SECTION 5D

Adaptations: Teaching a Mixed-Level Math Class

Introduction

Anyone who has taught any level of an adult education math class understands that to do so is to deal with a wide range of student abilities. This range can manifest itself both as a difference between students and also a difference between a given student's understanding of different topics in math. In addition to differences in content knowledge, adult learners in math classes can also come to us on either end of the spectrum (and everywhere in between) when it comes to their mindset, self-concept, comfort-level and their beliefs about mathematics.

Dealing with this reality is part of an adult education math teachers work. One strategy we might try is to bring extra work/handouts for the students who finish before everyone else. But if we do that, we find ourselves having to go over two handouts, one of which most of the class hasn't seen. This strategy also conveys to students that, in math, being fast is better than being slow, which reinforces damaging pre-conceived notions students have about what math is and what kind of a math person they are. It implies that doing your work quickly leads to reward, doing it slowly (read, at your own pace) leads to punishment and that math is about moving quickly to an answer.

Another strategy for teaching a mixed-level group is to give harder problems to your stronger students. Similar to giving them extra sheets, there are often unintended side effects to this. For one, it conveys to everyone in the class who you think are the "good math students" and who you think is not. It also can be very difficult to predict which content areas your students actually understand—you might have a student who is great with percents and fractions, who doesn't know anything about the coordinate plane. It also leaves you with multiple math problems to discuss with a class where no one has seen both problems.

We offer some other strategies that we have found effective in our own mixed-level classroom settings. Our challenge is to take the reality of mixed-level classes, and try to change it into a strength of the class. To do so requires two things: (1) Students involved in the same core activity and (2) making sure there is a valued role in the process for each student.

Teachers might define a good math student as having qualities like working hard, perseverence/doesn't give up, asking questions, learning from mistakes etc. Almost every adult math learner I've ever asked to define what makes a good math students says things like "always gets the answer right," "knows the rules," and "calculates quickly." Think of the implications of that—students define a good math student as being the one who already knows all the answers.

Choosing the Right Problem & Providing Different Levels of Support

This strategy is connected to asking support questions at the lower-level and providing extension problems for the more advanced students. We may decide that some students are more comfortable struggling, and go on longer before we ask support questions.

If we hand out a sheet with 25 problems that can all be answered by the same procedure, we'll often find ourselves in the situation where some students who are able to do the procedure and calculations quickly and will finish before we've even finished handing out the sheet. Now, we're being pulled in two directions, with the mixed-level characteristics of our class potentially creating tension.

A more effective option is to have students working on fewer problems, but richer problems. Ideally, we want our students to work on problems that can be solved using a wide range of problem-solving strategies. We also want to choose problems that can be described as having a "low entry and high ceiling." That means a problem that every student in class can access and begin to work on, but which can be extended for students who are ready for a further challenge. With problems that have this characteristic, we can create both support and extension questions. You will find examples of both in these in every lesson and teacher support component in the math section.

Here's a quick example of a low-entry, high-ceiling problem:

The sum of two numbers is 91 and the difference of the same two numbers is 85.

Can you figure out what the two numbers are?

Some students will require some support gaining "entry" into the problem. We want to ask them questions, as opposed to explaining, and we should only ask enough questions to get them moving. Try to wait at least 10 minutes, and give students some time to struggle through on their own before offering support through questioning.

SAMPLE SUPPORT QUESTIONS:

- *What are we looking for?*
- *What do we know about these two numbers?*
- *What does "sum" mean?*

- *What does "difference" mean?*
- *What could those two numbers be? How could you test those numbers?*
- *How do you know if your two numbers are correct or not?*

Some students will be able to get to work on the problem right away. They might finish before you are ready to stop the class from working. In this situation, we can ask students an extension question. Below are sample extension questions created by (1) changing the numbers, (2) adding a condition, (3) having students create their own problems and (4) encouraging students to look for patterns/structure.

- *The sum of two numbers is 33 and the difference of the same two numbers is 11. Can you figure out what the two numbers are?*
- *The sum of two numbers is 91 and the difference of the same two numbers is 1. Can you figure out what the two numbers are?*
- *The sum of two numbers is 24 and the difference of the same two numbers is 14. Can you figure out what the two numbers are?*
- *Can you write a similar problem with different numbers, so that I have to guess the two numbers?*
- *What if one of the numbers is negative? Is the sum of the two numbers always larger than the difference of the same two numbers?*
- *Can you create your own example where the difference is larger than the sum, and the sum is not zero?*
- *Can you describe a way that would allow you to find any two numbers if you are given the sum and difference of those two numbers?*

Then once it is time for the class to debrief on their work, students who completed the given problem (only) can share their strategies and methods. We can also decide if we want to have some of the students share their work on the extension questions—and we can include everyone in the discussion. For example, if a group is discussing the extension about a way to find any two numbers, given their sum and difference, we can ask the rest of the class processing questions—

- *Can we create a visual representation of that method?*
- *Can we test that procedure with different numbers?*
- *Why does that method work? / Will that method always work?*

Having Students Choose (and Write) the Right Problem (for Them)

Another strategy is to give students a choice of problems to work on. One of my favorite ways to do this is to not give students a problem at all. Instead, give them a mathematical situation. For example, you could give students the following:

> *Kirk and Cara (both adults) take their two kids and Kirk's niece (also a child) to the Newman Movie Theater. At the Newman, children's tickets are half the price of the adult tickets. Kirk and Cara paid $38.50 for all the tickets.*

Give students a few minutes to understand the situation. You might ask them to try to draw a visual representation of the situation. Then talk about the situation so that it makes sense to everyone. Then ask them to write a few questions that can be answered with the information given in the situation. You can say it doesn't need to be a question they necessarily need to know how to answer. Then the class shares out all their questions, either as a whole, in small groups or in pairs. Then you can have students choose which question they want to work on. If they finish early, they can choose another question related to the same problem. When you have students present, you might have them talk about why they chose the question they did, in addition to talking about their work and process.

As a heads-up, the first time I do this kind of activity with a class, there are often several students who get right to work on the "problem," even though there isn't one. I usually let them go for a bit, before asking them to share what question they are trying to solve, and pointing out that I didn't actually ask a question. Because students usually be working on a few different questions, it can be a nice opportunity to have every student reflect on the need to understand both the situation and the question posed in the problem before we start working.

Using Different Numbers

You can also use different numbers to keep students working on the same math content and problem. This can often draw out a wider range of problem-solving strategies. For example, consider these two versions of a problem:

Version 1: A bike shop has a total inventory of 36, some bicycles and some tricycles. Altogether, the bicycles and tricycles have a total of 80 wheels. How many of each type of bike are in the shop?"

Version 2: A bike shop sells bicycles and tricycles. The shop has 8 vehicles, some bicycles and some tricycles. All together, there are 18 wheels. How many bicycles and how many tricycles are there?

The Semester-Long Problem

Another strategy is to give your class a really challenging problem at the beginning of the semester—one that will take weeks to answer. You can present this problem to students as an independent project that they can present at the end of the semester. Of course, it doesn't have to be a semester—you'll want to pick a timeframe that makes sense within the structure of your program. When students complete the work of the day, they can take out their semester-long problem and continue working on it. You might even consider allowing them to ask you one question. You definitely won't get every student interested in this, which is fine, but it is a really nice example of math being a thoughtful, project based activity that can happen over a longer period of time. There are several resources where you might find problems of this kind. To get started, two websites are: nrich.maths.org and the Problem-of-the-Month offerings from insidemathematics.org. One example from *Inside Mathematics* is "Calculating Palindromes".

A Word on Groups

How we pair students in our math classes is an important question. The benefits of putting strong students together with slower students or keeping students of similar abilities together depends on the activity. It should not be an either/or situation. We want to think about each activity and figure out what makes the most sense for achieving the goals of the activity. A high/low level pairing can work really well for certain problems—especially if we want teaching to happen between students. It can also help students appreciate a wide range of problem-solving strategies—I've had faster students try to set up equations and not be able to, and then watch in awe as a member of their group comes up with an elegant strategy using a visual representation. For some kinds of activities, however, it can be frustrating for both students, especially when they are beginning to formulate an approach to a non-routine problem. We might want students to work from their own understanding and not be pulled into a faster student's orbit. The weaker student often loses out in the activity. We want to pair students so that they can support each other, but we also need to make sure both students have the space to grapple with the problem in their own way. Research suggests "tracking" students and always breaking them

into homogenous groups based upon perceived level can have negative effects, but every once in a while, for particular types of activities, it can be rewarding. One thing that can be helpful is to add a question about the group work to the problem debrief. After that math talk is done, ask students to discuss how they felt working in a group on that particular activity.

A Word on Pre-HSE and this Framework

The lesson plans and teacher supports were written for HSE students. Since every HSE class is a mixed-level group, you will find the strategies mentioned above throughout. For teachers working with basic education and low-level pre-HSE students, we suggest spending more time on the recommended warm-ups and routines, and Units 1, 2, 7, and 8. Many of the core problems and supplemental problems in Units 2, 3, 4, 5, and 6 are appropriate, but you should use them more as rich, low entry/high ceiling problems that are great for building students problem-solving and perseverance. You may not be able to develop the abstract and advanced function skills and concepts.