TEXAS MATH SOLUTION

Accelerated Grade 6

Module 2 Topic 3 Lesson 1

Many Ways to Measure

Teacher's Implementation Guide

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Many Ways to Measure

1

Using Ratio Reasoning to Convert Units

MATERIALS

None

Lesson Overview

Students deepen their understanding of converting units of measurement through the use of ratio reasoning and strategies for determining equivalent ratios. The term *convert* is defined, and students use approximate conversion rates to estimate measurement conversions before engaging with formal methods of converting. Converting among units of measurement in the same system is recast in terms of conversion ratios, which can also be called conversion rates.

Students use ratio reasoning and strategies to convert within and between the U.S. customary system and the metric system. Students use double number lines, ratio tables, and scaling up and down to convert units of measurement. They analyze Worked Examples of the different strategies. For scaling up and down, students explain why one conversion ratio is more appropriate than the other. Finally, students are introduced to unit analysis as a strategy for converting between units of measurement. They practice using unit analysis in problems about distance, money, and area. Students make choices about which strategy to use when converting between units of measurement.

Grade 6
Proportionality

(4) The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:

(H) convert units within a measurement system, including the use of proportions and unit rates.

Grade 7
Proportionality

(4) The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

(E) convert between measurement systems, including the use of proportions and the use of unit rates.

ELPS

1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E, 5.F

Essential Ideas

- When a smaller unit of measure is converted to a larger unit of measure, the larger unit of measure has fewer units.
- When a larger unit of measure is converted to a smaller unit of measure, the smaller unit of measure has more units.
- All of the strategies used to determine equivalent ratios (double number lines, ratio tables, scaling up and down) can be used to convert between units.
- Unit analysis is a strategy for converting units that ensures the correct calculations and units in the final result.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Customary to Whom?

Students determine which U.S. customary system units and which metric system units would be most appropriate for measuring a variety of objects or quantities.

Develop

Activity 1.1: Reasoning About Unit Conversions

Several estimates are provided to help students convert between metric and customary measures. Using these estimates, students will determine strategies to estimate different measurement conversions. Then, converting between units within the same system is recast in terms of conversion ratios, or rates, and students write common conversion statements as conversion ratios.

Activity 1.2: Using Double Number Lines to Convert Units

Students use a common ratio strategy to determine conversions within and across systems. They use a ruler with inches and centimeters, a real-world double number line, to convert from one unit to the other. Then they use double number lines to convert between other units.

Day 2

Activity 1.3: Using Ratio Tables and Scaling to Convert Units

Students use ratio tables to convert between pounds and ounces and between milliliters and liters. Students examine a Worked Example of scaling up to convert from pounds to kilograms, answering questions about why the rate $\frac{1 \text{ lb}}{0.45 \text{ kg}}$ was used instead of the rate $\frac{2.2 \text{ lb}}{1 \text{ kg}}$. Students use scaling up and down to convert measurements within and between systems.

Activity 1.4: Using Unit Analysis to Convert Units

Students analyze a Worked Example about unit analysis. They explain why multiplying by the conversion rate does not change the quantity being measured. They analyze student work to judge the reasonableness of two different answers using unit analysis. Students conclude that in unit analysis, the desired quantity should be placed in the numerator of the conversion rate before performing the multiplication. They then practice unit analysis to convert measurements within and between systems.

Demonstrate

Talk the Talk: Larger or Smaller?

Students answer questions about the process of converting units and how to judge the reasonableness of their results, depending on whether they are converting from smaller units to larger units or from larger units to smaller units. They choose which conversion strategy to use to solve conversion problems.

Getting Started: Customary to Whom?

Facilitation Notes

In this activity, students determine which U.S. customary system units and which metric system units would be most appropriate for measuring a variety of objects or quantities.

Direct students to complete the table in Question 1 with their partner or group. Discuss their answers as a class, noting the units that they used for each measurement. Some students may need to be reminded of the units used in each system to measure length, weight, and volume. Then have students complete Question 2 and justify their answers to the class.

Questions to ask

- Can you give an example of something that is about a mile in length?
- What container do you think of when you think of one gallon?
- Did you ever need to measure an ounce of anything? What was it?
- Did you ever need to measure exactly one inch? What was the situation?
- How do you know the weight of your object is approximately one ton?
- What would determine if ounces or pounds would best describe the weight of your math book?
- What would you estimate is the weight of a car?
- Did you ever need to measure exactly one centimeter? What was the situation?
- Did you ever need to measure a meter of anything? What was it?
- Do all car gas tanks have the same capacity?
- Which cars have the largest gas tank? Which cars have the smallest gas tank?
- What helped you to decide the most appropriate measurement for each item in Question 2?

Differentiation strategy

To assist all students, have common objects available (such as a gallon of milk, a can of vegetables, a 2-liter bottle of soda) to help students make estimates.

Summary

The U.S. customary system and metric system are useful when describing objects and quantities.

Activity 1.1
Reasoning About Unit Conversions

DEVELOP

Facilitation Notes

In this activity, several estimates are provided to help students convert between metric and customary measures.

Using these estimates, students will determine strategies to estimate different measurement conversions. Then, converting among units in the same measurement system is recast in terms of conversion ratios, or rates, and students write common conversion statements as conversion ratios.

Questions 1 through 5 are intended to build on intuition and provide a motivation for more formal conversion techniques. The focus is on reasoning about conversions, not computations.

Ask a student to read the introduction aloud and complete Question 1 as a class. Instruct students to work with a partner or in groups to review the estimates given and complete Questions 2 through 5. Share responses as a class. Direct students to work with a partner or in groups to review the information and complete Question 6. Share responses as a class

Questions to ask

- Does it make more sense to ask how many kilometers are in a mile, or to ask how many miles are in a kilometer? Explain.
- Does it make more sense to ask how many liters are in a gallon, or to ask how many gallons are in a liter? Explain.
- Does it make more sense to ask how many kilograms are in a pound, or to ask how many pounds are in a kilogram? Explain.
- How do inches compare to feet?
- How do meters compare to centimeters?
- One mile is approximately how many kilometers?
- One kilogram is approximately how many pounds?

- Are the conversions easier to understand when written in ratio language or expressed symbolically? Why?
- Are the conversions easier to understand when written in ratio language or written as a ratio in fractional form? Why?

Differentiation strategy

To extend the activity, ask students the value of each of the ratios in Question 6. Have a discussion as to why all the conversions are equal to 1.

Misconception

When making conversions, students sometimes overgeneralize, thinking a larger unit of measure means there will be a larger quantity of that unit and a smaller measure means that there will be a smaller quantity of that unit. Some students may grasp the correct interpretation when answering Questions 2 and 3, but lose the idea when they must answer Question 4. It may be useful to have students draw sketches to make sense of this concept, especially when converting between metric and customary measures. A meter/yard stick can perpetuate this misconception because students visualize them as the same size. If you have a meter/yard stick in your classroom, consider covering the inches over 36 to make it clearer to students that a yard is less than a meter. That may help them see more clearly that it will take more yards than meters to measure a given length.

Summary

Conversion ratios or conversion rates are used to compare two quantities that are measured in different units.

Activity 1.2

Using Double Number Lines to Convert Units

Facilitation Notes

In this activity, students use a common ratio strategy to determine conversions within and across systems. They use a ruler with inches and centimeters, a real-world double number line, to convert from one unit to the other. They then use double number lines to convert between other units.

Ask a student to read the introduction aloud. Have students work with a partner or in groups to complete Questions 1 through 5. Share responses as a class.

Questions to ask

- Which has a greater length, 1 centimeter or 1 inch?
- Is 2 inches less than 1 centimeter, or is 2 centimeters less than 1 inch?
- One cup of sugar is equal to how many tablespoons of sugar?
- One-fourth cup of sugar is equal to how many tablespoons of sugar?
- How do you think the intervals were selected for each number line?
- Two tablespoons is what part of a cup?
- How does the length of 1 meter compare to the length of 1 foot?
- How did you determine the length of the room in feet?

Misconception

Some students get confused when asked to measure in centimeters, but the ruler says "mm". Use this opportunity to revisit the terms interval and scale.

Differentiation strategy

For students who struggle reading a ruler, review that a whole on the inches side of the ruler is 16 intervals and a whole on the centimeter side of the ruler is 10 intervals. First they can write a fraction with 10 or 16 as the denominator, and then they can rewrite the fraction in lowest terms.

Summary

Double number line models can be used to in real-life scenarios to convert between measurement units.

Activity 1.3

Using Ratio Tables and Scaling to Convert Units

Facilitation Notes

In this activity, students use ratio tables to convert between pounds and ounces and between milliliters and liters. Students examine a Worked Example of scaling up to convert from pounds to kilograms, answering questions about why the rate $\frac{1\text{ lb}}{0.45\text{ kg}}$ was used instead of the rate $\frac{2.2\text{ lb}}{1\text{ kg}}$. Students use scaling up and down to convert measurements within and between systems.

Have students work with a partner or in groups to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- What values did you add to determine equivalent ratios?
- What values did you multiply or divide to determine equivalent ratios?
- Is it necessary to complete the table in order from left to right?
- Are there multiple ways to create the table?
- How is this table different from other tables you've completed in the past?
- Did you prefer working with the decimals or fractions? Why?
- What pattern do you recognize in the milliliter/liter table?

Misconception

Some students may think the equivalent to 6 ounces is $\frac{1}{3}$ pound, because they figured 4 ounces is $\frac{1}{4}$ pound and 8 ounces is $\frac{1}{2}$ pound, 6 ounces is in the middle of 4 and 8, so the equivalent of 6 ounces is $\frac{1}{3}$ (taking the middle number of the denominators). Disprove this method by using decimal equivalents. A correct method would be to use 4 ounces is $\frac{1}{4}$ pound, so 2 ounces is $\frac{1}{8}$ pound, by addition, 6 ounces is $\frac{3}{8}$ pound.

Ask a student to read the information after Question 4 aloud. Discuss the information provided in the conversion table. Review the Worked Example and answer Question 5 as a class.

Have students complete Questions 6 through 10 with a partner or in groups. Share responses as a class.

Differentiation strategy

Have each group solve one problem and present their work to the class. After students present, ask for different strategies to solve the same problem.

As students work, look for

- The selection of the correct conversion, but confusing the numerator and denominator.
- Conversion rates that have one correct unit of measure but not the other.

Questions to ask

- When setting up the proportion to convert gallons into liters, how did you decide what ratio to use?
- Did you convert meters into yards or yards into meters? Why?
- How did you convert meters into feet? Or feet into meters? Does it matter?
- When setting up the proportion to convert pounds into kilograms, how did you decide what ratio to use?

- When setting up a proportion to convert grams into ounces, how did you decide what ratio to use?

Summary

Ratio tables and scaling up and scaling down are strategies that can be used in real-life scenarios to convert between measurement units.

Activity 1.4
Using Unit Analysis to Convert Units

Facilitation Notes

In this activity, students analyze a Worked Example about unit analysis. They explain why multiplying by the conversion rate does not change the quantity being measured. They analyze student work to judge the reasonableness of two different answers using unit analysis. Students conclude that in unit analysis, the desired quantity should be placed in the numerator of the conversion rate before performing the multiplication. They then practice unit analysis to convert measurements within and between systems.

Ask a student to read the introduction aloud, then have students review the Worked Example individually. Have students work with a partner or in groups to complete Questions 1 through 3. Discuss as a class.

Questions to ask

- What do scaling up and unit analysis have in common?
- How are the two strategies different?
- What form of 1 is used in the unit analysis of this Worked Example?
- Does multiplying by a form of 1 change the quantity?
- Is it easier to solve this problem using scaling up or unit analysis? Why?
- In Christopher's work, should feet appear in the numerator or denominator? How do you know?
- Do units are eliminated when using unit analysis?
- What information is needed to use unit analysis as a strategy for unit conversion?

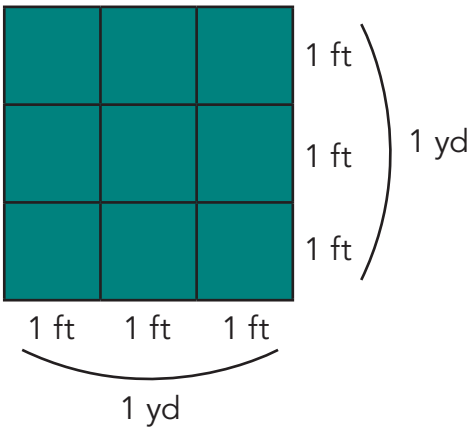
Have students work with a partner or in groups to complete Questions 4 through 6. Share responses as a class. Then have students complete Questions 7 through 9 and share responses as a class.

Questions to ask

- Did you use unit analysis to solve this problem? Why or why not?
- How did you know what unit conversion to use?
- Did you use scaling up or scaling down to solve this problem? Why or why not?

Misconception

Students often use the same conversion factor when using units or square units. They will encounter this situation in Question 9 part (b). To help students make sense of this situation converting square feet to square yards, draw this figure to scale on the board or tape it off on the floor for reference.



Summary

Unit analysis is a strategy that can be used in real-life scenarios to convert between measurement units.

DEMONSTRATE

Talk the Talk: Larger or Smaller?

Facilitation Notes

In this activity, students answer questions about the process of converting units and how to judge the reasonableness of their results, depending on whether they are converting from smaller units to larger units or from larger units to smaller units. They choose which conversion strategy to use to solve conversion problems.

Have students work with a partner or in groups to complete Questions 1 through 8. Share responses as a class.

Questions to ask

- Which conversion ratio is needed to convert gallons to liters?
- How is the conversion ratio is needed to convert gallons to liters different than the ratio needed to convert liters to gallons?
- Which conversion ratio is needed to convert ounces to pounds?
- Which conversion ratio is needed to convert kilometers to meters?
- Which conversion ratio is needed to convert inches to yards?

Summary

When a measurement with smaller units is converted to measurement with larger units, the number of units decrease. When a measurement with larger units is converted to a measurement with smaller units, the number of units increase. A strategy used to convert smaller units to larger units may differ from a strategy used to convert larger units to smaller units.

NOTES

1 Many Ways to Measure

Using Ratio Reasoning to Convert Units

WARM UP

Answer each question about a common measurement conversion.

1. How many inches are in 1 foot?
2. How many feet are in 1 yard?
3. How many grams are in 1 kilogram?
4. How many milliliters are in 1 liter?
5. How many centimeters are in 1 meter?
6. How many fluid ounces are in 1 cup?
7. How many quarts are in 1 gallon?
8. Which of the previous questions include units that are part of the U.S. customary system of measurement, and which include units that are part of the metric system?

LEARNING GOALS

- Use ratio reasoning with double number lines to convert measurement units.
- Use ratio reasoning with ratio tables to convert measurement units.
- Use scaling up or scaling down to convert and transform measurement units appropriately.
- Use unit analysis to convert and transform measurement units appropriately.

KEY TERM

- convert

In previous grades, you have worked with the U.S. customary system and the metric system of measurement. This year, you have also learned about ratios. How can you use ratio reasoning to convert from one measurement unit to another in order to solve problems?

Warm Up Answers

1. 12
2. 3
3. 1000
4. 1000
5. 100
6. 8
7. 4
8. U.S. customary: Questions 1 and 2, 6 and 7; metric: Questions 3 through 5

Answers

1.

U.S. Customary System	Metric System
feet	meters
inches	centimeters
miles	kilometers
pounds or ounces	kilograms or grams
fluid ounces	liters
gallons	kiloliters

2a. 15 pounds
2b. 55 liters
2c. 12 feet
2d. 3 meters

Getting Started

Customary to Whom?

In the U.S., customary units are primarily used for business, personal, and social purposes. Sciences, including the medical field, use the metric system.

You've learned about the relationships between inches and feet, feet and yards, quarts and gallons, meters and millimeters—to name a few.

1. Name a U.S. customary system unit and a metric system unit that would be an appropriate size to measure each object or quantity.

Object/Quantity	U.S. Customary System	Metric System
Your height		
Length of your pencil		
Distance from your school to the beach		
Weight of your math book		
Amount of water in a bottle		
Amount of water in a swimming pool		

2. Circle the most appropriate measurement for each item.

a. The weight of a dog
- 15 pounds
- 18 ounces
- 1 ton
- 25 fluid ounces

b. The amount of gas in a car's tank
- 50 milliliters
- 2 kiloliters
- 55 liters
- 12 kiloliters

c. The height of your classroom
- 90 inches
- 1 mile
- 2 yards
- 12 feet

d. The height of a basketball hoop
- 3 meters
- 70 centimeters
- 500 millimeters
- 1 kilometer

ELL Tip

In order to understand and successfully complete tasks in the U.S., English Language Learners will have to learn how to read and understand the U.S. customary system of measurement. Have English Language Learners share their home country's units of measurement for everyday tasks, such as building with wood, cooking, and reading distances on a map. Encourage students to know important formulas for quick converting and to find online conversion calculators for more complex conversions.

ACTIVITY
1.1

Reasoning About Unit Conversions

You can use more than one measurement to describe the same length, weight, or capacity. For example, you may say that a football field is 100 yards long or 300 feet long. You could also say that the football field is about 90 meters long. In each case, the lengths are the same—you just say them in different ways.

There are many situations in which you need to *convert* measurements to different units. To **convert** a measurement means to change it to an equivalent measurement in different units.

When you convert a measurement to a different unit, the size of the object does not change; only the units and the number of those units change.

1. **Name a situation in which converting one measurement to another would be necessary or useful.**

Before you start converting units, it is useful to estimate the number of units to expect in a conversion. A few estimates comparing common metric and U.S. customary measures are given.

- One meter is about the same length as one yard.
- One inch is about 2.5 centimeters.
- One kilometer is a little more than half of a mile.
- One foot is about 30 centimeters.
- One liter is about the same as one quart.
- One kilogram is a little more than 2 pounds.

Answer

1. Answers will vary.

Answers

2a. inches
2b. centimeters
2c. yards
2d. kilometers
2e. pounds

3. It will take more of the smaller unit of measurement to measure the same object.

4a. Approximately 274 miles
4b. Approximately 10 pounds

5. Sample answer. I used the estimates provided and then reasoned about which would be larger to determine if I should multiply or divide by 2.

Use the estimates given and your knowledge of metric and U.S. customary measures to answer each question.

2. The numeric value of which measurement will be greater?

a. The length of a table in inches or in feet

b. The length of a table in meters or in centimeters

c. The length of a table in meters or in yards

d. The distance from school to your house in miles or in kilometers

e. The weight of your math book in kilograms or in pounds

3. How did you decide which value would be greater in Question 2?

4. Estimate each measurement conversion.

a. The distance to Toronto is 548 km. About how many miles is that?

b. You order 5 kilograms of food pellets for your guinea pig. About how many pounds are you ordering?

5. Describe the strategies you used to estimate each measurement conversion in Question 4.

Because most conversions compare two quantities using multiplicative strategies, the conversion estimates provided and the conversions within systems that you already know can be written using ratio language. They can also be written symbolically in terms of equality.

Ratio Language	Symbolically
For every inch, there are approximately 2.5 centimeters.	1 in. ≈ 2.5 cm
For every meter, there is approximately 1 yard.	1 m ≈ 1 yd
For every foot, there are approximately 30 centimeters.	1 ft ≈ 30 cm
For every 12 inches, there is exactly 1 foot.	12 in. = 1 ft
For every 1 kilometer, there are exactly 1000 meters.	1 km = 1000 m

The ≈ symbol means that the two values are approximately equal.

When a conversion ratio is presented for use in converting between units of measure, it is often written as an equation: 12 in. = 1 ft. However, it can also be written as a ratio in fractional form: $\frac{12 \text{ in.}}{1 \text{ ft}}$.

Because conversions compare two quantities that are measured in different units, conversion ratios can also be called conversion rates.

6. Rewrite each common conversion using ratio language and as a ratio in fractional form.

a. 3 ft = 1 yd

b. 5280 ft = 1 mi

c. 1 lb ≈ 0.45 kg

d. 4 qt = 1 gal

e. 1 m = 100 cm

f. $\frac{1}{1000}$ m = 1 mm

Because these measurement conversions are ratios, you can use ratio reasoning to convert between units. For example, you can determine the number of miles in a 10-kilometer race or the number of fluid ounces in 500 milliliters of a solution.

Answers

6a. For every 3 feet, there is exactly 1 yard. $\frac{3 \text{ feet}}{1 \text{ yard}}$ or $\frac{1 \text{ yard}}{3 \text{ feet}}$

6b. For every 5280 feet, there is exactly 1 mile. $\frac{5280 \text{ feet}}{1 \text{ mile}}$ or $\frac{1 \text{ mile}}{5280 \text{ feet}}$

6c. For every 1 pound, there is approximately 0.45 kilogram.
$\frac{1 \text{ pound}}{0.45 \text{ kilogram}}$ or $\frac{0.45 \text{ kilogram}}{1 \text{ pound}}$

6d. For every 4 quarts, there is exactly 1 gallon.
$\frac{4 \text{ quarts}}{1 \text{ gallon}}$ or $\frac{1 \text{ gallon}}{4 \text{ quarts}}$

6e. For every 1 meter, there are exactly 100 centimeters.
$\frac{1 \text{ meter}}{100 \text{ centimeters}}$ or $\frac{100 \text{ centimeters}}{1 \text{ meter}}$

6f. For every $\frac{1}{1000}$ of a meter, there is exactly 1 millimeter.
$\frac{1 \text{ meter}}{1000 \text{ millimeters}}$ or $\frac{1000 \text{ millimeters}}{1 \text{ meter}}$

Answers

1. The top scale is centimeters and the bottom scale is inches. Centimeters are smaller than inches.

2a. $\frac{3}{8}$

2b. 2.5

2c. 2

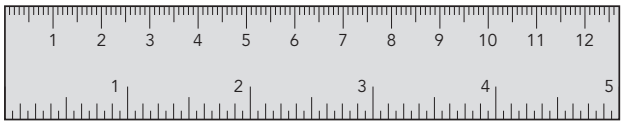
2d. 7.6

ACTIVITY 1.2

Using Double Number Lines to Convert Units

When you learned about ratios, you learned how to use double number lines to determine equivalent ratios. You can also use double number lines to convert from one unit to another.

Although you may not have realized it before, many rulers are set up as double number lines and can be used to convert between inches and centimeters.



1. Determine which scale represents inches and which represents centimeters. How did you decide? Label the scales on the ruler.

2. Use the ruler as a double number line to determine each approximate conversion.

a. 1 cm ≈ _____ in.

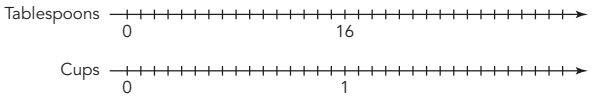
b. 1 in. ≈ _____ cm

c. 5 cm ≈ _____ in.

d. 3 in. ≈ _____ cm

You are baking cookies at your friend's house. After searching the cupboards and drawers, you cannot find the measuring cups, but you can find the tablespoon.

3. Use the double number line to determine how many tablespoons you need of each ingredient in the recipe.



a. 2 cups of sugar

b. $1\frac{3}{4}$ cups of flour

c. $\frac{1}{2}$ cup of raisins

4. Suppose you had found the cup but not the tablespoon. Use the double number line to determine how many cups you need if the recipe calls for 2 tablespoons of vanilla extract.

Answers

3a. 32
3b. 28
3c. 8
4. $\frac{1}{8}$

Answers

See below for double number line representation.
5a. 16.4 feet
5b. 13.12 feet
5c. 8.2 feet

You want to redecorate your bedroom and need to measure the room for new carpeting, paint, and a border on the walls. You realize that you have only a meter stick. You measure the room, but you need to know the dimensions in feet to purchase the materials. You record these measurements:

- The length of the room is 5 meters.
- The width of the room is 4 meters.
- The height of the room is 2.5 meters.

1 meter ≈ 3.28 feet

5. Use a double number line to determine the measurement of each dimension in feet.

a. length **b. width** **c. height**

ACTIVITY 1.3

Using Ratio Tables and Scaling to Convert Units

Answers

1. See table below.
2. Answers will vary.

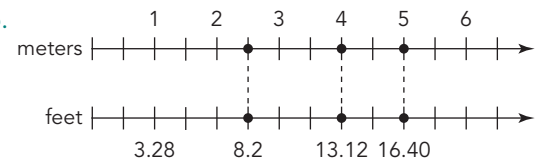
You can use ratio tables, as you did when determining equivalent ratios, as another strategy for converting units.

1. Complete the ratio table by converting between pounds and ounces.

Pounds	1	2		$1\frac{1}{4}$	$\frac{1}{2}$		
Ounces	16		4			6	40

2. What strategies did you use to determine the unknown values?

5.



1.

Pounds	1	2	$\frac{1}{4}$	$1\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$2\frac{1}{2}$
Ounces	16	32	4	20	8	6	40

3. Complete the ratio table by converting between milliliters and liters.

Milliliters	1000	100		50	1		575
Liters	1		0.5			0.01	

4. What strategies did you use to determine the unknown values?

Ratio tables are helpful tools for converting within a given system of measurement. Scaling up or down is a similar strategy for determining equivalent ratios that can be more easily used to convert from one unit of measurement to another.

You will use the common conversions shown in the table to convert between customary and metric systems.

Length	Mass	Capacity
1 in. = 2.54 cm	1 oz = 28.35 g	1 pt = 0.47 L
1 cm = 0.39 in.	1 g = 0.035 oz	1 L = 2.11 pt
1 ft = 30.48 cm	1 lb = 0.45 kg	1 qt = 0.95 L
1 m = 3.28 ft	1 kg = 2.2 lb	1 L = 1.06 qt
1 mi = 1.61 km		1 gal = 3.79 L
1 km = 0.62 mi		1 L = 0.26 gal
1 m = 39.37 in.		
1 in. = 0.0254 m		
1 m = 1.09 yd		

Most conversions that require moving between the U.S. customary and metric systems are approximations, so, in general, you will use conversion rates rounded to the nearest hundredth in your calculations.

Answers

3. See table below.

4. Answers will vary.

3.

Milliliters	1000	100	500	50	1	10	575
Liters	1	0.1	0.5	0.05	0.001	0.01	0.575

Answers

5. It is easier to scale up from 1 to 2.5 than to scale up from 2.2 to 2.5.
6. More; 60.64 L

NOTES

Scaling up or down is another strategy that you already know that can be used to convert between units.

WORKED EXAMPLE

You can use scaling up to determine how many kilograms are in 2.5 pounds. Because you want to determine the number of kilograms for a specific number of pounds, use the conversion rate 1 lb = 0.45 kg or $\frac{1 \text{ lb}}{0.45 \text{ kg}}$.

$$\frac{1 \text{ lb}}{0.45 \text{ kg}} = \frac{2.5 \text{ lb}}{? \text{ kg}} \longrightarrow \frac{1 \text{ lb}}{0.45 \text{ kg}} \overset{\times 2.5}{=} \frac{2.5 \text{ lb}}{1.125 \text{ kg}}$$

5. Why was the conversion rate $\frac{1 \text{ lb}}{0.45 \text{ kg}}$ used rather than the rate $\frac{2.2 \text{ lb}}{1 \text{ kg}}$?

Use scaling up or down to answer each question.

6. The school cafeteria has eight very large cans of tomato sauce for making pizza. Each can contains 2 gallons of sauce. Is there more or less than 50 L of sauce in these 8 cans?

ELL Tip

Give English Language Learners an opportunity to use academic language and make a distinction between formal and informal English by engaging in an activity called Radio Talk Show. Students create a talk show where they play the role of an expert. The "host" asks a "guest" questions. The guest, acting as a professional, explains the situation and methods for solving. Then other English Language Learners "call in" to ask the host and guest questions.

7. Tyrone, the quarterback for the Tigers Football team, can throw a football 40 meters. Jason, the quarterback for the Spartans, can throw a football 45 yards. Who can throw farther? How do you know?

8. Molly says that she is 1.5 meters tall. Shawna is 5 feet tall. Molly says that she is taller, but Shawna disagrees. Who is correct? Explain your reasoning.

9. Larry weighs 110 pounds, Casey weighs 98 pounds, Shaun weighs 42 kg, and Jamal weighs 52 kg. Place the boys in order from the least weight to the greatest weight using pounds and kilograms.

10. Karen has a gold bracelet that weighs 24 grams. She wants to sell the bracelet, but she needs a minimum of one ounce of gold to sell it. Can Karen sell her bracelet? Why or why not?

Answers

7. Jason
8. Shawna is correct.
9. Shaun, Casey, Larry, Jamal
10. No. She only has approximately 0.85 ounce.

Answers

1a. In scaling up, the form of 1 is $\frac{4.5}{4.5}$. In unit analysis, since 2.2 pounds = 1 kilogram, multiplying by the conversion ratio is the same as multiplying by 1.

1b. The expression kg $\times \frac{1}{\text{kg}}$ means kilograms divided by kilograms, which equals 1.

ACTIVITY 1.4

Using Unit Analysis to Convert Units

To use scaling up or down to convert one unit to another, you set up a proportion and use the conversion rate based on the given measurement that you are converting. In another strategy, *unit analysis*, you are multiplying by a form of 1 to rewrite the given measurement in a different unit.

In unit analysis, note how how the units are carried through all calculations. Units are divided out in the same way that factors can be divided out.

$$\cancel{\text{Given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$$

WORKED EXAMPLE

Determine the quantity in pounds that is equivalent to 4.5 kilograms.

Scaling Up

$$\frac{1 \text{ kg}}{2.2 \text{ lb}} \overset{\times 4.5}{=} \frac{4.5 \text{ kg}}{? \text{ lb}}$$

Unit Analysis

$$4.5 \text{ kg} \left(\frac{2.2 \text{ lb}}{1 \text{ kg}}\right)$$

$$\frac{4.5 \cancel{\text{kg}}}{1} \left(\frac{2.2 \text{ lb}}{1 \cancel{\text{kg}}}\right) = 9.9 \text{ lb}$$

$$\frac{1 \text{ kg}}{2.2 \text{ lb}} = \frac{4.5 \text{ kg}}{9.9 \text{ lb}}$$

$$4.5 \text{ kg} = 9.9 \text{ lb}$$

1. Analyze the Worked Example.

a. Both strategies used a form of 1 to determine the equivalent number of pounds in 4.5 kilograms. How is the form of 1 used in scaling up different from the form of 1 used in unit analysis?

b. Why are the labels for kilograms crossed out in the unit analysis strategy?

Christopher and Max want to determine the number of miles in 31,680 feet using unit analysis.

NOTES

Christopher

$31{,}680\ \text{ft}\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right) = 167{,}270{,}400\ \text{mi}$

Max

$31{,}680\ \text{ft} \cdot \frac{1\ \text{mi}}{5280\ \text{ft}} = 6\ \text{mi}$

2. Explain why Christopher's answer is not reasonable.

3. Explain what is different in how Christopher and Max set up their multiplication problem. What is important about how the units are arranged in the conversion rates?

Use unit analysis to convert each unit of measurement. Check to make sure your answer is reasonable.

4. A giraffe is 18 feet tall. How tall is the giraffe in inches?

Answers

2. Miles are longer than feet, so it should take fewer miles to equal a greater number of feet.
3. Max set up his conversion ratio so that the unit he is converting to is in the numerator and the unit he was starting with is in the denominator.
4. 216 inches

Answers

5. 14.5 feet
6. 96 feet
7. Answers may vary depending on the conversion factor students use.

Kilometers	Miles
5	3.1
10	6.2
20	12.4
21.091	13.1
100	62
3.864	2.4
180.32	112

5. A giraffe is 174 inches tall. How tall is the giraffe in feet?

6. The length of the school playground is 32 yards. How many feet long is the playground?

A marathon is a long-distance foot race with an official distance of 42.195 kilometers (26 miles and 385 yards) that is usually run as a road race. Larger marathons can have tens of thousands of runners. Most of these marathon runners are not professional marathoners but run to raise funds for various charities.

7. Although a marathon is a popular distance for a race, there are many other distances in which runners can train to race. Complete the table shown by writing the unknown measurements.

Race	Kilometers	Miles
Short Distance	5	
Medium Distance	10	
Medium Distance	20	
Half Marathon		13.1
Ultramarathon	100	
Ironman Triathlon Swim		2.4
Ironman Triathlon Bike		112

Conversion rates are also common in other contexts, like currency. During the 2016 Summer Olympics, the currency exchange rate between the U.S. dollar and the Brazilian real (pronounced "ray-all") was $1 US for every 3.17 BRL.

8. Alejandra's family went to the Rio de Janeiro Olympics and she budgeted $500 to spend while she was gone.

a. Write the conversion rate: _____ US = _____ BRL.

b. Did Alejandra budget more or less than 500 BRL? Explain.

c. How many BRL could she spend in Rio de Janeiro?

d. After Rio de Janeiro, Alejandra's family traveled to Mexico, where 1 BRL was equal to 5.92 pesos. If Alejandra had 295 BRL remaining, how many pesos did she have?

Answers

8a. 1 US = 3.17 BRL

8b. More. There are a little more than 3 BRL for each US$. Therefore, $500 converts to more than 500 BRL.

8c. 1585 BRL

8d. 1746.40 MXN

Answers

9a. 48 square yards

9b. 432 square feet. Convert 6 yards and 8 yards to feet and then multiply the values.

9. Emma is preparing to re-carpet her room. She measured the room to be 6 yards long and 8 yards wide. When she got to the carpet store, all of the measurements were in square feet.

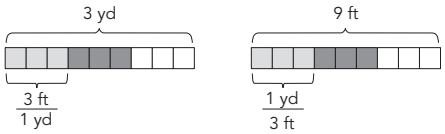
a. Determine how many square yards of carpet Emma needs to buy to re-carpet her room.

b. Determine how many square feet of carpet Emma needs to buy to re-carpet her room. How can you check your answer?

TALK the TALK

Larger or Smaller?

1. **Compare the two conversions. How are they similar? How are they different?**



2. **When you convert a measurement with smaller units to a measurement with larger units, does the number of units increase or decrease?**

3. **When you convert a measurement with larger units to a measurement with smaller units, does the number of units increase or decrease?**

NOTES

Answers

1. Both models show conversions between feet and yards. For both conversions, you multiply by 1. When you convert yards to feet and cross out the units, you are multiplying by $\frac{3}{1}$. When you convert from feet to yards and cross out the units, you are multiplying by $\frac{1}{3}$.
2. The number of units decreases.
3. The number of units increases.

Answers

4. To convert units, you must know the conversion ratio.
5. 45.48
6. 15
7. 380
8. 9

NOTES

4. What information is always needed to convert between measurement units?

For each conversion, explain which strategy you prefer to use and then convert the units.

5. 12 gal = _____ L

6. 240 oz = _____ lb

7. 0.380 km = _____ m

8. 324 in. = _____ yd

Assignment

LESSON 1: Many Ways to Measure

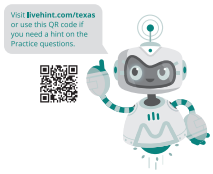
Write

Explain how to convert from one unit to another using ratio reasoning.

Remember

More than one unit can be used to describe the same length, weight, or capacity. To convert units means to change a measurement to an equivalent measurement in different units. You can use models, ratio reasoning, and unit analysis to convert units using conversion rates.

Practice



Use any strategy to convert between the specified units.

1. Janine will be traveling to Botswana, where the unit of currency is called the *pula*, which means "rain" in the local language. Suppose that $1 is equivalent to 7 pula.
   a. If Janine has $500 to spend in Botswana, how many pula will she have to spend?
   b. The safari lodge where she is staying in Chobe National Park costs 434 pula each night. What is the cost per night in dollars?
   c. When she goes to dinner at the safari lodge, the bill comes to 91 pula. How many dollars did Janine spend on dinner?
2. Jonah is going to the hardware store for his Uncle Frederick. He needs to buy 4 yards of electrical wire and 14 quarts of liquid nails.
   a. The store only sells wire by the foot. How many feet does Jonah need?
   b. The store only sells liquid nails by the gallon. How many gallons does Jonah need?
3. Jin Lee is volunteering at a zoo and is helping weigh a penguin's egg. The egg weighs 0.15 kilogram.
   a. Is this more or less than the average weight of 145 grams? Explain.
   b. If Jin Lee expands the penguin area to be about 500 meters wider than it is now, how many more kilometers wide is the area?
4. Harold is buying a new car. Some of the cars he has researched provide measurements in the U.S. customary system and some provide measurements in the metric system.
   a. One car manufacturer reports the mass of the car to be 3307 lb. How many kilograms is this?
   b. Another manufacturer recommends that the owner change the oil every 12,075 kilometers. After how many miles should the owner change the oil?
   c. Harold is a tall man and prefers cars with high ceilings. One car lists 43.3 inches of headroom and another car lists 99.3 centimeters of headroom. Which car has more headroom?
   d. He is concerned about the fuel tank capacity of the new car he wants to buy. He commutes a long distance to work every day and does not want to constantly be filling the tank. He finds 3 cars that he likes online. The Skyte has a fuel capacity of 19 gallons. The Madrid has a fuel capacity of 64.4 liters, and the Cougar has a fuel capacity of 63.6 quarts. Compare the fuel tank capacities of the cars using both gallons and liters. Order the cars from least to greatest fuel tank capacity.

Assignment Answers

Note: The answers provided are approximations. Check how students completed the conversion for slight variations in answers.

Write

Answers will vary.

Practice

1a. 3500 pula
1b. $62 per night
1c. $13 on dinner

2a. 12 feet
2b. 3.5 gallons

3a. More. It is 150 grams.
3b. 0.5 kilometer

4a. 1488.15 kilograms
4b. 7486.5 miles
4c. Car 1 has about 110 centimeters of headroom, which is greater than 99.3 centimeters.
4d. The fuel tank capacity of the Skyte is 19 gallons, or 72.01 liters. The fuel tank capacity of the Madrid is 16.74 gallons, or 64.4 liters. The fuel tank capacity of the Cougar is 15.9 gallons, or 60.26 liters. The cars, in order from least to greatest fuel tank capacity, are the Cougar, the Madrid, and the Skyte.

Assignment Answers

5a. Point A: 1.5 km; Point B: 2.31 km; Point C: 4.61 km; Point D: 3.5 km

5b. Point A: 0.93 mi; Point B: 1.43 mi; Point C: 2.86 mi; Point D: 2.17 mi

6. The zip line is about 78.5 yards long or 235.5 feet.

Stretch

1. 0.315 cubic meter
2. 315,000 cubic centimeters

Review

1. 300 girls
2. The flip-flops originally cost $15.

3a. 48 customers will need baseball equipment.

3b. 36 customers will need wrestling equipment.

4a. 0.08

4b. 3.725

5. A group of 4 campers must navigate through the forest using compasses, topographic maps, and other devices. They scatter and each of them travels to a different location. Using the clues below, determine how far it is from the start to each point on the map.
   - The distance to point A is 1.5 kilometers.
   - It is 0.5 more miles to get to point *B* from the start than to point *A*.
   - The total distance to points *A* and *D* from the start is 3.1 miles.
   - The distance from the start to point *C* is twice the distance from the start to point *B*.

   a. How many kilometers is it from the start to each location?

   b. How many miles is it from the start to each location?
6. A zip line activity is part of an obstacle course that a group of students must get through together. There are several zip lines on the course, the longest of which is about 72 meters long. How can this be stated using the most appropriate unit in the customary system? Show your work.

Stretch

Anthony measured the dimensions of a rectangular box to be 45 cm by 35 cm by 2 m.

1. Determine the volume of the box in cubic meters.
2. Convert the volume of the box to cubic centimeters.

Review

1. At Union Middle School, 99 girls, or 33% of the girls, play basketball. How many girls attend Union Middle School?
2. Kasey gets a 35% employee discount on anything she buys at The Foot Parade. If Kasey got a $5.25 discount on her new flip-flops, how much did they cost originally?
3. Mr. Hawkins manages a small store called Action Sporting Goods. He wants to make sure that his store is stocked with enough equipment for all of the community sports. He surveys 240 of his customers and asks them to choose the one sport that they're most likely to buy sports equipment for this season.

Sport	Percent of Responses
Basketball	30%
Baseball	20%
Football	35%
Wrestling	15%

   a. How many of the surveyed customers will need baseball equipment?

   b. How many of the surveyed customers will need wrestling equipment?

4. Estimate each quotient to the nearest whole number. Then calculate the quotient.

   a. 0.796 ÷ 9.95

   b. 23.84 ÷ 6.4

2 • TOPIC 3: Unit Rates and Conversions