NAME ______________________________ LAB PARTNERS ______________________________

Station Number ______________________________ ______________________________

Addition of Vectors: Translational Equilibrium

***Experiment* 2**

INTRODUCTION

For many physical quantities, the direction of a quantity is just as important as its magnitude. Quantities that require specification of both magnitude and direction are called vectors. Scalars, on the other hand, have magnitude only. Mass, length, and temperature are examples of scalar quantities. Displacement, velocity, and force are examples of vectors. The object of this experiment is to learn to add and subtract vectors by several methods. The vectors with which you will work are force vectors, but the methods you will learn apply to any vector. You will also learn about translational equilibrium.

THEORY

One method of adding vectors is to add them graphically. A vector is represented by a straight line with an arrowhead. The length of the straight line is proportional to the magnitude of the vector, and the direction of the vector is indicated by the arrowhead.

A simple example will serve to illustrate graphical vector addition. Suppose you walk 300 meters due east and then walk 400 meters northeast. (The direction northeast is 45° north of east.) Figure 2.1 shows one way to add these two displacement vectors. Vector **A** represents the 300-meter displacement, and vector **B** represents the 400-meter displacement to the northeast. Vector **R** represents the resultant (or sum) of vectors **A** and **B**. Measurements made with a ruler and a protractor give a magnitude of 648 meters and an angle of 26° north of east. The method of addition shown in Figure 2.1 is often called the triangle or polygon method of vector addition. If more than two vectors are added, Figure 2.2 shows the resultant. The resultant **R** joins the tail of the first vector to the arrowhead of the last vector. This process can be used for adding many vectors.

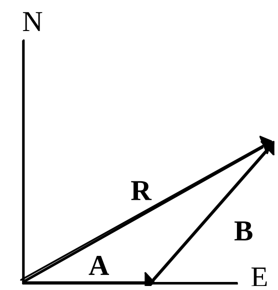

Figure 2.1 Graphical addition of two vectors A and B. The scale is 1/4 inch = 100 m

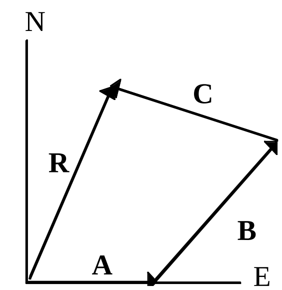

Figure 2.2 Polygon method of vector addition

The two vectors **A** and **B** shown in Figure 2.1 can be added graphically in yet another way. Figure 2.3 illustrates this technique, which is called the parallelogram method of vector addition. The tails of the two vectors are drawn at the origin so that the vectors form two sides of a parallelogram. The diagonal of the parallelogram is the resultant vector **R**.

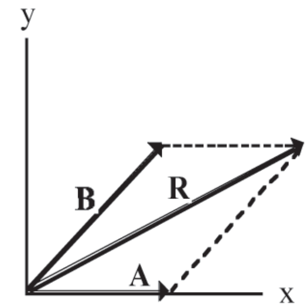

Figure 2.3 Parallelogram method of vector addition

Graphical methods of vector addition suffer from the obvious difficulty that accuracy is limited by the accuracy of the measuring devices and by the care taken to construct the drawings. This limited accuracy makes it necessary to develop an analytical method for adding vectors. This analytical method is based on trigonometry and geometry, and relies on finding the x- and y-components of each vector. To add vectors analytically, first find the x- and y-components of each vector, defined by

$$A_x = A\cos\theta \tag{1}$$

and

$$A_y = A\sin\theta. \tag{2}$$

Here A is the magnitude of the vector **A**, and θ is the angle between vector **A** and the +x axis. Figure 2.4 shows the construction of the x- and y-components of vector **A**. Since x-components and y-components of all vectors are now parallel to the x or y-axis, respectively, they can be added algebraically so that the resultant x- and y-components are given by

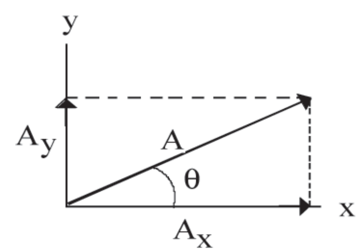

Figure 2.4 x- and y-components of vector

$$R_x = A_x + B_x + C_x + \cdots \tag{3}$$

and

$$R_y = A_y + B_y + C_y + \cdots, \tag{4}$$

where R_x and R_y are the x- and y-components of the resultant vector. By standard convention, x-components to the right are positive, and x-components to the left are negative. Similarly, upward y-components are positive, and downward y-components are negative. Since x-components are perpendicular to the y-components by definition, the Pythagorean theorem is used to find the magnitude R of the vector **R** so that

$$R = \left[R_x^2 + R_y^2\right]^{1/2}, \tag{5}$$

and the angle between the vector **R** and the +x axis is given by

$$\theta = \tan^{-1}\frac{R_y}{R_x}. \tag{6}$$

The addition of force vectors in this experiment makes use of the concept of translational equilibrium. If several forces act on an object, they may be replaced by a single resultant force that is the vector sum of all forces acting on the object. Translational equilibrium occurs when the resultant force on an object is zero.

EXPERIMENT NO. 2

1. Your instructor will assign you three masses, m_1, m_2, and m_3, and the angles at which to apply these masses on the force table. Top and side views of the force table are shown in Figures 2.5 and 2.6, respectively. The force **F** provided by each mass is given $\mathbf{F} = m\mathbf{g}$, where **g** is the acceleration due to gravity. Record the values for the three masses (in kg) and the angles at which they are applied on the force table. Record all angles as measured counter-clockwise with respect to the +x axis.

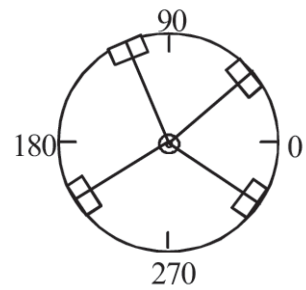

Figure 2.5 Top view of force table

m_1 = ________________ at an angle of ________________

m_2 = ________________ at an angle of ________________

m_3 = ________________ at an angle of ________________

Calculate the magnitude of the forces F_1, F_2, and F_3 caused by each of the masses.

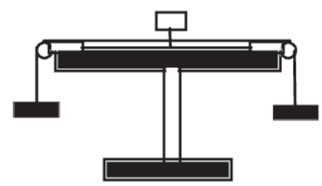

Figure 2.6 Side view of force table

F_1 = _____________ F_2 = ______________ F_3 = ______________

2. Select any two of the three forces and determine the magnitude and direction of the resultant vector using the parallelogram method. Show your construction on the graph paper provided. Be sure to record the scale you used on the drawing. Record the direction and the magnitude of the resultant vector and the equilibrant vector. The equilibrant vector is equal in magnitude but oppositely directed to the resultant vector. The vector sum of the resultant vector and equilibrant vector is zero. Please construct both the resultant and equilibrant vector on the graph paper provided and show detailed calculations. Record forces in Newtons.

Magnitude of resultant = ________________ Magnitude of equilibrant = ________________

Direction of resultant = ________________ Direction of equilibrant = ________________

3. Apply the two forces you selected and the equilibrant to the small ring on the force table. A peg in the center of the table keeps the ring from moving very far until equilibrium is attained. Be sure the connecting strings are kept in the grooves of the pulleys. Adjustments should be tested by removing the peg and pulling the ring slightly to the side and releasing it. If equilibrium has been established, the ring will return to the center of the force table. Record the magnitude and direction of the equilibrant vector, as determined using the force table.

Magnitude of equilibrant = _______________ Direction of equilibrant = ________________

4. Using the three forces from Part 1, determine the direction and magnitude of the resultant using the polygon method of vector addition. Show your diagram on the graph paper provided. State the scale.

   Magnitude of resultant = ________________ Direction of resultant = ________________

5. Check the measurement of Part 4 on the force table as you did in Part 3. Record the magnitude and direction of the equilibrant vector.

   Magnitude of equilibrant = ________________ Direction of equilibrant = ________________

   How should the resultant from Part 4 and the equilibrant from Part 5 be related? How are yours related?

6. Using the three forces from Part 1, calculate the magnitude and direction of the resultant vector analytically. Use Equations (1) – (4) to complete the table. Show your calculations in the space below the table.

Component	F_1	F_2	F_3	Resultant
x				
y				

Use Equations (5) and (6) to calculate the magnitude and direction of the resultant.

Magnitude of resultant = ________________ Direction of resultant = _______________

QUESTIONS

1. Draw three vectors that originate at the origin and lie in the first quadrant of an x-y coordinate system. Explain why the resultant of your vectors can or cannot be zero.

2. Vector **S** has a magnitude of 8.00 units and makes an angle of 46.0 degrees with respect to the +x-axis. Vector **T** has a magnitude of 6.00 units and makes an angle of 36.0 degrees measured clockwise with respect to the negative x-axis. Sketch the two vectors and calculate analytically the magnitude and direction of the resultant vector. What are the magnitude and direction of the equilibrant vector?