Adding Fractions

Adapted from MathisFun.com

There are 3 Simple Steps to add fractions:

- Step 1: Make sure the bottom numbers (the denominators) are the same
- Step 2: Add the top numbers (the numerators). Put the answer over the denominator.
- Step 3: Simplify the fraction (if needed).

Example 1:

$$\frac{1}{4} + \frac{1}{4}$$

Step 1. The bottom numbers (the denominators) are already the same. Go straight to step 2.

Step 2. Add the top numbers and put the answer over the same denominator:

$$\frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4}$$

Step 3. Simplify the fraction:

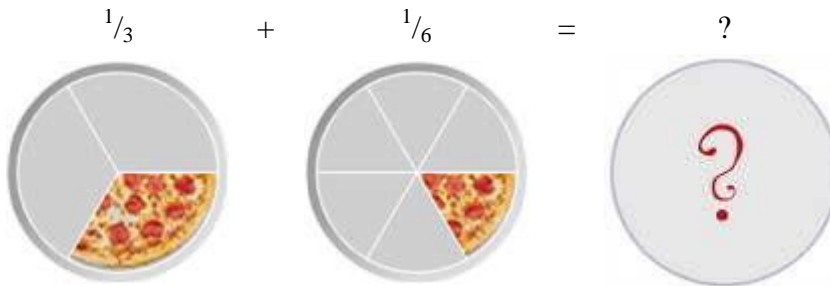$$\frac{2}{4} = \frac{1}{2}$$

In picture form it looks like this:

$$\frac{1}{4} \quad + \quad \frac{1}{4} \quad = \quad \frac{2}{4} \quad = \quad \frac{1}{2}$$

(If you are unsure of the last step see Equivalent Fractions.)

Example 2:

$$\frac{1}{3} + \frac{1}{6}$$

Step 1: The bottom numbers are different. See how the slices are different sizes?

$$^1/_3 \quad + \quad {}^1/_6 \quad = \quad ?$$

We need to make them the same before we can continue, because we **can't** add them like that.

The number "6" is twice as big as "3", so to make the bottom numbers the same we can multiply the top and bottom of the first fraction by **2**, like this:

$$\times 2$$
$$\frac{1}{3} = \frac{2}{6}$$
$$\times 2$$

Important: you multiply **both top and bottom** by the same amount, to keep the value of the fraction the same

Now the fractions have the same bottom number ("6"), and our question looks like this:
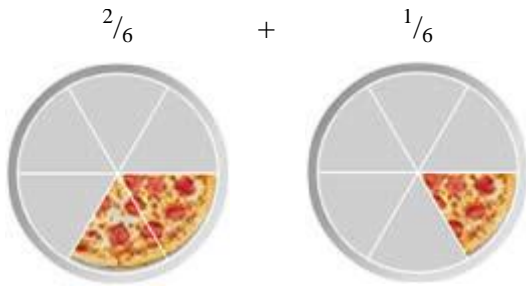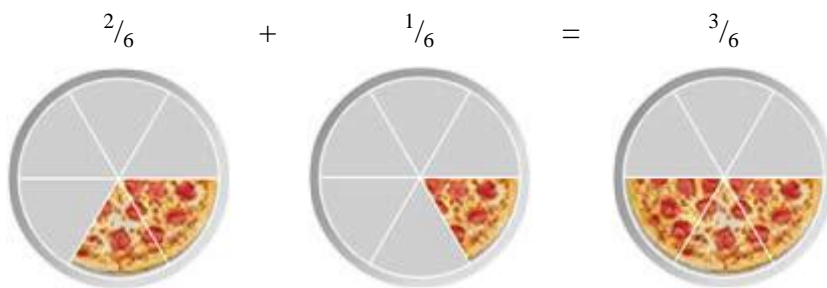
$^2/_6$ + $^1/_6$

The bottom numbers are now the same, so we can go to step 2.

Step 2: Add the top numbers and put them over the same denominator:

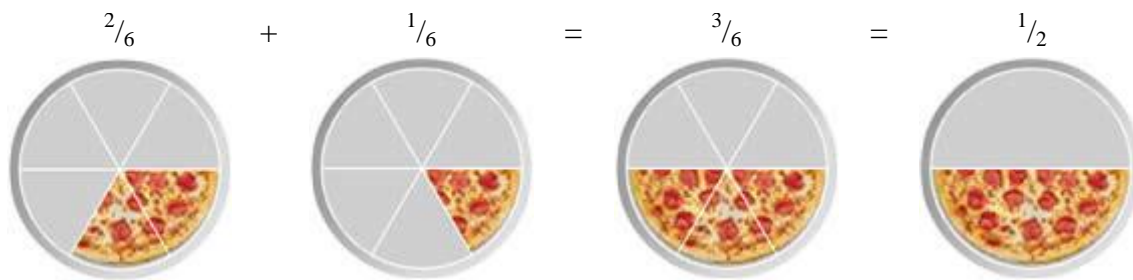$$\frac{2}{6} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6}$$

In picture form it looks like this:

$^2/_6$ + $^1/_6$ = $^3/_6$

Step 3: Simplify the fraction:

$$\frac{3}{6} = \frac{1}{2}$$

In picture form the whole answer looks like this:
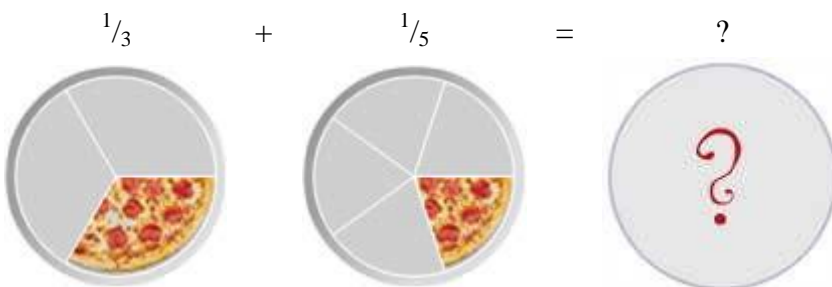
$$^2/_6 \quad + \quad ^1/_6 \quad = \quad ^3/_6 \quad = \quad ^1/_2$$

♫ *"If adding or subtracting is your aim,*
The bottom numbers must be the same!
♫ *"Changing bottom use multiply or divide,*
But the same to top must be applied,
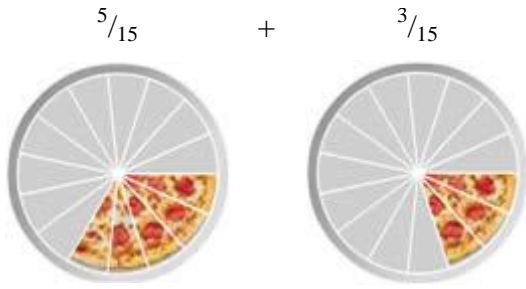♫ *"And don't forget to simplify,*
Before its time to say good bye"

Example 3:

$$\frac{1}{3} \quad + \quad \frac{1}{5}$$

Again, the bottom numbers are different (the slices are different sizes)!

$$^1/_3 \quad + \quad ^1/_5 \quad = \quad ?$$

But let us try dividing them into smaller sizes that **will each be the same**:

$^5/_{15}$ + $^3/_{15}$

The first fraction: by multiplying the top and bottom by 5 we ended up with $^5/_{15}$:

$$\times 5$$

$$\frac{1}{3} = \frac{5}{15}$$

$$\times 5$$

The second fraction: by multiplying the top and bottom by 3 we ended up with $^3/_{15}$:

$$\times 3$$

$$\frac{1}{5} = \frac{3}{15}$$

$$\times 3$$

The bottom numbers are now the same, so we can go ahead and add the top numbers:
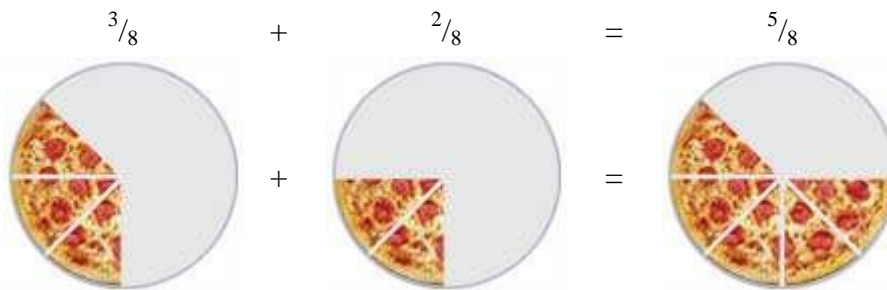
$^5/_{15}$ + $^3/_{15}$ = $^8/_{15}$
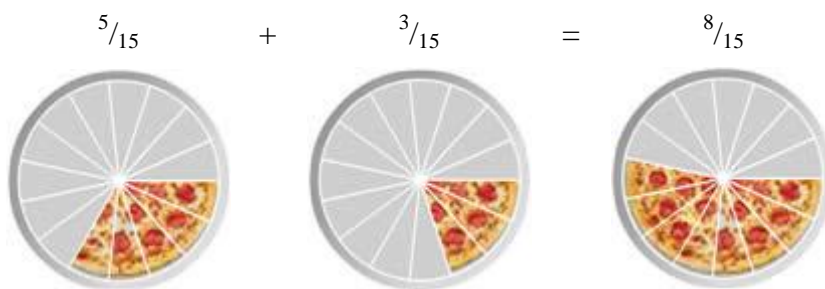
Adding Fractions with Different Denominators

But what if the **denominators** (the bottom numbers) are not the same? As in this example:

$^3/_8$ + $^1/_4$ =   ?

+

=
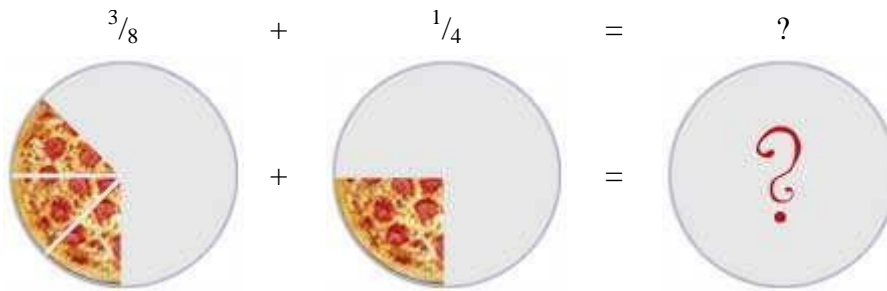
You must *somehow* make the denominators the same.

In this case it is easy, because we know that $^1/_4$ is the same as $^2/_8$:

$^3/_8$ + $^2/_8$ = $^5/_8$

+

=

In that example it was easy to make the denominators the same, but it can be harder ... so you may need to use either of these methods (finding the Least Common Denominator, or finding a Common Denominator) to make them the same (they both work, use whichever you prefer).

Subtracting Fractions

There are 3 simple steps to subtract fractions

- Step 1. Make sure the bottom numbers (the denominators) are the same
- Step 2. Subtract the top numbers (the numerators). Put the answer over the same denominator.
- Step 3. Simplify the fraction.

Example 1:

$$\frac{3}{4} - \frac{1}{4}$$

Step 1. The bottom numbers are already the same. Go straight to step 2.

Step 2. Subtract the top numbers and put the answer over the same denominator:

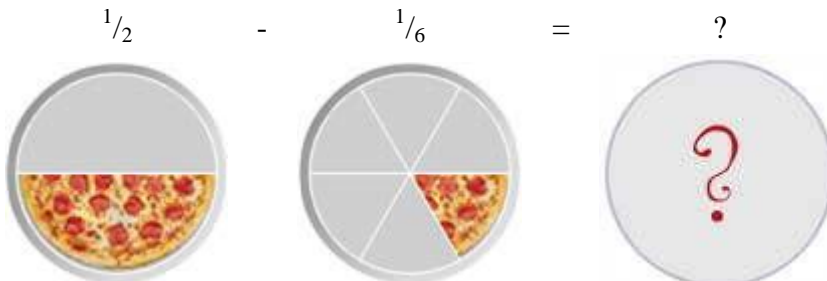$$\frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4}$$

Step 3. Simplify the fraction:

$$\frac{2}{4} = \frac{1}{2}$$
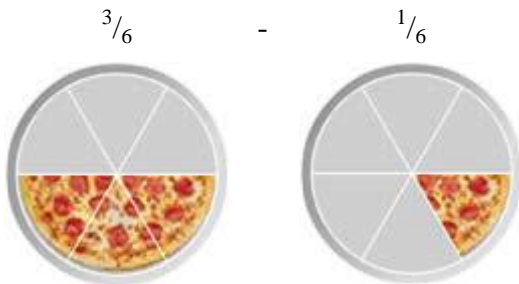
Example 2:

$$\frac{1}{2} - \frac{1}{6}$$

Step 1. The bottom numbers are different. See how the slices are different sizes? We need to make them the same before we can continue, because we **can't** subtract them like this:

$^{1}/_{2}$ - $^{1}/_{6}$ =   ?

To make the bottom numbers the same, multiply the top and bottom of the first fraction ($^1/_2$) by **3** like this:

$$\times 3$$

$$\frac{1}{2} = \frac{3}{6}$$

$$\times 3$$

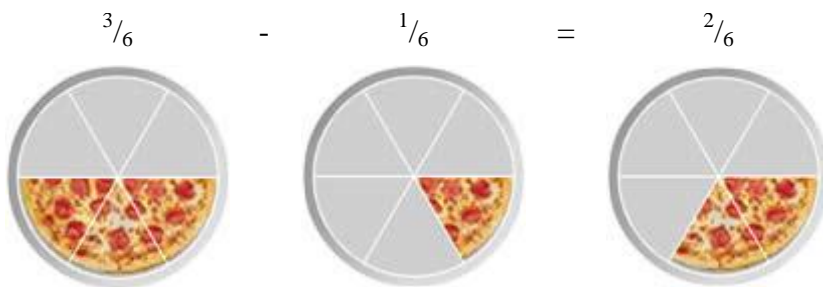And now our question looks like this:

$$^3/_6 \qquad - \qquad ^1/_6$$

The bottom numbers (the denominators) are the same, so we can go to step 2.

Step 2. Subtract the top numbers and put the answer over the same denominator:

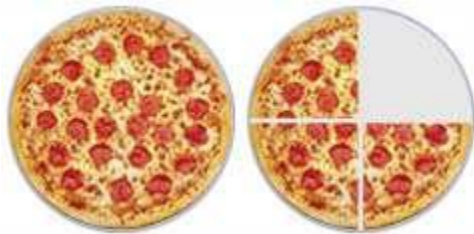$$\frac{3}{6} - \frac{1}{6} = \frac{3-1}{6} = \frac{2}{6}$$

In picture form it looks like this:

$$^3/_6 \qquad - \qquad ^1/_6 \qquad = \qquad ^2/_6$$

Step 3. Simplify the fraction:

$$\frac{2}{6} = \frac{1}{}$$

<u>Adding and Subtracting Mixed Fractions</u>

Quick Definition: A <u>Mixed Fraction</u> is a whole number and a fraction combined, such as 1 $^3/_4$.

1 $^3/_4$
(one and three-quarters)

To make it easy to add and subtract them, just convert to Improper Fractions first:

Quick Definition: An <u>Improper fraction</u> has a top number larger than or equal to the bottom number,

such as $^7/_4$ or $^4/_3$

*(It is "**top-heavy**")*

$^7/_4$
(seven-fourths or seven-quarters)

Adding Mixed Fractions

I find this is the best way to add mixed fractions:

- convert them to <u>Improper Fractions</u>
- then add them (using <u>Addition of Fractions</u>)
- then convert back to Mixed Fractions:

Example: What is 2 $^3/_4$ + 3 $^1/_2$?

Convert to Improper Fractions:

$$2\,^3/_4 = {}^{11}/_4$$

$$3\,^1/_2 = {}^7/_2$$

Common denominator of 4:

$${}^{11}/_4 \text{ stays as } {}^{11}/_4$$

$${}^7/_2 \text{ becomes } {}^{14}/_4$$
(by multiplying top and bottom by 2)

Now Add:

$${}^{11}/_4 + {}^{14}/_4 = {}^{25}/_4$$

Convert back to Mixed Fractions:

$${}^{25}/_4 = 6\,^1/_4$$

When you get more experience you can do it faster like this:

Example: What is $3\,^5/_8 + 1\,^3/_4$

Convert them to improper fractions:

$3\,^5/_8 = {}^{29}/_8$
$1\,^3/_4 = {}^7/_4$

Make same denominator: $^7/_4$ becomes $^{14}/_8$ (by multiplying top and bottom by 2)

And add:

$^{29}/_8 + {}^{14}/_8 = {}^{43}/_8 = 5\,^3/_8$

Subtracting Mixed Fractions

Just follow the same method, but subtract instead of add:

Example: What is $15\,^3/_4 - 8\,^5/_6$?

Convert to Improper Fractions:

$$15 \, {}^3/_4 = {}^{63}/_4$$

$$8 \, {}^5/_6 = {}^{53}/_6$$

Common denominator of 12:

$${}^{63}/_4 \text{ becomes } {}^{189}/_{12}$$

$${}^{53}/_6 \text{ becomes } {}^{106}/_{12}$$

Now Subtract:

$${}^{189}/_{12} - {}^{106}/_{12} = {}^{83}/_{12}$$

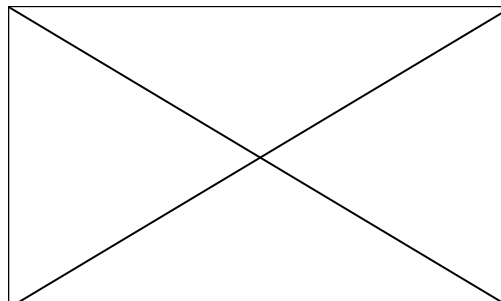Convert back to Mixed Fractions:

$${}^{83}/_{12} = 6 \, {}^{11}/_{12}$$

Multiplying Fractions

Multiply the tops, multiply the bottoms.

There are 3 simple steps to multiply fractions

1. Multiply the top numbers (the *numerators*).

2. Multiply the bottom numbers (the *denominators*).

3. Simplify the fraction if needed.

Example 1

$$\frac{1}{2} \times \frac{2}{5}$$

Step 1. Multiply the top numbers:

$$\frac{1}{2} \times \frac{2}{5} = \frac{1 \times 2}{} = 2$$

Step 2. Multiply the bottom numbers:

$$\frac{1}{2} \times \frac{2}{5} = \frac{1 \times 2}{2 \times 5} = \frac{2}{10}$$

Step 3. Simplify the fraction:

$$\frac{2}{10} = \frac{1}{5}$$

Example 2

$$\frac{1}{3} \times \frac{9}{16}$$

Step 1. Multiply the top numbers:

$$\frac{1}{3} \times \frac{9}{16} = \frac{1 \times 9}{} = \frac{9}{}$$

Step 2. Multiply the bottom numbers:

$$\frac{1}{3} \times \frac{9}{16} = \frac{1 \times 9}{3 \times 16} = \frac{9}{48}$$

Step 3. Simplify the fraction:

$$\frac{9}{48} = \frac{3}{16}$$

A Rhyme To Help You Remember

♫ *"Multiplying fractions: no big problem,*
Top times top over bottom times bottom." ♫
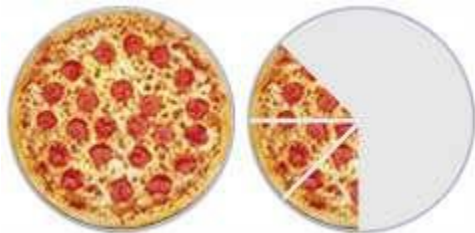
Multiplying Mixed Fractions

("Mixed Fractions" are also called "Mixed Numbers")

To multiply Mixed Fractions:

1. convert to Improper Fractions
2. Multiply the Fractions
3. convert the result back to Mixed Fractions

Example: What is $1^3/_8 \times 3$?

Think of Pizzas.

$1\ ^3/_8$ is 1 pizza and 3 eighths of another pizza.

First, convert the mixed fraction ($1 \, {}^{3}/_{8}$) to an an improper fraction (${}^{11}/_{8}$):

Cut the whole pizza into eighths and how many eighths do you have in total?

1 lot of 8, plus the 3 eighths = 8+3 = 11 eighths.
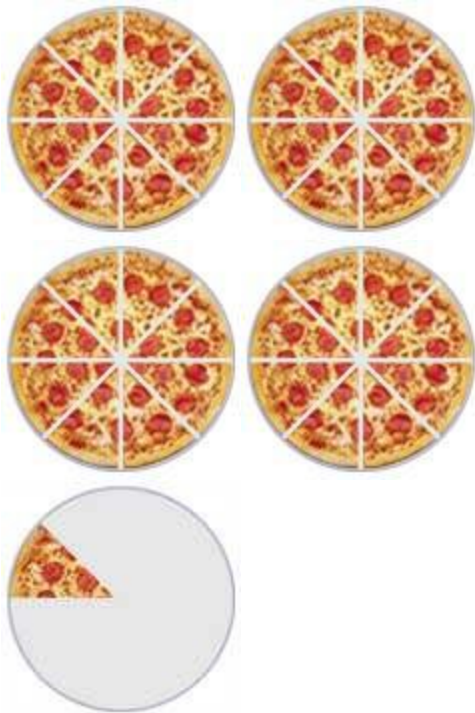
Now multiply that by 3:
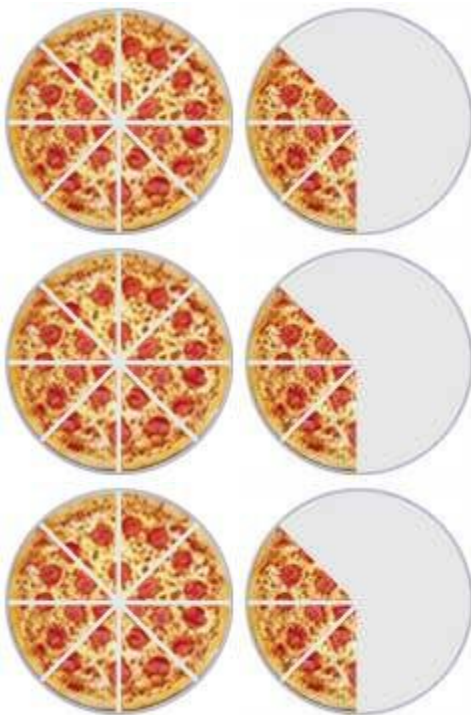
$1 \, {}^{3}/_{8} \times 3 = {}^{11}/_{8} \times {}^{3}/_{1} = {}^{33}/_{8}$

You have 33 eighths.

And, lastly, convert to a mixed fraction (only because the original fraction was in that form):

33 eighths is 4 whole pizzas (4×8=32) and 1 eighth left over.

And this is what it looks like in one line:

$$1\,^3/_8 \times 3 = \,^{11}/_8 \times \,^3/_1 = \,^{33}/_8 = 4\,^1/_8$$

Another Example: What is $1^1/_2$ x $2^1/_5$?

Do the steps from above:

1. convert to Improper Fractions
2. Multiply the Fractions
3. convert the result back to Mixed Fractions

Step, by step it is:

Convert both to improper fractions

$$1 \tfrac{1}{2} \times 2 \tfrac{1}{5} = \tfrac{3}{2} \times \tfrac{11}{5}$$

Multiply the fractions (multiply the top numbers multiply bottom numbers):

$$\tfrac{3}{2} \times \tfrac{11}{5} = \tfrac{(3 \times 11)}{(2 \times 5)} = \tfrac{33}{10}$$

Convert to a mixed number

$$\tfrac{33}{10} = \mathbf{3 \tfrac{3}{10}}$$

If you are clever you can do it all in one line like this:

$$1 \tfrac{1}{2} \times 2 \tfrac{1}{5} = \tfrac{3}{2} \times \tfrac{11}{5} = \tfrac{33}{10} = \mathbf{3 \tfrac{3}{10}}$$

One More Example: What is $3\tfrac{1}{4}$ x $3\tfrac{1}{3}$?

Convert both to improper fractions

$$3 \tfrac{1}{4} \times 3 \tfrac{1}{3} = \tfrac{13}{4} \times \tfrac{10}{3}$$

Multiply

$$\tfrac{13}{4} \times \tfrac{10}{3} = \tfrac{130}{12}$$

Convert to a mixed number (and [simplify](#)):

$$\tfrac{130}{12} = \mathbf{10 \tfrac{10}{12}} = \mathbf{10 \tfrac{5}{6}}$$

Once again, here it is in one line:

$$3 \tfrac{1}{4} \times 3 \tfrac{1}{3} = \tfrac{13}{4} \times \tfrac{10}{3} = \tfrac{130}{12} = 10 \tfrac{10}{12} = \mathbf{10 \tfrac{5}{6}}$$

This One Has Negatives: What is $-1\tfrac{5}{9} \times -2\tfrac{1}{7}$?

Convert Mixed to Improper Fractions:

$$1 \tfrac{5}{9} = \tfrac{9}{9} + \tfrac{5}{9} = \tfrac{14}{9}$$
$$2 \tfrac{1}{7} = \tfrac{14}{7} + \tfrac{1}{7} = \tfrac{15}{7}$$

Then multiply the Improper Fractions *(Note: negative times negative gives us a positive)* :

$$^{-14}/_9 \times {}^{-15}/_7 = {}^{-14\times-15}/_{9\times7} = {}^{210}/_{63}$$

I then decided to simplify next, first by 7 (because I noticed that 21 and 63 are both multiples of 7), then again by 3 (but I could have done it in one step by dividing by 21):

$$^{210}/_{63} = {}^{30}/_9 = {}^{10}/_3$$

Finally convert to a Mixed Fraction (because that was the style of the question):

$$^{10}/_3 = {}^{(9+1)}/_3 = {}^9/_3 + {}^1/_3 = 3\,{}^1/_3$$

<u>Dividing Fractions</u>

Turn the second fraction upside down, then just multiply.

There are 3 Simple Steps to Divide Fractions:

Step 1. Turn the second fraction *(the one you want to divide by)* upside-down (this is now a reciprocal).

Step 2. Multiply the first fraction by that reciprocal

Step 3. Simplify the fraction (if needed)

Example 1

$$\frac{1}{2} \div \frac{1}{6}$$

Step 1. Turn the second fraction upside-down (it becomes a **reciprocal**):

$$\frac{1}{6} \text{ becomes } \frac{6}{1}$$

Step 2. Multiply the first fraction by that **reciprocal**:

$$\frac{1}{2} \times \frac{6}{1} = \frac{1 \times 6}{2 \times 1} = \frac{6}{2}$$

Step 3. Simplify the fraction:

$$\frac{6}{2} = \mathbf{3}$$

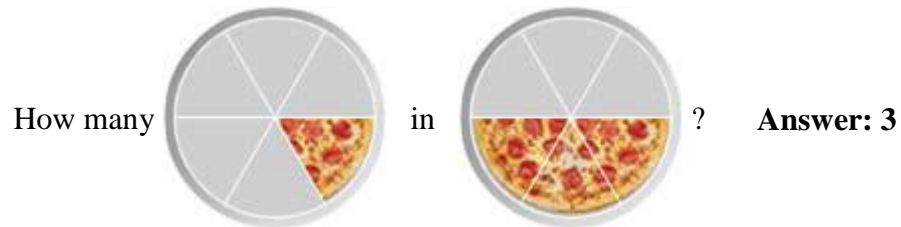**Does it make sense?**

$$\text{Does } \frac{1}{2} \div \frac{1}{6} \text{ really equal } \mathbf{3} \text{ ?}$$

You can change a question like "What is 20 divided by 5?" into "How many 5s fit into 20?"

In the same way our fraction question can become:

$$\frac{1}{2} \div \frac{1}{6}$$ ➡ How many $\frac{1}{6}$ in $\frac{1}{2}$?

Now look at the pizzas below ... how many "1/6th slices" fit into a "1/2 slice"?

How many in ? **Answer: 3**

So now you can see that $\frac{1}{2} \div \frac{1}{6} = \mathbf{3}$ really does makes sense!

Example 2

$$\frac{1}{8} \div \frac{1}{4}$$

Step 1. Turn the second fraction upside-down (the **reciprocal**):

$$\frac{1}{4} \text{ becomes } \frac{4}{1}$$

Step 2. Multiply the first fraction by that **reciprocal**:

$$\frac{1}{8} \times \frac{4}{1} = \frac{1 \times 4}{8 \times 1} = \frac{4}{8}$$

Step 3. Simplify the fraction:

$$\frac{4}{8} = \frac{1}{2}$$

And that is all you have to do.

♫ *"Dividing fractions, as easy as pie,*
Flip the second fraction, then multiply." ♫

But maybe you want to know **why** we do it this way...

Why Turn the Fraction Upside Down?

Well ... what Does a Fraction Do?

A fraction says to:
- multiply by the top number
- divide by the bottom number

$$\frac{3}{4} \begin{array}{l} \leftarrow \times 3 \\ \leftarrow \div 4 \end{array}$$

Example: $\frac{3}{4}$ means to cut into 4 pieces, and then take 3 of those.

So you:

- divide by 4
- multiply by 3

But when you **DIVIDE** by a fraction, you are asked to do the **opposite of multiply**...

So you:

- **divide** by the top number
- **multiply** by the bottom number

So dividing by $\frac{5}{2}$ is the same as multiplying by $\frac{2}{5}$

$$\div \frac{5}{2} \begin{array}{l} \leftarrow \div 5 \\ \leftarrow \times 2 \end{array} \quad \text{same!} \quad \times \frac{2}{5} \begin{array}{l} \leftarrow \times 2 \\ \leftarrow \div 5 \end{array}$$

Because:

Dividing by 5, then Multiplying by 2

is the same as

Multiplying by 2, then Dividing by 5

So instead of dividing by a fraction, it is easier to turn that fraction upside down, then do a multiply.