Advances in Engineering Research (AER), volume 143
6th International Conference on Energy and Environmental Protection (ICEEP 2017)

About the Finite Element Analysis for Beam-Hinged Frame

Duan Jin[1,a], Li Yun-gui[1]

[1] China State Construction Technical Center, Beijing, 101300, PR China

[a] duanjin78@126.com

Key words: structural simulation, DOFs coupling, DOFs condensation, finite element method

Abstract: In this paper, the method of DOFs condensation is adopted to simulate hinge node in finite element analysis. Other than adding additional overlapped nodes for the DOFs coupling method, the principle idea of DOFs condensation method is that each two elements connected by a hinge node do have a common node, which is exactly the hinge node, while the non-constrained DOFs of the hinge node would be condensed into the other constrained DOFs of the element. The procedure of condensation is much like the condensation of sub-structures. The detailed condensed stiffness matrices for beam elements is derived in this paper. A numerical example for a hinged frame is presented. The validity of the present theory would be demonstrated by comparing the present result and that of ABAQUS software.

Introduction

As a connection mode, hinge is widely used in structural engineering, especially for frame structures. In commercial software, such as ABAQUS, the most usually method to simulate hinge node is the coupling of degrees of freedom, or DOFs coupling for simplicity. Its principle idea is that each two elements connected by a hinge node don't have a common node while have two overlapped nodes. The two elements are connected by adding constraint equations for the two overlapped nodes. In other words, additional element nodes, or degree of freedom (DOF), would be introduced into the finite element model to simulate the hinge node.

In this paper, the method of DOFs condensation [1-6] is adopted to simulate hinge node in finite element analysis. Other than adding additional overlapped nodes for the DOFs coupling method, the principle idea of DOFs condensation method is that each two elements connected by a hinge node do have a common node, which is exactly the hinge node, while the non-constrained DOFs of the hinge node would be condensed into the other constrained DOFs of the element. The procedure of condensation is much like the condensation of sub-structures. The detailed condensed stiffness matrices for beam elements is derived in this paper. A numerical example for a hinged frame is presented. The validity of the present theory would be demonstrated by comparing the present result and that of ABAQUS software.

The DOFs condensation for hinged-connection restraint

The equivalent equation of beam element is as following [1-6]:

$$\mathbf{K}^e\mathbf{a}^e = \mathbf{P}^e \tag{1}$$

where， $\mathbf{a}^e$ represents the nodal displacement vector of the beam element

The degree of freedom (DOF) of the element could be split into two parts: non-constraint DOF and constraint DOF. The former one usually represents the lateral displacement angle and the torsional angle, denoted as $\mathbf{a}_i$. The latter one usually represents the axial and lateral displacement, denoted as $\mathbf{a}_b$. Decomposing the stiffness matrix **K** and load vector **P** according to that of displacement vector $\mathbf{a}^e$, Equation (1) could be transformed into the following one:

$$\begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{a}_b \\ \mathbf{a}_i \end{bmatrix} = \begin{bmatrix} \mathbf{P}_b \\ \mathbf{P}_i \end{bmatrix} \tag{2}$$

From the second line of above equation, the following could be derived:

$$\mathbf{a}_i = \mathbf{K}_{ii}^{-1}\left(\mathbf{P}_i - \mathbf{K}_{ib}\mathbf{a}_b\right) \tag{3}$$

Substituting Equation (2) into Equation (2), the condensed equation could be obtained as following:

$$\left(\mathbf{K}_{bb} - \mathbf{K}_{bi}\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib}\right)\mathbf{a}_b = \mathbf{P}_b - \mathbf{K}_{bi}\mathbf{K}_{ii}^{-1}\mathbf{P}_i \tag{4}$$

For simplicity, above equation could be simplified as following:

$$\mathbf{K}_{bb}^{*}\mathbf{a}_b = \mathbf{P}_b^{*} \tag{5}$$

Where

$$\mathbf{K}_{bb}^{*} = \mathbf{K}_{bb} - \mathbf{K}_{bi}\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} \tag{6}$$

$$\mathbf{P}_b^{*} = \mathbf{P}_b - \mathbf{K}_{bi}\mathbf{K}_{ii}^{-1}\mathbf{P}_i \tag{7}$$

The stiffness matrix for hinged-beam element

Generally speaking, the stiffness matrix **K** for a 3d beam element could be represented as following [1,2]:

$$\begin{pmatrix}
\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} & 0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} \\
0 & 0 & \frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & -\frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 \\
0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 \\
0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 & 0 & 0 & 0 & 0 & \frac{2EI_y}{l} & 0 \\
0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} & 0 & 0 & 0 & 0 & 0 & \frac{2EI_z}{l} \\
-\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & 0 & 0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} \\
0 & 0 & -\frac{12EI_y}{l^3} & 0 & 0 & 0 & 0 & 0 & \frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 \\
0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 \\
0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 \\
0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l}
\end{pmatrix} \tag{8}$$

Where I_y and I_z represent the principle moment of inertia, J represents the torsional moment of inertia

Substituting Equation (8) into Equation (6), the stiffness matrix for all kinds of hinged-beam element could be deduced. For example, the stiffness matrix for one lateral displacement angle hinge could be represented as Equation (9), and the matrix for two lateral displacement angle hinge could be represented as Equation (9), and the matrix for three dimension angle hinge (3D hinge) could be represented as Equation (10).

Here it should be pointed out that in coding program the condensation from Equation (2) to Equation (5) is not according to the Equation (4) while according to the Gauss-Jordan elimination method [2,7].

$$\begin{pmatrix}
\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 \\
0 & \frac{3EI_z}{l^3} & 0 & 0 & 0 & \frac{3EI_z}{l^2} & 0 & -\frac{3EI_z}{l^3} & 0 & 0 & 0 \\
0 & 0 & \frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & -\frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} \\
0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 \\
0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 & 0 & 0 & 0 & 0 & \frac{2EI_y}{l} \\
0 & \frac{3EI_z}{l^2} & 0 & 0 & 0 & \frac{3EI_z}{l} & 0 & \frac{3EI_z}{l^2} & 0 & 0 & 0 \\
-\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 & 0 \\
0 & -\frac{3EI_z}{l^3} & 0 & 0 & 0 & \frac{3EI_z}{l^2} & 0 & \frac{3EI_z}{l^3} & 0 & 0 & 0 \\
0 & 0 & -\frac{12EI_y}{l^3} & 0 & 0 & 0 & 0 & 0 & \frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} \\
0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & 0 \\
0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l}
\end{pmatrix} \quad (9)$$

$$\begin{pmatrix}
\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 \\
0 & \frac{3EI_z}{l^3} & 0 & 0 & 0 & \frac{3EI_z}{l^2} & 0 & -\frac{3EI_z}{l^3} & 0 & 0 \\
0 & 0 & \frac{3EI_y}{l^3} & 0 & -\frac{3EI_y}{l^2} & 0 & 0 & 0 & -\frac{3EI_y}{l^3} & 0 \\
0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l} \\
0 & 0 & -\frac{3EI_y}{l^2} & 0 & \frac{3EI_y}{l} & 0 & 0 & 0 & -\frac{3EI_y}{l^2} & 0 \\
0 & \frac{3EI_z}{l^2} & 0 & 0 & 0 & \frac{3EI_z}{l} & 0 & \frac{3EI_z}{l^2} & 0 & 0 \\
-\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 \\
0 & -\frac{3EI_z}{l^3} & 0 & 0 & 0 & \frac{3EI_z}{l^2} & 0 & \frac{3EI_z}{l^3} & 0 & 0 \\
0 & 0 & -\frac{3EI_y}{l^3} & 0 & -\frac{3EI_y}{l^2} & 0 & 0 & 0 & \frac{3EI_y}{l^3} & 0 \\
0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l}
\end{pmatrix} \quad (10)$$

$$\begin{pmatrix} \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & \frac{3EI_z}{l^3} & 0 & 0 & 0 & \frac{3EI_z}{l^2} & 0 & -\frac{3EI_z}{l^3} & 0 \\ 0 & 0 & \frac{3EI_y}{l^3} & 0 & -\frac{3EI_y}{l^2} & 0 & 0 & 0 & -\frac{3EI_y}{l^3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{3EI_y}{l^2} & 0 & \frac{3EI_y}{l} & 0 & 0 & 0 & -\frac{3EI_y}{l^2} \\ 0 & \frac{3EI_z}{l^2} & 0 & 0 & 0 & \frac{3EI_z}{l} & 0 & \frac{3EI_z}{l^2} & 0 \\ -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & -\frac{3EI_z}{l^3} & 0 & 0 & 0 & \frac{3EI_z}{l^2} & 0 & \frac{3EI_z}{l^3} & 0 \\ 0 & 0 & -\frac{3EI_y}{l^3} & 0 & -\frac{3EI_y}{l^2} & 0 & 0 & 0 & \frac{3EI_y}{l^3} \end{pmatrix} \quad (11)$$

Numerical example

To demonstrate the above theory, a numerical example is presented in the following. The original configuration and deformation of a right-angle-frame with a 3D hinge is shown in Figure 1. The cross-section of the frame is a circle with a 9mm inner radius and 11mm outer radius. Its Young's modulus is 2.0e11 Pa and its Poisson ratio is 0.3. The lateral rod is subjected a 1KN vertical concentrated load and the vertical rod is subjected a 3KN lateral concentrated load, see Figure 1. The deformation from the present method, i.e. DOFs condensation method, and from the ABAQUS software, i.e. DOFs coupling, are presented simultaneously in Figure 1. It is obvious that the result of the present method is identical with that of ABQUS software.

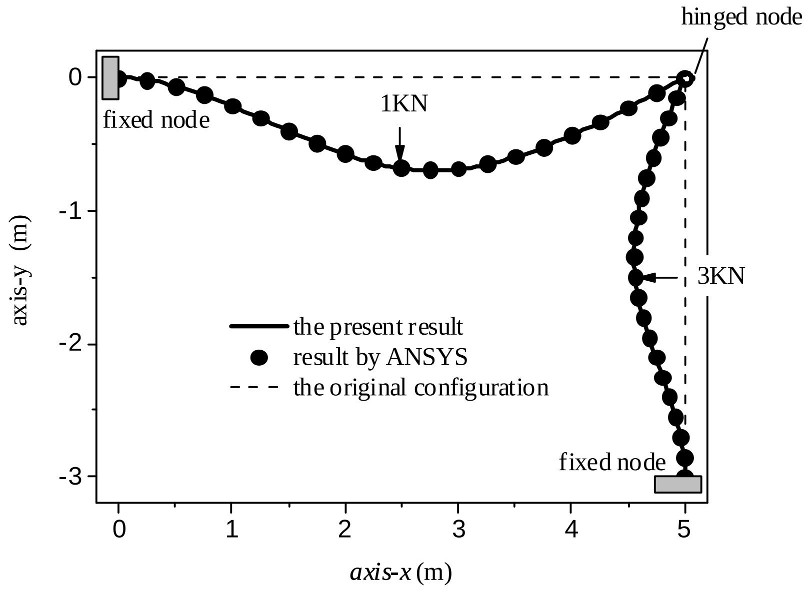

Figure 1 the deformation of the a right-angle-frame with a 3D hinge

Summary

In this paper, the method of DOFs condensation is adopted to simulate hinge node in finite element analysis. Other than adding additional overlapped nodes for the DOFs coupling method, the principle idea of DOFs condensation method is that each two elements connected by a hinge node do have a common node, which is exactly the hinge node, while the non-constrained DOFs of the hinge node would be condensed into the other constrained DOFs of the element. The procedure of condensation is much like the condensation of sub-structures. The detailed condensed stiffness matrices for beam elements is derived in this paper. A numerical example for a hinged frame is presented. The validity of the present theory would be demonstrated by comparing the present result and that of ABAQUS software.

References

[1] Bathe K J. Finite Element Procedures. NJ: Prentice-Hall, 1996

[2] Wang X C. Finite Element Method. Beijing, China: Tsinghua University Press, 2003 (in Chinese)

[3] Oden J T. Finite Element of Nonliear Continua. New York: McGraw-Hill, 1972

[4] Simo J C, Hughes T J R. Computational Inelasticity. New York: Springer, 1998

[5] Belytschko T, Liu W K, Moran B. Nonlinear Finite Elements for Continua and Structures. New York: Wiley, 2000

[6] Yang Y B, Kuo S R. Theory & Analysis of Nonlinear Framed Structures. New York: Prentice Hall, 1994

[7] Guan Z & Chen J L. Numerical Computation Method. Beijing, China: Tsinghua University Press, 1990 (in Chinese)