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ENTRY VELOCITIES AT MARS AND EARTH FOR SHORT TRANSIT TIMES

Reinald G. Finke

July 1993

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Propulsion systems composed of a Shuttle External Tank, appropriately modified for the purpose, with a rocket engine that is either an SSME or a NERVA could inject a gross personnel payload of 100,000 lb on a trans-Mars trajectory from Space Station "Freedom" with aerobraking at Mars with transit times of less than 70 days. Such transit times reflect a significant reduction from the 200-plus days generally considered. The 100,000-lb payload would include the mass of a hypothetical aerobrake for aerocapture at Mars. The entry velocities at Mars compatible with such transit times are greater than 21 km/sec, to be compared with previously stated constraints of 8.5 to 9.5 km/sec for nominal Mars entry velocity. Limits of current aerobrake technology are not well enough defined to determine the feasibility of an aerobrake to handle Mars-entry velocities for short-transit-time trajectories. Return from Mars to Earth on a mirror image of a 70-day outbound trajectory (consistent with a stay time of about 12 days) would require a Mars-departure velocity increment more than twice as great as that at Earth departure and would rèquire a correspondingly more capable propulsion system. The return propulsion system wou'd preferably be predeployed at Mars by one or more separate minimum-energy, 0.5-to-1.1-Mlb-gross-payload cargo flights with the same outbound propulsion systems as the personnel flight, before commitment of the personnel flight. Aerobraking entry velocity at Earth after such a transit time would be about 16 km/sec, to be compared with constraints set at 12.5 to 16 km/sec.

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ENTRY VELOCITIES AT MARS AND EARTH FOR SHORT TRANSIT TIMES

Reinald G. Finke

July 1993

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INSTITUTE FOR DEFENSE ANALYSES

FOREWORD

This paper represents the documentation of the analysis that was carried out as part of a self-initiated effort of several STD staff members (R.G. Finke, A. Hull, W. Jeffrey, P. Kysar, and R.S. Swanson) toward providing an IDA product of potential interest to NASA. The effort began in January 1990 after the Space Council announced that it would be soliciting new ideas from non-NASA contributors to help define NASA's Space Exploration Initiative. Interim results of this work were incorporated in presentations to Mr. Michael Weeks of NASA/HQ on September 11, 1990 and to the staff of the Synthesis Group on January 25, 1991.

The author wishes to thank the reviewers of this paper, H. Hagar, W. Jeffrey, R.C. Oliver, and R.S. Swanson, for their many suggestions for improvement of the exposition.

ABSTRACT

Propulsion systems composed of a Shuttle External Tank, appropriately modified for the purpose, with a rocket engine that is either an SSME or a NERVA could inject a gross personnel payload of 100,000 lb on a trans-Mars trajectory from Space Station "Freedom" with aerobraking at Mars with transit times of less than 70 days. Such transit times reflect a significant reduction from the 200-plus days generally considered. The 100,000-lb payload would include the mass of a hypothetical aerobrake for aerocapture at Mars. The entry velocities at Mars compatible with such transit times are greater than 21 km/sec, to be compared with previously stated constraints of 8.5 to 9.5 km/sec for nominal Mars entry velocity. Limits of current aerobrake technology are not well enough defined to determine the feasibility of an aerobrake to handle Mars-entry velocities for short-transit-time trajectories.

Return from Mars to Earth on a mirror image of a 70-day outbound trajectory (consistent with a stay time of about 12 days) would require a Mars-departure velocity increment more than twice as great as that at Earth departure, and would require a correspondingly more capable propulsion system. The return propulsion system would preferably be predeployed at Mars by one or more separate minimum-energy, 0.5-to-1.1-Mlb-gross-payload cargo flights with the same outbound propulsion system as the personnel flight, before commitment of the personnel flight. Aerobraking entry velocity at Earth after such a transit time would be about 16 km/sec, to be compared with constraints set at 12.5 to 16 km/sec.

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I. INTRODUCTION

In discussions of human flight to Mars, the observation is often made that the duration of the trip from one planet to the other ("transit time") should be made short to reduce the time of exposure of the crew to the rigors and hazards of space, such as low gravity, space radiation, confinement and isolation, and possible support-system malfunctions. However, as the transit time is shortened, there is an increase in the velocity increment (ΔV) demanded from the propulsion system. Inspection of the "rocket equation"

$$\Delta V = g_c \, I_{sp} \ln\left(\frac{M_{propellant} + M_{inert} + M_{payload}}{M_{inert} + M_{payload}} \right) ,$$

where g_c is the conversion of mass units to force units, I_{sp} is the propulsion system's specific impulse (thrust per unit mass flow rate) and M is mass, indicates that an increase in ΔV can be obtained by an increase in I_{sp} and/or an increase in the mass ratio (the quantity in parentheses).

It is often asserted (e.g., see Synthesis, 1991) that nuclear propulsion is necessary if short transit times are to be achieved. While nuclear propulsion has a greater I_{sp} than chemical propulsion, the nuclear rocket engine has a greater mass than the chemical rocket engine, increasing the inert mass (which is typically small in comparison with the propellant mass) and decreasing the mass ratio; further, the liquid-hydrogen propellant for the nuclear rocket engine has a lower density than the liquid-oxygen/liquid-hydrogen propellants of the chemical rocket engine, lowering the propellant mass, if propellant volume were constrained, and again decreasing the mass ratio. These countervailing trends in I_{sp} and mass ratio make the relative performance of nuclear propulsion with respect to chemical propulsion not intuitively obvious, but dependent on the actual values going into the rocket equation.

If an aerobrake could be employed instead of a propulsive maneuver to decelerate at Mars, the velocity decrement provided by the aerobrake could be regarded as an increase in the overall ΔV capability of the system, and the propellant saved by aerocapture at Mars could be added to the propellant of the main propulsion stage to be used for departure from Earth. This increase in departure propellant would ostensibly generate a higher departure speed and therefore a shorter transit time, and the main propulsion stage could be jettisoned after the departure impulse so that no insulation of cryogenic propellant(s) against boiloff during the months of outbound transfer would be needed. However, the mass of the aerobrake must come out of the payload mass, or, if payload mass is fixed, will increase the inert mass (an increase that is partially offset by the decreased tank-insulation requirement), decreasing the mass ratio and reducing the gain in departure ΔV produced by the gain in departure propellant. Again, as with nuclear propulsion, the relative performance with or without an aerobrake depends on the actual design values going into the rocket equation; a sufficiently great aerobrake mass could wipe out the benefit of increased propellant at Earth departure. If aerobrake mass increases with entry velocity (while entry velocity is increasing as transit time is shortened), then beyond some value of entry velocity an increase in entry velocity will lead to a greater increase in aerobrake mass than the mass of propellant required to cancel propulsively that increase in entry velocity. In a lack of understanding of heating rates and thermal protection systems at high entry velocities, an aerobrake designer could increase his aerobrake's entry-velocity capabilities simply by adding more ablative material, increasing the aerobrake's mass possibly unnecessarily.

The required aerobrake technology is set by the capability of the propulsion system to generate high departure ΔVs for short transit times. Low propulsion-system capability, compatible with minimum-energy transfer requirements set by planners who are over-concerned with the costs of generating high propulsive ΔV, leads to low aerobrake technology demands, and there is no motivation to advance aerobrake technology. On the other hand, if currently expressed constraints set on aerobrake entry velocities represent technology barriers, the designers of propulsion systems have no motivation to provide higher ΔVs.

The principal purpose of this analysis was to determine whether the limitation to achievement of short transit times is set by propulsion-system technology or by aerobrake technology. The approach taken here to resolving this question follows the procedure below:

We build on "existing" propulsion-system components to determine the resulting dependences of transit time and entry velocity on payload weight, i.e., we--

1. Define credible example propulsion systems,
2. Determine ΔV requirements that are to be satisfied by these propulsion systems for different transit times,

3. Find the dependence of transit time on payload for the defined propulsion systems, assuming that all the destination deceleration can be supplied by an aerobrake, with the aerobrake mass included in the payload mass, and

4. Find the dependence of entry velocity on transit time.

This approach allows us to determine the aerobrake entry-velocity capabilities that would be required in order to handle, without propulsive assist, Mars and Earth entries for short transit-time trajectories producible by currently available propulsion-system designs with payload (including aerobrake) masses covering a reasonable range of missions. These required entry-velocity capabilities can then be compared with current statements setting aerobrake entry-velocity limits at the two planets.

The determination of entry heating rates at high entry velocities at Mars is a subject of future experimentation and CFD computation, and the design of thermal protection systems capable of withstanding these heating rates may require advancement of the state of the art of refractory materials. The derivation of the dependence of aerobrake mass on entry velocity is beyond the scope of this Paper.

II. DISCUSSION

A. PROPULSION SYSTEMS

To determine whether a choice must be made between a chemical and a nuclear propulsion system on the basis of ability to achieve short transit times, we consider one example of each system. Both of these examples are composed of components of existing design to avoid questions of their achievability. Each makes use of the Shuttle's External Tank (ET), see Fig. 1, modified as needed, to carry the propellant(s), with a single rocket engine. The chemical engine chosen is the operational Space Shuttle Main Engine (SSME), and the nuclear engine is the Nuclear Engine for Rocket Vehicle Application (NERVA), a detailed solid-core-reactor paper design of the late 1960s.

The assumed values of engine thrust, mass and specific impulse for the SSME and NERVA are given in Table 1. The SSME characteristics for its existing area expansion ratio, ε, of 80:1 are from JSC, 1990. The NERVA engine mass is taken at the optimistic end of the range of estimates produced in design studies in the 1960s (cited in IDA, 1970), and the mass of a shield to protect the payload from the radiation from the operating reactor is not included in the engine mass, but could be viewed as part of the payload. The 850-sec specific impulse adopted here for NERVA is greater than the 825 sec projected in the 1960s for applications involving repeated or extended use (as approximated by a one-hour-plus burn time--see table) but less than the 900 sec projected at that time (op. cit.) for employment in a brief single-use mode. The degradation in specific impulse of the NERVA engine associated with running at less than operating temperature during the heat-up phase (IDA, 1970) is ignored. The combined mass of the operating subsystems, the manifolding, and the structure to attach the engine and the payload to the ET is taken here to be 13,000 lb, conservatively about twice the mass of those components determined by the DESIGN launch-vehicle synthesis model in IDA, 1966.

Below the SSME data in the table are given the properties of the existing ET carrying LO_2/LH_2 propellants, i.e., its inert mass, propellant volume, average propellant density at the specified mixture ratio (six pounds of LO_2 for each pound of LH_2) and the

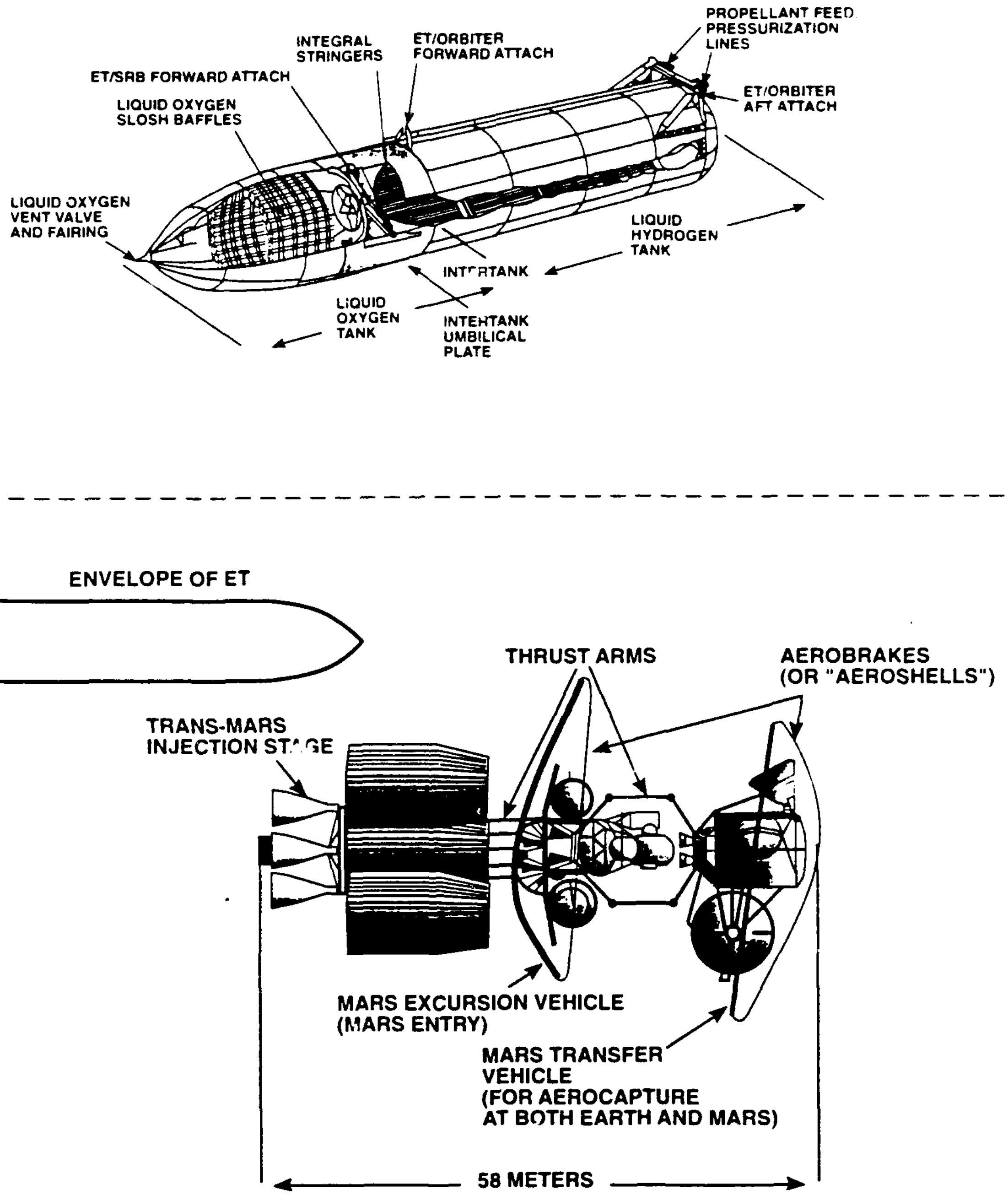


Figure 1. External Tank to Scale of Proposed Mars Transportation System (NASA, 1989)

mass of tanked propellants (from JSC, 1990). For the NERVA-powered system the propellant is LH_2 alone, so the tank mass is reduced by an estimated amount consistent with removal of the two adjacent tank ends in the intertank section, and the tank volume is increased by an amount estimated for the intertank region. The mass of the tanked LH_2 propellant is the product of the LH_2 density and the estimated volume of the modified monolithic tank.

Table 1. Assumed Properties of Propulsion Systems

Engine:	SSME	NERVA
Thrust, lbf	486,500	75,000
Engine Mass, lbm	7,000	15,000 (no shield)
Specific Impulse, sec	453.16 (ε = 80)	850* (825-900)
Thrust-Structure and Subsystems Mass, lbm	13,000	
ET Inert Mass, lbm	66,700	50,000 (est.)
ET Propellant Volume, ft^3	70,990	80,630 (est.)
Propellant Density, lbm/ft^3	22.54 (6:1)	4.42
Propellant Mass, lbm	1,600,000	356,400 (est.)
Burn Time, minutes at full thrust	25	67

* Ignoring startup cooling losses

B. TRANSIT TIMES

Departure of the trans-Mars vehicle from Earth is assumed to take place as shown in Fig. 2, with an impulse parallel with the circular orbital velocity V_{cE} of Space Station "Freedom". The value of V_{cE} is given in terms of the orbital altitude h by the relation

$$V_{cE} = \sqrt{\frac{GM_E}{R_E + h}} \tag{1}$$

where

GM_E = the gravitational constant times the mass of the Earth
= $3.98604 \times 10^5\ km^3/sec^2$

R_E = the equatorial radius of the Earth = 6378.149 km

h = the altitude of SSF's orbit = 416.7 km.

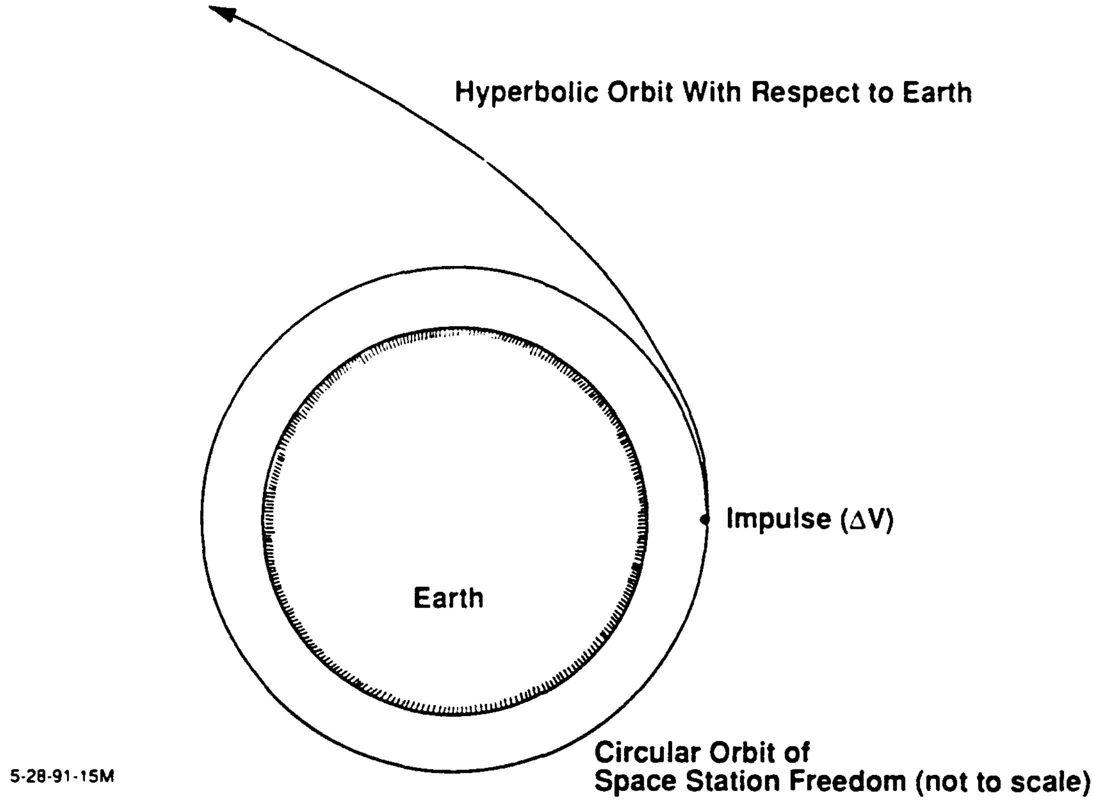


Figure 2. Departure from Earth Orbit

The departure-impulse ΔV from the SSF orbit is to raise the vehicle's initial total energy per unit mass in circular orbit, which is

$$E = \frac{1}{2} V_{cE}^2 - \frac{GM_E}{R_E + h} \quad , \tag{2}$$

to a value greater than zero (i.e., to make the velocity greater than the escape velocity). The excess energy will be such that there is a residual kinetic energy at great distances from the Earth in interplanetary space where the potential energy with respect to the Earth is essentially zero. The excess kinetic energy per unit mass is half the square of the vehicle's "hyperbolic excess velocity" V_{he}, its velocity to be added vectorially to the Earth's velocity V_E in orbit around the Sun to determine the vehicle's heliocentric transfer orbit.

The Earth's orbital velocity (29.785 km/sec) in an equivalent circular orbit, with radius equal to the semi-major axis of its elliptical orbit that has an eccentricity of 0.016272, is given by

$$V_E = \sqrt{\frac{GM_S}{AU}} \quad ,$$

where

GM_S = the gravitational constant times the mass of the Sun

= 1.32715×10^{11} km^3/sec^2

AU = the radius of the Earth's orbit (the "astronomical unit")

= 1.49599×10^8 km.

The velocity increment of the departure impulse is found from the energy equation (equation 2)

$$\frac{1}{2}(V_{cE} + \Delta V)^2 - \frac{GM_E}{R_E + h} = \frac{1}{2}V_{he}^2 \quad .$$

Replacing $GM_E/(R_E + h)$ by V_{cE}^2 per equation 1 and solving for ΔV gives

$$\Delta V = \sqrt{2V_{cE}^2 + V_{he}^2} - V_{cE} \quad . \qquad (3)$$

The types of heliocentric transfer trajectories considered here, Fig. 3, are examples of the two extremes: "perihelion at Earth" and "aphelion at Mars." The Hohmann and parabolic trajectories are special cases of the perihelion-at-Earth class. The Hohmann transfer is the minimum-energy trajectory, with perihelion at Earth and aphelion at Mars (the orbital eccentricity is about 0.2075), and the parabolic transfer involves a perihelion velocity just equal to that for escape from the Solar System at the Earth's distance from the Sun (the orbital eccentricity is 1.0). For the perihelion-at-Earth family, the vectors V_{he} and V_E are parallel, and the sum of the magnitudes constitutes the perihelion velocity V_p of the transfer orbit. The value of V_p to produce an orbital eccentricity e with perihelion at Earth is given by

$$V_p = V_E\sqrt{1 + e} \quad ,$$

so the value of the hyperbolic excess velocity to go into equation 3 is $(V_p - V_E)$, or

$$V_{he} = V_E(\sqrt{1 + e} - 1) \quad .$$

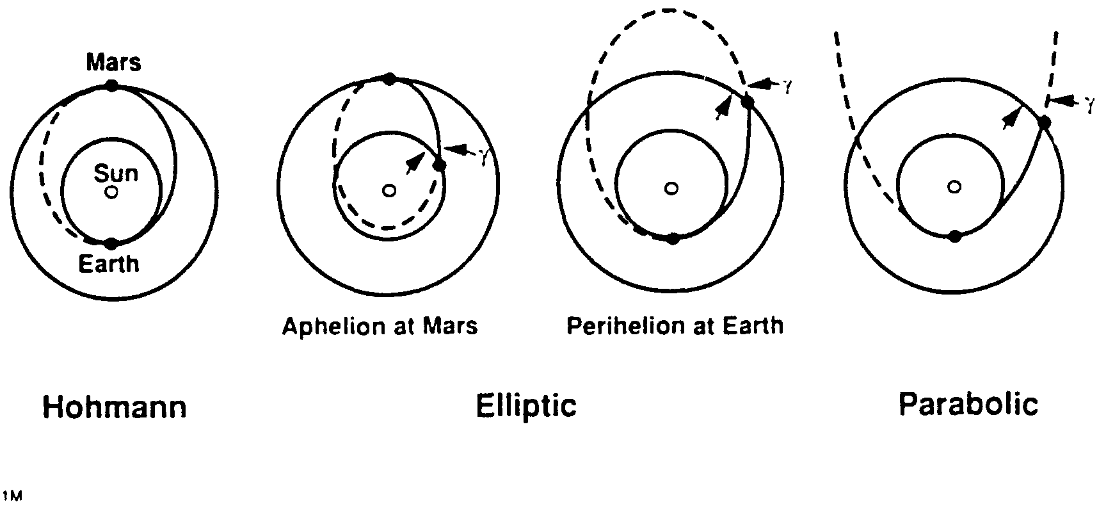
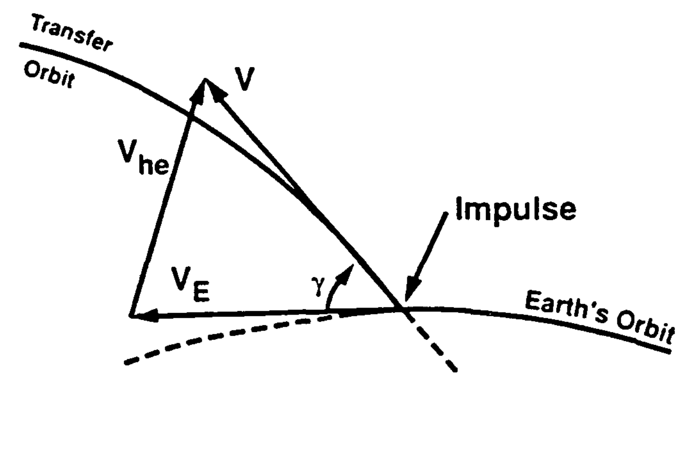


Figure 3. Types of Transfer Trajectories

The general ellipse for departure from Earth with aphelion at Mars (or for departure from Mars with perihelion at Earth) crosses the orbit of the planet of departure at a path angle γ (Fig. 3). Therefore determination of the hyperbolic excess velocity involves finding the vector difference between the planet's velocity, V_E in the case of the Earth, and the velocity V in the crossing elliptical orbit, according to the diagram--



So the cosine law gives

$$V_{he}^2 = V^2 + V_E^2 - 2VV_E \cos \gamma \ . \tag{4}$$

The velocity V at either of the two points that are at a radius r on an ellipse makes an angle γ with the circle that has that radius, and the angular momentum at the crossing point is the same as at an apsis (either of the extreme points on an ellipse, at the ends of the semi-major axis, where the velocity V_{aps} is perpendicular to the radius vector r_{aps}), or

$$Vr \cos\gamma = V_{aps} r_{aps} \quad . \tag{5}$$

The velocity at perihelion at Earth is $\sqrt{\frac{GM_S(1+e)}{r_{aps}}}$, where r_{aps} = AU, and the velocity at aphelion at Mars is $\sqrt{\frac{GM_S(1-e)}{r_{aps}}}$,where r_{aps} = AU × AUM and AUM is the ratio of the radius of Mars' orbit to that of the Earth's orbit, or 1.523691.

Conservation of energy says that the sum of the kinetic and potential energies at any point is the same as at an apsis, or

$$\frac{1}{2}V^2 - \frac{GM_S}{r} = \frac{1}{2}V_{aps}^2 - \frac{GM_S}{r_{aps}} \quad . \tag{6}$$

Substiuting the expressions relating V, γ and r from conservation of energy (equation 6) and angular momentum (equation 5) in the cosine law (equation 4) gives an expression for the value of the hyperbolic excess velocity in terms of the radii at the departure point (r, for either Earth or Mars) and at the apsis (r_{aps}, for either aphelion at Mars or perihelion at Earth), and the eccentricity e--

$$V_{he}^2 = \left(\frac{GM_S}{r_{aps}}\right)\left[\left(\frac{3r_{aps}}{r}\right) - \frac{r_{aps}}{a} - \left(\frac{2r_{aps}}{r}\right)\sqrt{\frac{a\left[1-e^2\right]}{r}}\right] \quad , \tag{7}$$

where the semi-major axis a = $r_{aps}/(1-e)$ for the apsis at perihelion and a = $r_{aps}/(1+e)$ for the apsis at aphelion.

From these values of the semi-major axis, we can use the following expressions to determine the period of revolution in the transfer orbit and the orbital angle from perihelion. If the semi-major axis is measured in astronomical units, i.e., as the ratio of its absolute value to the radius of the Earth's orbit, the period of revolution P in years is given by Kepler's third law as

$$P = a^{1.5} \quad .$$

The polar-coordinate equation for a conic section (where θ is measured from the perihelion and r is measured from a focus) is

$$r = \frac{a\left(1 - e^2\right)}{1 + e\cos\theta} \quad .$$

The orbital angle at which the radius is r is therefore found from

$$\cos\theta = \frac{\left[\frac{a\left(1 - e^2\right)}{r} - 1\right]}{e} \quad .$$

The time from perihelion to this orbital angle is given by the function TIME (Appendix A) as

$$t = \mathrm{TIME}(P, e, \theta) \quad .$$

Of course, the transit time with aphelion at Mars is half a period minus this time.

The transfer angle θ will be greater than the Earth's angular displacement in the transit time, i.e., the trans-Mars vehicle will get ahead of the Earth, if the transit time is less than 76.6 days, for the perihelion-at-Earth class of trajectories. In this short-transit-time case, the vehicle can wait at Mars the few days for the Earth to catch up in its synodic (relative) motion, and can return on the mirror image of the outbound trajectory, on a "sprint" mission. Figure 4 shows an example (from Koelle, 1961), a parabolic transfer with a 69.9-day transit time, in which the transfer vehicle gets ahead of the Earth by about six days and spends about twelve days at Mars before the mirror image of the outbound trajectory can be flown.

For any mirror-image transit time greater than 76.6 days, the waiting period ("stay time") in the vicinity of Mars for the next occurrence of the mirror image of the relationship of the planets at Mars arrival is given in Fig. 5. The values on the curve can be viewed intuitively as approximately a synodic period (780 days, i.e., the time interval of recurrence of identical transfer opportunities for our coplanar-circular-orbit model) less twice the excess in transit time over 76.6 days, e.g., ca. 580 days for a transit time each way of 176.6 days, versus 615 from the curve in Fig. 5. If "twice" is replaced with "1.68 times", the approximation gives agreement with values on the curve within three days for all transit times between 76.6 days and 176.6 days.

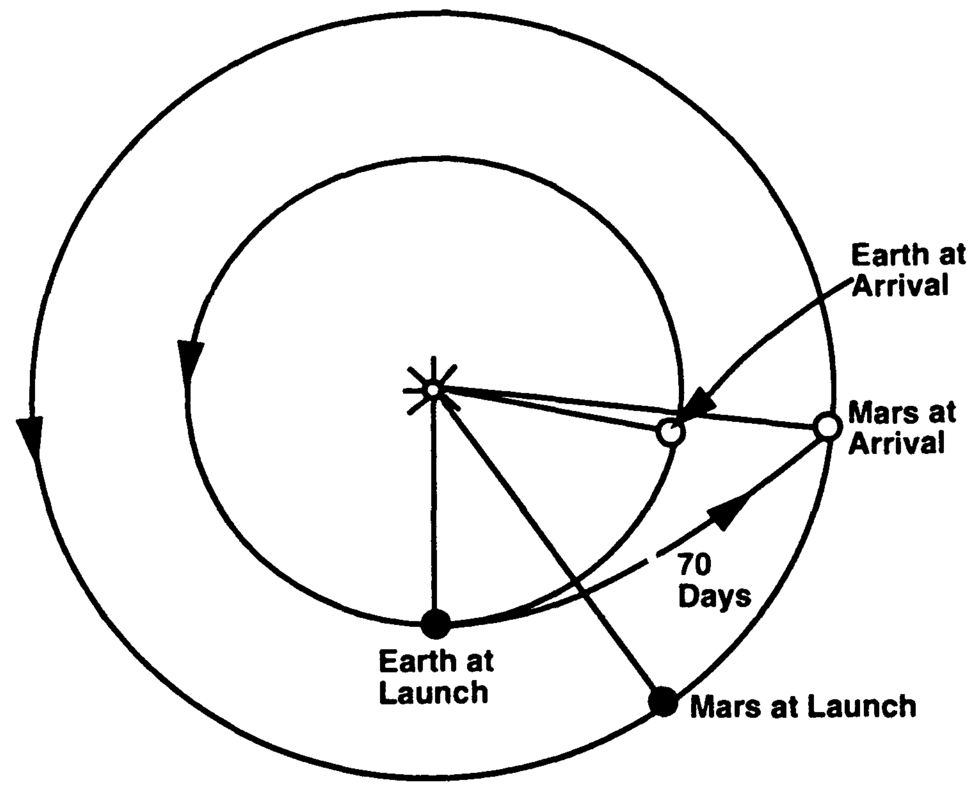


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Figure 4. Typical Parabolic Flight Profile to Mars
(illustrating "sprint" possibility).

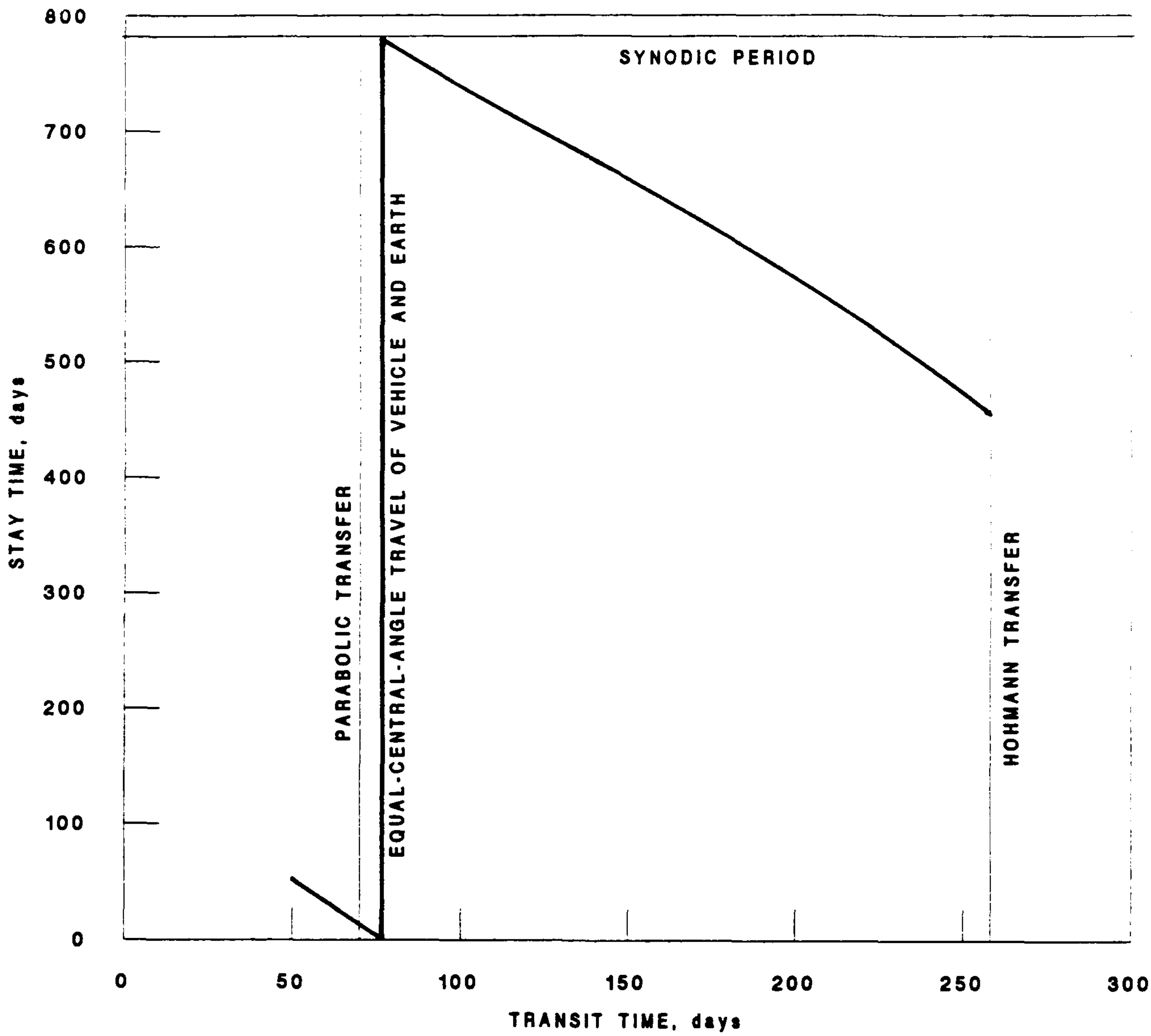

Figure 5. Stay Time at Mars between Outbound and Return Transfers with Equal Transit Times (coplanar circular orbits; perihelion at Earth)

For any assumed value of orbital eccentricity, one value of transfer ΔV and one value of transit time are obtained from the above relations. From the resulting pair of values of transfer ΔV and transit time, the dependence of ΔV on transit time can be plotted. Such a plot of the dependences of departure velocity on transit time for the two extreme trajectories is shown in Fig. 6, for transfer from Earth (from Space Station "Freedom") to Mars' orbit, with deceleration into orbit around Mars assumed to be accomplished by aerobraking and by using circularization propulsion to rendezvous with Phobos (the inner moon of Mars), say. (These ΔVs also represent the velocity increments saved by aerobraking at Earth return after such Mars-Earth transit times.) Note that the Earth-departure velocity increment for aphelion-at-Mars transfers is generally so much greater than for perihelion-at-Earth transfers, for the same transit time, that perihelion-at-Earth will almost certainly be the preferred outbound trajectory option. For the Hohmann transfer, the Earth-departure velocity increment is 3.566 km/sec and the transit time is 258.8 days; for parabolic transfer (e = 1), the Earth-departure ΔV is 8.758 km/sec with a transit time of 69.9 days.

For departure from Mars to return to Earth, the calculation of the velocity increment is similar to that for departure from Earth. The velocity of Phobos in its orbit (assumed circular) around Mars is

$$V_{cM} = \sqrt{\frac{GM_M}{R_M + h}} \ ,$$

where

GM_M = the gravitational constant times the mass of Mars

= 4.293×10^4 km^3/sec^2

R_M = the equatorial radius of Mars = 3392 km

h = the altitude of Phobos' orbit = 5988 km.

The heliocentric orbital velocity of Mars (24.129 km/sec) in an equivalent circular orbit, with radius equal to the semi-major axis of its elliptical orbit that has an eccentricity of 0.093370, is given by

$$V_M = \sqrt{\frac{GM_S}{AU \times AUM}} \ .$$

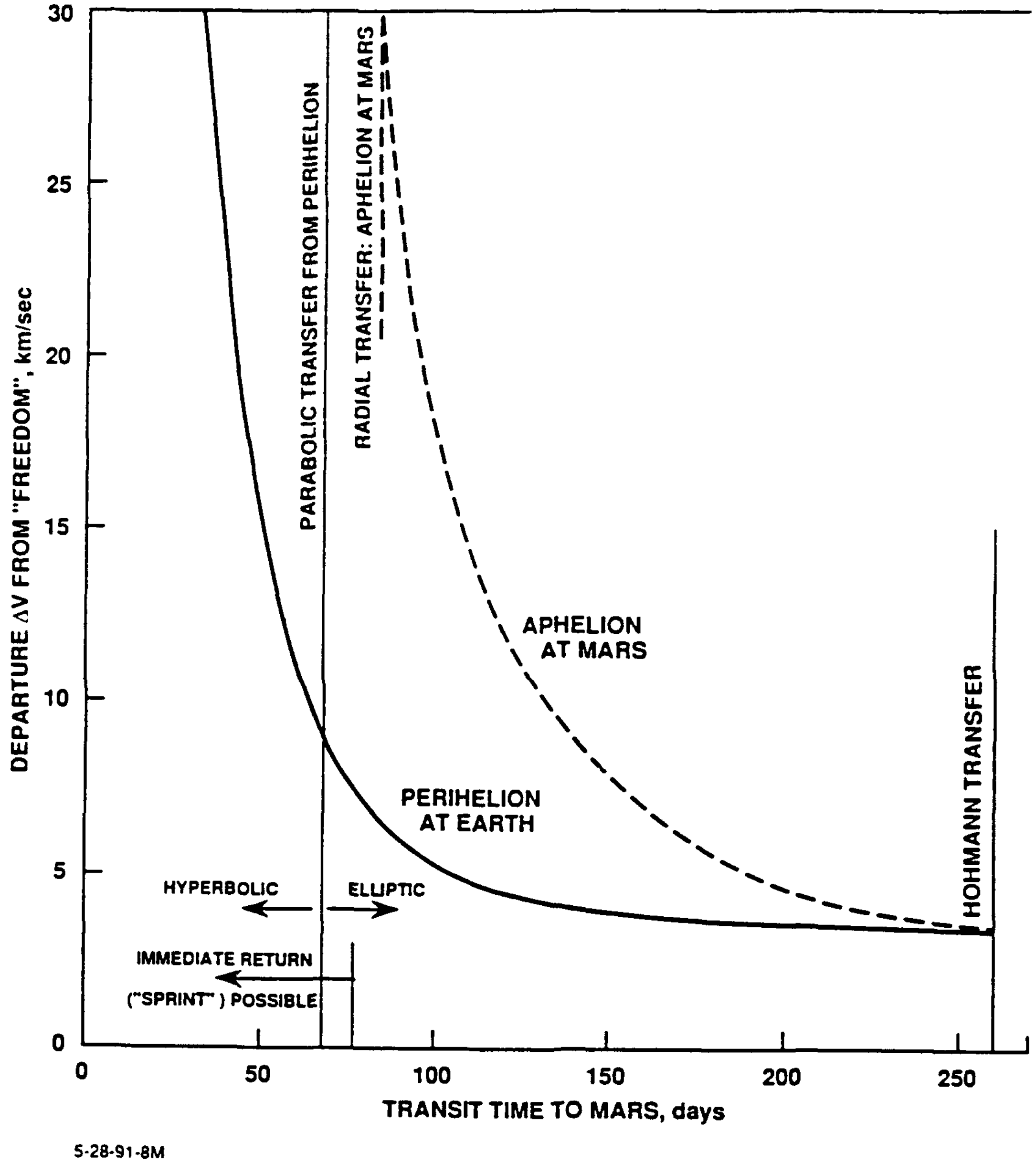

Figure 6. Velocity Requirement Versus Transit Time to Mars from Space Station "Freedom" to Mars' Orbit (coplanar circular orbits)

The velocity at aphelion at Mars for a heliocentric transfer orbit with eccentricity e is

$$V_a = V_M\sqrt{1-e} \quad ,$$

and the value of the hyperbolic excess velocity for Mars departure with aphelion at Mars is $(V_M - V_a)$, so

$$V_{he} = V_M\left(1 - \sqrt{1-e}\right) \quad .$$

The hyperbolic excess velocity for departure from Mars with perihelion at Earth is given by equation 7. And finally, the ΔV to depart from Phobos on a heliocentric transfer orbit with e as an implicit parameter is

$$V = \sqrt{2V_{cM}^2 + V_{he}^2} - V_{cM} \quad .$$

The transit time from Mars to Earth is the same as from Earth to Mars for the same eccentricity and the same assignment of an apsis, and the propulsive-deceleration velocity increment at arrival (if aerobraking is not used) is the same as the departure velocity increment at that terminus.

For return to the Earth's orbit from Mars, Fig. 7 shows the dependences of transfer velocity on transit time for the two extreme trajectories, for departure from Phobos with aerobraking at Earth. (These ΔVs also represent the Mars-arrival velocity increments saved by aerocapture at Mars.) The Mars-departure ΔV for aphelion-at-Mars transfers is less than that for perihelion-at-Earth transfers for transit times greater than about 95 days. However, aphelion-at-Mars transfers by their nature cannot achieve transit times less than 85 days (without going into a retrograde heliocentric orbit) because that time represents a straight fall from rest directly toward the Sun's center of attraction from the radius of Mars' orbit to the radius of Earth's, after completely stopping the velocity that had been imparted by Mars (and Phobos) to the transfer vehicle. For the Hohmann-transfer return, the Mars-departure ΔV is 1.882 km/sec, slightly over half the outbound Earth-departure Hohmann-transfer ΔV. In further comparison of Figs. 6 and 7, it can be seen that the departure ΔV from Mars for return to Earth (via an aphelion-at-Mars transfer) is greater than the outbound departure ΔV from Earth (via a perihelion-at-Earth transfer) for transit times each way less than about 140 days. Indeed, for a "sprint" mission (a transit time of 76.6 days or less), the return ΔV from Mars--now via a mandatory perihelion-at-Earth transfer--is more than *double* the outbound ΔV from Earth.

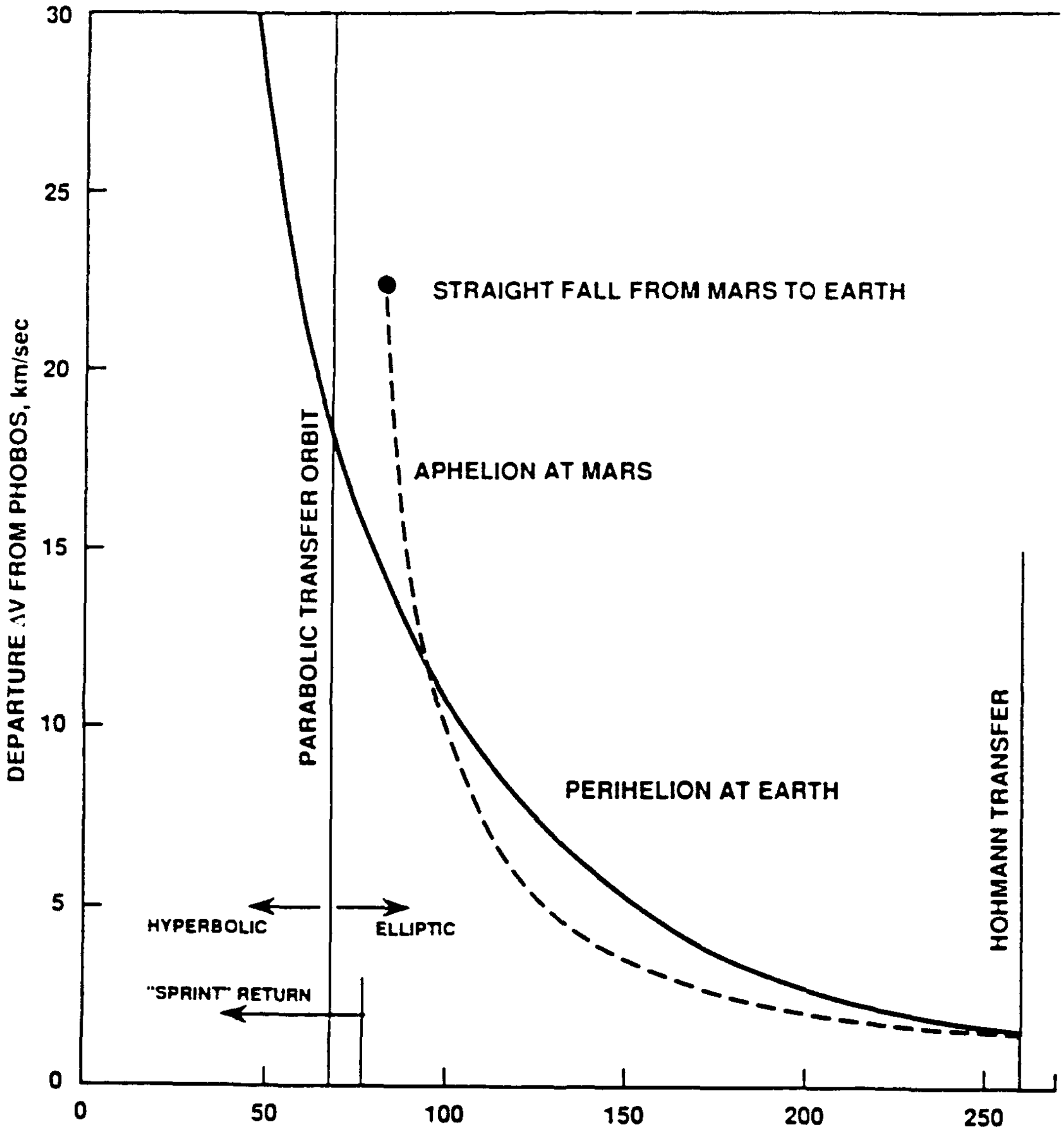

Figure 7. Velocity Requirement Versus Transit Time to Earth from Phobos to Earth's Orbit (coplanar circular orbits)

C. PROPULSION-SYSTEM PERFORMANCE

The ideal-velocity-increment capability (in gravity-free vacuum) of a propulsion system is given by the "rocket equation"

$$\Delta V = g_c I_{sp} \ln(M_o/M_f) \qquad (8)$$

where

g_c = conversion of mass units to force units

= 9.80665 kgmass-m/(sec^2-kgforce)

I_{sp} = vacuum specific impulse (thrust per unit mass flow rate), kgforce-sec/kgmass (or, simply, "sec")

M_o = initial mass = $M_p + M_t + M_{str} + M_{eng} + M_{PL}$

M_f = final mass = $M_o - M_p$

M_p = propellant mass

M_t = tankage mass

M_{str} = structure mass (to connect engine to tankage)

M_{eng} = engine mass

M_{PL} = payload mass.

An example calculation for the chemical-propulsion system with 100,000 lb of payload is as follows:

M_{PL} = 100,000 lb

M_{eng} = 7,000 lb

M_{str} = 13,000 lb

M_t = 66,700 lb

M_p = 1,600,000 lb,

so

M_f = 66,700 + 13,000 + 7,000 + 100,000 = 186,700 lb

M_o = $M_f + M_p$ = 1,786,700 lb

and from equation 8

$$\Delta V = 9.80665 \times 453.16 \ln(1{,}786{,}700/186{,}700)$$

$$= 10{,}037 \text{ m/sec} = 10.037 \text{ km/sec}.$$

During any extended period of time of acceleration from Space Station "Freedom," the propulsion system will climb slightly as its velocity grows beyond circular velocity. As it climbs, some of its kinetic energy is converted to potential energy and the velocity is less than it would have been if the acceleration had taken place at constant altitude (as with an ideal impulsive acceleration). While the potential energy is not lost to the vehicle, the rate of conversion of ΔV to energy, dE/dV, is proportional to V, so the efficiency of conversion of exhaust momentum to energy is reduced from its maximum value that would pertain for an altitude held constant at the initial value. This reduction from impulsive performance in the performance of low-thrust propulsion systems that burn for an appreciable fraction of an orbit (see Table 1) is called a "gravity loss," or "g loss." The calculated g loss for the example vehicle above with a thrust of 486,500 lb is 72 m/sec (from expressions in IDA, 1970).

Therefore the effective velocity increment from this example chemical-propulsion vehicle (with payload of 100,000 lb) for departure from SSF is 10,037 minus 72 = 9,965 m/sec. As shown by the curve in Fig. 6 for perihelion-at-Earth transfers, this departure ΔV from Earth will inject the payload on a transfer trajectory to Mars with a 65-day transit time.

For different payload masses, the relations above give different transit times. The resulting dependences of trans-Mars transit times on payload mass for the selected chemical and nuclear propulsion systems are plotted in Fig. 8. (The mass of an aerobrake is included in the payload mass of both systems, and the mass of a radiation shield is also included in the payload mass of the nuclear system.)

The results of Fig. 8 can be epitomized by the values of payload and transit time at the extremes of the curves: the first, at a payload, say 100,000 lb, that would characterize a mission to transport personnel on transit times near the shortest on the curves, and the second, at the maximum payload, representing inert cargo that could stand the longest transit time, that of the minimum-energy (Hohmann) transfer, i.e., about 250 days. These extremes for the two propulsion systems are listed in Table 2.

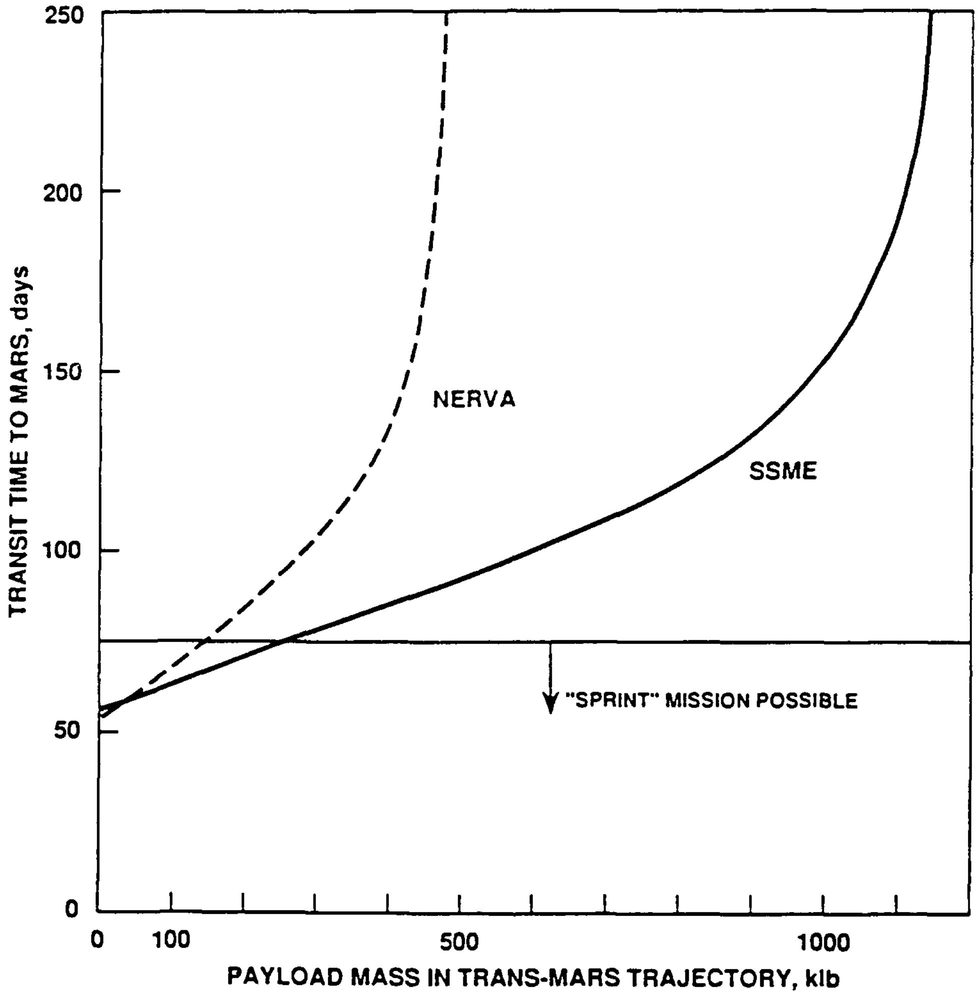

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Figure 8. Transit Time to Mars as a Function of Trans-Mars Payload for Single-ET, Single-Engine Propulsion Systems from "Freedom" (coplanar circular orbits; for aerobraking at Mars to Phobos, reduce plotted payload by aerobrake mass and propellant mass for circularization after aerocapture)

Table 2. Propulsion Systems' Capabilities (Injection from Space Station "Freedom" into Trans-Mars Trajectory with Aerocapture at Mars)

Engine	Gross Payload, klb	Transit Time, d	Total Mass, klb	Trans-Mars Costs, $/lb	Type Payload
SSME*	100	64.7	1,787	17,870	Personnel
SSME	1,134	249.8	2,821	2,490	Cargo
NERVA**	100	69.2	534	5,340	Personnel
NERVA	475	238.8	909	1,910	Cargo

* Space Shuttle Main Engine

** Nuclear Engine for Rocket Vehicle Application (with shield mass included in payload)

For transit times greater than about 58 days (with payloads above about 30,000 lb), the chemical propulsion system actually delivers *more* payload than does the nuclear. Comparisons between the two types of propulsion system are usually made at constant initial *mass*, for which the nuclear's greater I_{sp} delivers more final mass (including engine mass) to the same ΔV. This present comparison, on the basis of constant *volume*, is essentially reflecting the possibility that there may be a more stringent limit placed on the ability to ferry propellant tanks of large volume (that have the lowest structural mass per unit volume) to orbit than to ferry bulk propellant mass, which is more easily sub-dividable and reassemblable. In this present comparison, the factor-of-five propellant-density disadvantage (Table 1) of the nuclear propulsion system acts in its disfavor, reducing its available propellant mass, and therefore its payload mass beyond the 58-day crossover point in Fig. 8 for the assumed propulsion-system properties.

The total masses of the two propulsion systems at ignition are included in Table 2. For an assumed low-orbit delivery cost of $1000/lb, the total masses in klb become the orbital delivery costs of a vehicle in millions of dollars, e.g., $1.787B for the 100-klb-payload chemical vehicle, or $17,870/lb of payload on the 64.7-day trans-Mars trajectory. In this specific-delivery-cost figure of merit, the advantage of the nuclear propulsion system's greater performance is seen in its lower--by more than a factor of three--cost at constant payload, for the personnel-transport mission. (It does *not* have an advantage in transit time for that mission.) However, for the maximum-payload cargo-delivery mission, the cost advantage of the nuclear is reduced to only about 30 percent by the chemical propulsion system's payload advantage of 1,134 klb over 475 klb. Note incidentally that for the maximum-payload missions, the trans-Mars delivery costs (ignoring propulsion-system hardware costs as being small in comparison with Earth-to-orbit delivery costs) are only about 1.9 to 2.5 times the Earth-to-orbit delivery costs.

The assumptions used in this introductory analysis have tended to be favorable to the nuclear system. Any amplification of this comparison between nuclear and chemical propulsion systems should factor in (1) possible improvements in the chemical system, e.g., (a) increase in area expansion ratio of the SSME nozzle (to 400:1, say) appropriate to solely vacuum use and (b) reoptimization of the mixture ratio, and (2) the penalties to the nuclear system ignored here, e.g., (a) adding shield mass in the engine mass for personnel-transport missions and (b) including reactor startup cooling losses.

D. ENTRY VELOCITIES

The entry velocity at the (assumed non-rotating) destination planet, with no employment of propulsive deceleration, is approximated by the velocity at periapsis for a periapsis altitude that is just deep enough in the atmosphere for aerodynamic forces to become appreciable fractions of the force of gravity. The contribution of the motion of the atmosphere due to the planet's rotation depends on the inclination of the entry plane to the plane of the planet's equator; for Mars the motion of the atmosphere would add to or subtract from the entry velocity at most about 240 m/sec; for Earth, about 465 m/sec. These corrections are not included, but are small in comparison with the entry velocities.

If the altitude for aerobraking at Mars is h_{AB}, then the circular velocity at this periapsis altitude is

$$V_{cAB} = \sqrt{\frac{GM_M}{R_M + h_{AB}}} \quad .$$

If the velocity at periapsis is called the Mars entry velocity V_{ME}, the equality of the total energy per unit mass at periapsis to the hyperbolic excess energy can be written, by substituting the variables appropriate to Mars in equation 2, as

$$\frac{1}{2}V_{ME}^2 - \frac{GM_M}{R_M + h_{AB}} = \frac{1}{2}V_{he}^2 \quad ,$$

so, substituting V_{cAB}^2 for $GM_M/(R_M + h_{AB})$ and solving for V_{ME}, we have

$$V_{ME} = \sqrt{2V_{cAB}^2 + V_{he}^2} \quad .$$

From the values of V_{he} used in calculating departure velocities from Mars for various transfer trajectories, i.e., for various transit times, we can determine the dependence of Mars entry velocity on transit time. For h_{AB} set to 101 km (drag deceleration ≈ 1 Earth-g at escape velocity for W/C_DA^* = 10 lb/ft^2), the resulting dependence is shown in Fig. 9. Only the data for perihelion-at-Earth transfers are included; the Earth-departure-velocity requirements for aphelion-at-Mars transfers (Fig. 6) are deemed too high for them to be considered as viable options.

For entry at Earth, at a h_{AB} different from Mars, the circular velocity at this altitude is

$$V_{cAB} = \sqrt{\frac{GM_E}{R_E + h_{AB}}} \; .$$

and the Earth entry velocity V_{EE} is approximated, as for Mars, by

$$V_{EE} = \sqrt{2V_{cAB}^2 + V_{he}^2} \; .$$

From values of V_{he} for departure from Earth for different transit times, and for a h_{AB} now of 87 km (again drag deceleration ≈ 1 g at escape velocity for W/C_DA = 10 lb/ft^2), we get the dependences of Earth entry velocity on transit time shown in Fig. 10. Here both perihelion-at-Earth and aphelion-at-Mars transfers are included. However, the latter shows so much higher entry velocities than the former (similar to the disparity in Earth-departure velocities in Fig. 6), that any advantage in reduced Mars-departure velocity with aphelion-at-Mars transfers (Fig. 7) may be outweighed by their disadvantage in increased aerobrake stresses at Earth entry.

The magnitudes of the entry velocities plotted in Figs. 9 and 10 can be put in better perspective if particular values extracted from the figures and listed in Table 3 are compared. Included in the table for reference are the escape velocities at the selected entry altitudes and some limits set by NASA in Code Z, 1989 and NASA, 1989. The Code Z limits are stated as "Atmospheric entry speeds at (planet) shall be no greater than (value)," and the 90-Day-Study limits are phrased "... with the nominal entry velocity constrained to (value) at (planet)." The transit times implied by Figs. 9 and 10 for the NASA limits are indicated in parentheses. The reference LaRC, 1991 indicates that the "limits" set in the

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* W = mass; C_D = drag coefficient; A = cross-sectional area (perpendicular to the velocity vector).

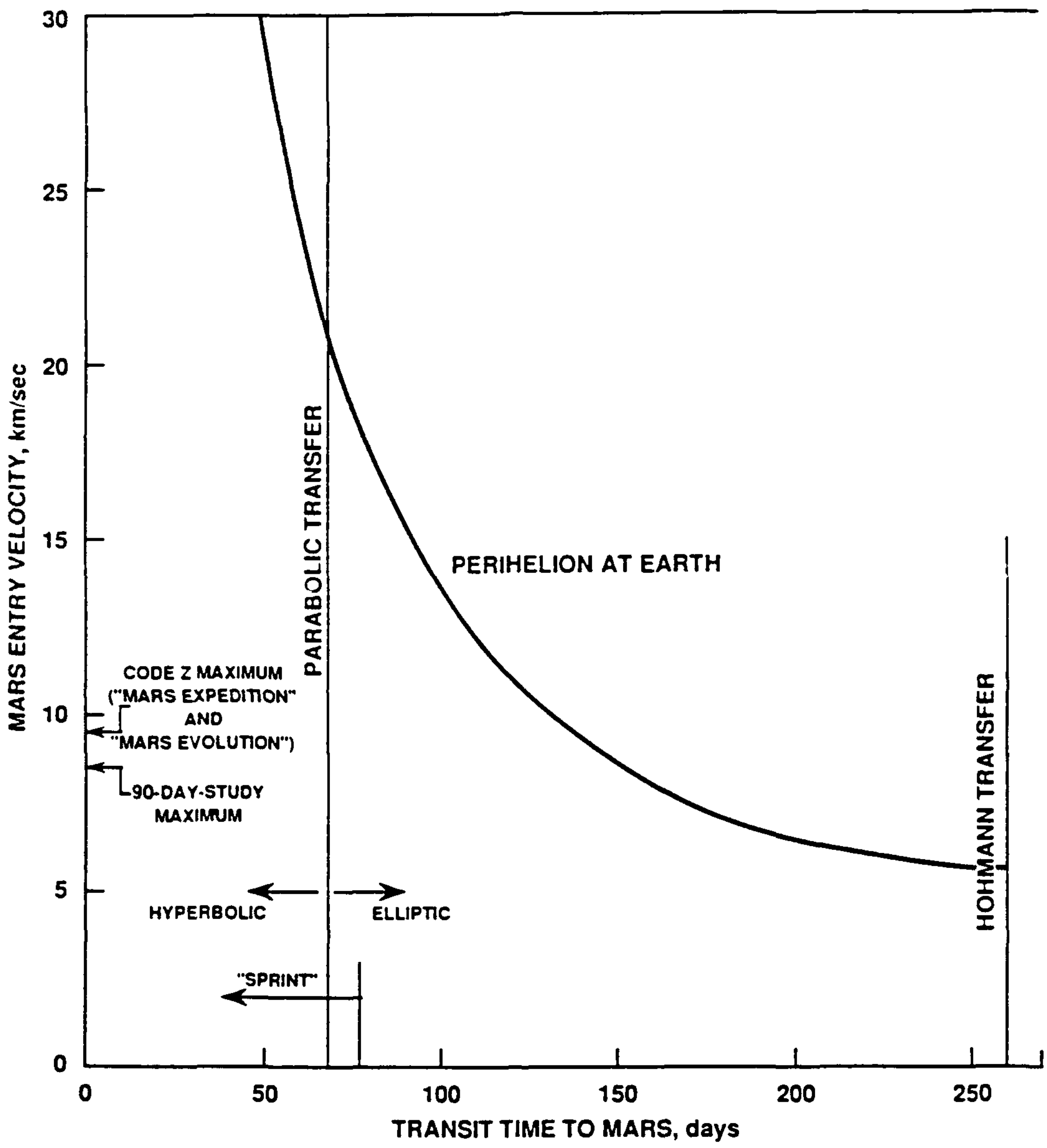

Figure 9. Mars Entry Velocity as Function of Transit Time to Mars (coplanar circular orbits; non-rotating atmosphere)

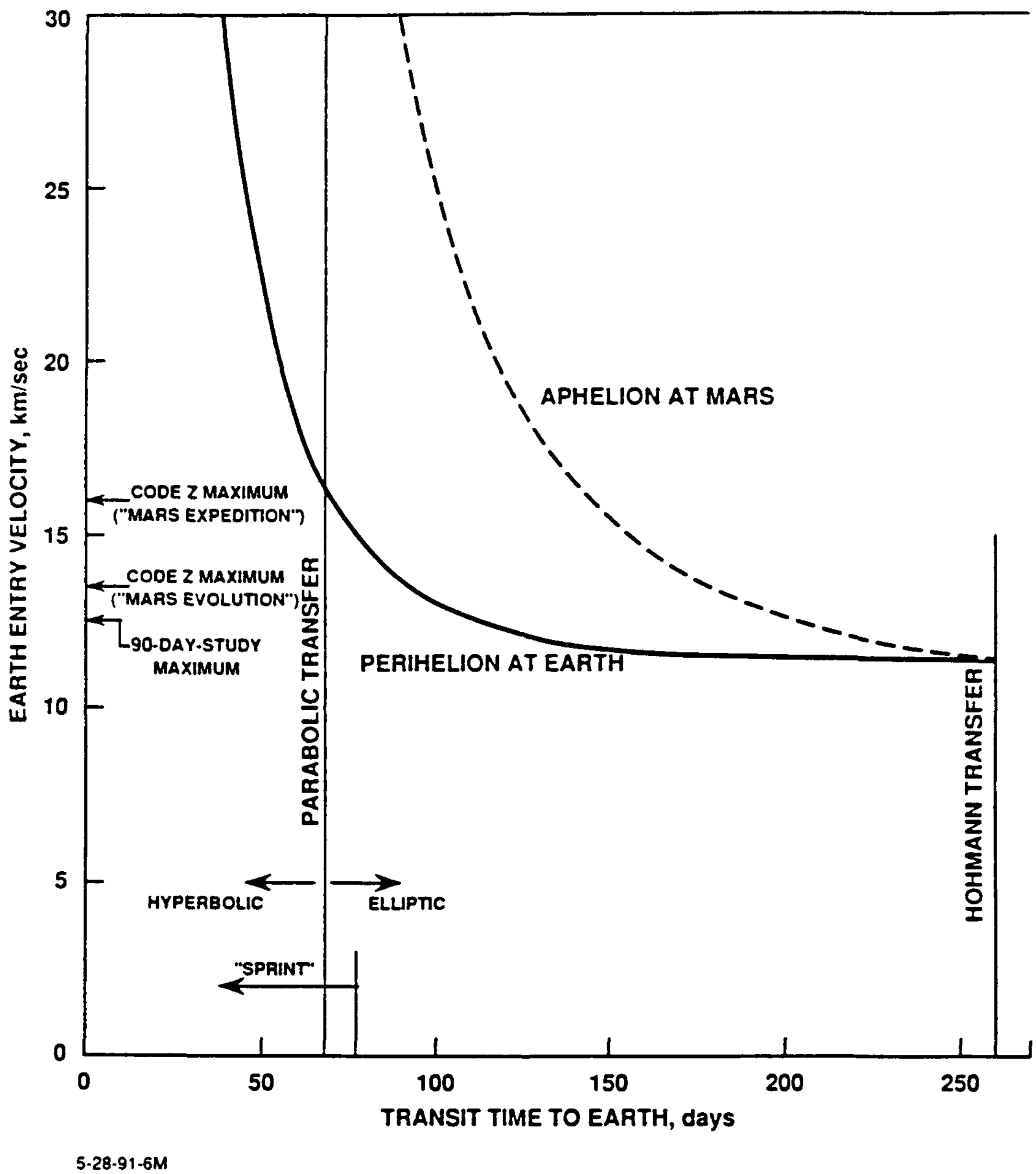

Figure 10. Earth Entry Velocity as Function of Transit Time to Earth (coplanar circular orbits; non-rotating atmosphere)

Table 3. Entry Velocities at Mars and Earth for Selected Transit Times (Mars entry altitude = 101 km; Earth entry altitude = 87 km)

Designation	E-M Transit Time, d	Mars Entry Velocity, km/sec	M-E Transit Time, d	Earth Entry Velocity, km/sec
(Escape Velocity)	–	(4.958)	–	(11.105)
Hohmann Transfer	258.8	5.621	258.8	11.488
Equal-Entry Crossover	106.0	12.9	106.0	12.9
"Sprint" Threshold	76.6	18.8	76.6	15.4
SSME, 100 klb PL	64.7	22.8	(64.7)	(17.8)
Code Z Limits				
"Mars Expedition"	(138)	9.5	(73)	16.0
"Mars Evolution"	(138)	9.5	(94)	13.5
90-day-Study Limits	(153)	8.5	(117)	12.5

two NASA reports may reflect expectation of 200-day transit times more than they do delineation of physical constraints; available literature (e.g., Walberg, 1985) does not define clear physical constraints.

The Hohmann-transfer entry velocities are only a little greater than the escape velocities. Below a 106.0-day transfer time (either way), indicated as "Equal-Entry Crossover" in the table, Mars entry velocities are greater than those at Earth; Mars entry velocities grow much faster as transit time is shortened than do those at Earth. At the threshold transit time of 76.6 days, below which a "sprint" mission becomes possible, the Mars entry speed grows to 18.8 km/sec (compared to 15.4 km/sec at Earth for the same transit time), and becomes 22.8 km/sec if the capabilities of the SSME-powered propulsion system with a 100,000-lb gross payload are utilized. These latter values are well above the NASA limits of 8.5 or 9.5 km/sec at Mars. The NASA limits at Earth are more compatible with short transit times.

III. OBSERVATIONS

Components for a propulsion system for transportation to Mars exist today either in the form of operational hardware or detailed paper design, utilizing a Shuttle External Tank (ET) and either a Space Shuttle Main Engine (SSME) or a Nuclear Engine for Rocket Vehicle Application (NERVA). The SSME-powered vehicle is capable of injecting a gross payload of 100,000 lb, representing a crew-transport mission, on a trajectory with a transit time of about 65 days from Space Station "Freedom" to intersection with the orbit of Mars. The propulsion system would provide the Earth-departure velocity increment, and a hypothetical aerobrake included in the payload would be used to slow the Mars-arrival velocity for aerocapture into Mars orbit. The Mars entry velocity for this 65-day personnel-transport mission would be almost 23 km/sec, well above limits of 8.5 or 9.5 km/sec set by NASA; the actual physical limits to aerobraking are not well defined and may only represent a transition from one rate of increase in aerobrake mass with entry velocity to a steeper rate.

For return to Earth, the Mars-departure velocity increment is greater than the Earth-departure velocity increment for transit times each way less than about 140 days. For a "sprint" mission (a transit time each way of 76.6 days or less) in which a nearly immediate return to Earth after a short stay time at Mars is possible, the Mars-departure velocity increment is more than double the outbound Earth-departure velocity increment. The Earth-entry velocity at return is less than the outbound Mars-entry velocity for transit times each way less than about 106 days, and is 15.4 km/sec for the 76.6-day sprint-threshold mission, to be compared with 18.8 km/sec at Mars. This 15.4-km/sec Earth entry is within the greatest limit of 16 km/sec stated by NASA, but is well above Apollo entry speeds of 11 km/sec.

The nuclear propulsion system delivers the 100,000-lb personnel-transport payload to trans-Mars injection (on a 69-day transfer) for less than one third the direct cost (ignoring hardware cost) of the chemical system, each using the propellant capacity of a single suitably modified ET. However, the chemical system injects almost two and a half times as much payload (over one million pounds) on a 250-day minimum-energy trajectory, cutting the nuclear system's cost advantage to only 30 percent less than the chemical system's. For maximum-payload missions, the here-estimated trans-Mars delivery costs are only about 1.9 (nuclear) to 2.5 (chemical) times the Earth-to-orbit delivery costs.

The results of this introductory analysis were intended to inspire further efforts to investigate, for example, aeroheating at high entry velocities, entry-velocity limitations of lifting entry, the trade-off of aerobrake weight versus propellant weight, and possible improvements and penalties in the characteristics of the nuclear and chemical propulsion systems.

REFERENCES

Code Z, 1989 — NASA Office of Exploration, *Study Requirements Document, 1989 Studies*, Doc. No. Z-2.1-002, March 3, 1989.

IDA, 1966 — Finke, R.G., et al. *Technologies and Economics of Reusable Space Launch Vehicles*, IDA Report R-114, February 1966.

IDA, 1970 — Finke, R.G. and R.C. Oliver. *Comparison of Chemical and Nuclear Propulsion for Lunar and Cislunar Missions*, IDA Paper P-687, presented at the 12th JANNAF Liquid Propulsion Meeting, October 1970.

JSC, 1990 — NASA/Johnson Space Center, *Shuttle Systems Weight and Performance, Status Report*, Doc. No. NSTS-09095-97, June 19, 1990.

Koelle, 1961 — Koelle, H.H. (editor), *Handbook of Astronautical Engineering*, p. 9-37, McGraw-Hill, 1961.

LaRC, 1991 — NASA/Langley Research Center, private communication from G.D. Walberg to R.S. Swanson, May 31, 1991.

NASA, 1989 — NASA/HQ, *Report of the Ninety-Day Study on Human Exploration of the Moon and Mars*, November 20, 1989.

Synthesis, 1991 — National Space Council/Synthesis Group, *America at the Threshold*, May 3, 1991.

Walberg, 1985 — Walberg, G.D. "A Survey of Aeroassisted Orbit Transfer," *Journal of Spacecraft and Rockets*, Jan-Feb 1985.

APPENDIX A

CALCULATION OF TIME OF ORBITAL MOTION: FUNCTION TIME(P, e, θ)

APPENDIX A
CALCULATION OF TIME OF ORBITAL MOTION: FUNCTION TIME(P, e, θ)

Problem: To determine the time t of the orbital motion through a central angle θ from the perigee, for an orbit with a period of revolution P and and eccentricity e (sometimes called the "Lambert Problem").

From Kepler's second law (the radius vector sweeps out equal areas in equal times), the time t from perigee to θ is the fraction of the period of revolution P that is the fraction of the total area of the orbit (πab for an elliptical orbit) swept out by the radius vector between perigee and θ, or

$$t = \frac{P}{\pi ab}\int_{0}^{\theta} \frac{1}{2} r^2 d\theta \quad ,$$

where, for a conic section, the radius r as a function of θ is given in terms of the semi-major axis a and the eccentricity e as

$$r = \frac{a(1-e^2)}{1 + e\cos\theta} \quad .$$

So, with the semi-minor axis $b = a\sqrt{1-e^2}$, the semi-major axis cancels out and the equation for t(θ) can be expressed in terms of the parameters P and e only, as

$$t = \frac{P(1-e^2)^{3/2}}{2\pi}\int_{0}^{\theta} \frac{d\theta}{(1 + e\cos\theta)^2} \quad .$$

With substitution of y for 1 + e cos θ, the integral becomes

$$\int_{0}^{\theta} \frac{d\theta}{(1+e\cos\theta)^2} \quad \text{---->} \int_{1+e\cos\theta}^{1+e} \frac{dy}{y^2\sqrt{(e^2-1) + 2y - y^2}} \quad .$$

This integral is in the form

$$\int \frac{dy}{y^2\sqrt{ay^2 + by + c}}$$

which is integral no. 172 (with n=2; involving in turn integral no. 169 for $c > 0$ and integral no. 170 for $c < 0$) from Burington's *Handbook of Mathematical Tables and Formulas*.

The integral therefore becomes (where the value of P is that obtained from the absolute value of a, per the main text, to pertain to either elliptical or hyperbolic orbits)

for $e < 1$ (elliptical orbits)

$$t = \frac{P}{2\pi}\left[\cos^{-1}\left(\frac{e + \cos\theta}{1 + e\cos\theta}\right) - \frac{\sqrt{1 - e^2}\ e\sin\theta}{(1 + e\cos\theta)}\right]$$

for $e > 1$ (hyperbolic orbits)

$$t = \frac{P}{2\pi}\left[\frac{\sqrt{e^2 - 1}\ e\sin\theta}{(1 + e\cos\theta)} + \ln\left(\frac{e}{\sqrt{e^2 - 1}}\right) - \ln\left\{\frac{e\sin\theta + \sqrt{e^2 - 1}}{(1 + e\cos\theta)} + \frac{1}{\sqrt{e^2 - 1}}\right\}\right]$$

The FORTRAN program TIME is appended.

ADDENDUM TO APPENDIX A

```
      FUNCTION TIME(PP, EE, THH)                            !10/1/91
      REAL*8 C, G, E, T1, P, PI, F, S, Q, TIME, ONE, HALF, THH
      REAL*8 FN, TWO
      P = PP
      E = EE
      ONE = 0.9999999999
      HALF = 0.4999999999
      TWO = 1.9999999999
      PI = 3.1415926536
      N = THH/PI + 1.
      FN = N/2
      T1 = ABS(TWO * FN * PI - THH)
      C = DCOS(T1)
      F = DSIN(T1) * DSQRT(DABS(ONE - E*E))/(ONE + E * C)
      IF(E .GT. ONE) THEN
        Q = (E * DSIN(T1) + DSQRT(E*E - ONE))/(ONE + E * C)
        S = ONE/DSQRT(E*E - ONE)
        TIME = HALF * P/PI * (E*F + DLOG(DABS(E*S)) - DLOG(DABS(Q+S)))
      ELSE
        G = (E + C)/(ONE + E * C)
        IF(DABS(G) .GT. ONE) G = DSIGN(ONE, G)
        TIME = HALF * P/PI * (DACOS(G) - E * F)
      END IF
      IF(TIME .LT. 0.) TIME = TIME + HALF * P
      TIME = (FN * P - (-1.)**N * TIME) * DSIGN(ONE, THH)
      RETURN
      END
```