $2\pi r$, where r is the radius measure. So the circumference of a unit circle is $2\pi(1)$ or 2π units. Therefore, an angle representing one complete revolution of the circle measures 2π radians. This same angle measures 360°. Therefore, the following equation is true.

$$2\pi \text{ radians} = 360°$$

To change angle measures from radians to degrees or vice versa, solve the equation above in terms of both units.

2π radians = 360°	2π radians = 360°
1 radian =	= 1°
1 radian is about	1 degree is about

These equations suggest a method for converting between radian and degree measure.

- **How do we convert between radian and degree measure?**

Examples: Convert the following to radian measure.

$50°$	$-120°$	$270°$

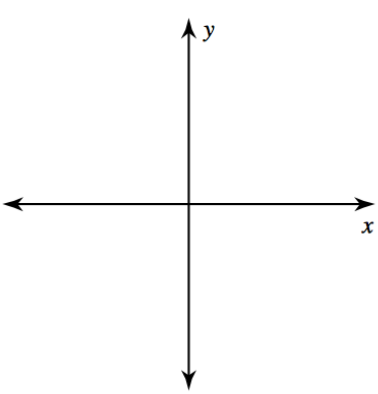
Examples: Convert the following to degree measure.

$\frac{\pi}{6}$	$-\frac{2\pi}{5}$	1.7π

1. Find, to the *nearest minute*, the angle whose measure is 3.45 radians.

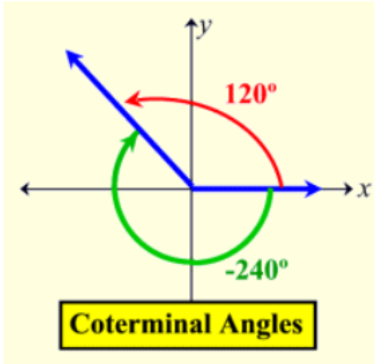
2. What is the radian measure, in terms of π, of the angle formed by the hands of a clock at 4:00 p.m.?

3. Sketch and label θ in standard position if $\theta = \frac{7\pi}{6.}$



PART IV: **Coterminal Angles**

Coterminal Angles- **angles in standard position that have the same terminal side**



Find one angle with positive measure and one angle with negative measure coterminal with each angle.

a. $240°$

b. $\frac{9\pi}{4}$

Regents questions

_____ 1. In which quadrant does a -285° angle lie?

(1) I
(2) II
(3) III
(4) IV

Explain your answer below.

_____ 2. Which angle is *not* coterminal with an angle that measures 300°?

(1) -420°
(2) -300°
(3) -60°
(4) 660°

_____ 3. Which angle *is* coterminal with an angle that measures -120°?

(1) -80°
(2) 60°
(3) 240°
(4) 580°

Explain your answer below.

____ 4. **State if the given angles are coterminal.**

$\frac{23\pi}{18}, \frac{11\pi}{6}$

A) No B) Yes

Explain your answer below.

____ 5. **Find a coterminal angle between 0 and 2π for each given angle.**

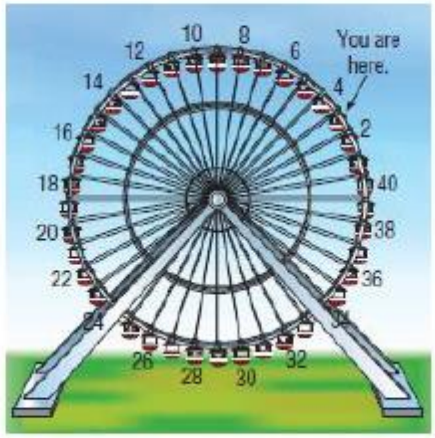
$-\frac{5\pi}{6}$

A) $\frac{5\pi}{3}$ B) $\frac{7\pi}{6}$

C) $\frac{\pi}{6}$ D) $\frac{5\pi}{6}$

Challenge

ENTERTAINMENT Suppose the gondolas on the Navy Pier Ferris wheel were numbered from 1 through 40 consecutively in a counterclockwise fashion. If you were sitting in gondola number 3 and the wheel were to rotate counterclockwise through 846 degrees, which gondola used to be in the position that you are in now?

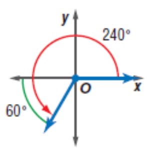


SUMMARY:

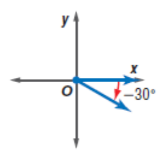
Example 1 Draw an Angle in Standard Position

Draw an angle with the given measure in standard position.

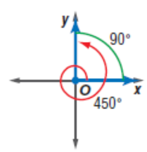
a. 240° $\quad 240° = 180° + 60°$
Draw the terminal side of the angle 60° counterclockwise past the negative x-axis.



b. $-30°$ The angle is negative.
Draw the terminal side of the angle 30° clockwise from the positive x-axis.



c. 450° $\quad 450° = 360° + 90°$
Draw the terminal side of the angle 90° counterclockwise past the positive x-axis.



Example 2 Convert Between Degree and Radian Measure

Rewrite the degree measure in radians and the radian measure in degrees.

a. 60°

$$60° = 60°\left(\frac{\pi \text{ radians}}{180°}\right)$$
$$= \frac{60\pi}{180} \text{ radians or } \frac{\pi}{3}$$

b. $-\frac{7\pi}{4}$

$$-\frac{7\pi}{4} = \left(-\frac{7\cancel{\pi}}{4}\ \cancel{\text{radians}}\right)\left(\frac{180°}{\cancel{\pi \text{ radians}}}\right)$$
$$= -\frac{1260°}{4} \text{ or } -315°$$

Example 4 Find Coterminal Angles

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

a. 240°

A positive angle is $240° + 360°$ or $600°$.

A negative angle is $240° - 360°$ or $-120°$.

b. $\frac{9\pi}{4}$

A positive angle is $\frac{9\pi}{4} + 2\pi$ or $\frac{17\pi}{4}$. $\quad \frac{9\pi}{4} + \frac{8\pi}{4} = \frac{17\pi}{4}$

A negative angle is $\frac{9\pi}{4} - 2(2\pi)$ or $-\frac{7\pi}{4}$. $\quad \frac{9\pi}{4} + \left(-\frac{16\pi}{4}\right) = -\frac{7\pi}{4}$

Exit Ticket

What is the radian measure of an angle whose measure is –420°?

1) $-\frac{7\pi}{3}$

2) $-\frac{7\pi}{6}$

3) $\frac{7\pi}{6}$

4) $\frac{7\pi}{3}$

Day 2 – Homework

Page 15 # 4 – 16 and Page 16 #'s 29, 33, 36, 38, 41, 42, 45, 54, 62 and 63

Check for Understanding

Concept Check

1. Name the set of numbers to which angle measures belong.
2. Define the term radian.
3. **OPEN ENDED** Draw and label an example of an angle with negative measure in standard position. Then find an angle with positive measure that is coterminal with this angle.

Guided Practice

Draw an angle with the given measure in standard position.

4. $70°$ 5. $300°$ 6. $570°$ 7. $-45°$

Rewrite each degree measure in radians and each radian measure in degrees.

8. $130°$ 9. $-10°$ 10. $485°$

11. $\frac{3\pi}{4}$ 12. $-\frac{\pi}{6}$ 13. $\frac{19\pi}{3}$

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

14. $60°$ 15. $425°$ 16. $\frac{\pi}{3}$

Application

ASTRONOMY For Exercises 17 and 18, use the following information. Earth rotates on its axis once every 24 hours.

17. How long does it take Earth to rotate through an angle of $315°$?
18. How long does it take Earth to rotate through an angle of $\frac{\pi}{6}$?

★ indicates increased difficulty

Practice and Apply

Draw an angle with the given measure in standard position.

19. $235°$ 20. $270°$ 21. $790°$ 22. $380°$

23. $-150°$ 24. $-50°$ ★ 25. π ★ 26. $-\frac{2\pi}{3}$

Rewrite each degree measure in radians and each radian measure in degrees.

27. $120°$ 28. $60°$ 29. $-15°$ 30. $-225°$

31. $660°$ 32. $570°$ 33. $158°$ 34. $260°$

35. $\frac{5\pi}{6}$ 36. $\frac{11\pi}{4}$ 37. $-\frac{\pi}{4}$ 38. $-\frac{\pi}{3}$

39. $\frac{29\pi}{4}$ 40. $\frac{17\pi}{6}$ ★ 41. 9 ★ 42. 3

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

43. $225°$ 44. $30°$ 45. $-15°$

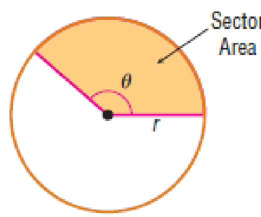
46. $-140°$ 47. $368°$ 48. $760°$

49. $\frac{3\pi}{4}$ 50. $\frac{7\pi}{6}$ 51. $-\frac{5\pi}{4}$

52. $-\frac{2\pi}{3}$ 53. $\frac{9\pi}{2}$ 54. $\frac{17\pi}{4}$

• 55. **DRIVING** Some sport-utility vehicles (SUVs) use 15-inch radius wheels. When driven 40 miles per hour, determine the measure of the angle through which a point on the wheel travels every second. Round to both the nearest degree and nearest radian.

GEOMETRY **For Exercises 56 and 57, use the following information.**
A *sector* is a region of a circle that is bounded by a central angle θ and its intercepted arc. The area A of a sector with radius r and central angle θ is given by $A = \frac{1}{2}r^2\theta$, where θ is measured in radians.



56. Find the area of a sector with a central angle of $\frac{4\pi}{3}$ radians in a circle whose radius measures 10 inches.

57. Find the area of a sector with a central angle of $150°$ in a circle whose radius measures 12 meters.

62. **QUANTITATIVE COMPARISON** **Compare the quantity in Column A and the quantity in Column B. Then determine whether:**

Ⓐ **the quantity in Column A is greater,**
Ⓑ **the quantity in Column B is greater,**
Ⓒ **the two quantities are equal, or**
Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B
$56°$	$\frac{14\pi}{45}$

63. Angular velocity is defined by the equation $\omega = \frac{\theta}{t}$, where θ is usually expressed in radians and t represents time. Find the angular velocity in radians per second of a point on a bicycle tire if it completes 2 revolutions in 3 seconds.

Ⓐ $\frac{\pi}{3}$ Ⓑ $\frac{\pi}{2}$ Ⓒ $\frac{2\pi}{3}$ Ⓓ $\frac{4\pi}{3}$

Warm - Up

Draw an angle with the given measure in standard position.

1. $160°$ **2.** $-\frac{5\pi}{4}$ **3.** $400°$

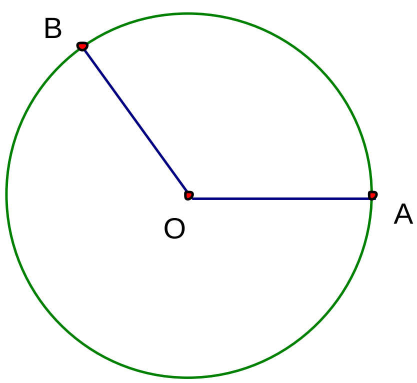
Rewrite each degree measure in radians and each radian measure in degrees.

4. $140°$ **5.** $-860°$ **6.** $-\frac{3\pi}{5}$ **7.** $\frac{11\pi}{3}$

Concept 1: Arc Length

To find the measure of an angle in radians when you are given the lengths of the arc and radius:

Measure of an angle in radians = $\frac{\text{length of the intercepted arc}}{\text{length of radius}}$

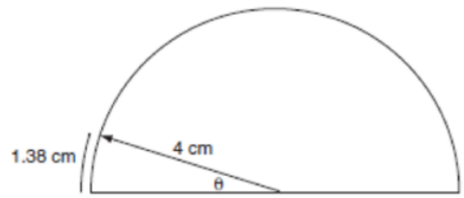


In general, if Θ is the measure of a central angle in radians, s is the length of the intercepted arc, and r is the length of a radius, then:

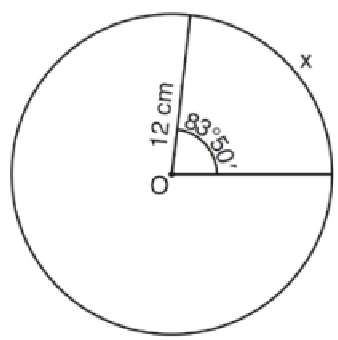
Examples

1) In a circle, a central angle of 3 radians intercepts an arc of 18 centimeters. What is the radius, in centimeters, of the circle?

2) As shown in the accompanying diagram, a dial in the shape of a semicircle has a radius of 4 centimeters. Find the measure of θ, in radians, when the pointer rotates to form an arc whose length is 1.38 centimeters.



3) Circle O shown below has a radius of 12 centimeters. To the *nearest tenth of a centimeter*, determine the length of the arc, x, subtended by an angle of 83°50'.



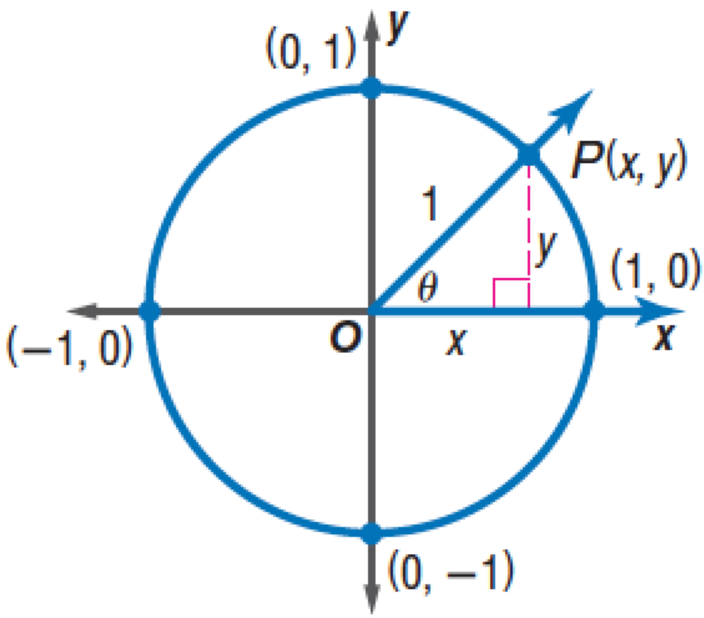
Concept 2: Unit Circle

UNIT CIRCLE

WHAT IS THE UNIT CIRCLE?

- A **unit circle** is a circle with a radius of one (a unit radius).
In trigonometry, the unit circle is centered at the origin.

- In the unit circle, the coordinates (*x*, *y*) can be rewritten as $(\cos\theta, \sin\theta)$



$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

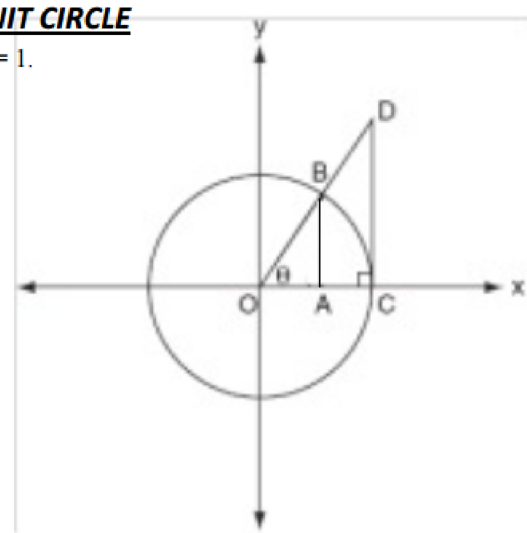
PRACTICE WITH THE UNIT CIRCLE

4. The accompanying diagram shows unit circle O, with radius OB = 1.

Which line segment has a length equivalent to sin θ?

(1) $\overline{OB}$ (3) $\overline{OD}$

(2) $\overline{CD}$ (4) $\overline{BA}$



5.

Which line segment has a length equivalent to tan θ?

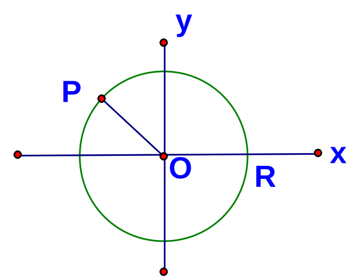
(1) $\overline{OB}$ (3) $\overline{OC}$

(2) $\overline{CD}$ (4) $\overline{OA}$

In questions 6 - 9, you are given the coordinates of point P, **where OP = 1, and** $m\measuredangle ROP = \boldsymbol{\theta}$.

Find a) sin θ   b) cos θ c)tanθ

6. P$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



sin θ = ______, because _____ = __-coordinate on the unit circle.

cos θ = ______, because _____ = __-coordinate on the unit circle.

tan θ =

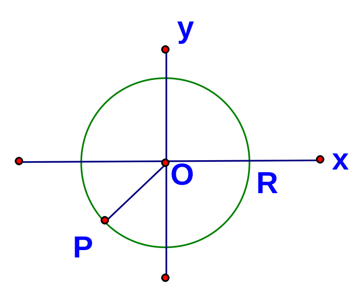
because _____ = $\frac{\square}{\square}$

csc θ = ______, because it's the reciprocal of ______.

sec θ = ______, because it's the reciprocal of ______.

cot θ = ______, because it's the reciprocal of ______.

7. P$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$



sin θ = ______, because _____ = __-coordinate on the unit circle.

cos θ = ______, because _____ = __-coordinate on the unit circle.

tan θ =

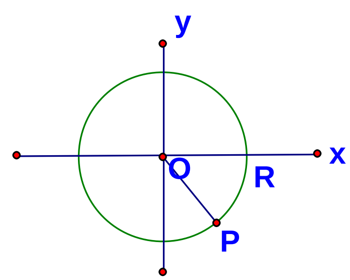
because _____ = $\frac{\square}{\square}$

csc θ = ______, because it's the reciprocal of ______.

sec θ = ______, because it's the reciprocal of ______.

cot θ = ______, because it's the reciprocal of ______.

8. P(.6, -.8)



sin θ =

cos θ =

tan θ =

csc θ =

sec θ =

cot θ =

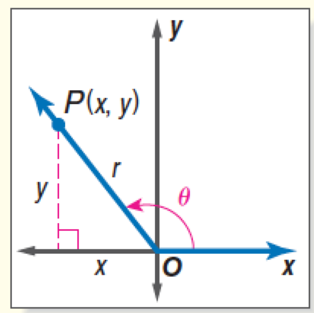
Concept 3: Points not on the Unit Circle

Key Concept *Trigonometric Functions, θ in Standard Position*

Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. Using the Pythagorean Theorem, the distance r from the origin to P is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

$\sin \theta = \frac{y}{r}$ $\qquad$ $\cos \theta = \frac{x}{r}$ $\qquad$ $\tan \theta = \frac{y}{x}, x \neq 0$

$\csc \theta = \frac{r}{y}, y \neq 0$ $\qquad$ $\sec \theta = \frac{r}{x}, x \neq 0$ $\qquad$ $\cot \theta = \frac{x}{y}, y \neq 0$



9) Find all 6 trigonometric function values of the angle formed by the point (-3, 4)

Draw each of the following points on a coordinate plane. Let θ be the angle in standard position that terminates at that point. Determine the sine, cosine, and tangent of θ.

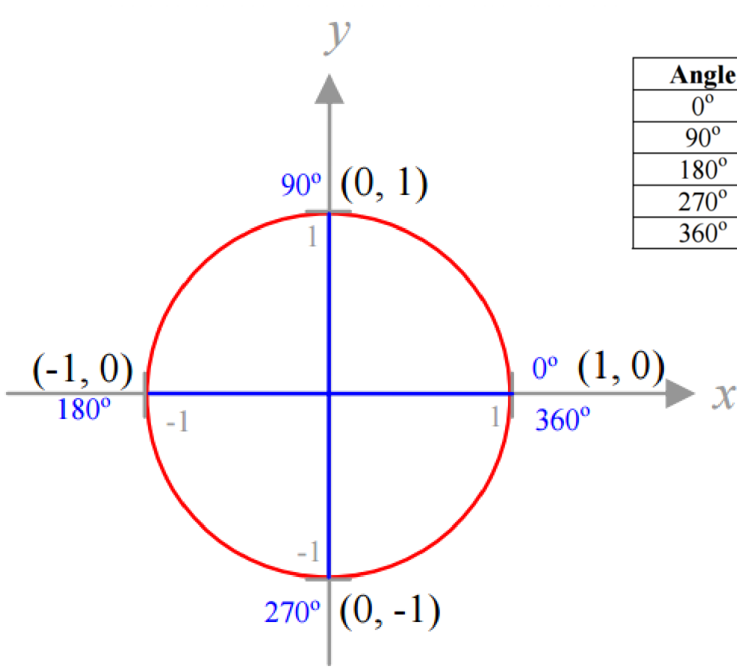
10. (5, 12)	11. (-8, 15)

Concept 4: Quadrantal Angles

<u>TRIGONOMETRY WITH QUADRANTAL ANGLES (DO NOT NEED TO MEMORIZE)</u>

	0°	**90°**	**180°**	**270°**	**360°**
sin θ					
cos θ					
tan θ					

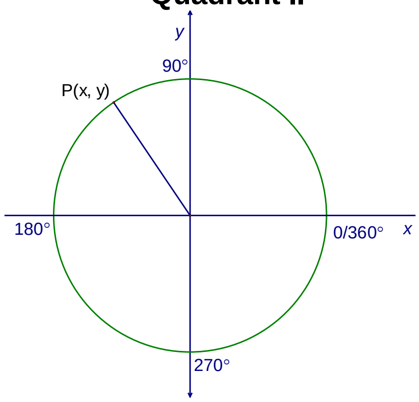
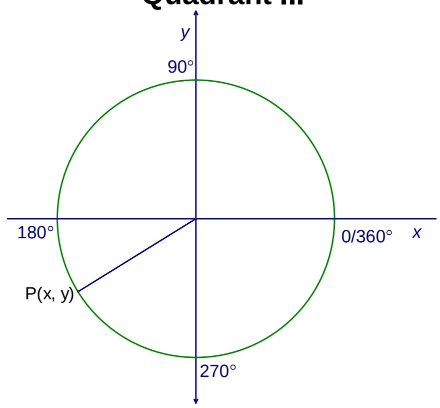
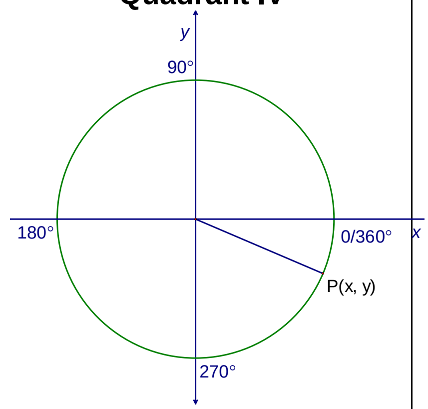
*****YOU CAN JUST PLUG THESE INTO YOUR CALCULATOR (in degree mode)*****



Angle	**Coordinates**
0°	
90°	
180°	
270°	
360°	

Concept 5: Signs of Trig Functions in the Quadrants

As point P(x, y) moves around the unit circle, and θ increases from 0° to 360°, x and y change signs, and thus the signs of sinθ, cosθ, and tanθ also change.

Quadrant II	Quadrant III	Quadrant IV
		
x is _____ and y is ______ ∴ $\cos\theta$ is ___ and $\sin\theta$ is ___. $\tan\theta$ is ___	x is _____ and y is ______ ∴ $\cos\theta$ is ___ and $\sin\theta$ is ___. $\tan\theta$ is ___	x is _____ and y is ______ ∴ $\cos\theta$ is ___ and $\sin\theta$ is ___. $\tan\theta$ is ___

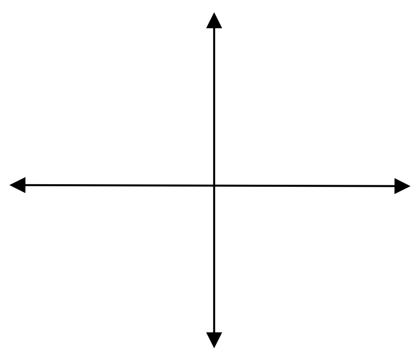
There is an easy way to remember the signs of sin, cos, and tan in the different quadrants.

_____ is/are + in QI

_____ is/are are + in QII

_____ is/are are + in QIII

_____ is/are are + in QIV



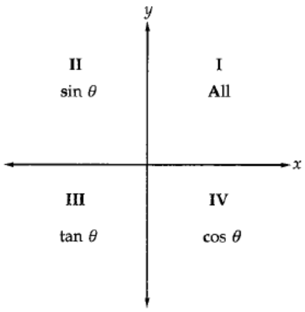
NOTE: * Reciprocal Functions have the same sign values as each other.*

Determine the sign (+/-) of trig functions on the coordinate plane.

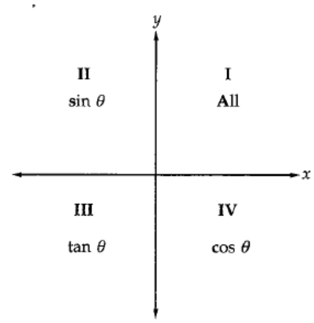
IMPORTANT: “>0” means “is positive”   “<0” means “is negative”

Example 12: In what quadrant(s) could θ be when…

a) $\sin\theta > 0 \; and \; \cos\theta > 0$



b) $\tan\theta > 0 \; and \; \cos\theta < 0$



c) If $\cos x = -\frac{4}{5}$ and $\tan x > 0$, then $\angle x$ terminates in Quadrant

1) I
2) II
3) III
4) IV

d) If $\cos x = -0.7$ and $\csc x > 0$, the terminal side of angle x is located in Quadrant

1) I
2) II
3) III
4) IV

e) If $\tan x = -\sqrt{3}$, in which quadrants could angle x terminate?

1) I and III
2) II and III
3) II and IV
4) III and IV

f) If $\sin\theta = \frac{1-\sqrt{17}}{4}$, then angle θ lies in which quadrants?

1) I and II, only
2) II and IV, only
3) III and IV, only
4) I, II, III, and IV

Concept 6: Let's put this all together!

Let point P be on the terminal side of θ. Draw a picture, and determine the sine, cosine, and tangent of the angle.

13. If $\sin\theta = \frac{12}{13}$, where θ is in Quadrant I, find $\cos\theta$ and $cot\theta$

14. If $\cos\theta = \frac{2}{3}$, where θ is in Quadrant IV, find $\csc\theta$ and $\tan\theta$.

15. If $\tan\theta = 3$, where θ is in Quadrant III, find $\sin\theta$ and $\sec\theta$.

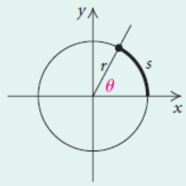
16. If $\sin\theta = \frac{5}{6}$, where θ is in Quadrant II, find $\cot\theta$ and $\sec\theta$.

SUMMARY

Radian Measure

The **radian measure** θ of a rotation is the ratio of the distance s traveled by a point at a radius r from the center of rotation, to the length of the radius r:

$$\theta = \frac{s}{r}.$$



When using the formula $\theta = s/r$, θ must be in radians and s and r must be expressed in the same unit.

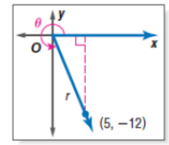
EXAMPLE 7 Find the measure of a rotation in radians when a point 2 m from the center of rotation travels 4 m.

Solution We have

$$\theta = \frac{s}{r} = \frac{4\text{ m}}{2\text{ m}} = 2. \quad \text{The unit is understood to be radians.}$$

Example 1 *Evaluate Trigonometric Functions for a Given Point*

Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point $(5, -12)$.



From the coordinates given, you know that $x = 5$ and $y = -12$. Use the Pythagorean Theorem to find r.

$r = \sqrt{x^2 + y^2}$ — Pythagorean Theorem

$= \sqrt{5^2 + (-12)^2}$ — Replace x with 5 and y with -12.

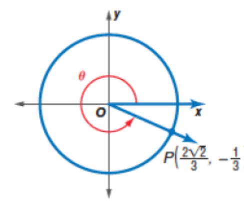
$= \sqrt{169}$ or 13 — Simplify.

Now, use $x = 5$, $y = -12$, and $r = 13$ to write the ratios.

$\sin\theta = \frac{y}{r} = \frac{-12}{13}$ or $-\frac{12}{13}$ $\qquad$ $\cos\theta = \frac{x}{r} = \frac{5}{13}$ $\qquad$ $\tan\theta = \frac{y}{x} = \frac{-12}{5}$ or $-\frac{12}{5}$

Example 1 *Find Sine and Cosine Given Point on Unit Circle*

Given an angle θ in standard position, if $P\left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$ lies on the terminal side and on the unit circle, find $\sin\theta$ and $\cos\theta$.



$P\left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right) = P(\cos\theta, \sin\theta)$,

so $\sin\theta = -\frac{1}{3}$ and $\cos\theta = \frac{2\sqrt{2}}{3}$.

Exit Ticket

If $\sin\theta$ is negative and $\cos\theta$ is negative, in which quadrant does the terminal side of θ lie?

1) I
2) II
3) III
4) IV

Day 3 – Homework

1. What is 235°, expressed in radian measure?

(1) 235π (3) $\frac{36\pi}{47}$

(2) $\frac{\pi}{235}$ (4) $\frac{47\pi}{36}$

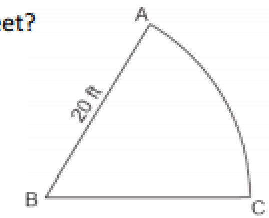
2. What is the number of degrees in an angle whose radian measure is $\frac{7\pi}{12}$?

3. Find, to the *nearest minute*, the angle whose measure is 2.75 radians.

4. An art student wants to make a string collage by connecting six equally spaced points on the circumference of a circle to its center with string. What would be the radian measure, in terms of π, of the angle between two adjacent pieces of string?

5. A sprinkler system is set up to water the sector shown in the accompanying diagram, with angle *ABC* measuring 1 radian and radius $AB = 20$ feet.

What is the length of arc *AC*, in feet?



1) 63

2) 31

3) 20

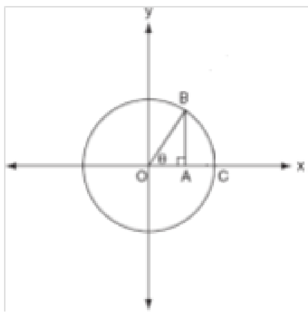
4) 10

6. In a circle of radius 8, find the length of the arc intercepted by a central angle of 1.5 radians.

7. Cities *H* and *K* are located on the same line of longitude and the difference in the latitude of these cities is 9°, as shown in the accompanying diagram. If Earth's radius is 3,954 miles, how many miles north of city *K* is city *H* along arc *HK*? Round your answer to the *nearest tenth of a mile*.

(Not drawn to scale)

8. The accompanying diagram shows unit circle O, with radius OB = 1.



Which line segment has a length equivalent to cos θ?

(1) $\overline{AB}$ (3) $\overline{OC}$

(2) $\overline{OB}$ (4) $\overline{OA}$

9. If $f(x) = \sin 2x + \cos x$, then $f(\pi) =$

(1) 1 (3) 0

(2) 2 (4) -1

10. If $\sec x < 0$ and $\cot x < 0$, in which quadrant does the terminal side of angle x lie?

1) I
2) II
3) III
4) IV

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

11. $(-15, 8)$ 12. $(-3, 0)$ 13. $(4, 4)$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

14. $\cos\theta = -\frac{1}{2}$, Quadrant II 15. $\cot\theta = -\frac{\sqrt{2}}{2}$, Quadrant IV

16. If $\sin\theta = \cos\theta$, in which quadrants may angle θ terminate?
1) I, II
2) II, III
3) I, III
4) I, IV

17. If $\sin x = -\frac{1}{3}$ and $\sin x \cos x > 0$, in which quadrant does angle x lie?
1) I
2) II
3) III
4) IV

18. An angle that measures $\frac{5\pi}{6}$ radians is drawn in standard position. In which quadrant does the terminal side of the angle lie?

19. If $f(x) = \sin^2 x$, then $f\left(\frac{\pi}{2}\right)$ equals
1) 1
2) $\frac{3}{4}$
3) $\frac{1}{2}$
4) $\frac{1}{4}$

20. If $f(x) = \cos 3x + \sin x$, then $f\left(\frac{\pi}{2}\right)$ equals
1) 1
2) 2
3) -1
4) 0

21. If θ is an angle in standard position and its terminal side passes through the point $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ on a ***unit circle***, find all 6 trigonometric functions.

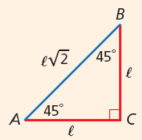
Day 4: SWBAT apply "Special" Angles to find the exact value of Trig Functions

Do Now: Recall the following theorems from Geometry:

Theorem 5-8-1 **45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, both legs are congruent, and the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

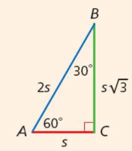
$AC = BC = \ell$ $\qquad$ $AB = \ell\sqrt{2}$



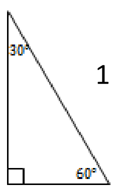
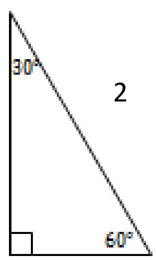
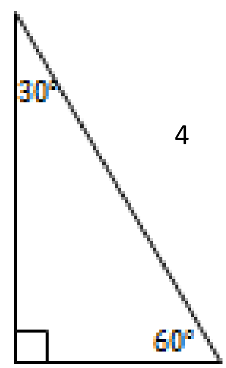
Theorem 5-8-2 **30°-60°-90° Triangle Theorem**

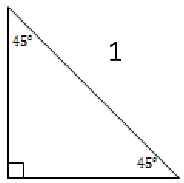
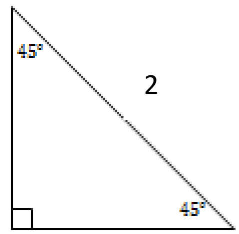
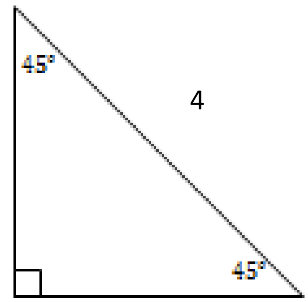
In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is the length of the shorter leg times $\sqrt{3}$.

$AC = s$ $\qquad$ $AB = 2s$ $\qquad$ $BC = s\sqrt{3}$



Complete the tables with a partner:
a) Find the lengths of the missing sides.
b) Find the sine, cosine and tangent of each acute angle in each triangle.
c) What relationship do you notice?

 1	 2	 4
sin 30^o = cos 30^o = tan 30^o = sin 60^o = cos 60^o = tan 60^o =	sin 30^o = cos 30^o = tan 30^o = sin 60^o = cos 60^o = tan 60^o =	sin 30^o = cos 30^o = tan 30^o = sin 60^o = cos 60^o = tan 60^o =
Conclusion:		

		
$\sin 45^{o} =$ $\cos 45^{o} =$ $\tan 45^{o} =$	$\sin 45^{o} =$ $\cos 45^{o} =$ $\tan 45^{o} =$	$\sin 45^{o} =$ $\cos 45^{o} =$ $\tan 45^{o} =$
Conclusion:		

Use these triangles to determine the following trigonometric values:

	$30°$	$45°$	$60°$
Sine			
Cosine			
Tangent			
Cosecant			
Secant			
Cotangent			

Putting it all together (only QI)

	$0°$	$30°$	$45°$	$60°$	$90°$
Sine					
Cosine					
Tangent					

How to construct this table:

- For Sines and Cosines only, write a denominator of "2" for each.
- For Sine, fill in the following numerators, left to right: $\sqrt{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}$.
- For Cosine, fill in the following numerators, left to right: $\sqrt{4}, \sqrt{3}, \sqrt{2}, \sqrt{1}, \sqrt{0}$.
- Simplify.
- Since tangent = sin/cos, each tangent box is sin/cos. Divide, and rationalize the denominators.

	$0°$	$30°$	$45°$	$60°$	$90°$
Sine					
Cosine					
Tangent					

Exact Values/Aprroximations

$\sin 60^\circ = \frac{\sqrt{3}}{2}$. **This is exact!**

$\sin 60^\circ \approx 0.8660254038$. **This is an approximation!**

Model Problems:

1. Find the exact value of (sin 30°)(cos 60°).	2. Find the exact value of $\csc^2 60^\circ$.
3. θ is an angle drawn in standard position and intersect a unit circle at point A. If the coordinates of point A are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, what is the smallest positive value of θ?	

Find the EXACT value of each expression.

a) $\cos 60° + 3 \tan 45°$	b) $\dfrac{\cos \frac{\pi}{3}}{\tan \frac{\pi}{3}}$
a) $\sin^2 45° + \cos^2 45°$	b) $2 \cos \frac{\pi}{6} + 4 \tan \frac{\pi}{3}$
c) $(\sec \frac{\pi}{4})(\cos \frac{\pi}{3})$	d) Let $f(x) = \csc 2x$. Determine $f\left(\frac{\pi}{6}\right)$
e) $2\sin \pi + \sec \frac{\pi}{2}$	f) $\dfrac{\cos 180^o - \sin 90^o}{\cot 45^o}$
g) If $f(x) = \csc x + \cot x$, find $f\left(\frac{\pi}{6}\right)$.	h) An acute angle is drawn in standard position. The coordinates of the terminal side are $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, what does the angle measure?

Exit ticket: The value of $2(\sin 30°)(\cos 30°)$ is equal to the value of:
(1) $\sin 60°$ (2) $\cos 60°$ (3) $\sin 90°$ (4) $\tan 30°$

Day 4 – Homework

1. If $f(x) = \tan\frac{x}{3} + \cos x$, what is $f(180°)$

2. Express as a single fraction the exact value of: $\cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$.

3. What is the value of $\cot(\frac{\pi}{3})$ in simplest radical form?

4. If θ is an angle in standard position and its terminal side passes through the point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on a unit circle, a possible value of θ is

(1) 30° (3) 120°

(2) 60° (4) 150°

5. If $f(x) = 2\cos\left(\frac{x}{6}\right)$, find $f(180)$.

6.

The value of $(\sin 60°)(\cos 60°)$ is

7.

Copy and complete the table.

θ	0°	30°	45°	60°	90°	180°	270°	360°
Radians								
Sin θ								
Cos θ								
Tan θ								

8. Find the exact value: $\dfrac{\cos^2 30^o + \sin 30^o}{\sec 60^0}$

9. Find, in *simplest radical form*, the exact value of

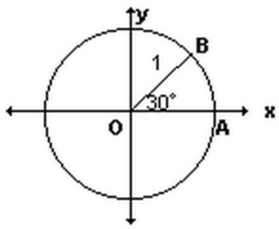
$\csc \dfrac{\pi}{3}$

10.	If $f(x) = \sin 2x + \cos x$, then $f(\pi) =$ (1) 1 (3) 0 (2) 2 (4) -1
11.	If the coordinates of point A are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ What is θ?
12.	If θ is an angle in standard position and its terminal side passes through the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ on a unit circle, a possible value of θ is (1) 30° (3) 120° (2) 60° (4) 150°

Day 5: **SWBAT apply Reference Angles to find Trig Values in All Quadrants**

Do Now:

1) In the diagram, the center of circle O is at the origin, radius $OB = 1$, and $m\angle AOB = 30$.

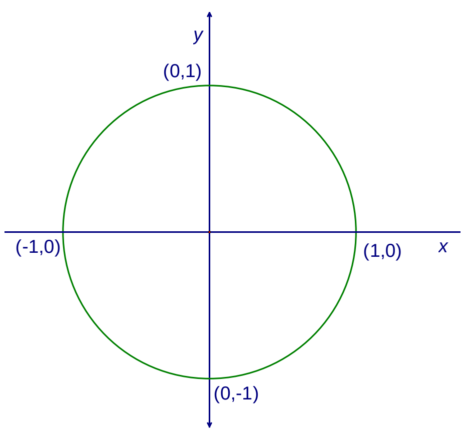


What are the coordinates of point B?

1. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
2. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
3. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
4. $(1, 1)$

2) a) Draw an angle of 150^o in standard position.

b) Draw a line parallel to the y-axis and perpendicular to the x-axis. What is measure of the angle formed?



How Do We Find and Graph Reference Angles?

$\sin 150 = \frac{1}{2}$

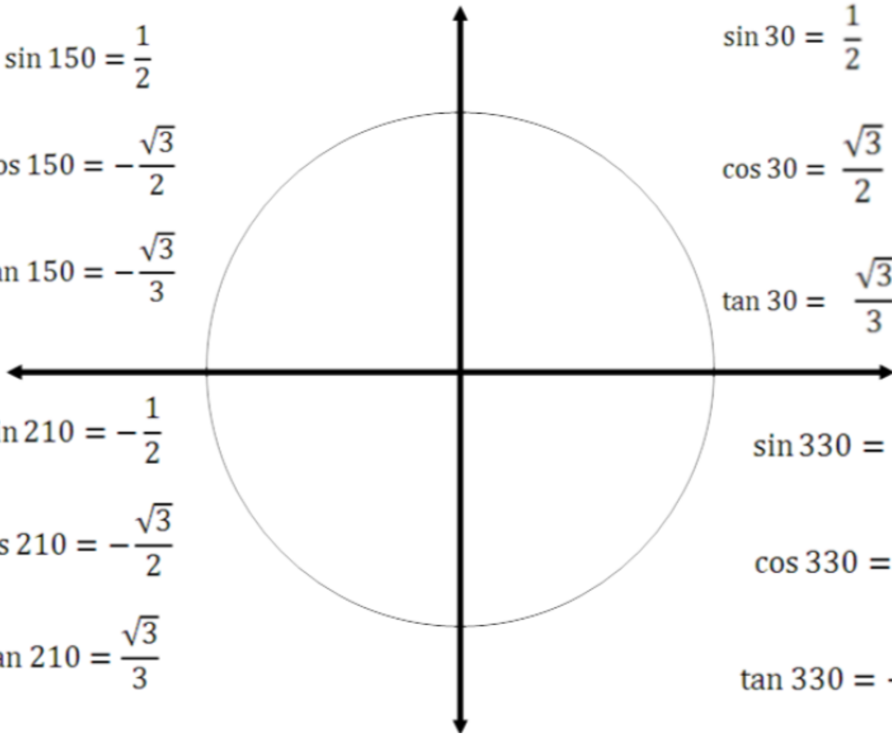
$\cos 150 = -\frac{\sqrt{3}}{2}$

$\tan 150 = -\frac{\sqrt{3}}{3}$

$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\tan 30 = \frac{\sqrt{3}}{3}$



$\sin 210 = -\frac{1}{2}$

$\cos 210 = -\frac{\sqrt{3}}{2}$

$\tan 210 = \frac{\sqrt{3}}{3}$

$\sin 330 = -\frac{1}{2}$

$\cos 330 = \frac{\sqrt{3}}{2}$

$\tan 330 = -\frac{\sqrt{3}}{3}$

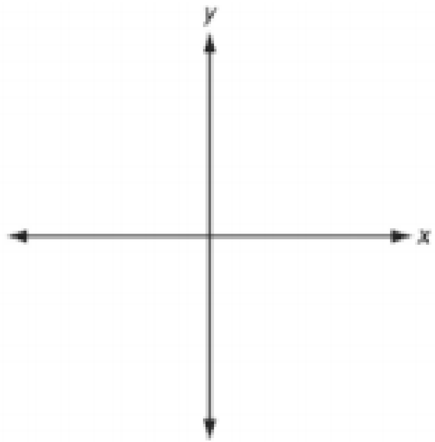
What is happening in this example above? Why?

What is a reference angle?

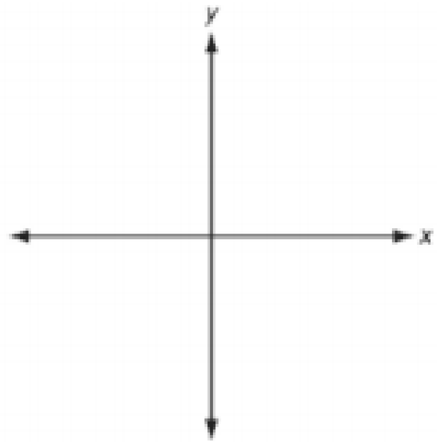
The reference angle is the positive acute angle formed by the terminal side of the given angle and the x-axis.

Practice: Find and graph the reference angle of each given angle in standard position.

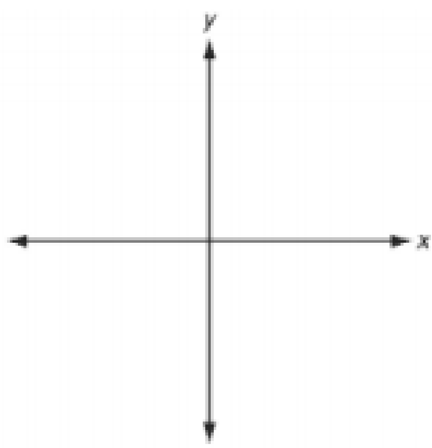
1. $50°$



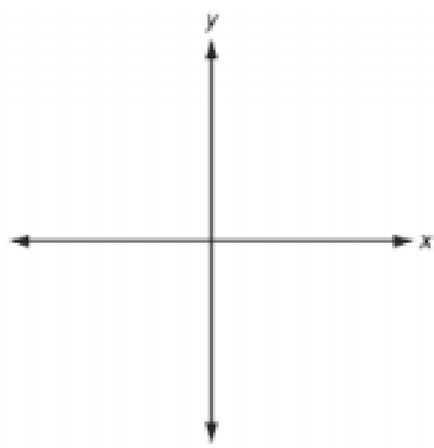
2. $105°$



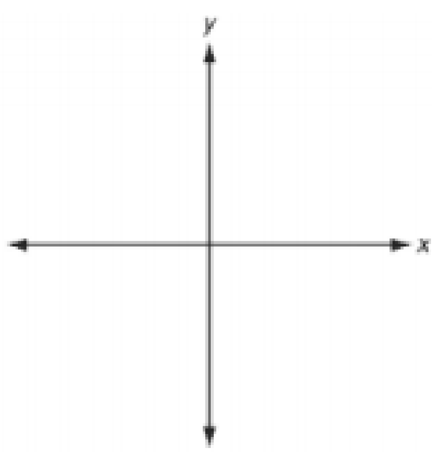
3. $260°$



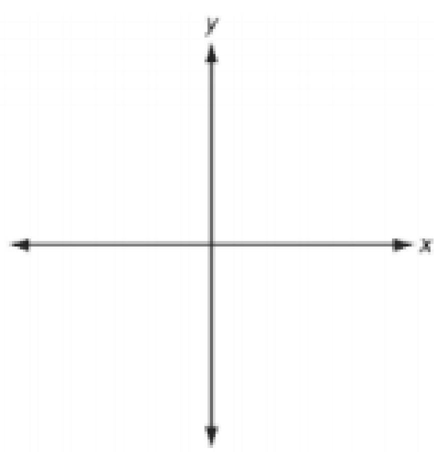
4. $300°$



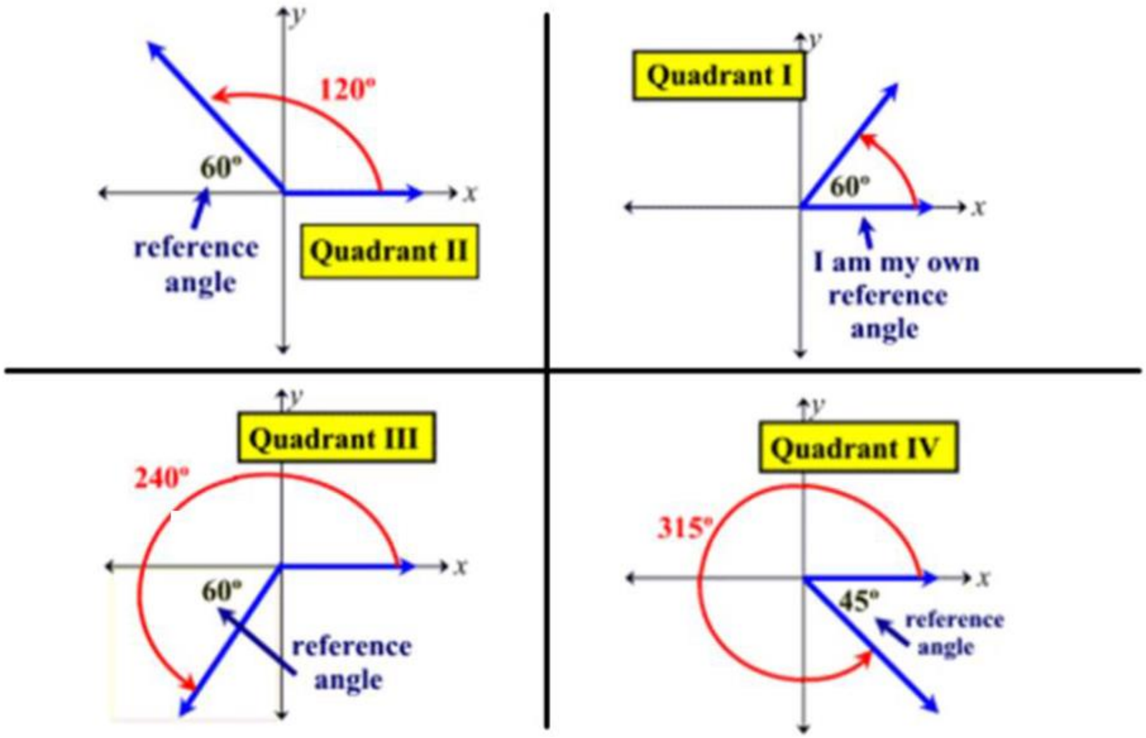
5. $-160°$



6. $555°$

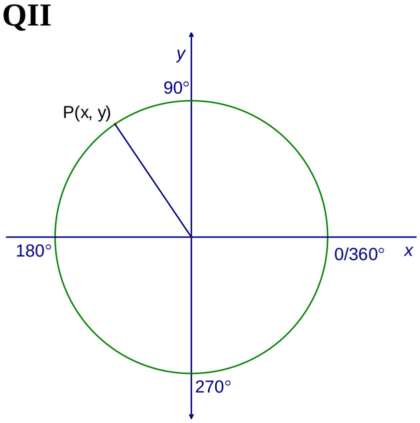
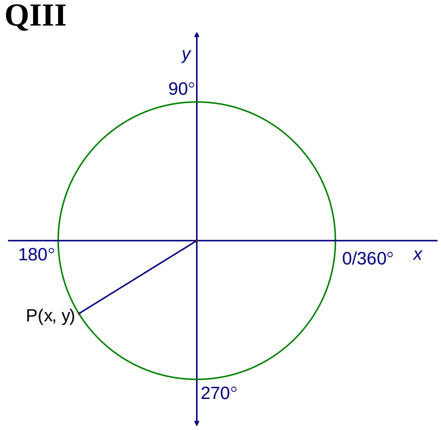
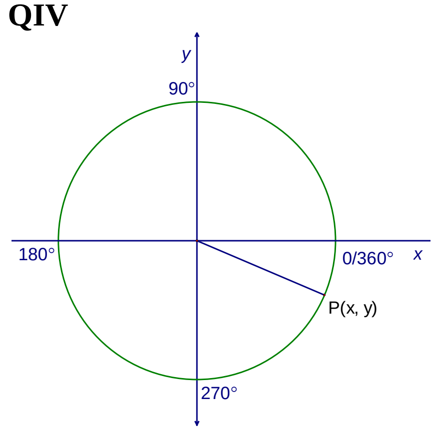


Refer to the diagram.   Fill in the chart below.



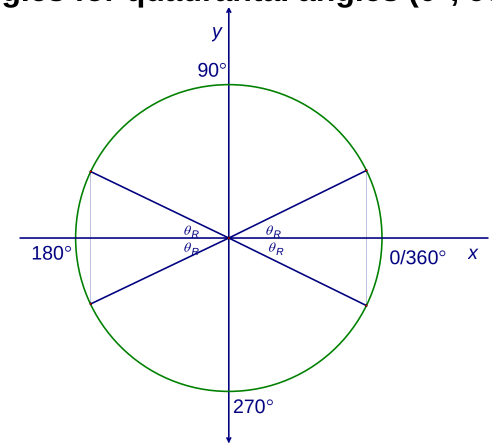
Reference angles look different in each quadrant. In QI, the reference angle for θ is θ itself. Every angle in QI is acute, so any angle in QI (θ_I) doesn't need a reference angle.

Reference angles for other quadrants

QII	**QIII**	**QIV**
		
In **QII,** θ_R= __________ .	In **QIII,** θ_R= __________ .	In **QIV,** θ_R= __________ .

REMEMBER: Reference angles are ALWAYS formed between the terminal side of the original angle and the x-axis. NEVER with the y-axis!!

Also, there are no reference angles for quadrantal angles (0°, 90°, 180°, 270°…)



Reference Angles

We already know that we can have trigonometric values of any angle, in any quadrant, and we've already determined what the signs (+/-) of each of them are. But we can also find the actual trig function values.

Model Problem: Find the exact value of cos 135.

a) Find the reference angle:

b) Express as the function of a positive acute angle:

c) Use your special angle values to find the exact value of the function:

Examples:

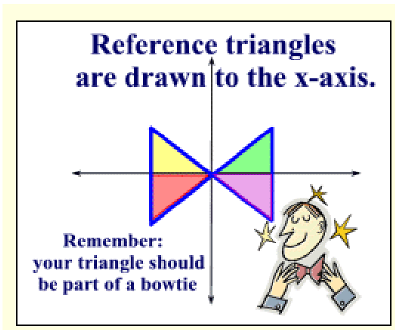
1. Find the exact value of $\cos(135°)$.	2. Find the exact value of $\sin\frac{5\pi}{3}$
3. Find the exact value of $\tan(-150°)$.	4. Find the exact value of $\sec\frac{7\pi}{6}$
5. Find the exact value of $\cot(300°)$.	6. Find the exact value of $\csc\frac{-11\pi}{6}$.

7. Find the exact value of csc 750^o	8. Find the exact value of tan $\left(-\frac{\pi}{2}\right)$.
9. Find the value of cot (-840^o).	10. Find the smallest positive angle drawn in standard position that intersects the unit circle at $\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$.
11. Express sin 225 as the function of a positive acute angle:	12. Express sec -80 as the function of a positive acute angle.

SUMMARY

If θ is the measure of an angle greater than 90° but less than 360°:

90°< θ < 180° Quadrant II	180° < θ < 270° Quadrant III	270° < θ < 360° Quadrant IV
sin θ = sin (180° - θ) cos θ = -cos (180° - θ) tan θ = -tan (180° - θ)	sin θ = -sin (θ-180°) cos θ = -cos (θ-180°) tan θ = tan (θ – 180°)	sin θ = -sin (360° - θ) cos θ = cos (360° - θ) tan θ = -tan (360° - θ)



Exit Ticket:

Expressed as a function of a positive acute angle, cos (−305°) is equal to

(1) −cos 55°

(2) cos 55°

(3) −sin 55°

(4) sin 55°

Day 5 - Homework

Find the exact value of each trigonometric function.

1) $\sin 765°$

2) $\tan 315°$

3) $\csc 930°$

4) $\csc 600°$

5) $\csc -480°$

6) $\tan -990°$

7) $\csc -\frac{\pi}{6}$

8) $\cot \frac{14\pi}{3}$

9) $\sin -210°$

10) $\sec \frac{\pi}{6}$

Find the reference angle.

11) $\frac{13\pi}{4}$

12) $-\frac{7\pi}{9}$

13) $\frac{28\pi}{9}$

14) $640°$

15) $-430°$

16) $335°$

17) Find the smallest positive angle drawn in standard position that intersects the unit circle at $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

18) Find the smallest positive angle drawn in standard position that intersects the unit circle at $\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$.

Answer Keys

Homework #1

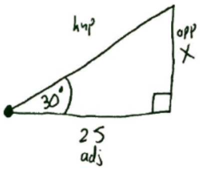
1. The angle of elevation from a point 25 feet from the base of a tree on level ground to the top of the tree is 30°. Which equation can be used to find the height of the tree?

1) $\tan 30° = \frac{x}{25}$

2) $\sin 30° = \frac{x}{25}$

3) $\cos 30° = \frac{x}{25}$

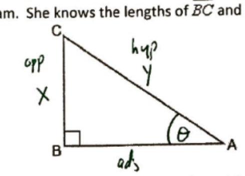
4) $30^2 + 25^2 = x^2$



$\tan\theta = \frac{opp}{hyp}$

$\boxed{\tan 30 = \frac{x}{25}}$

2. Cassandra is calculating the measure of angle *A* in right triangle *ABC*, as shown in the accompanying diagram. She knows the lengths of $\overline{BC}$ and $\overline{AC}$.



If she finds the measure of angle *A* by solving only one equation, which concept will be used in her calculations?

1) Pythagorean theorem

2) $\sin A$

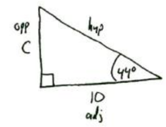
3) $\cos A$

4) $\tan A$

$\boxed{\sin\theta = \frac{opp}{hyp}}$

In figure 7, if $\theta = 44°$, what is the value of *c*?

(A) 6.94
(B) 7.19
(C) 9.66
(D) 10.36
(E) 13.90

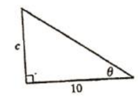


$\tan\theta = \frac{opp}{adj}$

$\tan 44 = \frac{c}{10}$

$c = 10 \cdot \tan 44$

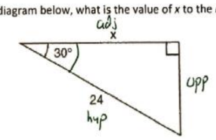
$c \approx 9.66$



Note: Figure not drawn to scale.

Figure 7

4. In the right triangle shown in the diagram below, what is the value of *x* to the *nearest whole number*?



$\cos\theta = \frac{adj}{hyp}$

$\cos 30 = \frac{x}{24}$

$x = 24 \cdot \cos 30$

$\boxed{x \approx 21}$

5a. 37 degrees

$$0.285 \text{ degrees} \cdot \frac{60 \text{ minutes}}{1 \text{ degree}} = 17.1 \text{ minutes}$$

$$0.1 \text{ minutes} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = 6 \text{ seconds}$$

Answer: $37°17'6''$

5b. 314 degrees

$$0.42 \text{ degrees} \cdot \frac{60 \text{ minutes}}{1 \text{ degree}} = 25.2 \text{ minutes}$$

$$0.2 \text{ minutes} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = 12 \text{ seconds}$$

Answer: $314°25'12''$

6a. $$82 \text{ degrees} + 42 \text{ minutes} \cdot \frac{1 \text{ degree}}{60 \text{ minutes}} = 82.7°$$

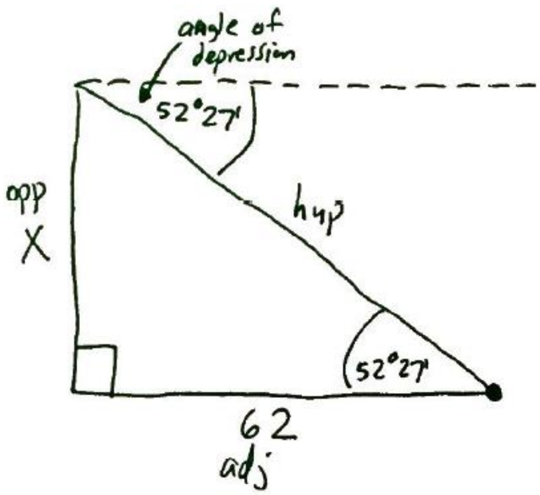
6b. $$213 \text{ degrees} + 15 \text{ minutes} \cdot \frac{1 \text{ degrees}}{60 \text{ minutes}} + 56 \text{ seconds} \cdot \frac{1 \text{ degree}}{3600 \text{ seconds}} \approx 213.266°$$

7. Use a calculator to determine the value of each trigonometric ratio:

a) $\sin 52°47'$	b) $\cos 79°15'45''$	c) $\cot 36°$
$\boxed{.796}$	$\boxed{.186}$	$\frac{1}{\tan 36} = \boxed{1.38}$

8. $\csc\theta = \frac{5}{2}$ and $\cos\theta = \frac{2}{3}$

9. A person measures the angle of depression from the top of a wall to a point on the ground. The point is located on level ground 62 feet from the base of the wall and the angle of depression is 52° 27′. How high is the wall, to the nearest tenth of a foot?



$$\tan\theta = \frac{opp}{adj}$$

$$\tan(52°27') = \frac{x}{62}$$

$$x = 62 \cdot \tan(52°27')$$

$$\boxed{x \approx 80.7}$$

10. A

Homework #2

Draw an angle with the given measure in standard position. 4–7. See margin.

4. $70°$ 5. $300°$ 6. $570°$ 7. $-45°$

Rewrite each degree measure in radians and each radian measure in degrees.

8. $130°$ $\frac{13\pi}{18}$ 9. $-10°$ $-\frac{\pi}{18}$ 10. $485°$ $\frac{97\pi}{36}$
11. $\frac{3\pi}{4}$ $135°$ 12. $-\frac{\pi}{6}$ $-30°$ 13. $\frac{19\pi}{3}$ $1140°$

Find one angle with positive measure and one angle with negative measure coterminal with each angle. 14–16. Sample answers are given.

14. $60°$ $420°, -300°$ 15. $425°$ $785°, -295°$ 16. $\frac{\pi}{3}$ $\frac{7\pi}{3}, -\frac{5\pi}{3}$

Rewrite each degree measure in radians and each radian measure in degrees.

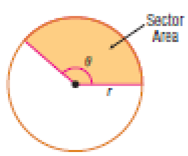
27. $120°$ $\frac{2\pi}{3}$ 28. $60°$ $\frac{\pi}{3}$ 29. $-15°$ $-\frac{\pi}{12}$ 30. $-225°$ $-\frac{5\pi}{4}$
31. $660°$ $\frac{11\pi}{3}$ 32. $570°$ $\frac{19\pi}{6}$ 33. $158°$ $\frac{79\pi}{90}$ 34. $260°$ $\frac{13\pi}{9}$
35. $\frac{5\pi}{6}$ $150°$ 36. $\frac{11\pi}{4}$ $495°$ 37. $-\frac{\pi}{4}$ $-45°$ 38. $-\frac{\pi}{3}$ $-60°$
39. $\frac{29\pi}{4}$ $1305°$ 40. $\frac{17\pi}{6}$ $510°$ ★ 41. 9 ★ 42. 3

Find one angle with positive measure and one angle with negative measure coterminal with each angle. 43–54. Sample answers are given.

43. $225°$ $585°, -135°$ 44. $30°$ $390°, -330°$ 45. $-15°$ $345°, -375°$
46. $-140°$ $220°, -500°$ 47. $368°$ $8°, -352°$ 48. $760°$ $400°, -320°$
49. $\frac{3\pi}{4}$ $\frac{11\pi}{4}, -\frac{5\pi}{4}$ 50. $\frac{7\pi}{6}$ $\frac{19\pi}{6}, -\frac{5\pi}{6}$ 51. $-\frac{5\pi}{4}$ $\frac{3\pi}{4}, -\frac{13\pi}{4}$
52. $-\frac{2\pi}{3}$ $\frac{4\pi}{3}, -\frac{8\pi}{3}$ 53. $\frac{9\pi}{2}$ $\frac{13\pi}{2}, -\frac{3\pi}{2}$ 54. $\frac{17\pi}{4}$ $\frac{25\pi}{4}, -\frac{7\pi}{4}$

55. **DRIVING** Some sport-utility vehicles (SUVs) use 15-inch radius wheels. When driven 40 miles per hour, determine the measure of the angle through which a point on the wheel travels every second. Round to both the nearest degree and nearest radian. 2689° per second; 47 radians per second

GEOMETRY For Exercises 56 and 57, use the following information.
A *sector* is a region of a circle that is bounded by a central angle θ and its intercepted arc. The area A of a sector with radius r and central angle θ is given by $A = \frac{1}{2}r^2\theta$, where θ is measured in radians.



56. Find the area of a sector with a central angle of $\frac{4\pi}{3}$ radians in a circle whose radius measures 10 inches. 209.4 in^2

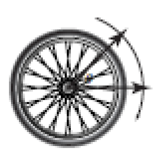
57. Find the area of a sector with a central angle of 150° in a circle whose radius measures 12 meters. about 188.5 m^2

62. **QUANTITATIVE COMPARISON** Compare the quantity in Column A and the quantity in Column B. Then determine whether:
Ⓐ the quantity in Column A is greater,
Ⓑ the quantity in Column B is greater,
Ⓒ the two quantities are equal, or
Ⓓ the relationship cannot be determined from the information given.

Column A	Column B
$56°$	$\frac{14\pi}{45}$

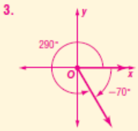
C

63. Angular velocity is defined by the equation $\omega = \frac{\theta}{t}$, where θ is usually expressed in radians and t represents time. Find the angular velocity in radians per second of a point on a bicycle tire if it completes 2 revolutions in 3 seconds. D



Ⓐ $\frac{\pi}{3}$ Ⓑ $\frac{\pi}{2}$ Ⓒ $\frac{2\pi}{3}$ Ⓓ $\frac{4\pi}{3}$

2. In a circle of radius r units, one radian is the measure of an angle whose rays intercept an arc length of r units.

3.


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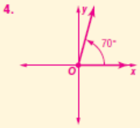

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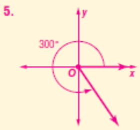

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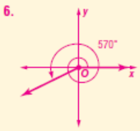

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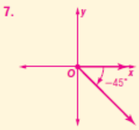


Homework #3

1. What is 235°, expressed in radian measure?

(1) 235π (2) $\frac{\pi}{235}$ (3) $\frac{36\pi}{47}$ (4) $\frac{47\pi}{36}$

$\frac{D}{R} = \frac{180}{\pi}$

$\frac{235}{R} = \frac{180}{\pi}$

$180R = 235\pi$

$\boxed{R = \frac{47\pi}{36}}$

2. What is the number of degrees in an angle whose radian measure is $\frac{7\pi}{12}$?

$\frac{D}{R} = \frac{180}{\pi}$

$\frac{D}{\frac{7\pi}{12}} = \frac{180}{\pi}$

$\pi D = 105\pi$

$\boxed{D = 105}$

3. Find, to the *nearest minute*, the angle whose measure is 2.75 radians.

$\frac{D}{R} = \frac{180}{\pi}$

$\frac{D}{2.75} = \frac{180}{\pi}$

$\pi D = 495$

$D = 157.5633937$

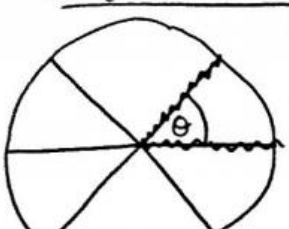
DMS

$157°33'48.217''$

above 30 seconds!

$\boxed{157°34'}$

4. An art student wants to make a string collage by connecting six equally spaced points on the circumference of a circle to its center with string. What would be the radian measure, in terms of π, of the angle between two adjacent pieces of string?



360° in a circle
+
6 equal slices

$\theta = \frac{360}{6} = 60°$

convert to radians!

$\frac{D}{R} = \frac{180}{\pi}$

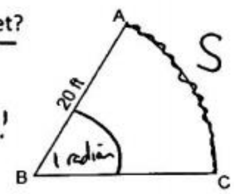
$\frac{60}{R} = \frac{180}{\pi}$

$180R = 60\pi$

$\boxed{R = \frac{\pi}{3}}$

5. A sprinkler system is set up to water the sector shown in the accompanying diagram, with angle ABC measuring 1 radian and radius $AB = 20$ feet.

What is the length of arc AC, in feet?



1) 63
2) 31
3) 20
4) 10

$S = r\theta$

θ must be in radians!

$S = 20(1)$

$\boxed{S = 20}$

6. In a circle of radius 8, find the length of the arc intercepted by a central angle of 1.5 radians.

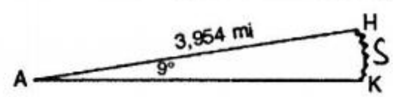
$S = r\theta$

θ must be in radians!

$S = (8)(1.5)$

$\boxed{S = 12}$

7. Cities *H* and *K* are located on the same line of longitude and the difference in the latitude of these cities is 9°, as shown in the accompanying diagram. If Earth's radius is 3,954 miles, how many miles north of city *K* is city *H* along arc *HK*? Round your answer to the *nearest tenth of a mile*.



(Not drawn to scale)

$S = r\theta$

θ must be in radians!

$\theta = 9°$
convert to radians

$\frac{D}{R} = \frac{180}{\pi}$

$\frac{9}{R} = \frac{180}{\pi}$

$180R = 9\pi$

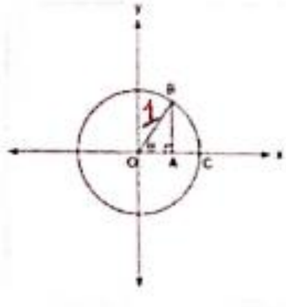
$\boxed{R = \frac{\pi}{20}}$

$S = (3954)\left(\frac{\pi}{20}\right)$

$S = 621.0928676$

$\boxed{S = 621.1}$

8. The accompanying diagram shows unit circle O, with radius $OB = 1$.



Which line segment has a length equivalent to $\cos\theta$?

(1) $\overline{AB}$ (3) $\overline{OC}$

(2) $\overline{OB}$ (4) $\overline{OA}$

$\cos\theta = \frac{x}{r} = \frac{OA}{OB} = \frac{OA}{1}$

9. If $f(x) = \sin 2x + \cos x$, then $f(\pi) =$

(1) 1 (3) 0

(2) 2 (4) -1

$f(\pi) = \sin 2\pi + \cos\pi$

$f(180) = \sin 360 + \cos 180$

$f(180) =$ look at "y" coordinate; look at "x" coord.

$f(180) = 0 + -1 = \boxed{-1}$

$\pi = 180°$

deg	point
90°	(0,1)
180	(-1,0)
270	(0,-1)
360	(1,0)

10. If $\sec x < 0$ and $\cot x < 0$, in which quadrant does the terminal side of angle x lie?

1) I
2) II
3) III
4) IV

S A
T C

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

11. $(-15, 8)$

$x=-15;\ y=8;\ r=?$

$r=\sqrt{x^2+y^2}=\sqrt{(-15)^2+8^2}=17$

$\sin\theta=\frac{y}{r}=\frac{8}{17}$; $\csc\theta=\frac{17}{8}$

$\cos\theta=\frac{x}{r}=\frac{-15}{17}$; $\sec\theta=\frac{17}{-15}$

$\tan\theta=\frac{y}{x}=\frac{8}{-15}$; $\cot\theta=-\frac{15}{8}$

12. $(-3, 0)$

$x=-3;\ y=0;\ r=?$

$r=\sqrt{(-3)^2+0^2}=3$

$\sin\theta=\frac{y}{r}=\frac{0}{3}=0$; $\csc=\frac{3}{0}=$ und

$\cos\theta=\frac{x}{r}=\frac{-3}{3}=-1$; $\sec\theta=\frac{3}{-3}=-1$

$\tan\theta=\frac{y}{x}=\frac{0}{-3}=0$; $\cot\theta=\frac{-3}{0}=$ und

13. $(4, 4)$

$x=4;\ y=4;\ r=?$

$r=\sqrt{4^2+4^2}=\sqrt{32}=\sqrt{16}\sqrt{2}=4\sqrt{2}$

$\sin\theta=\frac{y}{r}=\frac{4}{4\sqrt2}=\frac{1}{\sqrt2}\boxed{\frac{\sqrt2}{2}}$; $\csc\theta=\frac{\sqrt2}{1}=\boxed{\sqrt2}$

$\cos\theta=\frac{x}{r}=\frac{4}{4\sqrt2}=\frac{1}{\sqrt2}\boxed{\frac{\sqrt2}{2}}$; $\sec\theta=\frac{\sqrt2}{1}=\boxed{\sqrt2}$

$\tan\theta=\frac{y}{x}=\frac{4}{4}=\boxed{1}$; $\cot\theta=\boxed{1}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

S A
T C

14. $\cos\theta = -\frac{1}{2}$, Quadrant II

$\cos\theta=\frac{-1}{2}=\frac{x}{r}$

S A
T C

$\sin\theta=+$
$\csc\theta=-$
$\tan\theta=+$

$x=-1$
$r=2$
$y=?$

$\sqrt{x^2+y^2}=r$

$\left(\sqrt{(-1)^2+y^2}\right)^2=2^2$

$1+y^2=4$

$y^2=3$

$y=\sqrt3$

$\sin\theta=\frac{\sqrt3}{2}$; $\csc\theta=\frac{2}{\sqrt3}=\frac{2\sqrt3}{3}$

$\cos\theta=\frac{-1}{2}$; $\sec\theta=\frac{2}{-1}=-2$

$\tan\theta=\frac{\sqrt3}{-1}$; $\cot\theta=\frac{-1}{\sqrt3}=\frac{-\sqrt3}{3}$

$=-\sqrt3$

15. $\cot\theta = -\frac{\sqrt{2}}{2}$, Quadrant IV

$\sin\theta=-$
$\cos\theta=+$
$\tan\theta=-$

$\tan\theta=\frac{-2}{\sqrt2}=\frac{y}{x}$; $r^2=x^2+y^2$

$r^2=(\sqrt2)^2+(-2)^2$

$r^2=2+4$

$r^2=6$

$r=\sqrt6$

$\sin\theta=\frac{-2}{\sqrt6}=\frac{-2\sqrt6}{6}=-\frac{\sqrt6}{3}$; $\csc\theta=\frac{\sqrt6}{-2}$

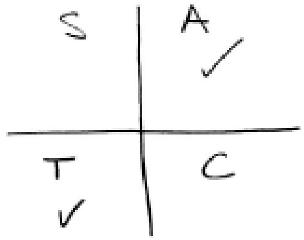
$\cos\theta=\frac{\sqrt2}{\sqrt6}=\frac{\sqrt{12}}{6}=\frac{2\sqrt3}{6}=\frac{\sqrt3}{3}$; $\sec\theta=\sqrt3$

$\tan\theta=\frac{-2}{\sqrt2}=\frac{-2\sqrt2}{2}=-\sqrt2$; $\cot\theta=-\frac{\sqrt2}{2}$

$\sin\theta = +$ $\cos\theta = +$ $\sin\theta = -$ $\cos\theta = -$

16. If $\sin\theta = \cos\theta$, in which quadrants may angle θ terminate?

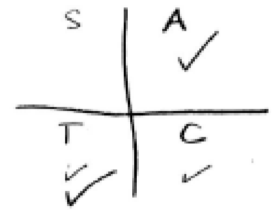
1) I, II
2) ~~II, III~~
3) I, III
4) I, IV



$- \; -$
$+ \; +$

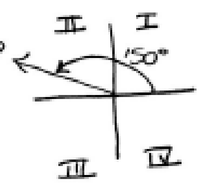
17. If $\sin x = -\frac{1}{3}$ and $\sin x \cos x > 0$, in which quadrant does angle x lie?

1) I
2) II
3) III
4) IV



18. An angle that measures $\frac{5\pi}{6}$ radians is drawn in standard position. In which quadrant does the terminal side of the angle lie?

$\frac{5\pi}{6} \rightarrow \frac{5 \cdot 180}{6} = 150° \rightarrow$



$\boxed{\text{II}}$

19. If $f(x) = \sin^2 x$, then $f\left(\frac{\pi}{2}\right)$ equals

1) 1
2) $\frac{3}{4}$
3) $\frac{1}{2}$
4) $\frac{1}{4}$

$\frac{\pi}{2} \rightarrow \frac{180}{2} \rightarrow 90°$

$f(90°) = (\sin 90)^2$

$f(90°) = (1)^2$

$f(90°) = 1$

Deg	pt
90	(0,1) ↑ $\sin\theta$

20. If $f(x) = \cos 3x + \sin x$, then $f\left(\frac{\pi}{2}\right)$ equals

1) 1
2) 2
3) -1
4) 0

$\frac{\pi}{2} \rightarrow \frac{180}{2} \rightarrow 90°$

$f(90) = \cos(3 \cdot 90) + \sin(90)$

$f(90) = \cos 270 + \sin 90$

↓ look at x-coor. ↓ look at y-coord.

$= 0 + 1$

Deg	pt
90	(0,1)
180	(-1,0)
270	(0,-1)
360	(1,0)

21. If θ is an angle in standard position and its terminal side passes through the point $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ on a ***unit circle***, find all 6 trigonometric functions.

$*$ unit circle $\rightarrow (x, y) = (\sin\theta, \cos\theta)$

$\sin\theta = y = \boxed{-\frac{1}{2}}$

$\cos\theta = x = \boxed{\frac{\sqrt{3}}{2}}$

$\tan\theta = \frac{y}{x} = -\frac{1}{2} \div \frac{\sqrt{3}}{2}$ → keep, change, flip

$-\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{-1}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$

$\csc\theta = -\frac{2}{1} = \boxed{-2}$

$\sec\theta = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

$\cot\theta = -\frac{\sqrt{3}}{1} = \boxed{-\sqrt{3}}$

Homework #4

1. $\tan\left(\frac{180}{3}\right) + \cos 180$

$\tan 60 + -1$

$\sqrt{3} - 1$

$\boxed{-1 + \sqrt{3}}$

2. $\cos 30° \cos 45° - \sin 30° \sin 45°$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

3. $\cot 60 = \frac{1}{\tan 60} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

4. $f(180) = 2\cos\left(\frac{180}{6}\right)$

$= 2\cos 30$

$= 2 \cdot \frac{\sqrt{3}}{2}$

$= \frac{\cancel{2}}{1} \cdot \frac{\sqrt{3}}{\cancel{2}}$

$= \boxed{\sqrt{3}}$

5. If θ is an angle in standard position and its terminal side passes through the point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on a unit circle, a possible value of θ is

(1) 30° (3) 120°

(2) 60° (4) 150°

$(x, y) = (\cos\theta, \sin\theta)$

$\cos\theta = x$ $\sin\theta = y$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\sin\theta = \frac{1}{2}$

$\theta = 30°$ $\theta = 30°$

6. $(\sin 60)(\cos 60)$

$\frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

$\boxed{\frac{\sqrt{3}}{4}}$

7.

θ	0°	30°	45°	60°	90°	180°	270°	360°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
Tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	0	undefined	0

8. $\dfrac{\left(\frac{\sqrt{3}}{2}\right)^2+\frac{1}{2}}{2} = \dfrac{\frac{3}{4}+\frac{1}{2}}{2} = \dfrac{\frac{5}{4}}{2} = \dfrac{5}{8}$

9. csc 60 = $\dfrac{2\sqrt{3}}{3}$

10. sin 2π + cos π = sin 360 + cos 180 = 0 + -1 = -1

11. sin θ = $\dfrac{\sqrt{2}}{2}$ and cos θ = $\dfrac{\sqrt{2}}{2}$; $\theta = 45^{\circ}$

12. sin θ = $\dfrac{\sqrt{3}}{2}$ and cos θ = $\dfrac{1}{2}$; $\theta = 60^{\circ}$

Homework #5

Find the exact value of each trigonometric function.

1) $\sin 765°$ $\dfrac{\sqrt{2}}{2}$

2) $\tan 315°$

-1

3) $\csc 930°$

-2

4) $\csc 600°$ $-\dfrac{2\sqrt{3}}{3}$

5) $\csc -480°$ $-\dfrac{2\sqrt{3}}{3}$

6) $\tan -990°$

Undefined

7) $\csc -\dfrac{\pi}{6}$

-2

8) $\cot \dfrac{14\pi}{3}$ $-\dfrac{\sqrt{3}}{3}$

9) $\sin -210°$ $\dfrac{1}{2}$

10) $\sec \dfrac{\pi}{6}$ $\dfrac{2\sqrt{3}}{3}$

Find the reference angle.

11) $\dfrac{13\pi}{4}$ $\dfrac{\pi}{4}$

12) $-\dfrac{7\pi}{9}$ $\dfrac{2\pi}{9}$

13) $\dfrac{28\pi}{9}$ $\dfrac{\pi}{9}$

14) $640°$

$80°$

15) $-430°$

$70°$

16) $335°$

$25°$

17) 120^o

18) 225^o