5

5

Algebra

Algebra rules my kitchen! Can algebra help you cook the perfect pancake?

A mathematics lecturer at an English university has published a formula she believes will create the perfect pancake. Dr Ruth Fairclough included every detail that could affect the quality of the pancake in her formula. She says that the most important factor, apart from the batter's recipe, is the frying pan's temperature.

The formula is:

$$P = 100 - \frac{10L - 7F + C(k - C) + T(m - T)}{S - E}$$

where
P = the pancake score
L = number of lumps in the batter
C = batter consistency
F = flipping score
k = ideal batter consistency
T = temperature of the pan
m = ideal temperature of the pan
S = length of time the batter stands before cooking
E = length of time the cooked pancake sits before being eaten.

The closer you get to 100, the more delicious the pancake. Yum!

Forum

Would you use this formula to make pancakes at home?

Would a chef in a restaurant use this formula?

Where would you use a formula around the home or in the garden? Who would use formulas in their work?

Why learn this?

Algebra is a mathematical language. It uses letters and symbols to communicate general rules found from patterns and to solve problems. Algebra can help engineers to calculate stress forces on bridges, architects to design environmentally friendly buildings, nurses to calculate the correct doses of medicine and accountants to calculate how much tax needs to be paid.

After completing this chapter you will be able to:

- write and simplify algebraic expressions using pronumerals
- use a flowchart to describe algebraic rules
- generate tables of values using rules given in words or algebraic formulas
- change rules expressed in words to algebraic formulas using pronumerals
- substitute values into formulas
- use algebra to solve problems
- use tables to plot points on a Cartesian plane
- use a rule to generate linear graphs
- interpret point and line graphs.

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R5.1

1 Find the value of the $\square$ in each of the following number sentences. If the $\square$ appears more than once in a number sentence, the same number must be used.

(a) $3 + 7 = \square + 5$ **(b)** $5 - 1 = 2 \times \square$ **(c)** $3 + \square + 4 = \square \times 8$

(d) $5 - 4 + \square = 5$ **(e)** $10 \div \square = 5$ **(f)** $\square \times \square = 16$

PP Worksheet R5.2

2 **(a)** Multiply each of the following by 11 and subtract 3 from your answer.

(i) 4 **(ii)** 10 **(iii)** 200

(b) Multiply each of the following by itself and then take 6 from your answer.

(i) 4 **(ii)** 10 **(iii)** 8

PP Worksheet R5.3

3 Write the following sentences using only numbers, the four operation signs (+, −, ×, ÷) and the equals sign (=).

(a) The sum of eleven and seventeen is twenty-eight.

(b) The difference between nine and seven is two.

(c) The product of three and four is twelve.

(d) The quotient of sixteen and eight is two.

PP Worksheet R5.4

4 Write the following sentences using only numbers, the four operation signs and brackets. Then, use the order of operations to find the answer.

(a) Take the number six, multiply it by two, and then subtract four from it.

(b) Add five to the number ten, then divide what you get by five.

(c) Multiply the sum of six and five by three.

(d) The difference between twenty-three and twenty-one is multiplied by twelve.

PP Worksheet R5.5

5 Decode the following by writing the letters contained in each grid square.

(a) The fastest living creature
E2 E4 A1 E4 B3 A1 D3 C2 E4
A3 A4 A2 C4 D2 C2

(b) The largest land carnivore
E3 D2 D4 D3 A4 E3 B4 E4 A4 A1

4	a	b	c	d	e
3	f	g	h	i	k
2	l	m	n	o	p
1	r	s	t	u	y
	A	B	C	D	E

Key Words

- axes
- Cartesian plane
- coefficient
- constant
- coordinates
- define
- equation
- evaluate
- expression
- formula
- like terms
- line graph
- linear graph
- ordered pair
- origin
- point graph
- pronumeral
- quadrant
- relationship
- substitute
- table of values
- terms
- unknown
- unlike terms
- variable

Pronumerals and variables

5.1

The meaning of 'x' (or 'n' or …)

Algebra is a language used by mathematicians to communicate mathematical ideas and information clearly. To become a mathematician, you need to learn the language in the same way that you learn any other language. You will begin this study of algebra by learning words and algebraic conventions (rules that everyone agrees to follow) that will help you to read, speak, write and understand mathematics.

Algebra is used to write general rules to describe patterns that we find with numbers and solve problems using these rules.

The word 'algebra' comes from *al-jabr*, an Arabic word meaning 'to put back together'.

Writing algebra

- **Pronumerals** are letters or symbols we use for numbers we don't know. Pronumerals represent (stand for) a number the same way that pronouns 'stand for' a noun.
- An **unknown** is the actual number that the pronumeral represents.
- A **variable** describes the unknown number if its value can change.

Using algebra, we can use x as a pronumeral to represent an unknown number of lollies in a packet. If the number of lollies in the packet can vary, x is called a variable.

If we are given 3 extra lollies, we now have $x + 3$ lollies.

If we buy another identical packet of lollies, we have $x + x$, or $2 \times x$ lollies.

If we have 2 identical packets and 3 extra lollies, we have $2 \times x + 3$ lollies in total.

When we have different variables, we need to use different pronumerals. For example, if we represent the number of boys in a class with the pronumeral b, we need to use a different pronumeral, such as g, to represent the number of girls in the class. The total number of students in the class is the sum of the two unknowns and can be written as $b + g$. Any two letters can be used to represent the number of boys and girls, so the number of students in the class could be written as $x + y$ or $p + r$.

Algebraic conventions

Mathematicians like to write algebra as simply as possible, so they have decided that it is okay to leave out the '×' sign between a number and a pronumeral and between different pronumerals. That means we can write $2 \times n$ as $2n$ and $x \times y$ as xy. It has also been agreed that a '÷' sign can be replaced by a fraction bar, so $n \div 2$ is written as $\frac{n}{2}$. Decisions like this are called algebraic conventions. They ensure that all mathematicians use the same mathematical language. Some conventions that we use in algebra are listed below.

- Leave out the '×' between a number and a pronumeral and between different pronumerals (e.g. $2 \times n$ is written as $2n$ and $x \times y$ as xy).
- Replace ÷ with a fraction bar (e.g. $n \div 2$ is written as $\frac{n}{2}$).
- Numbers are always written in front of the pronumeral (e.g. $x \times 2$ is written as $2x$).
- When multiplying more than one pronumeral, write the letters in alphabetical order (e.g. $f \times a \times c \times e$ is written as $acef$).
- Brackets show that addition or subtraction needs to be done before multiplication or division. They change the order of operations.
- Brackets can be left out when using a fraction bar; e.g. $(a + b) \div 2$ can be written as $\frac{a+b}{2}$.
- When a pronumeral is raised to a power, we use the same notation as we do for numbers (e.g. $2 \times 2 \times 2 \times 2 = 2^4$ so we write $c \times c \times c \times c = c^4$ and p squared as p^2).
- Numbers are usually written last in an expression (e.g. $6a + 7$).

When working with variables, the same words are used to describe the four operations as when working with numbers:

+ add, sum, total	− subtract, less than, minus
× multiply, product, lots of	÷ divide, quotient, share equally

The order of operations, including the use of brackets, is also the same.

Worked Example 1

Write the following situations using algebra.

(a) Simon has x fish in his aquarium. He buys 10 more. How many fish does he have now?

(b) There are r cards in one pack. How many cards are there in three identical packs?

(c) Sonia has n dollars in her bank account. She withdraws \$100. How much money is left in the account?

(d) There are d biscuits in a packet. Half of them are eaten. How many are left?

(e) A kitchen cupboard contains x plates and w bowls. What is the total number of plates and bowls in the cupboard?

Thinking	Working
(a) 'More' means add onto the unknown amount (add 10 to x).	(a) Number of fish now $= x + 10$.
(b) Multiplication is used to find 'lots of' the unknown amount (multiply r by 3).	(b) Number of cards in 3 packs $= 3r$.

(c) 'Withdraw' means subtract from the unknown amount (subtract 100 from n).	(c) Amount of money left in Sonia's account $= \$(n - 100)$.
(d) 'Half' means divide the unknown amount by 2 (divide d by 2).	(d) Number of biscuits left $= \frac{d}{2}$.
(e) 'Total' means add all the unknown amounts together (add x and w).	(e) Number of plates and bowls $= x + w$.

Worked Example 2

WE2

Write each of the following using algebra.

(a) The product of e, f and 7.

(b) c is divided by 4, then 1 is subtracted.

(c) The sum of x and 5 is multiplied by 3.

Thinking	Working
(a) 'Product' means multiply. Write the number first, then the pronumerals in alphabetical order with no multiplication signs between them.	(a) $7ef$
(b) Division is done first, then subtraction. Use a fraction bar to show division.	(b) $\frac{c}{4} - 1$
(c) Addition is done first, then multiplication. Brackets are needed to show this order of operations.	(c) $3(x + 5)$

5.1 Pronumerals and variables

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Answers page 648

Fluency

1 Write the following situations using algebra.

WE1

(a) There are p books in Jacob's locker. How many are left after Jacob takes out three for class?

(b) There are k people in the line for concert tickets. Nine more people join the end of the line. How many people are lined up now?

(c) There are v chocolates in one box. How many chocolates are there in 12 identical boxes?

(d) A group of four people won a prize of a dollars, which they shared equally. How much did each person receive?

(e) Kyle has d footy cards in his collection, while Khalid has e cards in his. If they combine their collections, how many cards do they have altogether?

WE 2

2 Write each of the following using algebra.

(a) The sum of x and 2.

(b) The product of 4 and y.

(c) t is subtracted from 5.

(d) The quotient when k is divided by 7.

(e) The product of h, 6 and g.

(f) d is multiplied by 10, then 8 is added.

(g) 8 is added to d, and the result is multiplied by 10.

(h) The sum of y and 9 is divided by 4.

(i) 3 less than 5 lots of e.

(j) The sum of r, s and t is divided by 3, then 11 is added.

(k) y is multiplied by itself, then 20 is added.

(l) x is squared, then multiplied by 8.

3 Nadine has 7 bags with x lollies in each bag, plus 9 extra lollies. The total number of lollies Nadine has altogether can be written using algebra as:

A $x + 7 + 9$ **B** $7x + 9$ **C** $9x + 7$ **D** $x^7 + 9$

4 The sum of z and 6 is divided by 2. When written using algebra it is:

A $z + 3$ **B** $\frac{z}{2} + 6$ **C** $z + 6 \div 2$ **D** $\frac{z + 6}{2}$

Understanding

5 Write each of the following using algebra. Use brackets where necessary.

(a) 1 is added to a number, n, and the result is multiplied by 5.

(b) 2 is added to a number, d, and the result is divided by 9.

(c) The sum of two different numbers, a and b, is multiplied by 12.

(d) 3 is subtracted from the product of two different numbers, m and n.

6 There are x people standing in line at the post office. Half of them leave, thinking they will come back later.

(a) How many people are in the line now?

(b) Three other people walk in and join the line. How many people are standing in the line now?

7 (a) What is the cost of d metres of timber, if it costs \$6 per metre?

(b) What is the cost of t kilograms of oranges, if they cost \$2.50 per kilogram?

8 What is the cost of hiring the community hall for x hours, if the rate is \$55 per hour, plus a one-off charge of \$300?

9 What is the cost of buying b movie tickets, at v dollars per ticket?

10 The cost of 1 desk is t dollars, and the cost of 1 chair is s dollars. 12 new desks and 24 new chairs are bought for a classroom.

(a) What is the total cost of the desks?

(b) What is the total cost of the chairs?

(c) What is the total cost of the furniture?

Reasoning

11 A purse contains n 5-cent coins and r 10-cent coins. There is no other money in it. Use algebra to write:

(a) the total value in cents of the 5-cent coins in the purse

(b) the total value in cents of the 10-cent coins in the purse

(c) the total value in cents of the money in the purse.

12 Emma makes and sells stationery gift packs. Her small pack has m pens, n pencils, q note pads and 1 eraser.

Her large pack has twice as many of each item. Use algebra to write:

(a) the total number of items in the small pack

(b) the total number of items in the large pack.

(c) Emma needs to make up 3 small and 2 large packs. Use algebra to write the number of pens that she will need altogether.

13 **(a)** The length of a rectangle is 5 cm longer than its width. If w represents the width of the rectangle:

(i) write the length of the rectangle using w as the variable

(ii) draw a rectangle and label each side using w as the variable.

(b) Write, in terms of w, the total distance around the rectangle (the perimeter).

Open-ended

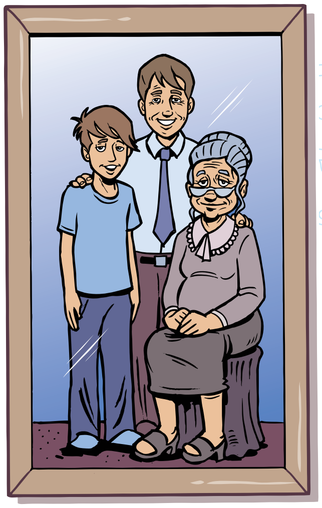
14 **(a)** Draw a diagram that could be represented by $3x + 5$. (Hint: Look at the 'lolly bag' diagrams at the start of this section.)

(b) What kind of diagram could you draw to show $\frac{x}{4}$? Draw a diagram, adding any necessary descriptions or labels.

15 Rory's grandma says she is twice as old as the combined ages of Rory and his dad. Rory is r years old and his dad is d years old.

(a) Write Grandma's age using algebra.

(b) Write some possible ages for Rory, his dad and his grandma, if Rory is in primary school.



5.2 Terms, expressions and equations

The language of algebra

Here are some more mathematical words that you need to know when using algebra.

Constants

A number by itself is called a **constant**.

Here are some examples of constants: 2, 9, -37, $\frac{2}{7}$, 4.6

Terms

Terms have one or more pronumerals, or may be just a number. mn means $m \times n$ and is the same as $n \times m$ or nm.The pronumerals are often multiplied by a number that is written first.

Here are some examples of terms: $5a$, $7q$, $3pr$, z, abc, 4, $\frac{3}{11}$, $-2.32x$

Coefficients

The number written in front of a pronumeral is called the **coefficient**. The sign of the number is included. For example, in $9x$, the coefficient of x is 9. In $9x - 3y$, the coefficient of y is -3.

Expressions

Expressions are made by adding terms. The expression $5a + 7q - 12$ has three terms.

Expressions can also consist of just one term, so $5a$ or 4 could each be called an expression.

In section 5.1, we wrote $x + 3$, $2x$ and $2x + 3$. These are called algebraic expressions. Here are some more examples of expressions: $5a + 7q$, $5a + 7q - 12$, $9xy - z$, $2(p - 7)$.

Equations

Equations contain an equals sign. They are made by writing two expressions equal to each other.

Equations have a left-hand side (LHS) and a right-hand side (RHS) on either side of the equals sign.

Here are some examples of equations: $9xy + z = 17$, $y = 6h$, $2(p - 7) = p + 10$

$3xy - 7z^2 + 5 = 16 + x$

- This is an equation because it has an equals sign.
- x, y and z are pronumerals that represent variables.
- 3 is the coefficient of xy and -7 is the coefficient of z^2.
- $3xy - 7z^2 + 5$ and $16 + x$ are expressions.
- $3xy$, $-7z^2$, 5, 16 and x are all terms.
- 5 and 16 are constants.

Representing situations using algebra

When we use algebra to describe practical situations, it is very important to **define** all the variables we are going to use so we know what they represent. For example, we may not know the cost per kilogram of apples and bananas, so they are the variables we need to define. We do know that we purchased 3 kg of apples and 2 kg of bananas and spent \$15.20. If we define the cost of apples/kg as x (\$) and the cost of bananas/kg as y (\$), then we can write:

- $3x$ as the cost of apples (\$)
- $2y$ as the cost of bananas (\$)
- $3x + 2y = 15.2$ as the equation
- $(3x + 2y)$ and 15.2 as the total cost of the fruit (\$).

In this situation, we have examples of:

a constant:	15.2	expressions:	$3x + 2y$, $3x$, $2y$, 15.2
pronumerals:	x, y	an equation:	$3x + 2y = 15.2$
terms:	$3x$, $2y$, 15.2		

Worked Example 3

For $3p + 4q - 8pq = 7 + 2r$

(a) state whether it is an equation or an expression

(b) identify the coefficient of pq

(c) list all the variables

(d) write down any constants

(e) list all the terms used.

Thinking	Working
(a) Is there an equals sign? If yes, then it is an equation.	(a) It is an equation.
(b) Look for the number in front of pq. Include the sign.	(b) Coefficient of pq is -8.
(c) Look for all the different letters used and list them. (Note that pq is a product of p and q.)	(c) p, q and r are the variables.
(d) Look for a number by itself.	(d) The constant is 7.
(e) Look for all the parts of the equation separated by addition or subtraction.	(e) $3p$, $4q$, $-8pq$, 7 and $2r$ are the terms.

Worked Example 4

For the following situation:

(a) identify the variables

(b) define each variable using the given pronumerals

(c) write an equation.

A farmer has a number of sheep, x, and a number of ducks, y, in a paddock. There are five times as many sheep as ducks.

Thinking	Working
(a) Identify the variables.	(a) The variables are the number of sheep and the number of ducks.
(b) Define the variables.	(b) Let $x =$ the number of sheep. Let $y =$ the number of ducks.
(c) Write the equation. (Multiply y by 5 to get x.)	(c) $x = 5y$

- An algebraic expression may have one term or it may be made up of two or more terms that are added or subtracted.
- An equation uses an equals sign to show that one expression has the same value as another expression.

5.2 Terms, expressions and equations

Navigator

Answers page 648

Q1, Q2, Q3, Q4 (a)–(d), Q5 (a) & (b), Q7	Q1, Q2, Q3, Q4 (b)–(e), Q5 (a)–(c), Q6, Q7	Q1, Q2, Q3, Q4 (c)–(f), Q5, Q6, Q7, Q8

Fluency

WE3

1 For each of the following:

  (i) state whether it is an equation or an expression

  (ii) identify the coefficient of x

  (iii) list all the variables

  (iv) write down any constants

  (v) list all the terms used.

(a) $2x + 3y - 7 + 4x^2$ **(b)** $32 - 4x = 3xy + 4z$

(c) $2xy - 3x^2 = 9 - 23x$ **(d)** $6 - x + 8y - 4xy$

WE4

2 For each of the following situations:

  (i) identify the variables

  (ii) define each variable using the given pronumerals

  (iii) write an equation.

(a) The number of boys, x, in the class is three times the number of girls, y.

(b) Fred is g years old. This is one less than three times James's age. James is h years old.

(c) At the market, an apple costs \$$m$ and a pear costs \$$n$. Two apples cost as much as three pears.

(d) A kilogram of potatoes costs \$$a$ and a watermelon costs \$$b$. Three kilograms of potatoes and a watermelon cost \$5.20.

(e) The distance, r, to Ranko's house is 5 km more than the distance, s, to Skye's house.

(f) You have v plants and w pavers. To make a path around a row of plants, you need to have twice as many pavers as you have plants plus 6 extra pavers.

3 Answer TRUE or FALSE for each of these statements.

(a) $6y$ is a term
(b) $7y - 9$ is an equation
(c) $y = 7x$ is a term
(d) ab is a term
(e) $r = 5t - 9$ is an expression
(f) $cd + 4$ is an expression
(g) $w = s - 5b$ is an equation
(h) $6g$ is an equation
(i) $rs = sr$
(j) $7 - 6xzy = 7 - 6yxz$

4 Which expression below matches the instruction given in each case?

(a) Choose a number and add any other number to it.

A $6 + a$ B $u + v$ C $c + 9$ D $a + b + 1$

(b) Choose a number and multiply it by any other number.

A $a - b$ B $7a$ C $4ab$ D ab

(c) Choose a number and multiply it by three, then subtract any other number.

A $3a - 1$ B $3n - y$ C $3m - 3$ D $3t$

(d) Choose a number and multiply it by two, add any other number, then subtract thirteen.

A $2f + 2d - 13$ B $2mn - 13$ C $2w + g - 13$ D $a + b - 13$

(e) Choose a number and add it to any other number, then multiply the answer by nine.

A $9xy$ B $9x + y$ C $9(x + y)$ D $9 + xy$

(f) Choose a number and multiply it by ten, choose any other number and multiply it by four, then add the two answers together.

A $w + v + 10 + 4$ B $14(v + w)$ C $10v + 4w$ D $40vw$

Understanding

5 (a) A bottle of mass 120 g contains 25 tablets. The total mass of the bottle and tablets is 195 g.

(i) If the variable is the mass of a tablet, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find the mass of a tablet.

(b) Half Mei Lee's age is the age she was 8 years ago.

(i) If the variable is Mei Lee's age, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find Mei Lee's age.

(c) In 10 years' time, Beth will be half her dad's age now.

(i) If one variable is Beth's age and the other other variable is her dad's age, use two pronumerals to represent these variables.

(ii) Write an equation you would use to find Beth's age now.

(d) In a ball game, a goal is worth 8 points and a penalty goal is worth 1 point. At a match, one team scored 100 points.

(i) Define the variables and represent them with pronumerals.

(ii) Using these pronumerals, write an equation you would use to show the team's score.

Reasoning

6 A family on holiday travelled a certain distance on the first day. The next day they were able to travel twice as far as on the first day. On the third day, they travelled 480 km further than on the first day.

(a) Define the variable.

(b) Write an expression to show the distance travelled on the second day.

(c) Write an expression to show the distance travelled on the third day.

(d) If the distance travelled on the third day was three times the distance they travelled on the first day, write an equation to show this situation.

Open-ended

7 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. A team has scored 114 points.

(a) Define any variables you need to use.

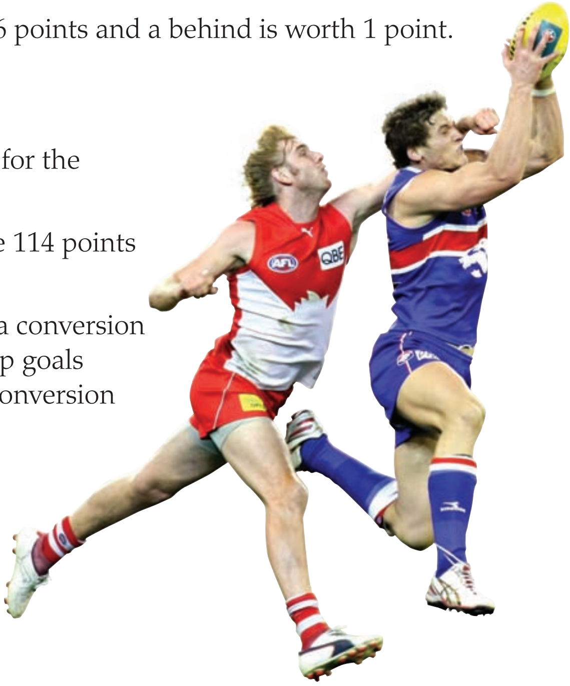
(b) Using these variables, write an equation for the number of points scored.

(c) Find at least three different ways that the 114 points could have been scored.

8 In Rugby Union, a try is worth 5 points and a conversion goal is worth 2 points. Penalty goals and drop goals are each worth 3 points. To be able to get a conversion goal you must first score a try.

(a) If the Waratahs win a match against the Brumbies by 23 points to 12, write down two different ways these points could have been scored.

(b) If P is the total number of points scored, t is the number of tries, g is the number of 2-point goals and p is the number of 3-point goals, write an equation to find the total number of points scored.



Outside the Square Puzzle

We all scream for ice-cream

Summer had arrived and four industrious people decided to sell ice-creams at their local beach.

In total, there were four flavours and each person had ice-cream in 2 different flavours.

- Mr Chocolate had vanilla.
- Mr Vanilla did not have mint.
- Mr Mint had strawberry but not chocolate, whereas Mr Strawberry did not have vanilla.
- One person with mint also had chocolate.
- One person with vanilla also had strawberry.
- One of the persons with chocolate had no mint.
- Neither of the persons with vanilla had chocolate.
- No person had two ice-creams of the same flavour, no two persons had the same two flavours and none had the same flavour as their name.

Can you tell who had which flavoured ice-cream?

Using rules

5.3

Relationships

One of the main uses of algebra is to describe a **relationship** between two or more variables. A relationship means that the variables are connected in some way so that changing the value of one affects the value of the other.

Consider the following examples. Identify the two variables (values that can change) and decide whether a relationship exists between them. Does changing the value of one cause the other to change?

The number of blocks of chocolate bought, and the total cost of the chocolate.

The number of houses being built, and the number of bricks the builder will require.

The number of students sharing a packet of lollies, and the number of lollies each one receives.

The number of animals in a zoo enclosure, and the amount of space each one has to move around in.

Rules and flowcharts

Relationships are described by a rule or set of instructions that tells you how to calculate one variable if you know the other.

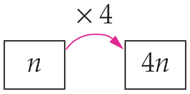
For example, if one block of chocolate costs $4, the rule for calculating the cost of several blocks of chocolate would be:

'Multiply the number of blocks of chocolate you are buying by 4'.

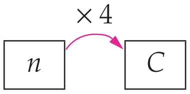
Flowcharts are step-by-step instructions for performing a task. They are used in industry to show the sequence of steps in building a product, such as a car on an assembly production line. Flowcharts are also used to write computer programs.

We can use a flowchart to represent the cost of the chocolate.

There are different ways of writing flowcharts, but, in mathematics, we usually write flowcharts this way:



or



Reading from left to right, this flowchart tells us: 'Take the number of blocks (n) and multiply them by 4 to get the total cost (C)'.

Rules using algebra

Rules can be written in words, but it is much quicker to use algebra. Written in algebra, a rule for the cost of the chocolate would look like this:

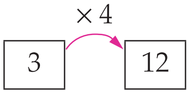
$C = 4n$

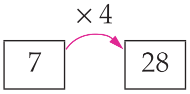
where C = total cost of the chocolate
and n = the number of blocks of chocolate bought at \$4 a block.

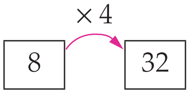
We can use a **table of values** to show the cost of buying different numbers of blocks of chocolate.

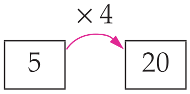
Number of blocks, n	3	7	8	5	0
Cost, C	12	28	32	20	0

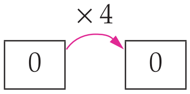
Each of the numbers in the top row of the table has gone through the flowchart (been multiplied by 4) to get the number in the bottom row.







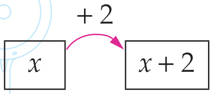




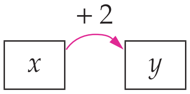
Creating rules

If we choose any two pronumerals, such as x and y, we can create different rules that connect them by constructing different flowcharts.

For example, the rule 'y is equal to x plus 2' would have the flowchart:



or



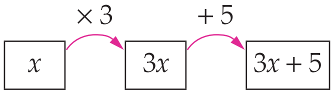
Written in algebra, the rule is: $y = x + 2$.

A 'table of values' for the rule could look like this:

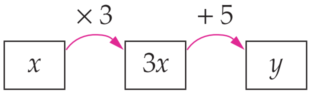
x	4	10	6	9	2
y	6	12	8	11	4

The numbers in the bottom row of the table (y-values) are the result of adding 2 to each of the numbers in the top row of the table (x-values).

Some rules are two-step rules; for example, the rule 'to get y, multiply x by 3, then add 5' would have a flowchart that looks like this:



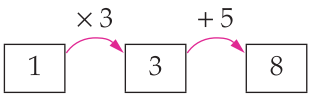
or



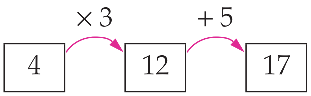
Written in algebra, the rule is: $y = 3x + 5$.

A table of values for this rule could look like this:

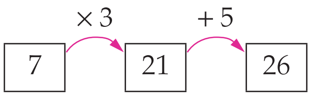
x	1	4	7	2
y	8	17	26	11



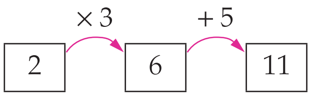
or $3 \times 1 + 5 = 8$



or $3 \times 4 + 5 = 17$



or $3 \times 7 + 5 = 26$



or $3 \times 2 + 5 = 11$

Worked Example 5

For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

(a) y is equal to x divided by 4.

x	20	16	4	12	10
y					

(b) 2 is added to the product of x and 5 to get y.

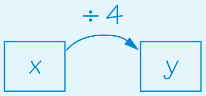
x	2	5	4	3	0
y					

Thinking

Working

(a) **1** Decide how many operations are performed on x, what operation(s) they are and in which order (one operation, ÷ 4). Draw the flowchart.

(a)



2 Write y on one side of the rule. On the other side, show the operations being performed on x ($\frac{x}{4}$).

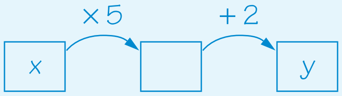
$y = \frac{x}{4}$

3 For each value of x in the table, apply the rule by passing it through the flowchart (divide it by 4).

x	20	16	4	12	10
y	5	4	1	3	2.5

(b) **1** Decide how many operations are performed on x, what operation(s) they are and in which order (× 5, then + 2). Draw the flowchart.

(b)



2 Write y on one side of the rule. On the other side, show the operations being performed on x ($5x + 2$).

$y = 5x + 2$

3 For each value of x in the table, apply the rule by passing it through the flowchart (multiply by 5, then add 2).

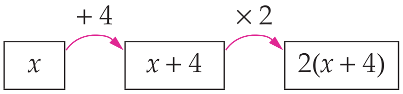
x	2	5	4	3	0
y	12	27	22	17	2

Using brackets

Brackets may be needed to show the correct order of operations.

For example:

If the rule is 'y is equal to twice the sum of x and 4', the flowchart would look like this:



The rule written in algebra would be $y = 2(x + 4)$.

Brackets are needed to show that the addition is to be done before the multiplication.

5.3 Using rules

Navigator

Answers page 649

Q1 Column 1, Q2, Q3, Q4, Q5 (a) & (b), Q6, Q8, Q10, Q12	Q1 Column 1, Q2, Q3, Q4, Q5 (c) & (d), Q6, Q7, Q8, Q10, Q13, Q14	Q1 Column 2, Q2, Q3, Q4, Q5 (e) & (f), Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency

WE5

1 For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

(a) y is equal to x plus 2.

x	13	11	7	28	1
y					

(b) To find y, subtract 5 from x.

x	6	18	9	85	5
y					

(c) To find y, double x.

x	2	3.4	10	11	101
y					

(d) y is equal to x divided by 3.

x	18	12	30	9	0
y					

(e) y is equal to multiplying x by 5, then subtracting 3.

x	4	2	0	5	20
y					

(f) To find y, add 5 to x, then divide by 10.

x	5	25	45	10	33
y					

(g) To find y, divide x by 2, then subtract 1.

x	10	6	24	9	15
y					

(h) y is equal to x multiplied by itself.

x	3	11	7	6	10
y					

2 Choose the correct algebraic rule given in each case.

(a) y is equal to the sum of x and twelve.

A $y = x + 12$ **B** $y + 12 = x$ **C** $y = 12x$ **D** $y = \frac{x}{12}$

(b) To get y, subtract 50 from x.

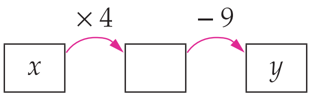
A $y = x - 50$ B $y = 50 - x$ C $y = x \times 50$ D $y = x + 50$

(c) To get y, add thirteen to x, then multiply by nine.

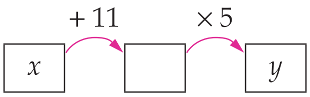
A $y = (x + 9) \times 13$ B $y = (x \times 13) + 9$ C $y = x + 13 \times 9$ D $y = 9(x + 13)$

3 Write the rule shown by each of these flowcharts. Make sure you use brackets where necessary.

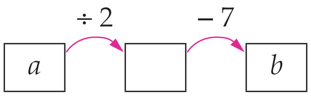
(a)



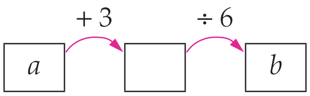
(b)



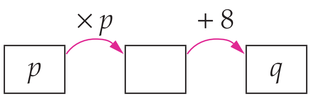
(c)



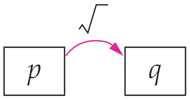
(d)



(e)



(f)



4 Rewrite each of these rules using algebra.

(a) To find y, subtract eighteen from x.

(b) y is equal to sixty multiplied by x.

(c) Divide x by seven to find y.

(d) To find y, add forty-three to x, then multiply by twenty.

(e) Multiply x by one hundred, then subtract fifty to find y.

(f) To find y, divide x by sixteen, then add thirteen.

(g) y is equal to x multiplied by itself.

(h) Subtract twelve from x, then divide by nine to find y.

(i) Multiply x by itself, then take away thirty-seven to find y.

Understanding

5 For the following rules, draw a flowchart and describe the rule using words.

(a) $y = 5x + 11$

(b) $y = \frac{x}{7} - 5$

(c) $b = 7(a + 6)$

(d) $b = \frac{a - 7}{12}$

(e) $d = \frac{e + 9}{4} - 13$

(the flowchart for this rule should have three steps)

(f) $d = \frac{e^2 - 1}{5}$

(the flowchart for this rule should have three steps)

6 Joe has some lollies. He gives half to his little brother and then eats four of the rest.

(a) Draw a flowchart to show how many lollies he has left. Use x for the number of lollies he starts with and y for the final number.

(b) Write the rule to describe the situation.

(c) Use the rule to find how many lollies Joe has now if he had 20 to start with.

7 Anita has some money in her wallet. Her mum gives her \$20 for doing some chores and she decides to spend half of the money she now has on a birthday present for her sister. She spends another \$4 on a birthday card.

- **(a)** Draw a flowchart you would use to find how much money she has spent. Use x for the initial amount in Anita's wallet and y for the amount she has spent.
- **(b)** Write a rule to describe the situation.
- **(c)** Use the rule to find how much money Anita spent if she had \$12 in her wallet initially.

8 Your Uncle Harry is twice your age, plus 5.

- **(a)** Show this information in a flowchart. Use m to represent your age, and H to represent Uncle Harry's age.
- **(b)** Write a rule to show this situation.
- **(c)** If you are 12 years old, how old is Uncle Harry?

9 To produce jeans it costs \$245 to set up the machine, and then \$7 for each pair of jeans. So, for one pair of jeans to be produced, it costs $\$245 + \$7 \times 1 = \$252$.

- **(a)** Using this information, fill in the table.

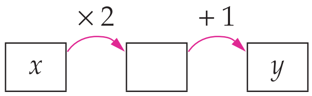
Number of pairs of jeans, n	10	50	150	200
Cost to produce the jeans, C				

- **(b)** Write a rule for the cost of producing the jeans. Use C for cost, and n for the number of pairs of jeans.

Reasoning

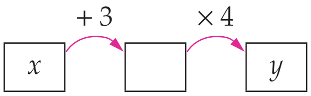
10 The following tables of values have only the y-values filled in. Work *backwards* along the flowchart to determine the values of x that were used, and complete the tables of values.

(a)



x					
y	5	11	15	9	21

(b)



x					
y	52	16	20	28	84

To work backwards along the flowchart, do the *opposite* operation to the one shown in the forwards direction.

11 For each of the following two rules:

- **(i)** y is equal to the sum of x and 3, which is then multiplied by 2
- **(ii)** to find y, multiply x by 2, then add 6

- **(a)** draw a flowchart.
- **(b)** Write the rule using algebra.
- **(c)** Copy and complete the table of values below.

x	2	5	3	4	7
y					

- **(d)** What do you notice about these two rules? Can you explain your observation?

Open-ended

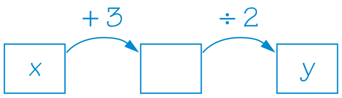
12 Construct your own table of values and complete it for the rule $y = \frac{x}{2} + 3$. Use at least five different values.

13 (a) Using two pronumerals (such as x and y), at least two of the following operations (+, −, ×, ÷) and some arrows, create at least three different flowcharts that show a rule connecting x to y. Use a different combination of operations for each flowchart.

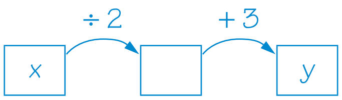
(b) Write the rules shown by your flowcharts using algebra.

14 Kim and Jai are working on the following question: 'Draw a flowchart that represents this rule: to obtain y, divide x by 2, then add 3'.

Kim's flowchart looks like this:



Jai's flowchart looks like this:



'Hey', said Jai. 'Yours looks different to mine. 'I must be wrong.'

'No, mate', said Kim. 'It's just a different way of writing the same thing.'

Is Kim's statement correct? Are they both right? If not, who has the correct flowchart, and why?

Outside the Square

Problem solving

Cutting string

Equipment required: 1 brain, 2 lengths of string, scissors

1 Imagine that you have a piece of string. How many pieces will you have if you cut the string once? If you cut one of the pieces again, how many pieces will you now have? Check your answer using some string and scissors, then complete the following table.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

2 This time, fold a piece of string in half each time before you cut it. How many pieces at each cut do you have now? *Remember*: Cut only one piece at each time.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

Strategy options

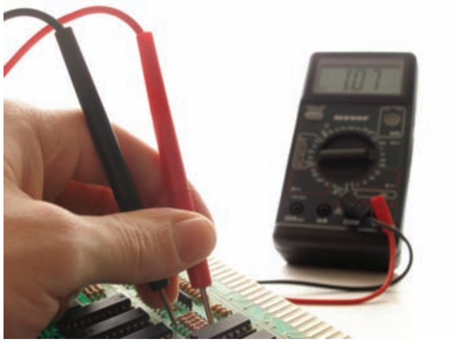
- Act it out.
- Look for a pattern.

5.4

Formulas and substitution

A **formula** is a mathematical rule that uses two or more variables. A formula is used to calculate the value of one variable when the value of the other variables are known. The plural of formula is formulas or formulae. The rules we have been working with so far can all be called formulas; however, we usually think of a formula as a rule we use in a practical situation. Here are some examples of practical formulas, and what they are used for:

$V = IR$



The voltage (V) of an electric circuit is found by multiplying the current (I) by the resistance (R).

$F = \frac{9C}{5} + 32$

To convert a temperature from Celsius to Fahrenheit, multiply the temperature in Celsius (C) by 9, divide by 5, and then add 32.

Substitution

In mathematics, we can **substitute** a number for a variable. This allows us to find a value for, or **evaluate**, the formula or expression.

Many team sports use substitution. How is it similar to the substitution we're doing here?

Worked Example 6

For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = 10x - 5, \quad x = 7$

(b) $y = \frac{x}{2} + 4, \quad x = 5$

Thinking	Working
(a) 1 Write the formula.	(a) $y = 10x - 5$
2 Replace the variable (x) in the formula with its given value (7). Insert the × sign between the coefficient and the number. State the value you will substitute for x.	$y = 10 \times 7 - 5, x = 7$
3 Calculate the value of the other variable (y).	$y = 70 - 5$ $= 65$
4 State the answer.	$y = 65, x = 7.$

(b)	**1** Write the formula.	(b) $y = \frac{x}{2} + 4$
	2 Replace the variable (x) in the formula with its given value (5).	$y = \frac{5}{2} + 4,\ t = 5$
	3 Calculate the value of the other variable (y).	$= 2\frac{1}{2} + 4$ $= 6\frac{1}{2}$ or 6.5
	4 State the answer.	$y = 6.5,\ x = 5$

When substituting numbers into a formula, write in any 'hidden' multiplication signs.

e.g. $b = 4a + 3$
$= 4 \times 5 + 3,\ a = 5$

Worked Example 7

(a) The cost of hiring a community hall is calculated using the formula $C = 65t + 200$, where C = cost (in dollars) and t = the hire period (in hours). Find the cost of hiring the hall for 4 hours.

(b) The area of a rectangle (in square metres) is given by the formula $A = lw$ where l = length (in metres) and w = width (in metres). Calculate the area of a rectangular room that is 4.5 m long and 3 m wide.

	Thinking	Working
(a)	**1** Write the formula.	(a) $C = 65t + 200$
	2 Replace the variable in the formula with its given value (replace t with 4, the number of hours). Insert a multiplication sign between the coefficients and the numbers (65 and 4).	$C = 65 \times 4 + 200,\ t = 4$
	3 Evaluate the formula by performing the multiplication and the addition.	$= 260 + 200$ $= 460$
	4 State the answer.	The cost of hiring the hall for 4 hours is $460.
(b)	**1** Write the formula.	(b) $A = lw$
	2 Replace the variables in the formula with their given values. (Replace l with 4.5 and w with 3.) Insert a multiplication sign between the numbers (4.5 and 3).	$A = 4.5 \times 3,\ l = 4.5,\ w = 3$
	3 Evaluate the formula by performing the multiplication.	$= 13.5$
	4 State the answer.	The area is 13.5 square metres.

5.4 Formulas and substitution

Navigator

Answers page 651

Q1 Column 1, Q2, Q3, Q4 Column 1, Q5 (a) & (b), Q6, Q7, Q8, Q10, Q12	Q1 Column 2, Q2, Q3, Q4 Column 1, Q5, Q6, Q7, Q9, Q10, Q11, Q12	Q1 Column 2, Q2, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE6

1 For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = x + 3,\quad x = 4$

(b) $y = x + 5,\quad x = \frac{1}{2}$

(c) $y = 2x - 7,\quad x = 3$

(d) $y = 3x - 1,\quad x = 8$

(e) $y = x - 1,\quad x = 3.6$

(f) $y = x - 5,\quad x = 30$

(g) $y = 25 - 3x,\quad x = 7$

(h) $y = 30 - 2x,\quad x = 1$

(i) $y = 4x,\quad x = \frac{1}{2}$

(j) $y = 8x,\quad x = 4$

(k) $y = 3(x + 5),\quad x = 1$

(l) $y = 6(x + 2),\quad x = 18$

(m) $y = \frac{x}{5},\quad x = 20$

(n) $y = \frac{x}{5},\quad x = 2.5$

(o) $y = 11(10 - x),\quad x = 4$

(p) $y = 12(10 - x),\quad x = 3$

WE7

2 **(a)** The cost of hiring power tools from the local hardware store is calculated using the formula $C = 35t + 60$, where C = cost (in dollars), and t = the hire period (in hours). Find the cost of hiring a sander for 6 hours.

(b) Body mass index (B) is one way of measuring a person's health, and is calculated by the formula $B = \frac{m}{h^2}$, where m is the person's mass in kilograms, and h is their height in metres. Calculate the body mass index of a runner who is 1.7 metres tall and weighs 64 kg. (Round your answer to two decimal places.)

(c) The average speed of a moving object (such as a car) can be calculated by the formula $s = \frac{d}{t}$, where s = speed (in kilometres per hour), d = distance (in kilometres) and t = time (in hours). Yoshi drives 210 km from Hobart to Launceston in 3 hours. Calculate her average speed.

3 Answer TRUE or FALSE for each of the following statements.

(a) If we substitute $a = 4$ into $b = 5a$ we get $b = 20$.

(b) If we substitute $a = 9$ into $b = a + 11$ we get $b = 20$.

(c) If we substitute $u = 4$ into $v = 6u + 1$ we get $v = 65$.

(d) If we substitute $q = 10$ into $k = 13q - 8$ we get $k = 122$.

(e) If we substitute $x = 6$ into $y = 4(x - 5)$ we get $y = 19$.

(f) If we substitute $x = 12$ into $y = 5(14 - x)$ we get $y = 10$.

4 Use each of the following rules to complete these tables of values.

(a) $b = 4a$

a	11	20	5	9	50
b					

(b) $y = 7x$

x	6	4	10	20	101
y					

(c) $n = 3m + 2$

m	1	2	10	6	5
n					

(d) $k = 4j + 7$

j	2	5	11	10	100
k					

(e) $q = 2p - 10$

p	11	15	10	20	100
q					

(f) $s = 8r - 2$

r	1	2	3	0	200
s					

(g) $v = 4(u - 1)$

u	5	3	1	201	6
v					

(h) $n = 3(m - 2)$

m	5	10	11	102	52
n					

5 **(a)** For the rule $m = 3x - 5$, when $x = 3$, m would be equal to:

A 1 **B** 2 **C** 3 **D** 4

(b) For the rule $y = \frac{x}{3} + 4$, when $x = 6$, y would be equal to:

A $\frac{10}{3}$ **B** $4\frac{1}{2}$ **C** 6 **D** 22

(c) For the rule $l = 3(n + 2) - 1$, when $n = 1$, l would be equal to:

A 5 **B** 6 **C** 8 **D** 9

Understanding

6 Sam's new fuel-efficient car uses 7 litres of petrol for every 100 km it travels.

(a) Write this as a formula to calculate the amount of petrol (p) that is needed to travel a distance (d) kilometres.

His father's 4WD uses 11 litres of petrol for every 100 km it travels.

(b) Write this as a formula to calculate the amount of petrol (f) that is needed to travel a distance of (d) kilometres.

(c) Use your formulas to calculate the number of litres each car uses to travel a distance of 400 km.

(d) Write a formula to calculate the cost (C), in dollars, of y litres of petrol if petrol costs $1.30/litre.

(e) Calculate how much more Sam's father spent on petrol than Sam to travel 400 km.

7 Jabbok, a mobile phone salesman gets \$350 per week and \$10 for every phone that he sells.

- **(a)** Write this as a formula with n representing the number of phones sold, and p representing the amount of money earned each week.
- **(b)** If 15 phones are sold in the first week, use your formula to find how much Jabbok earns this week.
- **(c)** If 20 phones are sold the next week, use your formula to find how much money Jabbok earns that week.

Reasoning

8 I buy three loaves of bread and a \$2.60 carton of milk. It costs me \$$D$.

- **(a)** If l represents the cost of a loaf of bread, write a formula to find the cost of my shopping (D) in terms of l.
- **(b)** Use the formula to evaluate D if $l = \$3.25$.

9 Jasmine is a caterer. She uses formulas to work out how much food she needs to feed different numbers of people.

- **(a)** To calculate the number of sausages she needs at a barbeque, Jasmine uses the following formula: 'Allow 2 sausages for every person and have an extra 10 sausages'. Write Jasmine's formula using algebra using n to represent the number of people and s to represent the number of sausages.
- **(b)** Another one of Jasmine's formulas for barbeques is: 'One bowl of salad will feed 8 people'. Write this formula using algebra. Use n to represent the number of people, and b for the number of bowls of salad.
- **(c)** Use the formulas you have written to determine the number of sausages and bowls of salad Jasmine will need to cater for 40 people.

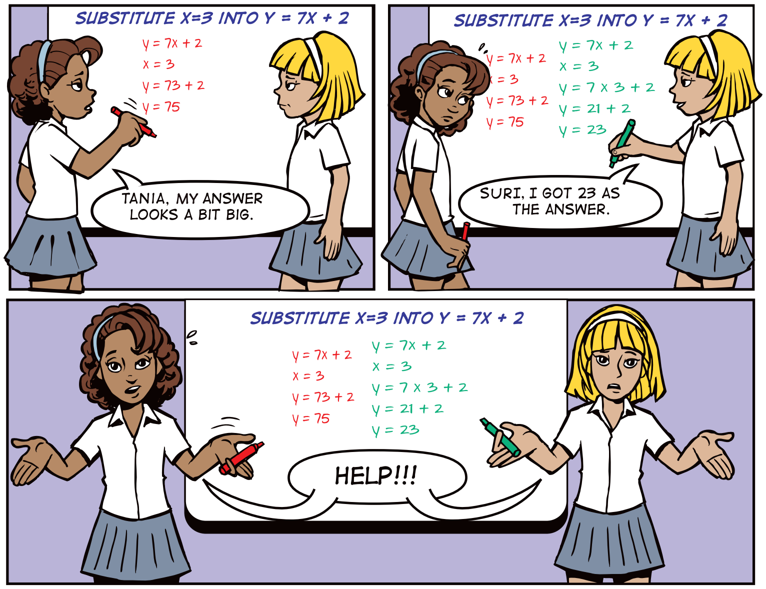
10 Show that the formulas $b = 3(a + 5)$ and $b = 3a + 5$ are different by substituting $a = 4$ into each of them. Explain how the two are different in terms of the order of operations.

Open-ended

11 Make up two different formulas connecting x and y that use addition and multiplication. Use them to complete the following table for each one.

x	7	20	13	101
y				

12



Which of the two students has substituted correctly? What mistake has the other student made? Give them some advice so they can avoid a similar mistake in the future.

Outside the Square Game

Dicey formulas

Equipment required: 2 brains, 1 die

How to win:
Be the first to 60.

How to play:

1 Each person is given a formula.

2 You roll the die in turn. After rolling the die the first time, you must substitute the value shown on the die into your formula and write down the answer.

3 On each later throw, calculate the value from the formula as before, and add this to your previous total. Your opponent checks your answer and if it is wrong you lose a turn.

4 Your game ends when you think that you can get no closer to 60.

5 The other player may keep going until they decide they can get no closer to 60.

6 The winner is the person who gets to exactly 60, or, failing that, the person with the total nearest to, but less than, 60. If you score more than 60 you have lost, so you must decide when to stop.

Three sets of formulas are given:

Game 1:
Person 1, $A = 3n + 2$;
Person 2, $B = 3n - 2$

Game 2:
Person 1, $A = 3n + 4$;
Person 2, $B = 3(n + 2)$

Game 3:
Person 1, $A = 6 - n$;
Person 2, $B = 7 - n$

As a challenge, play this game where you have to get exactly 60. You are able to not include results if you wish. Here, you need to make sure that the last number you need to total 60 is possible to get using your formula when $n = 1$, 2, 3, 4, 5, or 6.

(Use 2 dice and be the first to 150 or more.)

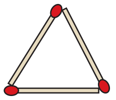
5.5 Patterns and rules

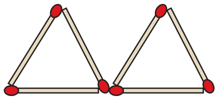
A strategy we often use in problem solving is to look for a pattern in our results. Algebra can help us describe a pattern, which can often be written as a general rule, or formula. We can then use the formula to solve problems without having to draw out endless patterns.

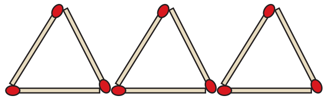
Worked Example 8

WE 8

(a) Here is a matchstick pattern of triangles. Copy and complete this table of values by continuing the pattern.







Number of triangles (t)	1	2	3	4	5
Number of matches (m)					

(b) Find a general rule that connects the number of triangles in the pattern (t) to the number of matches used (m). Write the rule in words and in algebra.

(c) Use your rule to find the number of matches required to make 100 triangles.

Thinking

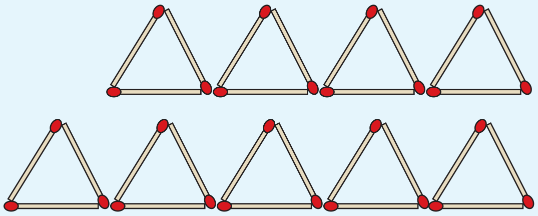
Working

(a) **1** Construct the table of values. Fill in any information you know from the pattern provided.

(a)

Number of triangles (t)	1	2	3	4	5
Number of matches (m)	**3**	**6**	**9**		

2 Continue the pattern by adding matches to create more triangles.



3 Count the number of matches used for the continued pattern (formed with four and five triangles) and fill in the table.

Number of triangles (t)	1	2	3	4	5
Number of matches (m)	3	6	9	**12**	**15**

(b)	**1**	Identify the number of matches that are being added on each time to make a new shape (3). (This number is also being added on to each number in the second row of the table.) This is the multiplication factor in the rule (× 3).	(b) $m = 3 \times t$
	2	Write the rule that links the two variables (t and m) in the table, in words and in algebra.	The number of matches is 3 times the number of triangles. $m = 3t$
(c)	**1**	Substitute $t = 100$ into the rule, and evaluate.	(c) $t = 100$ $m = 3 \times 100$ $m = 300$
	2	Write the answer in words.	300 matches are needed to make 100 triangles.

In the above Worked Example, we see that one 'lot' of 3 matches is added every time to make a new triangle and the next number in the sequence. Therefore, we are multiplying the number of triangles by 3 to find the number of matches.

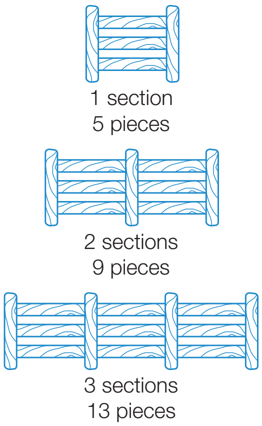
When trying to find a pattern, look for the number that is being added or subtracted every time. This will tell us what to multiply by in our rule.

In some cases, the above procedure may not give us the complete rule. We must then look for the number that is required to complete the first pattern. We then need to add on this number to make our rule work; e.g. $m = 3t + 1$; $m = 3t + 2$; $m = 3t + 3$ etc. Worked Example 9 shows this in detail.

Always try to understand how the rule works by looking at the pattern that has been formed.

Worked Example 9

Frederico is building a fence around Farside Farm. The fences are made up of pieces of timber as shown.



(a) Copy and complete this table of values by continuing the pattern.

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9			

(b) Find a general rule that connects the number of sections of fence (F) to the number of pieces of timber (P) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of pieces of timber required for a fence made up of 50 sections.

Thinking | **Working**

(a) 1 Construct the table of values. Fill in any values you know from the pattern provided. Continue the pattern to complete the table.

(a)

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9	13	17	21

(b) 1 Identify the number of pieces of the pattern that are needed to complete the new section (4). (This number is also being added on to each number in the second row of the table.) This number is the multiplication factor in the rule (× 4).

(b) 4 pieces are being added to make a new section. Therefore, $4F$ is in our rule.

2 When we multiply F by 4, we do not have the numbers in the second row, so we now look for a number to add or subtract. How many pieces of the pattern were needed to start the first section? This number is added on to the rule. (1 piece of timber.)

$4F + 1$

3 Write the rule that links the variables, both in words and in algebra.

The number of pieces is 4 times the number of sections plus 1 extra piece.

$P = 4F + 1$

(c) 1 Substitute the value of F into the formula ($F = 50$).

(c) $P = 4F + 1$

$= 4 \times 50 + 1$

2 Evaluate.

$P = 200 + 1$

$= 201$

3 Write the answer in words.

To make a fence of 50 sections, 201 pieces of timber are needed.

5.5 Patterns and rules

Navigator

Answers page 651

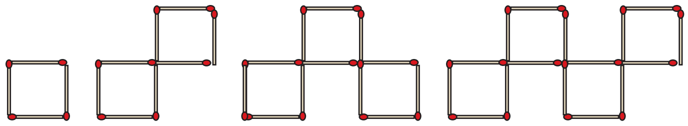
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q12	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

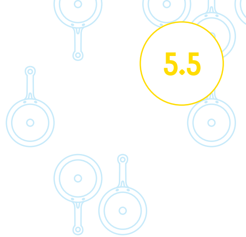
Equipment required: Centimetre grid paper for Question **12**

Fluency

WE8

1 **(a)** Here is a matchstick pattern of squares. Copy and complete this table of values by continuing the pattern.





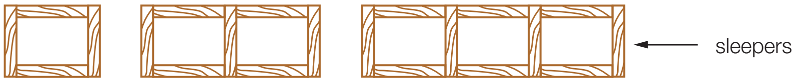
Number of squares (s)	1	2	3	4	5	6	7	8
Number of matches (m)								

(b) Find a general rule that connects the number of squares in the pattern (s) to the number of matches used (m). Write the rule in words and in algebra.

(c) Use your rule to find the number of matches required to make 100 squares.

2 Larissa, a landscape gardener, uses sleepers to divide up gardens into separate beds as shown.

WE9



1 garden bed
4 sleepers

2 garden beds
7 sleepers

3 garden beds
10 sleepers

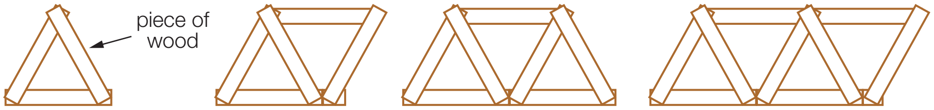
(a) Copy and complete this table of values by continuing the pattern.

Number of garden beds (B)	1	2	3	4	5
Number of sleepers (S)	4	7	10		

(b) Find a general rule that connects the number of garden beds (B) to the number of sleepers (S) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of sleepers required to divide the garden into a section made up of 21 beds.

3 Clarence the carpenter has been working on a new restaurant. The owners want a triangular woodwork design, like the one shown below, running across the walls.



1 triangle
3 pieces of wood

2 triangles
5 pieces of wood

3 triangles
7 pieces of wood

4 triangles
?

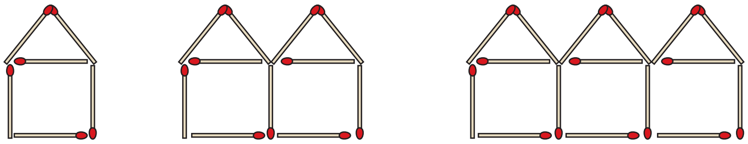
(a) Copy and complete this table of values by continuing the pattern.

Number of triangles (T)	1	2	3	4	5
Number of pieces of wood (P)	3	5	7		

(b) Find a general rule that connects the number of triangles to the number of pieces of wood required.

(c) Use your rule to find how many pieces of wood are required to make a total of 203 triangles.

4 Here is a matchstick pattern of houses.



1 house
6 matches

2 houses
11 matches

3 houses
? matches

(a) Copy and complete the table below.

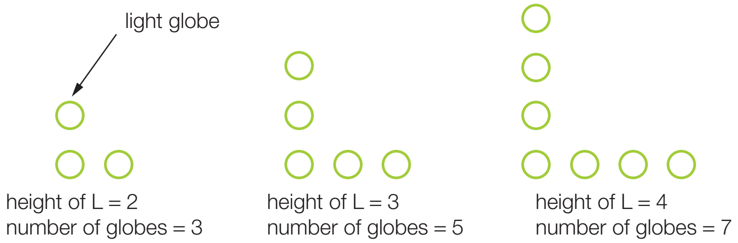
Number of houses (h)	1	2	3	4	5	6	7	8
Number of matches (m)	6	11						

(b) Find the general rule that connects the number of matches to the number of houses.

(c) Use your rule to find the number of matches required to build 20 houses.

Understanding

5 Lightworks International Co., designers and manufacturers of large illuminated advertising signs, want to put a giant L made up of individual globes onto their largest building. They have already made some small Ls on some of their other buildings.



(a) Construct and complete a table of values for this pattern. Use the three patterns given here to begin your table, then continue the pattern for the next two Ls in the pattern.

(b) Find the rule that connects the height of the L to the number of globes required. Let H = height of the L and G = number of globes.

(c) Use your rule to find how many globes you would need to make an 'L' with a height of 120.

6 A rule is given as $L = 3n + 1$. Which statement is not true?

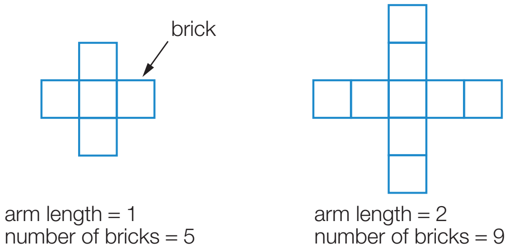
A When $n = 3$, $L = 10$.

B When $n = 4$, $L = 12$.

C When $n = 5$, $L = 16$.

D When $n = 6$, $L = 19$.

7 The White Cross charity organisation has a white cross as its emblem. Their buildings all have a white cross built into the brickwork using white bricks. Some smaller versions of the cross are shown.



(a) Draw the next two white crosses in the pattern.

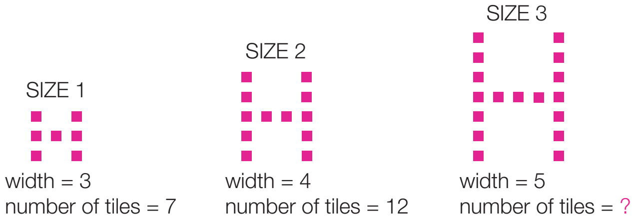
(b) Construct and complete a table of values connecting the arm length (a) and the number of white bricks (b) for these four different-sized crosses.

(c) Extend your table for arm lengths of 5 and 6.

(d) Find the rule that connects the two pronumerals.

(e) Find how many white bricks would be needed to make a cross with an arm length of 52.

8 Mr Harrison has decided that he wants to use tiles to form large letter Hs, which are built into the brickwork of the walls of his company offices. He must work out how many tiles are needed to make different-sized Hs.



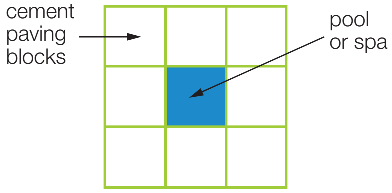
(a) Draw the next two Hs in the pattern.

(b) Construct and complete a table of values connecting the width of the H (w), and the total number of tiles (t).

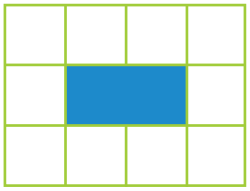
(c) Find the rule for these five different sizes that connects the width of the H (w) to the total number of tiles (t).

(d) Find the number of tiles needed to make an H that is 12 tiles wide.

9 Plunge & Co., the Pool Paving People, specialises in large square cement paving blocks to surround swimming pools and outdoor spas, like the ones shown below.



length of spa = 1
number of paving blocks = 8



length of spa = 2
number of paving blocks = 10

(a) Draw the next two spas and pavers in the pattern.

(b) Construct and complete a table for values of up to 4 spa lengths, connecting the spa length (*l*) with the number of pavers used (*p*) for these four different spa sizes.

(c) Find a rule without brackets connecting the two pronumerals.

(d) Explain your rule by using diagrams of spas of different lengths. Which paving blocks does the constant represent? Where does the coefficient of the pronumeral representing spa length come from?

(e) Find a rule using brackets that also works.

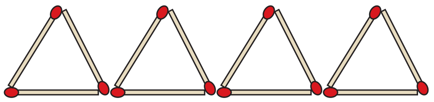
(f) Plunge & Co. has recently been asked by a swimming club to build a single-lane lap pool to help train its long-distance swimmers. It has been worked out that the lap pool is 345 paving blocks in length. Find out how many paving blocks will be needed to pave around the whole pool.

(g) For another job, Plunge & Co. uses 40 paving blocks to surround an outdoor spa. What is the length of this spa?

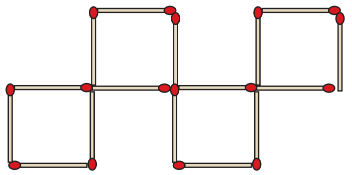
Reasoning

10 In each of the following matchstick patterns, the diagrams show the pattern after the fourth set of matchsticks has been added.

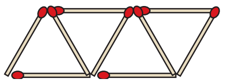
(i)



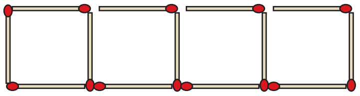
(ii)



(iii)



(iv)

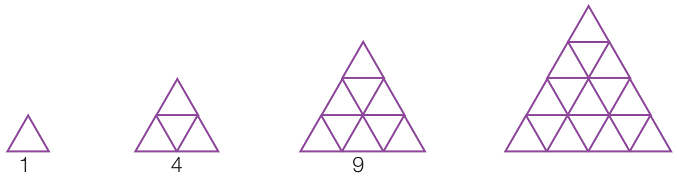


(a) Write a general rule for each of the patterns if one more set of matchsticks is added to the pattern each time. Use s = number of shapes and m = number of matchsticks as pronumerals.

(b) Patterns **(i)** and **(iii)** both add a new triangle every time. However, the rules are different. Patterns **(ii)** and **(iv)** both add a new square every time. However, the rules are also different. How are patterns **(i)** and **(ii)** different from the patterns in **(iii)** and **(iv)**? How is this difference shown in the rule for each pattern?

Not all patterns grow by a consistent amount each time. For the following question, complete the table and describe how the sequence of numbers formed in the bottom row of the table increases as extra elements of the pattern are added.

11 **(a)** Count *all* the small triangles in the fourth large triangle.



(b) How many small triangles do you think there will be in the fifth large triangle?

(c) Copy and complete the following table. Try to find the pattern to help you.

Large triangle	1	2	3	4	5	6
Number of triangles	1	4	9			

Open-ended

12 Choose one of the following letters: P, T, A, X, or Y. Create a design for the letter from small centimetre squares, and use it to create a pattern of letters that gradually increases in size. Try to find a rule that describes your pattern.

13 Shreyas has decided to make a closed matchstick shape with 6 matchsticks. He then adds matchsticks to make a pattern that has a rule $m = 5s + 1$ where m is the number of matchsticks and s is the number of repeated shapes.

(a) Construct a shape that Shreyas may have started with.

(b) Shreyas changes his shape and adds matchsticks to make a new pattern. He finds that the rule is now $m = 4s + 2$. Draw a second shape that he may have used.

Outside the Square

Puzzle

Puzzling tables

The table contains multiple patterns, across each row and down each column.

Copy the table and write the correct number in each blank space.

Describe the pattern for each row and for each column.

	Column 1	Column 2	Column 3	Column 4
Row 1	4	6	8	10
Row 2	7	10.5	14	
Row 3		15	20	25
Row 4	13		26	32.5
Row 5	16	24		40

Patterns like this often exist in tables of information. This table gives the cost to send parcels, up to 20 kg, by air.

Different parts of the table have different patterns. A pattern may only work in part of a row or column. Describe the patterns that you can find.

Weight	Zone A NZ	Zone B Asia/Pacific	Zone C USA/Canada/ Middle East	Zone D Rest of World
AIR				
Up to 250 g	\$7.20	\$8.40	\$9.55	\$11.35
Over 250 g up to 500 g	\$11.00	\$13.40	\$15.70	\$19.30
Over 500 g up to 750 g	\$14.80	\$18.40	\$21.85	\$27.25
Over 750 g up to 1000 g	\$18.60	\$23.40	\$28.00	\$35.20
Over 1000 g up to 1250 g	\$22.40	\$28.40	\$34.15	\$43.15
Over 1250 g up to 1500 g	\$26.20	\$33.40	\$40.30	\$51.10
Over 1500 g up to 1750 g	\$30.00	\$38.40	\$46.45	\$59.05
Over 1750 g up to 2000 g	\$33.80	\$43.40	\$52.60	\$67.00
Extra 500 g or part thereof	\$4.00	\$5.20	\$7.45	\$9.95

Postal Charges 2009, Australia Post website.