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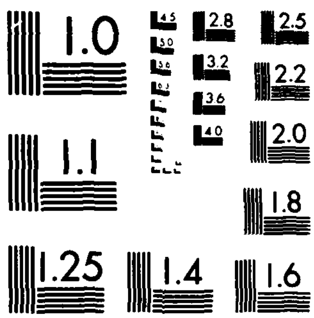

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ELECTRON BEAM-INDUCED ELECTROMAGNETIC WAVES IN A MAGNETOSPHERIC PLASMA

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Scientific Report No. 1

15 May 1983

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-83-0085	2. GOVT ACCESSION NO. AD-A130116	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ELECTRON BEAM-INDUCED ELECTRO-MAGNETIC WAVES IN A MAGNETOSPHERIC PLASMA		5. TYPE OF REPORT & PERIOD COVERED Scientific Report No. 1
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) D. E. Donatelli T. S. Chang*		8. CONTRACT OR GRANT NUMBER(s) F19628-81-K-0032
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Physics Boston College Chestnut Hill, MA 02167		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2311G1BA
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory Hanscom AFB, Massachusetts 01731 Monitor/David A. Hardy/PHG		12. REPORT DATE 15 May 1983
		13. NUMBER OF PAGES 20
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

*Space Physics Center, MIT, Cambridge, MA 02135

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Electromagnetic Waves; Geostationary altitude; electron gyrofrequency; magnetized plasma;

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A theoretical interpretation of waves detected by the SCATHA satellite near geostationary altitude during electron beam operations is presented. The observations consist of emission bands at or between harmonics of the electron gyrofrequency, combining "background" and beam-induced emissions. Beam-induced emissions often broaden and/or intensify the emission band. A magnetic field component is detected along with the electric field component for most of the emissions. Therefore, these emissions are assumed to be electromagnetic waves.

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The linearized Vlasov equation is used to derive the dispersion relation for electromagnetic waves in a warm, homogeneous, uniformly magnetized plasma. The ambient plasma is represented as a Maxwellian distribution and the electron beam as a delta-function in velocity space. The dispersion relation is solved analytically for non-resonant, perpendicularly propagating, electromagnetic waves. Using observational data to quantitatively define the plasma parameters, unstable solutions are found at frequencies consistent with those of the observed emissions.

INTRODUCTION

The question of whether or not electromagnetic cyclotron harmonic waves can be excited in the magnetosphere near geostationary altitude, was raised upon examination of the data from the VLF broadband measurements on the SCATHA satellite during a severe spacecraft charging event (Donatelli et al., 1983). During this event strong emission bands were detected at or near the first two harmonics of the 2 kHz electron gyrofrequency. These emission bands appeared on both the electric and magnetic wave field detectors. While the SCATHA electron gun was in operation at varying current/energy levels attempting to discharge the satellite, there were frequency shifts in the emission band and consistent magnetic field components. This observation prompted the present theoretical study to determine whether electromagnetic electron cyclotron harmonic waves could be generated in a plasma representative of local conditions.

Theoretical and observational studies to date have explained observations analogous to those of the SCATHA data in terms of electrostatic emissions. Using the Harris dispersion relation, Fredricks (1971) showed that non-resonant electrostatic waves could be excited in a plasma in which the electron perpendicular velocity distribution has a narrow region of positive slope, i.e. $\partial f_0/\partial v_\perp > 0$. The restriction on the velocity distribution was relaxed by Young et al. (1973) to the requirement that a distribution of cold and warm electrons have a velocity distribution that is non-monotonic in $V_\perp$. They showed that this provides sufficient free energy for driving instabilities. The ratio of cold to hot plasma density, above which instability will not occur, was established by Ashour-Abdalla et al. (1975). The non-convective and convective nature of the instability is shown to be controlled by the temperature ratio of the cold to hot electron populations (Ashour-Abdalla and Kennel, 1978). Electrostatic upper hybrid waves observed on ISEE-1 have also been explained by this theory (Kurth, et al. 1979). Sentman et al. (1979) show examples of two-component electron distribution functions constructed from ISEE-1 observations of low energy magnetospheric electrons that occur *simultaneous with* the detection of electrostatic emissions between the electron cyclotron frequency and the upper hybrid resonance.

Electromagnetic emissions within this frequency range, with the exception of the whistler mode, have not been reported prior to the SCATHA observations. Ohnuma et al. (1981) have shown that electromagnetic cyclotron harmonic waves may be generated in a dense plasma; i.e. one in which $\omega_{pe} >> \Omega_{ce}$, where ω_{pe} and Ω_{ce} are the electron plasma and gyro- frequencies, respectively. Although their results were applied to laboratory plasmas, this condition may prevail in the vicinity of SCATHA during beam operations.

Here the problem will be approached in a manner suggested by the work of Tataronis and Crawford (1970). They used the quasi-static approximation in deriving a dispersion relation for a warm magnetoplasma and proceeded to examine under what conditions unstable electrostatic cyclotron harmonic waves (Bernstein modes) exist. They looked at several types of distribution functions and examined ranges of plasma parameters for finding unstable modes. They show that if the analytic solution *to the dispersion relation undulates about zero for a given distribution* function, non-convective mode coupling instabilities occur. These occur between specific ranges of the two ratios, the plasma frequency/electron gyrofrequency and the gyroradius/wavelength, for each harmonic pair. For two component distributions, the density and velocity ratios of the two components must also be considered. Similar solutions are sought here for electromagnetic, cyclotron harmonic waves (extraordinary and ordinary modes).

In the next section the electromagnetic dispersion relation for a warm, uniformly magnetized, homogeneous plasma is examined. Analytical solutions will be presented for the three perpendicularly propagating modes, the Bernstein, extraordinary, and ordinary modes, using a two component distribution function. The ambient plasma is represented as a Maxwellian; the electron beam is represented as a delta-function in velocity space. Sample calculations show that instabilities may exist for both the Bernstein and the extraordinary modes. The frequencies and wave numbers of the excited modes vary with the beam-to-ambient-plasma density and velocity ratios.

THEORY

The dispersion relation is derived using the linearized Vlasov equation and Maxwell's equations (in cgs units):

$$[\partial/\partial t + \underline{v} \cdot \underline{\nabla} + ((\underline{v} \times \underline{B}_0)/c) \cdot \underline{\nabla}_{\underline{v}}]f_{j1} = - (q_j/m_j)[\underline{E}_1 + (\underline{v} \times \underline{B}_1)/c] \cdot \underline{\nabla}_{\underline{v}} f_{jo} \quad (1)$$

$$\left.\begin{aligned} \underline{\nabla} \times \underline{E}_1 &= -c^{-1}\, \partial\underline{B}_1/\partial t \\ \underline{\nabla} \times \underline{B}_1 &= c^{-1}\, \partial\underline{E}_1/\partial t + (4\pi q_j n_{j1}/c) \int \underline{v} d\underline{v} f_{j1} \\ \underline{\nabla} \cdot \underline{E}_1 &= 4\pi q_j n_{j1} \int d\underline{v} f_{j1} \\ \underline{\nabla} \cdot \underline{B}_1 &= 0 \end{aligned}\right| \quad (2)$$

The equilibrium electron velocity distribution is $f_{j0}(v_\perp, v_\parallel)$ where $\perp$ and $\parallel$ refer to components perpendicular and parallel to the external magnetic field, $B_0 = B_{0z}$. There is assumed to be no external electric field, $E_0 = 0$. The perturbation electric and magnetic fields are $\underline{E_1}$ and $\underline{B_1}$, respectively. The perturbed particle distribution and density are f_{j1} and n_{j1}, respectively, where particle species is indicated by the subscript j. The charge and mass for each particle species are q_j and m_j, respectively; c is the velocity of light in a vacuum.

The plasma is assumed to be infinite, spatially homogeneous and uniformly magnetized, since variations in time and space may be neglected. Equations (1) and (2) are solved by introducing a Fourier transform in space, a Laplace transform in time, and integrating along unperturbed particle orbits. This leads to the nine element dielectric tensor, from Krall and Trivelpiece (1973):

$$\begin{vmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{xy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{vmatrix} \begin{vmatrix} E_x \\ E_y \\ E_z \end{vmatrix} = 0 \qquad (3)$$

where the elements of the dielectric coefficient are ($Im(\omega)>0$):

$$D_{xx} = 1 - (c^2 k_\parallel^2/\omega^2) - (2\pi/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \chi_j J_N^2 N^2 \Omega_j^2/k_\perp^2$$

$$D_{xy} = - (2\pi i/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \chi_j J_N J_N' v_\perp N\Omega_j/k_\perp$$

$$D_{xz} = (c^2 k_\parallel k_\perp/\omega^2) - (2\pi/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \Lambda_j J_N^2 v_\parallel N\Omega_j/k_\perp$$

$$D_{yx} = - D_{xy}$$

$$D_{yy} = 1 - [c^2(k_\parallel^2 - k_\perp^2)/\omega^2] - (2\pi/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \chi_j (J_N')^2 v_\perp^2$$

$$D_{yz} = (2\pi i/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \Lambda_j J_N J_N' v_\perp v_\parallel$$

$$D_{zx} = (c^2 k_\parallel k_\perp/\omega^2) - (2\pi/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \chi_j J_N^2 v_\parallel N\Omega_j/k_\perp$$

$$D_{zy} = -(2\pi i/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \chi_j J_N J_N' v_\parallel v_\perp$$

$$D_{zz} = 1 - (c^2 k_\perp^2/\omega^2) - (2\pi/\omega) \sum_j \sum_N \omega_{pj}^2 \; II \; \Lambda_j J_N^2 v_\parallel^2$$

where:

$$II \equiv \int_{-\infty}^{\infty} dv_\parallel \int_0^\infty dv_\perp 2v_\perp/(N\Omega_j + k_\parallel v_\parallel - \omega$$

$$\chi_j \equiv [1-(k_\parallel v_\parallel/\omega)](\partial f_{jo}/\partial v_\perp^2) + (k_\parallel v_\parallel/\omega)(\partial f_{jo}/\partial v_\parallel^2)$$

$$\Lambda_j \equiv (N\Omega_j/\omega)[(\partial f_{jo}/\partial v_\perp^2) - (\partial f_{jo}/\partial v_\parallel^2)] + (\partial f_{jo}/\partial v_\parallel^2)$$

For simplicity in the above derivation, the wave vector $\underline{k}$ is defined as follows:

$$\underline{k} = \underline{k}_\perp + \underline{k}_\parallel = k_\perp \underline{x} + k_\parallel \underline{z}.$$

The definition of other terms are:

ω = wave frequency

ω_{pj} = plasma frequency = $(4\pi n_j q_j^2/m_j)^{1/2}$

Ω_j = particle gyrofrequency = $q_j B_o/m_j c$

J_N = Nth order Bessel function of the first kind

$k_\perp v_\perp/\Omega_j$= argument of the Bessel function

J_N' = derivative of J_N with respect to the argument.

There is no separation of electrostatic and electromagnetic modes in the dispersion tensor (3). However, by considering parallel propagation ($k_\perp = 0$) and perpendicular propagation ($k_\parallel = 0$) separately, it is possible to simplify the dispersion tensor and examine some characteristic wave modes. Setting $k_\perp = 0$ in (3) singles out the modes that propagate parallel to the equilibrium magnetic field. These modes are:

(1) The longitudinal, electrostatic, Landau damped, Langmuir waves and ion acoustic waves.

(2) The weakly damped transverse electromagnetic Alfven waves at frequencies below the ion cyclotron frequency; the Whistler waves at $\omega \approx (1/2)\Omega$ and the cyclotron waves which occur at $\omega = \Omega$. The cyclotron waves are strongly damped for $\omega = k_\parallel c$, which is also the band in which spontaneous emission of radiation occurs.

By taking $k_\parallel = 0$, the characteristic waves that propagate perpendicular to the equilibrium magnetic field are singled out. These modes are:

(1) the almost purely longitudinal, nearly electrostatic Bernstein modes that correspond to the electrostatic waves discussed at the beginning of this section.

(2) The transverse, electromagnetic, cyclotron-harmonic waves which include the extraordinary mode with $\underline{E} \perp \underline{B}_0$ and the ordinary mode with $\underline{E} \parallel \underline{B}_0$. These modes are normal modes of a high density, magnetized plasma ($\omega_p >> \Omega_e^2$), (Ohnuma et al., 1981) and are excited near ion and electron cyclotron harmonics. The SCATHA observations are within the range of these emissions. It will be determined if these modes are unstable within the local beam-background plasma.

For $k_\parallel = 0$, the dispersion tensor (3) reduces to:

$$\begin{vmatrix} D_{xx} & D_{xy} & 0 \\ -D_{xy} & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{vmatrix} = 0 \qquad (4)$$

At the frequencies of oscillation considered here, ions may be regarded as providing a charge neutral background. The approximation $\omega_p^2 << k^2c^2$ is also valid since:

$$\omega_p^2/k^2c^2 < \omega_p^2 v_b^2/\omega^2 c^2 < \omega_p^2 v_b^2/\Omega^2 c^2 < 0.1$$

where: v_b = beam velocity.

Since the D_{xy} terms are second order in ω_p^2/k^2c^2, they may be neglected. Then (4) may be approximately solved for the three eigenmodes:

(1) $D_{xx} = 0$: Bernstein mode

(2) $D_{yy} = 0$: extraordinary mode

(3) $D_{zz} = 0$: ordinary mode

where the elements of the dispersion matrix reduce to:

$$D_{xx} = 1 - (4\pi\omega_p^2/k^2) \sum N^2\Omega^2/\omega(N\Omega - \omega) \int dv_\parallel \int dv_\perp \, v_\perp J_N^2 \, \partial f_o/\partial v_\perp^2 \tag{5}$$

$$D_{yy} = 1 - (k^2c^2/\omega^2) - 4\pi\omega_p^2 \sum (\omega(N\Omega - \omega))^{-1} \int dv_\parallel \int dv_\perp \, v_\perp^3 (J_N')^2 \, \partial f_o/\partial v_\perp^2 \tag{6}$$

$$D_{zz} = 1 - (k^2c^2/\omega^2) - 4\pi\omega_p^2 \sum (\omega(N\Omega - \omega))^{-1} \int dv_\parallel \int dv_\perp \, v_\perp \, v_\parallel^2 \, J_N^2 [N\Omega/\omega \, (\partial f_o/\partial v_\perp^2 - \partial f_o/\partial v_\parallel^2) - \partial f_o/\partial v_\parallel^2)] \tag{7}$$

Unstable solutions are now sought for each of the eigenmodes using the SCATHA data to evaluate the necessary parameters.

ANALYTICAL SOLUTIONS

The SCATHA observations presented in Donatelli et al. (1983) for 24 April 1979, indicate that beam-injected electrons may create a dense plasma where electromagnetic cyclotron harmonic instabilities are generated. At the time of interest the magnetospheric plasma was made up of two populations: a low-energy component with a temperature about 300 eV, and a high-energy component about 25 keV (Mullen et al., 1981). The ejection energy of the beam from the electron gun was either 50 eV or 150 eV. After passing through the satellite sheath the beam electrons had energies of 1-3 keV due to acceleration through the vehicle potential. Emission bands were detected consistently at or near the first and second harmonic of the electron gyrofrequency, Ω, which was about 2 kHz. From the data, beam-to-background velocity ratios are estimated. A ratio of 2-3 is reasonable for the "artificial" beam (electrons from the beam systems on SCATHA), and a ratio of 9-10 for the natural beam (injections of high energy magnetosperic electrons). The density ratios are assumed to be greater than one, both for the natural beam (Mullen et al., 1981) and the "artifical" beam. Although the current and radius of the artificial beam are known, the effective beam density is not, since the electrostatic forces between beam electrons contribute to rapid spreading as does the external magnetic field (Gendrin, 1973). Furthermore, at these low emission energies the electron beam cannot be highly focussed.

The dimensionless variables to be used in these solutions are defined as follows:

$$s^2 \equiv 2KTk^2/m\Omega^2 = v_T k^2/\Omega^2$$

$$x_b \equiv v_{\perp b}/v_T$$

where:

$$v_{\perp b} = v_b \sin \phi_b;$$

ϕ_b = pitch angle of the electron beam, $15^0 < \phi < 165^0$.

Then:

$$sx_b = kv_{\perp b}/\Omega = (kv_b/\Omega)\sin\phi_b$$

and:

$$0.26\,(kv_b/\Omega) < sx_b < kv_b/\Omega$$

where sx_b is the argument of the Bessel function in equations (5), (6), and (7).

The ambient density is between 0.5 and 1.0 cm^{-3}. If the density is set at 1.0 cm^{-3}, the value of the following non-dimensionalized parameters are:

$$(\omega_b/\Omega)^2 \approx 20$$

$$(\omega_b/kc)^2 \approx 0.1$$

The distribution function is approximated as follows:

$$f_o = n_p(m/2\pi KT)^{3/2}\exp[-(m/2KT)(v_\parallel^2 + v_\perp^2)] + (n_b/2\pi v_{\perp b})\,\delta(v_\parallel - v_{\parallel b})\,\delta(v_\perp - v_{\perp b}) \tag{8}$$

where:

T = temperature (°K)

K = Boltzmann constant

n_p = low energy ambient electron density

n_b = density of electron beam

This distribution function is a Maxwellian combined with a ring distribution in velocity space and including motion parallel to the magnetic field. The Maxwellian represents the ambient plasma, and the delta-function represents the beam, with the ring distribution describing the portion of the monoenergetic beam electrons moving in the plane perpendicular to the magnetic field, uniformly distributed in gyrophase angle. Tataronis and Crawford (1970) conducted a numerical study of the propagation characteristics of perpendicularly propagating electrostatic waves in a combined ring-Maxwellian distribution. They state that instability occurs if the analytic function for the dispersion relation is undulatory about zero. Their results show that mode coupling is a feature of the ring distribution leading to strong non-convective instability. Combining the ring and Maxwellian distributions leads to non-convective instability with higher growth rate and lower instability threshhold than for the ring distribution alone. The electromagnetic dispersion relation may be solved for each of the three eigenmodes of perpendicularly propagating waves. Solutions to the dispersion relation may be found as functions of x_b and n_b/n_p, using preceding definitions.

A. The Bernstein Mode

The dispersion relation for the Bernstein mode, obtained by substituting equation (8) into equation (5) and integrating, is the following:

$$D_{xx} = 1 - (4\omega_p^2/\Omega^2) \sum N^2\Omega^2 A_{Nx}/(\omega^2 - N^2\Omega^2) = 0 \qquad (9)$$

where:

$$A_{Nx} = s^{-2}\exp(-s^2/2) I_N(s^2/2) + (n_b/n_p s x_b) J_N J_N'$$

I_N is the modified Bessel function of the first kind with argument $s^2/2$. The prime denotes the derivative of the Bessel function with respect to the argument. $\sum$ is now the summation from N=1 to ∞. Since A_{Nx} is undulatory about zero, unstable solutions are anticipated (Tataronis and Crawford, 1970). In Appendix A it is shown that in using a three-term approximation to equation (9), unstable solutions are found for:

$n_b/n_p = 2$ and $x_b = 2$

such that:

$$\omega/\Omega = 1.7 + 0.9i \tag{10}$$

and:

$$n_b/n_p = 4.5 \text{ and } x_b = 10$$

such that:

$$\omega/\Omega = 1.8 + 1.1i \tag{11}$$

The Bernstein Mode is nearly a pure electrostatic mode. The solutions are equivalent to those obtained by Tataronis and Crawford (1970) using the electrostatic approximation to the dispersion relation. They found unstable solutions in this frequency range with the growth rate, γ, a finite fraction of the real part of the frequency, ω_r. In equation (10), with $\omega_r = 1.7\ \Omega$, $\gamma = 0.5\omega_r$; for equation (11), $\gamma = 0.6\omega_r$. In both equations (9) and (10) ω_r is in the range of emissions detected by the SCATHA broadband receiver (Donatelli et al., 1983). However, this mode is not expected to have the observed magnetic field component. the presence of a magnetic component requires the existence of electromagnetic extraordinary and/or ordinary modes.

B. The Extraordinary Mode

The dispersion relation for the extraordinary mode is obtained by substituting equation (8) into equation (6) and integrating to obtain:

$$D_{yy} = 1 - (k^2c^2/\omega^2) - (2\omega_p{}^2A_o/\omega^2) - 4\omega_b{}^2 \sum A_{Ny}/(\omega^2 - N^2\Omega^2) \tag{12}$$

where:

$$A_{Ny} = (N^2/s^2)\exp(-s^2/2)I_N(s^2/2) + (n_b/n_p)[(J_N')^2 + (sx_b/2)J_N'J_N'']$$

The function A_{Ny} is undulatory about zero, therefore, meeting the instability condition of Tataronis and Crawford (1970). Using a four-term approximation to the dispersion relation, instabilities are found for (see Appendix B):

$n_b/n_p = 10$ and $x_b = 9$

such that:

$\omega/\Omega = 1.5 + 0.2i$

and:

$n_b/n_p = 10$ and $x_b = 3$

such that:

$\omega/\Omega = 1.8 + 0.6i$

Here it is shown that unstable solutions exist within the desired frequency range. The larger beam-to-background density ratios required to support the extraordinary mode are consistent with the work of Ohnuma et al. (1981).

C. The Ordinary Mode

The dispersion relation for the ordinary mode is obtained by substituting equation (8) into equation (7) and integrating to obtain:

$$D_{zz} = 1 - (k^2c^2/\omega^2) - (A_{oz}\,\omega_p^2/\omega^2) - 2\omega_p^2 \sum A_{Nz}/(\omega^2 - N^2\Omega^2) \qquad (13)$$

where:

$$A_{Nz} = \exp(-s^2/2)I_N(s^2/2) + (n_b/n_p)J_N^2$$

Since A_{Nz} is always positive, no unstable solutions are anticipated.

CONCLUSIONS

The analytical solutions presented here show that electromagnetic non-resonant instabilities may be excited in a plasma represented as a Maxwellian background with a monoenergetic beam of electrons. The electromagnetic dispersion relation was solved for $k_\parallel = 0$, using the approximation $\omega_p^2 << k^2c^2$. These simplifications permit the elements of the dispersion relation to be separated and solved as three distinct eigenmodes of perpendicularly propagating waves:

1. Bernstein Mode; $\underline{E} \perp \underline{Bo}$, $\underline{k} \parallel \underline{E}$
2. Extraordinary Mode; $\underline{E} \perp \underline{Bo}$, $\underline{k} \perp \underline{E}$
3. Ordinary Mode; $\underline{E} \parallel \underline{Bo}$, $\underline{k} \perp \underline{E}$

In the first two cases it is shown that cyclotron harmonic modes may couple between the first two harmonics of the electron gyrofrequency, exciting non-convective instabilities with growth rates, γ, that are a finite fraction of ω_r. In the third case no unstable solutions exist. Sample solutions presented for the Bernstein and extraordinary modes are shown to depend on the ratios n_b/n_p and $x_b = v_{\perp b}/v_T$. These ratios must be greater than one.

The ratio, n_b/n_p, for the nearly electrostatic Bernstein mode can be compared quantitatively to the α of Tataronis and Crawford (1970) and the ratio N_C/N_H of Ashour-Abdalla et al. (1975) by considering the Maxwellian portion of the electron distribution as the "cold" component and the delta-function as the "hot" or "ring" component. The results for $n_b/n_p = 4.5$ and $x_b = 10$ agrees with the results for $\alpha = 0.2$ and $N_C/N_H = 0.2$. These are values associated with non-convective electrostatic instabilities at frequencies between the first two harmonics of the electron gyrofrequency.

For the extraordinary mode a larger density ratio, $n_b/n_p \approx 10$, is required to excite instabilities, consistent with the results of Ohnuma et al. (1981). These instabilities may be excited for velocity ratios of 3 and 9, representing ratios of the "artificial" and "natural" electron beam densities, respectively, to the ambient density. These electromagnetic instabilities are non-convective, non-resonant, with large growth rates. They may be excited by electron beams, given sufficient beam-to-ambient density and velocity ratios. The full range of parameters over which they may occur will be explored through numerical calculations. For further understanding of the relationship of these waves to effects observed in the SCATHA data and in the magneto-sphere, the full electromagnetic dispersion relation, including the k∥ terms, must be solved for conditions pertaining to space vehicles in space plasmas.

ACKNOWLEDGEMENTS

To William J. Burke for many helpful discussions. This work was supported in part by AFGL Contract F19628-81-K-0032.

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APPENDIX A

Solution for Bernstein Modes:

The dispersion equation for the Bernstein modes is:

$$D_{xx} = 1 - (4\omega_p^2/\Omega^2) \sum N^2\Omega^2 A_{Nx}/(\omega^2 - N^2\Omega^2) \qquad \text{(A-1)}$$

where:

$$A_{Nx} = s^{-2}\exp(-s^2/2) I_N(s^2/2) + (n_b/n_p s x_b) J_N(s x_b) J_N'(s x_b)$$

This may be approximated:

$$D_{xx} = 1 - (4\omega_p^2/\Omega^2)[(A_1\Omega^2/(\omega^2 - \Omega^2)) + (4A_2\Omega^2/(\omega^2 - 4\Omega^2))] \qquad \text{(A-2)}$$

Setting $D_{xx} = 0$ leads to the fourth order equation:

$$\omega^4/\Omega^4 - [5 + (4\omega_p^2/\Omega^2)(A_1+4A_2)]\,\omega^2/\Omega^2 +$$
$$4[1 + (4\omega_p^2/\Omega^2)(A_1 + A_2)] = 0 \qquad \text{(A-3)}$$

This may be solved as a quadratic, then transformed to polar coordinates, to obtain:

$$\omega/\Omega = (Kr)^{1/2}[\cos(\theta/2 + m\pi) \mp i\sin(\theta/2 + m\pi)], \quad m = 0,1 \qquad \text{(A-4)}$$

where:

$$K = [5 + (4\omega_p^2/\Omega^2)(A_1 + 4A_2)]/2$$

$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1}(y/x)$$

$$x = 1$$

$$y = [[16(1 + (4\omega_p^2/\Omega^2)(A_1 + A_2))/(5 + (4\omega_p^2/\Omega^2)(A_1 + 4A_2))^2] - 1]^{1/2}$$

For instability, the following condition must be satisfied:

$$0 < [5 + (4\omega_p^2/\Omega^2)(A_1 + 4A_2)]^2 < 16[1 + (4\omega_p^2/\Omega^2)(A_1 + A_2)] \qquad \text{(A-5)}$$

A_1 and A_2 are evaluated as functions of n_b/n_p using tabulated values of Bessel functions from Abramowitz and Stegun (1970) for the approximate range of the argument, as defined by the constraints on sx_b. Substituting these and the value of ω_p/Ω into the inequality (A-5), determines the constraints on n_b/n_p.

For $sx_b = 3.6$; $x_b = 20$; $n_b/n_p = 2$:

$A_1 = 0.0464$; $A_2 = -0.0132$.

Substituting in equation (A-4) and solving for θ and Kr:

$$\omega/\Omega = 1.95(0.89 + 0.45i) = 1.73 + 0.886i \qquad \text{(A-6)}$$

A second solution is found for $sx_b = 3.2$, $x_b = 10$. Then with $n_b/n_p = 4.5$; $A_1 = 0.078$, $A_2 = -0.0223$; and

$$\omega/\Omega = 1.83 + 1.14i$$

APPENDIX B

Solution for Extraordinary Modes:

The dispersion equation for the extraordinary mode is:

$$D_{yy} = 1 - (k^2c^2/\omega^2) - (2\omega_p{}^2A_o/\omega^2) - 4\omega_p{}^2 \sum A_{Ny}/(\omega^2 - N^2\Omega^2) \qquad \text{(B-1)}$$

where:

$$A_{Ny} = (N^2/s^2)\exp(-s^2/2)I_N(s^2/2) + (2n_b/n_p)[(J_N'(sx_b))^2 + sx_bJ_N'(sx_b)J_N''(sx_b)]$$

This may be approximated:

$$D_{yy} = 1 - (k^2c^2/\omega^2) - (2\omega_p{}^2A_o/\omega^2) - 4\omega_p{}^2[(A_1/(\omega^2 - \Omega^2)) + (A_2/(\omega^2 - 4\Omega^2))] \qquad \text{(B-2)}$$

Setting $D_{yy} = 0$ leads to the sixth order equation:

$$(\omega^4 - 5(\omega\Omega)^2 + 4\Omega^4)[(\omega^2/k^2c^2) - 1 - (2\omega_p{}^2A_o/k^2c^2)] - (4\omega_p{}^2/k^2c^2)[\omega^4(A_1+A_2) - (\omega\Omega)^2(4A_1+A_2)] = 0 \qquad \text{(B-3)}$$

This may be reduced to a fourth order equation by using the approximation $\omega^2/k^2c^2 << 1$:

$$[1 + (2\omega_p{}^2A_o/k^2c^2) + (4\omega_p{}^2/k^2c^2)(A_1+A_2)]\,(\omega/\Omega)^4 - [5(1 + (2\omega_p{}^2A_o/k^2c^2)) + (4\omega_p{}^2/k^2c^2)(4A_1+A_2)]\,(\omega/\Omega)^2 + 4[1 + (2\omega_p{}^2A_o/k^2c^2)] = 0 \qquad \text{(B-4)}$$

This may be solved as a quadratic, then transformed to polar coordinates, analogous to the solution for the Bernstein mode of Appendix A, to obtain:

$$\omega/\Omega = (Kr)^{1/2}[\cos(\theta/2 + m\pi) \pm i\sin(\theta/2 + m\pi)], \quad m = 0,1 \qquad \text{(B-5)}$$

where:

$$E = [5(1 + 2\omega_p^2 A_0/k^2c^2) + (4\omega_p^2/k^2c^2)(4A_1+A_2)]/$$
$$2[1 + (2\omega_p^2 A_0/k^2c^2) + (4\omega_p^2/k^2c^2)(A_1+A_2)]$$

$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1}(y/x)$$

$$x = 1$$

$$y = [[16(1 + 2\omega_p^2 A_0/k^2c^2)[1 + (2\omega_p^2 A_0/k^2c^2) + (4\omega_p^2/k^2c^2)(A_1+A_2)]/$$
$$[5(1 + 2\omega_p^2 A_0/k^2c^2) + (4\omega_p^2/k^2c^2)(4A_1+A_2)]^2] - 1]^{1/2}$$

For instability, the following condition that must be satisfied:

$$0 < [5(1 + 2\omega_p^2 A_0/k^2c^2) + (4\omega_p^2/k^2c^2)(4A_1+A_2)]^2$$
$$< 16(1 + 2\omega_p^2 A_0/k^2c^2)[1 + 2\omega_p^2 A_0/k^2c^2 + (4\omega_p^2/k^2c^2)(A_1+A_2)] \quad \text{(B-6)}$$

A_N and A_{N+1} are evaluated as functions of n_b/n_p using the tabulated values of Bessel functions from Abramowitz and Stegun (1970) for the appropriate range of the argument as defined by the constraints on sx_b. Substitute these and the value of ω_p^2/k^2c^2 into the inequality (B-6) to obtain the constraints on n_b/n_p.

For $sx_b = 4.0$; $x_b = 9.0$; $n_b/n_p = 10$:

$A_0 = 1.05$, $A_1 = -0.714$, $A_2 = 2.72$. Substituting and solving for θ and Kr:

$$\omega/\Omega = 1.55(0.99 + 0.14i) = 1.53 + 0.22i \quad \text{(B-7)}$$

A second solution is found for $sx_b = 4.4$; $x_b = 3$. Then with $n_b/n_p = 10$: $A_0 = 3.06$, $A_1 = -2.4$, $A_2 = 2.79$, and

$$\omega/\Omega = 1.8 + 0.6i$$