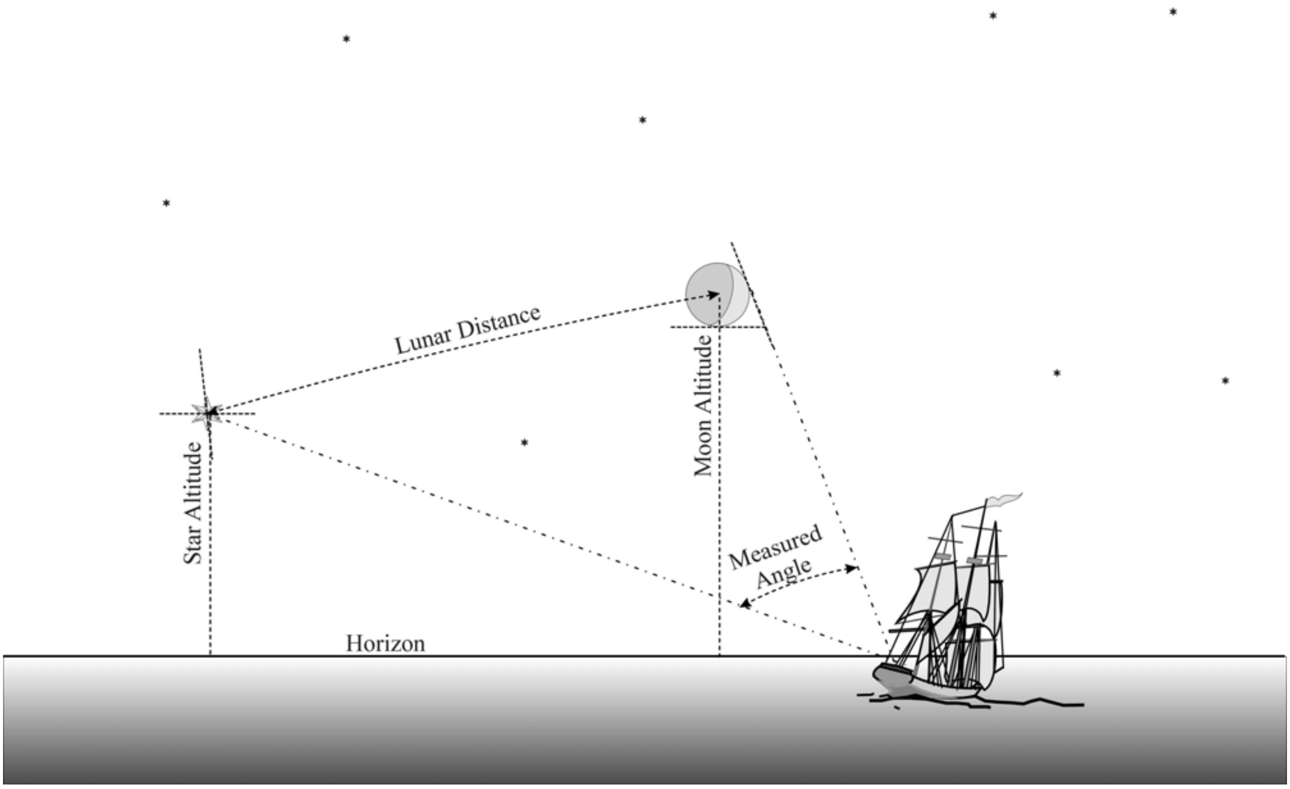
About Lunars



Observation of Lunar Distance at sea. (Sketch by Clive Sutherland).

George Huxtable. 2002
NavList

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About Lunars, Part 1.

Part 1 explains what a Lunar Distance is, how it is measured and "cleared" to obtain a Corrected Lunar Distance.

Part 2 explains how to obtain tabulated lunar distances, so that the GMT can be deduced. Some examples are given.

Part 3 will come at a future date. It will discuss the possibility of calculating the altitudes required with a lunar distance, instead of measuring them: also the question of observing Moon altitudes in place of lunar distances. It will show how to obtain local apparent time and use it with GMT to obtain longitude, as early navigators had to do.

INTRODUCTION.

I am aware that this multi-part posting is over-wordy and rambling, and really needs much tightening up. It's a try-out for NAV-L subscribers, ahead of putting the thing out in another format. So I would welcome readers' comments, particularly about any sections they have found difficult or impenetrable; preferably via the NAV-L list.

It is, no doubt about it, a complex matter.

I am grateful for many discussions with list-member Steven Wepster, and with Catherine Hohenkerk of H.M. Nautical Almanac Office. Any errors are entirely my own, however.

It's rather too much to hope that what follows will be entirely free of mistakes, and I would be grateful to have them pointed out.

I should make it clear that I don't claim to be an experienced observer of lunars at sea; they are few and far between. I would be most interested to hear from anyone that is.

USEFUL READING.

I recommend Charles H Cotter's, "The history of nautical astronomy", (London, 1968) which has all the technical details anyone might wish for, but is hard to find. For an easier read, try Derek Howes' contribution to "the Quest for Longitude", ed. Andrewes, (Harvard,1996). This is a beautiful but expensive volume, stuffed with interesting information.

I have recently become aware of "Self-Contained Celestial Navigation with H.O.208" by John S. Letcher, Jr., 1977. This has a useful section on lunar navigation, with some novel and interesting ideas, well explained. I would not agree with every word he says, though.

WHAT ARE LUNAR DISTANCES?

The phrase "Lunar Distance" confuses many. It's nothing to do with the distance of the Moon. It's the angle-in-the-sky, to be measured with a sextant, between the direction of the Moon, as seen by an observer on Earth, and some other body visible in the sky. That measurement is often abbreviated as "taking a lunar".

The purpose of a lunar is to obtain the Greenwich Mean Time, from anywhere on Earth. I will stick to the old abbreviation GMT, but for those that prefer to be more modern, it's indistinguishable in practice from UT.

A lunar uses the precise position of the Moon in the sky, in relation to other bodies, to provide the Mean Time at Greenwich. With a really trustworthy chronometer, or an accurate quartz watch, on board, there is no need for a lunar. In these days of radio signals, against which the watch or chronometer can be checked, and GPS, which gives a precise position in a flash, lunars are an anachronism. Why bother, indeed?

You have to put yourself back into the 18th or 19th century, times when chronometers didn't exist, or were ruinously expensive. If a tiny speck of grit got into the works of your one chronometer, and its rate changed, then it could lead you into danger. There were no radio time signals to put you right: you were on your own. If you had a second chronometer, and they started to disagree, how could you tell which was in error? Really three chronometers were needed, to be safe. Few mariners could afford such an expensive luxury.

Well into the 1800s, then, mariners would use lunars instead of a chronometer (or if they had one, as a check on its going). The last exponent of the lunar art was probably Joshua Slocum, sailing "Spray" around the world single-handed in 1895-1898.

The user of lunars needed a pre-calculated table of lunar distances, which Nautical Almanacs carried until 1906. Because lunars had by then fallen into disuse, these tables were dropped. That was the end of lunars.

In those days of longhand calculation, the complex motion of the Moon made accurate prediction of its position a very tedious matter. Nowadays, with a pocket calculator, portable computer, or modern almanac, you can do in seconds what once took almanac-calculators hours. It is now possible for you to obtain lunar predictions for yourself. Lunars are back in business again, for anyone who wishes to follow-up the early days of astronavigation.

HOW DOES IT WORK?

If you go out on a clear night, when you can see the Moon against a background of stars, take a good look and remember just where it is placed against those stars. Now go out on the next night and look again, and you will see that the Moon has shifted very noticeably against those same stars, toward the East. The movement in 24 hours is about 12 degrees. Why? Because the Moon travels right around the star background, 360 degrees, in the course of its motion round the Earth in a month. It's the fastest-moving object in the sky, with respect to the stars, by far.

Even over an hour, if you observe closely, you will see that the Moon has moved by its own diameter, roughly speaking. And another observer, elsewhere on Earth, who can see the Moon at the same time, will see it in the same position against the star background (give or take some important corrections). So observers, anywhere in the world, by observing where the Moon is in the sky and making those corrections, can synchronise their watches to an agreed time scale, that of Greenwich Mean Time, or GMT. They are using the exact position of the Moon in the sky as a clock that anyone in the world can read.

The position of the Moon can also, more conveniently, be measured against the Sun, and also against the planets. All these move much more slowly against the stars than the Moon does.

There are some snags, however-

1. The Moon is a clock that moves only very slowly. In an hour, it moves only 1/2 degree or so. IF you can measure its position to 1 minute of arc, this corresponds to 2 minutes of time. This is about the best accuracy that you can realistically hope for. If you have some sort of timekeeper, you could then set it, from the Moon, to that precision, two minutes of time, and use it with an altitude to determine your longitude.

The trouble is, a timekeeper that's only been set within two minutes is not a very good one. Because the whole sky appears to move around at 15 degrees in an hour, in that 2-minute-of-time error interval it moves 30 minutes of arc. As a result, your longitude has been determined to a precision of only 30 minutes of arc, or 30 miles at the equator. After experience with GPS, you may wonder whether such a crude position is any use at all. It should be clear, now, how important it is to achieve the utmost precision in the lunar distance observation.

In the late 1700s, navigators were delighted when lunars became available. Until then, they had only dead-reckoning, which could put them thousands of miles out on a long voyage with unknown ocean currents. Cook thought that the 30-mile accuracy of a lunar longitude was all that navigators would ever require (how wrong he was!)

2. How do you measure, precisely, the position of the Moon in the sky as it travels round its orbit? Unfortunately, the sky isn't marked out with convenient lines of declination and right-ascension. At a fixed observatory, a quadrant could be set precisely in the ground, and the Moon position could be read off. But that was no use to a mariner.

This was resolved in the 1760s by the contribution made by Maskelyne, with Mayer. He invented the first pre-computed tables, giving at 3-hour intervals, the predicted angles, to be compared with angles measured in the sky, between the Moon and other objects. For each day, he would select a few convenient objects, some to the East of the Moon, some to the West, which were near to the Moon's path across the sky, and at a convenient angle from the Moon for measurement. The Sun was included when its angle to the Moon was in the range 40 to 120 degrees. Around New Moon, the Moon was invisible for a few days, so lunar distances were then useless, and those days were omitted from the tables.

HOW IS A LUNAR DISTANCE MEASURED?

It is presumed that anyone reading this will be familiar with the use of a sextant to measure the altitude of a body in the sky, make the necessary corrections, and obtain a position line.

The difference with a lunar is that the observer is not measuring an altitude, straight up from the horizon. Instead, he wants the angle between the Moon and another body, up in the sky. The other body is often the Sun, but it might be a planet or one of those stars.

For the method to work properly, the other body has to be somewhere near to the path of the Moon around the sky. The Moon is always within 5 degrees or so of the ecliptic (which is the path of the Sun). So the Sun always meets that requirement, as do the bright planets. Only a limited number of stars are acceptably close to the ecliptic, and lunars are limited to using a selection from these stars, which are **Altair, Fomalhaut, Hamal, Aldebaran, Pollux, Regulus, Antares, Spica, and Markab**.

For now, we will consider the Sun as the other body, but bear in mind that it might be a planet or a star.

The horizon plays no part in the lunar distance observation itself, and the view, straight through the sextant, is instead used to observe the fainter of the bodies observed (usually the Moon). The brighter one (usually the Sun) is viewed in the index mirror. To see them both, the sextant may have to be tilted at what seems like a most unnatural angle, pointing up in the air, and over on its side, even upside-down. This is something that just has to be accepted. The index is then adjusted until both bodies can be seen. This is a rather hit-and-miss affair, and is a test of the lunar observer's skill, especialy in rough weather. It is much easier to get both bodies in view if the angle between them can be estimated roughly beforehand. Of course, the shades have to be used appropriately.

Once the two bodies are both in view, it's necessary to bring the images of the limbs of Sun and Moon together so that they just (and only just) "kiss". The observer rocks the sextant a bit so the two bodies just skim past each other, in a similar way to taking an altitude against the horizon. Their limbs just touch,as they pass, with no overlap. This part of the operation demands the utmost possible precision. The two discs close, or part, very slowly, so it's not practical to wait for the kiss to occur, but to achieve it by turning the sextant knob. The moment when this occurs is the moment for which GMT will be obtained, so that time should be recorded using the best timepiece available. At the end, it's the error in this timepiece that will be determined.

The sextant reading should be noted, to the utmost accuracy possible. On a big ship in calm weather, or from land, it may be possible to observe this lunar distance to a small fraction of a minute of arc. This is rather a forlorn hope from a small craft, however.

Usually (depending on its phase) the Moon shows a fuzzy, shadowed limb, and a crisp, rounded limb, which is always the limb closest in angle to the Sun. The Moon limb that is measured to is always the crisp round limb. Sometimes, with a star-or-planet lunar, that will be the further limb, measured "across" the Moon disc from your object, not the nearer. It's important to note which, because it affects whether the correction for Moon semidiameter is to be added or subtracted.

To increase accuracy, it's recommended to repeat measurements of lunar distance at very short intervals, and average the angles and the times. If an odd number of measurements is taken at equal intervals, this becomes easy. The index error of the sextant should be checked well, preferably before and after.

Because the sextant may need to be pointing up in the air, it may be useful for the observer to lean back on a seat, or even lie down. However, it's necessary for his head and the sextant to remain free to move, to compensate for the motion of the vessel. He has to find a position where both bodies he measures are unobscured by sails, but it doesn't matter if a sail comes across some of the angle between them.

There are many corrections required to be made to the measured lunar distance angle, and they all need to be made as precisely as possible. For this purpose, it is necessary to measure the altitudes, above the horizon, of the Moon and the other body involved, at the same average time, or nearly so, as the lunar distance was measured.

In the Navy, this could require three observers with three sextants altogether, and some ceremony. However, others in more straitened circumstances can make do with a single observer making all observations in a carefully timed sequence: first Sun alt., then Moon alt., then several lunar distances, then Moon alt., then Sun alt. In this way, the averaged altitude observations can be at the same averaged time as the lunar distances.

The corrections are such that the Sun altitude is not needed to great accuracy, but the Moon altitude should be measured with precision.

For the Sun altitude, the lower limb can be aligned with the horizon, as usual with any Sun altitude observation, and then semidiameter will later be added. For the Moon altitude, the upper or lower limb has to be chosen, whichever is the round limb, and the fuzzy shadowed limb avoided. The observer must note whether upper or lower was chosen to align with the horizon, because the Moon semidiameter will later be subtracted, or added, respectively, to the observed altitude.

Now all the observations have been made and noted down, and the calculations can start.

CALCULATIONS REQUIRED.

Get ready for quite a lot of mathematics now. It's an unavoidable part of working lunars.

The problem is this. The compilers of the lunar distance tables do not know where on Earth a navigator is going to be who is using them. The best they can assume is that he is at the centre of a transparent Earth, ("geocentric"), in which case parallax and refraction would always be zero. However, a real navigator will be somewhere on the Earth's surface, so he will have to make allowance for the effects of parallax and refraction before the tables can be any use to him.

Parallax and refraction shift only the altitude of the Moon and the Sun (or other body), hardly at all the azimuth. However, the lunar distance is generally in a direction that is skewed to those shifts. So the way it is affected by those shifts is rather complex, and requires a knowledge of the altitudes of the centres of the two bodies, the combined correction for (refraction + parallax) for each, and the measured lunar distance angle.

Refraction corrections are in general not very large (a very few minutes at most, provided altitudes do not go below 10 degrees). It's still important to correct for refraction as exactly as possible. In comparison, the parallax correction for the Moon can be immense (up to 1 degree) and changes rapidly through the day: it's a dominant factor.

It's necessary to aim for the highest possible standard of precision. In perfect conditions, by averaging many measurements, one might aim to get a measured lunar distance to, say, 0.2 or 0.3 minutes. To retain this precision, the many correction terms should be estimated, where possible, to within 0.1 minutes, and the calculations carried out to that same accuracy. The Nautical Almanac tabulates positions of Sun and Moon to 0.1 min

If printed tables of logs, trig functions, and log trig functions are to be used, then 5-figure tables are required. Generally, scientific pocket calculators will provide sufficient accuracy. If a computer program is being used, which offers a choice between single- and double-precision, double-precision should be chosen. If no such choice is offered, then it is likely that the program is inherently precise enough.

It's up to you whether to do your calculations in degrees-and-minutes, or in decimal degrees. It depends somewhat on whether you are using tables or else a computer or calculator. Beware, although some calculators offer conversions between the two, some get it wrong, especially for negative angles.

INITIAL STEPS.

First, average the lunar distance measurements and the altitudes, in such a way that the averages correspond to the same moment of time.

Correct all three sextant observations for scale calibration errors (from the certificate in the sextant box) and index errors. Correct the Sun and Moon altitudes for dip and semidiameter (subtracting if upper-limb). Correct the observed lunar distance by adding the semidiameters of the Moon and (if it's used) the Sun (Moon semidiameter may instead have to be subtracted, in the case of a star or planet, if its distance was measured "across" the Moon).

For the semidiameter of the Sun, you can take the daily figure (in minutes) from the almanac, and convert to degrees as required. For the Moon semidiameter, make a very rough guess at GMT (within a couple of hours will do), look up Moon's Horizontal Parallax (HP), tabulated each hour in the Nautical Almanac, convert to degrees, and multiply it by .2724. Then multiply by (1 + Sin (alt)/55), which is the "augmentation of Moon's semidiameter". This is better than simply using a tabulated value for the Moon's semidiameter.

This work results in three quantities, d (apparent lunar distance between Sun and Moon centres); also s (apparent Sun-centre altitude) and m (apparent Moon-centre altitude), measured up from the true horizontal. Altitude s may apply to a star or planet, instead of to the Sun.

The apparent lunar distance d is slightly affected by the effects of refraction and greatly affected by the effect of parallax (mainly that of the Moon). However, because of the skewed angle that the lunar distance makes in the sky, these effects combine in a rather complex manner. Allowing for these combined effects is what's known as "clearing the lunar distance". It provides the true lunar distance, which we denote D.

CLEARING THE LUNAR DISTANCE.

Cotter goes through many methods for this operation, and mentions many more. He refers to a paper published in 1797 which described forty such methods! I propose to refer to Young's method only (J.R.Young, Practical Astronomy, 1856).

Correct both observed altitudes, m and s, for refraction, using whatever method suits you best. If atmospheric conditions are abnormal, correct refraction for temperature and/or pressure beforehand. The refraction correction to the apparent altitude is always a SUBTRACTION. The altitudes must also be corrected for parallax. For the Moon, where getting its parallax right is very important indeed, look up the current value of Horizontal Parallax as before, apply "Reduction of the Moon's horizontal parallax" by multiplying it by (1- (sin(lat))^2/300), multiply by cos(alt), and that is the parallax correction, which always INCREASES the apparent altitude (and always greatly exceeds the refraction). The symbol ^2 means "to the power 2", or more simply "squared".

For the Moon, then, the corrections are such that M will always exceed m, and for the Sun (or other body) S will always be less than s.

So now we have the apparent and true altitudes of the Sun (s and S) and the Moon (m and M), and the apparent lunar distance d. We require the true lunar distance D, in which the effects of refraction and parallax have been allowed for.

For this, Young gives-

cos D = (cos d + cos (m+s))*(cos M*cos S)/(cos m*cos s) - cos (M+S)

or D = acos ((cos d + cos (m+s))*(cos M*cos S)/(cos m*cos s) - cos (M+S))

where acos means "arc cos" or "the angle whose cosine is"

The derivation is given in Cotter page 213.

Because it relies in finding an arc-cos to obtain D, this will be rather imprecise around lunar distances of 0 deg and 180 deg, but that's not a real drawback for our application.

Note that all angles should be in degrees unless your program will work only in radians, in which case it's necessary to convert by multiplying or dividing by 180/pi appropriately.

Also note that some programs such as Quickbasic do not provide functions such as acos (arc cos) and asin (arc sine). However, atn (arc tan) is provided. Instead these functions must be obtained indirectly using

acos(x) = atn ((sqr(1-x*x))/x)

asin(x) = atn (x/(sqr(1-x*x)))

where "sqr" means "square root of".

At last, this provides a corrected lunar distance, D, which can be compared with tabulated lunar distances to obtain Greenwich Time.

This is the end of part 1. Part 2 explains how to obtain tabulated lunar distances, so that the GMT can be deduced. Some examples are given. Part 3 will come at a future date. It will discuss the possibility of calculating the altitudes required with a lunar distance, instead of measuring them: also the question of observing Moon altitudes in place of lunar distances. It will show how to obtain local apparent time and use it with GMT to obtain longitude, as early navigators had to do.

About Lunars, part 2.

Part 1 explained what a Lunar Distance is, how it is measured and "cleared" to obtain a Corrected Lunar Distance.

Part 2 explains how to obtain tabulated lunar distances, so that the GMT can be deduced. Some examples are given.

Part 3 will come at a future date. It will discuss the possibility of calculating the altitudes required with a lunar distance, instead of measuring them: also the question of observing Moon altitudes in place of lunar distances. It will show how to obtain local apparent time and use it with GMT to obtain longitude, as early navigators had to do.

TABULATED LUNAR DISTANCES.

Until 1906, the Nautical Almanac carried tables of predicted lunar distances, at 3-hour intervals, for selected bodies. The navigator, having obtained a corrected lunar distance (as detailed in part 1), now had to enter the lunar distance table to find two predicted lunar distances for that body, 3 hours apart, which bracketed his observed lunar distance. At that point, he knew that the GMT of his observation was somewhere within the three-hour interval between those entries.

The process of finding the exact GMT time of the observation is one of inverse linear interpolation, which is simpler than it sounds. Given a tabulated lunar distance D1, at a GMT time T1 (in whole hours), earlier than the observation, and a distance D2, at a time T2, (in whole hours), later than the observation, then assuming a linear change, the rate of change over the period is (D2-D1) / (T2-T1) degrees per hour.

If the observed lunar distance at time T (as yet unknown) is the measured, corrected, quantity D, as determined in part 1, we know that

(D-D1) / (T-T1) = (D2 - D1) / (T2-T1), by simple proportion.

This assumes that the rate-of-change of lunar distance remains constant over the period of up to 3 hours. So we can deduce-

T = T1 + (T2-T1)*(D - D1) / (D2 - D1), where T2-T1 can be chosen as 1, 2, or 3 whole hours, giving the GMT time of the observation in hours.

This is the Inverse Linear Interpolation formula.

The lunar distances are always treated as positive values, but they may be increasing or decreasing with time, so the sign of their difference has to be treated properly. Also, when the period in question crosses over midnight, the step of 24 hours must be allowed for in a sensible way, of course.

Now, here's the problem. Those tables have not been published for 100 years, nearly. Without them, how is an observer to calculate D1 and D2?

CALCULATING PREDICTED LUNAR DISTANCES FOR YOURSELF.

The basic data required for the calculation is the GHA (Greenwich Hour Angle) and dec. (declination) of the Moon, and the same for the Sun (or other body), at the two chosen times, T1 and T2. These numbers can come from the following sources-

1. The Nautical Almanac provides predictions, at every exact hour of GMT, of the GHA (Greenwich Hour Angle) and Dec (declination) of the Sun and Moon, Venus, Mars, Jupiter, Saturn, and (for predicting star positions) Aries, all to 0.1 minutes of arc.

2. Many navigators have access to a computer program which calculates these quantities, just for the asking, at any specified moment, GMT.

3. Others have written their own program (mine runs, painfully slowly, on a Casio pocket calculator), using the basic astronomical data provided by Jean Meeus, Astronomical Algorithms, 1998.

Any of these sources will provide, for the Moon, Dec.m and GHA.m and for Sun or other body, Dec.s and GHA.s, at any integral hour, GMT.

Finding, at any time, the angular distance between the predicted positions of the two bodies, in degrees, is identical with familiar problems in spherical trig: obtaining the zenith distance of a body seen by an observer, or finding the great-circle distance between two points on the (assumed spherical) Earth's surface.

The Predicted Lunar Distances, that is, the angles D1 and D2 between the Moon and other body, in degrees, at the two times T1 and T2, are calculated using-

Predicted Lunar Distance = acos ((sin Dec.m*sin Dec.s) + (cos Dec.m * cos Dec.s * cos (GHA.m - GHA.s)))

(For comments on symbols and on limitations of some computer trig functions, see part 1)

Again, use 5-figure tables, or double-precision arithmetic, to preserve the required accuracy, to 0.1 minutes. Remember to treat North declinations as positive, and vice versa.

For anyone preferring to use trig tables and logs, rather than electronic calculation, I would recommend the cosine-haversine method (as for zenith distance), to be found in Norie's tables. If care is taken with interpolation, these can be worked to 0.1 minutes accuracy.

I know of no precomputed altitude tables that are appropriate for calculating the Predicted Lunar Distance. In general, they don't work to sufficient accuracy, and only cover a range of 90 degrees for the result.

What has just been described is the range of tools available for calculating the Predicted Lunar Distances, D1 and D2, at chosen times T1 and T2. The task that Maskelyne set himself in 1766 was to precompute and tabulate those distances at 3-hour intervals for a useful selection of bodies. Now you have to use one of the methods above instead, which may seem complicated, but are not a large part of the overall complexity of taking a lunar.

HOW ARE THE TIMES T1 and T2 TO BE CHOSEN?

First, the navigator has to make a guess at the GMT of his lunar observation, to within an hour or so. This should not be difficult, but if it turns out to have been a bad guess, it can be readjusted later.

Two times T1 and T2 should be chosen, at integral hours GMT, which should bracket the estimated GMT of the observation. They should not be more than three hours apart, or the assumption of linearity will be endangered. The Dec and GHA of the Moon, and of the Sun or other body, should be calculated, or extracted from the tables, at those times, and the two distance-angles calculated, D1 and D2. If these do not bracket the measured, corrected, lunar distance D, different hours are to be chosen for T1 and T2, using common-sense. Apply the Inverse Linear Interpolation formula, shown above.

T= T1 + (T2 - T1)*(D - D1) / (D2 - D1), with times measured in hours.

This will, at last, provide the GMT at the time of the observation.

Once GMT is known, it can be compared with the time that was noted on the deckwatch to provide a correction (to within a couple of minutes) to that deckwatch, for as long as it continues to provide reliable time.

It can be used to check our ship's chronometer (if one is carried) in the same way.

It can be used directly to calculate the altitude and azimuth of one or both of the observed bodies, and then obtain one or more position lines.

In Cook's time, position-line navigation had not been invented. Instead, navigators thought about latitude and longitude as two quite-separate quantities. Latitude, of course, was easy. Longitude was obtained from the difference between the local time by the Sun, and GMT, allowing for the "equation of time".

Part 3 will explain how this was done, and will pick up a few more points about lunars.

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At this point, it would be useful to introduce some actual lunar-distance measurements, taken out in the Atlantic Ocean. These were made, not by me, but by my e-mail friend Steven Wepster, the only man I know who actually takes lunar distances from a small craft (8 metres).

FIRST EXAMPLE.

Date, 2001 April 02. Estimated time, something between 1700 and 1800 GMT. DR position, 35 deg N, 015 deg 30 min W. Sun lunar taken at 17:36:43 by the watch. What is the error of that watch?

Choose T1 to be 15:00, and T2 to be 18:00 GMT (these are the same times Maskelyne would have chosen, times at 3-hour intervals which bracket the estimated time of the observation.).

Start off by calculating the Predicted Lunar Distance for each of those times.

From the 2001 Nautical Almanac on 2 April- at T1, 15:00 GMT

Sun dec (Dec.s) = N 05 deg 06.4 min, GHA.s = 044 deg 07.6 min.
Moon dec (Dec.m) = N 21 deg 59.8 min, GHA.m = 295 deg 17.9 min.

Converting these angles to decimal degrees, and using the Predicted Lunar Distance formula given above, I get from a pocket calculator D1 = 105.350 deg for the lunar distance, as a decimal-degree quantity. In degrees and minutes it is 105 deg 21.0 min.

Using the cosine-haversine formula and Norie's tables, predicts a lunar distance of 105 deg 20.9 min. So that shows very satisfactory agreement between the different methods

Now do the same for T2, 3 hours later at 18:00 GMT.

Sun dec (Dec.s) = N 05 deg 09.3 min, GHA.s = 089 deg 08.1 min.
Moon dec (Dec.m) = N 21 deg 46.2 min, GHA.m = 338 deg 31.0 min.

Using the calculator I get D2 = 107.000 deg, or 107 deg 00.0 min, for the predicted LD. Calculated by cosine-haversine from Norie's, this is 107 deg 00.1 min. Again, excellent agreement between the two methods.

You can see that in three hours, the Moon has moved further from the Sun by 1.65 degrees, or 0.55 degrees (33 minutes of arc) in each hour.

I have shown two alternative methods, but you simply have to choose one, whichever method is more appropriate for you.

I have shown the Sun and Moon predicted positions extracted from the Nautical Almanac, but instead, if you have the gear and the program, these quantities could be computed from first principles instead, at the appropriate times. This would avoid transcription errors. My own pocket calculator, programmed to use all the harmonic terms in Meeus, generally agrees with the Nautical Almanac, sometimes showing a difference of 0.1 minutes of arc. But it's dreadfully slow, taking 5 minutes for each Moon prediction! If you could do the job must faster than that, you could precalculate and print-out lunar distance tables, at 3-hour intervals, covering long periods, to rival anything Maskelyne could do.

But if all you need to do is to measure the occasional lunar distance, my suggestion is that it's more sensible to make that measurement, then just compute the predictions at two times, within three hours, that bracket the time of the measurement, as part of the data-reduction process.

To summarise what has been achieved so far, we have calculated two values of Predicted Lunar Distance, 3 hours apart. Each is a value for the angle between the centres of the Sun and Moon, as if they were viewed from an observer at the centre of a transparent Earth. Our observed, corrected, lunar distance ought to lie between those values, if we have estimated GMT correctly.

We have substituted our own calculation in the place of the Lunar Distance tables (which are no longer available), just as we set out to do.

THE NEXT STEP: DEALING WITH THE SEXTANT READINGS.

We have not, yet, even considered the sextant observations that were made at this watch-time of 17:36:43 on 2 April 01..

Assume that the raw sextant readings have already been corrected for any known sextant calibration errors and for index errors.

The measurements were taken in the following sequence: Sun LL., Moon UL., Lunar Distance, Moon UL., Sun LL. (LL means Lower-Limb, etc). If the mean times of the Sun and Moon altitudes didn't correspond to the watch-time of 17:36:43, corresponding small adjustments were made to the altitudes. Assume all this has been done.

First thing to do is to correct the observed Sun and Moon altitudes for dip and for semidiameter. The dips may differ slightly, as the bodies may have been observed from different parts of the vessel, with different height-of-eye.

Observed Sun Altitude, lower limb above horizon. 20 deg 56.6 min. Subtract dip of 2.2 min. Add semidiam. (16.0 min from almanac). So centre of Sun is 21 deg 10.4 min above the true horizontal. This is s.

Observed Moon Altitude, upper limb above horizon, 50 deg 10.7 min. Subtract dip of 1.8 min. Subtract Moon semidiameter (subtract because upper-limb was measured). Calculate semidiameter from Moon HP (from the Almanac, for that hour) * .2724 *(1+sin (alt)/55). As the HP was 59.4 min, this gives a SD for the Moon of 16.4 . The result is that the Moon centre is 49 deg 52.5 min above the true horizontal. This is m.

Next stage is to make refraction-and-parallax corrections, to s and m, to obtain the corrected altitudes S and M, as follows-.

For the Sun, take the altitude s of 21 deg 10.4min. Subtract the refraction appropriate to the altitude, 2.4 min. Add the Sun parallax. The Sun horizontal parallax is only 0.15 min, and the parallax itself is then 0.15*cos(alt), so working to an accuracy of 0.1 min, a sensible rule might be: take the Sun Parallax as 0.1 min for altitudes up to 70 deg, and above that, as zero. We then get the corrected Sun altitude S to be 21 deg 8.1 min.

For the Moon, take the altitude m of 49 deg 52.5 min, and subtract the appropriate refraction for that altitude, 0.8 min.

Add the Moon parallax. Compared with everything else this is an immense, dominant, correction. We have already observed that the Moon's HP, for that hour, from the almanac, was 59.4 min. As noted in part 1, this should be multiplied by cos(alt), and also by the Reduction (1-(sin(lat)^2)/300), ending up with a parallax of 38.2 min. So the corrected Moon altitude M is 50 deg 29.9min.

What was all that work for? For the lunar itself, we aren't going to need those altitudes any further. We worked out those altitudes solely because they enter into the correction for the Lunar Distance, as will be seen.

NOW TO DEAL WITH THE LUNAR DISTANCE, AT LAST!

After correction for sextant errors, the Observed Lunar Distance between the near limbs of the Sun and Moon was 106º 50.5’. There is no dip to consider because the horizon is not involved in the measurement.

Because we want the angle between the centres, not the limbs, the semidiameters of Sun and Moon must be added to the observed lunar distance. The SD of the Sun for that day was obtained from the Almanac as 16.0 min.

For the Moon (see part 1) we take the HP at the estimated GMT, of 59.4 min. From this, as shown above, the augmented SD of the Moon for that hour was calculated to be 16.4 min. So we get the apparent angle between the centres of Moon and Sun to be 107 deg 22.9 min. This is the angle d which we now have to put through the "clearing" process, correcting it for refraction and parallax, to obtain the cleared Lunar distance D, as seen by a geocentric observer, to compare with predictions.

The formula needed (from part 1) is Young's method-

D = acos ((cos d + cos (m+s))*(cos M*cos S)/(cos m*cos s) - cos (M+S))

We have all the necessary angles, summarised below, to put into that equation and obtain D.-

```
d = 107 deg 22.9 min or 107.381 deg
m = 49 deg 52.5 min or 49.875 deg
M = 50 deg 29.9 min or 50.498 deg
s = 21 deg 10.4 min or 21.173 deg
S = 21 deg 08.1 min or 21.135 deg
(m+s) = 71.048 deg
(M+S) = 71.633 deg
```

The end result is that D = 106.8209 deg, or 106 deg 49.3 min.

CLEARING THE LUNAR DISTANCE, LONGHAND.

The formula above is fine for electronic computation, but quite unsuitable for longhand calculation using logs. The trouble is that some of the quantities may go negative, and the log of a negative number is meaningless.

In the era of lunar distances, the navigator relied on 5-figure logs and trig tables. The many different ways devised for clearing the Lunar distance were mostly devoted to expressing it in such a way that logs could be used for the solution, sometimes using auxiliary tables designed for the purpose.

If readers find the need to do this clearance longhand using logs, my suggestion is to use Borda's method, to be found in Cotter. On request, I will spell it out for the list.

THE FINAL STEP, TO GET GMT.

We are nearly there, having ended up with a cleared Lunar Distance of 106.821 deg. Look back at the predicted Lunar Distances that were calculated for 15:00 GMT (105.350 deg.), and for 18:00 GMT (107.000 deg.), and it is clear that our lunar distance fits nicely somewhere in the gap between them. That means that we estimated sensible times for calculating those predictions. If that had turned out not to be so, we would have to choose different times and calculate new predictions.

Now the Inverse Linear Interpolation formula is needed, from the beginning of Part 2, as follows.

T = T1 + (T2-T1)*(D - D1) / (D2 - D1)

where T2-T1 can be chosen as 1, 2, or 3 whole hours, giving the GMT time of the observation in hours.

T1 = 15 hrs., T2 = 18hrs.
D1 = 105.350 deg., D2 = 107.000 deg., and D = 106.821 deg.

This gives T = 17.6745 hrs GMT or 17:40:28 GMT. THE ANSWER!!!

As the deck-watch read 17:36:43 GMT at the moment of lunar distance measurement, the conclusion is that it is slow by 3 min 45 sec.

And you might well think: what a complex way to determine the time!

==============================

I have worked through the above example step by step, in somewhat painful detail, to illustrate what has to be done. If any reader finds an error, or disagrees with any point, or simply fails to understand, please contact me, preferably via the NAL-L list, and I will do my best.

Now, for anyone that is still with me, here is another example for you to work out for yourself. This is another observation made at sea by Steven Wepster.

2001 April 08, DR position 44N 012W. Lunar between Moon and Mars. Deckwatch time 03:22:05 GMT.

Observations corrected for index error.
Moon observed altitude (LOWER limb) 31 deg 14.8 min, dip 2.5 min
Mars observed altitude 17 deg 50.4 min, dip 2,2 min
Lunar distance, Mars to Moon FAR limb 65 deg 11.9 min (i.e. measured ACROSS Moon).

Some hints-

This measurement was made at almost exactly full Moon. The Moon edge was therefore clear and sharp all round and any limb could have been used. Which limb was actually used is stated above. Think hard (draw a picture) about which way you need to correct, using the Moon semidiameter, to obtain the lunar distance between centres.

Mars is treated as a point so has a semidiameter of zero. Consider the Mars observation as requiring a parallax correction of 0.2 min. of arc. This can be checked from page A4 at the beginning of the Nautical Almanac, under "additional corrn" for Mars: this is the parallax correction. This correction was greatest in June 2001, when Mars was closest to the Earth, at opposition.

See if you can work out the GMT of the observation, and therefore the error of the deckwatch.

In case you don't possess a 2001 Nautical Almanac, here are some relevant quantities you might need-

```
2001 April 08
03:00 GMT
Moon dec S 02 deg 31.5 min Moon GHA 042 deg 49.3 min
Mars dec S 23 deg 08.3 min Mars GHA 338 deg 55.7 min
06:00 GMT
Moon dec S 03 deg 13,2 min Moon GHA 086 deg 17.9 min
Mars dec S 23 deg 08.6 min Mars GHA 024 deg 00.5 min
Moon horizontal parallax from 03:00 is 59.3 minutes, not changing
over 3-hour period.
```

==================================

FURTHER CORRECTIONS, INDEED.

You may think you have seen enough in the way of corrections, but I am aware that there are further additional corrections which I have neglected, partly because I am unsure about the details.

One is due to further effects of the Earth's ellipticity on the parallax of the Moon, which can cause the apparent position of the Moon to shift in both altitude and azimuth, by amounts up to 0.2 min., depending on the Moon's azimuth.

Another is caused by differences in the refraction at the upper and lower limbs, for both Sun and Moon, which makes both bodies appear to be slightly elliptical. When accounting for semidiameters in correcting a lunar distance, it is assumed that both bodies are round. There is a small correction to be made to put this right, which has effect mainly at low altitudes.

There may well be even more corrections which could be made. I would be grateful to any listmember who can supply information about these matters.

==================================

JUST THINK....

By now you should have a good idea about the complexity of the lunar-distance procedures. Right through the lunar-distance era, navigators have had to go though the same calculations described here, with the exception of the predicted lunar distances: they could get those from an almanac, and you can't.

If you go back in time to the early and middle 1800s, on any day there would be literally thousands of ships plying the oceans (far, far more than today). Mostly, their navigators would be struggling with lunar distances, with a quill pen or a slate, under oil-lamps or candles at night, with only log tables to ease their arithmetic. If they made an arithmetical slip, there was little to check it against. It could, literally, put them on the rocks: and often did.

They were, generally, men of little education, who had been trained to go through the motions of these calculations by rote. My guess is that only a few really understood what they were doing.

They were in a dangerous trade, and most ships ended their days from the hazards of the sea rather than from old age. To some extent, that must have been true of the mariners as well.

As I have wrestled with the lunar task that they had to face, I have appreciated, more and more, their achievements. I hope you agree.

I have also come to appreciate the work of Maskelyne, who set up the whole procedure with Mayer's help. They stood on the shoulders of their predecessors: mainly French, but not forgetting Newton.

==================================

Part 1 explained what a Lunar Distance is, how it is measured and "cleared" to obtain a Corrected Lunar Distance.

Part 3 will come at a future date. It will discuss the possibility of calculating the altitudes required with a lunar distance, instead of measuring them: also the question of observing Moon altitudes in place of lunar distances. It will show how to obtain local apparent time and use it with GMT to obtain longitude, as early navigators had to do.

About Lunars, part 3

3.1 CHANGE OF PLAN.

In part 2, there was a prediction of what part 3 would contain. However, I have recently had some useful feedback from listmembers, (especially Steven Wepster) which shows that rather more remains to be said about measuring and correcting the lunar distance. So part 3 will fill in some of those gaps.

Part 4, which should appear in the next few weeks, will discuss the possibility of calculating the altitudes required with a lunar distance, instead of measuring them: also the question of observing Moon altitudes in place of lunar distances. It will show how to obtain local apparent time and use it with GMT to obtain longitude, as early navigators had to do.

3.2 UNDUE EMPHASIS ON ACCURACY IN MOON ALTITUDES.

In part 1, under "How is a lunar distance measured?", I stated "The corrections are such that the Sun altitude is not needed to great accuracy, but the Moon altitude should be measured with precision." This overstated things.

An error of a degree in Moon altitude will, at the most, give rise to an error of 1 minute in the parallax correction of the lunar distance, therefore the accuracy required in Moon altitude is no better than to 20 minutes or so. This is useful in night measurements of Moon-star or Moon-planet lunars, when the horizon has to picked out from the ripply reflection of moonlight below the Moon.

For the refraction of Sun, stars, or planets, when altitudes are as low as 10 deg., altitude measurements to 20 min. of arc are needed, but from 20 deg up, 1 degree will suffice. Again, for stars or planets, this is helpful when the night-horizon is hard to make out.

The conclusion is that the requirements for accuracy of the altitude measurements are usually rather easy to meet. This leaves the observer to concentrate all his precision on measuring the lunar distance.

3.3 UPSIDE-DOWN SEXTANT?

In part 1, under "How is a lunar distance measured?", I said-

"The horizon plays no part in the lunar distance observation itself, and the view, straight through the sextant, is instead used to observe the fainter of the bodies observed (usually the Moon). The brighter one (usually the Sun) is viewed in the index mirror. To see them both, the sextant may have to be tilted at what seems like a most unnatural angle, pointing up in the air, and over on its side, even upside-down."

I should have added here that if you find the "sextant upside-down" posture particularly awkward (as many do), it will do no great harm to swap the two images over, in the two light-paths, and use the sextant in a more natural attitude. This presumes that your sextant offers a suitable choice of shades.

3.4 REFRACTION CORRECTIONS. (see parts 1 and 2)

In clearing a lunar distance, it's necessary to separate the refraction corrections from those for dip and for semidiameter. In many tables these will have been combined, sometimes with Moon parallax also, to minimise the arithmetic for a normal sextant altitude sight. Such a combined refraction table is not useful for lunars. You can find the refraction correction, on its own, for any body, from the "mean refraction" table in Norie's, or in the star correction table in the pull-out card in the Nautical Almanac.

3.5 INVERSE LINEAR INTERPOLATION FOR TIME (see part 2).

This unwieldy phrase describes the process of finding the exact GMT corresponding to a lunar distance, given predicted values of LD at two times. Although I stated that those times, T1 and T2, could be chosen at intervals 1, 2, or 3 hours apart, the example quoted presumed an interval of 3 hours. This is the maximum interval that's compatible with the assumption of a linear change, and is the interval on which the old lunar-distance tables were based. That 3-hour interval was chosen to minimise the size of the tables and to minimize the amount of calculation in their making (in the days when that mattered).

If you are calculating lunar distances for yourself, it will do no harm to choose a shorter interval than 3 hours, if you can be sure that the GMT will end up within that interval.

3.7 ABOUT SOME PROBLEMS IN CLEARING THE LUNAR DISTANCE

In "About Lunars. part 1." it was shown how to obtain a corrected Lunar Distance, D, using Young's method.

It included the following comment, about Young's method-

"CLEARING THE LUNAR DISTANCE, LONGHAND.

The formula above is fine for electronic computation, but quite unsuitable for longhand calculation using logs. The trouble is that some of the quantities may go negative, and the log of a negative number is meaningless.

In the era of lunar distances, the navigator relied on 5-figure logs and trig tables. The many different ways devised for clearing the Lunar distance were mostly devoted to expressing it in such a way that logs could be used for the solution, sometimes using auxiliary tables designed for the purpose.

If readers find the need to do this clearance longhand using logs, my suggestion is to use Borda's method, to be found in Cotter. On request, I will spell it out for the list."

================ (end of quote)

Bill Noyce responded by saying-

And I would like to take him up on his offer to spell out Borda's method for clearing the distance longhand. Young's formula seems to require converting out of logs to do additions.

==================

For modern navigators, the availability of calculators and on-board computers has made lunars more accessible to the rest of us, and that is the approach I would recommend. But readers are entitled to choose a technique using tables and logs, the traditional method that was employed right through the era of lunars.

There are several problems that crop up when trying to use Young's method using tables, rather than a calculator. Borda offers an alternative, but still involves popping in and out of logs, and more than once!

However, I promised Borda's method, on request, and Bill Noyce has indeed requested it, so here goes-

3.8 BORDA'S METHOD

This is more-or-less as quoted by Cotter. The quantities m, M, s, S, d, are given, as follows.

- d (observed lunar distance between centres)
- m (observed moon-centre altitude above true horizontal)
- M (true Moon-centre altitude above true horizontal: it has been corrected for parallax and refraction)
- s (obs Sun/body-centre altitude above true horizontal)
- S (true Sun/body-centre altitude above true horizontal: it has been corrected for parallax and refraction)

The result D, will be the true lunar distance, which has been corrected for parallax and refraction.

Firstly, an angle A has to be calculated. A is just an angle that's used as an intermediate step in the calculation. Whether it has any physical reality, I rather doubt. Obtain A as follows-

First work out and write down (m+s+d)/2 and (m+s-d)/2

log cos A = {log cos((m+s+d)/2) + log cos((m+s-d)/2) +log cos M + log cos S + log sec m + log sec s}/2

This involves working out all those 6 log cos and log sec terms, adding them up, and dividing by two.

Having calculated a value for the right-hand side of this equation, search for that value in the log cos table and find out what angle it corresponds to. This is angle A. You are now "out of logs" and back in the world of ordinary numbers, for a time.

Now work out and write down A + (M+S)/2, and A ~ (M+S)/2.

where "~" means "subtract the smaller from the larger"

then go back into logs again, obtaining D from

log sin (D/2) = {log sin (A + (M+S)/2) + log sin (A ~ (M+S)/2)}/2., So having calculated a value for the right-hand side of the equation, search for that value in the log sin table and find out what angle it corresponds to. Double the result. That gives you D, the corrected lunar distance, all corrections made.

That second term of that last equation differs from what is given in Cotter, who puts a minus sign, which I have changed to "~". Following his notation would lead you into trying to obtain the log of a negative quantity. I think he has got that wrong.

The way I have rewritten that expression now conforms with the following explanation in words of how to tackle the problem, as follows (from Cotter, but somewhat modified)-

1. Find M, S, m, s, and d, as above.

D is the true lunar distance (in which the effects of parallax and refraction have been allowed for) that we wish to find.

2. Place under one another the apparent distance d and the apparent altitudes m and s: and take half their sum, L. From the half-sum L, subtract the apparent distance d. Under this place the true altudes M and S.

3. Take from tables log cosines of L, L-d, M, and S, log secant m, and log secant s. Add these six quantities and divide by 2. The result is the log cosine of A. So look up this quantity in the log cosine table, and find what angle corresponds to it. This is A.

4. Take half the sum of the true altitudes M and S. Call this B. Find the sum of, and the difference between, A and B. Add the log sines of the sum and the difference. Divide by 2. The result is the log sine of half the true lunar difference, that is D/2. So look up that result in the log sine table, find the angle that corresponds to it, and double it to obtain the corrected lunar distance D.

(Cotter also missed out an important word in his paragraph 4, which I have reinstated.)

3.9 BORDA'S METHOD: AN EXAMPLE

Let's try Borda's method for real using the numbers below, for Steven Wepster's Sun-lunar in the Atlantic on 2001 April 02, just as were used in part 2 with Young's method.

I will stick to the degrees-and-minutes notation rather than decimal degrees, as that's what is needed for looking up the tables.

d=107 º 22.9 ‘ (observed lunar distance between centres)
m= 49 º 52.5 ‘ (observed moon-centre altitude above true horizontal)
M= 50 º 29.9 ‘ (true Moon-centre altitude above true horizontal)
s= 21 º 10.4 ‘ (obs Sun/body-centre altitude above true horizontal)
S= 21 º 08.1 ‘ (true Sun/body-centre altitude above true horizontal)

Following the written-out instructions in 3.8, we have now completed step 1.

now for steps 2 and 3

```
d 107deg 22.9
m 049deg 52.5 log sec = 10.19080
s 021deg 10.4 log sec = 10.03036
```

```
sum (d+m+s) 178deg 25.8
half-sum L 089deg 12.9 log cos = 8.13673
L-d -018deg 10.0 log cos = 9.97779 (same as log cos +18deg 10.0)
M 050deg 29.9 log cos = 9.80353
S 021deg 08.1 log cos = 9.96976
          sum = 58.10897
          Halve = 29.05448
          cast off 20= 9.05448
          from log cos table, Angle A = 083deg 29.4min
```

next, step 4.

```
M (copied) 050deg 29.9
S (copied) 021deg 08.1
sum 071deg 38.0
half-sum, =B 035deg 49.0
A (copied) 083deg 29.4
sum, A and B. 119deg 18.4 log sin = 9.94052
diff., A and B. 047deg 40.4 log sin = 9.86884
          sum = 19.80936
          halved = 9.90468
          from log sin table, Angle D/2 = 053deg 24.6
          so Angle D = 106deg 49.2
```

This result, of 106deg 49.2min, for the corrected lunar distance, can be compared with the value obtained (in part 2) for D using Young's method with a calculator, which was 106deg 49.3. So rather good agreement: nothing to complain about there!

However, few navigators would find clearing the lunar distance by Borda's method to be a lot of fun, I admit.

3.10 AN EASIER APPROACH TO CLEARING THE LUNAR DISTANCE.

The two methods considered so far, Young's and Borda's, are "mathematically exact". Both these methods end up by multiplying the observed lunar distance d by a calculated factor, which is always very near 1. The observed distance d can be of the order of 100 deg or so. To obtain the answer D to within 0.1 min, that multiplying factor of about 1 has to be known to 1 part in 100,000 or thereabouts. In other words the error in that multiplying factor should be within .00001. That is why 5-figure logs are required, and every step in the calculation made with scrupulous attention to accuracy.

But what if we can correct d, to obtain D, by ADDING a correction to it, rather than using a multiplier? The correction is never more than about 60 minutes, so to make that correction to provide D to 0.1 minutes, it's only necessary to evaluate that added correction to 0.1 in 60 minutes. An error of 1 part in 1,000 of that added amount now becomes acceptable. That is an accuracy that might even be approached by the careful wielder of a slide-rule that shows trig functions.

Throughout the 19th century, much effort went into devising a solution to clearing the lunar distance which involves such an added correction, rather than a multiplying factor. The trouble has been that such solutions have involved an approximation to the geometry, not an exact solution. They rely on the fact that both angles are small, the parallax and the refraction. In practice, those angles always are acceptably small. If observations are made only when altitudes exceed 10 degrees, refractions are always less than 5.5 min. Parallax of the Moon never much exceeds 60 minutes. If we make an additional rule that lunar distances are measured only when they exceed 10 degrees the requirements are met for good accuracy of the "approximate solution".

3.11 LETCHER'S METHOD.

In "Self-contained celestial navigation using H.O.208" (1977), John S Letcher describes such an approximate method for clearing the lunar distance. The way he proposes for its use would be suitable only for lunar distances up to 90 degrees, whereas any lunar observer would certainly require its application to angles up to 120 degrees. I will therefore give a modified calculation method that will cover the full range of useful sextant angles.

I am not sure where this method comes from, as Letcher does not quote any reference, nor does he show how it is derived from first-principles. It might be his own invention, perhaps. To me, it seems very clever. I will christen it "Letcher's method" until we find out more about its provenance. If anyone recognises it from another publication, I would be interested to learn.

Let's investigate Letcher's Method.

We will assume that all sextant observations have been corrected for index error.

As before, it starts with the observed lunar distance d, after correction for semidiameters. This is to be corrected by adding, separately,-

a) The correction P, for the combined parallaxes of the Moon and the Sun, which may be a positive or negative amount, never greater than about 60 min.

b) The correction R, for the refractions of Moon and Sun, always positive, never more than 11 min.

Both P and R need to be calculated to within 0.1 min.

These two corrections require knowledge of-

- d, the observed lunar distance between centres (i.e. corrected for semidiameters).
- m, the observed altitude of the Moon's centre above the true horizon (i.e. corrected for dip and semidiameter)
- s, the observed altitude of the centre of the Sun (or other body), above the true horizon (i.e. corrected for dip and, if necessary, semidiameter).
- HP, the Moon's horizontal parallax, in minutes of arc, at the hour of observation.
- The HP of the Sun (or other body) is ignored, thus losing a bit of precision.

It is no longer necessary (as it was in the other methods) to make parallax and refraction corrections to m and s individually, as M and S are no longer needed. These corrections are taken into account automatically, as part of the clearance procedure. That is partly why the method is so much simpler.

Let's go through it.

First, obtain B = (cos d sin m - sin s) / sin d (B is just an intermediate angle, used in the computation).

Then the parallax correction in minutes is-

P = HP*B + (HP)^2*[(cos m)^2 - B^2)]/ (6900*tan d)

P may be a positive or negative number, in minutes, to be added or subtracted from d.

The second term of P is usually a very small quantity, but still needs to be evaluated.

Now for the refraction term R. This, very cleverly, includes its own built-in calculation of the way refraction changes with altitude.

R = .95* (sin s/sin m + sin m/sin s - 2*cos d) / sin d ,in minutes of arc.

So the end result is D = d + P + R , giving the Corrected Lunar Distance.

LETCHER: WORKED EXAMPLE

Let's compare this method with the others, using Steven Wepster's Atlantic observations of 2001 Apr 02 once again.

The inputs we need are, as before-

- d = 107.382 deg (observed lunar distance between centres)
- m = 049.875 deg (observed moon-centre altitude above true horizontal)
- s = 021.173 deg (obs Sun or body-centre altitude above true horizontal)
- HP = 59.4 min.

By pocket calculator we get-

- B= -0.6178
- so P = -36.7 - 0.1 = -36.8 min
- R = 3.2 min

As d (in minutes) is 107deg 22.9, then for the Corrected Lunar Distance,

D= d + P + R, or 107deg 22.9 - 36.8 + 3.2

Therefore D = 106deg 49.3 by Letcher's method.

===============

This should be compared with 106 deg 49.2, by Borda's method using tables, and 106deg 49.3 using Young's method with a calculator. Of these, I would choose the last, 106deg 49.3, as being the more precise. Really, what better agreement could anyone wish for, between three different methods?

However, don't expect to always obtain that same precision using Letcher's method. He concedes that by ignoring some of the contributions such as the Sun's parallax and the ellipsoidal figure-of-the-Earth, errors may combine up to a total of 0.3 minutes. Even so, when compared with the likely errors inherent in measuring the lunar distance, due to the motion of a small craft, a possible error of 0.3 minutes in the clearing of it may be very acceptable. My vote would go to Letcher. But it's your choice...

My thanks to Bill Murdoch for alerting me to this useful Letcher publication. If you can find a secondhand copy, I think it would be a worthwhile buy. It was written in the days when scientific calculators existed but were uncommon; hence the emphasis on using tables.

=================

Part 4, which should appear in the next few weeks, will discuss the possibility of calculating the altitudes required with a lunar distance, instead of measuring them: also the question of observing Moon altitudes in place of lunar distances. It will show how to obtain local apparent time and use it with GMT to obtain longitude, as early navigators had to do.

About Lunars, Part 4b. (24 Jan 2004)

This is an amended version of "About Lunars, Part 4a, which appeared in early 2003. And that, in turn was amended from the original Part 4. These updates reflect a changing understanding, to which my friends and fellow-arguers of the Nav-L mailing list have contributed greatly. (I should add, however, that remains some disagreement among those listmembers about my concluding paragraphs of section 4.4b.) The views expressed here, and any errors, are my own.

This new Part 4b is intended as a complete replacement for the earlier Part 4a, which I suggest you remove and trash.

The change is in an amended section 4.4b in place of 4.4a, which contained errors about the effects of "parallactic retardation".

Pointers to earlier parts of "About Lunars" can be found at Arthur Pearson's useful website <www.lunardistance.com>. Concluding parts to this series (part 5 and possibly 6) have been promised for some time but have not yet appeared, I'm sorry to say.

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This is part 4a of a series "About Lunars".

It will deal with-

- Bruce Stark's Lunar Tables
- The effects of lunar parallax.
- The possibility of calculating altitudes from predictions, rather than measuring them.
- The possibility of calculating "lunars" from measured altitudes rather than observed lunar distances.
- Lunar measurements on-land.

4.1 BRUCE STARK'S LUNAR TABLES.

Thanks to the kindness of list member Bruce Stark, I am now the owner of his "Tables for clearing the Lunar Distance, and finding GMT by Sextant Observation". A long title, perhaps, but it defines exactly what these tables do. You can get a copy via at $37 + carriage.

With a current Nautical Almanac, to give the coordinates (dec and GHA) of the Moon and other-body, the tables allow the prediction of lunar distances at two times, one hour apart, on-the-hour, which you hope will bracket your GMT. The tables then allow you to correct ("clear") a sextant observation of lunar distance between the Moon and other-body for the effects of parallax and refraction, and calculate GMT from the way that this observation fits between the two predicted lunar distances. No other tables are required; everything is supplied in Bruce's book. It uses logarithms throughout, but in a way that the user is hardly aware of it.

I have been very impressed by the ingenuity with which Bruce has modified the trig and the logs to make everything as simple as possible for the navigator, and avoid pitfalls that might otherwise lurk in his path. Bruce's system avoids the need for any interpolations, yet maintains a precision to 0.1 minutes, as far as I can judge by limited spot-checks. He has allowed for small effects that are often neglected. Blank calculation forms (for photocopying) are supplied, and these are essential. Examples are given for the user to check his working.

Captain Cook would have been delighted to have a copy of Bruce's lunar tables on board. He was provided with precomputed lunar distances in the Nautical Almanac of his day, which made his task a bit simpler than ours.

I have one criticism, which Bruce and I have discussed, as follows. The user has to go through a series of steps, which are well defined, but nowhere are the underlying principles and equations EXPLAINED. This might be acceptable to an old-style navigator who is prepared to follow a rote; modern individuals with that mindset will nowadays all be using GPS. Anyone who is measuring lunars today will be doing it (to some extent, at least) out of intellectual curiosity, as has become clear from the correspondence with list members. For them, Bruce's tables would, in my own opinion, be rather more satisfying if background explanation were added.

4.2 FINDING THE LONGITUDE

The user of Bruce's tables (or of any of the other lunar techniques discussed so far), ends up with a measure of GMT. He is put into the same position as is the owner of a chronometer. If he does carry a chronometer, then he is enabled to check its going. But even after all that he does not yet have a longitude.

Using his now-known GMT, the modern navigator can establish position lines for objects he can observe in the sky, from their measured and predicted altitudes. By choosing suitable objects he can cross position lines to provide his latitude and longitude. Those objects might be (but don't have to be) the same ones that were used for the lunar distance.

In earlier times he would have to establish his longitude, by determining his Apparent Local Time from a Sun (or star) observation, and compare that with the known GMT to determine longitude. This process would call for a known latitude, readily measured at noon, and followed-up since by the "reckoning".

These matters will be considered in part 5.

4.3 A SURPRISING OBSERVATION. (Well, it surprised me!)

With some friends, I was examining a series of land-based Sun-lunars that they measured last summer, over a period of about 1 hour, at a time when the Moon was quite high in the sky, and drawing a plot of the observed (uncleared) lunar distance against time. It turned out to be a rather straight line, as expected. But instead of the expected slope of about 30 arc-minutes per hour, the observed lunar distance was changing at just over 20 arc-minutes per hour. Why so much slower? The answer lies in the way the Moon's correction for parallax changes so rapidly with time.

4.4b EFFECTS OF THE CHANGING PARALLAX OF THE MOON.

As long as altitudes are above 10 degrees, refractions are no more than a few arc-minutes, and for the purpose of this argument we can neglect them. In contrast, parallax makes an enormous difference to the apparent position of the Moon, up to 60 arc-minutes.

As explained in an earlier part, all nautical tables have been calculated from the point-of-view of an imaginary observer at the centre of a transparent Earth. A real observer on the surface of the Earth sees the Moon, against its background of stars, from a different perspective, depending on just where he is. The difference in apparent angle is the parallax. For the Moon, the parallax is so large because it is much closer to the Earth than any other object in the sky. When the Moon is overhead, its parallax is zero. When the Moon is on the horizon, it has its maximum value (the Horizontal Parallax) of somewhere near 1 degree. In between, it varies as cos(altitude). Parallax always makes the Moon appear lower in the sky.

For simplicity, imagine a navigator near the equator, and the Moon with near-zero declination. This is the worst-case scenario, when parallax has its greatest effect, and it is also simple to visualise. The Moon will rise in the East, pass over his head six hours later, and set in the West. Parallax will depress the apparent Moon at Moonrise by about 1 deg, then as the Moon altitude increases, the parallax will decrease until at the moment it is directly overhead, the parallax is zero. After the Moon passes "over the top", parallax starts to increase again, but now it is in the opposite direction, still depressing the Moon's position, more and more, but now pushing it down to the Westward, until finally the Moon sets in the West, with parallax at its maximum value of about 1 degree again. The Sun follows a similar path, but a few hours apart.

So it's reasonably clear, if you think about it and draw a diagram, that parallax increases the apparent speed of the Moon in its path across the sky, from East to West, because the Moon appears to travel an extra 2 degrees all told. The rate at which the Moon is pushed by parallax becomes greatest when it's at it's highest, overhead, at about 15 arc-minutes per hour, at a time when the parallax itself is zero.

But what about the movement of the Moon with respect to the other bodies, Sun, stars, planets? Because they are all so much further away than the Moon, the effect of parallax on the position of those bodies is quite negligible (for this argument). Parallax doesn't move them, we approximately presume.

So as the Moon and the other bodies pass across the sky, more-or-less together, parallax shifts the apparent Moon westwards, by up to 15 arc-minutes per hour if it's at its zenith, with respect to everything else.

Now remember what we are trying to measure to obtain GMT. It's the position of the Moon with respect to the background of Sun and stars. And with respect to that background, the Moon is moving, always Easterly, at about 30 arc-minutes an hour, give or take a bit. However, we have just worked out above that parallax causes the APPARENT Moon, as seen by an observer on the Earth's equator, to be shifted Westerly at a rate of up to 15 arc-minutes per hour, with respect to its true position. So with respect to the star background, the Moon, when directly above the observer, has lost half of its apparent velocity, because of the changing parallax. And that apparent motion of the Moon is what a lunar observer uses to determine GMT from the Moon's position.

We might call this effect "parallactic retardation" of the apparent Moon. I have never seen it discussed in an English-language text. If anyone else has, I would be interested to learn. Steven Wepster has found a discussion in a German text by Carl Bremiker, writing in-"Zeitschrift fur Vermessungswesen", Berlin, 1875, pages 59 to 79. The relevant part of the text is on pages 77 to 79. If anyone is interested enough to ask for it, I can email a copy of the German text of these pages, and an amateur translation into English.

I have described this effect in simplified terms, by ignoring the tilt of the Earth's equator to the ecliptic - quite an approximation!.

This effect can slow the apparent motion of the Moon with respect to the background of stars (etc), and slow it very significantly. Although it's the position of the apparent moon (with respect to that background) that we measure, as a starting-point to a lunar distance calculation, that observation next has to be "cleared". The clearing process combines a number of operations, which are jumbled together in a way that rather hides what's going on. It allows for the effects of Moon parallax and the parallax (if any) of the other body; the refraction in the light-path from each body, and any misalignment of the other body from the path of the Moon. Of these, the Moon parallax component is by far the largest, and is changing most quickly as the Moon passes across the sky.

It so happens that the correction for the changing parallax of the Moon is just sufficient to make up for the parallactic retardation, such that the end result of the cleared lunar-distance is, effectively, the reasonably-steady motion of the true Moon across the sky. This isn't an accident, of course: parallactic retardation is due to the effect of parallax on the apparent Moon, and the parallax part of the clearing process is to compensate for that. Thus, as the apparent Moon travels across the sky, part of the corrected motion of the true Moon (which can be as low as half of it) arises from the measured motion of the apparent Moon, and the rest (up to half of it) arises from the clearing process. It becomes clear why the clearing process is so necessary, and why it's so important to get it right.

Fortunately, knowing the altitude of the Moon (and to a lesser extent the other body), the clearing corrections can be made to high accuracy. The resulting overall error in the cleared lunar distance is then dominated by the error in measuring the angle of the apparent lunar distance. The error in that angular measurement is unaffected by the parallactic retardation. So, though the parallactic retardation has a major effect of the apparent motion of the Moon across the sky, it does not affect the overall accuracy of a lunar distance. This is in contrast with the conclusions of an earlier posting, in "About Lunars", part 4a, which in this respect were wrong.

4.5a LUNARS WITHOUT ALTITUDES

Every method for "clearing", or correcting, the lunar distance requires a knowledge of the altitudes of the Moon and the other-body involved, obtained at the same moment (or nearly so) as the lunar distance between them was measured. Sometimes, however, it's impossible to measure those altitudes: for example, when the horizon can't be seen because the night is black. Because the altitudes are required only to make corrections, they are not required to very high accuracy. For the most demanding case, that of the Moon's altitude, errors up to 6 arc-minutes or so are acceptable. Such an error would contribute no more than 0.1 arc-minutes to the error in correcting the lunar distance. Even so, without a good view of the horizon, such accuracy in the measured altitude may be unachieveable.

However, if the Local Apparent Time (LAT) is known, at the moment of the lunar distance observation, it's possible to deduce those altitudes from Almanac data, with sufficient accuracy, instead of measuring the altitude of the Moon and the other-body.

It's likely that the Local Apparent Time will have been obtained in any case, at the moment of observing the lunar distance, as a necessary step in obtaining the observer's longitude. Local apparent time is usually obtained from two altitudes of the Sun, one around noon, the other several hours from noon, in the morning or afternoon of the same day. Other techniques, and other bodies, may be used for the same purpose.

Once the Local Apparent Time has been measured, it can be converted to Local Mean Time (LMT), and used to set, or correct, the ship's clock or deckwatch. Changing Apparent Time to Mean Time isn't difficult: it's only a matter of applying the Equation of Time as a correction, of up to 15 minutes or so. (The difficult bit is deciding which direction to make the correction, which we will discuss later in this series.

Fom then on, the time given by that timepiece can be used over the following (or preceding) hours or days, for as long as its timekeeping and the constancy of its rating can be trusted. In that interim. any movement of the vessel in the East or West direction, from the dead-reckoning, should be used to correct the LMT. For each degree of ship's Westing since the LMT was measured, the LMT should be delayed by 4 minutes of time.

So, given a reliable measure of LMT, from reading the ship's clock. let's consider the question of calculating the altitude of a celestial body, at the same moment as the lunar distance observation, from data given in the Nautical Almanac.

To calculate the altitude of a body, the quantities required are the observer's latitude, the declination of the body, and the Local Hour Angle between the body and the observer. This is familiar ground to any astro-navigator: obtaining a calculated altitude, often referred to as Hc.

sin alt = sin lat sin dec + col lat cos dec cos LHA

Any navigator worth his salt, with clear sky around noon, will have no difficulty in obtaining a good value for his latitude, obtaining Sun declination from the Almanac. Then, over a limited period, he can keep track of changes in his latitude, to some extent, by dead reckoning of his North-South travel. At the moment of observing the lunar distance, we can assume that the vessel's latitude is known to within a couple of arc-minutes, perhaps a bit more. Maybe the latitude at the moment of the lunar can be deduced from the Sun's altitude at the following noon, rather than the preceding one.

This leaves the declination and Local Hour Angle (LHA) of the body to be obtained, using the Almanac. A serious problem arises here. Although the Local Mean Time is known, the Greenwich Mean Time isn't (the first aim of a lunar observation is usually to determine GMT). However, GMT is the time that a navigator must use to look up coordinates of the two bodies in an almanac, at the moment of the lunar. Where does the navigator get his GMT from, for entering the almanac? Here's how-

The difference between the Local Mean Time and Greenwich Mean Time is the Westerly longitude, expressed in hours at 15° per hour.

I think of longitude, measured Westerly, as Westitude, increasing Westerly from 0 to 360 degrees. A longitude of 1° E would have a Westitude of -1°, or (same thing) 359°. Some authorities have chosen an East-is-positive convention for longitude, but, with Meeus, I prefer it the other way, in the same direction as Hour Angles, which are always Westerly.

At any moment, then, GMT = LMT + Westitude(in hours) . At 12 hrs local mean time in Washington, Greenwich mean time will be about 5 hrs (75°) later, or 5pm. All this is straightforward stuff.

To start with, the navigator has to make a GUESS at his longitude. It doesn't need to be a very precise guess: within, say, 3° (which corresponds to 12 minutes of time) will be perfectly adequate. If his guess is worse, it will show up later. Using that guess gives an estimated GMT with which he can enter and interpolate the almanac tables for dec. and GHA of the two bodies. We hope (but don't insist) that this estimated GMT will be within 12 minutes of the truth.

Looking up the declination for the two bodies is quite straightforward. Declinations of all bodies change slowly enough that even for the Moon (the worst case) an error in Greenwich time of 12 minutes results in an error of only 3 arc-minutes in the Moon dec. No problem there.

The real difficulty arrives in assessing the GHA of the two bodies. All hour angles change rapidly, at roughly 15° per hour, the rate of rotation of the Earth. So a possible error in our estimate of GMT of 12 min of time would give rise to an error of 3° in the interpolated GHA of the body. Such an error would make the GHA prediction quite useless for calculating the altitude of either of the bodies involved.

The difficulty is overcome by a clever trick, for which I think Maskelyne should get the credit: he described it in his British Mariner's Guide 0f 1762. It goes like this-

For each of the bodies in question, consider not just its own GHA, but also the GHA of the Sun, both taken from the almanac. Although these are both increasing rapidly, at nearly 15° per hour, the difference between them, GHA (body) - GHA (Sun), changes much more slowly: in the worst-case, for the Moon, at about 30' per hour. This is the amount by which the body is West of the Sun. If we have estimated the GMT to within 12 minutes of time, then we will know the Westing of the body from the Sun to within about 6 arc-minutes. We already have an accurate value for LHA Sun, based on measurement.

We take: LHA (body) = LHA (Sun) + (GHA (body)- GHA (Sun))

So now we have the LHA of the body to within about 6 arc-minutes, if our longitude guess was within our target range. Taken with our measured lat and the dec of the body from the almanac, this provides all the necessary information to predict the altitude of the body, as described above, within about 6 arc-minutes. And that calculated altitude then allows the lunar distance to be "cleared", to within about 0.1 arc-minute, and a new value of GMT calculated from the lunar observation. The important factor here is that our initial "guess" of longitude, which we hoped was within 3°, or 12 minutes of time, contributes an error to the resulting value of GMT, of no more than 12 seconds of time, if we were within our target: a perfectly acceptable result, which has reduced the error of our initial guess by a factor of 60.

If the difference between the guessed value of GMT and the final result happens to exceed 12 minutes of time, this indicates that the improved GMT value should be substituted for the earlier guess, and then a reiteration should shrink the resulting error by another factor of 60. There will never be a need for further iteration after that.

The procedure I have described isn't quite as Maskelyne proposed. In his day, the position East-West around the sky was measured in terms of Right Ascenscion, not GHA, which was defined in the opposite direction, from a different reference-point..Also, in his day, almanacs used Apparent Time, and not Mean Time, as their argument, so there was no need in those days to correct with the Equation of Time.

4.6 LONGITUDES WITHOUT LUNAR DISTANCES?

Why is it necessary to measure lunar distances at all? It's an unfamiliar and awkward process to many navigators. Can the same information be obtained just by the familiar process of measuring altitudes? This question arose recently on the Nav-L mailing list, and I will tackle it here by quoting (edited) versions of Chuck Griffith's question and my own reply. Chuck asked-

Consider an alternative approach to finding GMT. Why can't we observe the altitude of the moon and one other body and, using our assumed latitude, solve for the meridian angle of both bodies. The difference between the two angles should change by the rate at which the moon moves through the sky faster than another body. If that's true, can't we find the meridian angle between the two bodies for the even hours, say on either side of what time we think it is, and use the same inverse linear interpolation approach to find the time of our sight? Of course, I can think of a couple issues with this approach worth discussion. First, this only works when the altitude of the moon and the other body change reasonably with time, i.e., we can't do it when either body is close to being a meridian sight. Second, we need both altitudes simultaneously. I think this could be solved by alternately observing one body then the other several times and graphing the sights so that we could derive an averaged simultaneous altitude from the graph.

My response is as follows-

The question is a very fair one. It has been asked before, however; starting in 1674. Francis Chichester, the famous single-handed circumnavigator, proposed such a method in 1966, and a spate of publications followed, on similar lines. These were answered in an authoritative article by David Sadler, then director of HM Nautical Almanac Office, in the RIN's "Journal of Navigation", 31, 2 May 1978, page 244, entitled "Lunar Methods for 'Longitude Without Time' ".

From my point of view, the drawback of Sadler's article is that it is illustrated by a diagram of such devilish cunning and complexity that I am quite unable to make head or tail of it. If any reader manages to penetrate its mysteries, I would be grateful for an explanation.

It's important to bear in mind that in any measurement that uses the Moon's motion to provide time and hence longitude, accuracy in determining the Moon's position is all. This follows from the fact that each minute-of-arc error results in an error in the final position of the vessel of 30 minutes of arc (which near the equator corresponds to 30 miles) or sometimes more. It is FAR more demanding that the normal run of astronavigation.

The main virtue of a lunar is that the all-important measurement in which so much accuracy is required, the angle-in-the-sky between the Moon and the Sun (or other body) does not involve the horizon AT ALL. True, the altitudes of Sun and Moon do have to be measured up from the horizon as an auxiliary measurement, but this is only to get a correction to a correction, and an imprecise value for those altitudes will be perfectly adequate.

Why is the accuracy so degraded whenever the horizon is involved?

First, if there's any haziness in the air, the first thing to become indistinct is the line of the horizon.

Second, even if the horizon is really sharp, it isn't exactly a well-defined straight line (except in millpond conditions), especially from a small craft. The horizon-line is made up from the peaks of overlapping waves and swell, and the vessel, too, is riding on those waves. The observer does what he can by timing his shots when he judges his vessel to be on the top of its "heave", but it is inevitably a compromise.

Third, even if the horizon is both sharp and straight, its angle can be affected by anomalous refraction, which causes the dip to vary from its predicted value. Air layers at different temperatures near the horizon can cause the sun's image to be distorted as it rises and sets, and can in extreme cases cause mirage effects when a distant vessel is observed as floating well above the horizon, sometimes even inverted. Where none of these objects is there to give a clue to the odd behaviour of light on its path from skimming the horizon to the observer's eye, anomalous dip may nevertheless be present, quite unsuspected and undetectable. An error in dip of 1 minute may be quite usual, and 2 or 3 minute errors can also occur occasionally. There is no way for the observer to correct for it. (Special instruments to measure the dip-of-the-moment have been devised but are very uncommon).

These errors may present no real problems in normal astronavigation. After all, what significance has an error of 2 or 3 miles in an astro position? However, to the lunar observer, where any such errors are multiplied 30 times or more in calculating his longitude, they are intolerable.

When objects lie in opposite parts of the sky, the Moon to the East, say, and the Sun to the West, such horizon errors would actually add when comparing the positions of the two objects.

This was well-known to eighteenth-century navigators, who accepted the practical and arithmetical difficulties of measuring lunar distance up in the sky, rather than altitudes up from the horizon, to cling on to all the precision that they possibly could.

4.7 LUNARS IN YOUR BACKYARD.

It isn't necessary to go to sea to observe lunar distances. Because the angle between Moon and other-body doesn't require a horizon, it can be measured from on-land, and that is indeed a good way to practice, when the observer is not being distracted and shaken-up by the motion of a vessel.

However, the required auxiliary measurements, the altitudes of the Moon and the other-body, above the horizontal, do need to be made. If you don't live with a view of the sea (or a large lake) then you have to create your own horizon. This is easy to do.

What's needed is a pool of reflecting liquid in a tray, placed on the ground. In the days of explorers, they would carry a few pounds weight of liquid mercury for this purpose. Nowadays, old black engine-oil, or a dollop of black treacle (molasses in American, I understand) will do the job. The viscosity helps, in limiting ruffling of the surface by the wind. If wind is a problem, surround the tray with a wind-break, or even cover it over with a sort-of garden 'cloche', in an inverted vee, made of glass which you have checked for flatness beforehand. A good method for making that check is to look with the sextant at the two views of a distant object with the index set to zero, as if checking index-error. Then interpose the glass plate into one view-line from the sextant (but not both), move the plate about, and check that there is no shift between the images. If shift is observed, all is not lost: just average two measurements between which the glass has been turned through 180 in its plane (not overturned).

Look down with your sextant (the straight-through view via the half-silvered mirror) at the reflected image of the object in the pool, and in the index mirror view the object directly, up in the sky. The angle shown on the sextant will be angle between the object and its reflected image, or twice its altitude above the true horizon. Because a normal sextant can measure only to 120 degrees (sometimes a few degrees more), then the maximum altitude that can be measured in this way is limited to 60 degrees (or slightly over).

The sextant angle, as measured, should be corrected for any sextant error shown in the box, and for index error, and then halved. No dip correction should be made.

After noting the observed altitudes in this way for both bodies, corrections for refraction and for parallax must be made (unless Letcher's method is being used which includes these corrections automatically). If your altitude is significantly above sea level, make the appropriate corrections to refraction caused by the reduced pressure, and also, if necessary, any abnormal temperature. Note the corrected altitudes, and then proceed as for any other lunar distance calculation.

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This is the end of part 4b. Further parts will cover the final step of obtaining a ship's longitude from a knowledge of GMT, whether this was derived from a lunar or from a chronometer: and will also detail some of the changes in the almanac, and in the thinking about celestial positions, that have occurred over the many years since lunars first appeared. That will conclude this series, unless further questions arise that need answering.

George Huxtable. 2002
NavList http://www.irbs.com/lists/navigation/ http://fer3.com/NavList
contact George Huxtable, at george@hux.me.uk or at +44 1865 820222 (from UK, 01865 820222) or at
1 Sandy Lane, Southmoor, Abingdon, Oxon OX13 5HX, UK.

Ref:

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