AP©

AP Physics 1 and 2

Adding Inquiry to AP® Physics 1 and 2 Investigations:

A Teacher's Guide with Four Sample Labs

CollegeBoard

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The College Board
New York, NY

About the College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success – including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators and schools.

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The College Board strongly encourages educators to make equitable access a guiding principle for their AP programs by giving all willing and academically prepared students the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial and socioeconomic groups that have been traditionally underserved. Schools should make every effort to ensure their AP classes reflect the diversity of their student population. The College Board also believes that all students should have access to academically challenging course work before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved.

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Introduction

AP Physics 1 and AP Physics 2 are designed as yearlong college-level courses with each course requiring a minimum of 25% of instructional time to be spent engaged in laboratory work. Additionally, students must be provided with an opportunity to engage in a minimum of 14 labs that collectively address the foundational physics principles in each course, with at least half of the labs conducted in a guided or open inquiry format.

Inquiry-based laboratory investigations are integral to the AP Physics 1 and 2 courses because they provide opportunities for students to apply the seven science practices (defined in the curriculum framework) as they identify the questions they want to answer, design experiments to test hypotheses, conduct investigations, collect and analyze data, and communicate their results. As a result, students are able to concentrate on understanding concepts and developing the reasoning skills essential to the study of physics.

This guide will help you devise inquiry-based instructional approaches for AP Physics 1 and 2 labs in the 2014-15 school year. It also includes the four previously released sample labs (two for AP Physics 1 and two for AP Physics 2) that you can use as models as you prepare your syllabus. Teachers are encouraged to develop their own inquiry-based labs that address the learning objectives in the Curriculum Framework. You should also consider supporting your physical laboratory work with interactive, online simulations, such as PhET simulations developed by the University of Colorado, Boulder.

To assist and support teachers in this process, the College Board operates the online AP Teacher Community (https://apcommunity.collegeboard.org/), which provides opportunities for collaboration and the sharing of resources and ideas.

To come: A complete teacher's manual

The College Board is currently developing the *AP Physics 1 and 2 Inquiry-Based Lab Investigations: A Teacher's Manual* to be published in February 2015. This manual will include everything in this Guide to Inquiry plus an additional 12 inquiry-based labs (see list below), for a total of 16 labs.

These labs will be examples of the kind of investigations that students should engage in, but are not to be considered required investigations; and while they will cover the foundational physics principles for each of the two courses, they are not intended to be the only investigations that students should be engaged in during the course of study in AP Physics. The investigations will illustrate a variety of approaches and the different levels of guidance and support that you can use when implementing inquiry-based laboratory work.

None of the labs provided will require a significant investment in new equipment. However, since an AP lab should provide an experience equivalent to that of a college laboratory, you should make every effort to provide a range of experiences – from experiments students contrive from plumbing pipe, string and duct tape, to investigations in which students gather and analyze data using calculator or computer-interfaced equipment.

Inquiry-based investigations to be included in the upcoming Teacher's Manual

AP Physics 1 Lab Investigations	AP Physics 2 Investigations
1D and 2D Kinematics	Boyle's Law
*Newton's Second Law	Fluid Dynamics
Circular Motion	RC Circuits
*Conservation of Energy	*Magnetism
Impulse and Momentum	Electromagnetic Induction
Harmonic Motion	*Geometric Optics
Rotational Motion	The Particle Model of Light
Mechanical Waves	
Resistor Circuits	

*This lab is also one of the four sample labs.

Chapter 1: About This Guide

The AP Physics 1 and Physics 2 algebra-based courses are designed to promote student learning of essential physics content and foster development of deep conceptual understanding through an inquiry-based model of instruction. The instructional approaches utilized in this guide are informed by several decades of research on student learning and knowledge construction, especially with regard to physics principles. (Further discussion of inquiry-based instructional approaches is found in Chapter 2.)

In this inquiry-based model, students learn by engaging in the seven *AP Science Practices* that develop their experimental and reasoning skills. By engaging in the science practices students begin to see that the study of physics is much more than just gaining knowledge about our physical world, but also requires practices that are "used to establish, extend, and refine that knowledge" over time (NRC, 2012). The science practices (summarized in Chapter 3 and more fully described in the curriculum framework) enable students to make predictions of natural phenomena, develop and refine testable explanations, and use established lines of evidence and reasoning to support claims.

To create a model of excellence for the lab component in AP science courses, the College Board, in conjunction with the Lab Vision Team and Physics Lab Development Team, worked to create an innovative vision and approach to lab investigations. Both teams of subject matter experts consisted of master AP Physics teachers and higher education faculty members as well as experts in the field of inquiry-based instructional design, quantitative skill application, and lab investigations.

The four previously released sample laboratory investigations, presented again in this guide, are examples of the kind of investigations that students should engage in, but are **not to be considered mandatory**; nor should it be assumed that any of these investigations would be specific targets for assessment on AP examinations. The sample investigations are simply models that teachers can use when implementing inquiry-based laboratory work. Teachers are encouraged to develop their own inquiry-based investigations that meet the same cognitive objectives.

Goals of Investigations in AP Physics 1 and AP Physics 2

Inquiry-based laboratory experiences support the AP Physics 1 and AP Physics 2 courses and curricular requirements by providing opportunities for students to engage in the seven science practices as they design plans for experiments, make predictions, collect and analyze data, apply mathematical routines, develop explanations, and communicate about their work.

The sample investigations in this guide provide examples of investigations which support recommendations by the National Science Foundation (NSF) that science teachers should include opportunities in their curricula for students to develop skills in communication, teamwork, critical thinking, and commitment to life-long learning (NSF 1996, NSF, 2012, AAPT 1992). Investigations in the style of those in this guide should engage and inspire students to investigate meaningful questions about the physical world, and should align with the best practices described in *America's Lab Report* (http://www.nap.edu/catalog.php?record_id=11311), a comprehensive synthesis of research about student learning in science laboratories from the National Research Council. Teachers should feel free to use any investigations that capture the spirit of these examples.

How Inquiry-Based Investigations support the AP Physics 1 and 2 Curriculum Framework

The AP Physics 1 and AP Physics 2 courses, equivalent to the first and second semesters of a typical introductory, algebra-based, college physics course, emphasize depth of understanding over breadth of content. By delivering the content across two full-year courses, students will have more time to engage in inquiry-based learning experiences to develop conceptual understanding of content while at the same time building expertise in the science practices.

The AP Physics exams will assess students' abilities to apply the science practices to the learning objectives in the curriculum framework. These science practices and learning objectives can be addressed by the sample labs in this guide and other inquiry-based labs that teachers may choose. This instructional approach to laboratory investigations typically takes more time than simple verification/confirmation labs; however the reduced amount of content covered in each course will allow teachers to meet the curricular requirement that 25% of course time must be devoted to "hands-on laboratory work with an emphasis on inquiry-based investigations".

The sample labs in this guide are intended to serve as models to help teachers develop their own teacher-guided or student-directed, inquiry-based labs that address the learning objectives in the curriculum framework. To assist and support teachers in this process, the College Board operates the online AP Teacher Community (https://apcommunity.collegeboard.org/), which provides opportunities for collaboration and sharing of resources and ideas. There are multiple strategies that can be applied to modify traditional confirmation investigations into guided inquiry labs, as further discussed in Chapter 2. Regardless of the approach, the goal is to engage students in the investigative process of science and allow them to discover knowledge for themselves in a self-reflective, safe, organized manner.

How the Sample Investigations in This Guide Connect to the AP Physics 1 and 2 Curriculum Framework

The key concepts and related content that define the AP Physics 1 and AP Physics 2 courses are organized around seven underlying principles called the big ideas, which address (1) Properties of Objects and Systems (2) Fields and Interactions (3) Object Interactions and Forces (4) System Interactions and Changes (5) Conservation Laws (6) Waves and Wave Models (7) Probability, Complex Systems and Quantum Systems. The big ideas, as described in the curriculum framework, encompass the core scientific principles, theories, and processes modeling physical interactions and systems. For each big idea, enduring understandings are identified, which incorporate the core concepts that students should retain from the learning experience.

Learning objectives for each big idea detail what students are expected to know and be able to do. Because content, inquiry and reasoning are equally important in AP Physics 1 and AP Physics 2, each learning objective in the curriculum framework combines content with inquiry and reasoning skills as described in the science practices.

Each of the four sample investigations in this guide is structured to align to one or more learning objectives from the AP Physics 1 or AP Physics 2 course, and specifies the big idea(s), enduring understandings, learning objectives, and science practices most relevant to and/or addressed by the various activities in that investigation. Although each experiment may address one primary learning objective, there is often significant overlap of the learning objectives within a given enduring understanding, and often across different big ideas.

There is no particular sequence required for the labs in each course, and teachers may choose whatever learning sequence makes sense for them and their students. It is often desirable to have students gain experience with phenomena at the beginning of a topic, to help them build a conceptual model that describes the phenomena, but it can also be desirable to have students use a model to predict the outcome of an experiment and then design the experiment to test their prediction. As suggested by *America's Lab Report*, by the AAPT position paper, *The Role of Labs in High School Physics* (http://www.aapt.org/Resources/policy/roleoflabs.cfm), and by a recent research summary published in *Science* (De Jong et al, 2013), it is highly desirable to integrate laboratory work to align with work students are doing in other parts of the course. Students will gain greater conceptual understanding from a learning sequence that fully integrates laboratory work with other course content.

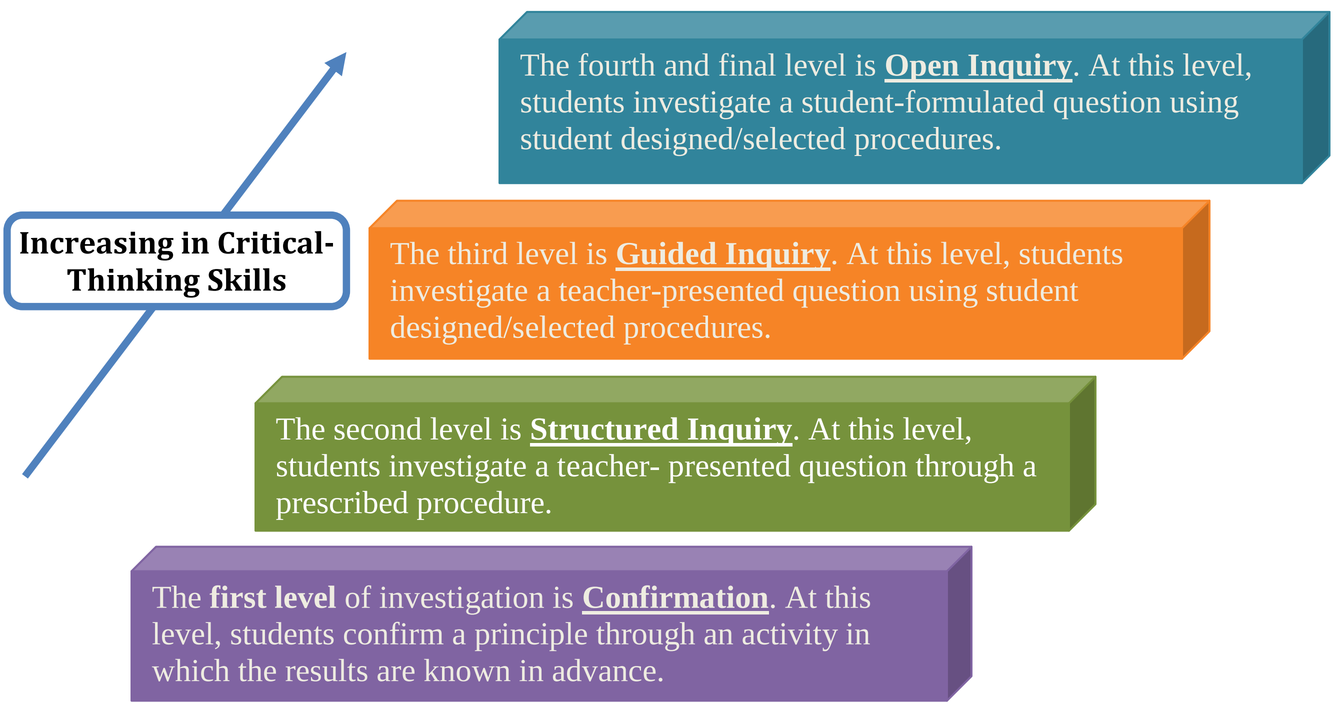
Chapter 2: Creating an Inquiry-Based Learning Environment

"The problem is to provide students with enough Socratic guidance to lead them into the thinking and the forming of insights but not so much as to give everything away and thus destroy the attendant intellectual experience(I deliberately us the word 'guidance' to imply a distinction between this mode and the more conventional modes of providing instructions and answers)." (Arons, 1993) [Arons, A.B. 1993. "Guiding Insight and Inquiry in Introductory Physics Laboratories," Phys.Teach. 31(5): 278.]

Integrating Inquiry-Based Learning

Although laboratory work has often been separated from classroom work, research shows that experience and experiment are often more instructionally effective when flexibly integrated into the development of concepts. When students build their own conceptual understanding of principles of physics, their familiarity with the concrete evidence for their ideas leads to deeper understanding and gives them a sense of "ownership" of the knowledge they have constructed.

Scientific inquiry experiences in AP courses should be designed and implemented with increasing student involvement to help enhance inquiry learning and the development of critical-thinking and problem solving skills and abilities. Adaptations of Herron's approach (1971) [Herron, M.D. (1971). The nature of scientific enquiry. School Review, 79(2), 171- 212] and that of Rezba, Auldridge, and Rhea (1999) [Rezba, R.J., T. Auldridge, and L. Rhea. 1999. Teaching & learning the basic science skills.]define inquiry instruction for investigations in four incremental ways:



Typically, the level of investigations in an AP classroom should focus primarily on the continuum between guided inquiry and open inquiry. However, depending on student familiarity with a topic, a given laboratory experience might incorporate a sequence involving all four levels or a subset of them. For instance, students might first carry out a simple confirmation investigation that also familiarizes them with equipment, and then proceed to a structured inquiry that probes more deeply into the topic and gives more practice with equipment. They would then be presented with a question and asked to design/select their own procedure. A class discussion of results could then lead to student-formulated questions that could be explored differently by different groups in open inquiry.

The idea of asking questions and inquiry is actually natural to students. However, in the classroom setting it may not seem natural to them as they may have developed more teacher-directed procedural habits and expectations in previous lab courses. As students experience more opportunities for self-directed investigations with less teacher guidance, they will become more sophisticated in their reasoning and approach to inquiry. The teacher can promote inquiry habits in students throughout the course—during class and in the laboratory—by handing over more of the planning of experiments and manipulation of equipment over to students.

Getting Students Started with Their Investigations

There are no prescriptive "steps" to the iterative process of inquiry-based investigations. However, there are some common characteristics of inquiry that will support students in designing their investigations. Often, this simply begins with using the learning objectives to craft a question for students to investigate. Teachers may choose to give students a list of materials they are allowed to utilize in their design or they may require that students request the equipment they feel they need to investigate the question. Working with learning objectives to craft questions may include:

- The teacher selects learning objectives from the curriculum framework that relate to the subject under study, and which may set forth specific tasks, in the form of "Design an experiment to…"

  For Example:

  > **Learning Objective 3.B.3.2:** The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force.
  >
  > Students are asked to – Design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force.

- The teacher rephrases or refines the learning objectives that align to the unit of study to create and inquiry-based investigation for students.

  For Example:

  > **Learning Objective 3.B.3.1:** The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties.
  >
  > Students are asked to – Make predictions about what properties determine the motion of a simple harmonic oscillator then design an experiment that tests your predictions and allows for analysis of the dependence of the motion on those properties.

After students are given a question for investigation, they may:

- Refine the question posed by the teacher so the question or purpose best fits the experimental design.
- Plan in small groups prior to lab day to determine how they will manipulate the equipment to accomplish the goal and how they will process the data. (This may involve some initial play with the equipment to inform their plan). Students should record their predictions and assumptions prior to collecting data.
- Conduct the experiment and then develop and record their analysis. The analysis should include a discussion of their prior predictions and assumptions as well as possible sources of uncertainty.
- Present their findings either in a written report or as an oral report to the teacher and class for feedback and critique on their final design and results. Students should be encouraged to critique and challenge one another's claims based on the evidence collected during the investigation. (See Chapter 5 for further information on scientific argumentation strategies).

Students should be given latitude to make design modifications or ask the teacher for additional equipment appropriate for their design. It is also helpful for individual groups to "report out" to the class on their basic design to elicit feedback on feasibility. With the teacher as a guide, student groups proceed through the experiment, with the teacher allowing the freedom to make mistakes—as long as those mistakes don't endanger students or equipment or lead the group too far off task. Students should also have many opportunities to have "post-lab" reporting so that groups can hear of the successes and challenges of individual lab designs.

Demonstrations:

Can demonstrations occasionally count as inquiry laboratory experience if necessary? In the high school classroom, where equipment can be limited, teachers can compensate by implementing "Interactive Lecture Demonstrations" (Thornton and Sokoloff, 1997) [D.R. Sokoloff and R.K. Thornton, "Using Interactive Lecture Demonstrations to Create an Active Learning Environment", Phys. Teacher, 35, 340 (1997)]. Such demonstrations can be effectively used as low-tech alternatives, by having students make and record their predictions, assist in carrying out the demonstration, record and evaluate the data, and present their conclusions. With the right guiding questions, even larger classes can be effectively engaged in such demonstration experiments, and demonstration experiments may be designed to involve students in several different levels of inquiry.

Simulations:

There are now a large variety of well-designed simulations available for physics (e.g., PhET) that can be used to allow students to investigate areas such as solar-system dynamics in a virtual laboratory where students can modify masses and orbital parameters in the simulation to explore and analyze relationships among variables. In a recent review, several studies have shown that a course sequence that includes both real and virtual laboratory experiments may be more effective than either alone (Ton de Jong et al, 2013). Simulations in addition to hands-on lab investigations can greatly benefit students, but simulations alone should not be considered a substitute for labs and should not represent a significant amount of authentic lab investigation time.

Creating a Safe Environment for Student Investigations

Giving students the responsibility for the design of their own laboratory experience requires special responsibilities for the teacher. To ensure a safe working environment, teachers must provide "up front" the limitations and safety precautions necessary for potential procedures and equipment students may use during their investigation. The teacher should also provide specific guidelines prior to students' discussion on investigation designs for each experiment so that those precautions can be incorporated into final student-selected lab design and included in the background or design plan in a laboratory record. It may also be helpful to print the precautions that apply to that specific lab as "safety notes" to place on the desk or wall near student workstations. A general set of safety guidelines should be set forth for students at the beginning of the course. The following is an example of possible guidelines a teacher might post.

- Before every lab, make sure you know and record the potential hazards involved in the investigation and the precautions you will take to stay safe.
- Before using equipment, make sure you know the proper use to acquire good data and avoid damage to equipment.
- Know where safety equipment is located in the lab, such as fire extinguisher, safety goggles, and first aid kit.
- Follow the teacher's special safety guidelines as set forth prior to each experiment. (Students should record these as part of their "design plan" for a lab.)
- When in doubt about the safety or advisability of a procedure, check with the teacher before proceeding.

The teacher should interact constantly with students as they work to observe safety practices and anticipate and discuss with students problems that may arise. A teacher walking among student groups, asking questions and showing interest in students' work allows the teacher to "keep the pulse" of what students are doing as well as maintain a watchful eye for potential safety issues.

Material and Equipment Use

A wide range of equipment may be used in the physics laboratory, from generic lab items such as meter sticks, rubber balls, springs, string, metal spheres, calibrated mass sets, beakers, glass and cardboard tubes, electronic balance, stopwatches, clamps, and ring stands to items more specific to physics, such as tracks, carts, light bulbs, resistors, magnets, and batteries. Successful guided inquiry student work can be accomplished both with simple, inexpensive materials and with more sophisticated physics equipment such air tracks, force sensors, and oscilloscopes. However, remembering that the AP lab should provide experience for students equivalent to that of a college laboratory, the teacher should make every effort to provide a range of experiences – from those experiments students contrive from plumbing pipe, string and duct tape to experiments in which students gather and analyze data using calculator or computer-interfaced equipment.

Chapter 3: The Role of the Science Practices

Students begin their study of physics with some prior knowledge based on their experiences in the physical world, knowledge that serves them well in particular contexts. This knowledge is often piecemeal, may be unarticulated, and is often not able to be explicitly organized into any broad coherent scheme. Some of their ideas are scientifically accurate, while others may be partially correct or incorrect according to our present collective understanding of natural phenomena. Research in physics education has shown that merely telling students about concepts in physics has little effect on their conceptual understanding.

The role of inquiry-based physics laboratory investigations is to provide students with the opportunity to design and carry out organized investigations of the physical world; to analyze their observations in an attempt to find coherent patterns that can serve as a basis for developing conceptual and mathematical models of phenomena; and ultimately, to organize and consolidate their understanding of these models within the theories of physics. Laboratory investigations provide students with opportunities to experience and observe phenomena by engaging in science practices, whereby they design and carry out organized investigations of phenomena in order to build models and test predictions stemming from the models they have constructed. Through this process students begin to value the reasoning skills associated with the construction of knowledge within the scientific community.

The Ten Key Points about Science

Throughout the study of the history and philosophy of science, there are ten key points that have emerged about the development of scientific knowledge over time. In total, these points lead to one key conclusion: that science is not a body of theories and laws, but rather an approach to understanding observations that allows us to make sense of the world around us. If we think about these key points, they can help us understand the reasons for using inquiry in the physics laboratory.

These key points about the nature of science (as modified from McComas, 2004) are:

- Scientific knowledge is tentative but durable.
- Laws and theories serve different roles in science and are not hierarchal relative to one another.
- There is no universal step-by-step scientific method.
- Science is a highly creative endeavor, grounded by theory.
- Scientific knowledge relies heavily, but not entirely, on observation, experimental evidence, rational arguments, creativity, and skepticism.
- Scientific progress is characterized by competition between rival theories.
- Scientists can interpret the same experimental data differently.
- Development of scientific theories at times is based on inconsistent foundations.
- There are historical, cultural, and social influences on science.
- Science and technology impact each other, but they are not the same.

The science practices that align to the concept outline of the curriculum framework capture important aspects of the work that scientists engage in, at the level of competence expected of AP Physics students.

AP Physics teachers will see how these practices are articulated within each learning objective and therefore allow laboratory investigations and instruction to emphasize both content and scientific practice.

The Seven Science Practices

The following is an abridged version of the seven science practices for physics. The complete version can be found in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

Science Practice 1: The student can use representations and models to communicate scientific phenomena and solve scientific problems. The real world is complex. When physicists describe and explain phenomena they simplify real objects, systems, and processes to make the analysis manageable. Models are then used to predict new phenomena. Inherent in physicists' invention and use of models is the use of representations, such as pictures, motion diagrams, force diagrams, graphs, energy bar charts, ray diagrams and mathematical representations such as equations. Laboratory investigations provide opportunities for students to explore and develop skills in the use of representations and models to communicate their thinking about phenomena.

1.1. The student can *create representations and models* of natural or man–made phenomena and systems in the domain.

1.2. The student can *describe representations and models* of natural or man–made phenomena and systems in the domain.

1.3. The student can *refine representations and models of natural or man–made phenomena and systems* in the domain.

1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

1.5. The student can *re-express key elements of natural phenomena across multiple representations* in the domain.

Science Practice 2: Students can use mathematics appropriately. Physicists use mathematical representations to describe and explain phenomena, as well as to analyze physical situations and solve problems. When students work with these representations we want them to understand the connections between the mathematical description, the physical phenomena, and the concepts represented in the mathematical description. When using equations or mathematical representations, students need to be able to justify why using a particular equation or graph to model an experimental result is appropriate and to be aware of the conditions and limits of applicability of the equation to describe their results.

2.1. The student can *justify the selection of a mathematical routine* to solve problems.

2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena.

2.3. The student can *estimate numerically quantities that describe* natural phenomena.

Science Practice 3: Students can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course. Research scientists pose and answer meaningful questions — a point that students may miss if they are taught in a way that makes it seem that science is about compiling and passing down a large body of known facts. Thus, helping students learn how to pose, refine and evaluate scientific questions, while recognizing that there are other questions that cannot be answered by scientific practices is an important instructional and cognitive goal, albeit a difficult skill to learn. Experience with experimental investigations should help students to pose, refine and evaluate scientific questions.

3.1. The student can *pose scientific questions.*

3.2. The student can *refine scientific questions.*

3.3. The student can *evaluate scientific questions.*

Science Practice 4: The student can plan and implement data-collection strategies in relation to a particular scientific question. Scientific questions can range from broad to narrow, from determining what factors might be of influence to determining the detailed mechanisms of the phenomena under study. Data can be collected from controlled investigations, from careful scientific observations, from work by others, from historic reconstruction of archived records. All data has some degree of uncertainty both in precision and accuracy. Students need to discuss the pros and cons of different experimental procedures with attention paid to control of variables and evaluation of the reliability of the data they collect.

4.1. The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

4.2. The student can *design a plan* for collecting data to answer a particular scientific question.

4.3. The student can *collect data* to answer a particular scientific question.

4.4. The student can *evaluate sources of data* to answer a particular scientific question.

Science Practice 5: Students can perform data analysis and evaluation of evidence. Students often think that each graph they produce should connect the data points, or that the best-fit function is always linear. They need to engage in opportunities that help them see the value in constructing a best-fit curve for data that does not fit a linear relationship. They should understand that data points are really intervals whose size depends on experimental uncertainty. They need to learn to both observe and interpret patterns in data, and try to explain the patterns in terms of physics principles, or to revise and extend their knowledge based on data that consistently and repeatedly exhibit patterns that they did not expect.

5.1. The student can *analyze data* to identify patterns or relationships.

5.2. The student can *refine observations and measurements* based on data analysis.

5.3. The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Science Practice 6: The students can work with scientific explanations and theories.
Scientific explanations may specify a cause-and-effect relationship between variables representing physical quantities or describe a mechanism through which a particular phenomenon occurs. A scientific explanation, purporting to account for observed phenomena, needs to be able to make predictions about new phenomena that are experimentally testable. A coherent set of models explaining a wide variety of phenomena in a unified fashion constitutes a theory. Students should be able to use models and theories to make testable claims about phenomena and justify their claims as they engage in scientific arguments from evidence. Students need to recognize that even a good theory with broad application may have limits to its validity and needs to be replaced or extended as part of the process of knowledge construction. Looking for experiments that can reject explanations and claims is at the heart of the process of science.

6.1. The student can *justify claims with evidence.*

6.2. The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

6.3. The student can *articulate the reasons that scientific explanations and theories are refined or replaced.*

6.4. The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

6.5. The student can *evaluate alternative scientific explanations*.

Science Practice 7: The student is able to connect and relate knowledge across various scales, concepts, and representations in and across the domains. Students should have the opportunity to transfer their learning across disciplinary boundaries so that they are able to link, synthesize, and apply the ideas they learn across the sciences and mathematics. Learning research indicates that having opportunities to apply major ideas in multiple contexts facilitates transfer, helping students to see how big ideas cut across traditional disciplinary boundaries. Thus after learning about conservation laws in the context of mechanics, students should be able to extend these ideas to other contexts and at other scales and see where they are appropriate to apply and where they might not apply.

7.1. The student can *connect phenomena and models* across spatial and temporal scales.

7.2. The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Assessing the Science Practices

The four sample investigations in this guide include rubrics that may be used to assess students' understanding of the science practices specific to that particular lab. Each rubric provides teachers with guidance on the range of student understanding and proficiency for each science practice. The rubrics provided in the sample investigations align directly to the investigation's learning objectives. However, the very nature of inquiry-based investigation often elicits numerous other science practices that teachers may also want to assess.

Accordingly, a sample rubric that serves for all of the science practices is set out in the Appendix, which can be used to assess student understanding in any lab a teacher might choose to use.

Chapter 4: Experimental Data Analysis

In the laboratory neither the measuring instrument nor the measuring procedure is ever perfect; consequently, *every* experiment is subject to experimental error. A reported result which does not include the experimental error is incomplete. The only numbers that are valid to report without experimental error are discrete quantities that you can count, for example, the number of students in your class. Values measured from a measuring instrument such as a balance or meter stick are not discrete quantities and have experimental error associated with them.

In this chapter, we will discuss kinds of measurement error, and how to determine and calculate errors, or *uncertainties*. In addition, we will discuss linearizing data, best practices for graphing, and linear fits used to determine unknown quantities.

Kinds of Error and Implications for Precision and Accuracy

Experimental errors are generally classified under two broad categories: systematic errors and random errors.

Systematic errors include errors due to the calibration of instruments and errors due to faulty procedures or assumptions. When reporting results in scientific journals, one might have to go to great lengths to assure that one's meter sticks and clocks, for example, have been accurately calibrated against international standards of length and time. However, even if an instrument has been properly calibrated, it can still be used in a fashion which leads to systematically wrong (always either high or always low) results. If instruments are calibrated and used correctly, which you might hope for in at least your most basic labs, then you can expect accurate results; but even the most basic measurements might include things such as parallax errors in measuring length, or human reaction time errors with a stopwatch, creating inaccuracies in results. Another common example of a systematic error in the physics lab is to assume air resistance is not a factor for a falling body, which we do in textbooks all the time, but which makes real results inaccurate.

Random errors include errors of judgment in reading a meter or a scale and errors due to fluctuating experimental conditions. Because no instrument is perfectly precise, even if measurement conditions are not fluctuating, careful measurements of the same quantity by the same person will not yield the same result over multiple trials. For many measurements, environmental fluctuations (e.g., the temperature of the laboratory or the value of the line voltage) will necessarily give different results each time, and there will also be fluctuations caused by the fact that many experimental parameters are not exactly defined. For example, the width of a table top might be said to be 1 meter, but close examination would show that opposite edges are not precisely parallel and a microscopic examination would reveal that the edges are quite rough. The table does not have a width in the same sense that you can talk about the number of students in your class: you precisely know how many students are missing from class today, but even if you are measuring it correctly (without systematic error), the table's width cannot be precisely known.

When the systematic errors in an experiment are small, the experiment is said to be *accurate*. Accuracy is a measure of how close you are to the accepted answer.

When the random errors in an experiment are small, the experiment is said to be *precise*. Precision tells you how well you know the answer you have determined; it is how sure you are of your measurement, or alternatively your *uncertainty* of your measurement, regardless of whether the measurement is accurate or correct.

For example, if you are doing an experiment to measure the acceleration of a cart on a horizontal track due to an attached falling mass, and you idealize the situation to where there is no friction, the measured acceleration is going to be systematically less than the theoretical acceleration if friction is the only force unaccounted for (the track could also not be level and that would be another systematic error.) With systematic errors, the measured result is always different from the expected result in the same way each time (in this example, the measured result is always *less* than the expected result.) In addition, there will be random errors in this experiment: if the acceleration is being determined from distances measured on the track and times measured by stopwatches or photogates, the stopwatches, photogates and meter sticks have some limit to their precision, and no matter how accurate the instruments are, every time the measurements might be a little bit high *or* a little bit low because exact measurements cannot be made. But the measurements will randomly fall on either side of the mean result.

Systematic errors are avoidable or can be accounted for, and any time you determine that there is one you should do your best to eliminate it or account for it. Random errors, on the other hand, are reducible in easy ways (using more precise measuring instruments), but are unavoidable. The sensitivity of your measuring instrument determines the ultimate precision of any measurement. Since random errors are unavoidable, we should never use a measuring instrument so crude that these random errors go undetected. We should always use a measuring instrument which is sufficiently sensitive so that duplicate measurements do not yield duplicate results. Recommended procedure is that you report your readings of analog scales and meters by estimating the last digit as best you can. Digital meters will generally come with a stated precision by the manufacturer.

Significant Digits

When doing data analysis calculations, calculators and computers typically give more digits than are significant. A typical calculator may give an eight or ten digit answer. Students need to think about their answers, and think about how many of those digits are actually meaningful, or *significant*. Experimentally measured numbers need to be expressed to the correct number of significant digits, determined by the error, or the precision, to which they are known.

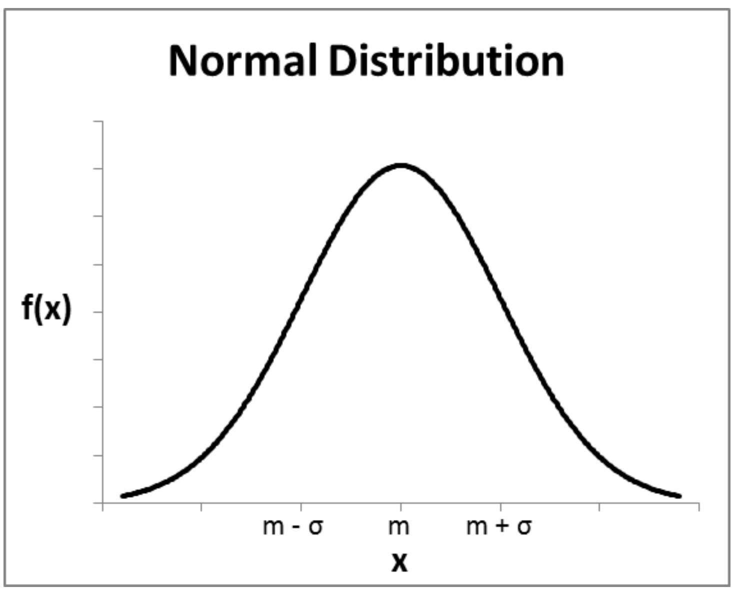
When counting significant digits, leading zeroes are not significant: $0.0045\,\mathrm{m} = 4.5*10^{-3}\,\mathrm{m} = 4.5\,\mathrm{mm}$, and each of these measures have two significant figures. But trailing zeroes are significant, and tell us how precise something is: 0.5 s is less precise than 0.500 s. The first (one significant digit in the tenths place) implies that the time measurement is known within about a tenth of a second; while the second (three significant digits, with the least significant being in the thousandths place) implies it is known to about a millisecond.

When you quantify random error in a measurement, as you should always do in the lab, the error tells us about how unsure you are of each digit. Since the error, or precision, is the uncertainty in the measurement, it determines the number of significant digits in the measurement. Let's say you determine with some calculations the speed of a student walking at constant speed in the lab. Your calculator may give you eight digits, so you find that $v = 3.0342842 \pm 0.17092203\,\mathrm{m/s}$. What this uncertainty tells you is that you are unsure of the tenths place to ± 0.1, of the hundredths place to ± 0.07, and so on. If you are already unsure of the tenths place (the most significant digit in this uncertainty), we are generally not concerned with an additional uncertainty in a less significant digit such as the hundredths place. The biggest uncertainty is in the largest decimal place, or most significant digit of the uncertainly, so the rest of the error is extraneous: you should always round errors to one significant digit. So we would have $v = 3.0342842 \pm 0.2\,\mathrm{m/s}$. In addition, since we are saying we are unsure of the tenths place, having a measurement in the hundredths place is not very meaningful, so we need to *round the measurement to the same decimal place as the uncertainty*. So the correct answer is $v = 3.0 \pm 0.2\,\mathrm{m/s}$. Note that the rounding of the measurement is to the same decimal place as the error, not the same number of significant digits, so in this case the measurement has two significant digits while the error has one.

Distribution of Measurements

Given the pervasiveness of random error, one of the tasks of the experimentalist is to estimate the probability that someone who performs an apparently identical experiment will obtain a different result. Subsequent measurements will not be expected to give the same answer, but multiple measurements will be distributed in such a way that we can make a good estimate of what we think is the correct answer. If we made a lot of measurements of one quantity, and the error was truly random, we would expect to make as many measurements that are higher than the correct value as measurements that are lower. And we would expect to get fewer measurements that are further away from the correct value than those that are closer. For example, let's say that lab groups from your class are at an amusement park trying to measure the speed of a roller coaster car at a certain point on its track, and the average of all their measurements is 21.5 m/s. We do not expect any of the individual measured values to necessarily be the same due to random errors, but we would expect it to be as likely that a student measures 22 m/s as 21 m/s. On the other hand, we would expect it to be a lot less likely for them to measure a value much farther away – such as 15 m/s – we expect to see fewer of those measurements than measurements that are closer to the average. If many measurements are made, we can expect to see a pattern emerge.

For example, if we were to plot the frequency of measurements vs. the value of the measurement, we most often expect to get a distribution that approximates the following:



This distribution occurs so frequently for measurement that it is given two names: the Gaussian or the Normal distribution. Remember, we are talking about making multiple measurements of a single quantity here – x is simply the value of the measurement we make (e.g. the width of the table) and f(x) is the number of times we come up with a particular number. The distribution of answers will usually look like this Normal distribution no matter what we are measuring, as long as we make a large number of measurements. Note that if we make only two or three measurements, there is no way we could get a distribution of measurements that looks this way, but once we take about ten or more measurements that are influenced entirely by random error, the Normal distribution starts to emerge. Statistical theory allows us to determine an equation describing the Normal distribution:

$$f(x) = \frac{e^{-\left[\frac{(x-m)^2}{2\sigma^2}\right]}}{\sqrt{2\sigma^2}}$$

Equation 1

This equation is not one we expect students to use, but the parameters m and σ are important to understand. The function is characterized by these two parameters: the *mean*, m, which tells us where the peak of the curve falls along the x axis, and the *standard deviation*, σ, which is a measure of how wide the curve is. The mean is our best guess of the correct value if only random errors impact the measurements, and the standard deviation is a measure of how close we expect any given measurement to be to the mean.

Error Analysis

Estimates of the Mean and Standard Deviation

So what does statistical theory tell us about being able to determine the value of a measurement? The method is different depending on whether we take a very large number of measurements (thousands), or just a few. It is often, if not usual, that when making scientific measurements, we only make a few measurements, so we will not get exactly the Normal distribution which would allow us to very precisely determine a measured value. Given a sample, a finite number of N measurements, we would like to *estimate* the mean and standard deviation of a series of measurements of values denoted x_i. The estimate of the mean, *M*, is given by a familiar procedure:

$$M = \sum_{i=1}^{N} \frac{x_i}{N}$$

Equation 2

Similarly, when we have a set of measurements from a sample, the estimate of the standard deviation, *S*, is:

$$S = \sqrt{\sum_{i=1}^{N} \frac{(x_i - M)^2}{N-1}}$$

Equation 3

Of course, most data analysis programs such as Excel or Logger Pro have these functions built in so that you do not have to do the sums manually.

Estimating Precision

Precision is a measure of how uncertain you are about your measurement given random fluctuations. If you are making a single measurement with an analog instrument, the precision can be estimated from how well you think you can read the instrument. Typically that might be as much as estimating tenths of the smallest division. For example, using a meter stick with millimeter divisions, for a single measurement, you might be able to estimate the measurement to the next decimal place, tenths of a millimeter. Using a digital measuring device, which gives an exact measurement as output, the precision should be given in the instruction manual.

If you are making a single measurement with an analog instrument, the precision can be estimated from how well you think you can read the instrument. Typically that means estimating half of the smallest division. For example, using a meter stick with millimeter divisions, for a single measurement, you should be able to estimate the measurement to the nearest half a millimeter. Using a digital measuring device, which gives an exact measurement as output, the precision should be given in the instruction manual.

But what if you are making multiple measurements of a single quantity? Now we ask the question: "What is the probability that my estimate of the mean, *M*, based upon a small number of measurements, will fall close to the true mean, *m*, which is based on a large (ideally, infinite) number of measurements?" What we want to estimate here is not the standard deviation (spread) of the individual measurements, but the standard deviation of estimated means. For example, if multiple lab groups were to do similar experiments and each took a relatively small number of data points, none would be expected to measure the true mean, *m*, but each would be expected to measure a different estimated mean M. From statistical theory, if we know how much the estimated mean values are expected to be spread out, then we know the correct value is likely to be in that spread. The estimated spread of the estimated mean measured values is called the *standard error*, abbreviated *SE*.

Theory tells us that a good estimate of the standard deviation in measured mean values, *SE*, is

$$SE = \frac{S}{\sqrt{N}}.$$

Equation 4

Notice that our estimate of the mean, *M*, and standard deviation, *S*, might not change appreciably as the number of measurements, *N*, is increased, because as the numerator increases proportionally, so does the denominator (the wider the spread of values x_i, the more are needed before *M* and *S* become approximately constant with additional measurements). Alternatively note that the standard error, *SE*, gets smaller with larger *N*.

The standard error is a measure of the precision, or uncertainty, when you take multiple measurements of a quantity. You can state it as absolute uncertainty, or as a percent uncertainty. For example, if you measure the acceleration of your cart on a track system to be $a = 1.21\,\mathrm{m/s^2}$, and you determine the standard error to be $SE = 0.06\,\mathrm{m/s^2}$, then the acceleration can be written with uncertainty to be $a = 1.21 \pm 0.06\,\mathrm{m/s^2}$, or $a = 1.21\,\mathrm{m/s^2} \pm 7\%$.

Methods of Reducing Random Errors

Since *SE* gets smaller as the number of measurements, *N*, gets larger, you can always reduce your error in an experiment by making additional measurements. It is important to know that this estimate of the standard error is an approximation that is only valid for small *N* and does not mean that the standard error goes to zero as *N* gets large. There is always a finite, non-zero uncertainty for every measurement.
In addition, note that if the measuring instrument is so crude as to give the same value every time, with no standard deviation, that does not mean your measurement is infinitely precise, it means that your measuring instrument is either not precise enough or is being used incorrectly. You can always reduce random errors by choosing an appropriate measuring instrument for the experiment. A more precise

instrument will generally help to reduce random errors, but it must be appropriate to the purpose: if you are measuring the length of a room, a more precise caliper will probably create more error than a less precise tape measure since the caliper was not designed to measure as large a distance.
Since the stopwatches we use in lab are typically all of similar precisions, then if making a time measurement, it is typically better to measure for a longer time to reduce human reaction time errors and precision errors. For example, if you are measuring the speed of a platform rotating at constant speed, it is better to time how long it takes to rotate for ten rotations (and appropriately divide by ten) than just one rotation. If you are doing an experiment, though, that is changing over time, you need to balance measuring for enough time versus how much the other changes will impact the experiment.

Propagation of Error

Let's say we want to measure the area of a tabletop and report the result with uncertainty, since we cannot make an exact measurement.

Assume that you have measured the length and width as:

$$L = 1.763 \pm 0.004\,\text{m}, \text{ and } W = 0.759 \pm 0.003\,\text{m}$$

where the uncertainties in the length and width come from standard error calculations.

If we wish to report the area of the table with uncertainty, we must allow for the possibility that errors in the independent measurements of *L* and *W* might offset one another. The correct procedure according to statistical theory is as follows:

First, calculate the uncertainty in *A* caused by the uncertainty in *L* only, using the mean value of *W* but the maximum value of *L*. The uncertainties in a quantity are denoted by the Greek letter Δ. Call the resulting uncertainty ΔA_L :

$$\Delta A_L = (L + \Delta L)*W - L*W\,.$$

Note that in this case, we are using $A = L*W$, but this can be applied to any function of one or more variable.

In our example, this result is

$$\Delta A_L = 1.767\,\text{m}*0.759\,\text{m} - 1.763\,\text{m}*0.759\,\text{m} = 0.003\,\text{m}^2$$

Next, calculate the uncertainty in *A* caused by the uncertainty in *W* only, using the mean value of *L* in the calculation. Call this result ΔA_W :

$$\Delta A_W = L*(W + \Delta W) - L*W\,.$$

In our example, this result is

$$\Delta A_W = 1.763\,\text{m}*0.762\,\text{m} - 1.763\,\text{m}*0.759\,\text{m} = 0.005\,\text{m}^2$$

and statistics tells us to combine the uncertainties to find that the net uncertainty in Area is given by the following:

$$\Delta A = \sqrt{\Delta A_L{}^2 + \Delta A_W{}^2}$$

Thus in our example,

$$\Delta A = \sqrt{0.003^2 + 0.005^2} = 0.006\,\text{m}^2$$

So that the determined area with correct uncertainty is $A = 1.338 \pm 0.006\,\mathrm{m}^2$.

This procedure is a generally valid one. In general, for a function $f(G,H)$ of two measured variables with means *G* and *H*, and uncertainties Δ*G* and Δ*H*, the error in *f* due to *G* is

$$\Delta f_G = f(G + \Delta G, H) - f(G,H),$$

Equation 5a

and the error in *f* due to *H* is

$$\Delta f_H = f(G, H + \Delta H) - f(G,H).$$

Equation 5b

The error in *f* is then

$$\Delta f = \sqrt{\Delta f_G{}^2 + \Delta f_H{}^2}.$$

Equation 6

The function can be of one, two, three or more variables and can involve any arithmetic operation.

Measurements of a Dependent Variable

When measuring, for example, the constant acceleration of an object, since $v = v_0 + at$, you could choose to start the object from rest ($v_0 = 0$) and then measure the velocity, *v*, after the same amount of time, each time, and average all your values of v/t to get a value of *a*. This method may lack accuracy and precision versus measuring the velocity as a function of time for varying times. When taking measurements, you can often reduce error by instead of measuring the same thing over and over again, varying an independent variable and measuring a dependent variable. By varying the time over which we take data, we are exploring a range of values, and a more complete graphical analysis can reduce the random and systematic errors in the experiment.

The Error Distribution and Confidence

Earlier we stated that the standard error, *SE*, is a measure of precision. Again, statistical theory tells us what the standard error means to us, and it can be used to express our confidence in our estimate of the mean as follows:

If we add or subtract the standard error to the mean, we will get a possible range of values. This is usually written as the interval $M \pm SE$. Statistics tells us that if we are only making a very small number of measurements, we should expect the likelihood that the true mean (the "correct" value) is in that interval is 60%. The interval determined by $M \pm SE$ is called a "60% confidence interval".

For your purposes, 60% confidence might not be good enough. So you can choose to make the interval bigger, making it more likely that the correct value is contained. If you add or subtract two standard errors to the mean, $M \pm 2 * SE$, for small numbers of measurements, we should expect the "correct" value to be in that interval 90% of the time, or with a confidence of 90%. Obviously, this can be taken to extremes, and you can keep adding more SE's to your range of precision. You will be more confident you have the correct answer in your range, but your answer with uncertainty will get less precise. You need to balance needs of precision with needs of confidence, and that is typically choosing to report results to 60 or 90 percent confidence.

Your answer is accurate if it is either equal to, or encompasses, the expected value. Of course, you cannot know if it is accurate if you do not have an expected answer. You would assume accuracy, but still only to the confidence level that you are measuring, if you believe you have accounted for all systematic errors. For some measurements, we have expected values for what we are trying to measure. Let's assume we are using 90% confidence: if we do have an expected value, and if your 90% confidence interval for the measured quantity does not agree with the expected value, then you should investigate the accuracy and the systematic errors that may have been present in your experiment. If there are no systematic errors, it is possible for the answer to be inaccurate. Statistics state that 10% of correctly performed measurements will give an incorrect estimate of the mean within the 90% confidence interval.

Comparing Results to Each Other or to an Expected Value

Percent Difference and Percent Error

If two lab groups measure two different values for an experimental quantity, they may be interested in how they compare to each other, and this is often expressed as a percent difference, defined as the absolute value of the difference divided by the mean times 100:

$$\%difference = \left| \frac{value1 - value2}{\frac{1}{2}\left(value1 + value2\right)} \right| * 100$$

Equation 7

When you have an expected or theoretical value that you want to compare to a measured value, this is often expressed as a percent error, defined as the absolute value of the difference divided by the expected value:

$$\%error = \left| \frac{measured\ value - expected\ value}{expected\ value} \right| * 100$$

Equation 8

Note that when the expected value is very small, approaching zero, the percent error gets very large, and is undefined when the expected value is zero. % error may not be a very useful quantity in these cases.

The Null Hypothesis

Using what we know about combining uncertainties, we can compare a measured value with an accepted value, or two measured values with each other, to determine whether they are similar enough to be considered equal. Let's suppose you were measuring the speed of a cart at an instant. Let's also suppose that someone else has measured it and told you that the speed was $34.0 \pm 0.3 \mathrm{cm/s}$. Suppose your measurement was $33.4 \pm 0.4 \mathrm{cm/s}$. Can we say that there is a significant difference between these two speeds? Another way to phrase this question is: is the difference between these two figures due to random effects only?

At first glance you might suspect that the two figures do not differ significantly, since the difference between them is 0.6 cm/s and this difference is less than the sum of the confidence intervals, which is 0.7

cm/s. This simple conclusion is incorrect because of statistics - the odds don't favor your being too low on one measurement while simultaneously being too high on the other. As we saw when combining errors above, statistical theory tells us that the difference between the two figures, if due to random effects only, can be expected to be no larger than the square root of the sum of the squares of the individual uncertainties: $\sqrt{(0.3\text{cm/s})^2 + (0.4\text{cm/s})^2} = 0.5\text{cm/s}$.

This is an example of using statistical methods to test a *null hypothesis***.** In the present case, our hypothesis might be stated in the form of a question: is the difference between the two values equal to zero (is $v_1 - v_2 = 0$) ?

When we subtract these two numbers, the presence of random error makes it unlikely that we would get exactly zero, but statistically, they might still be measuring the same value. Since both numbers have an uncertainty associated with them, we have to answer this question by asking whether the number 0 is contained in the appropriate interval when you subtract the two quantities. For our example, $34.0 \pm 0.3\text{cm/s} - 33.4 \pm 0.4\text{cm/s} = 0.6 \pm 0.5\text{cm/s}$ does not include 0. In the present case, since the two values for the speed differ by more than can be accounted for due to random effects only, the conclusion that the two speeds are the same is probably false.

Suppose that your two numbers for the speeds agreed with one another within the random uncertainties or even exactly. This would not "prove" the hypothesis that the speeds are equal, as the agreement might be a statistical anomaly. Even if experiments agree with one another, we have to admit the possibility that continued improvement in the precision of the experiment might ultimately lead to detecting a statistically significant difference. This illustrates an important philosophical principle concerning experimental results. Although we can be reasonably confident that a given hypothesis is false (i.e., two measurements or measurement and theory differ by more than we can account for due to random effects only), we can never prove with equal assurance that it is true, that two values are equal.

Graphs

Graphs are often an excellent way to present or to analyze data. When making graphs, there are a few guidelines you should follow to make them as clear and understandable as possible:

- Graphs should be labeled completely and appropriately
  - Each axis should be labeled with the quantity plotted including units
  - Each axis should include a reasonable number of tick marks at even intervals, and should include a scale
  - Typically, graphs should be labeled with a meaningful title or caption
  - If a legend is needed, the legend should be meaningful (e.g. Excel automatically includes a legend that often does not add any information unless the legend is edited)
- A typical problem with graphs is fitting them into a space that makes it difficult to see the trends in the data, so the graph should be of such a design that trends can be observed in the data
  - Generally, you want to fill more than one-half of space vertically and horizontally (which means that the scale will not always start at zero). A particular exception to this is a set of data that indicates a horizontal line, or a line with zero slope. Because you expect random error in real data, if you make the y-axis scale such that the data fills the page vertically, it will not look like a horizontal line. In the case where the slope is near zero, analysis of the data will be improved by

looking at the data with a significantly larger scale to determine if it actually looks like a horizontal line.

- The graph should be big enough to see, typically at least 1/8 of a page.

Graphical Analysis: Linearizing Data

Most often when you take data in an experiment, you intend to do one of three things:

1. You want to verify that the data has a certain relationship. For example, you want to determine if a body is moving with a constant acceleration, so you plot position versus time data for the body to see if it is a parabola.
2. You expect that your data has a certain relationship and want to determine some parameter. For example, you have position and time data for constant-acceleration motion of a body, and you want to determine the acceleration.
3. You are trying to find an unknown relationship between two variables. For example, you take position versus time data for a body, but you do not know what kind of motion it is, and you may ask, is position related to time with a linear, square, exponential, power law, or some other relationship?

The Equation of a Straight Line

When you make a plot on x-y axes, a straight line (line with constant slope) is the simplest relationship that data can have. Representing a straight line with a function on x-y axes only requires two arbitrary parameters, m and b, such that

$$y = m * x + b$$

Equation 9

Because there are only two parameters in the linear function, it is the easiest function to use as a model, the meaning of the parameters are most clear (slope and y-intercept), and the parameters can always be worked out with a best fit line with very little manipulation compared to a higher-order function. Be cautious, though, to only model something with a linear function if indeed it is linear.

The slope is a measure of how the *y* variable changes with changes in *x*, $m = \Delta y / \Delta x$. Be careful when estimating slopes from best-fit lines: the slope should be determined from the best-fit line, *not* by taking two of the data points and subtracting their *y* and *x* values.

The *y*-intercept is where the line crosses the *y*-axis (where $x = 0$). It is often interpreted as the initial value of the function, assuming the function starts when $x = 0$.

Linearizing data so that you can do a straight-line fit is an important data analysis technique. Even if the data you take do not have a linear relationship, if you have a model for it, you can often figure out what to do to linearize it.

For example, in physics we often start with the quadratic relationship between position in one dimension, y, and time, t, for constant acceleration: $y = \frac{1}{2}at^2$. When y vs. t is graphed, it yields a parabola. If instead we set the x-axis variable equal to t^2, so y is graphed vs. t^2, the model is $y = \frac{1}{2}ax$, and should yield a straight line with slope $m = \frac{1}{2}a$.

A more general technique of linearizing data is to do a *log-log* plot of data. If the data is exponential, $y = Ae^{bx}$ or a power law, $y = ax^n$, taking the log of both sides of the relationships will linearize them. So if you take the log (typically base 10 or natural log) of both of your *x* and *y* data sets, you can determine the unknown parameters:

Exponential: $\ln(y) = \ln(Ae^{bx}) = \ln(A) + bx.$ If you plot $\ln(y) vs. x,$ the data will approximate a line with y-intercept $\ln(A),$ and slope $b.$

Power Law: $\log(y) = \log(ax^n) = \log(a) + n*\log(x).$ If you plot $\log(y) vs. \log(x),$ the data will approximate a line with y-intercept $\log(a)$ and slope $n.$

Note that not all functions can be linearized. If you simply take our first example of the quadratic time dependence of position with time, and add an unknown initial speed, $y = \frac{1}{2}at^2 + v_0t$, there are now too many unknown parameters multiplied by different combinations of variables, and the equation cannot be linearized without knowing one parameter ahead of time, such as the initial speed.

Graphical analysis of Linear Data

Now that you have linearized your data, a straight line fit to the data will yield parameter values and allow you to determine unknown quantities. For example, in the first example in the previous section, a determination of the slope allows you to determine the acceleration of the object.

The Least Squares Fit of a Straight Line

As discussed above, in an experiment there is often a linear relationship between the measured variables, or you can linearize them. For example, the velocity of an object in free fall changes linearly with time, in the absence of air resistance. When we plot a set of data and find that it approximates a straight line, the next question is how to find the slope and the intercept of the line that seems to provide the best fit. If we were actually to measure speed versus time for a falling object that is not impacted significantly by any forces other than gravity, the data would be scattered around a straight line. We do not expect to find the data looks exactly like a straight line because we know that the presence of random error causes this scatter away from the ideal line. We can find approximate values for slope and intercept by using a straight edge to draw a line which appears to "split the difference" between the scattered points: the line should have as many points above and below the line. A more exact answer is given by a statistical analysis, which is described here. You will often be able to use a computer or graphing calculator to do this analysis.

The process of finding the best-fit line proceeds as follows: We do an experiment where we have our two variables y and x that depend on each other (e.g. measure speed of a freely falling body as a function of time, or maybe it is some more complicated combination of variables as described in linearizing data, above). If there were no random errors present, all of our experimental results would fall exactly on a line given by Equation 9, $y = m*x + b$.

Equation 8 is our model for the data. If we expect our data to have this relationship, then we must have a theoretical equation which takes this form, where *y* and *x* are variables and *m* and *b* are undetermined constants. Note, though, that if your data does not look linear, perhaps it is not!
As an example, let's take the expected physical relationship for the velocity versus time for an object undergoing constant acceleration:

$$v = v_0 + at.$$

Equation 10

If we are measuring velocity at varying times, the relationship between *v* and *t* should be a linear one: this equation looks a lot like the equation for the straight line, above. If we have *N* pairs of data points, and our data consists of time values, *t*, that we put on the x-axis, and each data point is called x_i, and velocity values, *v*, that we put on the y-axis, where each value is denoted y_i, then we would expect it to look like a straight line. We need to determine the best-fit line from these values in order to determine the initial speed v_o (the y-intercept) and acceleration, *a* (the slope).

Assuming only random errors are present, based on the mathematics of the Normal distribution discussed earlier, in order to maximize the probability that we have the correct fitted line, we have to minimize the sum of the squares of the deviations of the measured points from the fitted line. For data points that each have approximately equal absolute random error (equally *weighted*), this minimum value occurs when the slope, *m*, and y-intercept, *b*, are given by

$$m = \frac{N\sum(x_i y_i) - \sum x_i \sum y_i}{N\sum\left(x_i^2\right) - (\sum x_i)^2},$$

Equation 11

$$b = \frac{\sum y_i - m\sum x_i}{N}.$$

Equation 12

The uncertainties in these values, Δm and Δb, are given by

$$\Delta m = \sqrt{\frac{N\sum\left(y_i - mx_i - b\right)^2}{(N-2)[N\sum\left(x_i^2\right) - (\sum x_i)^2]}},$$

Equation 13

$$\Delta b = \sqrt{\frac{\sum\left(x_i^2\right)\sum\left(y_i - mx_i - b\right)^2}{(N-2)[N\sum\left(x_i^2\right) - (\sum x_i)^2]}}.$$

Equation 14

The procedure can be carried out systematically by a computer or graphing calculator using regression or best-fit line analysis. In order to use a computer program intelligently, keep in mind the following points:

1. You must enter at least three data points or the least squares procedure will not work.
2. You need at least five data points to determine approximate uncertainties for the slope and y-intercept.
3. No matter how wildly scattered your data may be, or even if the variables are not linearly related, the computer can always come up with a slope and an intercept which is a best-fit in the least squares sense. It's a good idea to make at least a rough plot of your data to be sure that your chosen method of plotting does yield something close to a straight line. An example you may want to show your students is the set of data and graphs known as the Anscombe quartet of graphs which all have the same linear "best fit" line, but have plots that are wildly different. See Wikipedia https://en.wikipedia.org/wiki/Anscombe%27s_quartet.

You can use a computer to make calculations of the above equations, or use built-in functions to determine slope, y-intercept, and uncertainty in each. As well as determining the unknown parameters and their errors, you should also be sure to plot the data and the best fit line to be sure it looks as expected.

Weighted Least Squares

Instead of minimizing the sum of the squares of the differences between each data point and the fitted line, we could minimize the *weighted* sum of the squares. This can be done when data quality varies—when we know some data with more confidence than others. Each data point should be weighted by the inverse square of its standard error. If each data point y_i has individual standard errors, e_i, then in order to minimize the weighted sum of the squares of the distance from each data point to the fitted line, the slope and y-intercept become

$$m = \frac{\sum \frac{1}{e_i^2} \sum \frac{x_i y_i}{e_i^2} - \sum \frac{x_i}{e_i^2} \sum \frac{y_i}{e_i^2}}{\sum \frac{1}{e_i^2} \sum \frac{x_i^2}{e_i^2} - (\sum \frac{x_i}{e_i^2})^2},$$

Equation 15

$$b = \frac{\sum \frac{x_i y_i}{e_i^2} - m \sum \frac{x_i^2}{e_i^2}}{\sum \frac{x_i}{e_i^2}},$$

Equation 16

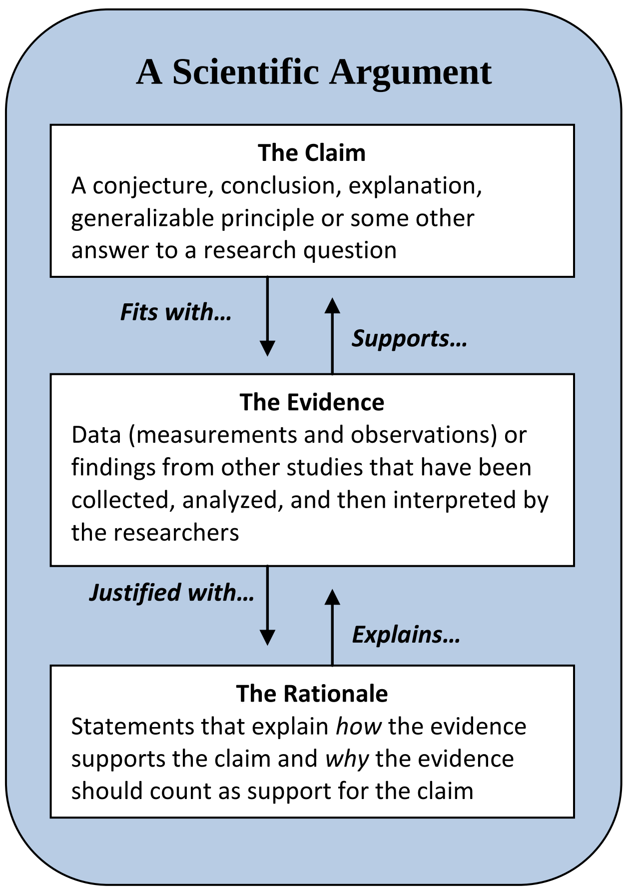
While these expressions might look daunting, they only involve a few different sums, and can be implemented in a spreadsheet without too much difficulty.

Chapter 5: Written, Verbal, and Visual Communication

Engaging Students in Scientific Argumentation

Teachers should guide students to develop the mindset that there are no “wrong” answers to questions—instead responses are steps toward developing the best explanation possible. Responses become valid only as they are supported by physics concepts and explanations. The physics concepts are in turn supported by evidence students collect during their investigations. To foster the inquiry process, teachers should cultivate a classroom environment in which students have the opportunity to be “wrong” without embarrassment, an environment in which students can offer explanations with confidence that they will be taken seriously. Once explanations are proposed, then teachers should guide students to final conclusions through a process of open scientific discussion and argumentation based on evidence.

The goal is to enable students to build skills in constructing arguments from evidence in order to defend their conclusions. Laboratory experience, as distinguished from simple, everyday experience, must involve active engagement of students’ minds as well as their hands. Experience becomes experiment when students are in a situation that requires them to articulate their observations, build mental models and draw conclusions from their observations, and then make and test the predictions from their models. Students should then be able to construction claims based on their investigations that are supported by evidence. They should also be able to provide explanations as to how their evidence supports their claims about their observations.



Model developed by Sampson & Grooms, 2008; Sampson, Grooms, & Walker, 2009

Communicating Scientific Evidence from Investigations

Students need to be given time to consider their explanations and the opportunity to discuss and respond—allowing the ensuing discussion to evolve as students develop their own understanding. Students should have opportunities to convey their evidence from investigations in the laboratory in several ways. Laboratory work should be recorded directly in written form (e.g., a bound journal or portfolio) in “real time.” Teachers may choose to have the analysis of this work completed in the journal or developed in written form as a formal report. Reporting results can also be verbal, as individual students or student groups report their observations and conclusions to a larger group for discussion and feedback.

Chapter 6: Making AP Physics 1 and 2 Inclusive for All Learners

As a teacher, you should anticipate having students with special needs in your course, and should plan to meet the individual needs of those students in order to support them in being successful in the course. This chapter provides guidance regarding issues that are particularly pertinent to special-needs students in the guided-inquiry physics laboratory.

Safety

The most important consideration for teachers is always the safety of their students in the laboratory. Teachers may need to make special efforts in order to ensure that students with special needs can work effectively and safely in the lab. The inclusion of students with special needs can be successful when teachers have sufficient information about students' needs, have the proper materials to assist students in the lab (as needed), and receive support from professionals who specialize in the special needs of particular students. In some cases, the teacher will need to spend more time with special-needs students in the laboratory. Thus, the total number of students the teacher can adequately supervise may be smaller, so teacher/studio ratio is particularly important. Teachers may need to have additional professionals in the laboratory to be able to guide and manage all students safely. Special-needs students may need specialized equipment or other aids to support their work in the lab. A team of professionals (counselors, science teachers, special-education teachers, and school administrators) should discuss class size, specialized equipment, and other issues pertinent to the requirements of the special-needs student prior to laboratory work, and the teacher must ensure that recommendations are followed. The teacher can help the team to identify risks that might arise from a student's special needs in the specific context of the physics laboratory.

Accommodations

Both physical and nonphysical accommodations that enhance learning can be made for students with special needs. The most common special needs relate to (1) vision, (2) mobility, (3) autism spectrum, (4) learning and attention, (5) hearing, and (6) health. Consultation with educational professionals who specialize in the particular special needs of the student is important. Awareness of organizations such as DO-IT (Disabilities, Opportunities, Internetworking, and Technology) can provide teachers with information about working in the laboratory/classroom with students with special needs. Many students with learning issues have individualized education programs (IEPs) that can guide the accommodations.

You may want to consider including the following suggestions:

- **Students with vision impairments** might benefit greatly from enhanced verbal descriptions and demonstrations. Lab equipment can be purchased with Braille instructions, promoting independent participation for visually impaired students. Students with visual challenges might also benefit from preferential seating that allows them to see demonstrations more easily. If possible, you should provide students with raised-line drawings and tactile models for illustrations. You might also consider using technology to increase accessibility to the lab experience. For example, video cameras attached to a computer or television monitor can be used to enlarge images.
- **Students who have mobility challenges** may need a wheelchair-accessible laboratory. You should keep the lab uncluttered and make sure that aisles are wide enough for wheelchair

movement. Students often can see a demonstration better if a mirror is placed above the instructor. Lab adaptations are available for students with mobility problems to assist them in most lab activities. You will need to know a student's limitations before planning a successful lab experience.

- **Students with autism spectrum disorders** (including Asperger's syndrome and pervasive developmental disorder) may have a range of communication and impulsive behavior challenges requiring accommodations and close monitoring in the laboratory setting to ensure a safe and supportive learning environment. These students' particular challenges and needs are highly individualized. Guidance and support from appropriate professionals is particularly important for preparing teachers to meet their needs. An educational aide or support staff member working with the student in the lab is sometimes helpful, as a lower student–educator ratio is often beneficial and may, in some cases, be called for in the student's IEP.
- **Students with hearing difficulty** might benefit from preferential seating near you when demonstrations are given. It is also helpful to provide hearing-impaired students with written instructions prior to the lab, and to use instructor captioning when showing videos and DVDs.
- **Students who have learning and attention special needs** may require a combination of oral, written, and pictorial instruction. Scaffolding instruction increases learning, and safety issues and procedural instructions may need to be repeated. Having audio-taped instructions may be helpful to allow students to hear them as often as needed for comprehension. Some students who have special needs related to attention need frequent breaks to allow them to move around and refocus. Providing students with preferential seating to avoid distractions is also helpful. Students with reading and writing challenges often require more time to prepare for lessons and to complete the follow-up activities. Students with learning and attention challenges sometimes benefit greatly from the use of technology, such as scanning and speaking pens that help with reading. Other students might benefit from using laptops to take notes during class.
- **Students with health issues,** such as asthma, allergies, or insulin-dependent diabetes, may benefit from certain accommodations. Care should be taken to avoid risking a student's health because of exposure to chemicals or allergens such as noxious gases or vapors, latex gloves, or food components (e.g., milk or egg proteins, peanuts) while conducting laboratory investigations. Students with asthma or allergies may benefit from wearing a mask designed for physical laboratory use. Teachers should be aware of any student requiring epinephrine administration (e.g., an Epi-Pen) in the case of an allergic reaction.

Universal Design

Creating a laboratory environment that is universal in design means creating one that is accessible to students both with and without special needs. By creating such an environment, you should address most concerns and accommodations for students with special needs and, at the same time, improve learning opportunities for *all* students in the lab. The teacher should be proactive whenever possible in implementing accommodations, including the following:

- Providing both written and oral directions
- Giving students adequate time to prepare for labs and to complete follow-up activities
- Making the aisles wide enough for wheelchairs
- Installing a mirror above the area where demonstrations are performed
- Using tables that can be adjusted for height

Supporting English Language Learner Students

AP Physics teachers should be prepared to accommodate English language learner (ELL) students in their courses. Teachers can employ a number of strategies to support such students; many of these strategies will benefit all students, not just ELL students. Examples include:

- Using printed pictures and graphics (e.g., pictures of lab glassware) to support English text in curricular materials and lab handouts;
- Teacher demonstrations of basic procedures and techniques;
- Video clips showing laboratory techniques; and
- Multimedia simulations of chemical phenomena.

Another idea to consider is pairing students with less developed English language skills with another student who speaks their first language and has more developed English language skills, though of course this is not a substitute for teacher supervision and support. Close teacher monitoring and prompting in the lab will further help students who appear confused or "on the wrong track" during inquiry activities, and will prevent any potential safety hazards from arising.

Developing a Community of Learners

Teachers must foster the creation of a learning environment that includes and respects *all* students. For example, creating cooperative learning groups provides students with the opportunity to share their knowledge and skills and to learn from each other. This is particularly advantageous for special-needs students. Teachers may find it helpful to talk with students to discover firsthand what accommodations they need to implement to make their students' lab experience successful. By modeling attitudes and behaviors expected from students, teachers can develop activities that help *all* students build meaningful academic and personal relationships.

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AP Physics 1 Sample Lab # 1: Newton's Second Law

What Factors Affect the Acceleration of a System?

Central Challenge

In this lab the students will investigate the relationship among the net force exerted on an object, its inertial mass, and its acceleration. Students should have already completed the study of kinematics and Newton's First Law.

Background

Newton's Laws are the basis of classical mechanics and enable us to make quantitative predictions of the dynamics of large-scale (macroscopic) objects. These laws, clearly stated in Isaac Newton's book the *Principia* over 300 years ago, explain how forces arising from the interaction of two objects affect the motion of objects.

Newton's First Law states that an object at rest remains at rest and an object moves in a straight line at constant speed unless the object has a net external force exerted on it.

Newton's Second Law states that when a next external force is exerted on an object of mass *m,* the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

The mass of an object in Newton's second law is determined by finding the ratio of a known net force exerted on an object to the acceleration of the object. The mass is a measure of the inertia of an object. Because of this relationship, the mass in Newton's second law is called inertial mass, which indicates how the mass is measured.

Newton's laws of motion are only true in frames of reference that are not accelerating, known as inertial frames.

In this lab students will discover how the acceleration of an object is related to its mass and the force exerted on the object. Students will use their experimental results to derive the mathematical form of Newton's second law.

Real World Application

There are numerous real-world applications of Newton's Second Law that can spark student interest.

Students can research their favorite sport and apply the concepts learned in this investigation to understand how the magnitude of the acceleration varies when a force is exerted on objects of different mass like golf balls, tennis balls and baseballs.

Another application could be the physics of cars. Students think the engine makes the car move, but why does it not work on ice? Because it takes an external force exerted on an object by another object to create an acceleration, so the tires must push back on the ground and the ground pushes forward on the tires and the car goes forward. That is, the external force provides the acceleration to the car.

In this investigation the students will use a modified Atwood's machine. Atwood's machines are systems with two masses suspended over a pulley by a cable and provide for a constant acceleration of any value required (see Figure 1). Some students might be interested in investigating a real life application of this technology such as an elevator and its counterweight.

Inquiry Overview

This entire investigation is structured as guided inquiry. The students will be presented with the question: "What factors affect the acceleration of a system?"

After observing the demonstrations suggested in Part I of the investigation, the students will be guided to discover the factors to be investigated. The students will also design the procedure of the investigation, and the data collection strategy.

Students might need some guidance with the analysis of data and the construction of graphs. More specifically, students might be confused about how to merge the results of the two parts of the investigation to answer the overall lab question.

The Investigation section below offers specific guiding questions to support the students in the design and interpretation of their experiments. Part II of the investigation is divided into two separate activities. The first is limited to the relation of acceleration to force and the second is limited to the relation of acceleration to mass.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 1

Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding 1.A

The internal structure of a system determines many properties of the system.

Learning Objective 1.C.1

The student is able to design an experiment for collecting data to determine the relationship between the net force exerted on an object, its inertial mass, and its acceleration. (**Science Practice 4.2)**

Big Idea 3

The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.A

All forces share certain common characteristics when considered by observers in inertial reference frames.

Learning Objective 3.A.2.1

The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. (**Science Practice 1.1)**

[Note: In addition to those science practices listed in the learning objectives above, the following science practices are also addressed in the various lab activities: 4.1, 4.3, 5.1, and 5.3. Also, students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Skills and Practices Taught/Emphasized In this Investigation

Science Practices	**Activities**
1.1. The student can create representations and models of natural or man-made phenomena and systems in the domain	Students will produce multiple representations of the data in the form of graphs and diagrams as follows: Graphs of the data: • acceleration versus force • acceleration versus mass Force diagrams that represent the forces exerted on the objects
***4.1.** The student can justify the selection of the kind of data needed to answer a particular scientific question	Students will identify the quantities that need to be measured in order to determine the acceleration of the system.
4.2. The student can design a plan for collecting data to answer a particular scientific question	Students will design a procedure to investigate the relationships among the net force exerted on an object, its inertial mass, and its acceleration.
***4.3.** The student can collect data to answer a particular scientific question	Students will gather the following data: • net force and acceleration when the total mass is kept constant • total mass and acceleration when the net force is kept constant
***5.1.** The student can analyze data to identify patterns or relationships	Students will analyze the graphs to identify the relationship between the variables
***5.3.** The student can evaluate the evidence provided by data sets in relation to a particular scientific question	Students should be able to articulate an operational definition of Newton's Second law based on the evidence presented by the graphs.

*See note above.

Equipment and Materials

***For each group of 3 to 4 students*:**

- Dynamics track
- Cart
- Assorted masses
- Mass hanger and slotted masses
- Low-friction pulley
- String
- Meterstick
- Stopwatch

If you do not have a dynamics track, then any flat, smooth surface, perhaps even the lab tables themselves, will work just fine. The carts should have wheels with a small rotational inertia and low friction bearings.

Data acquisition using motion detectors or photogates is recommended when available, as it helps reduce experimental procedural errors. Another option is to record a video of the motion of the cart and use video analysis software to analyze the motion.

Timing and Length of Investigation

Teacher preparation: 15–20 minutes

This time is needed to prepare the demos and set out equipment from which students may choose for their investigation.

Pre-lab discussion/activities: 30 minutes

It is advisable to conduct the activities and pre-lab discussion in one class or lab period.

Student investigations: 1.75–2 hours

Design of procedure: 20–30 minutes

Data collection: 30 minutes

Data analysis: 60 minutes

You may assign as homework the design of the data collection procedures. Students should gather the materials and do their own setup for their investigations. At the beginning of the lab period, ask for volunteers to present their draft procedures to the class, and solicit feedback from the various groups.

Post-lab discussion/presentations: 30 minutes

Total time: 3–3.25 hours

[Note: This investigation is designed to enable a deeper understanding of Newton's Second Law and therefore it might take more time than investigations performed in the context of the previous AP Phyiscs B course.]

Safety

There are no major safety concerns for this lab. However, pay attention to high speeds of carts, masses flying off carts, masses hitting feet of students and student fingers being squeezed when stopping a cart at the pulley when a high proportion of mass is on the hanger. Also, to keep students and equipment from being damaged, restrict the total slotted mass. General lab safety guidelines should always be observed.

Preparation and Pre-Lab

Pre-Lab Activities

The following activities are optional and could be conducted to assess student's prior knowledge, skill levels, and understanding of key concepts. Set up the modified Atwood machine and pose questions such as those suggested below in this 4 part pre-lab session:

Part 1:

What will a graph of the cart's velocity (v) vs. time (t) look like after the system is released from rest?

After making and discussing their predictions, students could carry out an experiment, using a motion detector to record v vs. t or using video capture, in which case the students will have to put some thought into how to produce the velocity vs. time graph. But the main point of this part is for students to see and make sense of the conclusion that the slope of the v vs. t graph is constant.

Part 2:

(a) If the cart's mass is increased; will the new* v *vs.* t *graph look the same or different from the graph in part 1?

(b) If the hanging mass is increased, will the new* v *vs.* t *graph look the same or different from the graph in part 1?

Again, these are qualitative questions, but students can obtain quantitative data to answer them. As usual with these kinds of qualitative questions, the lab works well if students first make and discuss their predictions before designing and carrying out the experiments.

Part 3:

If both the cart's mass and the hanging mass are doubled, will the new* v *vs.* t *graph look the same or different from the graph in part 1?

Part 4:

What if the cart is moving initially?

What will the velocity vs. time graph look like, compared to the graph from Part 1, if the cart at $t = 0$ is given a brief push away from the pulley? Will the graph be the same? If not, what will be different?

Some students may spontaneously have the idea of doing another trial where the cart is given a brief push towards the pulley — and it would be great for them to try that! They should be able to identify that the y-intercept in the velocity-time graph represents the initial velocity of the cart.

Common Student Challenges

Some of the common challenges that students have regarding Newton's First Law is the idea that forces are required for motion with constant velocity. When observing the demonstrations the students need to recognize that the velocity of the object is changing as a result of the net force exerted on the object. It should be clear that the net force determines an object's acceleration, not its velocity. To counter this student misconception, you can use a motion detector and a force probe to study the motion of a cart being pulled by a mass hanging from a string which passes over a pulley (as shown in the Investigation section below). Simultaneously graph the force on the cart and the motion of the cart. The students should be directed to notice the shape of the force graph (horizontal line) and acceleration graph are the same, but the velocity vs. time graph is a line with a positive slope. A constant forward force produces an increasing velocity, and a constant acceleration.

Students might not see the connection between Newton's Laws and kinematics, so it is important for them to recognize Newton's Second Law as "cause and effect." It is important to present Newton's Second Law in its operational form of $\vec{a} = \frac{\Sigma \vec{F}}{m}$ as the commonly used $\Sigma \vec{F} = m\vec{a}$ leads some students to believe that the product of mass and acceleration, '*ma*', is a force.

A specific student challenge in this investigation is to recognize that both the cart and the falling mass are included in the total inertial mass of the system being affected by the gravitational force on the falling mass. During the investigation, all masses to be used as falling masses should be placed in the cart when not pulling the cart. Students will be tempted to have the cart on table and replace the falling mass with a different falling mass that is on the lab table. This, in effect, changes the total mass being pulled. This is a good opportunity to have students discuss the meaning of "system." The system that is being accelerated is the cart and falling mass.

Another specific student challenge is the role of friction of the cart and the pulley as well as the rotational inertia of the wheels of the cart and the pulley. These can be ignored when conducting the investigation, for sufficient hanging mass, but should be discussed at some point in the analysis of results.

The Investigation

Part I

In the first part of this activity the students observe a number of demonstrations that include variations of an object being accelerated.

A modified Atwood's machine with a system consisting of a cart and a hanger with slotted masses like the one shown in the diagram below is a suitable set up:

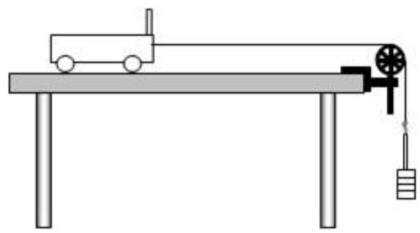

Figure 1

Examples include a demonstration where the total mass of the system is kept constant and the net force is varied and a demonstration where the net force is kept constant and the total mass of the system is varied. Instructors could use any available lab equipment that allows for a variation of the force exerted on the object with added masses. Ask the students these three questions:

1. "What do you observe?"
2. "What can you measure?"
3. "What can you change?"

A guided discussion should yield some of the following answers to the questions:

1. The cart-mass hanger system is accelerated
2. Quantities that can be measured include: the mass of the cart, the mass of the hanger, distance traveled by the cart, distance traveled by the hanger and the slotted masses, the time of travel, etc.
3. Quantities that can be changed are the net force on the system and the total mass of the system

Students may have difficulty identifying the net force exerted on the system. Drawing free-body diagrams might help in determining that the net force on the system is equal to the gravitational force by the earth on the hanger and slotted masses. Some students will indicate that a force of kinetic friction is exerted on the cart.

Part II

After the discussion students are instructed to design two data collection strategies to determine how two factors affect the acceleration of the system: the net force on the system and the total mass of the system.

Activity 1: Students design procedures that include calculation of the acceleration when the total mass of the system is kept constant and the net force is varied.

Activity 2: Students design procedures that include calculation of the acceleration when the total mass of the system is varied and the net force is kept constant.

***A few tips*:**

- Students should be discouraged from trying to combine the two activities into one.
- Students should be careful to keep the string parallel to the track throughout the data collection.
- The length of the string connecting the cart to the mass hanger should allow the mass hanger to reach the floor just before the cart reaches the pulley.
- Make sure that the string does not rub against anything, such as the pulley mount.

Extension

An extension to this lab is to investigate the effect of friction on the acceleration of the cart.

Alternative investigations that use dynamics concepts can be provided as challenges. Two examples of this type of activities have been published in *The Physics Teacher*, a publication from the American Association of Physics teachers (see Supplemental Resources section below):

- "Turning a Common Lab Exercise into a Challenging Lab Experiment: Revisiting the Cart on an Inclined Track"
- "Time Trials — An AP Physics Challenge Lab"

Another engaging extension activity consists of having the students apply the concepts learned in this investigation to their favorite sports. Students could do short presentations in the class or they could create a poster with their findings if time for presenting is a constraint.

The Science360 Video Library, sponsored by the National Science Foundation, gathers the latest science videos provided by scientists, colleges and universities, science and engineering centers. Two of their offerings are recommended resources for students to explore: "Science of NFL Football" and the "Science of the Summer Olympics: Engineering In Sports" (see the Supplemental Resources). Each of these resources consists of a 10-part video series produced in partnership with NBC Learn.

Analyzing Results

How the students analyze their results will depend on how they decided to make measurements and complete the calculations. Some students may use a stopwatch to measure the time of the acceleration over a fixed distance. These students would then use the equations of constant acceleration motion to calculate the acceleration. Other students may choose to use motion sensors to plot the velocity vs. time for the cart. In that case, they would use the slope of the graph for the acceleration.

The sources of experimental uncertainty depend on the equipment used as the precision is limited by the apparatus resolution. In this investigation, uncertainty might be related to the measurements of time, and/or length and/or mass. Students can minimize the uncertainties by taking measurements in multiple trials and average the results. Valuable resources to support you and your students in the measurement of experimental uncertainties are included in the Supplemental Resources section.

The development of mathematical models from graphs of acceleration versus force and acceleration versus mass should be an expectation of this investigation. In order to determine the relationship between net force and acceleration, and between total mass and acceleration, the students should plot a graph with independent variable on the horizontal axis and dependent variable on the vertical axis. If students are not familiar with linearization methods (see Linearization Exercises in the Supplemental Resources section), they could be guided as they linearize the acceleration versus mass graph.

The use of multiple representations in this lab is highly recommended as it leads to a deeper conceptual understanding of Newton's Second Law. The lab report should include verbal descriptions of their observations as well as labeled free-body diagrams of the forces exerted on the system.

Sample qualitative graphs for this lab are:

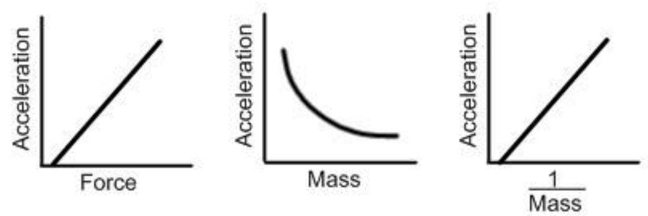

Figure 2

Some guiding questions that will help the students interpret their graphs generated in Part II of the investigation are:

Activity 1:

How does your data indicate if the acceleration was proportional to the force?

Students can determine the relationship between the acceleration and the force from the graph. A straight line represents a direct variation between the acceleration and the net force.

What does the slope of the acceleration versus force graph represent?

The slope of the acceleration versus force graph represents the mass of the system.

What is the algebraic relationship between acceleration and net force in this system?

The algebraic relationship between acceleration and net force is expressed as $a \propto \Sigma F$

Note. You may want to point out to students that the graph does not go through zero. This accounts for the frictional force between the cart and the surface.

Activity 2:

How does the data indicate if the acceleration was inversely proportional to the mass?

Students can determine the relationship between the acceleration and the mass from the graph. A hyperbola represents an inverse variation between the acceleration and the mass.

What does the slope of the acceleration versus the inverse of the mass represent?

The slope of the acceleration versus the inverse of the mass graph represents the net force of the system.

What is the algebraic relationship between acceleration and mass in this system?

The algebraic relationship between acceleration and mass is expressed as: $a \propto \frac{1}{m}$

As part of the analysis, students could find the percent difference between the theoretical value of the acceleration from one configuration of the masses using the free-body diagram of the system and the experimental value.

Note. Percent Difference: Applied when comparing two experimental quantities, E1 and E2, neither of which can be considered the "correct" value. The percent difference is the absolute value of the difference over the mean times 100.

Assessing Student Understanding

By the end of the investigation the students should be able to:

- Articulate that the acceleration of an object is directly proportional to the net force: $a \propto \Sigma F$
- Articulate that the acceleration is inversely proportional to the mass: $a \propto \frac{1}{m}$
- Determine a relationship between arbitrary combinations of mass, force and acceleration using dimensional analysis.
- Calculate the proportionality constant, *k*, for the relationship derived from dimensional analysis:
  $$a = k\frac{\Sigma F}{m}$$
- Obtain a proportionality constant value of 1.0.
- Identify the sources of experimental uncertainty and ways to minimize experimental uncertainties.

Assessing the Science Practices

Science Practice	An attempt	On the path to proficiency	Nearly proficient	Proficient
SP 1.1. The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Provides incorrect graphical representations of the relationship between acceleration and the net force and/or between acceleration and mass.	Creates flawed or incomplete graphical representations of the relationship between acceleration and the net force and/or between acceleration and mass.	Creates graphical representations of the relationship between acceleration and the net force and between acceleration and mass. The graphs may not fully reflect all aspects of the relationships among the variables.	Creates accurate and appropriate graphical representations of the relationship between acceleration and the net force and between acceleration and mass.
SP 4.1. The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Provides generally weak justification for the relevance of the variation of mass and/or net force in the system justification includes minimal reasoning and evidence.	Provides justification for the relevance of the variation of mass and/or net force in the system with occasional and/or minor errors; justification may be correct but lacks completeness.	Provides accurate justification for the relevance of the variation of mass and net force in the system with only an occasional or minor error.	Provides accurate and detailed justification explaining relevance of the variation of mass and net force in the system.

Science Practice	An attempt	On the path to proficiency	Nearly proficient	Proficient
SP 4.2. The student can *design a plan* for collecting data to answer a particular scientific question.	Presents an incomplete data collection plan to answer the question. Makes errors in identifying the variables (independent, dependent and controlling).	Designs a data collection plan to answer the question via quantitative measurements of acceleration. Measurements may not be clearly defined or articulated. Acknowledges need to consider estimated error. Accurately identifies independent, dependent, and controlling variables with few errors as follows: 1. Determination of the acceleration when the total mass of the system is kept constant and the net force is varied 2. Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.	Designs an appropriate data collection plan to answer the question via quantitative measurements of acceleration. Measurements may lack complete details. Identifies equipment (balance and meterstick and stopwatch or motion detector or photogates). Identifies appropriate data sources and estimated error. Accurately identifies and describes independent, dependent, and controlling variables as follows: 1. Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. 2. Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.	Designs an effective data collection plan to answer the question via well selected quantitative measurements of acceleration providing rationales for all choices. Accurately evaluates uncertainty in measurements. Effectively explains equipment selection for acquiring data (balance and meterstick and stopwatch or motion detector or photogates). Accurately explains different sources of error in data. Accurately identifies and explains independent, dependent, and controlling variables, and justifies choices as follows: 1. Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. 2. Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.

Science Practice	An attempt	On the path to proficiency	Nearly proficient	Proficient
SP 4.3. The student can *collect data* to answer a particular scientific question.	Collects relevant, but significantly inadequate data to determine the relationship among the acceleration, the net force and the inertial mass of the system. Provides observations and/or record keeping which are incomplete and/or inadequate for answering a particular question. Selects inappropriate mathematical routines; measurements contain many errors.	Collects appropriate data to determine the relationship among the acceleration, the net force and the inertial mass of the system. Provides observation logs and record keeping that contain several errors. Selects appropriate mathematical routines and provides measurements with few errors or only a single significant error.	Collects appropriate and adequate data to answer some aspects of the relationship among the acceleration, the net force and the inertial mass of the system with only minor errors in the precision of observation, record keeping and use of tools and conditions. Selects appropriate mathematical routines and provides measurements with only few minor errors.	Collects appropriate data to fully determine the relationship among the acceleration, the net force and the inertial mass of the system with precision of observations, accuracy of records, and accurate use of scientific tools and conditions. Accurately applies mathematical routines and appropriately uses measurement strategies.
SP 5.1. The student can *analyze data* to identify patterns or relationships.	Identifies a few legitimate pattern(s) in data, though these may be irrelevant to determine the relationship among the acceleration, the net force and the inertial mass of the system. Identifies some mathematical routines that are appropriate. Identifies some the sources of experimental error but does not suggest ways to minimize the uncertainties.	Identifies the most obvious patterns within data, relative to the relationship among the acceleration, the net force and the inertial mass of the system, with some errors and inaccuracies. Selects appropriate mathematical routines but makes some application errors. Identifies some the sources of experimental error and suggests ways to minimize the uncertainties.	Identifies most patterns within data relative to the relationship among the acceleration, the net force and the inertial mass of the system with only an occasional minor error. Selects appropriate mathematical routines and applies them with only minor errors. Correctly identifies most the sources of experimental error and suggests ways to minimize the uncertainties.	Comprehensively describes the patterns and relationships within data, relative to the relationship among the acceleration, the net force and the inertial mass of the system. Accurately applies appropriate mathematical routines. Correctly identifies all the sources of experimental error and suggests ways to minimize the uncertainties.

Science Practice	An attempt	On the path to proficiency	Nearly proficient	Proficient
SP 5.3. The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.	Fails to recognize or provide a connection to the relationship between the acceleration and the inertial mass of the system, and the relationship between the acceleration and the net force of the system.	Provides a connection, but the generalization of the relationship between the acceleration and the inertial mass of the system and/or the relationship between the acceleration and the net force of the system is not correct.	Provides a connection but no justification is offered, or a justification is offered but it is vague regarding the relationship between the acceleration and the inertial mass of the system and/or the relationship between the acceleration and the net force of the system. Attempted to represent the proportionalities among acceleration, net force and inertial mass as an equation; rearrange and solve for the constant of proportionality *k*.	Provides a connection along with a clear justification, such as the calculation of the proportionality constant (*k*) for the relationship derived from dimensional analysis to determine the relationship between the acceleration and the inertial mass of the system and the relationship between the acceleration and the net force of the system.

Supplemental Resources

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"Newton's Three Laws Of Motion." Science360. http://science360.gov/obj/video/642db496-d506-432e-85b4-4e38f75d9142/newtons-three-laws-motion (accessed December 27, 2013).

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Amato, Joseph and Williams, Roger. "Turning a Common Lab Exercise into a Challenging Lab Experiment: Revisiting the Cart on an Inclined Track" *Physics Teacher* 48, no. 5 (2010): 322

Jones, David. "Time Trials — An AP Physics Challenge Lab" *Physics Teacher* 47, no. 5 (2009): 342

AP Physics 1 Sample Lab # 2: Conservation of Energy

How does the compression of a spring affect the motion of a cart?

Central Challenge

In this investigation, students will experiment with the concept of the conservation of energy by qualitatively investigating the relationship between elastic potential energy and gravitational potential energy. The students will take a spring loaded cart and release it so that it travels up a ramp. In addition making observations and measurements, they will make predictions as to what would happen if the angle of the ramp changed. Secondly, students will experiment quantitatively with the relationship between the compression of the spring and the gravitational potential energy of the Earth-cart system. They will do this by repeating measurements of the cart on the ramp for different compressions of the spring.

Background

The gravitational potential energy (GPE) of an Earth-cart system can be calculated with the equation GPE = mgh. Total energy for a closed system is conserved and so the decrease in the spring potential energy (SPE) is equal to the gain in the GPE as the cart moves up the incline.

Conservation of energy is the hallmark organizing principle in all sciences. As the total energy of a closed system remains constant, a loss of one form of energy must be equal to a gain in another form of energy. Potential energy of a system is due to the interactions and relative positions of its constituent objects. Energy transferred into or out of a system can change the kinetic, potential, and internal energies of the system. Energy transfers within a system can change the amount of kinetic energy in the system and the amount of potential and internal energy, or the amount of different types of potential energy. These transfers of energy can be seen in many instances: amusement parks, electric generators, fluid flow dynamics and heating.

Real World Application

In this lab, students will find that the loss of spring potential energy is equal to the gain in kinetic energy of a cart. In turn, that kinetic energy then decreases as the gravitational potential energy increases. Operators of trains and trucks use these principles for emergency stops. At the train station, a huge spring is compressed to bring the train to rest should the brakes fail. Similarly, a truck driver might use an uphill ramp on the side of a road to bring the truck to rest. In the case of the train, the loss in kinetic energy is equal to the gain in the spring potential energy. In the case of the truck, the loss in kinetic energy is equal to the gain in gravitational potential energy. In both cases, some energy is converted into thermal energy.

People seeking thrills jump off bridges secured by a bungee cord. In this case, the energy transformations include a loss of gravitational potential energy and a gain of kinetic energy. The kinetic energy then decreases and is accompanied by an increase in the spring potential energy. Once again, some energy is converted into thermal energy.

In designing amusement park or carnival rides, it is also necessary to apply the principle of conservation of mechanical energy. For example, to build a roller coaster one must accurately predict the speed at the top of a loop to insure that the ride is safe.

Inquiry Overview

This lab is divided into three different parts. Each part engages the student in guided inquiry activities.

In Part I, a spring loaded cart is placed on an incline and the cart's motion is observed once the spring is released. Students are asked to design their own experiment to test how the angle of the ramp changes the motion of the cart for the same compression of the spring.

In Part II, students are asked to design their own experiment to determine how changes in the compression of the spring change the amount of increase of the gravitational potential energy of the Earth-cart system.

In Part III, students are asked to consider how to improve their experimental design to take into account overlooked aspects of the earlier experiments. As an extension, they can also begin a new experiment where the transfer of energy out of the Earth-cart system changes the compression of the spring.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 5:

Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding

5.B: The energy of a system is conserved.

Learning Objective:

5.B.3.1: The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy. (Science Practices 2.2, 6.4, and 7.2)

[Note: In addition to those science practices listed in the Learning Objectives above, the following science practices are also addressed in the various activities in this Lab: 3.1, 4.1, 4. 3, 4.4, 5.1 and 6.1. In addition, students should be keeping artifacts (lab notebook, portfolio, etc) that may be used as evidence when trying to get lab credit at some institutions.]

Skills and Practices Taught/Emphasized In this Investigation

Science Practices	Activities
2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena.	Part II: Students must find the mathematical relationship between the compression of the spring and the gain in gravitational potential energy. Since this is not a linear relationship, students will have to find alternative means of graphing and analyzing the data to secure a linear relationship (i.e. plotting the square of the compression vs the gain in GPE in the case of many data points.) The students with four data points or more should be able to show that the relation between compression of the spring and the energy the spring can provide a cart is not linear. They should also show that a quadratic relationship is supported by the data.
***3.1** The student can *pose scientific questions.*	Part I: Students make observations of a cart going up a ramp and pose a question about how the angle of the incline will change the motion. Part II: Students must pose questions about the relationship between the compression of the spring and the gain in gravitational potential energy of the Earth-cart system.
***4.1.** The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Part II: Students must decide on how to measure the compression of the spring and the change in gravitational potential energy. They must also decide on the number of trials required.
***4.3** The student can <u>collect data</u> to answer a particular scientific question	Parts I, II and III: Students must collect data as they design their own experiments and/or engage in the different data collection activities.
***4.4.** The student can *evaluate sources of data* to answer a particular scientific question.	Part III: Students consider the role that friction played in their experimental design and data collection.
***5.1.** The student can *analyze data* to identify patterns or relationships	Part II: Students must decide if their data better fits a linear model or a quadratic model.
***6.1.** The student can *justify claims with evidence.*	Part I: The students create a claim regarding the motion of the cart up different inclines (e.g. more time, more distance, more speed, more height) and then have to use their experimental evidence to support or refute their claim.
6.4. The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	Part I: Although a cart on a steeper slope will travel at a different acceleration, a different distance, and for a different elapsed time, the Earth-cart system will gain an identical amount of GPE. This allows the student to use the theory of conservation of energy to make claims and predictions about the investigation.

7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.	Part II: The relationship between the compression of the spring and the gain in height leads to an understanding of the conservation of energy where the compression of the spring is related to the spring potential energy and the gain in height corresponds to a gain in gravitational potential energy. Part III: The conservation of energy principle (an enduring understanding) does not result in constant total energy in this experiment. Students can recognize that this is due to the fact that the system is not closed since there are losses of energy due to friction.

*See Note above.

Equipment and Materials

Each group will need:

- Low friction dynamics cart with spring bumper (or spring loaded plunger cart)
- Ramp
- Meterstick
- Stopwatch
- Assorted Masses
- Books or blocks (to create incline)
- Poster-size whiteboards for sharing group work

Timing and Length of Investigation

Teacher preparation:

10-15 minutes for set up

Part I:

Procedural time: 20 minutes (this includes pre-lab time)

Post-Lab Presentation/Discussion: 20 minutes (or allow 5-10 minutes per group)

Part II:

Pre-Lab/Demonstration Discussion: 10-15 minutes

Procedural time: 40 minutes

Post-Lab Presentation/Discussion: 40 minutes (or allow 5-10 minutes per group)

Part III:

Discussion 15 minutes

Prcedural time to repeat experiments: 30 minutes

Post-lab presentation: 20 minutes

Total time: 3.5-4 hours.

Please note that these times assume familiarity with the equipment and approaches.

Safety

Students must be reminded that the carts should not be on the floor where someone could slip on one. They should also consider how the spring-loaded cart could hurt someone if the plunger released near the body, especially the eye. All general lab safety guidelines should always be observed.

Preparation and Pre-Lab

Part I of this investigation will serve to determine students' prior knowledge regarding the change in the gravitational potential energy of the Earth-cart system. This will also serve as the pre-lab for Part II.

Common Student Challenges

Part I:

Students should observe that changing the angle of the ramp will change the distance traveled, the acceleration of the cart, and the elapsed time to reach the top. The students will have to design a way in which to accurately measure the distances that the cart travels since the cart is only at its peak for a moment. Changing the angle will not have a large effect on the height above the ground that the cart reaches. It will not be obvious to many students why the most important variable is the one variable (height) that does not change, or why it does not change.

Part II:

Since Part I should confirm that the gravitational potential energy gained by the cart-Earth system was always the same for the same compression, students should be comfortable with using the final gravitational potential energy as the quantity for the initial elastic potential energy. As students vary the compression distance, the observation should be that the cart's final height is directly related to the compression; however the relationship will not be linear. If students have only two possible compressions, they should try to look for a mathematical pattern with the two data points (linear or not linear). If there are multiple compressions permitted with the apparatus, then students should make a graph and find that it is not linear.

The Investigation

Part I: Introducing the apparatus and experimental design

Introduce this part of the investigation by setting up a demonstration with a spring-loaded cart on an inclined ramp (see Figure 1). Release the cart and have the students observe the motion.

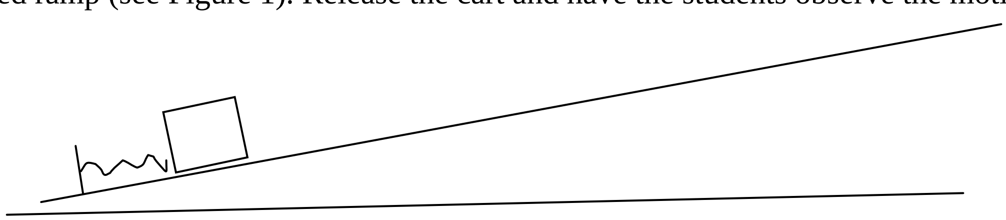


Figure 1

Prompt students:

If the cart were to be "shot" up a steeper vs. a shallower ramp, describe how its motion will change.

Note that you should not ask how the height changes, since that limits your ability to find out everything that a student is thinking about concerning the change. You can expect some students will focus on greater height, greater distance or greater time. Others may say that the cart will go a different amount up the slope while others may say that it reaches the same height and still others may say that it will take more or less time to reach the top. All are suitable responses and all can be developed into experimental designs.

Guide students:

Instruct students to first make and justify their predictions individually, then discuss those predictions in small groups, then have a whole-class discussion (do NOT reveal the "right" answer). Next, have students design an experiment with the cart and ramp to investigate the question above. Each group should discuss their design and findings, and prepare to present them to the class (individual poster-size whiteboards are great for this). As a whole class, discuss the results. If there was enough friction that it affected the results, you may need to bring it in to the discussion here. If there were negligible friction, the final height achieved would be the same in either case. However, since the distance travelled to reach the same height is larger on the smaller angle ramp, friction usually means it will not go as high. If not careful, students will use this observation to support the wrong conclusion.

In reviewing the experimental design, you should discuss whether multiple measurements should have been made for each angle and, if so, how many measurements would be sufficient. You should also ask if one angle change was sufficient or if multiple angle changes should have been made.

If this did not come up in the class discussions, in reviewing the experimental designs and results, you should raise the question of the role of friction in the experiment. If there was much more friction, how would the results have changed.

Part II: Applying the principle of conservation of energy

In this part of the investigation, students explore their understanding of energy and energy conservation.

Background:

Traditionally, students have learned that the principle of conservation of energy states that energy can neither be created nor destroyed, and the total energy of a closed system remains constant. They should have also learned that the gravitational potential energy of the cart-Earth system can be calculated with the equation GPE = mgh. You can then remind them that if energy is indeed conserved, then the work on the spring from compressing it must give it some spring potential energy (SPE).

Energy exists in the compression of the spring (spring potential energy SPE), in the movement of the cart (kinetic energy KE) and in the Earth-cart system (gravitational potential energy GPE).

Ask students:

As a way of testing student understanding of this principle for this part of the investigation, have the students answer the question below:

For each of the following four locations of the cart shown in Figure 2 below, what is the magnitude of the SPE, KE and GPE at that location? Specifically, which is large, which is small, which is zero?

Location 1: block is next to fully compressed spring

Location 2: spring is no longer compressed; block is in touch with spring

Location 3: block is halfway up the ramp

Location 4: block is at peak distance along the ramp

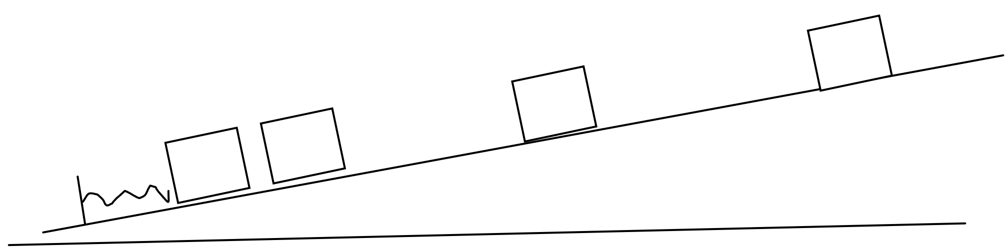


Figure 2

The students should recognize that the SPE must then be equal to the GPE of the cart-Earth system after the cart gets to its peak position and no longer has any kinetic energy KE. Ask the students if these statements are consistent with what they found in Part I of the investigation, and to explain how they are, or are not. If friction were eliminated, would the new expected experimental results be consistent with this energy explanation?

Guide students:

Introduce this part of the investigation by repeating the demonstration with a spring-loaded dynamics cart on an inclined ramp. Release the cart and have the students observe the motion. Describe to the students that we can change the SPE by compressing the spring different amounts. Some apparatus allows two possible compressions while others allow for more possible compressions.

Ask students to design an experiment to investigate how the energy (SPE) stored in the spring depends on the distance by which it is compressed. Specifically, if you increase the compression by a factor of 2, what happens to the SPE?

Part II (A): Qualitative Investigation of Potential Energy

Instruct students to design an experiment to qualitatively describe the relationship between compression of the spring and the gravitational potential energy. Students should be prepared to present a convincing argument and defend their results. Again, have small groups create a presentation to be shared with the whole class (individual poster-size whiteboards work well).

Part II (B): Quantitative Investigation of Potential Energy

Instruct students to design an experiment where they collect data in order to quantitatively support their claim. Students should complete their experiment and share their results with the class.

Part III: Improving the experimental design

There are a number of potential experimental errors. If students did not take these into account as they conducted their experiments in Part II, they should now consider them.

1. What role does friction play in the experiment? How can you minimize or take into account the frictional effects?
2. If the spring could only be compressed by two values (or if the spring could be compressed for multiple values), how would your experiment change?
3. How does the amount of compression of the plunger change the manner in which you measure the distance the cart moved and/or the maximum height?

Extension

There are a number of possible extensions to this investigation that students can choose from as well as extensions that they can create on their own.

1. How would the results change if the angle of the ramp were to change?
2. Should the experiment be done at multiple angles?
3. Which angle produces the most reliable results?
4. Do the wheels have an impact on the experimental results? Would the experiment work better with large wheels or small wheels?
5. Does the mass of the cart affect the experimental results? Which mass car would produce the most reliable results?

A more complex extension would be to have the cart descend the ramp and hit the spring. With this set-up, students can investigate how much the spring compresses. They can also investigate at which point the cart is traveling the fastest.

Analyzing Results

Part I:

Having students report on large individual whiteboards is ideal. Since this investigation is qualitative in nature, students need only present their general findings. As small groups present, be sure to call particular attention to the presentations that include convincing data (especially graphic data). There are a number of variables that could have been studied (velocity, distance traveled, height attained, and elapsed time). If these have not been investigated by any group, you can ask them for their predictions and an explanation for that prediction.

If no team chose to investigate the height attained (and you did not encourage a team that identified height as a variable to measure it), then it will be necessary to have them do so now. Some students may wonder why you did not just tell them at the outset that height is the important variable instead of letting them "waste time" on variables that, in effect, are not as helpful. But doing so would have prevented you from being able to tap into the students' sense of what variables matter and what should determine their design of the experiment; it might also have mislead them into thinking that their variables were just as valuable as height attained. Telling the students which variables to study would limit the inquiry-based methodology we are encouraging.

In reviewing the experimental design and results, you should once again discuss whether multiple measurements should have been made for each angle and, if so, how many measurements would be sufficient. You should also ask if one angle change was sufficient or if multiple angle changes should have been made.

At this point you should also raise the question of the role of friction in the experiment: if there was much more friction, how would the results have changed?

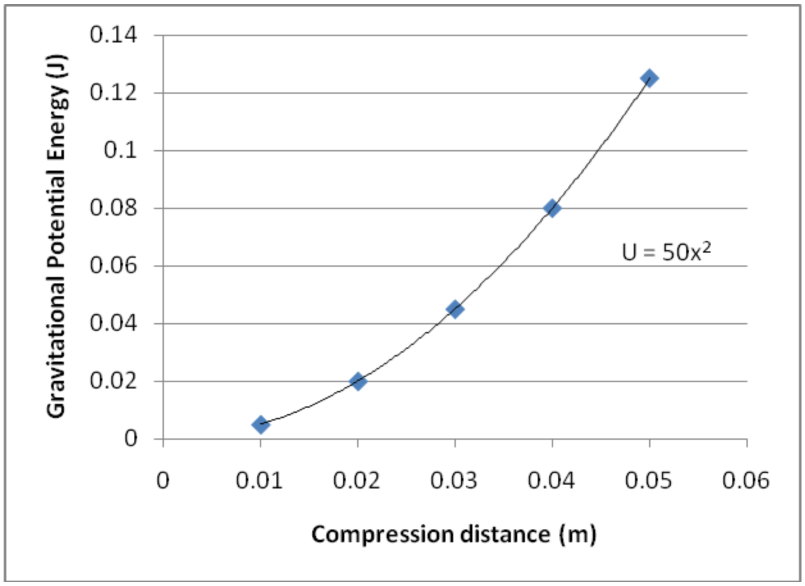
Part II:

If the apparatus has only two settings for the spring compression, that will prevent a graph from being useful. Students can still investigate if twice the compression changes the GPE by a factor of 2 or more than 2. If the apparatus allows for multiple spring compressions, then students should consider the value of making a graph.

Students should create a presentation that will provide a convincing argument supporting their findings. Have students present and discuss what was observed. Each presentation should be followed by questions from the other groups challenging the experimental technique and asking how different factors were taken into account. Students should be encouraged to come up with alternative interpretations of the data. While the whiteboard is useful for displaying procedure, data, and graphs in a way that can be easily shared, students should use a graphing program (calculator or computer software) to evaluate the trend-line; and if linear, include the equation of the line with their graph.

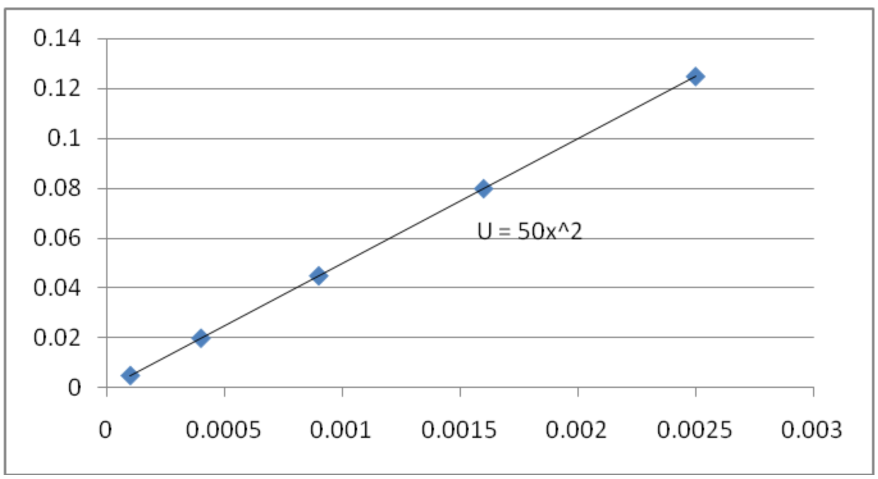
Students should include enough detail so that other groups could perform their experiment. This includes the mass of the cart, description of the ramp, measurement of the angle of the ramp, and how the compression of the spring and the final height (for GPE calculation) were measured.

When the compression distance is varied, students should observe that height increases, but the relationship is not linear, as shown in Graph 1. This function behaves as $y = x^2$, so plotting the compression distance squared vs. the gravitational potential energy will yield a linear relationship.



Graph 1*:* Gravitational Potential Energy vs. Compression Distance

An example of how to graph the compression distance vs cart height is shown above in Graph 1, and an example of how the linearized data would appear is shown below in Graph 2:



Graph 2*:* Gravitational Potential Energy vs. Compression Distance ^2

Students more familiar with approaches to making a graph linear may choose to make a log-log plot of the gravitational potential energy vs. the compression distance. They will find that the log-log plot is linear and that the slope is equal to 2 which can be interpreted as the quadratic relationship. Teachers will have to decide on whether graphs should be completed by hand or using a computer (spreadsheet or graphing program) or calculator.

With fewer than four data points, it is not possible to disprove a linear relationship graphically. With few data points, even if more than four, the graphs may not reveal the relationship that gravitational potential energy is proportional to the square of the compression distance. This can lead you to have the students investigate the uncertainties inherent in each of their measurements. What is the uncertainty in your measurement of height? How does this lead to uncertainty in the calculation of gravitational potential energy? Similarly, what is the uncertainty in the measurement of the spring compression?

Part III:

This part speaks to subtleties in the interpretation of experimental results. As extensions, students can perform additional experiments and/or explain how they would respond to these questions and/or how they would design experiments to test them.

Students should record the final product of the experiments either in their lab journal, portfolio, or on a white-board display. Have students examine the best examples and give an opportunity to move around the room and record the general procedure, data and graph, and discuss the results.

Assessing Student Understanding

Part I:

At the end of this part of the investigation, students should be able to make the following statement regarding the transfer of energy from the spring to the cart:

For a given compression of the spring, the energy transferred from the spring to the cart-Earth system produced a consistent height traveled by the cart regardless of the angle of the incline.

Part II:

Given a reminder about the calculation of GPE and the assumption that energy is conserved, students should be able to explain the energy decrease in the spring was equal to the energy gain by the cart-Earth system. At the end of this part of the investigation, students should be able to make the following statements regarding the transfer of energy from the spring to the cart:

When the compression of the spring is increased, the resulting height traveled by the cart increases non-linearly.

A doubling of the compression more than doubled the maximum height of the cart.

Students should be able to conclude that it is a quadratic relationship. They should be able to recognize that compressing the spring changed the value of the spring's potential energy SPE. Students should also see that a quadratic relationship between spring compression and SPE could account for the experimental results.

Assessing the Science Practices

Science Practice	An Attempt	On the way to Proficiency	Nearly Proficient	Proficient
2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena.	Part 2: Using a few data points, the student can graph the compression vs. the gain in GPE. Using only two data points, (due to limitations of the apparatus), the student can illustrate that an increase in the compression of the spring increases the energy the spring can provide a cart. Several errors may be present in the illustration. The student can explain the quantities expressed by variables in the equation. No calculations of SPE and GPE are attempted.	Part 2: Using multiple data points, the student can create a graph of the compression vs. the gain in GPE and determine that it is not linear. Using only two data points (due to limitations of the apparatus), the student can illustrate the relationship between compression of the spring and the energy the spring can provide a cart is not linear. Several errors may be present in the illustration. The student can identify the values needed to calculate the SPE and the GPE from the data. Attempted calculations contain several errors.	Part 2: Using multiple data points, the student can create a new graph of the square of the compression vs. the gain in GPE and determine the equation for this straight line. Using only two data points (due to limitations of the apparatus), the student can illustrate the relationship between compression of the spring and the energy the spring can provide a cart is not linear. The student can calculate the SPE and the GPE from the data.	Part 2: Using multiple data points, the student can create a new graph of the square of the compression vs. the gain in GPE and determine the equation for this straight line as well as the significance of the slope and y-intercept. Using only two data points (due to limitations of the apparatus), the student can illustrate that the relationship between compression of the spring and the energy the spring can provide a cart is not linear. The student can calculate the SPE and the GPE from the data.
3.1. The student can *pose scientific questions*	Part I: The student makes an incomplete claim regarding angle size and distance traveled. Major errors are present. The student attempts to identify the relationship between the compression of the spring and how it may affect	Part I: The student can make a claim regarding angle size and distance traveled. Several errors are present. The student can make a statement regarding an increase in the spring compression and the increase in gravitational potential energy. The student can pose scientific questions	The student can make a claim regarding angle size and distance traveled. The student can make a quantitative statement about the ratio of the compression of the spring, GPE and the measured height. The statement contains minor errors.	Part I: The student can make a claim regarding angle size and distance traveled, and will provide a quantitative estimate for its justification. Part 2: A student can make a quantitative statement about the ratio of the compression of the spring, GPE and the measured height. The student can pose scientific questions

	the height that the cart attains. Several errors in logic are present.	based on their claim.	The student can pose scientific questions based on the claim or quantitative statement.	based on the translation of their claims and quantitative statements.
4.1. The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Part 2: The student can describe the type of data being collected.	Part 2: The student identifies that measurements of the compression of the spring must be made along with the change in gravitational potential energy. The student can explain why multiple trials and measurement readings are made.	Part 2: The student demonstrates how to best measure the compression of the spring and the change in gravitational potential energy. The student can explain why at least three trials should be taken for each compression of the spring.	Part 2: The student demonstrates how to best measure the compression of the spring and the change in gravitational potential energy, and provides justification for measuring each. The student can explain why at least three trials should be taken for each compression of the spring and how more will be needed if the data has too much spread.
4.4. The student can *evaluate sources of data* to answer a particular scientific question.	The student can make a statement regarding the presence of friction. Some errors may be present.	The student can articulate that there is a transfer of energy. The student can describe the impact of friction on the data.	The student can identify that the transfer of energy is due to the work done by frictional forces. The student can explain how the results would differ if friction were somehow eliminated.	The student can identify and describe that the transfer of energy is due to the work done by frictional forces. The student can explain how the results would differ if friction were somehow eliminated. The student can describe the relationship between friction and the energy considerations of the experimental design.
5.1. The student can *analyze data* to identify patterns or relationships.	The student will observe that the data are not linear, but cannot demonstrate why.	The student will observe that the data are not linear and that a change of axes could produce a linear relationship.	The student observes that the data are not linear and that a change of axes could produce a linear relationship. The student observes the graph to be quadratic and draws a new graph with the square of the compression	Part 2: The student can demonstrate that the data are not linear and that a change of axes could produce a linear relationship. The student observes the graph to be quadratic and draws a new graph with the square of the compression distance on the x-axis.

			distance on the x-axis. The student will be able to draw the regression line.	Using the regression line, the student, can write an equation (for this line) and determine the spring constant. The student can demonstrate how a quadratic relationship is supported by the data.
6.1. The student can *justify claims with evidence.*	The student will make a claim.	The student can make a claim and provide insufficient evidence. The evidence will be based on a statement referring to possible data.	The student will make a claim regarding the motion of the cart up different inclines, and provide experimental evidence and reasoning to support or refute the claim. Minor errors are present. The student's evidence is based on experimental data.	Part 1: The student will make a claim regarding the motion of the cart up different inclines, and provide experimental evidence and reasoning to support or refute the claim. The student's evidence is based on experimental data. The student's reasoning includes the concepts of energy transfer and the role of frictional forces.
6.4. The student can *make claims and predictions about natural Phenomena* based on scientific theories and models.	Part 1: The student can state the principle of the conservation of energy with minor errors, and identifies that spring potential energy and gravitational potential energy are both present in the system.	Part 1: The student can state the principle of conservation of energy and identifies that the spring's compression is one measure of energy and that the height the cart attains, represents the gravitational potential energy from the data.	Part 1: The student can define the principle of conservation of energy and explains how the spring's compression can be used to calculate the spring potential energy. The student identifies that the height the cart attains, can be used to calculate the gravitational potential energy from the data. Calculations are attempted with several errors.	Part 1: The student can apply the conservation of energy and can explain how the spring's compression can be used to calculate the spring potential energy. The student uses the height the car attains to calculate the gravitational potential energy from the data.

7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.	Part 2: With several errors in logic, the student articulates the relationship that exists between spring potential energy, kinetic energy, and gravitational potential energy. . The student can identify that energy losses occur.	Part 2: The student can connect the concepts of spring potential energy, the kinetic energy and the gravitational potential energy to the big idea of conservation of energy with minor errors. The student can state where each energy is a maximum. The student can describe the sources of energy losses.	Part 2: The student can connect the concepts of spring potential energy, the kinetic energy and the gravitational potential energy to the big idea of conservation of energy. With minor errors, the student can track the total energy, the spring potential energy, the kinetic energy and the gravitational potential energy at many of the points on the incline. The student can explain where energy losses occur and/or what energy has not been accounted for in the experiment.	Part 2: The student can connect the concepts of spring potential energy, the kinetic energy and the gravitational potential energy to the big idea of conservation of energy. The student can track the total energy, the spring potential energy, the kinetic energy and the gravitational potential energy at all points on the incline. Part 3: The student can explain where energy losses occur and/or what energy has not been accounted for in the experiment. The student can provide upper limits to the loss of energy and make reasonable predictions of how the system would behave if the frictional forces were eliminated.

Supplemental Resources

The Hyperphysics website provides a basic explanation of the energy stored in a spring. http://hyperphysics.phy-astr.gsu.edu/hbase/pespr.html (accessed Feb. 7, 2014).

The Zona Land Education website provides a good definition of gravitational potential energy. It shows a basic derivation of the equation from work. There are also sample problems to solve. http://zonalandeducation.com/mstm/physics/mechanics/energy/gravitationalPotentialEnergy/gravitationalPotentialEnergy.html (accessed Feb. 7, 2014).

The Physics Classroom website outlines many applications of the conservation of energy. http://www.physicsclassroom.com/class/energy/u5l1b.cfm (accessed Feb. 7, 2014).

Froehle, Peter and Charles, Miller. (2012). Student Misconceptions and the Conservation of Energy. *Physics Teacher*, 50 (6), 367 – 368.

Science Practice	An Attempt	On the way to Proficiency	Nearly Proficient	Proficient
			The student can explain where energy losses occur and/or what energy has not been accounted for in the experiment.	The student can provide upper limits to the loss of energy and make reasonable predictions of how the system would behave if the frictional forces were eliminated.

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AP Physics 2 Sample Lab #1: Magnetism

How can we investigate magnetic fields?

Central Challenge

This investigation encourages students to explore the magnitude and direction of the magnetic field of magnets, current-carrying wires, and Earth, both qualitatively and quantitatively, using magnets, compasses, iron filings, and (optional) magnetic field probes.

Background

This investigation can be used to either generate or enhance students' knowledge of a vector or field-line representation of the magnetic field based on qualitative measurements. Once the vector nature of the field has been explored qualitatively, the students can make quantitative measurements of the magnitude of the magnetic field. The investigation also explores the concept of superposition, as applied to the magnetic field. While there is not significant numerical analysis, vector fields is an area of quantitative difficulty for students, and this will help in developing semiquantitative representations.

Representations of these fields are important to the skills that students need to develop in the course. This activity develops the pattern of magnetic field vectors tangent to concentric circles around a current-carrying wire; the dipole pattern of field vectors around a bar magnet are needed representations. It also helps develop the needed representations of magnetic materials as containing magnetic domains that are themselves little magnets.

Real World Application

Magnets are used in many real-life applications, such as burglar alarms, doorbells, loudspeakers, and electromagnetic motors and generators. Since the invention of the compass, the earth's magnetic field has long been used for navigation and finding direction (even birds and magnetotactic bacteria use the earth's magnetic field to navigate or orient themselves). Archeomagnetic dating is also used to study the past history of the earth's magnetic field. Other applications include MAGLEV, or magnetically levitated trains, and MRI machines. Investigations into magnetic fields will help connect students to these real-life applications.

Inquiry Overview

This guided inquiry based investigation allows students to explore the magnetic fields all around us, such as that of Earth, and the fields of permanent magnets and current-carrying wires. With proper guidance, this investigation can be implemented before students study magnetism in class. However, the investigation is also quite useful after the concept and vector nature of magnetic fields (along with some mathematical representations) have been introduced, in order to give a laboratory experience with qualitative observations and the design of procedures for measuring a magnetic field. The investigation also enhances the understanding of superposition of fields.

Part I is a qualitative investigation of magnetic fields that takes between 50 and 70 minutes of instructional time, depending on whether it is used as a basic introduction. If it is a basic introduction a little more time will be required, as students explore new concepts. If this is a first lesson on magnetism, Parts I and II should be separated by a lesson developing a quantitative model for the magnetic field of a wire.

- In Parts I (A) and (B), students explore some basics properties of magnetic fields. The instructor wanders among the students, asking questions to help the students focus on relevant phenomena.
- In Part I (C), students are asked to design experiments to explore an area of common misconception: the cause of magnetic attraction. The questions guiding the inquiry are not directly student generated, but the methods to answer those questions are significantly under students' control. Some particular techniques, such as the use of a Faraday cage, must either be directly suggested or can be motivated by a recent review of properties of electric fields.
- In Part I (D), students receive more guidance to allow them to develop a representation of the magnetic field inside of a magnet. In Part I (E), students are prompted to make the observations needed to allow them to create a representation of the magnetic field due to a current-carrying wire. In order to ensure sufficient teacher guidance in subpart (D) while keeping subpart (E) mostly student guided, it may be best to provide two separate stations with set ups for each activity so that students can rotate between them. Alternatively, subpart (D) can be carried out as a demonstration with the class. If this is a first exploration, and the concepts are new, whole-class demonstration is recommended.

In between the implementation of Parts I and II, students should have a lesson developing quantitative models for magnetic fields, or a reading and homework, if Part I was the students' introduction to magnetic fields. If class time is an issue, planning the investigation for Part II can be given as homework. Students could then make brief whiteboard presentations to the group discussing their chosen methods.

In Part II, students carry out a quantitative exploration of magnetic fields. This can take from 50–90 minutes, depending on the extensions pursued and the equipment available to the students. Again, the major guiding questions are given to them, but students must figure out exactly how to do this activity, what measurements need to be made, and how the data should be analyzed. The activity develops scientific practices beyond those associated with the learning objectives related to magnetic fields.

Connections to the AP Physics 2 Curriculum Framework

Big Idea 2: Fields existing in space can be used to explain interactions.

Enduring Understanding 2.D: A magnetic field is caused by a magnet or a moving electrically charged object. Magnetic fields observed in nature always seem to be produced either by moving charged objects or by magnetic dipoles or combinations of dipoles and never by single poles.

Learning Objective 2.D.2.1: The student is able to create a verbal or visual representation of a magnetic field around a long straight wire or a pair of parallel wires. (**Science Practice 1.1**)

Learning Objective 2.D.3.1: The student is able to describe the orientation of a magnetic dipole placed in a magnetic field in general and the particular cases of a compass in the magnetic field of the Earth and iron filings surrounding a bar magnet. (**Science Practice 1.2)**

Learning Objective 2.D.4.1: The student is able to use the representation of magnetic domains to qualitatively analyze the magnetic behavior of a bar magnet composed of ferromagnetic material. (**Science Practice 1.4)**

[Note: In addition to those science practices listed in the learning objectives above, the following science practices are also addressed in the various lab activities: 2.2, 4.2, 4.3, 5.1, and 7.1. Also, students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Skills and Practices Taught/Emphasized In this Investigation

Science Practice(s)	Activity
1.1. The student can *create representations and models* of natural or man–made phenomena and systems in the domain.	In Part I of the investigation, students learn to create representations for a variety of magnetic fields.
1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	In Part II, students create representations for the fields and use these representations to determine what measurements are necessary.
***2.2** The student can *apply mathematical routines* to quantities that describe natural phenomena.	The extension is quantitative and requires a number of calculations.
***4.2** The student can *design a plan* for collecting data to answer a particular scientific question.	In Part I (B) and (C) and Part II, students design plans to collect data, although in Part I, the data are qualitative.
***4.3** The student can *collect data* to answer a particular scientific question.	In each part of the activities, students make observations that allow them to answer scientific questions.
***5.1** The student can *analyze data* to identify patterns or relationships.	In Part II, students carry out a semiquantitative analysis to understand addition of vector fields. They must analyze data to determine magnetic field in the extension.
***7.1** The student can *connect phenomena and models* across spatial and temporal scales.	In Part I (D), students relate the field of a set of magnets to the behavior of magnetic domains in order to understand the field inside a magnet.

*See Note above.

Equipment and Materials

For each lab group generally:

- 6–10 small compasses
- three or more bar and horseshoe magnets of various sizes and shapes
- 6–10 1 $^3/_8$ " unmarked bar magnets (it is very important to have small magnets available)
- one container, such as a salt shaker, filled with iron filings
- a sheet of paper, transparency, plastic zipper bag, or sheet protector
- one 3" × 3" sheet of magnetically sensitive film (optional, for increased variety of observations)
- one pith ball or paper clip hung on an insulated string
- one rubber rod or PVC pipe (for making charged rods)
- one glass or acrylic rod
- one rabbit fur or other material (for negatively charging the rods or PVC pipe)
- one silk (for positively charging the glass or acrylic rods)
- one Styrofoam cup
- one piece of aluminum foil, large enough to cover a Styrofoam cup
- battery holders
- 10–15 copper wires (16–18 gauge) with alligator clips
- one switch
- 6–10 pieces of string to hang magnets
- one Magnaprobe (optional - a small alnico bar magnet mounted in a gimbal. It rotates in 3D so that all x, y and z co-ordinates can be mapped.)
- one magnetic field probe

For the extension:

- one linear variable resistor
- one ammeter

For Part 1(D) and 1(E) of the investigation (each set up as a single station, students may bring the compasses from one to the other):

- 20 ceramic bar magnets 1 $^7/_8$" × $^7/_8$" × $^3/_8$" that can be stacked to form a large rectangular magnet
- one flat piece of wood or cardboard (approximately 6 × 6 inches) with a hole in the middle for wire to pass through to serve as a platform for compasses
- one clamp
- one rod stand to support clamps to hold the cardboard or wood
- 8–10 1.5V D-cell batteries or a power supply to send current through the wire
- two 22-gauge wires to attach power supply to wire (optional)

Timing and Length of Investigation

Teacher preparation: 15–20 minutes

This is the time needed to set up the equipment, assuming everything is available at your school.

Student investigations: 2–2.5 hours

Part I: Qualitative Investigations

50–70 minutes (depending on if this part of the investigation serves as an introduction to magnetic fields).

Part II: Quantitative Investigations

60–90 minutes (depending on if the design of the data collection procedures is assigned as homework).

Design of procedure: 30 minutes

Data collection: 30 minutes

Data analysis: 20–30 minutes (depending on if they have access to magnetic probes or must measure Earth's magnetic field relative to the field of a long straight wire)

Post-lab discussion/presentations: 30 minutes

Total time: 2.5–3.5 hours

Safety

There are no specific safety concerns for this lab. General lab safety guidelines should always be observed.

Preparation and Pre-Lab

While students are generally very familiar with magnets, they will often hold some common beliefs that are not consistent with the scientific view. You could have an introductory discussion with students to address these misconceptions, but if time allows, it is probably more productive to allow students to do the qualitative Part I of this activity as an exploration in order to gain a more scientific view. This requires careful observation and questioning as the students investigate.

If you wish to pretest students on their understanding, you will find several good questions to pose in references 4, 7, 8, and 12 in the Supplemental Resources section, and reference 5 has two examples of what homework questions might look like early in the students' study of magnetic fields. References 9 and 10 provide on line simulations students can use to explore magnetic fields further.

Common Student Challenges

This lab is useful because by carrying out the qualitative and quantitative observations and measurements, students will, through their observations, overcome the challenges many still face after instruction. Magnetism is a subject in which some prior beliefs exist that should be altered, and this exploration also serves as a place to help students in several areas that they often have difficulty: dealing with superposition of vectors, graphing data, and interpreting graphs.

In particular, Part I (C) provides students the opportunity to demonstrate that magnetic poles are not and do not behave as clusters of positive and negative charges. Throughout all of Part I students who have not yet studied magnetism will have the opportunity to observe magnetic behavior and begin to build mental models of the magnetic field for the first time. They will develop a better understanding of the strength and the direction of the field inside a magnet; how a measured field is the superposition of the existing fields; and of the strength and direction of Earth's magnetic field. If they have studied magnetism, they will come to understand some of the challenges of the material through the laboratory observations.

Magnetic field explorations are often very interesting to students, as the macroscopic observations of the noncontact magnetic force are often exciting. You may find that students want to just play and observe magnetic effects — a little of this is encouraged before serious observations and data taking begins. After working through the lab, students should have a better understanding of magnetic fields and have improved their measurement skills. They should also be able to answer the guiding questions found in the Analyzing Results section.

If students are still struggling with particular challenges during the lab, such as the field inside a magnet or the superposition of fields, ask probing questions at that point. In the later parts of the lab, ask students explicitly about what they are graphing and why, and what information can be gained from the graph.

The Investigation

Part I: Qualitative Investigations of Magnetic Fields

You should start by hanging 1 $^{3}/_{8}$" unmarked bar magnets from the ceiling and letting them come to equilibrium. Discuss the meaning of "north-seeking" pole and "south-seeking" pole in terms of a magnet, and ask the students to label the poles. Then give them a compass and discuss which is the "north" end and which is the "south" end. This should lead to a discussion of the difference between magnetic and geographic north. Once the students are clear about how the poles of a compass are labeled, they can use the compass to explore the magnetic behavior of other systems, including bar magnets and current-carrying wires.

Part I (A): Exploring the magnetic field of the Earth

In this part students will explore qualitatively Earth's magnetic field. One way to do this activity is to use the small magnets hung from the ceiling at various points in the room. Magnaprobes, which allow rotation in three dimensions, can also be used. The magnets hanging from the ceiling or the Magnaprobes should all point in the same direction. One note of caution: in some buildings, the steel in the walls is an iron alloy that will attract a magnet. Magnets near the steel will not point in the direction of Earth's field, which should either be avoided or used for discussion. The concept of superposition of the field of permanent magnets and Earth's field can also be explored here. To do this, ask the students to explore these two questions:

- ***How does the direction of a small magnet suspended in Earth's field change when a permanent magnet is brought nearby?***
- ***How close does it have to be to see the superposition? When does one field dominate over the other?***

Part I (B): Exploring the magnetic field of a magnet

In this next activity you may choose to have the students explore the behavior of compasses placed at different locations and distances away from a bar (or other shaped) magnet. Ask the students if they have information on the existence of a magnetic field and its strength or direction based on their observations. Be sure to ask them to observe how quickly or slowly the compasses "lock" into position and if that gives them any information on the strength of the field.

They can also use iron filings or magnetically sensitive film to explore the magnetic field (see references 1-3 in the Supplemental Resources section for examples). The iron filings will not help with the direction of the magnetic field, but they will help with the strength of the field and the concept of field lines (if introduced). If students use iron filings, make sure they place any magnet to be studied either under a sheet of paper or transparency or inside a sheet protector or plastic zip-lock bag, so that the filings do not come into direct contact with the magnet (the sheet of paper might create the best contrast to see the pattern created by the filings). Magnetically sensitive film can help with the direction of the field that is perpendicular or parallel to the film. Have students draw a picture using field vectors at 6–10 points in space around the magnet to indicate the strength of the field at various locations around the magnet. It is useful to explore the fields of different shaped magnets and combinations of magnets (e.g., between like and unlike poles).

Part (C): Exploring magnetic poles

In this activity the students develop experiments to determine if magnetic poles are (or behave as) positive and negative charges. [Note: If students need a refresher on electrostatics before beginning this section, you might choose to either review the operational definition of charge and the behavior of objects in the presence of a charge, or to provide guidance to each group as they explore various options.]

Ask the students to design experiments that would either support or prove incorrect the hypothesis that the north and south ends of a magnet act as a cluster of positive or negative charges. They should write out their predictions for positive and negative charges and for north and south poles in simple situations. For example, they could predict the attraction or repulsion of a charged (or uncharged) pith ball by a charged rod, and predict the behavior of the pith ball (charged or uncharged) near a north or south magnetic pole. Once they have completed the predictions, they carry out the experiments. A charged pith ball will attract equally to both ends of a metallic magnet, so make sure students using such magnets test their predictions at both poles.

Direct students to explore the interaction of a charged pith ball or paper clip hung inside a Faraday cage with a charged rod outside the Faraday cage, then with a magnetic pole outside the Faraday cage. A simple example of a Faraday cage is a paper cup covered in foil. The students should observe the behavior of both north and south magnetic poles and positively and negatively charged rods. They can also observe the behavior with just the Styrofoam cup (not coated with foil). They should perform enough experiments so that they consistently observe (and come to understand) that north and south poles are not a cluster of positive and negative charges. It is imperative that they make sure the pith ball or paper clip is not too close to the upper edge of the Faraday cage, or the shielding will not be effective. If you did not cover the Faraday cage in your discussions on electric fields and conductors, this is a good time to ask the students what they predict the electric field to be inside a conductor, and develop an explanation of the behavior of the Faraday cage.

Part I (D): Determining magnitude and direction of a magnetic field (superposition of magnetic fields)

In this activity the students are challenged to determine the magnitude and direction of the field inside a magnet. [Note: Students might need significant direction with this activity in order to relate the phenomena they observe with the superposition of fields due to multiple magnets. This may be most effectively done as a demonstration with strong student involvement followed by a class discussion. Another option would be to have each group cycle through this station (in parallel with students at the current-carrying wire station (see Part I (E) below), which will not need as much teacher direction) and have each group create a whiteboard representation of the results of their exploration. This way you can check that they have developed the correct understanding while they are still in the activity.]

One way to approach this part of the investigation is to use the concept of superposition of magnetic fields. First, ask students to observe the superposition of fields outside of magnets by observing the change in direction of compass needles near a magnet when a second or third magnet is brought close to the first. If done as a whole group, use a compass that is clear on both sides and place it on an overhead along with the magnets; or use any compass with an Elmo-like projection system. Then, have students stack the 20 ceramic bar magnets to simulate a bar magnet that can be "broken" so as to enable them to "see" the field inside. The direction of the field inside can be determined by the following process:

1. With the stack of magnets in the shape of a bar magnet lying horizontally on a table, remove half of the stack and replace it with a small compass at the end of the remaining stack.
2. Observe the direction of the field caused by the half of the magnet on the table.
3. Remove the compass and replace the half that was removed.
4. Remove the other half and replace it with the compass at the end of the stack now on the table. (This will allow an observation of the direction of the magnetic field at approximately the same position caused by the second half of the stack.)
5. By superposition, the field inside the total stack, with both halves together, must be the sum of the fields caused by each stack independently.
6. To help students see that the field is stronger when both halves are together, be sure students observe that the field varies from straight out from the end of the bar magnet

more quickly (for magnets placed perpendicular to Earth's field) as they weaken the magnet, and relate this to the net force on the needle.

Students determine that the field inside of a magnet points south to north and that it should be quite strong. This type of analysis is often presented in textbooks in a discussion of magnetism and magnetic domains at the microscopic level, but this activity provides a way to analyze magnetic fields macroscopically. More details and pictures of a setup can be found in a number of available teaching materials, such as the Magnetism section of the book *Tutorials in Introductory Physics* by the University of Washington Physics Education group or online (see references 4 and 5, respectively, in the Supplemental Resources section).

Part I (E): Exploring the magnetic field of a current-carrying wire

Now students will observe qualitatively the field of a current-carrying wire using compasses.

[Note: Since this activity might require a relatively high current and use a significant amount of table space, it is beneficial to set up a separate station for it and let the student groups rotate through as they complete the previous activities.]

Setup a circuit with the D-cell batteries or a power supply, a switch, and low resistance. Pass the wire vertically through a hole in a piece of cardboard or a piece of wood, supported by clamps and a stand, which the compasses can rest on. For images of a setup, see Resources 5 and 7. The switch should only be closed for brief periods as observations are being made.

Students should note the direction of the magnetic field when there is no current through the wire. When the switch is closed, the compasses will point in the direction of the magnetic field of the wire (the current must be large enough to make Earth's field irrelevant). The switch should be held down just long enough to see the effect and then opened, or the batteries will run down quite quickly. The batteries or power supply can then be turned around in order to observe the direction of the magnetic field when the direction of the current is reversed. Encourage students to draw a picture of the wire using magnetic field vectors at various points around the wire to indicate the strength of the field, which as previously observed is qualitatively determined by how quickly the compass aligns with the field. Ask students to draw both side and end views of the wire. [Note: with fewer batteries, or as batteries run down, the field may not be strong enough to make qualitative observations of the decrease in the magnetic field with distance from the wire.]

Part II: Quantitative Investigations of Magnetic Fields

In this part of the investigation students measure the magnitude of the magnetic fields of various combinations of magnets, using a magnetic field sensor. This should be done at various distances from the magnets. Different sensors work in different ways, so it is important that you properly instruct your students in their use.

While measuring the magnitude of a magnet's magnetic field it is necessary to take Earth's field into account, in order to obtain just the field of the magnet. Before the students design their data collection procedures (either as homework or at the beginning of this activity), ask them to figure out different ways to do this. One option is to take measurements on both sides of the magnet: the first on the side where the direction of the Earth's field is opposite that of the magnet's field, and the second on the side where the direction of Earth's field is in the same direction as the magnet's field. The Earth's field will cancel out when the two measurements are added together, yielding just twice the magnitude of the desired field. Other ways to obtain the field of the magnet include zeroing the sensor far from the magnet; fixing the sensor in one location and zeroing it, then moving the magnet closer to it at different locations; or taping the sensor to the table, then moving the bar magnet's N end to various distances along a line that is normal to the sensor surface.

[Note: Before proceeding, ask for volunteers to present their draft procedures to the class, and solicit feedback from the various groups.]

During the investigation, the students should take measurements in the following manner and sequence:

1. Take measurements surrounding one bar magnet.
2. Repeat the same measurements for a different bar magnet.
3. Place the first magnet on top of the second and repeat the same measurements.
4. Reverse the orientation of the top magnet and repeat the measurements.

Instruct students to create a vector field representation to semi-quantitatively justify the results of their measurements. Additional magnets can be stacked to further vary the field. Ask students to make predictions about what they expect to measure before taking additional measurements.

Extension

Once students have studied the form of the equation for the magnetic field of a straight wire in class, as an extension they can measure the horizontal component of Earth's field using a current-carrying wire as a reference (finding the magnetic field from the equation for a long current-carrying wire at a given distance and current) by determining the direction of Earth's magnetic field, placing the wire such that the horizontal field produced by the wire is perpendicular to Earth's field, and varying the current (measured by an ammeter) in the wire until a compass placed next to the center of the wire (so the long wire approximation is most valid) is deflected to an angle 45 degrees from Earth's field. Students can plan the details of this measurement and for other angles. This is just an example: at 45 degrees the magnitude of the two fields are equal so the analysis is simplified greatly.

One possible source of error here is that the surface on which the experiment is carried out may contain materials that can become magnetized. Discussing the impact this would have on their measurements helps students achieve the AP Physics 2: Algebra-Based goal of being able to discuss what happens when some parameter of a physical situation is changed. It always helps to ask students to remember to make careful observations and consider their control variables.

Analyzing Results

The qualitative results can be analyzed by having the students draw magnetic field vectors or magnetic field lines for the various magnets or current-carrying wires they are observing. They can also discuss the strength and direction of the field at various points away from a magnet or current-carrying wire. It is particularly useful to ask students explicit questions about challenging topics. The guiding questions for this lab:

- Based on qualitative laboratory explorations, what evidence do we have of the existence of magnetic fields, and how can the magnitude and direction of the fields be represented by either field lines or vectors?
- Are magnetic poles the same as positive and negative charges, and what experiments could be done to demonstrate this?
- How can we use the concept of superposition of fields to determine the strength of the magnetic field inside a magnet?
- How can we qualitatively and quantitatively measure the strength and determine the direction of Earth's magnetic field?
- Are permanent magnets the only way to create a magnetic field?
- Do stationary or moving charges give rise to a magnetic field?
- Do we have to account for Earth's magnetic field when measuring the magnetic field of magnets or current-carrying wires?
- How could we use the magnetic field of a current-carrying wire to measure the magnetic field of the earth?

Assessing Student Understanding

By the end of the investigation, the students answer the questions that guide the investigation. Students should understand and be able to articulate how to determine the magnitude and direction of the magnetic field at a point in space qualitatively, and how to make a quantitative measurement of the strength of the magnetic field.

They should understand and/or be able to:

- Articulate that magnetic poles are not clusters of positive or negative charge
- Visualize the magnitude and direction of the magnetic field due to a magnet and due to a current-carrying wire at various points in space and be able to draw pictures using magnetic field vectors to represent the field at those points in space
- Conceptualize the magnitude and direction of Earth's magnetic field at various points in space and be able to experimentally measure the strength of the field and determine its direction.
- Articulate the superposition of magnetic fields.
- Use the superposition of magnetic fields to design an experiment to measure an unknown field, given a reference field.

[Note: Some questions useful for assessment can be found in references 4–10 in the Supplemental Resources section.]

Assessing the Science Practices

Science practice	An attempt (1)	On the path to proficiency (2)	Nearly Proficient (3)	Proficient (4)
1.1. The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	The representation is present but it is difficult to understand AND more than one main feature of the phenomenon is not represented; OR more than one necessary detail is missing; OR the model does not account for the most important features of the phenomenon or it is not experimentally testable.	The representation is present but it is difficult to understand; OR more than one main feature of the phenomenon is not represented; OR more than one necessary detail is missing; OR the model does not account for the most important features of the phenomenon or it is not experimentally testable.	The representation is comprehensible but does not show all of the main features of the phenomenon; OR one necessary detail is missing; OR the model accounts for the most important features of the phenomenon and is experimentally testable but the limitations and assumptions are not mentioned.	The representation has the following properties: 1. Appropriateness and correctness of the kind of representation used to represent the magnetic field of a current-carrying wire (drawing magnetic field vectors that form concentric circles and have no component toward the wire). 2. Inclusion in the visual representation of features representing essential aspects of the phenomenon (the magnitude of the magnetic field is proportional to the magnitude of the current in the long, straight wire; the strength of the field decreases as the distance from the wire increases [qualitative dependence]; the direction of the field is determined by a right-hand rule). 3. Appropriateness in the level of detail of the features of the representation (indicating the subtraction or addition of field's different sources). 4. Accuracy in the labeling of the elements of the representation (labeling the current and

Science practice	**An attempt (1)**	**On the path to proficiency (2)**	**Nearly Proficient (3)**	**Proficient (4)**
				the magnetic field and indicating how the magnitude of the field decreases with distance from the wire or wires). 5. Causal explanation expressed in the representation (i.e. that the magnetic field is caused by the current and decreases as the distance from the current increases). 6. Correct superposition of the magnetic field to find the direction of net field.
1.2. The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.	The student extracted information from the representation but focused on irrelevant features AND while describing a model the student omitted multiple important aspects.	The student extracted information from the representation but focused on irrelevant features; OR while describing a model the student omitted multiple important aspects.	The student extracted relevant information from the representation but one important piece of information is missing; OR while describing a model the student made one important omission.	The description includes: 1. A compass needle is a permanent magnetic dipole with “north” and “south” polarity. 2. Iron filings in a magnetic field become induced magnetic dipoles. 3. Dipole orientation is anti-parallel with field direction. 4. The magnetic north of a compass needle points toward the magnetic south of Earth, which is near Earth’s geographic north pole.
1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	The student can utilize a domain model but cannot accurately use it to predict how the bar magnets together act like a single magnet.	The student can utilize a domain model to predict how the fields of two bar magnets add or how bar magnets in the same direction together act like a single magnet but has trouble	The student can utilize a domain model and uses it to predict how the bar magnets together act like a single magnet but some feature of the prediction is missing or inaccurate.	The use of the model includes: 1. Accuracy of the representation of the domain model of ferromagnetic materials (bar magnets have magnetic fields and are composed of small

Science practice	An attempt (1)	On the path to proficiency (2)	Nearly Proficient (3)	Proficient (4)
		considering multiple magnets or cases when all domains are not aligned.		regions called domains, which each have their own internal magnetic field; the atomic-scale structure can be represented by arrows or smaller bar magnets that indicate the directional nature of magnets even at small scales; domains are magnetic dipoles, thus there is no beginning or end to the magnetic field — it is a continuous loop for both a domain and a bar magnet). 2. Accuracy of the translation between the structure of a bar magnet and the magnetic domain representation. 3. Accuracy of the predictions based on the magnetic domain representation (if a bar magnet is broken in half, both halves are magnetic dipoles and have magnetic fields; no magnetic north pole has ever been isolated from a south pole).
2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena (extension).	The student attempts to create a mathematical representation, but significant errors in the use of symbols, equations, etc., are present.	The student uses appropriate terms or symbols in developing an equation (mathematical representation) but there is an error in the form of the equation.	The student is able to accurately mathematically represent the form of the appropriate magnetic field for the method chosen and the vector relationship between the reference field and the field under study.	The student is able to substitute values into the developed equation. The student correctly solves by manipulating the equation.

Science practice	An attempt (1)	On the path to proficiency (2)	Nearly Proficient (3)	Proficient (4)
4.2. The student can *design a plan* for collecting data to answer a particular scientific question.	The student made an attempt at designing a plan to collect data; however, this plan was not aligned to the question or problem being investigated. The plan contained significant misalignments and errors.	The student is able to come up with some aspects of a plan to collect data in response to teacher questions but student produced plan is not adequate to answer question.	For Part I, student designs an appropriate collection plan to answer question, with appropriate included qualitative measures	For Part II, student designs an appropriate collection plan to answer question, with appropriate included qualitative measures and adequacy of justification for the selection of equipment.
4.3. The student can *collect data* to answer a particular scientific question.	The student attempts the collection of data; but the data is not appropriate for the investigation and the collection strategies contain major errors.	The student is able to implement some aspects of his or her plan to collect data but the data was not taken with sufficient care or contained some mistakes or unnecessary inaccuracies.	The student is able to implement all parts of the plan to collect data but not with sufficient care; OR the student collects data with appropriate care but not all parts of the planned collection. The weaknesses in data collection weaken the conclusion that can be drawn from the data.	In each part of the activity, students make observations that allow them to answer scientific questions.
5.1. The student can *analyze data* to identify patterns or relationships.	The student attempts to use data, but fails to identify any patterns and illustrates many errors when representing vectors.	The student can use the direction of the compass to identify the direction of a field of a single magnet but cannot use a vector representation to show addition of vectors.	The student can identify the directions from the compass and apply the vector representation to add fields of individual magnets but makes some errors.	The student is able to identify the patterns in the data relevant to the question of the relationships between individual fields and a resultant field or between the direction of field of interest and direction of reference field in the extension. The student uses appropriate terminology to describe the relationships.

Science practice	An attempt (1)	On the path to proficiency (2)	Nearly Proficient (3)	Proficient (4)
7.1. The student can *connect phenomena and models* across spatial and temporal scales.	The student makes an attempt to connect the external magnetic field to the concept of internal magnetic domains, but many errors are present.	The student is able to articulate that magnets can be made up of smaller magnets but in terms of physical magnets, not at the scale of magnetic domains.	The student is able to articulate a relationship between the external observed field and the internal structure of the magnet as being multiple magnetic domains but in terms of all of the domains being aligned within the magnet.	The student is able to articulate the relationship of the external observed field to the internal structure of the magnet as being made of multiple magnetic domains, relating this to the addition of many magnets and that the magnetic field represents the direction of the net field, so that all the individual magnets do not have to be aligned.

Supplemental Resources

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5. Beth Thacker, "PHYS 1404," Beth Thacker Physics Faculty Teaching Page. Accessed April 15, 2013, http://www.phys.ttu.edu/~batcam/Courses/sem2.html.

6. David P. Maloney, "Charged poles?" *Physics Education* 20.6, (1985) 310-316.

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8. D.P. Maloney, T.L. O'Kuma, C.J. Hieggelke, & A. Van Heuvelen, "Surveying students' conceptual knowledge of electricity and magnetism," *American Journal of Physics, 69*, S12 (2001).

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10. PhET Interactive Simulations, University of Colorado at Boulder, "Magnets and Electromagnets," accessed Feb 6, 2014, http://phet.colorado.edu/en/simulation/magnets-and-electromagnets

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AP Physics 2 Sample Lab # 2: Geometric Optics

How do we find the focal length of a thin lens?

Central Challenge

In this investigation the students are given a converging lens and asked to make measurements that can be graphed to find the focal length of that lens.

Background

Lenses are formed by shaping glass or plastic into one of two general shapes. Lenses that are convex on both sides, and thus thicker in the middle than on the edges, take rays that are incident on the lens parallel to each other and converge them to a single point, called the *focal point.* These are called *converging lenses*. Lenses that are concave on both sides, and thus thinner in the middle and thicker on the edges, take rays that are incident on the lens parallel to each other and diverge them as if they came from a single focal point. These are called *diverging lenses* (in fact, in air, any lens that is thicker in the middle, for example a plano-convex lens, is a converging lens; and any lens that is thinner in the middle, for example a plano-concave lens, is a diverging lens).

Snell's law governs the way that rays are refracted through each lens. The path of light is reversible, so light rays that are incident through the focal point will be refracted parallel to the *principal axis,* a line that passes through the lens, perpendicular to the plane of the lens. We can use these facts to trace specific rays from an object through the lens to determine, qualitatively, where the image is formed.

We define the following terms:

- *image distance* - the distance from the image to the center of the lens
- *object distance* - the distance from the object to the center of the lens
- *focal length* - the distance from the focal point to the center of the lens

We can use the geometry of similar triangles to derive an equation that relates the image distance, the object distance and the focal length (Equation 1). This is known as the lens equation.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Equation 1: Lens Equation

Real World Application

Both diverging and converging lenses are used to correct vision and many students are curious about how their glasses (or contact lenses) work. During your unit on optics, you can include a discussion about how the two types of lenses are used to correct vision, and how the focal length of each lens relates to how good (or poor) an individual's vision is. In particular, you could show how converging lenses are used for reading glasses. The focal length is long enough that the object distance is always less than the focal length, and the image produced is virtual, enlarged and upright. Students could use ray boxes (which create parallel beams of light) to examine how their own glasses refract the light and determine for themselves whether they are diverging or converging.

You could also discuss how diverging lenses, which are used to correct near-sightedness, make virtual images that are always closer to the lens than the object. If you want to expand upon this activity, you could look into multi-lens systems. Multi-lens systems have many applications including telescopes, binoculars, microscopes and other optical instruments.

Inquiry Overview

This lab is designed to provide students a guided-inquiry experience: you give them the question to answer, and they design the experimental procedure. After the students have gained experience using the lens equation, present them with a converging lens of "unknown" focal length. The students will then design an experiment to determine the focal length of the converging lens, make a graph, and use the graph to determine the focal length.

If some students complete the above task in significantly less time than their peers, you can provide them with a diverging lens and ask them to find its focal length. As the diverging lens does not form real images, they will have to use the converging lens in a system with the diverging lens to create a real image and measure image and object distances. Alternatively, you could require all students to find the focal length of both lenses.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 6: Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.

Enduring Understanding 6.E: The direction of propagation of a wave such as light may be changed when the wave encounters an interface between two media.

Learning Objective 6.E.5.1: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the refraction of light through thin lenses. (Science Practice 1.4, 2.2)

Learning Objective 6.E.5.2: The student is able to plan data collection strategies, perform data analysis and evaluation of evidence, and refine scientific questions about the formation of images due to refraction for thin lenses. (Science Practice 4.1, 5.1 and 5.2)

[Note: Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Skills and Practices Taught/Emphasized In this Investigation

Science Practices	Activities
1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	The students will form an image using an illuminated object, a converging lens and a screen.
4.1. The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	The student will measure image distance and object distance from the lens.
2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena. **4.1.** The student can *justify the selection of the kind of data* needed to answer a particular scientific question. **5.1.** The student can *analyze data* to identify patterns or relationships.	The students will compare the lens equation to their data and determine what to plot to get a straight line that can be used to determine the focal length.
2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena. **4.1.** The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Students will draw a best fit line to the data and extrapolate to find the intercept, the reciprocal of which is the focal length.
1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively. **2.2.** The student can *apply mathematical routines* to quantities that describe natural phenomena. **5.2.** The student can *refine observations and measurements* based on data analysis.	(Optional) Students will use a converging lens in a system with a diverging lens to determine the focal length of the diverging lens.
***4.3** The student can *collect data* to answer a particular scientific question.	Throughout the lab, students make observations that allow them to answer scientific questions.

*This science practice is in addition to those listed in the learning objectives connected to this lab.

Equipment and Materials

For each group of 2-4 students:

- Light source such as a clear lamp with a filament or a candle (either wax or battery operated).
- Converging lenses, focal length 15–25 cm
- Lens holders
- Meter sticks
- Index cards for screen (5”x7” or larger)
- (Optional) Diverging lens

Timing and Length of Investigation

Teacher Preparation Time – 15 minutes

The set-up time for this investigation is minimal and it should only take you about 10 minutes to retrieve the equipment from storage and make it available to the students.

Student Investigation – 90 minutes

This time will be needed for an introductory presentation of the question, and for your students to design the experiment and collect the data.

Extension (Diverging Lenses) - 60 minutes

This time will be needed for students to work through a double-lens system to create a real image. They will need this much time to design their set-up, make their measurements and calculate the focal length of the diverging lens.

Post-Lab Discussion – 45 minutes

The following day, you can engage the students in a discussion of the precision and accuracy of their results. In particular, you can tell them the focal length specifications for each groups’ lenses, have them compute a percent difference, and then engage in a discussion about the sources of uncertainty.

Total time: 2.5-3 hours.

Safety

If wax candles are used as a light source, be aware of the danger of the student’s hair or clothing catching fire. Students with long hair should pull it back and fasten it so it doesn’t get near the candle. Students should secure any loose clothing (shawl, sweater, etc.) so it doesn’t hang near the flame. Other than that, there are no safety concerns with this lab. General lab safety guidelines should always be observed.

Preparation and Pre-Lab

This investigation should follow treatment of Snell's Law and refraction. You can transition to convex lenses by having the students consider how a ray is refracted at each surface of a converging lens. Demonstrate for the students how a ray parallel to the principal axis is converged through the focal point. Inexpensive laser levels (or other laser pointers) can be used to create a bright ray that the students can see as the ray is refracted through a piece of Plexiglas shaped like a converging lens. Several of these rays will converge to a focal point on the far side of the Plexiglas. Alternatively, if available, a Black Board Optics kit may be used, or ray boxes that produce parallel rays although these produce smaller effects and need to be used in smaller groups.

Then the students should be given instruction in ray tracing. Demonstrate ray tracing for the students for a converging lens forming both a real image and a virtual image, and for a diverging lens. Then provide them the chance to practice on their own. You can refer them to the Northwestern Virtual Physics Lab web site (see Supplemental Resources) where they can move the object and see the corresponding location of the image for both kinds of lenses.

You can either derive the lens equation for them from geometric principles and a ray diagram, assign them to read it from the text, or just simply present them with the equation and the conditions for its application. The students should be given several practice problems for homework in the application of the lens equation.

Once the students are familiar with ray tracing, the lens equation and its use in problem solving, you can then present them with the question and the converging lens.

Common Student Challenges

Students will probably easily understand that they need to use the lens to create an image, and then measure the object distance and the image distance. They may need to be reminded that when using the lens equation, these distances are measured from the lens, not from each other. They may also need to be encouraged to make sure that their image is well-focused and sharp.

Students probably will also need to be reminded to measure the image distance several times and take an average. Students are not very good at taking multiple independent measurements, and thus will need to be reminded of techniques they can use to do this, such as having each group member take their own measurement of the image distance.

They will also probably have some difficulty determining what to plot on each axis of their graph and how to use it to determine the focal length. Each group should be allowed to struggle with this a bit, and you should refrain from having a whole group discussion before the groups get a chance to plot their graphs. If they struggle, you could encourage each group to write out the equation relating image, distance, object distance and focal length (i.e. the lens equation) and to write out the generic form of the equation for a straight line ($y=mx+b$). They should then be encouraged to consider what the dependent variable is, what the independent variable is, and make comparisons between the two equations.

The Investigation

You will task each group to design an experiment to determine the focal length of a converging lens by taking measurements and creating a graph. Lenses with focal lengths between 15 cm and 25 cm work well. You can either provide the students the equipment they need, or have them decide what they will need and ask for it. If they get stuck at this initial stage, ask them which equations they know that relate to focal length, and prompt them to explain that equation and all the terms in it. This should help them zero in on what they want to measure.

The students will hopefully decide to form real images with a light source and screen, and then measure the image distance and object distance. Ask them to identify what the independent variable and dependent variable are. Once they have done this, ask them to consider how to make multiple measurements of the dependent variable for each chosen value of the independent variable. You may need to suggest to them that each member of their team should independently locate the image and record the image distance for each object position. That way they will have several independent values of the dependent variable. They can then take the average of these values, and use them to determine the uncertainty in their measurements. The average of the image distance should be used for the graph. The students should make measurements of the dependent variable (image distance) for at least five different values of the independent variable.

Once the students have completed the data collection portion of this lab, they should consider how to plot the data in order to determine the focal length. As mentioned above in the section on common student challenges, this is usually difficult for students and they may need significant guidance.

One approach is to graph the inverse of the image distance on the vertical axis and the inverse of the object distance on the horizontal axis. This should produce a straight line with a slope equal to -1. The focal length can then be found as the inverse of the y-intercept. However, since the lens equation is symmetric with respect to object and image distance, the students can reverse which quantities are plotted on each axis, and the inverse of the focal length will still be the intercept. In fact, it will be both the x-intercept and the y-intercept.

Another approach is to algebraically rearrange the lens equation and plot the product of the image distance and the object distance on the vertical axis, and the sum of the two on the horizontal axis. When plotted this way, the focal length is the slope of the graph.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{f} = \frac{d_i + d_o}{d_i d_o}$$

$$d_i d_o = f(d_i + d_o)$$

A third approach would be to have the students plot the image distance versus the object distance and use the asymptotes to find the focal length. When the data is plotted this way, the graph is asymptotic both to the vertical line at $d_o = f$ and to the horizontal line $d_i = f$. If data of a great enough range is taken, these asymptotes will be easier to determine. However, this is not the most precise way to find the focal length, and students, after attempting this method, should be encouraged to consider one of the two previous methods.

Extension

Often there is one group of students that finishes far ahead of the other groups. This group should be asked to find the focal length of a diverging lens, which, by itself, only produces virtual images. Alternatively, you could ask all the students to consider how they could find the focal length of a diverging lens. To produce a real image, a system of two lenses, one converging and one diverging, is needed. Each group of students could determine the exact details of the how they would arrange two lenses to create a real image, the location and size of which can be measured and used to calculate the focal length of the diverging lens.

Typically, the simplest arrangement of lenses for this task is to place the object next to the diverging lens, a converging lens on the other side of the diverging lens, and the screen (where the real image will be focused) on the far side of the converging lens, as shown in Figure 1 below. The focal points of the diverging lens are labeled *f*, and the focal points of the converging lens are labeled *f'*. The virtual image formed by the diverging lens acts as the object for the converging lens. The final image distance is measured from the screen to the converging lens. Given the focal length of the converging lens, the students can then calculate the location of the intermediate image formed by the diverging lens. This intermediate image is serving as the object for the converging lens.

Using the object distance (for the converging lens) and the distance between the lenses, the students can calculate the location of the intermediate image. Once the students know the location of the intermediate image formed by the diverging lens, they can use that, and the original object distance from the diverging lens, to calculate the focal length of the diverging lens.

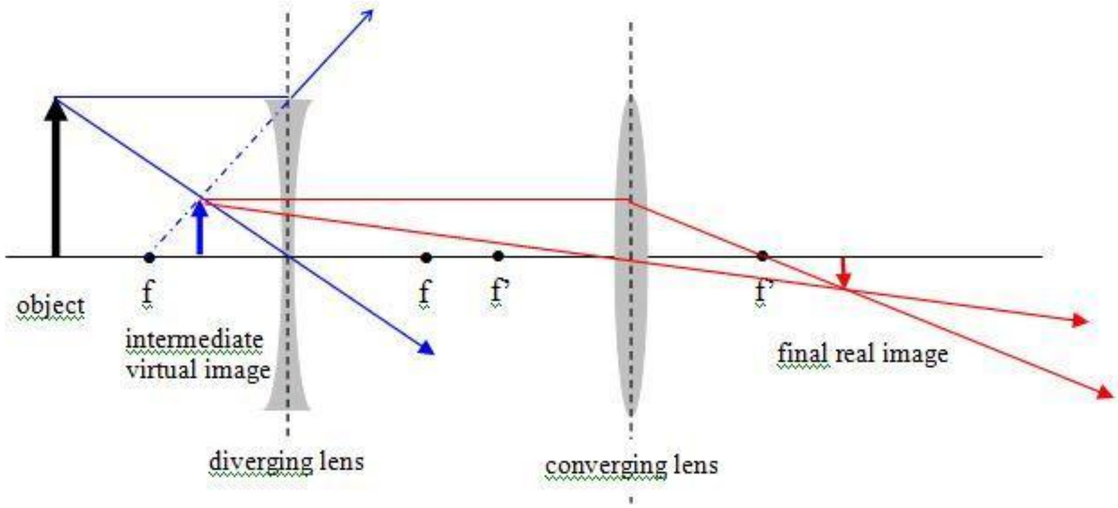

Figure 1

In drawing a ray diagram for a double-lens system, it is typical to simply address the image formation process by one lens at a time. In Figure 1, the rays shown in blue are refracted first by the diverging lens to form the intermediate virtual image. The rays which were traced to form this image are not useful for finding the final, real image. So the intermediate virtual image is used as the object for the converging lens. The rays shown in red, parallel to the principal axis and through the center of the converging lens, are drawn from the intermediate image to the converging lens, and shown as NOT refracted at the diverging lens. This is a convention used for the purpose of using the ray tracing process with two lenses. In fact, those rays originate with the object (path not shown), strike the diverging lens at the location shown, and then follow the parallel or center paths (between the lenses) toward the converging lens.

It probably is not practical to repeat the process of drawing the graph, but the students could be asked to move the lenses or the object until a different real image is formed and repeat the calculation to verify their first result.

Analyzing Results

If you decide to give the students lenses all with the same focal length, you might want the groups to post their results on whiteboards to share in a class discussion. Or, you can task each group (or individual) to prepare a formal written report of the procedure, diagrams (both of the physical set up used and ray tracing diagrams), data and conclusions. The use of multiple represesentations (mathematical, graphical and ray diagrams) helps reinforce the concepts which the students are studying in this lab. Perhaps you will choose to have a class discussion and then have each group turn in a formal report with their results.

Students should be asked to compare their measured value of the focal length to the actual focal length of the lens (as specified by the manufacturer). If all the students are given lenses of the same focal length, then they should be asked to compare their results to other groups' results and comment on which group's result was closest to the actual value. This comparison of the group's various values should lead to a comparative discussion of the techniques used in measurements. Students should discuss what measurement techniques led to more precise and accurate measurements.

You should then lead them in a discussion of the uncertainty in their image distance measurement. Some guiding questions you could ask include:

- What was the measured uncertainty in the image distance?
- How much variation was there between the measurements made by each member of the group?
- What is the percent uncertainty in the image distance measurement?
- What is the percent uncertainty in the object distance measurement? (They may have to estimate this if they only measured it once.)
- Is the true value of their focal length (as specified by the manufacturer) within the range indicated by their uncertainties? In other words, if their image distance measurement has an uncertainty of 5%, is their measured value within 5% of the true value?
- What could be done to improve the precision of their measurements?
- How do the measurement uncertainties affect the value of the focal length? E.g., if the image distances measured are all too small, what effect would that have on the calculation of the focal length?

If you want to further explore the behavior of lenses, you could introduce the Lens-Maker's equation (available at http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/lenmak.html) and discuss how the manufacturer creates a lens of a specified focal length, and what the uncertainties are in this focal length.

Assessing Student Understanding

At the conclusion of this investigation the students should be able to:

- articulate the difference between a virtual and a real image
- use a converging lens to form a real image
- articulate why they plotted the inverse of the image distance as a function of the inverse of the object distance, referring to the lens equation
- justify why the focal length is the inverse of the intercept
- estimate the uncertainty in all measurements which were made several times

Assessing the Science Practices

Some of the practices below need to be observed in a formative manner, as the students are performing the measurements. For example, Science Practices 4.1 and 5.2 are best assessed while the students are in the lab in a formative way rather than as summative assessments.

Science practices	An attempt (1)	On the path to proficiency (2)	Nearly Proficient (3)	Proficient (4)
1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	The student draws an incomplete or incorrect ray diagram.	The student draws the rays accurately for a converging lens, but does not accurately locate the image.	The student can draw a ray diagram for a converging lens accurately, but had difficulty with a diverging lens. The student can accurately locate the image if it is real, but has difficulty ray tracing to find virtual images.	The student can accurately draw a ray diagram to locate the image for both a converging and diverging lens, indicating the correct refraction for each ray and accurately locating the image. The student can identify when an image is real or virtual.
2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena.	The student plots an incomplete or incorrect graph.	The student plots the image distance and the object distance and uses a data point to calculate the focal length.	The student plots the inverses of the image and object distances, but then uses the slope to calculate the focal length.	The student can use the lens equation to determine which quantities should be plotted to determine the focal length, and then accurately calculate the focal length as the inverse of the intercept.
4.1. The student can *justify the selection of the kind of data needed* to answer a particular scientific question.	The student does not know how to accurately measure the image or object distances.	The student knows that the image distance and the object distance need to be measured, but does not know how to use them to calculate the focal length.	The student explains why the inverse of the image and object distances are plotted, and knows to calculate the inverse of the slope, but there is no discussion of the assumptions inherent in the model.	The student explains why she/he is measuring the distance from the object to the lens and the image to the lens, and the justification is clear, easy to follow, and correct in terms of the lens equation. The student explains how the data will be plotted to answer the question and provides mathematical models and the models are justified. There is a discussion of assumptions inherent in each model and what to do to validate them.

Science practices	An attempt (1)	On the path to proficiency (2)	Nearly Proficient (3)	Proficient (4)
5.1. The student can *analyze data* to identify patterns or relationships.	The student does not know how to use the image distance and the object distance data to get a focal length.	The student plots the dependent variable versus the independent variable and draws a best fit line to the data (which is not straight), but does not know how to calculate the focal length	The student accurately plots the inverse of the image distance versus the inverse of the object distance, and draws a best fit line. The student then uses the slope to calculate the focal length.	The the student accurately plots the inverse of the image distance versus the inverse of the object distance. The student draws a best fit line to the data. The student extrapolates the best fit line until it intersects the axis. The student calculates the inverse of the intercept and reports this as the focal length.
5.2: The student can *refine observations and measurements* based on data analysis.	The student cannot adjust the position of the screen until a clear crisp image is seen.	The student cannot explain how to determine if data needs to be retaken or observations need to be refined. Many of the images are slightly out of focus. The student cannot estimate the uncertainty in the image or object distance measurements.	The student ignores wildly deviant data points in the drawing of the best fit line. Some of the images formed are slightly out of focus. The student can estimate uncertainties in the image and object distances but is not clear on how they affect the focal length calculation.	The student notices if data points seem not to be consistent with the rest of the data set or do not lie near a best fit line through the rest of the data. The student repeats any observations that seem to be inaccurate. The student can accurately locate a clearly formed image. The student can estimate the uncertainty in the object and image distance measurements and explain how they impact the calculation of the focal length.

Supplemental Resources

1. Northwestern Virtual Physics Laboratory: http://groups.physics.northwestern.edu/vpl/optics/lenses.html (accessed Feb.7, 2014).
   This java applet allows the students to move the lens and the object and show the primary rays refracted through the lens (either converging or diverging) to form the image. Real images are shown in blue, and virtual images are shown in green.
2. PhET: http://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html (accessed Feb.7, 2014). The Geometric Optics simulation at PhET also allows the students to move the object and show where the image location is. In addition, it allows the students to change the refractive index of the material in the lens, and change the radius of curvature of the lens as well as the position of the object above or below the principal axis of the lens.

3. Seeingan Old Lab in a New Light: Transforming a Traditional Optics Lab into Full Guided Inquiry ,Tim Maley, Will Stoll and Kadir Demir, Phys. Teach. 51, 368 (2013)
4. Dr. Arthur Eisenkraft: Video 6. Physics of Optics. Teaching High School Science." http://www.learner.org/resources/series126.html (accessed Feb.7, 2014).

Appendix

Rubrics for Science Practices in AP Physics 1 and 2 Investigations

Rubrics for Science Practices in AP Physics 1 and 2 Investigations

Science Practice	An Attempt	On the Path to Proficiency	Nearly Proficient	Proficient
1.1. The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Creates flawed or incomplete representations/models to represent familiar phenomena.	Creates representations /models that generally represent familiar phenomena. The models may not fully reflect all aspects of phenomena. The model is not experimentally testable (either involves some artificial explanations or the experiments are difficult to perform).	Creates accurate and appropriate representations/models used to represent familiar phenomena. The model accounts for the most important features of the phenomenon and is experimentally testable but the limitations and assumptions are not explained.	Creates accurate and appropriate representations/models used to represent novel phenomena. The model accounts for the most important features of the phenomenon and is experimentally testable and limitations of the model (or inherent assumptions) are clearly explained.
1.2. The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.	Includes errors when articulating links among representations or models and the natural phenomena or mechanisms they represent. Uses generally inaccurate language and definitions within descriptions associated with elements of the models or representations.	Accurately articulates a few links among representations or models and the natural phenomena or mechanisms they represent. Includes some inaccuracies in language and definitions within descriptions associated with elements of the representations/models.	Accurately articulates some links among representations or models and the natural phenomena or mechanisms they represent. Uses accurate language and/or definitions in descriptions associated with elements of the representations/models. Extracts relevant information from the representation.	Accurately articulates most links among representations or models and the natural phenomena or mechanisms they represent. Uses accurate language and definitions in descriptions associated with elements of the models/representations. Addresses relevancy of the description to the goal of the representation/model. Extracts all relevant information from the representation.
1.3. The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.	Makes significant errors in refining representations or models of phenomena. Uses incomplete definitions and/or language.	Refines representations or models of phenomena with some errors and inaccuracies in definitions and language.	Correctly refines, with occasional or minor errors on complex problems, representations or models of phenomena, using accurate definitions and language. Accurately identifies nearly all deficiencies of given representations/models and explains how the revised models address these deficiencies.	Effectively refines representations or models of phenomena, using accurate and precise definitions and language. Comprehensively identifies deficiencies of given representations/models and explains how the revised models address these deficiencies.
1.4. The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.	Uses representations and models to analyze situations or solve problems qualitatively and quantitatively, but analysis and/or problem solving strategies contain multiple inaccuracies and errors.	Uses representations and models to analyze situations or solve problems qualitatively and quantitatively, but with some significant errors and inaccuracies, either in analysis or problem solving strategies.	Accurately uses representations and models to analyze situations or solve complex problems qualitatively and quantitatively, with occasional or minor errors in analysis or problem solving.	Accurately uses representations and models to analyze situations or solve complex problems qualitatively and quantitatively without any errors. May manipulate a representation/model as an alternative to manipulation of equations and/or numerical data.
1.5. The student can *re-express key elements of natural phenomena across multiple representations* in the domain.	Re-expresses a very limited number of elements of natural phenomena across multiple representations in the domain with many errors and inaccuracies.	Re-expresses key elements of natural phenomena across multiple representations in the domain, and links elements of one representation with another familiar representation with some significant errors and inaccuracies.	Re-expresses key elements of natural phenomena across multiple representations in the domain, and links elements of one representation with another familiar representation with occasional or minor errors.	Re-expresses key elements of natural phenomena across multiple representations in the domain. Appropriately links elements of one representation with another familiar representation without any errors. The representations address different (but important) aspects of the phenomenon.

Rubrics for Science Practices in AP Physics 1 and 2 Investigations

Science Practice	An Attempt	On the Path to Proficiency	Nearly Proficient	Proficient
2.1. The student *can justify the selection of a mathematical routine* to solve problems.	Provides justification for the selection of mathematical routines that may bear little relevance to the routine. Uses language and terminology, but with major errors and inaccuracies.	Provides justification for the selection of mathematical routines that lacks some evidence, reasoning and/or key factors. Uses language and terminology that includes a few errors and inaccuracies.	Provides relevant justification for the selection of mathematical routines, but precise detail is lacking. Uses accurate language and terminology.	Provides relevant and detailed justification for the selection of mathematical routines. Uses accurate and precise language and terminology.
2.2. The student can *apply mathematical routines* to quantities that describe natural phenomena.	Selects and applies mathematical routines, but the selections are inappropriate or the applications contain major errors.	Selects and applies appropriate mathematical routines in new contexts, but with some inconsistency and/or errors.	Selects and applies appropriate mathematical routines in new contexts, but with occasional minor errors on complex problems.	Appropriately and accurately selects and applies mathematical routines in new contexts and in simple to complex problems.
2.3. The student can *estimate numerically quantities* that describe natural phenomena.	Estimates quantities that describe phenomena through the use of mathematical routines; the estimates contain some errors or are not always relevant to the description.	Estimates quantities that describe phenomena through the use of appropriate mathematical routines on familiar and/or simple problems.	Estimates quantities that describe phenomena through the use of appropriate mathematical routines; the estimates contain occasional minor errors on complex problems.	Correctly estimates quantities that describe phenomena through the use of appropriate mathematical routines on complex problems.
3.1. The student can *pose scientific questions.*	Poses scientific questions using incorrect or imprecise language and terminology, resulting in a lack of clarity in linking the question to its purpose. Incorrectly identifies many of the aspects of the phenomena being queried.	Poses scientific questions using appropriate language and terminology with some inconsistency, errors, and/or inaccuracies. Identifies essential aspects of the phenomenon being queried.	Poses scientific questions using appropriate language and terminology, with occasional minor errors. Links the question to existing knowledge and purpose.	Poses scientific questions using precise language and terminology. Links the question to existing knowledge and purpose with clarity and detail. Poses scientific questions which extend thinking about a concept, relationships between concepts, causal mechanism and/or phenomena.
3.2. The student can *refine scientific questions.*	Modifies scientific questions, but with little positive effect in removing ambiguity or clarifying focus. Provides some justification for refining questions. However, justification lacks reasoning and/or evidence and includes major inaccuracies.	Reduces ambiguity and/or improves focus in refining scientific questions. Provides simple justification for refining questions. However, justification lacks reasoning and use of complete evidence.	Removes ambiguity and/or clarifies focus in refining scientific questions. Provides justification for refining the question. Justification includes some reasoning and/or evidence.	Removes ambiguity, fully clarifies and/or limits focus in refining scientific questions. Provides appropriate justification for refining questions, including appropriate reasoning and evidence.
3.3. The student can *evaluate scientific questions.*	Provides an evaluation of scientific questions that lacks justification for inclusion of the scientific questions are in the scope of an investigation or domain, and that includes inaccuracies.	Evaluates scientific questions for inclusion in the scope of either an investigation or domain, but justification is unclear. May use incomplete and/or inaccurate evidence and/or faulty reasoning.	Identifies basic evaluation criteria; explains the relevance of the selected criteria with only a few errors; evaluates and justifies scientific questions for inclusion in the scope of an investigation using some evidence and/or appropriate reasoning.	Identifies evaluation criteria, explains the relevance of selected evaluation criteria, and evaluates, with convincing justification, scientific questions for inclusion in the scope of an investigation and domain.

Rubrics for Science Practices in AP Physics 1 and 2 Investigations

Science Practice	An Attempt	On the Path to Proficiency	Nearly Proficient	Proficient
4.1. The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Provides generally weak justification for data selection; justification includes minimal reasoning and evidence.	Provides justification for data selection with occasional and/or minor errors; justification may be correct but lacks completeness and/or reference to relevance.	Provides accurate justification for data selection with only an occasional or minor error.	Provides accurate and detailed justification for data selection, explaining relevance of the selected variables and of the data.
4.2. The student can *design a plan* for collecting data to answer a particular scientific question.	Presents an incomplete data collection plan to answer a particular scientific question; includes appropriate data sources but makes insufficient distinction between qualitative and quantitative measures. Makes errors in identifying the variables (independent, dependent, and controlling) and/or misuses language or scientific terminology.	Designs a data collection plan to answer a particular scientific question via qualitative or quantitative measures; measures may not be clearly defined or articulated. Acknowledges need to consider sources of error. Accurately identifies independent, dependent, and controlling variables with few errors and/or misuse of language or scientific terminology.	Designs an appropriate data collection plan to answer a particular scientific question via qualitative and/or quantitative measures; measures may lack complete details. Identifies the selected observation schedule, units of measurement, and tools. Identifies appropriate data sources and sources of error. Accurately identifies and describes independent, dependent, and controlling variables.	Designs an effective data collection plan to answer a particular scientific question via well selected qualitative and quantitative measures, providing rationale for all choices. Accurately evaluates and explains sources of error. Effectively explains tool selection for acquiring data and conditions for their use. Accurately identifies and explains independent, dependent, and controlling variables, and justifies choices.
4.3. The student can *collect data* to answer a particular scientific question.	Collects relevant, but significantly inadequate data to answer a particular scientific question. Provides observations and/or record keeping which are incomplete and/or inadequate for answering a particular question. Selects inappropriate mathematical routines; measurements contain many errors.	Collects appropriate data to answer a particular scientific question. Provides observation logs and record keeping that contain several errors; however, the use of tools and conditions is adequate and appropriate for the most part. Selects appropriate mathematical routines and provides measurements with a few errors or a significant error.	Collects appropriate and adequate data to answer some aspects of a particular scientific question with only minor errors in the precision of observation, record keeping and use of tools and conditions. Selects appropriate mathematical routines and provides measurements with only a few minor errors.	Collects appropriate data to fully answer a particular scientific question with precision of observations, accuracy of records, and accurate use of scientific tools and conditions. Accurately applies mathematical routines and appropriately uses measurement strategies for the question and data sources.
4.4. The student can *evaluate sources of data* to answer a particular scientific question.	Inconsistently identifies legitimate data sources to answer a particular scientific question, frequently failing to recognize the incomplete or inappropriate nature of the data. Identifies and selects appropriate mathematical routines for answering the question, but selections lack justification.	Makes only minor errors in identifying legitimate data sources to answer a particular scientific question, and/or fails to recognize that some selected data sets are incomplete or inappropriate. Selects and justifies appropriate mathematical routines for answering the question. Evaluates uncertainty in the data.	Identifies fully legitimate data sources to answer a particular scientific question, and appropriately identifies the data set as complete. Selects and applies appropriate mathematical routines. Estimates the percent uncertainty based on the largest source of uncertainty (instrumental or random) and articulates efforts to minimize uncertainty.	Identifies fully legitimate sources of data to answer a particular scientific question and justifies the completeness of the data set. Selects, justifies, and/or applies appropriate mathematical routines. Evaluates, explains, and estimates the percent uncertainty based on the largest source of uncertainty (instrumental or random).

Rubrics for Science Practices in AP Physics 1 and 2 Investigations

Science Practice	An Attempt	On the Path to Proficiency	Nearly Proficient	Proficient
5.1. The student can *analyze data* to identify patterns or relationships.	Identifies one or more legitimate pattern(s) in data, though these may be irrelevant to a scientific question being asked. Identifies some mathematical routines that are appropriate.	Identifies the most obvious patterns within data, relative to a scientific question being asked, with some errors and inaccuracies. Selects appropriate mathematical routines but makes some application errors.	Identifies most patterns within data, relative to a scientific question being asked, with only an occasional, minor error. Selects appropriate mathematical routines and applies them with only minor errors.	Comprehensively describes the patterns and relationships within data, relative to a scientific question being asked. Accurately applies appropriate mathematical routines.
5.2. The student can *refine observations and measurements* based on data analysis.	Identifies relevant data, but applies mathematical routines that fail to improve the data collection plan.	Accurately identifies relevant data, but selects incomplete and/or inappropriate refinements to observations and/or measurements. Selects and applies mathematical routines with at least one significant error.	Accurately identifies relevant data and makes appropriate refinements to the observations and/or measurements. Appropriately selects mathematical routines; application includes only minor errors.	Accurately identifies relevant data and makes appropriate and comprehensive refinements to the observations and/or measurements. Appropriately selects and accurately applies mathematical routines.
5.3. The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.	Makes some judgment errors about how data are used as evidence in relation to a particular scientific question and about appropriate data collection methods. Selects and applies mathematical routines with some significant errors.	Relates evidence provided by data sets to a particular scientific question, but fails to fully align the evidence to a question or to the data collection methods used. Selects and applies mathematical routines with at least one significant error.	Evaluates evidence provided by data sets by justifying the appropriateness of the data, but provides only basic insight into how data relate to a particular scientific question and the data collection methods used. Appropriately selects mathematical routines; application includes only minor errors.	Evaluates evidence provided by data sets by effectively justifying the appropriateness of the data as they relate to a particular scientific question and the data collection methods used. Appropriately selects and accurately applies mathematical routines.
6.1. The student can *justify claims with evidence.*	Identifies some appropriate evidence in support of claims, but connections drawn between evidence and claims are generally weak.	Identifies and aligns evidence with claims. Some evidence is inappropriate or fails to support the claims. Accurately differentiates between a claim and the evidence that supports it.	Accurately identifies and aligns most but not all available and relevant evidence with claims it supports. Provides a clear justification for the selection and/or exclusion of evidence. Considers data from multiple sources.	Accurately identifies and aligns a comprehensive array of evidence with claims the evidence supports. Provides a substantive justification for the selection and/or exclusion of evidence. Considers data from multiple sources and provides appropriate rationales for the selection and exclusion of evidence.
6.2. The student can *construct explanations of phenomena based on evidence* produced through scientific practices.	Describes phenomena with limited reference to evidence produced through scientific practice. Uses flawed language and terminology.	Provides explanations of phenomena based on some evidence produced through science practices, using basic logic. Identifies some links between evidence and claims and uses scientific language with few significant errors.	Provides explanations of phenomena based on evidence produced through scientific practices; links evidence and claims; uses precise and accurate scientific language with only minor or occasional errors.	Provides comprehensive explanations of phenomena based on evidence produced through scientific practices; presents a logical argument that links evidence and claims; uses scientific language precisely and accurately.

Rubrics for Science Practices in AP Physics 1 and 2 Investigations

Science Practice	An Attempt	On the Path to Proficiency	Nearly Proficient	Proficient
6.3. The student can *articulate the reasons that scientific explanations and theories are refined or replaced.*	Articulates a limited number of reasons why familiar scientific explanations and theories are refined or replaced, but offers inaccurate or incomplete reasoning or justification for why they were refined or replaced.	Articulates general reasons why scientific explanations and theories are refined or replaced, and uses scientific language and terminology with few errors and inaccuracies. Describes why revisions are necessary, but not why they might represent an improvement.	Articulates the appropriate reasons that scientific explanations and theories are refined or replaced, and uses appropriate and accurate scientific language with only minor or occasional errors. Descriptions of why revisions are necessary may be incomplete.	Articulates the reasons why scientific explanations and theories are refined or replaced, and appropriately explains why particular revisions were necessary.
6.4. The student can *make claims and predictions about natural phenomena* based on scientific theories and models.	Describes scientific theories and models in making claims and predictions about phenomena, but without appropriate reasoning and/or justification. Some connections are inappropriate or inaccurate. Articulates the scope of claims or predictions with some errors.	Draws directly from scientific theories and models in making claims and predictions about phenomena, with incomplete justification or reasoning. Describes, with some limitations, connections between the model and the claim or prediction and the model and the phenomena. Accurately describes the scope of the claim or prediction.	Draws directly and appropriately from scientific theories and models in making claims and predictions about phenomena, with appropriate justification and reasoning. Explains, with few errors, connections between the model and the claim or prediction and the model and the phenomena.	Draws convincingly from scientific theories and models in making claims and predictions about phenomena, with appropriate justification and reasoning. Explains the connections between the model and the claim or prediction and the model and the phenomena.
6.5. The student can *evaluate alternative scientific explanations.*	Evaluates the appropriateness or accuracy of contradictory evidence or alternative explanations. Evaluation contains some errors.	Evaluates alternative scientific explanations with limited consideration of either the alternative's strengths or its weaknesses based on evidence. Evaluation of the strengths or weaknesses may be flawed.	Evaluates alternative scientific explanations with consideration of both the alternative's strengths and its weaknesses, though the evaluation of either the strengths or weaknesses may be flawed.	Evaluates alternative scientific explanations with consideration of both the alternative's strengths and its weaknesses, appropriately based on available evidence.
7.1. The student can *connect phenomena and models* across spatial and temporal scales.	Articulates a general account of phenomena described at one scale and some relationships at another scale, but makes some errors in attempts to connect phenomena and models.	Connects phenomena and models across spatial or temporal scales, relating specific variables across finer or greater spatial and/or temporal scales. Cites relationships among variables that are not fully valid.	Connects phenomena and models across spatial and/or temporal scales. With few or occasional errors, describes the impact on phenomena or models caused by a change at one scale.	Connects phenomena and models across both spatial and temporal scales. With an occasional or minor error, fully explains how a change at one scale affects phenomena or models at another scale.
7.2. The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.	Makes statements linking concepts or phenomena, but with some errors and inaccuracies. Makes claims about how a change in one phenomenon might affect another, but with an incomplete consideration of evidence.	Compares features of phenomena that are related, making connections across concepts and among contexts, but not necessarily across big ideas and/or enduring understandings. Predicts, with basic reasoning or justification, how a change in one phenomenon might affect another.	Connects concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas with only occasional or minor errors. Predicts, with reasoning and justification, how a change in one phenomenon might affect another with only occasional or minor errors.	Connects concepts in and across domain(s) to accurately generalize or extrapolate in and/or across enduring understandings and/or big ideas. Predicts, with appropriate reasoning and detailed justification, how a change in one phenomenon might affect another.

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