EARTH-IMPACT PROBABILITY COMPUTATION OF DISRUPTED ASTEROID FRAGMENTS USING GMAT/STK/CODES

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There is a nationally growing interest in the use of a high-energy nuclear disruption/fragmentation option for mitigating the most probable impact threat of near-Earth objects (NEOs) with a short warning time. Consequently, this paper investigates the orbital dispersion and impact probability computation problem of a disrupted/fragmented NEO using several computer programs, called General Mission Analysis Tool (GMAT) developed by NASA, AGI's Satellite Tool Kit (STK), and Jim Baer's Comet/asteroid Orbit Determination and Ephemeris Software (CODES). These tools allow precision orbital simulation studies of many fragmented bodies, with high-fidelity visualizations. Various mathematical models for impact probability computation are examined and compared to JPL's Sentry, which is a highly automated NEO collision monitoring system. For example, we obtained an impact probability of 4.2E-6 for asteroid Apophis on April 13, 2036, which is very close to 4.3E-6 predicted by JPL's Sentry system. Our research effort of exploiting various commercial software such as GMAT, STK, and CODES will result in a robust software system for assessing the consequence of a high-energy nuclear disruption mission for mitigating the impact threat of hazardous NEOs.

INTRODUCTION

Asteroids and comets have collided with the Earth in the past and are predicted to do so in the future. These collisions have a significant role in shaping Earth's biological and geological history, most notably the extinction of the dinosaurs 65 million years ago. Another event is the 1908 Tunguska impact in Siberia, which released an explosion energy equivalent to approximately five to seven megatons of TNT. This explosion had enough power to destroy a 25 km radius of forest. It has been estimated that an impact from the asteroid 99942 Apophis would release approximately 900 megatons of energy, over 130 times the Tunguska event.[1] The results of a collision of this magnitude in a highly populated area would be catastrophic.

This paper is the first step of developing an interface that combines the research efforts in the Asteroid Deflection Research Center with high-fidelity commercial software. The interface is called the Asteroid Mission Design Software Toolbox (AMiDST) to be used for validating and enhancing research in computational astrodynamics using Graphics Processing Units (GPUs), Yarkovsky effect modeling, GN&C algorithms, and nuclear fragmentation modeling. These core research areas will connect through the use of the AMiDST which utilizes JPL's Horizons, CODES, GMAT, STK along with AGI Components, MATLAB, and GMV's CLEON software as illustrated in Figure 1.

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The AMiDST is being developed to be used for the design and analysis of real deflection/disruption missions in the future, which will require reliable, high-fidelity, precision orbital modeling and simulations. The commercial software toolbox incorporates all aspects of the mission and creates a foundation for innovation and validation of asteroid missions. Figure 1 depicts the overall research goal while this paper is the first step of developing the AMiDST.

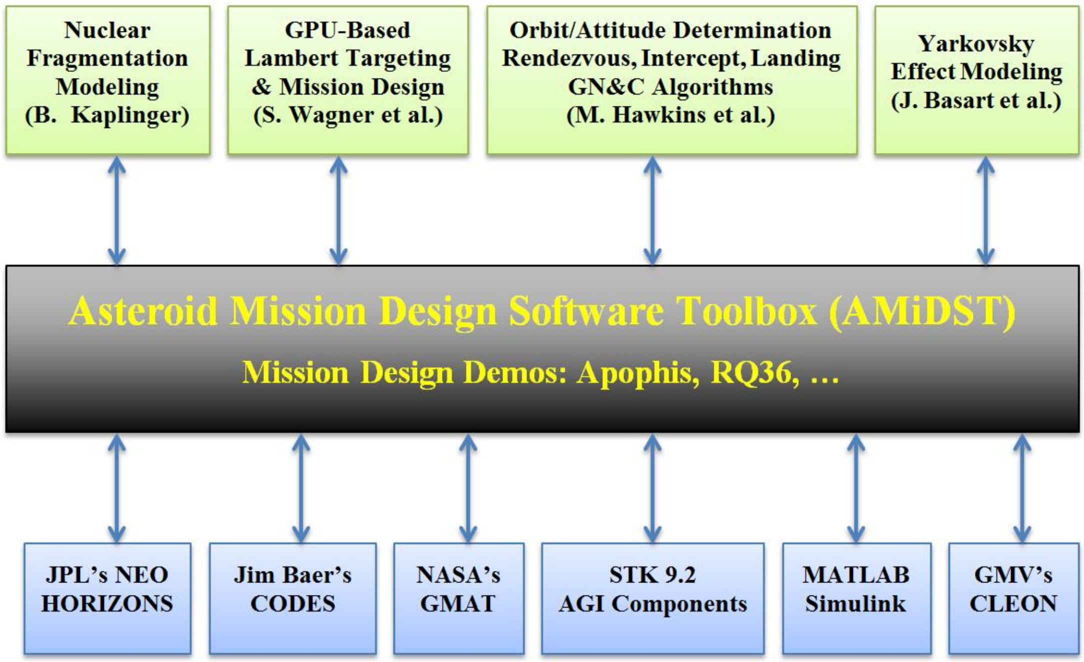


Figure 1. Illustration of the Asteroid Mission Design Software Toolbox (AMiDST).

Given short warning times, high-energy disruption missions, such as delivering a nuclear explosive device to an asteroid, are needed to properly disperse the asteroid fragments from hitting the Earth.[2] Orbital dispersion simulation and analysis results show that fragmenting and dispersing a hazardous NEO could lower the total mass impacting the Earth.[3] This could be beneficial in situations where some impacting mass is inevitable, or where the resulting fragments will be small enough to burn up in Earth's atmosphere.[2] Having an impact probability assessment of such a disruption mission will prove whether Earth is safe from multiple fragments. This paper presents software validation, trajectory analysis, and the implementation of Monte Carlo simulations to find the impact probability of a hazardous NEO. The tools used to find the impact probability can be easily modified to study the consequences of disrupted asteroid fragments from high-energy nuclear disruption missions of Earth-threatening bodies, with short warning time. To accurately estimate the impact probability of disrupted asteroid fragments, commercial mission analysis software are utilized, which include NASA's General Mission Analysis Tool (GMAT), AGI's Satellite Tool Kit (STK), Jim Baer's Comet/asteroid Orbit Determination and Ephemeris Software (CODES), and JPL's Horizons.

There has been a great deal of discussion over the probable impact threat posed by the asteroid Apophis. This possible threat makes Apophis an ideal reference model to study. On April 13, 2029, the asteroid Apophis will have a close-encounter with Earth, in which the asteroid will pass below geostationary orbit.[4] Apophis could pass through a keyhole and impact the Earth on April 13th, 2036. Keyholes are very small regions of the first encounter target-plane causing a resonant return impact with the Earth if an NEO passes through it.[4] Due to the uncertainty in observations, gravitational effects, and other forces, orbit trajectories of small bodies are very difficult to predict in long-term periods. The impact probability of Apophis in 2036 is currently estimated as four in one million, according to JPL's Sentry system.[1]

Apophis was first discovered in 2004. Tables 1 and 2 show the physical parameters and orbital elements respectively of Apophis from JPL's Apophis orbit solution #144. Throughout this study, Apophis is used as a reference asteroid and these orbital elements are used in each simulation. The overall research objective is met but with much difficulty due to the nature of the problem, the software inflexibility, and orbit differences in each software. These differences have been noted and documented, but further investigation of these differences is required.

Table 1. Physical Parameters of Apophis taken from JPL.[1]

Physical Parameters	Value
Rotational Period (hrs)	30.5
Mass (kg)	4.5E10
Diameter (m)	270
Absolute Magnitude H	19.7
Albedo	0.33

Table 2. Orbital elements of Apophis at Epoch 2455800.5 (2011-Aug-27.0) TDB.[1]

Orbital Elements	Value
Semi-Major Axis (AU)	0.92230
Eccentricity	0.19108
Inclination (deg)	3.3319
Ω (deg)	204.4304
ω (deg)	126.4245
M_o (deg)	287.5823

TRAJECTORY ANALYSIS OF APOPHIS USING STK/GMAT/CODES

Introduction

STK is a graphical user interface software for modeling and analyzing several applications including flight paths and logistics, communications, satellites in specific orbits, Earth satellites, defense applications, and more.[5] With high-fidelity visualizations, STK has a multitude of features that trump most physics-based computer programs. A communication interface between STK and MATLAB can be established to utilize functions and script files created in MATLAB while exploiting STK features such as propagators, orbit properties, and post-processing components. Propagators that manage interplanetary missions include the High Precision Orbit Propagator (HPOP) and Astrogator.[5] Astrogator is a highly-versatile component of STK used to study detailed mission scenarios, including the detailed parameterization of the spacecraft subsystems (dry and wet mass, mass flow rates, pressures of fuel tanks, etc.). The high-fidelity visualization that STK provides for conceptual mission design is invaluable to space applications.[5]

GMAT is a space trajectory optimization and mission analysis software developed by NASA, the space community, and the open source community. GMAT's primary goal is to "research, develop, verify, and transfer new technologies in space trajectory optimization and mission design."[7] GMAT is completely open-source allowing for user extension and personalization. MATLAB, minimization, and optimization plug-ins have been released and developers are currently working on a plug-in between GMAT and the MATLAB Orbital Dynamics Toolbox. A built-in scripting language included with GMAT allows for mission creation and modification. The user is able to create their own propagators from a list of integrators and force models. GMAT contains a new but powerful visualization tool for modeling and analyzing mission concepts with emphasis on space missions around Earth or in the Solar System. The Mac and Linux releases of GMAT 2011a are currently in the Alpha stage in the design process while the Windows release is in the Beta.

CODES is a graphical user interface program written by Jim Baer. CODES calculates a variety of small body characteristics such as orbits, optical observations, and physical parameters using a precise n-body numerical integrator. It can use topocentric or geocentric ephemeris based on user-specification or import minor bodies from the Minor Planet Center (MPC) database.[8] Additionally, it allows for linear and non-linear analysis of collisions/near misses between minor planets and major planets. Calculation of the state vector or orbital elements with covariance matrices can be obtained using an n-body simulator accounting for solar radiation pressure, gravity harmonics, and relativistic effects.

JPL's Solar System Dynamics Group provides an on-line ephemeris computation service that provides flexible information about solar system objects. JPL's Horizons has three different access methods; telnet, e-mail, and web-browser. The web interface provides access to a small subset of program functions with an interactive GUI.[9] The system provides access to highly accurate ephemerides for solar system objects including over 560,000+ asteroids and comets, 9 planets, the sun, natural satellites, spacecraft, and more.[9,10] Close-approaches by asteroids and comets to the planets, Ceres, Pallas, and Vesta, can be identified along with the encounter uncertainties and impact probabilities. Orbital uncertainties and covariance matrices can be computed for asteroids and comets. The underlying planet/satellite ephemerides and small-body osculating elements are the same ones used at JPL for radar astronomy, mission planning and spacecraft navigation.[9]

Apophis Orbit Simulation Using STK 9.2

In STK, the HPOP is used to propagate interplanetary trajectories with a solar radiation pressure model, user-specified selection of third-body gravity, object rotation, and high-fidelity visualizations. The HPOP can use several different integrators for numerical analysis. The default integrator for HPOP is the Runge-Kutta-Felhberg (RKF) of order 8 to estimate the local error in the method of order 7. The RKF 7(8) is an integration technique utilizing polynomial functions to approximate the ordinary differential equations.[11] The RKF 7(8) has a variable step size for integration. If the difference between two solutions is outside the bounds of the error tolerance, set by the user, then the solution is estimated again with a decreased step size.[11] Another type of integration scheme that employs a variable step size is the Gragg-Bulirsch-Stoer (GBS) method. This method utilizes rational functions as fitting functions for the Richardson extrapolation instead of polynomial functions.[12] It is also combined with a modified midpoint method which is extremely important in the accuracy of the solution. This method also has a variable step size. Lastly, a Gauss-Jackson (GJ) integration scheme is used to propagate an orbit. The Gauss-Jackson fixed, multi-step predictor-corrector method is widely accepted in numerical integration problems for astrodynamics.[13]

The default minimum and maximum step sizes for both the RKF and the GBS integrator are 1 second and 86400 seconds, respectively. The default error tolerance is 1.0E-13. A change to these default values is listed in Table 3. In order to compare STK to other commercial software, a reasonable compromise was reached for the minimum and maximum step size and error tolerance. For planetary flyby sequences, a minimum step size is most likely used due to the large gravity terms acting on the body. An average speed of an asteroid around the sun is approximately 30 km/s. If the default minimum step size of 1 second is chosen, the distance traveled during one time step is 30 km. Thus if a minimum step size is used during these times, a propagation of 3 meters per step is sufficient. When studying the effects of Earth's gravity on a small asteroid body, a rigorous selection of a step size is important and a 1 second step size is not acceptable. These values listed in the table are used throughout all the STK test cases in this study.

Table 3. Integration parameters in STK 9.2.

Integration Method	Minimum Step-Size (Sec.)	Maximum Step-Size (Sec.)	Error Tolerance
RKF	0.0001	15,000	1.0E-11
GBS			
GJ	15,000	N/A	N/A

The solar radiation pressure model was included based on Apophis's physical parameters such as the slope parameter, albedo, spin rate, size, and mass. The solar radiation pressure model is assumed to be a spherical model with properties reflecting those shown in Table 1. Third-body gravity includes all 9 planets and Earth's moon from ephemeris file DE-421. The Yarkovsky model and relativistic effects were not included in this study.

A comparison is made against the three different integration methods with the same force models. The epoch position and velocity is chosen from JPL's Horizons on January 1, 2029 in the J2000 Ecliptic Coordinate System. STK then propagated the object multiple times, each with a different integration method. The differences are calculated against one another and the results are shown in Table 4. Based on the propagation prior to the close-approach with Earth, it was found that all integrators are able to produce exactly the same results. However one and one-half month later, the Gauss-Jackson integration scheme fails completely due to its fixed step size. If a smaller window of time is to be analyzed with an extremely small step-size, a Gauss-Jackson integrator is as reliable as the other integrators. Since the study of the asteroid is much more than two months, a Gauss-Jackson is not used. Also, from the differences, the Gragg-Bulirsch-Stoer method is nearly identical to the RKF 7(8). This is important in comparing the other mission analysis software since both GMAT and STK have the Gragg-Bulirsch-Stoer method but not the same RKF order method. Due to excessively long computation time, the Gauss-Jackson fixed step-size is not further reduced.

Table 4. Difference between integration schemes in STK at Epoch of January 1, 2029 from Horizons.

	Date	X (km)	Y (km)	Z (km)	$\dot{X}$ (m/s)	$\dot{Y}$ (m/s)	$\dot{Z}$ (m/s)
RKF - GBS	1-Mar-29	0	0	0	0	0	0
	13-Apr-29	9.95E-2	2.67E-2	2.21E-2	1.20E-2	2.0E-3	2.0E-3
	1-Jun-29	0.20	6.84E-4	7.94E-2	0	0	0
RKF - GJ	1-Mar-29	0	0	0	0	0	0
	13-Apr-29	5.77E-2	1.02E-2	1.14E-2	7.00E-3	1.00E-3	1.00E-3
	1-Jun-29	3.63E+6	1.16E+6	5.51E+5	838.12	642.39	81.16
GBS - GJ	1-Mar-29	0	0	0	0	0	0
	13-Apr-29	0.16	3.69E-2	3.35E-2	1.90E-2	3.00E-3	3.00E-3
	1-Jun-29	3.63E+6	1.16E+6	5.51E+5	838.12	642.39	81.16

Apophis Orbit Simulation Using GMAT

GMAT's main strength over other software choices is its versatility. Its scripting ability is easy to use and edit without any knowledge of computer languages. The MATLAB plug-in allows an expansion of the user's ability to personalize each mission. Rather than choosing from a list of possible propagators, the user is able to create his/her own by choosing the propagation settings such as integrator scheme, step size, error tolerance as well as the gravitational bodies, and the non-gravitational forces including drag, solar radiation pressure, and more.

In GMAT there are seven different n-body numerical integrators to choose from. These include the Runge-Kutta 5(6), Runge-Kutta 6(8), Runge-Kutta 8(9), Prince-Dormand 4(5), Prince-Dormand 7(8), Bulirsch-Stoer, and the Adams-Bashforth-Moulton method. Each of these integrators employ variable step sizes and are well tested by the GMAT team. The Runge-Kutta (RK) method, as explained earlier, is a single-step method employing a series of coupled variables to solve differential equations.[14] The Prince-Dormand (PD) method as used in PD 78 and PD 45 is a form of an expanded RK method.

Unfortunately, the Bulirsch-Stoer method could not complete the propagation test within the accuracy limit for long periods of time (more than 7 years). Therefore, this method is considered only as a verification tool but was not used for impact probability simulations. Adams-Bashforth-Moulton, on the other hand is an implicit linear multistep method that iteratively solves the differential equation. The Adams-Moulton integration method is used as a differential corrector for the Adams-Bashforth integration allowing for multistep integration.[15] The Adams-Bashforth-Moulton method also could not complete the propagation within the accuracy limit.

The propagation test comprised of using several different integrators each having the same step sizes and error tolerances. The planetary bodies use ephemeris data from DE-405. Apophis solution #144 is used as the starting conditions listed in Table 2. Each case propagated from August 2011 until April 13, 2036. The 2029 close-approach distance was found as well as the 2036 distance. Table 5 shows the results of the integration test cases.

Each propagator's nominal close-approach distance in 2029 and 2036 is compared to distinguish which integrator performs best. After running each integrator, no difference was seen in the 2029 close-approach. However, after further review of STK and CODES, the Runge-Kutta 8(9) was chosen as the default integrator for simulations. Additionally, after careful evaluation and testing, a maximum step of 15,000 seconds, a minimum step of 1E-4 seconds, and an accuracy of 1E-11 were chosen.

Table 5. Comparison of GMAT Integration Methods using Apophis.

Integration Method	2029 Distance, km	2036 Distance, km	Runtime, sec
RK 89	37,344.48	20,323,355.5	88.92
RK 68	37,344.48	20,323,302.7	50.21
PD 78	37,344.48	20,323,313.9	70.44

Once the integration method is selected, integration parameters can be chosen for the initial step size, accuracy limit, minimum step size, maximum step size, and maximum step attempts. Variable step size calculations change the step size within the bounds set by the minimum and maximum step size variables to achieve the given accuracy limit. If the accuracy is determined to be outside the acceptable range then the integration is repeated with a smaller step size. This is repeated until the accuracy is obtained, the minimum step size is used, or the number of attempts equals the maximum step attempts. If the accuracy cannot be reached, an error is displayed and the program stops. If the accuracy is reached, the integrator moves onto the next step and repeats the process.

GMAT has several different options regarding which gravitational bodies the user wants. Gravitational bodies include the sun, 9 planets, Earth's moon, several large asteroids, and 300 known asteroids. After studying the effects of these bodies it was determined that including all the bodies is not necessary. Many of the smaller bodies have such a small effect on the orbit of an asteroid that they can be neglected. Taking out these bodies improves computation time allowing for more simulations to be run.

For the reference asteroid Apophis, simulations including various heavenly bodies are run to determine at what point their effect becomes negligible. After running these tests, which can be seen in Table 6, the sun, 9 planets, Earth's moon, Ceres, Vesta, Pallas are determined to be the bodies with the largest influence on Apophis. Including other bodies increased the computation time with no significant change in the close-approach distance. Ceres, Vesta and Pallas are added to GMAT by importing their SPICE files, but all the other bodies came built-in with GMAT.

Table 6. Comparison of GMAT Results with Different Gravitational Bodies.

	Sun Only	Sun, Earth, & moon	Sun, 9 Planets, moon, Ceres, Vesta, & Pallas	Sun, 9 Planets, & 300 Asteroids
2029 Distance, km	3,317,819	5,420,063	37,344.5	37,345.1
Runtime, sec	34.94	40.00	45.17	72.12

Another important gravitational force that must be considered for any accurate model is non-spherical gravity. This is accounted for within the integration model with the use of .cof potential files. Gravitational potential files for Venus, Earth, Earth's moon and Mars are built into GMAT, but it also allows for users to add other bodies manually. Repeated tests for the asteroid Apophis revealed that only the Earth's non-spherical gravity has a significant effect. This is possibly because of the 2029 close-approach to the Earth. Future simulations will include Earth as a non-spherical gravitational model.

Other non-gravitational forces can also be included in the propagation. Currently, these include drag and solar radiation pressure (SRP), but the GMAT support team plans on expanding this to include both relativistic and Yarkovsky effects. GMAT includes high-fidelity models, not only for atmospheric drag but also F10.7 drag, and magnetic drag of the Earth. The SRP model can be applied using the albedo, spin rate, size and mass of the asteroid. Both drag and SRP forces are included in the Monte Carlo simulation described later.

Apophis Orbit Simulation Using CODES

CODES is a graphical user interface program used to calculate a variety of information using observations, an N-body numerical integrator, and physical parameters. First, the program asks to designate the type of object, whether it be an asteroid or comet. Afterwards, the user can import a body from the Minor Planet Center (MPC), load in observations, or manually specify an orbit. If observations are chosen the program allows evaluation of the observations and propagates the object with a best-fit orbit using n-body mechanics. These observations can then be compared to positions of known minor planets.

One reason that makes CODES credible is the N-body numerical propagator and force models. The N-body propagator has three options: include all 9 planets and Earth's moon, or the 9 planets, moon, plus Ceres, Pallas, and Vesta, or the 9 Planets, moon, and 300 asteroids.[8] The planetary bodies use ephemeris data from DE-405 while the 300 asteroids use BC-405. Having a rigorous propagator increases computation time and accuracy. The force models include solar radiation pressure, relativistic effects, and gravity harmonics. The numerical integrator utilizes a Dormand-Prince embedded Runge-Kutta 7(8) method.

In this paper, the N-body propagator including the 9 planets, Earth's moon, plus Ceres, Pallas, and Vesta was used for CODES simulation runs. Once the orbit has been propagated, the user can extract the heliocentric ecliptic J2000 orbital elements or state vector at epoch. The orbital elements and state vector can also be propagated to a new epoch with a new covariance matrix. This is important when trying to study an asteroid's orbit further in the future without having to propagate the state vector from the original epoch each time.

Lastly, the collision and/or near miss tool can be used to check for any collisions or near misses to all major planets or Earth. The tool is broken down into a linear and nonlinear analysis. The linear analysis propagates the nominal state vector and covariance matrix to the specified end date. Using N-body mechanics and force models, CODES checks the distance between the object and distance to a solar system body (Sun, planets, and Earth's moon).[8] If the calculated distance is under the specified distance then a near miss is predicted. CODES uses the state vector and covariance matrix to estimate the probability of collision. On the other hand, if the calculated distance is less than the radius of the solar system body, then a collision has occurred and is reported. CODES uses a bisection method that determines the exact date and time of the collision.[8] All of these near misses and collisions are reported in a text file at the end of the simulation.

The nonlinear analysis can also be used. The analysis is primarily used for multiple near misses that result in sensitive orbit trajectories. A nonlinear analysis handles a user-specified number of variant state vectors either distributed normally about the nominal epoch state vector or uniformly along the Line-of-Variations (LOV).[8] The LOV is the major axis of the epoch state vector uncertainty ellipse. Once these state vectors are created, they are then propagated in the same manner as the linear method. If a near miss is found, a Monte Carlo simulation model is created by introducing virtual asteroids to extensively examine the close-approach distance and impact probability.

Unfortunately for our study, the nonlinear analysis, which is required to determine the impact probability of a small body, is not currently setup to handle a specified or imported nominal state vector. Instead, it is required to have both the observations and best-fit orbit with covariance matrix. The nonlinear analysis involves adding normally distributed noise to each observation, calculating the resulting orbit and then propagating it.

Using CODES with a specified orbit from Apophis solution #144 as listed in Table 2, a linear collision analysis was ran. The n-body propagator was chosen as well as the physical parameters were set as shown in Table 1. Table 7 shows the results of the linear analysis simulation. It accurately depicts the close-approach in 2029 with a near miss of 5.97 Earth radii or 38,078 km. Due to the linear analysis and the sensitivity of the close-approach distance in 2029, the near misses, past 2029, should be checked using a nonlinear analysis. All the miss distances are represented in Earth radii.

The original intention of using CODES was to validate the impact probability estimation made

Table 7. Collision/near miss results of a linear analysis for 50 years of Apophis with Epoch on August 27, 2011.

year	mm	dd	hh	mm	secs	Nominal Miss Distance (er)	Impact Probability
2013	01	09	11	42	11.4	2,267.18	0.0
2021	03	06	01	13	43.7	2,642.21	0.0
2029	04	13	21	45	08.9	5.97	0.0
2036	03	26	05	37	38.6	7,859.87	0.0
2037	09	23	01	42	34.2	4,525.3	0.0
2044	07	31	02	52	16.3	2,585.68	0.0

by JPL's Sentry. Once found, the next step is to study the effects of fragments, from a nuclear disruption mission, with an impacting trajectory 15 days prior to impact Earth, and then run a Monte Carlo simulation scenario of the impacting object to determine the impact probability of having multiple fragments hit the Earth. However, due to the inflexibility of CODES, this type of collision assessment cannot be accurately found using a specified orbit with covariance matrix. Future upgrades for CODES, to name a few, include determination of impact on a planet using latitude and longitude, additional asteroid perturbations, Yarkovsky effect, and implementation of JPL's DE-406 planetary ephemeris.

Comparison of Orbit Simulation Results for Apophis

Validation is the first priority to ensure reliable results from the mission analysis software. By testing each software and their respective integrator/propagator using identical initial conditions, validation is achieved. Each program is tested using the same position and velocity vector taken from JPL's Horizons in the J2000 Ecliptic Coordinate System using the same integrator, step size, and error tolerance. Between STK and GMAT a variable step size Gragg-Bulirsch-Stoer integrator scheme is used for propagation. The force models for each are also identical in nature including the solar radiation pressure and third body gravity. JPL's Horizons does not use the same integrator scheme but an Adams-Krogh integrator, named DIVA, with a variable order and variable step size.[10] It contains highly-accurate force and dynamical models used to provide conclusive ephemeris information. For small bodies, JPL's Horizons uses integrated gravitational point-mass equations of motion and can be extended to include solar radiation pressure, Yarkovsky effect, gravity harmonics, and relativistic effects.[10] Although GMAT, STK, and Horizons do not all share the common propagator and properties, similarities can still be drawn among them.

After individual software propagation, the position and velocity vectors are recorded at predefined times. These test cases assess each program's ability to calculate Apophis's close-encounter in 2029 for two months as well as deep-space propagation for several years. There are two types of cases that are used to gather information. Case 1 focuses on the close-approach distance between the asteroid and Earth. The case starts one month prior to the close-approach in 2029 and ends one month after. Case 2 examines the deep-space propagation of the asteroid from one month after the close-approach in 2029 until a possible impact on April 13, 2036. The positions and velocities within each program are recorded and checked against JPL's Horizons.[16] Tables 8 and 9 show the differences in the position and velocity vector components for each case at the specified date.

Case 1 starts with an epoch on March 13, 2029 using position and velocity vectors of Apophis from Horizons. This information is then entered into STK and GMAT. Once propagated, the position and velocity vector on April 13 and May 13 of 2029 are recorded. These differences shown in Tables 8 and 9 show differences between STK and Horizons and GMAT and Horizons. It should be noted that both have a relatively large difference on May 13, however, the difference of both are relatively of equal value. Thus, STK and GMAT match closely with one another for planetary close flybys.

In Case 2, the difference between STK and Horizons is greatly significant and it should be noted as an undesirable difference. Case 2 starts with an epoch date of May 13, 2029 with position and velocity vectors taken from Horizons. After one year of deep-space propagation, the difference between GMAT and Horizons is very small. This implies that the propagation results from GMAT and Horizons are similar for deep space propagation but differ slightly in planetary close flybys. On the other hand, the propagation results from STK and Horizons differ considerably on May 13, 2030. The error between them increases as the propagation continues. A difference of 33,000 km is not acceptable for scientific purposes in the study of accurate impact probability computation.

Table 8. Case 1: Difference Between Horizons and STK/GMAT for Epoch of March 13, 2029.

	Date	X (km)	Y (km)	Z (km)	$\dot{X}$ (m/s)	$\dot{Y}$ (m/s)	$\dot{Z}$ (m/s)
STK	13-Apr-29	11.49	10.42	5.80	0.25	0.13	0.06
	13-May-29	69,138.30	213,142.12	91,981.88	16.01	85.53	32.55
GMAT	13-Apr-29	20.98	6.03	5.29	0.28	0.12	0.06
	13-May-29	68,385.27	213,523.94	89,183.43	15.69	85.75	31.55

Table 9. Case 2: Difference Between Horizons and STK/GMAT for Epoch of May 13, 2029.

	Date	X (km)	Y (km)	Z (km)	$\dot{X}$ (m/s)	$\dot{Y}$ (m/s)	$\dot{Z}$ (m/s)
STK	13-May-30	771.03	4,610.49	175.21	0.76	0.06	0.01
	13-Apr-36	25,000.73	20,351.78	1,105.38	4.71	4.45	0.08
	13-May-36	33,482.19	4,856.95	689.32	1.36	7.15	0.23
GMAT	13-May-30	0.06	27.29	0.42	4.92E-3	1.55E-5	9.19E-5
	13-Apr-36	38.27	39.58	2.03	8.18E-3	1.16E-2	3.22E-4
	13-May-36	52.03	4.37	0.89	2.06E-3	1.52E-2	5.46E-4

Figure 2 shows the radial position differences of Apophis with respect to Horizons using STK and GMAT. STK uses the RK 7(8) method and GMAT uses the RK 8(9) method with identical minimum and maximum step sizes and error tolerances. This comparison uses the same time period as Case 2 along with the same ephemeris data files, DE-421. It is worth noting that there is an increasing sinusoidal difference between STK and GMAT with Horizons for long-term propagation. However, STK shows a much earlier radial position difference which then increases dramatically. GMAT follows a similar trend but with a much smaller difference amplitude for the same propagation time.

Further investigation of this propagation error in STK is conducted using different integrators, variable step sizes, fixed step sizes, higher error tolerances, and planetary ephemeris files. The maximum step size of the Gragg-Bulirsch-Stoer method, is set at 3,750 seconds, one-fourth of the initial maximum step size and the new position vectors are recorded. The difference is taken

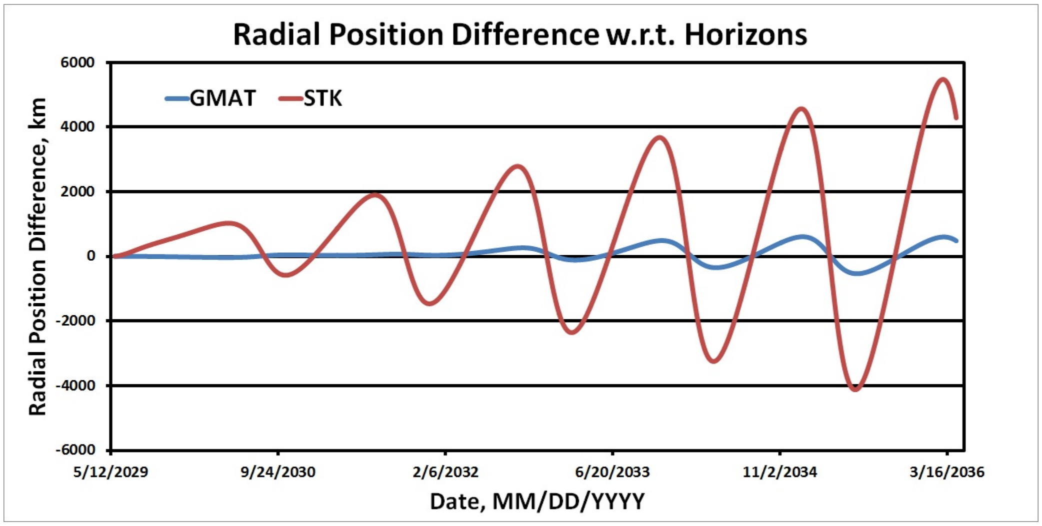


Figure 2. Radial Position Differences of Apophis w.r.t. JPL's Horizons.

from both the GBS methods using different maximum step sizes in STK. The result showed little difference between the new and the old step sizes. Finally, the Gauss-Jackson method was also used in Case 2 with a step size of 60 seconds. On May 13, 2030, the Gauss-Jackson method and GMAT's GBS method resulted in a larger difference than shown in Table 9. Solar radiation pressure model was also turned off in both STK and GMAT. However, the resulting change was a 10-30 km difference from the original orbit in both software. These differences have been noted but examination of the error between the two programs is still continuing. AGI headquarters has heard these differences through customer support and an investigation has begun.

IMPACT PROBABILITY MODELING

Knowledge of an asteroid's position comes from a series of visual, radar and Doppler observations. From these observations a nominal orbit and uncertainty ellipsoid is determined. The uncertainty ellipsoid is the volume inside which the asteroid must exist. The target asteroid's actual position could be anywhere within the uncertainty ellipsoid but statistically it is likely to be at the nominal location. Many methods use a series of Virtual Asteroids (VAs) to determine impacting probabilities. Each VA represents one possible location and orbit of the target asteroid of interest.

One consequence of this positional uncertainty is long-term trajectory propagation error. A probability distribution can then be used to estimate the impact probability (IP) of an asteroid hitting a major body. This impact probability value is important for determining the impact risk of an asteroid to a major body.

Monte Carlo Method

The Monte Carlo (MC) method is widely used for determining impact probability. This method randomly samples a pool of Virtual Asteroids (VAs) from the uncertainty ellipsoid. Since the definite position and orbit of the asteroid is not precisely known, each VA represents a possible position and orbit where the asteroid could exist. Each VA has its own set of six orbital elements which include the semi-major axis, eccentricity, inclination, longitude of the ascending node, argument of periapsis, and mean anomaly angle (a, e, i, Ω, ω, M_o). The VAs are propagated until the end date in which the final position is recorded. A statistical tool uses these recorded positions to determine the probability of impact. The MC method, is widely used to simulate unknown, complex physical models such as the orbits of small bodies in the Solar System.

Virtual Asteroid Generation and Propagation

The first step in determining the impact probability using the Monte Carlo method is to create the VAs. A pool of random numbers lying within the interval [0,1] is generated for this purpose. Both GMAT and STK have MATLAB plug-ins allowing for the user to use built-in MATLAB random number generators.

The pool of $6*N$ random numbers is used to sample N VAs from the six-dimensional uncertainty space (a, e, i, Ω, ω, M_o). Each orbital element is independently and randomly sampled from a Gaussian distribution as shown in Figure 3.

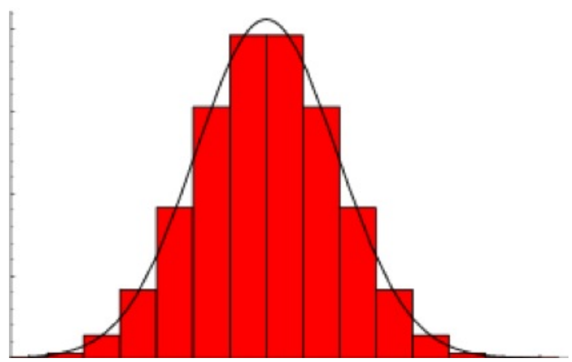


Figure 3. Example of a Gaussian Distribution.

$$a = \sigma_a \sqrt{2} \left(\text{erf}^{-1} \left(2 * \text{RAND} - 1 \right) + \mu_a \right) \tag{1}$$

The Gaussian distribution of orbital elements is achieved by taking the inverse cumulative distribution function of the random number pool.[17] The inverse cumulative distribution function, as described by Equation 1, is a function of the mean, standard deviation, and the random number sampled from the pool.

This results in the creation of one of the six orbital elements for a VA. Repeating the process five more times defines a VA. Values closer to the mean are favored to create a Gaussian distribution in the pool. This results in a mean and standard deviation equal to the values entered into the MC. In this case the mean and standard deviation are equal to those from Apophis solution #144 as provided in Table 10.[1]

A scatter plot of the virtual asteroids' semi-major axis, eccentricity, and inclination for the Apophis Monte Carlo simulation is provided in Figure 4. Each red dot represents one of the 1000 virtual asteroids created each having its own set of six orbital elements. Each axis shows the variance from the nominal values. The nominal VA is represented by a blue dot located at (0,0,0).

At this point there are some checks that can be done to prevent spurious data. The first of these is a test of the random number generator. The easiest way to do this is to find the mean and standard

Table 10. Orbital elements of Apophis at Epoch 2455800.5 (2011-Aug-27.0) TDB from Apophis Solution #144.[1]

Orbital Elements	Mean Value (μ)	Standard Deviation (σ)
Semi-Major Axis (AU)	0.922300	7.674E-9
Eccentricity	0.191076	3.6429E-8
Inclination (deg)	3.331960	1.5069E-6
Ω (deg)	204.43041	3.0196E-5
ω (deg)	126.42447	3.0819E-5
M_o (deg)	287.5823	3.0636E-5

deviation of all the random numbers generated. For a large pool, the mean and standard deviation values should be approximately equal to 0.5 and 0.34134, respectively. In the case where these values do not match, a new pool of random numbers is generated.

Another method to detect dubious data, is to compare the mean and standard deviation of all the VA's six orbital elements. In this case, they are compared with JPL's Small Body Database Apophis's Orbital Elements.[1] Having too few VAs results in a difference between the calculated and compared mean and standard deviation values.

The pool of VAs are then propagated until they reach perigee in 2036. This is the closest point to the Earth the VAs reach in 2036. Each VA's Cartesian position at perigee is recorded for use in post-processing. Post-processing for determining the impact probability begins once all positions are recorded.

Post-processing

The simplest post-processing method is executed by dividing the number of VAs that hit the Earth, also called Virtual Impactors (VIs), by the total number of VAs used in the calculation. The main limitation of such a method is that the number of VAs required is roughly equal to the inverse of the impact probability.[18] For an asteroid with an impact probability of approximately 1.0E-6, at least one million VAs would be needed in order to validate this claim. This is computationally expensive and difficult to manage; thus, alternative methods have been developed in the literature using an impact probability model of the form:

$$\mathrm{IP} \;=\; \iiint\limits_{V_\oplus} \mathrm{PDF}(x,y,z)\;\mathrm{d}x\;\mathrm{d}y\;\mathrm{d}z \qquad (2)$$

One such method of decreasing the necessary number of simulations is to use a statistical approximation. The first step in doing this is to find the three-dimensional probability density function (PDF) of the asteroid's close-approach-position. The impact probability of Apophis becomes the integral of the PDF over the volume of the Earth as defined by Equation 2. This expression can then

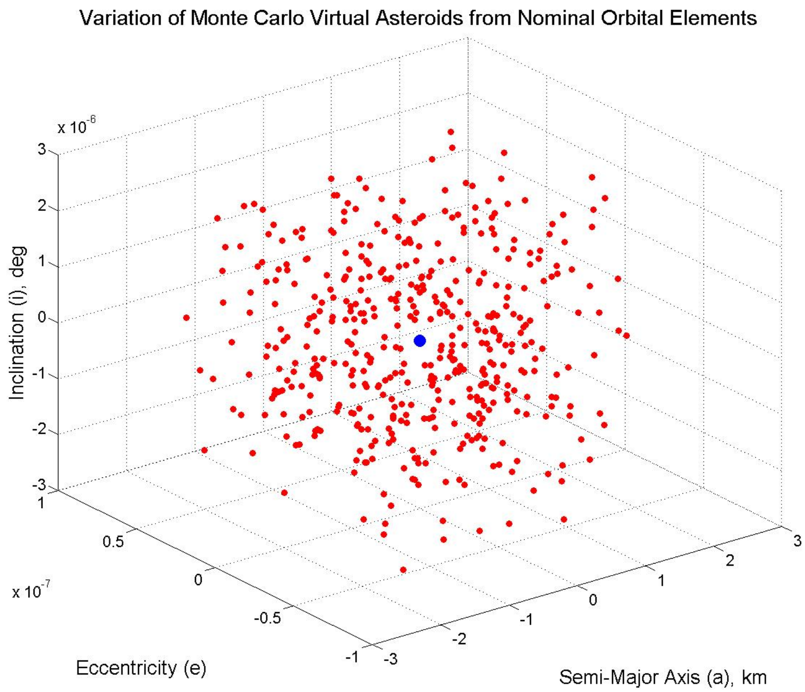


Figure 4. Semi-Major Axis - Eccentricity - Inclination Scatter Plot of VAs created by a Monte Carlo Model.

be simplified by converting it to spherical coordinates, as follows:

$$\mathrm{IP} = \int_0^{r_\oplus} \mathrm{PDF}(r)\,\mathrm{d}r = \mathrm{CDF}(r_\oplus) - \mathrm{CDF}(0) \tag{3}$$

$$= \frac{1}{2}\left[\mathrm{erf}\left(\frac{r_\oplus - \mu}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{0-\mu}{\sigma\sqrt{2}}\right)\right] \tag{4}$$

where CDF denotes the cumulative density function, μ is the mean distance from the Earth, σ is the standard deviation of the perigee distances from the Earth, and $r_\oplus$ is the radius of the Earth.

This process results in a number between 0 and 1 corresponding to the probability of impact. The MC method must be repeated with various pools of VAs to ensure accuracy. If the difference between the IPs predicted from each run is outside an acceptable range, then this implies that a larger pool of VAs is required. Having a larger pool of VAs increases computation time but results in a more accurate impact prediction.

Impact Probability of Apophis

The most basic application of various software options is finding the asteroid's nominal close-approach distance to the Earth. This value corresponds to the most probable close-approach distance and is of great use in determining how dangerous a potential impactor is to Earth. For Apophis, a comparison of the close-approach distance is made by each software starting with the same initial conditions. The conditions are set to reflect Apophis solution #144 listed in Tables 1 and 2. Starting in August 2011 and propagated to April 13, 2029, the minimum distance was found. Table 11 shows the miss distance of Apophis on April 13, 2029 during its closest approach using JPL's Small Body Database, NEODyS-2, CODES, GMAT and STK. This distance is within geosynchronous orbital altitude and, though it will not hit the Earth, it may pose a danger to Earth's satellites prompting the need for continuous observations.

NEODyS-2 presents information for near-Earth asteroids including orbital elements, physical parameters, a covariance matrix, and a risk table with a convenient Web-based interface. The risk table is calculated using an original software called OrbFit with collaboration with NEODyS/CLOMON2 team and JPL's Sentry team. NEODyS-2 is comparable and similar to JPL's NEO website. NEODyS-2 also includes close-approach tables, ephemeris files, observational data, and possible impact solutions. Table 11 shows the close-approach distance of Apophis in 2029. Figure 5 shows the 2029 close-approach of Apophis generated by GMAT.

Table 11. Nominal Miss Distance of Apophis on April 13, 2029 and Impact Probability in 2036 calculated by each software.

	JPL's Sentry	NEODyS-2	CODES	GMAT	STK
2029 Distance, km	38,067	38,342	38,078	37,859	49,211
Impact Probability	4.3E-6	4.3E-6	-	4.2E-6	-

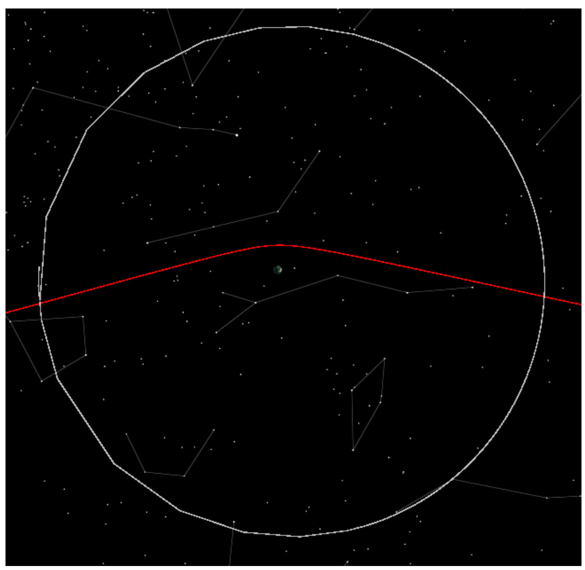


Figure 5. Nominal Trajectory of Apophis with a Close Earth Flyby on April 13, 2029, created by GMAT.

Another use of various software options is determining the impact probability for an asteroid. This impact probability number is important for determining the danger that a specific asteroid poses to Earth. The asteroid Apophis is used here for comparison. In 2036 there is a small chance that Apophis will impact the Earth. This is dependent on the asteroid going through a small 'keyhole' during the 2029 close flyby. Passing through this keyhole results in a 7/5 orbital resonance causing an impact seven years later on April 13, 2036.

The MC simulation is written in GMAT's scripting language. A pool of 6000 random numbers is created using an Inverse Transform Sampling Pseudo-Random Number Generator. The random number distribution is checked and found to have a mean equal to 0.5 and a standard deviation equal to 0.34134. This confirms that the random numbers are evenly distributed.

The pool of 6000 random numbers were then turned into orbital elements creating 1000 VAs. The distribution is confirmed to be equal to the values listed in Table 10.[1] Each set of six orbital elements is assigned to a VA, and the VA is propagated until the 2036 perigee condition. Each state vector is recorded at perigee into a report file. The created report file is then imported into Excel for post-processing. Using built-in Excel functions and Equations 3 and 4, the impact probability is then computed. A comparison of computed impact probabilities from different sources is provided in Table 11. It can be noticed that we obtained an impact probability of 4.2E-6 for asteroid Apophis on April 13, 2036, which is very close to JPL's impact probability value of 4.3E-6.

Impact Probability Concept as Applied to Disrupted Apophis Fragments

If an NEO on an Earth-impacting course can be detected with a mission lead time of at least several years, the challenge becomes mitigating its threat. When the time to impact exceeds a decade, the velocity perturbation needed to alter the orbit is small ($\approx$ 2 cm/s).[3] When the time to impact is short, the necessary velocity change may become very large, and the use of a nuclear subsurface explosion may become inevitable.[3] A common concern for such a powerful nuclear option is the risk that the deflection or disruption mission could result in fragmentation of the NEO, which could rather substantially increase the damage upon its Earth impact. Therefore, it is important to develop a computational tool to determine the impact probability of the disrupted fragments from a nuclear subsurface disruption mission.

A conservative estimation of the impacting mass for a worst-case mission scenario with a lead time of 15 days before impact is studied for a nuclear subsurface explosion with a shallow burial ($<$ 5 m) for a test case of Apophis in Refs. 2 and 3. At detonation, the energy source region expands, creating a shock that propagates through the body resulting in fragmentation and dispersal. The mass-averaged speed of the fragments after 6 seconds was near 50 m/s with a peak near 30 m/s.[2] As a result only 0.2% of the initial mass resulted in impacting the Earth if the explosion direction is aligned along the inward or outward direction of the orbit, i.e., perpendicular to NEO's orbital flight direction.[3] Such a sideways push is known to be optimal when a target NEO is in the last orbit before the impact. Obviously, the impact mass can be further reduced by increasing the intercept-to-impact time or by increasing the energy level of nuclear explosives (i.e., higher yields).[3]

A simulation of a nuclear disruption mission of a fictitious Earth-impacting Apophis trajectory is considered. Furthermore, the impact probability of the orbital dispersal fragments from this nuclear disruption mission 15 days before impact is extensively examined. Table 12 displays the modified state vector of an Earth-impacting Apophis at an Epoch date of March 29, 2036 in the J2000 Ecliptic Coordinate System. This sample test case is used as a reference which demonstrates the development of the AMiDST.

Table 12. Modified State Vector of Apophis at Epoch of March 29, 2036, in a Collision Course with Earth on April 13, 2036.

X (km)	Y (km)	Z (km)	$\dot{X}$ (km/s)	$\dot{Y}$ (km/s)	$\dot{Z}$ (km/s)
-155,168,152.5446	-23,546,881.5797	-1,508,530.2557	9.7751	-28.1744	1.1376

The fragmentation model consists of an impulsive velocity perturbation which is applied to each fragment. Typically, velocity perturbations of fragments from disrupted asteroids are non-Gaussian with a tail of high-velocity ejecta. However in this sample test case, the magnitude of the velocity perturbations is assumed to form a Gaussian distribution sampled randomly, with a mean value of 50 m/s and a standard deviation of 10 m/s. The velocity perturbation direction is sampled to favor directions perpendicular to Apophis's velocity vector. For simplicity, the velocity perturbation directions are set in the spherical coordinate system, with the origin at the asteroid's body and the x-axis along the velocity vector. By using the same Gaussian Monte Carlo method, the right ascension is sampled with a mean value and standard deviation of 90° and 45°, respectively. The declination is then sampled between 0° and 360°, randomly. This process is repeated to create N fragments each having a random velocity magnitude and direction.

STK and GMAT are used to carry out the calculations of this disruption model. STK was chosen for the illustration of this study due to its superior visualization tools and Astrogator feature. Due to the short time frame of this simulation and little deep space propagation, STK's performance for this simulation study is not affected by the discrepancies noted earlier.

A sample test case of delivering a nuclear explosive device (NED) to a fictitious Apophis impact is simulated and the results are illustrated in Figures 6-9. The simulation starts with the interception by a spacecraft carrying a NED to the threatening asteroid Apophis as depicted in Figure 6. The spacecraft intercepts Apophis on March 29, 2036, 15 days before the fictitious impact with Earth on April 13, 2036.

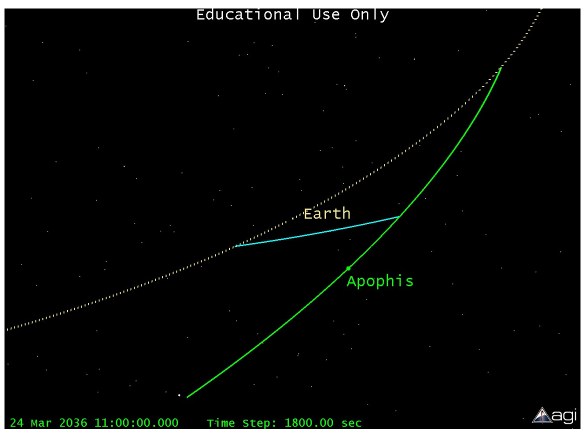
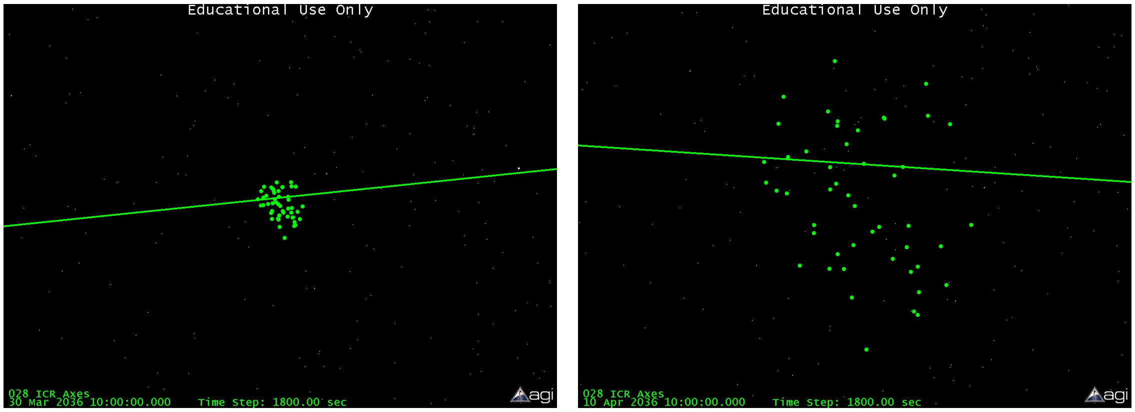


Figure 6. Nuclear Disruption Mission for the Fictitious Impact of Apophis on April 13, 2036.

A dispersion cloud of fragments is created to simulate the consequence of a nuclear disruption of Apophis.[2] This cloud is created by imposing a velocity perturbation individually on each fragment. In this visual sequence of events only 50 fragments were considered. The resulting orbital dispersion forms a debris cloud as shown in Figure 7.

Each fragment is then propagated until April 30, 2036 using the Astrogator feature in STK. Figure 8 shows these fragments being propagated as they approach Earth. If a fragment reaches an altitude of 120 km above the Earth, it is assumed that the fragment has impacted Earth. This conservative model results in a safe estimation of the impact probability. Figure 9 shows the debris cloud after Earth passes through. The debris cloud is then broken due to Earth's gravity and each fragment's miss distance is recorded to be analyzed in post-processing.



(a) 10 Hours After Explosion (b) 13 Days and 10 Hours After Explosion

Figure 7. Orbital Dispersion of Fragments after Nuclear Explosion of Apophis.

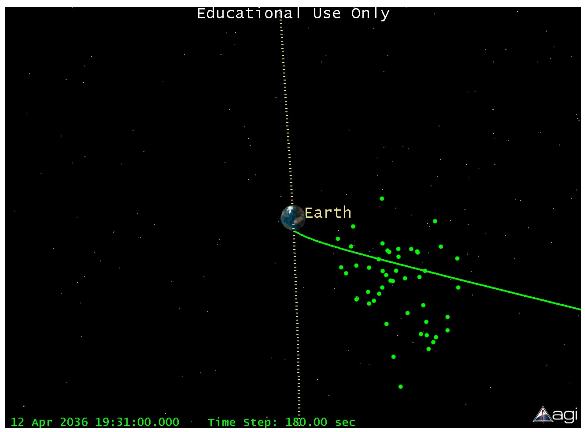


Figure 8. Debris Cloud Approaching the Earth. The size of each fragment shown here is not to scale.

The same sample model is used as reference to find the impact probability of 2000 fragments instead of 50. The recorded miss distances are used to estimate the Gaussian probability density function of the fragments' potential location in three-dimensional space. Integrating over the volume of the Earth solves for the probability of impact for each fragment. Table 13 shows the results of this sample test case, which is in agreement with the results of Refs. 2 and 3.

Table 13. Statistical Results of Nuclear Disruption of Apophis Using 2000 Fragments.

Mission Analysis Software	Mean Miss Distance (km)	Standard Deviation (km)	Impact Probability
STK	47,549	14,328	1.6E-3
GMAT	47,123	14,474	9.0E-4

This simple model was used to demonstrate the AMiDST's initial capabilities of finding the impact probability of disrupted fragments resulting from a nuclear disruption mission. The impact probability analysis of the disrupted asteroid fragments under a variety of fragmentation conditions will be needed when planning a nuclear disruption mission. This application can encompass multi-

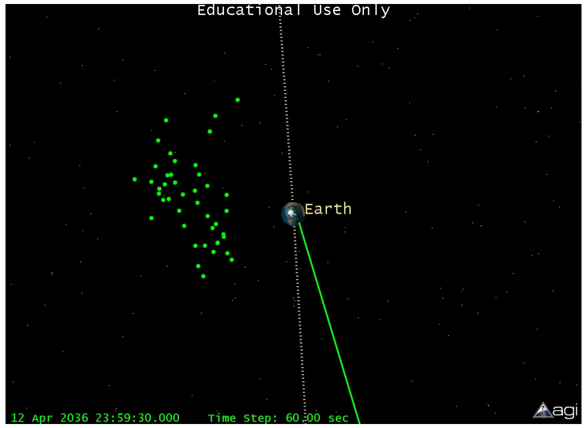


Figure 9. Orbital Dispersion of Fragments After Passing through the Earth Target Plane.

ple intercepting dates, NED sizing, and impacting trajectories, to determine an optimal solution for a disruption mission.

A more realistic fragmentation model will replace the simple model used in this paper as future research is undertaken by incorporating the study result described in Ref. 19. The next step for AMiDST is to develop the capability to enter in various statistics resulting from a nuclear fragmentation simulation using an advanced hydrodynamic code to generate N fragments in an orbital dispersion cloud. This will allow an accurate representation of the impact probability of a worst-case scenario for nuclear disruption missions.

Summary

Future research efforts include a more accurate nuclear fragmentation model and extending the applications to the asteroid 1999 RQ36. The nuclear fragmentation model is under current research efforts of the Asteroid Deflection Research Center in collaboration with the Lawrence Livermore National Laboratory. The asteroid RQ36 is NASA's new target for an unmanned spacecraft to collect samples and return them to Earth. RQ36 is an asteroid with a diameter of approximately 560 meters and an estimated impact probability of 2.8E-04 on September 2182.[19] This high impact probability makes it a potential target for a deflection mission.

CONCLUSION

In this study STK, GMAT, CODES, and JPL's Horizons have been exploited to find the close-approach distance of a reference asteroid Apophis, calculate the impact probability of the asteroid, and extend the simulation to handle a fragmentation model to study the consequences of high-energy nuclear disruption missions. The impact probability computation process has been setup through the use of a Monte Carlo method. Through post-processing, the use of a statistical model can estimate the impact probability of one body or a fragmented body. The ultimate goal of this research is the development of an interface that integrates the research efforts in the Asteroid Deflection Research Center with commercial astrodynamics software to be used by practicing engineers and researchers for real mission planning and design.

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