ACT MATH SECTION CHEAT SHEET

ALGEBRA:

ZERO PRODUCT PROPERTY: If ab=0, then a=0, b=0, or a=b=0.

SYSTEMS OF EQUATIONS

How to evaluate the NUMBER of possible solutions:

When solving down a system in the form ax+by=c and dx+fy=g using elimination or substitution:

One Solution	Zero Solutions	Infinite Solutions	
	If you get a statement that is nev- er true (such as 5=6), you have no solutions.	If you get two values that always equal each other, (i.e. x=x or 0=0) you have infinite solutions.	
Compare slopes! Put both equation	ns into v=mx+b form, and compar	e m's (slopes)	

Compare slopes: 1 at both equations into y=mx+b form, and compare in s (slopes)				
One Solution	Zero Solutions	Infinite Solutions		
Different slopes (intercepts don't	Same slopes (parallel lines), different	Same slope, same intercept.		
matter).	y-intercepts.			

FOIL AND FACTORING:

- A monomial is a single product such as $4x, 7x^3$, or $8n^2$.
- A binomial has two elements added together such as 4x + 3 or $5n^3 + 3n$.
- A polynomial has multiple elements added together such as $5n^3 + 3n^2 + 7n + 2$ or $5x^2 + 2x + 4$.
- Difference of Squares: The product of the difference (a-b) and the sum a+b is equal to a squared minus b squared, $(a-b)(a+b) = a^2 - b^2$.
- Square of a Sum: $(a+b)^2 = a^2 + 2ab + b^2$.
- Square of a Difference: $(a-b)^2 = a^2 2ab + b^2$.

SLOPE FORMULA: For points (x_1, y_1) and (x_2, y_2) , $m = \frac{y_2 - y_1}{x_2 - x_1}$. Slope is RISE OVER RUN.

SLOPE-INTERCEPT FORM: *y*=*mx*+*b*

Where *m* is the **slope** of the line, and *b* is the **y-intercept** of the line at point (0,b).

MIDPOINT FORMULA: The midpoint of two coordinate points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{(x_1+x_2)}{2},\frac{(y_1+y_2)}{2}\right)$$

DISTANCE FORMULA: Given two points, (x_1, y_1) and (x_2, y_2) , the distance between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

SPEED AND RATES DISTANCE FORMULA: d=rt**WORK FORMULA:** w=rt**COMBINED WORK FORMULA:** $w = (r_1 + r_2)t$ CROSS MULTIPLICATION: $\frac{a}{b} \times \frac{c}{d}$ ad = bc

QUADRATICS AND POLYNOMIALS:

- VERTEX FORM: the vertex form of a parabola is: $f(x) = a(x-b)^2 + k$
 - The vertex of the parabola in this form is (h,k)
 - When a is positive, the parabola opens upwards, and the minimum is (h,k).
 - When a is negative, the parabola opens downwards, and the minimum is (h,k).
- FACTORED FORM: The factored form of a polynomial usually takes the form:

$$f(x) = a(x-n)(x-m)$$

- When *a* is positive, the parabola opens upwards.
- When *a* is negative, the parabola opens downwards.
- *n* and *m* are zeros/x-intercepts of the equation and pass through the x-axis (horizontal axis)

The midpoint of 2 zeros (n and m respectively) is the x-value of the vertex.

- STANDARD FORM: the standard form of the parabola has the general form: $f(x) = ax^2 + bx + c$ b
 - $-\frac{b}{2a}$ is the x-value of the vertex. The y-value of the vertex can be found by plugging in this value for x and solving for y (or f(x)).
 - The vertex is always either the maximum or the minimum of the graph.
 - When *a* is positive, the parabola opens upwards.
 - When *a* is negative, the parabola opens downwards.
 - The sum of the two roots is $-\frac{b}{a}$ (not necessary to know)
 - The product of the two roots is $\frac{c}{c}$ (not necessary to know)

$$-b\pm\sqrt{b^2-4ac}$$

- QUADRATIC FORMULA: $x = \frac{1}{2a}$
- FINDING SOLUTIONS USING THE DISCRIMINANT: Given that $f(x) = ax^2 + bx + c$, the discrimi-

nant is defined as $b^2 - 4ac$ (the argument under the root in the quadratic equation):

- When this value is positive, there are two real roots.
- When this value is 0, there is one real root.
- When this value is negative, there are no real roots (but there are two imaginary roots).

RADICAL AND EXPONENT RULES:

EXPONENT RULES	RADICAL RULES
Zero Power Rule: $a^0 = 1$	Fractional exponent conversion: $\sqrt[6]{a^b} = a^{\frac{b}{c}}$
Negative exponent rule: $a^{-b} = \frac{1}{a^b}$	Product of radicals: $\sqrt[6]{a}\sqrt[6]{b} = \sqrt[6]{ab}$
Power of a power: $(a^b)^c = a^{bc}$	
Product of powers: $(a^b)(a^c) = a^{b+c}$	
Power of a product: $(ab)^c = a^c b^c$	
Power of a quotient: $\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$	
Quotient of powers: $\frac{a^b}{a^c} = a^{b-c}$	

THE DEFINITION OF A LOGARITHM: $\log_c a = b$ means that $c^b = a$

COMMON LOGARITHMS: $\log x = \log_{10} x$

NATURAL LOGARITHMS: $\ln x = \log_e x$

CHANGE OF BASE FORMULA: For all positive numbers *a*, *b*, and *c*, where $b \neq 1$ and $c \neq 1$:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

PROPERTIES OF LOGARITHMS:

- Power Property: $a \log x = \log_b x^a$
- Product Property: $\log_a x + \log_a y = \log_a xy$
 - Quotient Property: $\log_a x \log_a y = \log_a \frac{x}{y}$

DEFINITION OF LOGARITHMS:

- $n^{\log_n} = a$
- $\log_x x^n = n \log$
- Logarithm of the base: $\log_x x = 1$

CONICS:

- Circle Equation: $(x-b)^2 + (y-k)^2 = r^2$ with center point at (b,k) and radius of r
- Ellipse Equation: $\frac{(x-b)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ with center point at (h,k) and the distance from that center to the end points of the major and minor axes denoted by *a* and *b*.

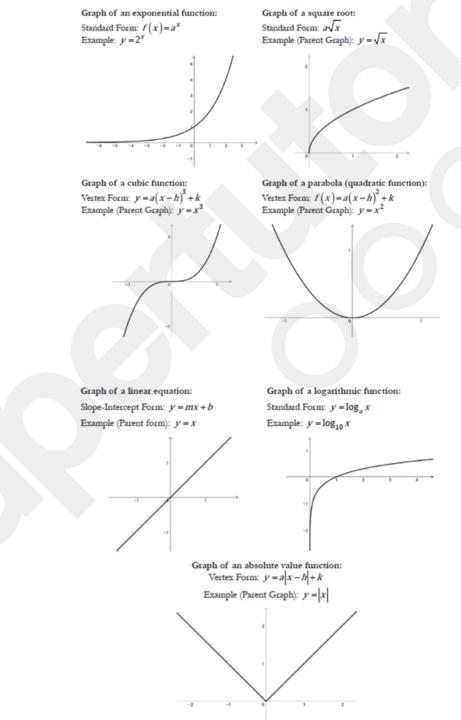
$$\frac{(x-b)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \text{ or } \frac{(y-k)^2}{b^2} - \frac{(x-b)^2}{a^2} = 1$$

• Equation of a Hyperbola:

- Conic Equation Types: A conic section of the form $Ax^2 + By^2 + Cx + Dy + E = 0$, in which A and B are both not zero is:
 - A circle if A=B.
 - A parabola if AB=0.
 - An ellipse if $A \neq B$ and AB > 0.
 - A hyperbola if AB < 0.

GRAPH BEHAVIOR:

Types of graphs:



- Horizontal Shift: For ALL functions, if you replace all instances of x in a function with "(x-b)" you'll find that the graph moves "b" units to the right.
- Vertical Shift: If you replace all instances of y in a function with "(y-k)" you'll find that the graph moves "k" units upward.

END BEHAVIOR:

Degree: Even	Degree: Even		
Leading Coefficient: positive	Leading Coefficient: negative		
End Behavior: $f(x)$ approaches $+\infty$ at both ends of the	End Behavior: $f(x)$ approaches $-\infty$ at both ends of the		
graph (upward facing)	graph (downward facing)		
Domain: all reals	Domain: all reals		
Range: all reals ≥ maximum	Range: all reals ≤ maximum		
Example: $y = x^2$	Example: $y = -2x^6 + 3x^5 + 4x$		

Degree: Odd	Degree: Odd		
Leading Coefficient: positive	Leading Coefficient: negative		
End Behavior: At graph left, $f(x) \rightarrow -\infty$. At graph right,	End Behavior: At graph left, $f(x) \rightarrow +\infty$. At graph right,		
$f(x) \rightarrow +\infty$ (upward sloping)	$f(x) \rightarrow -\infty$ (downward sloping)		
Domain: all reals	Domain: all reals		
Range: all reals	Range: all reals		
Example: $f(x) = x^3$	Example: $f(x) = -3x^5 - 2x$		

TRANSLATIONS AND REFLECTIONS:

- Reflectional symmetry: the property a figure has if half of the figure is congruent to the other half over an axis.
- Rotational symmetry: also known as radial symmetry—the property a figure has if it is congruent to itself after some rotation less than 360°.

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NUMBERS & STATS:

KEY MATH TERMS (Most important only, see chapters for more & for divisibility rules)

INTEGER: A number that is a whole number and includes no fraction or decimal parts. Integers can be negative, zero or positive. (Examples: -400, -7, 0, 1, 3, 56, 230).

ZERO: A number that is **even** and **neither positive nor negative**. Also, an integer. Numbers cannot be divided by zero, and zero cannot be the value of the denominator (bottom) of a fraction. Sometimes, this word is also used to indicate an x-intercept of a polynomial (when y is zero and x is a number).

PRIME NUMBER: A number for which the only factors are itself and 1. IMPORTANT: (one) is **NOT** a prime number!

LEAST COMMON MULTIPLE: The least common multiple of two or more numbers, often abbreviated as LCM, is the smallest whole number that has those two or more numbers among its factors. For example, the LCM of 2 and 3 is 6, since it is the smallest whole number that has both 2 and 3 among its factors.

GREATEST COMMON FACTOR: The greatest common factor of two or more numbers, often abbreviated as GCF, is the largest whole number that is a factor of all the numbers. For example, the GCF of **18** and **24** is **6** because **6** is the largest whole number that is a factor of **18** and **24**.

KNOW THE FOLLOWING RULES OF HOW ODD & EVEN NUMBERS WORK:

An ODD times an ODD is an ODD: (3)(3)=9An EVEN times ANYTHING is EVEN: 2(17)=34; 4(6)=24An ODD plus an ODD is EVEN: 7+5=12An EVEN plus an EVEN is EVEN: 4+8=12An EVEN plus an ODD is ODD: 2+5=7An EVEN exponent creates a positive solution: $(-2)^4 = 16$ An ODD exponent keeps the sign (+ or -) the same: $(-3)^3 = -27$

PERCENTS

TO CALCULATE PERCENT OF INCREASE OR DECREASE

<u>New Number – Original Number</u> Original Number

TO APPLY PERCENT INCREASE OR DECREASE

 $Original Amount + \frac{Percent Increase}{100} (Original Amount) = Total Amount$

SEQUENCES:

FORMULA FOR THE n^{th} TERM OF AN ARITHMETIC SEQUENCE $a_n \!=\! a_1 \!+\! \left(n\!-\!1\right)\! d$

Where a_n is the n^{th} term in the sequence, a_1 is the first term in the sequence, n is the number of the term in the sequence, and d is the common difference.

FORMULA FOR THE n^{th} TERM OF A GEOMETRIC SEQUENCE

To find the n^{th} term in a geometric sequence, where a_n is the n^{th} term in the sequence, a_1 is the first term in the sequence, and r is the common ratio (what we multiply each subsequent term by):

$$a_n = a_1 r^{n}$$

AVERAGES (MEAN, MEDIAN, MODE, RANGE)

Mean (Average)

MEAN and AVERAGE are the same thing: the average value of a set of numbers, found by adding all of the numbers in a set and then dividing by the number of items in the set (below, left).

THE AVERAGE FORMULA

$$AVERAGE = \frac{SUM \ OF \ ALL \ ITEMS}{NUMBER \ OF \ ITEMS}$$
 or $SUM = (average)(\# \ of \ items)$

Median

If you line up a set of numbers in **numerical (chronological) order,** and you find the number physically in the middle of this list, that number is called the MEDIAN. If there are an even number of items in a list, then average the two middle values.

Mode

The mode occurs most often in a set. I like to think that **MO**de and **MO**st both start with "**MO**" to remember this one. You can have multiple modes.

Range

The range of a set of values is the difference between the greatest value and the least value.

Standard Deviation

Essentially standard deviation measures how tightly packed a set of data is about the mean.

AVERAGE RATE FORMULA

AVERAGE (ELEMENT A) per (ELEMENT B) = $\frac{TOTAL \ OF \ ALL \ ELEMENTS \ A}{TOTAL \ OF \ ALL \ ELEMENTS \ B}$

AVERAGE SPEED FORMULA

Average Speed = $\frac{TOTAL \ DISTANCE}{TOTAL \ TIME}$

ARRANGEMENTS & PROBABILITY

THE FUNDAMENTAL COUNTING PRINCIPLE

If there are m ways to do one thing, and n ways to do another thing, and assuming that each "thing" is unique (or that the order of these things matters), there are m times n ways to do both.

FACTORIAL (DEFINITION)

If *n* is a positive integer, then n! = n(n-1)(n-2)(n-3)...

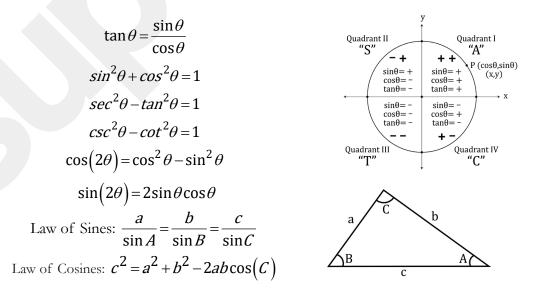
Example: $5 \times 4 \times 3 \times 2 \times 1$ is written 5! and is read 5 factorial.

By definition, 0!=1.GENERAL PERMUTATIONS
(distinct items that are dependent events, order matters)COMBINATIONS
(order doesn't matter, unique elements) $n P_r = \frac{n!}{(n-r)!}$ $n C_r = \frac{n!}{r!(n-r)!}$ where n is the # of items taken r at a time.Where n is the # of choices taken r at a time.

BASIC PROBABILITY FORMULA

Number of possible desired outcomes Number of total possible outcomes

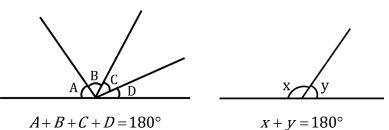
TRIG IDENTITIES & FORMULAS



There are π radians in 180 degrees, or 2π radians in 360 degrees: $2\pi = 360^{\circ}$

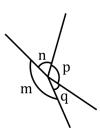
GEOMETRY

STRAIGHT LINES AND CIRCLES

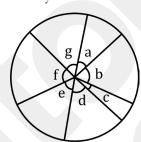


Straight lines represent 180°. Anytime you see two or more angles popping out to one side of a straight line, these angles must sum to 180°.

The sum of all the angles formed by lines that all converge at a single point (like spokes of a bicycle, or a "circle" of angles) is 360°. Similarly, the interior angles of a circle always sum to 360°.



 $m+n+p+q=360^\circ$



 $a + b + c + d + e + f + g = 360^{\circ}$

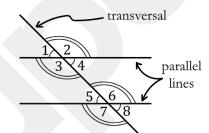
R°

VERTICAL ANGLES

Q and R are vertical angles, as are S and T.

The vertical angles theorem states that vertical angles are congruent; above, angle Q and R equal each other, and angle S and T equal each other.

PARALLEL LINES THEOREM



 $\angle 1 = \angle 4 = \angle 5 = \angle 8$ (small angles)

 $\angle 2 = \angle 3 = \angle 6 = \angle 7$ (big angles)

Any small angle plus any big angle = 180°

SOHCAHTOA

Trigonometric function	Abbreviation	Acron	iym	Is equa	ıl to	
Sine	sin	SOI	Η	O pposite leg divided	d by H ypotenuse	
Cosine	cos	CAI	Η	Adjacent leg divided	l by H ypotenuse]
Tangent	tan	TO	А	O pposite leg divided	d by A djacent leg	
	$\sin(\theta) = \frac{opp}{hypo}$	oosite tenuse	tan(e	9)= <u>opposite</u> adjacent	$sec\theta = \frac{1}{\cos\theta} = \frac{1}{\theta}$	hypotenuse adjacent
adjacent	$\cos(\theta) = \frac{adj}{hypo}$	acent otenuse	cotθ	$=\frac{1}{\tan\theta}=\frac{adjacent}{opposite}$	$csc\theta = \frac{1}{\sin\theta} = \frac{1}{\theta}$	hypotenuse opposite

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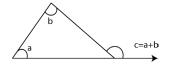
TRIANGLES

TRIANGLE SUM THEOREM

The sum of the three angles in any triangle must equal 180°.

EXTERIOR ANGLE THEOREM

The exterior angle of a triangle is equal to the sum of the remote interior angles. a+b=c, where c is the exterior angle and a and b are remote interior angles.



Right Triangle One angle is **90** degrees. The longest side is a hypotenuse. The shorter two sides are legs.

Obtuse Triangle One angle is greater than 90 degrees.

Acute Triangle All angles are less than **90** degrees.

Scalene Triangle All sides are unequal. Therefore, all angles will be unequal as well.

Isosceles Triangle At least two sides are congruent. The angles opposite those sides are also congruent.

Equilateral Triangle All sides are congruent. All angles of an equilateral triangle will be **60** degrees.



hypotenuse

120°

leg

90°

leg

J30°

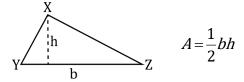
TRIANGLE SIMILARITY:

AA: If two angles are congruent to two angles of another triangle, then the triangles must be similar. **SSS**: If all the corresponding sides of two triangles are in the same proportion, then the two triangles must be similar.

SAS: If one angle of a triangle is congruent to the corresponding angle of another triangle and the lengths of the sides adjacent to the angle are proportional, then the triangles must be similar.



TRIANGLE AREA

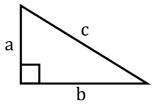


Where b is the measurement of the base of the triangle and

h is the measurement of the height of the triangle.

The **perimeter** of a triangle is the sum of its sides.

Perimeter $= s_1 + s_2 + s_3$



PYTHAGOREAN THEOREM

For any right triangle with side lengths a, b and c, where c is the longest side (opposite 90 degrees), or hypotenuse:

 $a^2 + b^2 = c^2$

PYTHAGOREAN TRIPLES

A Pythagorean triple consists of three positive integers a, b, and c such that $a^2 + b^2 = c^2$. Such a triple is commonly written (a,b,c). Some well-known examples are:

$$(3,4,5),(5,12,13),(8,15,17), \& (7,24,25).$$

Special Triangle: 45-45-90

Х

 $x\sqrt{2}$

45

Х

Special Triangle: 30-60-90

 $x\sqrt{3}$

TRIANGLE INEQUALITY THEOREM,

TRIANGLE WITH SIDES A, B, C



TRIANGLE CONGRUENCY:

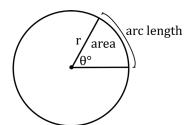
2x

ASA: If two angles are congruent to two angles of another triangle, and the side between them is congruent, then the triangles must be congruent.

SSS: If all the corresponding sides of two triangles are congruent, then the two triangles must be congruent.

SAS: If one angle of a triangle is congruent to the corresponding angle of another triangle and the lengths of the sides adjacent to the angle are congruent, then the triangles must be congruent.

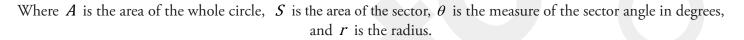
CIRCLES



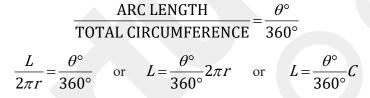
AREA OF A SECTOR FORMULA

SECTOR AREA θ°

$$\overline{\text{TOTAL AREA}} = \overline{360^{\circ}}$$
$$\frac{S}{\pi r^2} = \frac{\theta^{\circ}}{360^{\circ}} \quad \text{or} \quad S = \frac{\theta^{\circ}}{360^{\circ}} \pi r^2 \quad \text{or} \quad S = \frac{\theta^{\circ}}{360^{\circ}} A$$

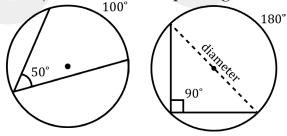


ARC LENGTH FORMULA

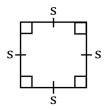


Where L is the arc length, C is the circumference of the circle, θ is the measure of the sector angle in degrees, and r is the radius.

Inscribed angles (formed by two chords) are half the corresponding arc measure.

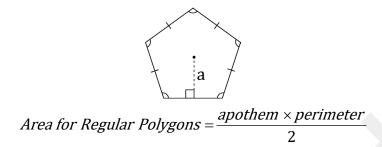


POLYGONS

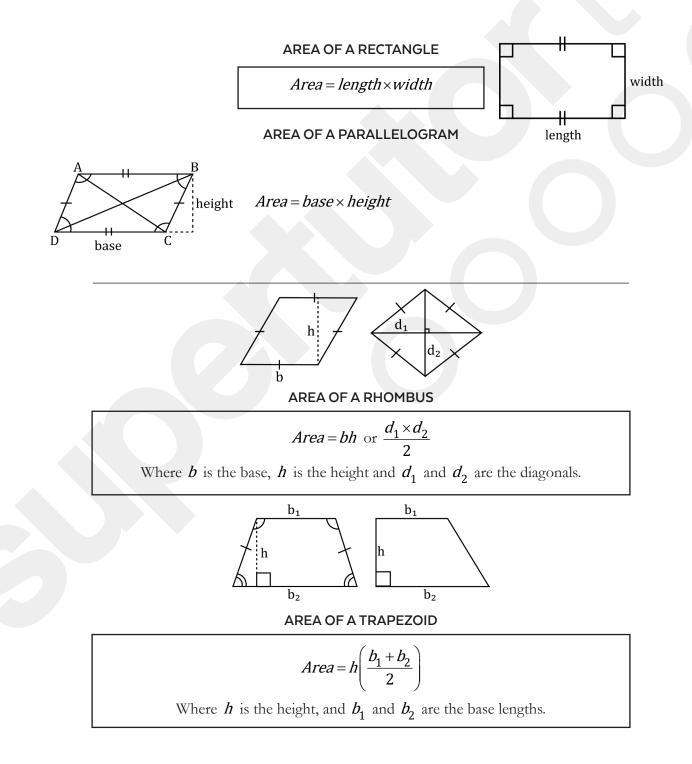


AREA AND PERIMETER OF A SQUARE

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Where the apothem (a) is the length of the distance from the center of the shape to a side at a right angle.



SUM OF INTERIOR ANGLES

