

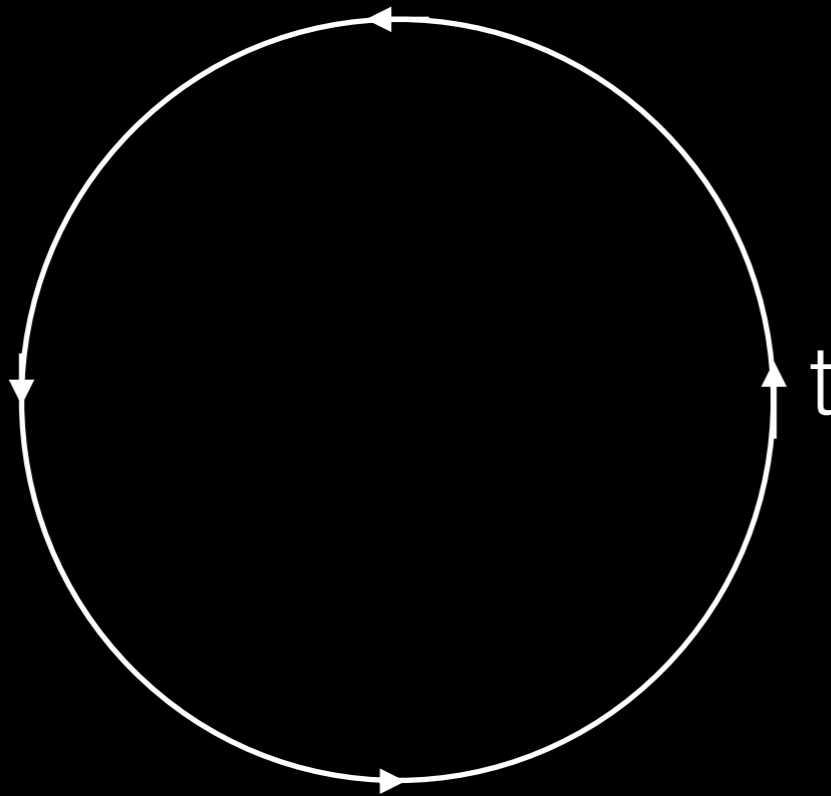
**T**raversable  
**A**causal  
**R**etrograde  
**D**omains  
**I**n  
**S**pacetime

Ben Tippett, UBCO  
CCGRRRA 2016

# “Time Travel”

## Closed Timelike Curves

Timelike curves which are closed



“The Time Machine”  
*HG Wells 1895*

# Popular Geometries with Closed Timelike Curves

- Gödel 1949
  - *angular momentum cosmology*
- Tipler 1974
  - *infinite rotating cylinder*
- Tomimatsu Sato 1973
  - *weird rotating kerr thing*
- Friedman, Morris *et al.* 1990
  - *wormholes + twin paradox*

# Piecemeal Refutation of Closed Timelike Curves

- Gödel 1949
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**Asymptotically  
weird**

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**How do we even  
make this thing?**

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# Unphysicality of Morris-Thorne Wormhole

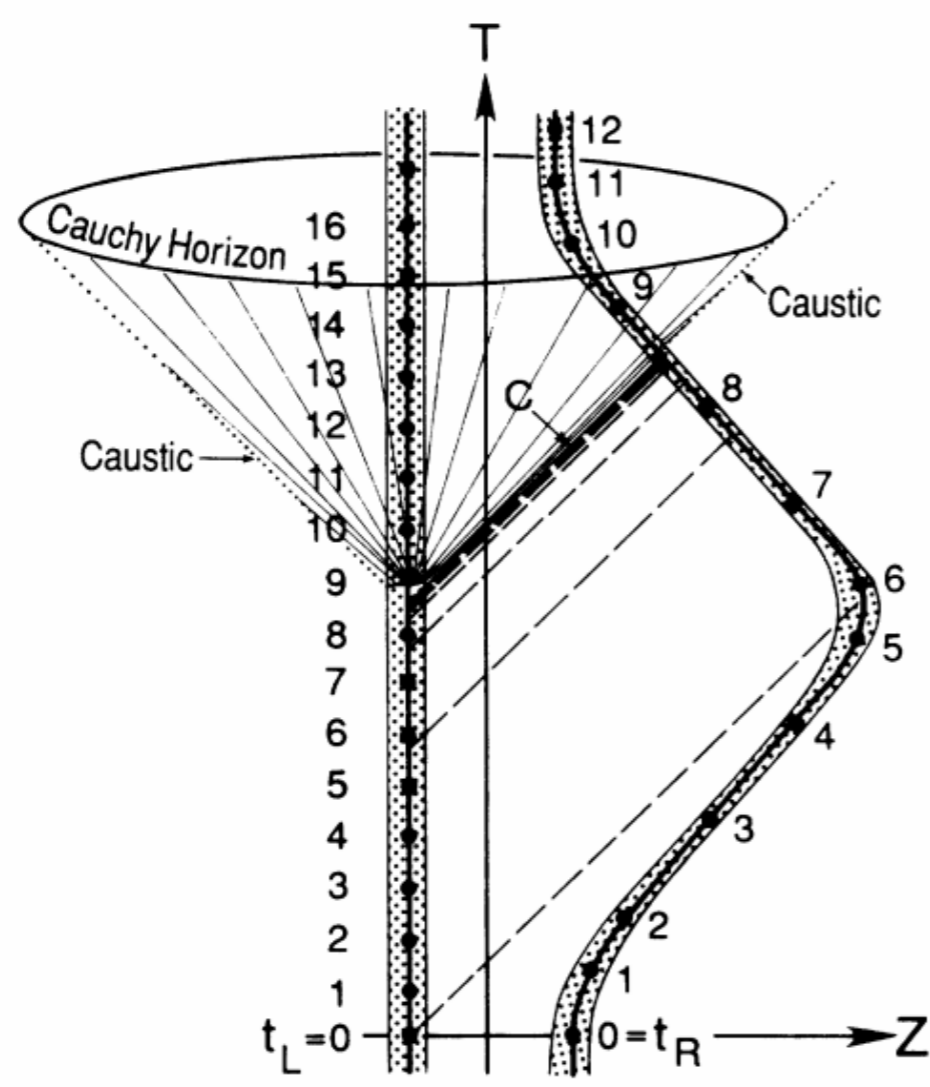


FIG. 2. Spacetime diagram for conversion of a wormhole into a time machine.

Morris, Thorne, Yutsever 1988

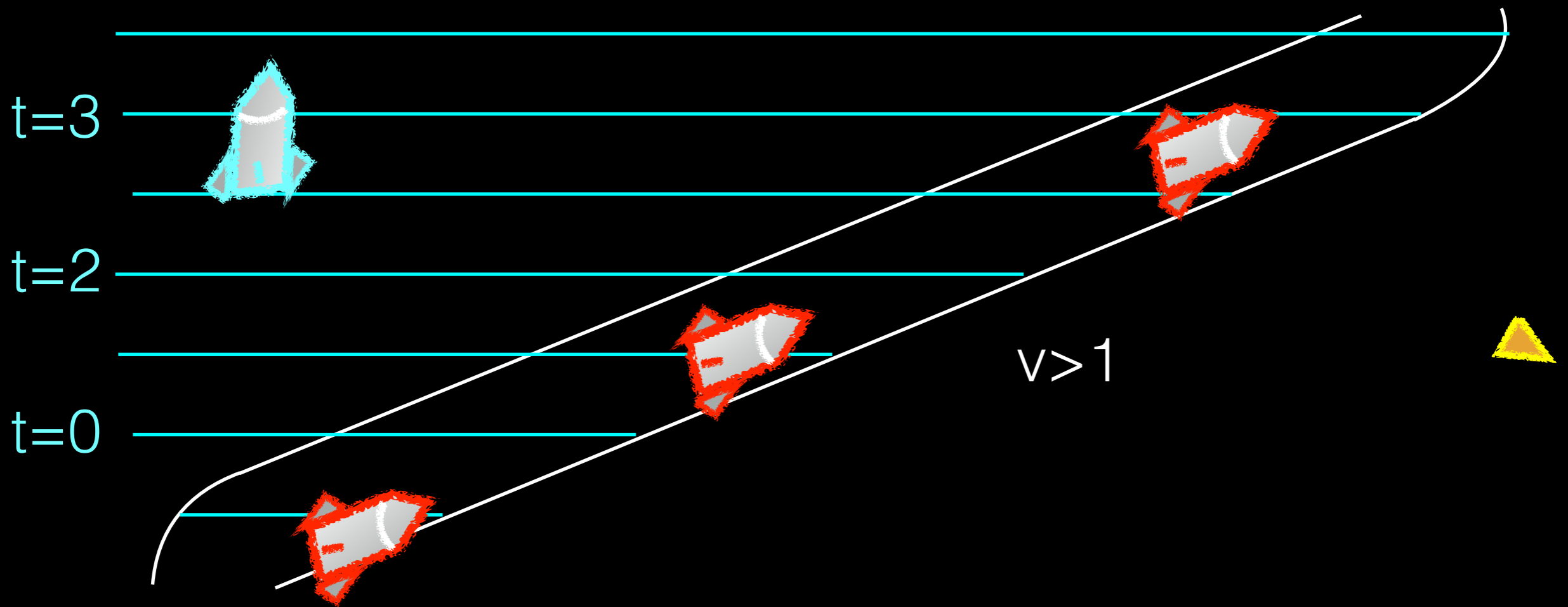
- Violates Topological Censorship
  - Friedman, Schleich, Witt 1993
  - **Exotic Matter Required**
- Compactly Generated Cauchy Horizon
  - **Unstable to perturbations**

# General Refutations of CTC

- Manifold is **not** time Orientable
  - Convenient assumption
  - **ad hoc**
- Chronology Protection Conjecture
  - Hawking 1991
  - **Compactly Generated Cauchy Horizons are unstable**
  - **Some CTC are CGCH free**
- Spacetimes with CTC and no CGCH violate energy conditions or are geodesically incomplete
  - Maeda, Ishibashi, Narita 1998

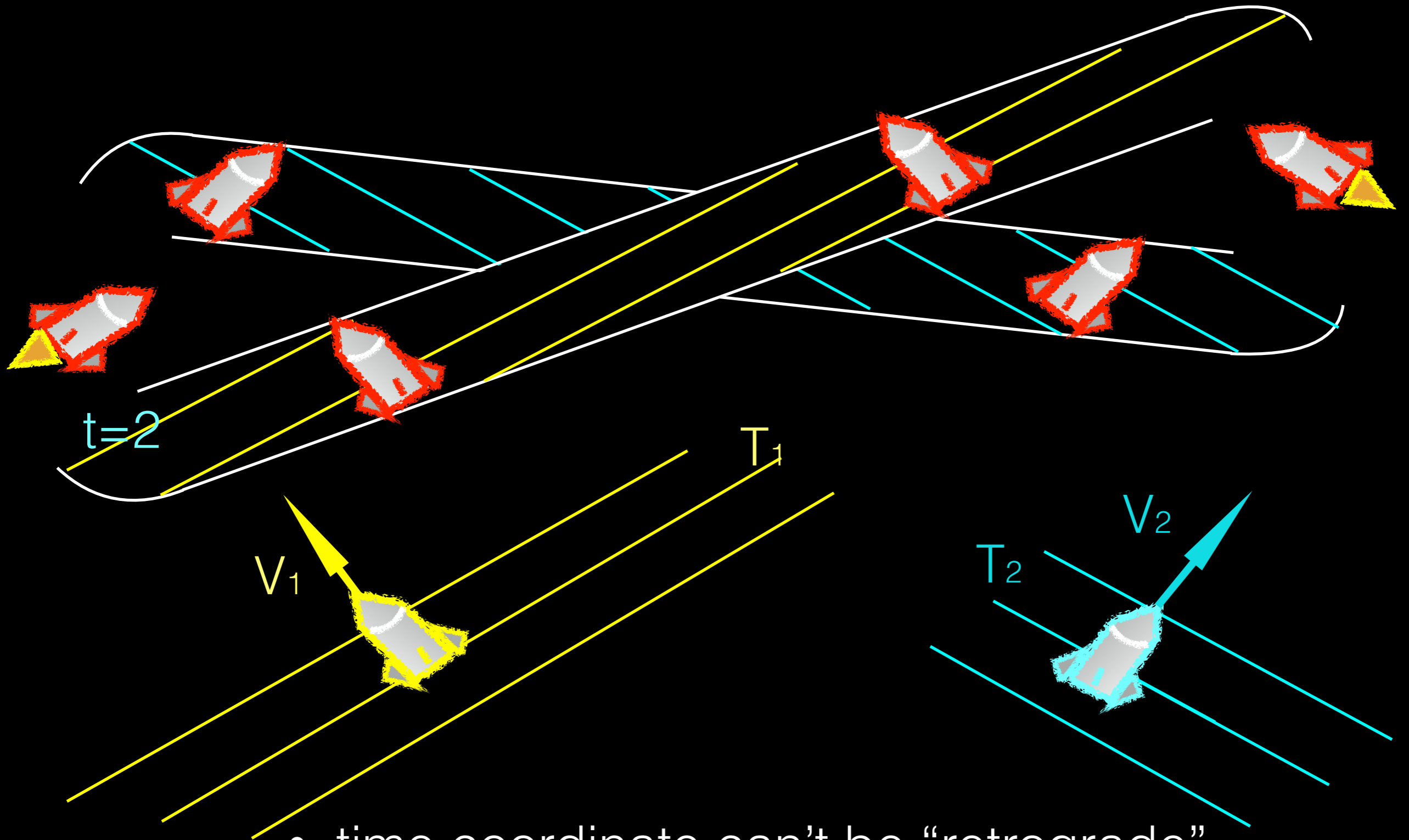


# Alcubierre Warp drive



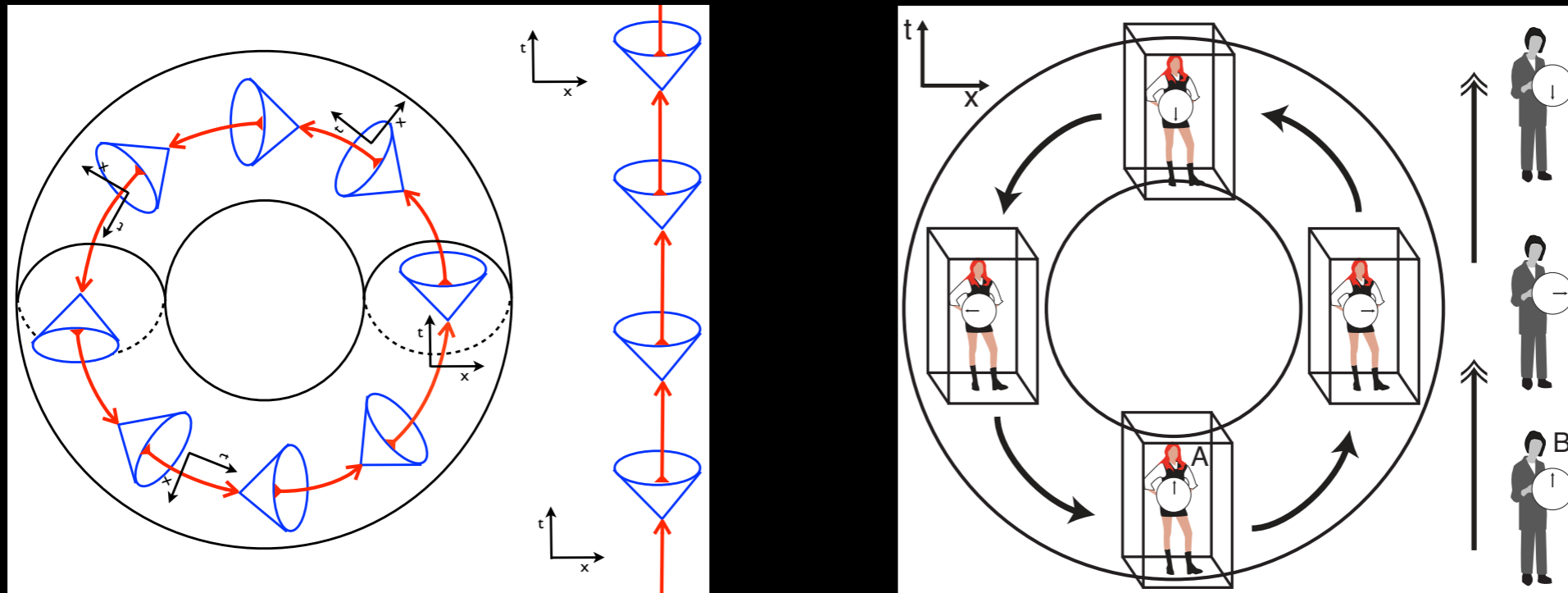
- Vacuum inside and outside
- “Bubble” geometry
- observer inside, comoving with walls has same time coord as an external observer
- Violates Classical Energy conditions

# 2 Warp bubbles=CTC



- time coordinate can't be "retrograde"

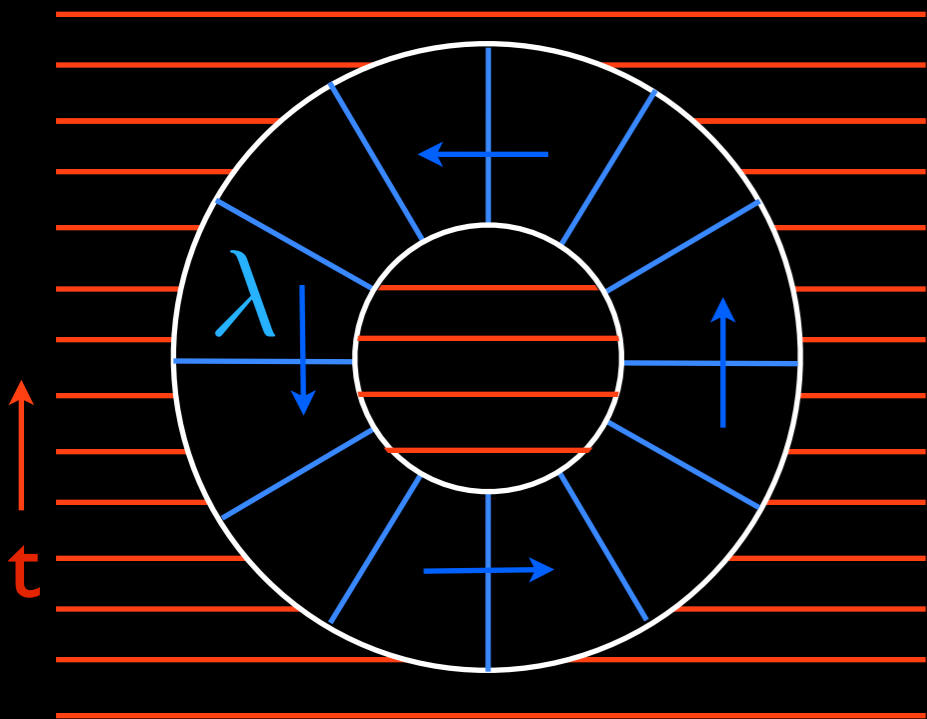
# Causal structure of a “Time Machine” bubble



Toroidal Traversable Acausal  
Retrograde Domain In Spacetime  
(T.A.R.D.I.S)

# Bubble Geometry

$$ds^2 = \left[ 1 - h(x^a) \left( \frac{2t^2}{x^2 + t^2} \right) \right] (-dt^2 + dx^2) + h(x^a) \left( \frac{4xt}{x^2 + t^2} \right) dxdt + dy^2 + dz^2$$



**Outside Bubble:** Minkowski Vacuum  
 $h(x)=0$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

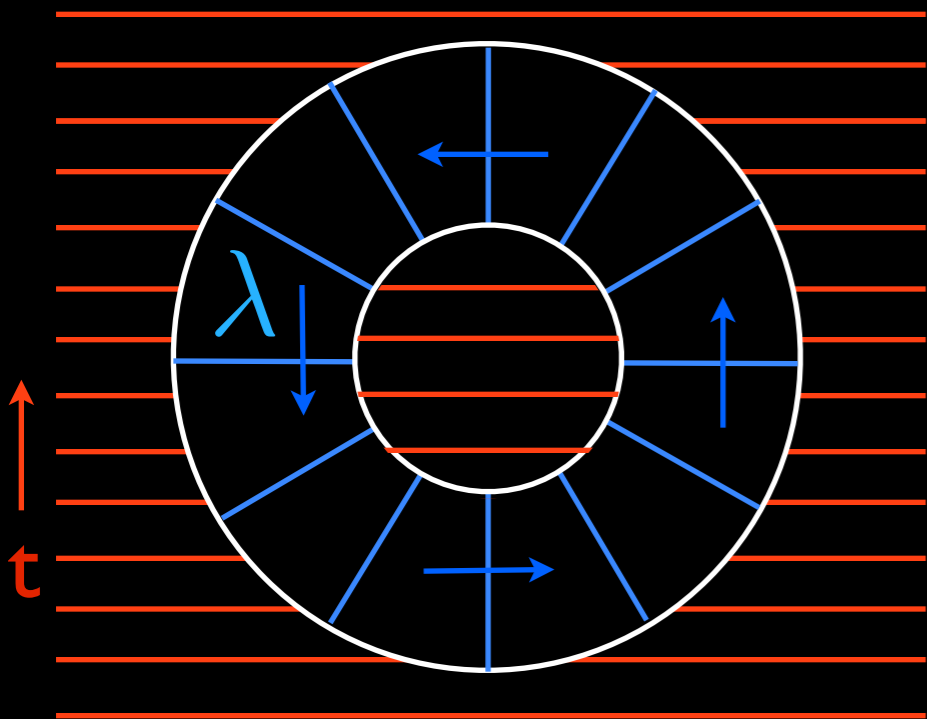
**Inside Bubble:** Rindler Vacuum  $h(x)=1$

$$ds^2 = \left( \frac{x^2 - t^2}{x^2 + t^2} \right) (-dt^2 + dx^2) + \left( \frac{4xt}{x^2 + t^2} \right) dxdt + dy^2 + dz^2$$

$$t = R \sin(\lambda) , \quad x = R \cos(\lambda) \quad \longrightarrow \quad ds^2 = -R^2 d\lambda^2 + dR^2 + dy^2 + dz^2$$

# Bubble Geometry

$$ds^2 = \left[ 1 - h(x^a) \left( \frac{2t^2}{x^2 + t^2} \right) \right] (-dt^2 + dx^2) + h(x^a) \left( \frac{4xt}{x^2 + t^2} \right) dxdt + dy^2 + dz^2$$



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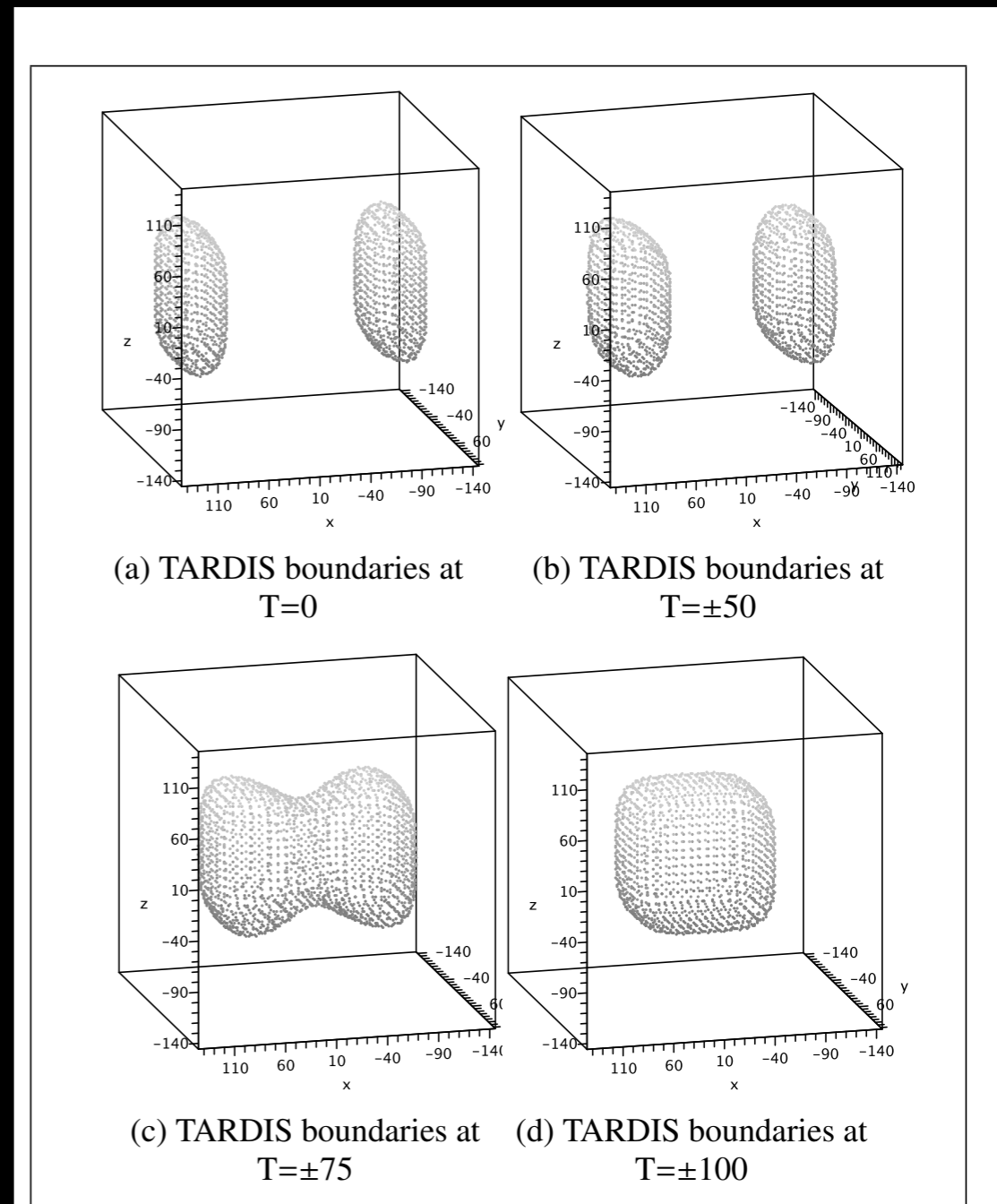
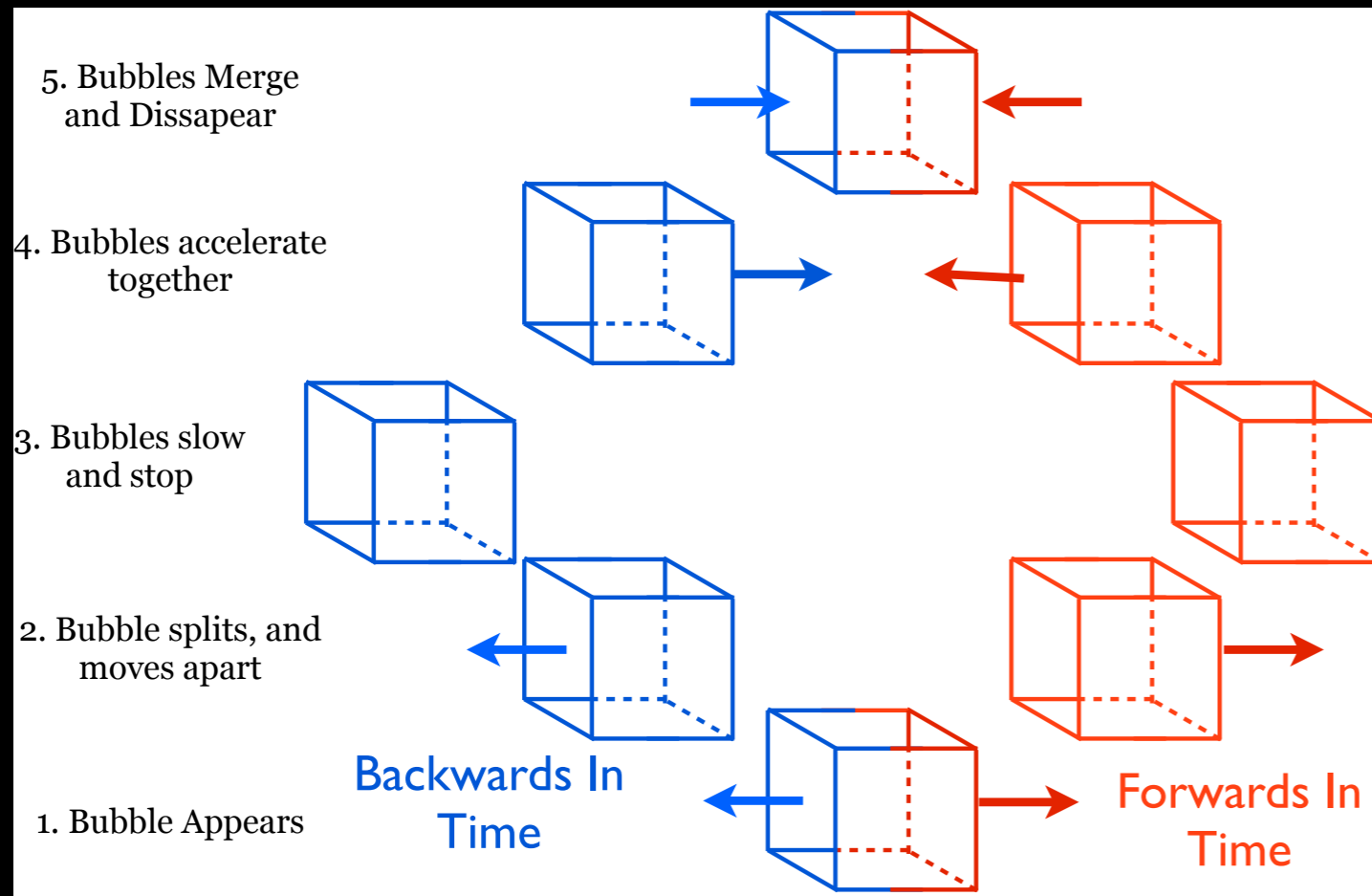
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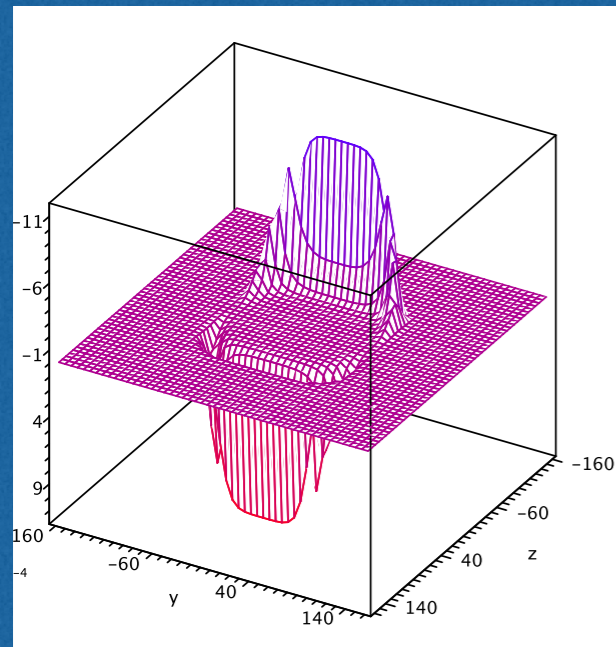
$$t = R \sin(\lambda) , \quad x = R \cos(\lambda) \quad \longrightarrow \quad ds^2 = -R^2 d\lambda^2 + dR^2 + dy^2 + dz^2$$

# From The Outside:

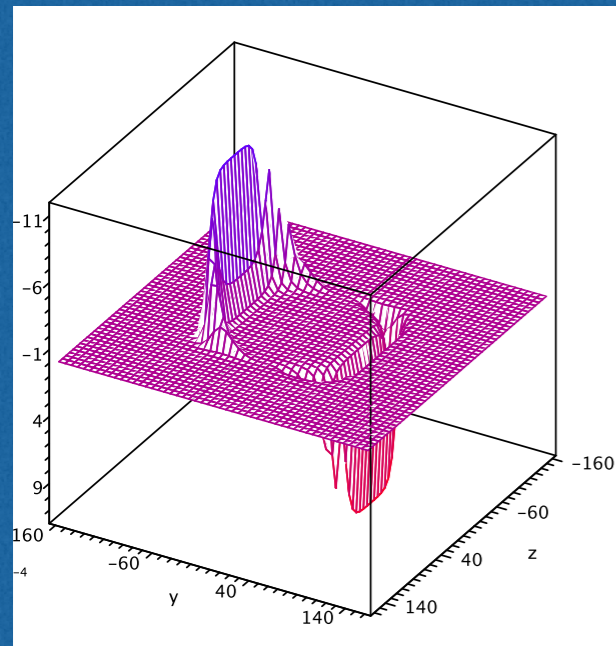
$$ds^2 = \left[ 1 - h(x^a) \left( \frac{2t^2}{x^2 + t^2} \right) \right] (-dt^2 + dx^2) + h(x^a) \left( \frac{4xt}{x^2 + t^2} \right) dxdt + dy^2 + dz^2$$



# Stress Energy Tensor

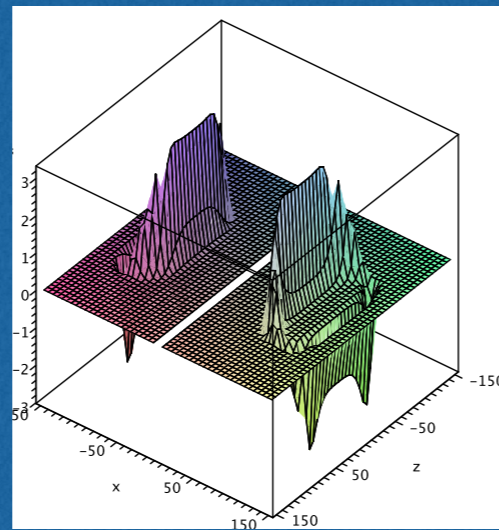


(a)  $G_{xz}$  along the slice  $t = 0$ ,  $x = 100$

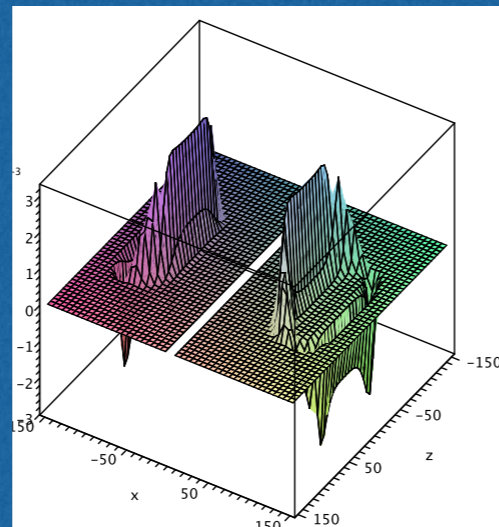


(b)  $G_{yy}$  along the slice  $t = 0$ ,  $x = 100$

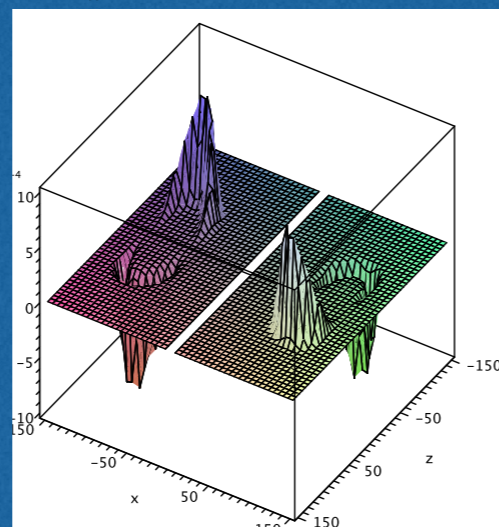
Figure 9: Nonzero terms of the stress energy tensor along the slice  $x = 100$ ,  $t = 0$ .



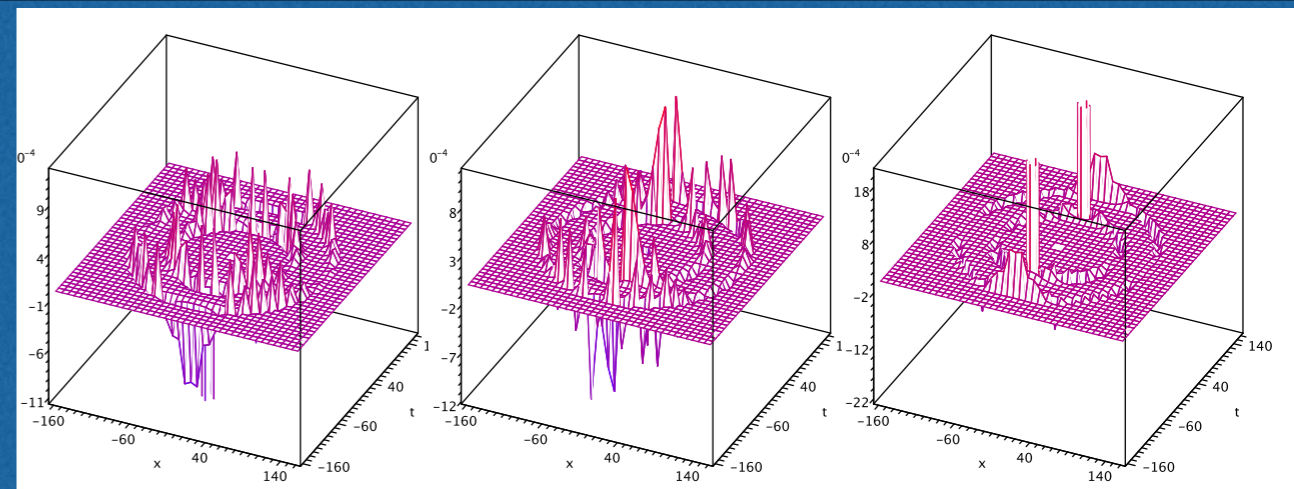
(a)  $G_{zz}$  along the slice  $y = 0$ ,  $t = 0$



(b)  $G_{yy}$  along the slice  $y = 0$ ,  $t = 0$



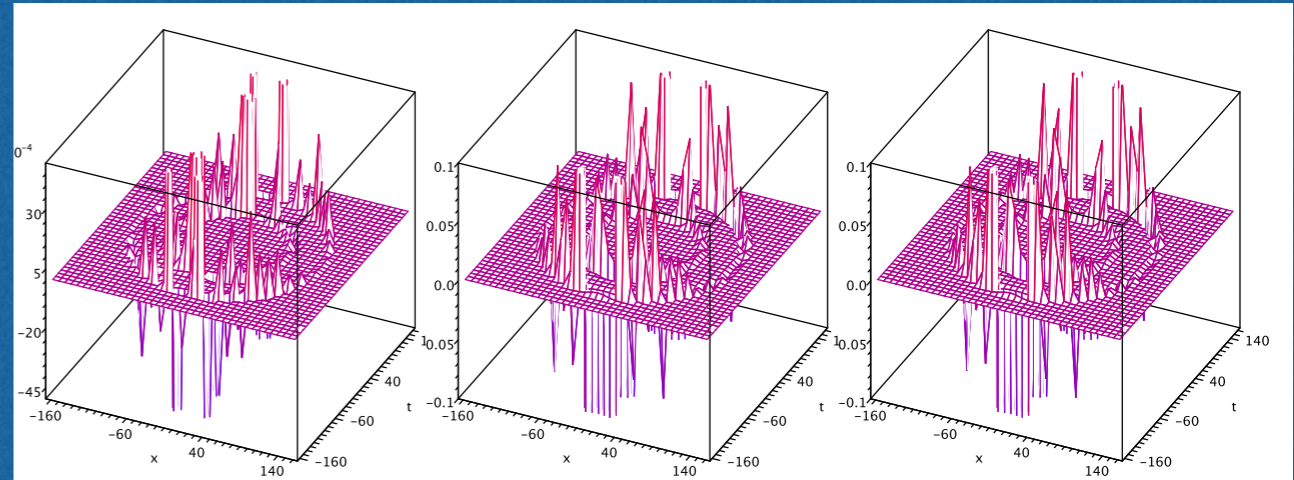
(c)  $G_{xz}$  along the slice  $y = 0$ ,  $t = 0$



(a)  $G_{tt}$  along the slice  $z = 30$ ,  $y = 0$

(b)  $G_{tt}$  along the slice  $z = 30$ ,  $y = 0$

(c)  $G_{xx}$  along the slice  $z = 30$ ,  $y = 0$



(d)  $G_{xz}$  along the slice  $z = 30$ ,  $y = 0$

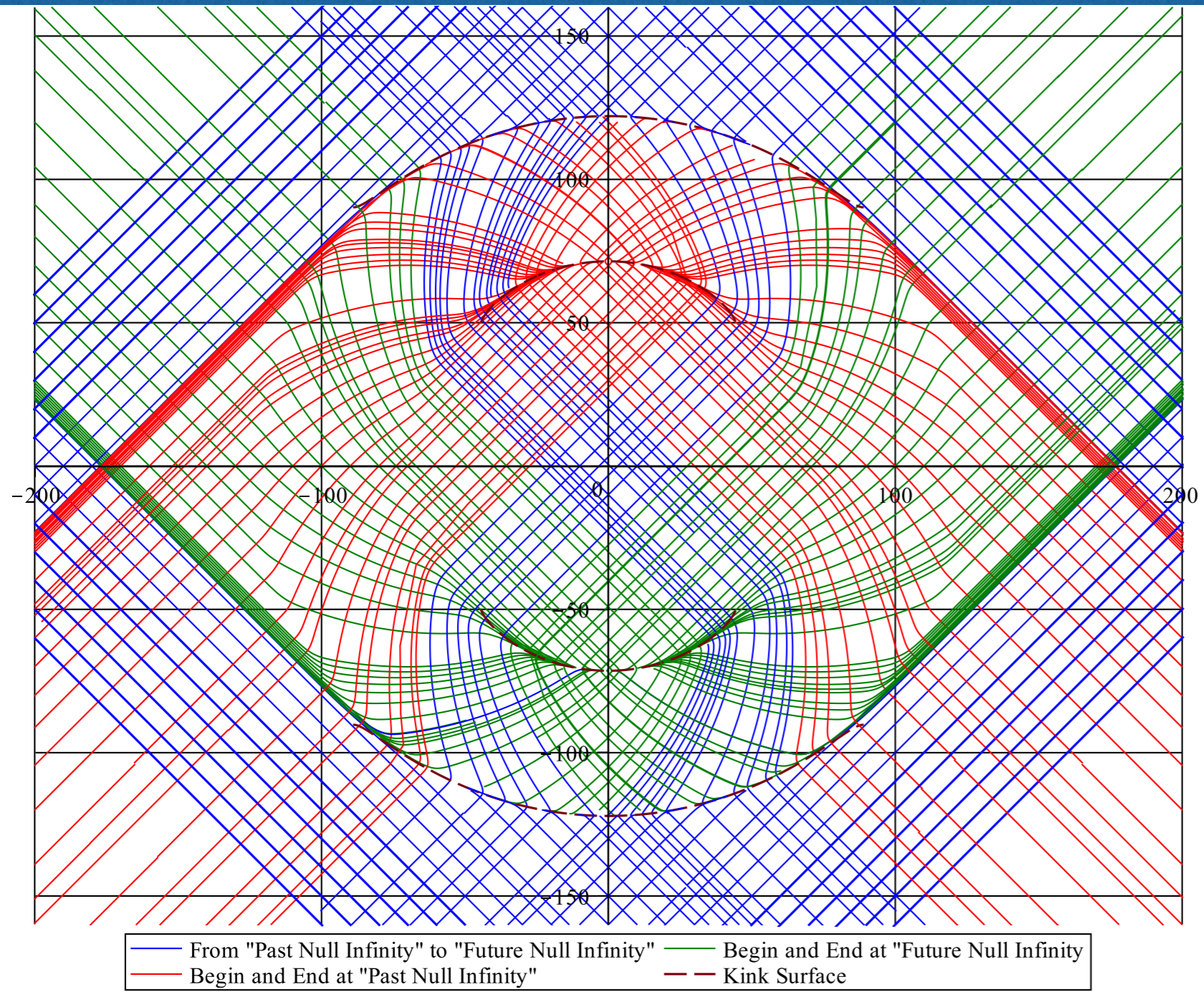
(e)  $G_{yy}$  along the slice  $z = 30$ ,  $y = 0$

(f)  $G_{zz}$  along the slice  $z = 30$ ,  $y = 0$

Figure 11: Nonzero elements of the stress energy tensor along the slice  $y = 0$ ,  $z = 30$ .

- Violates CEC
- Curvature Singularities

# Causal Structure: Null Geodesics



Light cones tilt

Cauchy Horizons

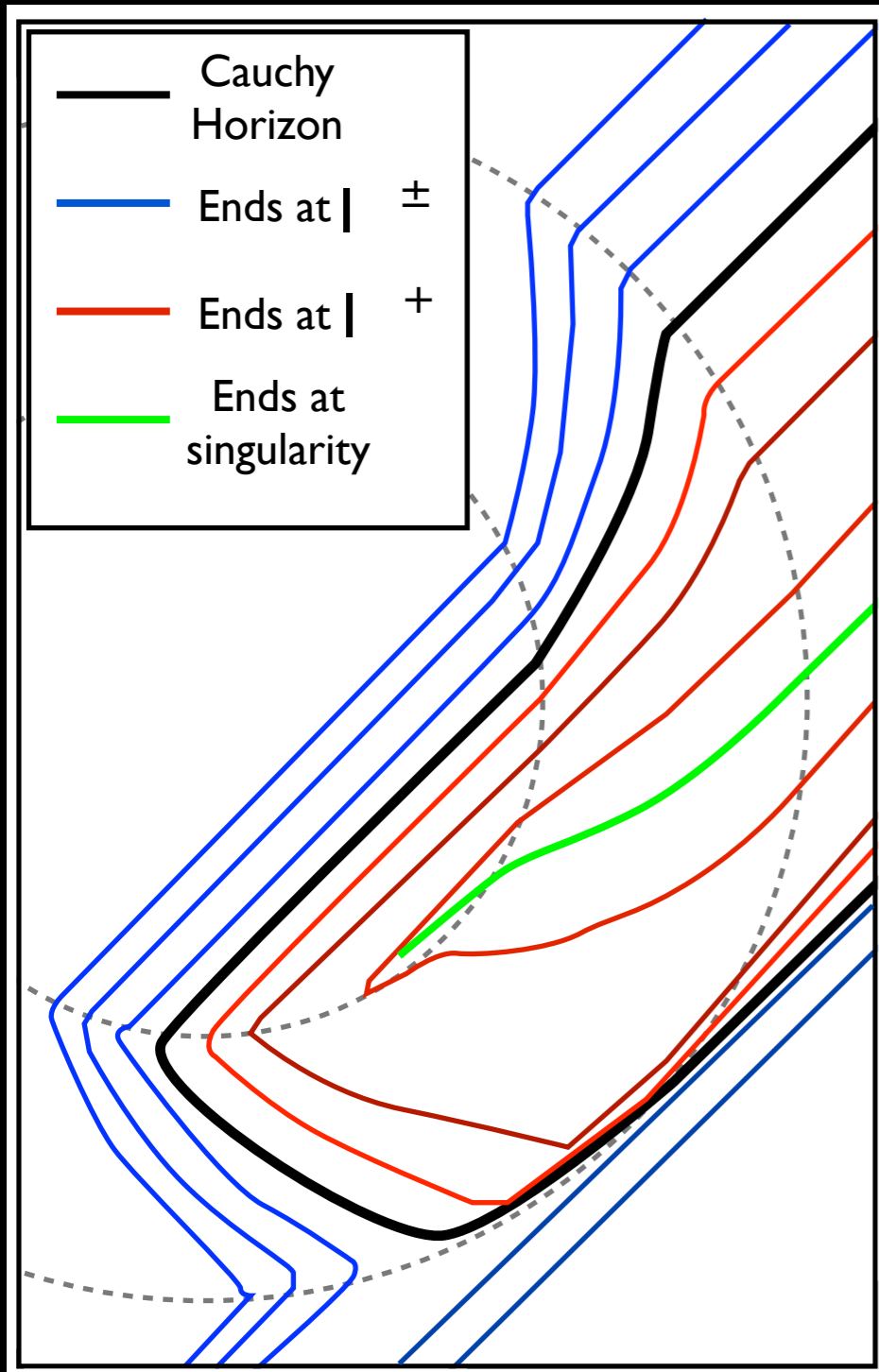
Manifold Non-Orientable

CH not compactly generated

• (CPC does not apply)

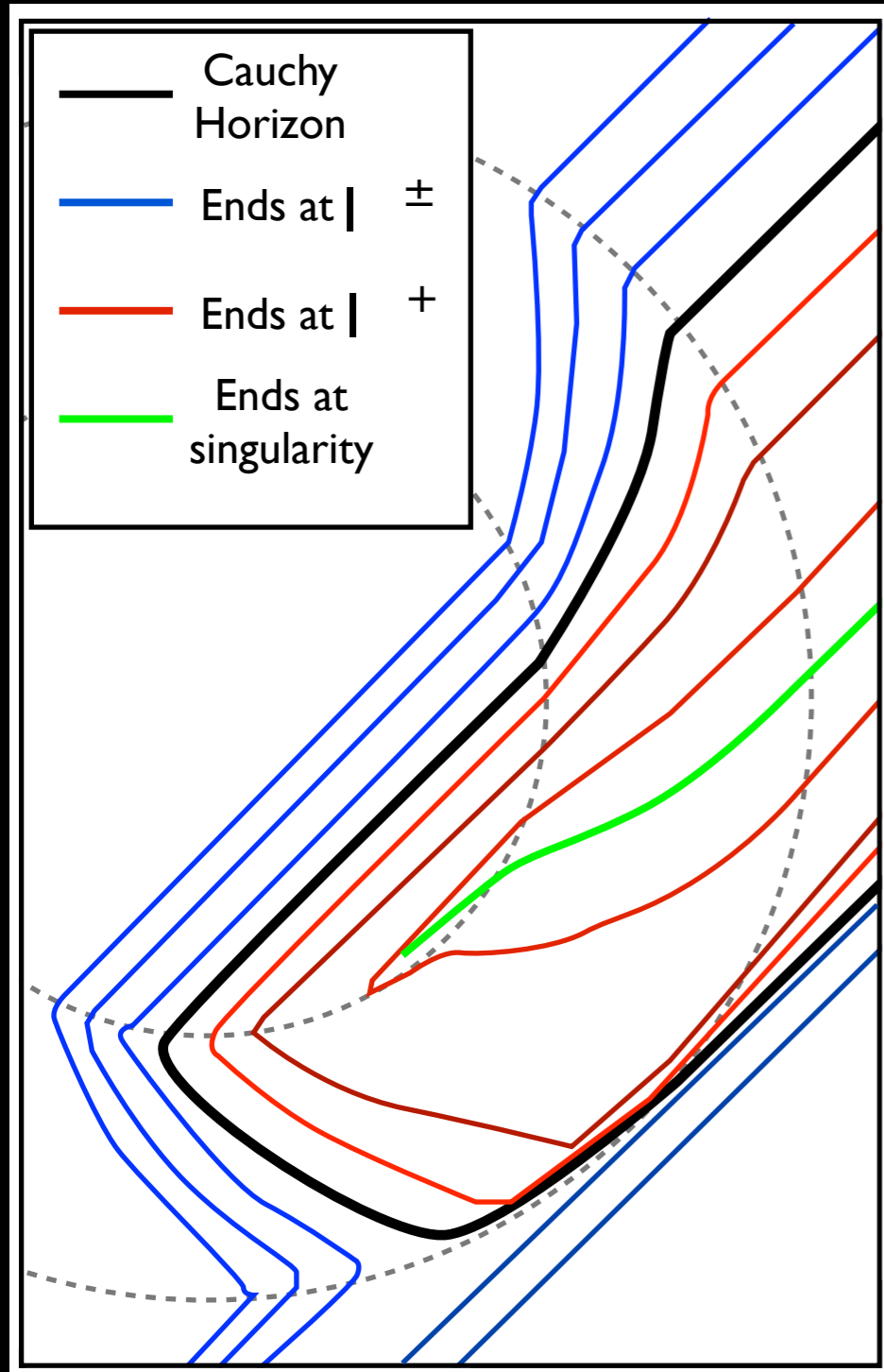


# Causal Structure: Geodesically Incomplete



- Incomplete null geodesics terminate at curvature singularities
- Some of these incomplete curves generate Cauchy Horizon

# Conclusion: Fun!



- Bubble geometry has all desired properties
  - Non Orientable manifold
- Physically Problematic:
  - Violates Classical EC
  - Curvature Singularities
  - Geo. Incomplete
- Why aren't these issues present in the Warp Drive CTC geometry?