BAB II KEGIATAN PEMBELAJARAN

A. Limit Fungsi Aljabar

A.1. Teorema Limit Fungsi Aljabat Pada Titik Tertentu

Pada penyelesaian limit fungsi harus menghidari nilai-nilai tak tentu, diantaranya adalah $\frac{0}{0}, \ \frac{\infty}{\infty}, \ \frac{a}{0}, \ \infty, \ -\infty$

Berikut beberapa teorema penyelesaian limit fungsi aljabar

Teorema 1

Limit Fungsi Konstan

Jika $f(x)$ adalah fungsi konstan dan $a \in R$, maka berlaku :

$$\lim_{x \to a} f(x) = f(x)$$

Contoh Soal :

1. $\lim_{x \to 2} 5 = 5$
2. $\lim_{x \to 5} b = b$
3. $\lim_{y \to 2} (3x - 2) = 3x - 2$

Teorema 2

Subtitusi Langsung

Jika $f(x)$ adalah fungsi aljabar dan bukan fungsi konstan, $a \in R$ dan $f(a) \in R$ maka :

$$\lim_{x \to a} f(x) = f(a)$$

Contoh Soal :

1. $\lim_{x \to 1} (x^2 + 2x - 1) = ...$

   Penyelesaian

   $$\lim_{x \to 1} (x^2 + 2x - 1) = 1^2 + 2(1) - 1$$
   $$= 2$$

2. $\lim_{x \to 5} (3x^2 - 4x) = ...$

   Penyelesaian

   $$\lim_{x \to 5} (3x^2 - 4x) = 3(5)^2 - 4(5)$$
   $$= 75 - 20$$
   $$= 55$$

3. $\lim_{x \to 3} \frac{\sqrt{3x^2 + 9}}{3} = ...$

Penyelesaian

$$\lim_{x\to 3}\frac{\sqrt{3x^2+9}}{3}=\frac{\sqrt{3(3)^2+9}}{3}$$
$$=\frac{\sqrt{27+9}}{3}$$
$$=\frac{\sqrt{36}}{3}$$
$$=2$$

4. $\lim_{p\to -2}\left(-5p^3-6p^2\right)^{-\frac{1}{2}}=..........$

Penyelesaian

$$\lim_{p\to -2}\left(-5p^3-6p^2\right)^{-\frac{1}{2}}=\lim_{p\to -2}\frac{1}{\sqrt{-5p^3-6p^2}}$$
$$=\frac{1}{\sqrt{-5(-2)^3-6(-2)^2}}$$
$$=\frac{1}{\sqrt{40-24}}$$
$$=\frac{1}{\sqrt{16}}$$
$$=\frac{1}{4}$$

Jika pada hasil subtitusi langsung menghasilkan nilai-nilai tak tentu $\left(\frac{0}{0},\quad \frac{\infty}{\infty},\quad \frac{a}{0},\quad \infty,\quad -\infty\right)$ terutama pada bentuk $\lim_{x\to a}\frac{f(x)}{g(x)}$.

Teorema 3

Faktorisasi

Jika $(x-a)p(x)$ adalah faktor dari $f(x)$ dan $(x-a)q(x)$ adalah faktor dari g(x), $a \in R$ dan $p(a).q(a) \neq 0$ maka :

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{\cancel{(x-a)}p(x)}{\cancel{(x-a)}q(x)} = \frac{p(a)}{q(a)}$$

Contoh Soal :

1. $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - x - 2} = \ldots\ldots\ldots\ldots$

   Penyelesaian

   $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - x - 2} = \frac{2^2 - 3(2) + 2}{2^2 - 2 - 2} \Rightarrow \frac{0}{0}$. Ternyata jika kita subtitusikan langsung menghasilan nilai tak tentu maka kita gunakan teorema 3. maka :

$$\begin{aligned}
\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - x - 2} &= \lim_{x \to 2} \frac{(x-2)(x-1)}{(x-2)(x+1)} \\
&= \lim_{x \to 2} \frac{(x-1)}{(x+1)} \\
&= \frac{(2-1)}{(2+1)} \\
&= \frac{1}{3}
\end{aligned}$$

2 $\lim_{x\to 4}\frac{x^2-16}{x-4}=\ldots\ldots\ldots.$

Penyelesaian

$$\begin{aligned}\lim_{x\to 4}\frac{x^2-16}{x-4}&=\lim_{x\to 4}\frac{(x-4)(x+4)}{x-4}\\&=\lim_{x\to 4}(x+4)\\&=8\end{aligned}$$

3. $\lim_{x\to 1}\frac{x^4-4x+3}{x^2-1}=\ldots\ldots\ldots.$

Penyelesaian

$$\begin{aligned}\lim_{x\to 1}\frac{x^4-4x+3}{x^2-1}&=\lim_{x\to 1}\frac{(x-1)(x^3+x^2+x-3)}{(x-1)(x+1)}\\&=\lim_{x\to 1}\frac{(x^3+x^2+x-3)}{(x+1)}\\&=\frac{(1^3+1^2+1-3)}{(x+1)}\\&=\frac{0}{2}\end{aligned}$$

4. $\lim_{x\to 2}\left(\frac{6-x}{x^2-4}-\frac{1}{x-2}\right)=\ldots\ldots\ldots.$

Penyelesaian

$$\begin{aligned}\lim_{x\to 2}\left(\frac{6-x}{x^2-4}-\frac{1}{x-2}\right)&=\lim_{x\to 2}\left(\frac{6-x}{x^2-4}-\frac{1(x+2)}{(x-2)(x+2)}\right)\\&=\lim_{x\to 2}\left(\frac{4-2x}{x^2-4}\right)\\&=\lim_{x\to 2}\left(\frac{-2(x-2)}{(x+2)(x-2)}\right)\\&=\frac{-2}{(2+2)}\\&=-\frac{1}{2}\end{aligned}$$

Teorema 4

Perkalian Bentuk Sekawan

Jika $f(x)$ atau $g(x)$ salah satunya atau keduanya merupakan fungsi dalam bentuk akar dan $a \in R$ maka:

a. Jika $f(x) = a - \sqrt{x-c}$ maka

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{a-\sqrt{x-c}}{g(x)} \cdot \frac{a+\sqrt{x-c}}{a+\sqrt{x-c}}$$

b. Jika $g(x) = a - \sqrt{x-c}$ maka :

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f(x)}{a-\sqrt{x-c}} \cdot \frac{a+\sqrt{x-c}}{a+\sqrt{x-c}}$$

c. Jika $f(x) = a - \sqrt{x-c}$ dan $g(x) = b - \sqrt{x-d}$ maka :

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{a-\sqrt{x-c}}{b-\sqrt{x-d}} \cdot \frac{a+\sqrt{x-c}}{a+\sqrt{x-c}} \cdot \frac{b+\sqrt{x-d}}{b+\sqrt{x-d}}$$

Contoh Soal :

1. $\lim_{x\to 0} \frac{x}{2-\sqrt{4-x}} = \ldots\ldots\ldots\ldots$

Penyelesaian

$$\begin{aligned}
\lim_{x\to 0} \frac{x}{2-\sqrt{4-x}} &= \lim_{x\to 0} \frac{x}{2-\sqrt{4-x}} \cdot \left(\frac{2+\sqrt{4-x}}{2+\sqrt{4-x}}\right) \\
&= \lim_{x\to 0} \frac{x\left(2+\sqrt{4-x}\right)}{4-(4-x)} \\
&= \lim_{x\to 0} \frac{x\left(2+\sqrt{4-x}\right)}{x} \\
&= \lim_{x\to 0} \left(2+\sqrt{4-x}\right) \\
&= 4
\end{aligned}$$

2. $\lim\limits_{x \to -1} \dfrac{x^2 - 1}{\sqrt{8-x} - 3} = \ldots\ldots\ldots.$

<u>*Penyelesaian*</u>

$$\lim_{x \to -1} \frac{x^2 - 1}{\sqrt{8-x} - 3} = \lim_{x \to -1} \frac{x^2 - 1}{\sqrt{8-x} - 3} . \left(\frac{\sqrt{8-x} + 3}{\sqrt{8-x} + 3} \right)$$

$$= \lim_{x \to -1} \frac{(x^2 - 1)(\sqrt{8-x} + 3)}{8 - x - 9}$$

$$= \lim_{x \to -1} \frac{(x-1)(x+1)(\sqrt{8-x} + 3)}{-(x+1)}$$

$$= \lim_{x \to -1} -(x-1)(\sqrt{8-x} + 3)$$

$$= -(-2)(6)$$

$$= 12$$

3. $\lim\limits_{x \to 4} \dfrac{\sqrt{x+5} - \sqrt{2x+1}}{x - 4} = \ldots\ldots\ldots.$

<u>*Penyelesaian*</u>

$$\lim_{x \to 4} \frac{\sqrt{x+5} - \sqrt{2x+1}}{x - 4} = \lim_{x \to 4} \frac{\sqrt{x+5} - \sqrt{2x+1}}{x - 4} . \left(\frac{\sqrt{x+5} + \sqrt{2x+1}}{\sqrt{x+5} + \sqrt{2x+1}} \right)$$

$$= \lim_{x \to 4} \frac{(x+5) - (2x+1)}{(x-4)(\sqrt{x+5} + \sqrt{2x+1})}$$

$$= \lim_{x \to 4} \frac{-(x-4)}{(x-4)(\sqrt{x+5} + \sqrt{2x+1})}$$

$$= \lim_{x \to 4} \frac{-1}{(\sqrt{x+5} + \sqrt{2x+1})}$$

$$= \frac{-1}{6}$$

4. $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{\sqrt{x-4}} = ...$

Penyelesaian

$$\lim_{x \to 4} \frac{x^2 - 16}{\sqrt{x-4}} = \lim_{x \to 4} \frac{x^2 - 16}{\sqrt{x-4}} \left(\frac{\sqrt{x-4}}{\sqrt{x-4}} \right)$$
$$= \lim_{x \to 4} \frac{(x^2 - 16)(\sqrt{x-4})}{x-4}$$
$$= \lim_{x \to 4} \frac{(x-4)(x+4)(\sqrt{x-4})}{x-4}$$
$$= \lim_{x \to 4} (x+4)(\sqrt{x-4})$$
$$= 8.0$$
$$= 0$$

5. $\lim\limits_{x \to p} \dfrac{x\sqrt{x} - p\sqrt{p}}{\sqrt{x} - \sqrt{p}} = ...$

Penyelesaian

$$\lim_{x \to p} \frac{x\sqrt{x} - p\sqrt{p}}{\sqrt{x} - \sqrt{p}} = \lim_{x \to p} \frac{x\sqrt{x} - p\sqrt{p}}{\sqrt{x} - \sqrt{p}} \left(\frac{x\sqrt{x} + p\sqrt{p}}{x\sqrt{x} + p\sqrt{p}} \right) \left(\frac{\sqrt{x} + \sqrt{p}}{\sqrt{x} + \sqrt{p}} \right)$$
$$= \lim_{x \to p} \left(\frac{x\sqrt{x} - p\sqrt{p}}{\sqrt{x} - \sqrt{p}} \right) \left(\frac{x\sqrt{x} + p\sqrt{p}}{\sqrt{x} + \sqrt{p}} \right) \left(\frac{x\sqrt{x} + p\sqrt{p}}{\sqrt{x} + \sqrt{p}} \right)$$
$$= \lim_{x \to p} \left(\frac{x^3 - p^3}{x - p} \right) \left(\frac{x\sqrt{x} + p\sqrt{p}}{\sqrt{x} + \sqrt{p}} \right)$$
$$= \lim_{x \to p} \frac{(x-p)(x^2 + xp + p^2)}{x - p} \left(\frac{x\sqrt{x} + p\sqrt{p}}{\sqrt{x} + \sqrt{p}} \right)$$
$$= \lim_{x \to p} (x^2 + xp + p^2) \left(\frac{x\sqrt{x} + p\sqrt{p}}{\sqrt{x} + \sqrt{p}} \right)$$
$$= \frac{6p^2\sqrt{p}}{2p\sqrt{p}}$$
$$= 3p$$

A.1. Teorema Limit Fungsi Aljabat Tak Hingga

Pada prinsip penyelesaian limit tak hingga sama seperti pada penyelesaian limit pada titik tertentu, yaitu harus menghidari nilai-nilai tak tentu $\left(\frac{0}{0}, \frac{\infty}{\infty}, \frac{a}{0}, \infty, -\infty,\right)$.

.

Teorema 5

Membagi dengan variabel tertinggi

Jika *f*(*x*) dan g(*x*) adalah merupakan fungsi aljabar maka nilai dari $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)}$ adalah dengan cara membagi semua unsur/suku dengan variabel dari *f*(*x*) atau pun g(*x*) yang merupakan pangkat tertinggi.

$$\lim_{x\to\infty} \frac{ax^n - bx^m}{dx^n - r} = \lim_{x\to\infty} \frac{\frac{ax^n}{x^n} - \frac{bx^m}{x^n}}{\frac{dx^n}{x^n} - \frac{r}{x^n}} = \frac{a}{d}, \Rightarrow$$

Jika n merupakan pangkat tertinggi

Contoh Soal :

1. $\lim\limits_{x\to\infty} \frac{2x^2 - x + 5}{x^2 - 3x + 2} = ...$

Penyelesaian

$$\lim_{x\to\infty} \frac{2x^2 - x + 5}{x^2 - 3x + 2} = .\lim_{x\to\infty} \frac{\frac{2x^2}{x^2} - \frac{x}{x^2} + \frac{5}{x^2}}{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2}}$$

$$= \lim_{x\to\infty} \frac{2 - \frac{1}{x} + \frac{5}{x^2}}{1 - \frac{3}{x} + \frac{2}{x^2}}$$

$$= \frac{2 - \frac{1}{\infty} + \frac{5}{\infty^2}}{1 - \frac{3}{\infty} + \frac{2}{\infty^2}}$$

$$= \frac{2 - 0 + 0}{1 - 0 + 0}$$

$$= 2$$

2. $\lim_{x \to \infty} \left(\frac{3x}{x-1} - \frac{2x}{x+1} \right) = ...$

<u>Penyelesaian</u>

$$\lim_{x \to \infty} \left(\frac{3x}{x-1} - \frac{2x}{x+1} \right) = \lim_{x \to \infty} \left(\left(\frac{3x}{x-1} \right)\left(\frac{x+1}{x+1} \right) - \left(\frac{2x}{x+1} \right)\left(\frac{x-1}{x-1} \right) \right)$$

$$= \lim_{x \to \infty} \left(\frac{x^2 + 5x}{x^2 - 1} \right)$$

$$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2} + \frac{5x}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$$

$$= 1$$

3. $\lim_{x \to \infty} \frac{(3x-2)^3}{(4x+3)^3} = ...$

<u>Penyelesaian</u>

$$\lim_{x \to \infty} \frac{(3x-2)^3}{(4x+3)^3} = \lim_{x \to \infty} \frac{(3x-2)(9x^2 - 12x + 4)}{(4x+3)(16x^2 + 24x + 9)}$$

$$= \lim_{x \to \infty} \frac{27x^3 - 54x^2 + 36x - 8}{64x^3 + 144x^2 + 108x + 27}$$

$$= \frac{27}{64}$$

4. $\lim_{x\to\infty} \frac{x}{\sqrt{x^2-2x-1}} = ...$

Penyelesaian

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2-2x-1}} = \lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2-2x-1}}{x}}$$

$$= \lim_{x\to\infty} \frac{\frac{x}{x}}{\sqrt{\frac{x^2-2x-1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{1-0-0}}$$

$$= 1$$

5. $\lim_{x\to\infty} \frac{5x-3x^2+6}{3x^3-8} = ...$

Penyelesaian

$$\lim_{x\to\infty} \frac{5x-3x^2+6}{3x^3-8} = \lim_{x\to\infty} \frac{\frac{5x}{x^3}-\frac{3x^2}{x^3}+\frac{6}{x^3}}{\frac{3x^3}{x^3}-\frac{8}{x^3}}$$

$$= \frac{0-0+0}{3-0}$$

$$= 0$$

Teorema 6

Limit tak hingga dengan perkalian sekawan

Jika $f(x)$ dalam bentuk akar maka nilai dari $\lim_{x\to\infty} f(x)$ adalah dengan cara mengalikan dengan bentuk sekawan dari $f(x)$.

$$\lim_{x\to\infty} \sqrt{ax^n - b} - \sqrt{cx^n - d} = \lim_{x\to\infty}\left(\sqrt{ax^n - b} - \sqrt{cx^n - d}\right)\times\frac{\left(\sqrt{ax^n - b} + \sqrt{cx^n - d}\right)}{\left(\sqrt{ax^n - b} + \sqrt{cx^n - d}\right)}$$

Contoh Soal :

1. $\lim_{x\to\infty} \sqrt{x^2 - 1} + \sqrt{1 - x^2} = ...$

Penyelesaian

$$\lim_{x\to\infty} \sqrt{x^2 - 1} + \sqrt{1 - x^2} = \lim_{x\to\infty}\left(\sqrt{x^2 - 1} + \sqrt{1 - x^2}\right)\times\frac{\sqrt{x^2 - 1} - \sqrt{1 - x^2}}{\sqrt{x^2 - 1} - \sqrt{1 - x^2}}$$

$$= \lim_{x\to\infty}\frac{\left(x^2 - 1\right) - \left(1 - x^2\right)}{\sqrt{x^2 - 1} - \sqrt{1 - x^2}}$$

$$= \lim_{x\to\infty}\frac{2x^2 - 2}{\sqrt{x^2 - 1} - \sqrt{1 - x^2}}$$

$$= \frac{2}{0}$$

2. $\lim_{x\to\infty} \sqrt{x^2+2x} - \sqrt{x^2-4x} = ...$

Penyelesaian

$$\lim_{x\to\infty} \sqrt{x^2+2x} - \sqrt{x^2-4x} = \lim_{x\to\infty} \left(\sqrt{x^2+2x} - \sqrt{x^2-4x}\right) . \left(\frac{\sqrt{x^2+2x} + \sqrt{x^2-4x}}{\sqrt{x^2+2x} + \sqrt{x^2-4x}}\right)$$

$$= \lim_{x\to\infty} \frac{\left(x^2+2x\right) - \left(x^2-4x\right)}{\sqrt{x^2+2x} + \sqrt{x^2-4x}}$$

$$= \lim_{x\to\infty} \frac{6x}{\sqrt{x^2+2x} + \sqrt{x^2-4x}}$$

$$= \lim_{x\to\infty} \frac{\frac{6x}{x}}{\sqrt{\frac{x^2+2x}{x^2}} + \sqrt{\frac{x^2-4x}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{6}{\sqrt{1}+\sqrt{1}}$$

$$= 3$$

3. $\lim_{x\to\infty} \sqrt{x^2+6x} - (x-4) = ...$

Penyelesaian

$$\lim_{x\to\infty} \sqrt{x^2+6x} - (x-4) = \lim_{x\to\infty} \left(\sqrt{x^2+6x} - (x-4)\right) \left(\frac{\sqrt{x^2+6x} + (x-4)}{\sqrt{x^2+6x} + (x-4)}\right)$$

$$= \lim_{x\to\infty} \frac{\left(x^2+6x\right) - \left(x^2-8x+16\right)}{\sqrt{x^2+6x} + (x-4)}$$

$$= \lim_{x\to\infty} \frac{14x-16}{\sqrt{x^2+6x} + (x-4)}$$

$$= \lim_{x\to\infty} \frac{\frac{14x}{x} - \frac{16}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{6x}{x^2}} + \frac{x}{x} - \frac{4}{x}}$$

$$= \frac{14}{2}$$

$$= 7$$

Teorema 7

Limit tak hingga dengan cara quantum

Jika *f*(*x*) adalah fungsi dalam bentuk $f(x) = \sqrt{ax^2 + bx + c} - \sqrt{px^2 + qx + r}$ dengan:

- $a = p$
- Pangkat tertinggi dari variabel kedua suku adalah 2
- Operasi pengurangan

maka berlaku :

$$\lim_{x \to \infty} \sqrt{ax^2 + bx + c} - \sqrt{px^2 + qx + r} = \frac{b - q}{2\sqrt{a}}$$

Contoh Soal :

1. $\lim_{x \to \infty} \sqrt{2x^2 + 2x + 2} - \sqrt{2x^2 - 4x + 5} = \ldots\ldots\ldots\ldots$

   Penyelesaian

$$\lim_{x \to \infty} \sqrt{2x^2 + 2x + 2} - \sqrt{2x^2 - 4x + 5} = \frac{b - q}{2\sqrt{a}}$$

$$= \frac{2 - (-4)}{2\sqrt{2}}$$

$$= \frac{6}{2\sqrt{2}}$$

$$= \frac{6}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3}{2}\sqrt{2}$$

2. $\lim_{x \to \infty} x\left(\sqrt{25 - \frac{10}{x}} - \sqrt{25 + \frac{10}{x}}\right) = \ldots\ldots\ldots\ldots$

   Penyelesaian

$$\lim_{x\to\infty} x\left(\sqrt{25-\frac{10}{x}}-\sqrt{25+\frac{10}{x}}\right)=\lim_{x\to\infty}\left(x\sqrt{25-\frac{10}{x}}-x\sqrt{25+\frac{10}{x}}\right)$$

$$=\lim_{x\to\infty}\left(\sqrt{x^2\left(25-\frac{10}{x}\right)}-\sqrt{x^2\left(25+\frac{10}{x}\right)}\right)$$

$$=\lim_{x\to\infty}\left(\sqrt{25x^2-10x}-\sqrt{25x^2+10x}\right)$$

$$=\frac{b-q}{2\sqrt{a}}$$

$$=\frac{-10-10}{2\sqrt{25}}$$

$$=\frac{-20}{10}$$

$$=-2$$

3. $\lim_{x\to\infty}\sqrt{x^2-5x}-(x-2)=..........$

Penyelesaian

$$\lim_{x\to\infty}\sqrt{x^2-5x}-(x-2)=\lim_{x\to\infty}\sqrt{x^2-5x}-\sqrt{(x-2)^2}$$

$$=\lim_{x\to\infty}\sqrt{x^2-5x}-\sqrt{x^2-4x+4}$$

$$=\frac{b-q}{2\sqrt{a}}$$

$$=\frac{-5-(-4)}{2\sqrt{1}}$$

$$=\frac{-1}{2}$$

B. Limit fungsi Trigonometri

Pada prinsipnya penyelesaian limit fungsi trigonometri sama dengan penyelesaian fungsi aljabar, yakni menghindari nilai-nilai tak tentu.

Teorema 8

Teorema dasar limit fungsi trigonometri

- $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1 \quad \Rightarrow \quad \lim\limits_{x \to 0} \frac{x}{\sin x} = 1$
- $\lim\limits_{x \to 0} \frac{\sin ax}{ax} = 1 \quad \Rightarrow \quad \lim\limits_{x \to 0} \frac{ax}{\sin ax} = 1$
- $\lim\limits_{x \to 0} \frac{\tan x}{x} = 1 \quad \Rightarrow \quad \lim\limits_{x \to 0} \frac{x}{\tan x} = 1$
- $\lim\limits_{x \to 0} \frac{\tan ax}{ax} = 1 \quad \Rightarrow \quad \lim\limits_{x \to 0} \frac{ax}{\tan ax} = 1$

Contoh Soal

1. $\lim\limits_{x \to 0} \frac{\sin 3x}{\tan x} = \dots\dots\dots\dots$

Penyelesaian

$$\begin{aligned}\lim_{x \to 0} \frac{\sin 3x}{\tan x} &= \lim_{x \to 0} \frac{\sin 3x}{\tan x} . \frac{3x}{3x} . \frac{x}{x} \\ &= \lim_{x \to 0} \frac{\sin 3x}{3x} . \frac{x}{\tan x} . \frac{3x}{x} \\ &= 1.1.3 \\ &= 3\end{aligned}$$

2. $\lim\limits_{x \to 0} \frac{\tan x}{x^2 + 2x} = \dots\dots\dots\dots$

Penyelesaian

$$\lim_{x\to 0}\frac{\tan x}{x^2+2x}=\lim_{x\to 0}\frac{\tan x}{\left(x^2+2x\right)}\cdot\frac{x}{x}$$

$$=\lim_{x\to 0}\frac{\tan x}{x}\cdot\frac{x}{x(x+2)}$$

$$=\lim_{x\to 0}\frac{1}{(x+2)}$$

$$=\frac{1}{2}$$

3. $\lim_{x\to 0}\frac{x\left(\cos^2 6x-1\right)}{\sin 3x.\tan^2 2x}=........$

Penyelesaian

$$\lim_{x\to 0}\frac{x\left(\cos^2 6x-1\right)}{\sin 3x.\tan^2 2x}=\lim_{x\to 0}\frac{x\left(\left(1-\sin^2 6x\right)-1\right)}{\sin 3x.\tan^2 2x}$$

$$=\lim_{x\to 0}\frac{x\left(-\sin^2 6x\right)}{\sin 3x.\tan^2 2x}$$

$$=\lim_{x\to 0}\frac{-x\sin^2 6x}{\sin 3x.\tan^2 2x}$$

$$=\lim_{x\to 0}\frac{-x\sin^2 6x}{\sin 3x.\tan^2 2x}\cdot\frac{3x}{3x}\cdot\frac{(6x)^2}{(6x)^2}\cdot\frac{(2x)^2}{(2x)^2}$$

$$=\lim_{x\to 0}\frac{3x}{\sin 3x}\cdot\frac{\sin^2 6x}{(6x)^2}\cdot\frac{(2x)^2}{\tan^2 2x}\cdot\frac{-x(6x)^2}{3x(2x)^2}$$

$$=\lim_{x\to 0}1.1.1\frac{-36x^3}{12x^3}$$

$$=-3$$

Teorema 9

Dalil L' Hopital

Jika $f'(x)$ adalah merupakan turunan dari $f(x)$ dan $g'(x)$ adalah turunan dari $g(x)$ maka berlaku :

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)} = \frac{f''(x)}{g''(x)} = \frac{f'''(x)}{g'''(x)}$$

Contoh Soal :

1. $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2} = ...$

Penyelesaian

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2} = \lim_{x \to 2} 2x - 5$$

$$= -1$$

2. $\lim_{x \to 3} \frac{x - \sqrt{2x + 3}}{x^2 - 9} = ...$

Penyelesian

$$\lim_{x \to 3} \frac{x - \sqrt{2x+3}}{x^2 - 9} = \lim_{x \to 3} \frac{x - (2x+3)^{\frac{1}{2}}}{x^2 - 9}$$

$$= \lim_{x \to 3} \frac{1 - \frac{1}{\sqrt{2x+3}}}{2x}$$

$$= \frac{\frac{2}{3}}{6}$$

$$= \frac{2}{3} \cdot \frac{1}{6}$$

$$= \frac{1}{9}$$