

Additive thinking: developing flexible strategies

Stage 3

Overview

Learning intention

Students learn to select and apply effective strategies and incorporate reasoning when offering solutions to problems involving additive thinking.

Syllabus outcomes

The following teaching and learning strategies will assist in covering elements of the following outcomes:

- MA3-1WM describes and represents mathematical situations in a variety of ways using mathematical terminology and some conventions
- MA3-2WM selects and applies appropriate problem-solving strategies, including the use of digital technologies, in undertaking investigations
- MA3-3WM gives a valid reason for supporting one possible solution over another
- MA3-5NA selects and applies appropriate strategies for addition and subtraction with counting numbers of any size

Year 5 NAPLAN item descriptors

The following Year 5 NAPLAN item descriptors can be used by teachers to co-construct success criteria and include the actions of Working Mathematically to clarify expectations, for example: reason, represent and model.

- calculates the difference between two four-digit numbers
- solves a multi-step problem involving adding and subtracting whole numbers
- solves a multi-step word problem
- solves a problem involving addition and subtraction
- solves a problem involving addition and subtraction of whole numbers
- solves a word problem by adding whole numbers

National Numeracy Learning Progression guide

What are additive strategies?

Additive strategies encompass a student's ability to manipulate numbers in additive situations. As students develop an understanding of whole number and the operations of addition and subtraction, they transition from counting strategies to using more flexible strategies to solve problems. When applying additive strategies, students may manipulate numbers using their part-part-whole knowledge, renaming and partitioning of numbers (including using place value knowledge) and the inverse relationship between addition and subtraction.

Students may demonstrate these behaviours as they develop increasing confidence with choosing and using flexible strategies to solve additive problems:

AdS7 Flexible strategies with two-digit numbers

- applies knowledge of 10 as a unit to add and subtract 2 two-digit numbers (jump strategy, split strategy or compensation)
- manipulates tens and ones flexibly for addition and subtraction (to add 45 and 37, adds the tens on 45 ... 55, 65, 75, then partitions the 7 into 5 and 2, adds the 5 to make 80 and 2 more to make 82)
- uses part-whole knowledge of numbers to 20 to calculate two-digit addition and subtraction (when finding $53 - 27$, recognises the subtraction within as $13 - 7$ and regroups the 53 into 40 and 13 and the 27 into 20 and 7)

AdS8 Flexible strategies with three-digit numbers and beyond

- manipulates hundreds, tens and ones flexibly to add and subtract 2 three-digit numbers (to add 250 and 457, ungroups the 250 into 2 hundreds and 5 tens, responds 457 plus 2 hundred is 657, plus 50 is 707)
- manipulates place value of numbers flexibly in regrouping for addition of three-digit numbers and beyond (when adding 650 and 550, regroups 650 as 600 and 50, adds 50 to 550, then doubles 600)
- manipulates place value of numbers flexibly in regrouping for subtraction (when solving $3000 - 260$, treats the 3000 as 2700 and 300 to aid in mental calculation)
- regroups for subtraction involving trading or exchange of units with different place values
- chooses and uses multiple strategies for solving everyday problems involving addition and subtraction

Overview of teaching strategies

Purpose

These numeracy teaching strategies support teaching and learning from Stage 2 to Stage 5. They are linked to NAPLAN task descriptors, syllabus outcomes and numeracy learning progressions.

These teaching strategies target specific literacy and numeracy skills and suggest a learning sequence to build skill development. Teachers can select individual tasks or a sequence to suit their students.

What works best

Explicit teaching practices involve teachers clearly explaining to students why they are learning something, how it connects to what they already know, what they are expected to do, how to do it and what it looks like when they have succeeded. Students are given opportunities and time to check their understanding, ask questions and receive clear, effective feedback.

This resource reflects the latest evidence base and can be used by teachers as they plan for explicit teaching.

Teachers can use assessment information to make decisions about when and how they use this resource as they design teaching and learning sequences to meet the learning needs of their students.

Further support with [What works best](#) is available.

Differentiation

When using these resources in the classroom, it is important for teachers to consider the needs of all students, including [Aboriginal](#) and EAL/D learners.

EAL/D learners will require explicit English language support and scaffolding, informed by the Enhanced [EAL/D enhanced teaching and learning cycle](#) and the student's phase on the [EAL/D Learning Progression](#). Teachers can access information about [supporting EAL/D learners](#) and [literacy and numeracy support](#) specific to EAL/D learners.

Learning adjustments enable students with disability and additional learning and support needs to access syllabus outcomes and content on the same basis as their peers. Teachers can use a [range of adjustments](#) to ensure a personalised approach to student learning.

[Assessing and identifying high potential and gifted learners](#) will help teachers decide which students may benefit from extension and additional challenge. [Effective strategies and contributors to achievement](#) for high potential and gifted learners helps teachers to identify and target areas for growth and improvement. A [differentiation adjustment tool](#) can be found on the High potential and gifted education website.

Using tasks across learning areas

This resource may be used across learning areas where it supports teaching and learning aligned with syllabus outcomes.

Literacy and numeracy is embedded throughout all K-10 syllabus documents as general capabilities. As the English and mathematics learning areas have a particular role in developing literacy and numeracy, NSW English K-10 and Mathematics K-10 syllabus outcomes aligned to literacy and numeracy skills have been identified.

Considerations

Language and vocabulary

As students are provided opportunities to experience concepts, teachers can also build understanding of mathematical vocabulary and communicating skills. Teachers can help build students' confidence and capabilities by making complex mathematical ideas visible to students through drawings, diagrams, enactment, gestures and modelling. Making intentional connections between various representations and experiences with mathematical language helps build an understanding of important vocabulary whilst also building conceptual understanding.

Talk moves

Classroom talk is a powerful tool for both teaching and learning. Rich, dialogic talk supports students in making sense of complex ideas and builds classroom communities centred around meaning-making. 'Talk moves' are some of the tools a teacher can use to support rich, meaningful classroom discussion.

Some of the talk moves include:

- wait time
- turn and talk
- revoicing
- reasoning
- adding on
- repeating
- revise your thinking.

Additional [resources to support talk moves](#) is available on the [Literacy and numeracy website](#).

Number talks

Number talks are a powerful teaching routine centred on short, intentional classroom conversation about a purposefully crafted problem that is solved using a broad range of mental strategies. Their general goal is to build fluency and sense-making through meaningful communication, problem solving and reasoning. They provide regular opportunities to develop number sense and mathematical reasoning through exploring, using and building confidence in additive and multiplicative strategies.

Suggested structure for an open-sharing number talk:

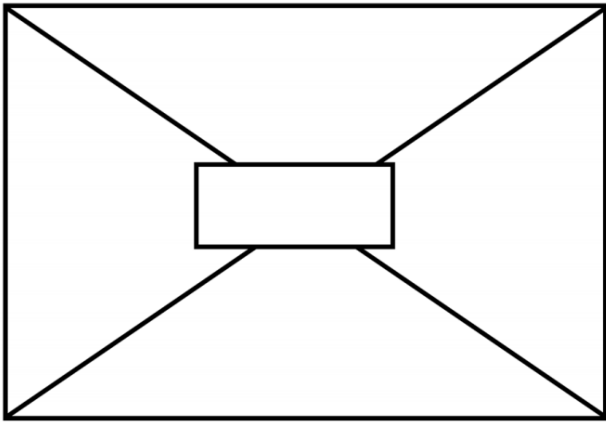
1. A teacher determines the next learning goal for students and finds/designs a problem connected to that learning need.
2. The teacher (and their colleagues) consider and discuss possible responses from students and plan formative assessment strategies, questioning and how to use a broad range of tools to represent the possible ideas student may raise (for example enactment, diagrams, models, etc.)
3. The carefully designed problem is posed to all students within the class.
4. Thinking time is allowed for students to consider the different strategies they would use to solve the problem.
5. Readiness to share is indicated by individual students raising a thumb unobtrusively against their chests (and raising one or more fingers if they think of other solutions).
6. Students are provided opportunities to turn and talk, sharing their ideas with other students sitting nearby.
7. The teacher listens to students as they talk, moving about the class inviting particular students to share their thinking more broadly, intentionally selecting and sequencing conversation that will best support the purpose of the number talk.
8. Thinking is collected and discussed. The teacher may seek a variety of answers without comment, then discuss them as a class. Or the teacher may invite one student at a time to explain their thinking.
9. The teacher supports students to make connections between ideas and to other learning experiences.
10. The teacher concludes the open-sharing number talk by connecting back to the purpose of the task, making explicit the mathematical goal of the conversation.

Two versus two

For most games, we recommend small groups of 4 students, working in pair of 2 (2 versus 2). This gives students the opportunity to discuss mathematical ideas, strategies and understanding with their team mates as well as their opponents.

Think board

[Think boards](#) can be used to make connections between different mathematical concepts or for students to visually represent their understandings and strategies in a range of ways.



Tools and resources to support learning

These tools and resources can be used throughout the tasks:

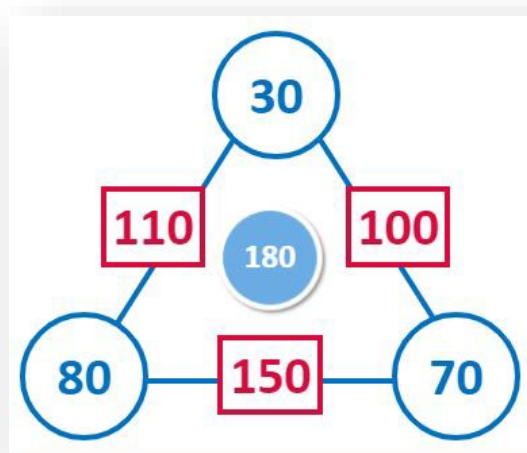
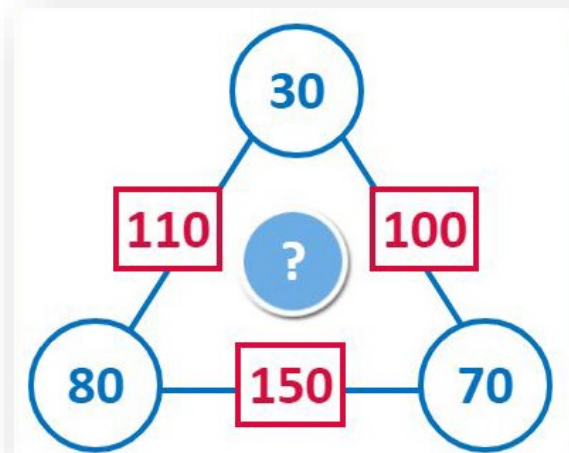
- playing cards
- white boards and markers
- Unifix cubes or centicubes
- counters
- dice (with various faces)
- dominos
- rekenrek (you can [learn how to make your own rekenrek](#) with students)
- multi-attribute blocks (MAB).

Tasks

What do you notice and wonder?

This task can be used as a number talk and aims to promote rich discussion about flexible strategies when approaching numbers. This can be used as a formative assessment strategy and can be used as a whole-class activity or in a small group.

1. Ask students to observe the following problem and share with a partner what they notice and wonder. What could the number in the centre be?
2. Show the second image with the number in the centre and have students continue to notice. These numbers can be changed to reflect the topic being explored. Teachers may like to support student thinking by using questions designed to extend student thinking. For example
 - a. What do you notice about the numbers in the top circle and bottom left circle? Then, let's look at the number in red along the line that connects those two circles. What do you notice now?
 - b. Can you see this on other places within the puzzle?
 - c. Could that information help you work out what goes in the centre?



Students may have noticed such things as:

- all even numbers
- all multiples of 10
- circles are 2-digit numbers
- rectangles are 3-digit numbers
- the rectangle is the sum of the two adjoining circles, for example, $110 = 30 + 80$
- part-whole relations, inverse operations and turn-around facts, for example, $30 + 70 = 100$; $70 + 30 = 100$; $100 - 70 = 30$; $100 - 30 = 70$
- reasoning about what the question mark might be – when '180' is revealed. The 3 circles = 180. Each circle and opposite rectangle = 180.

Variation

- To support students who found the task challenging, teachers can annotate the image with addition symbols when reviewing it with the students.

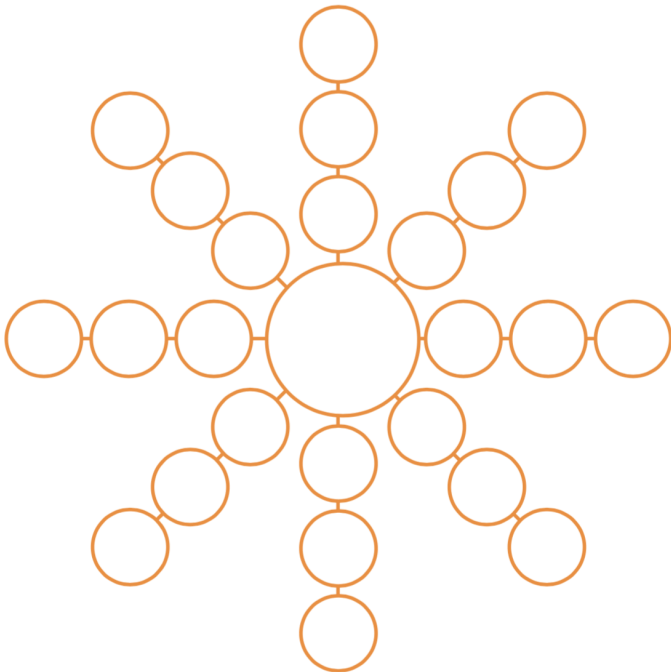
Reference: [Arithmagons activity at nrich.org](https://www.nrich.org/Arithmagons-activity)

Addition wheel

This activity supports students to develop a variety of strategies to use when solving number problems. Knowing the relationship between addition and subtraction can help students solve number problems. This could be played as a two-player versus two-player game to encourage dialogue.

You will need

- [addition wheel](#) template (refer to Appendix 1)



How to play

1. Students each have a copy of the addition wheel (see Appendix 1) or click on link above.
2. Ask the students to record a number in the centre of the wheel.
3. Have students record number combinations between the spokes of the addition wheel, radiating out from the centre. The numbers in the spokes add up to make the total shown in the middle.
4. Have students share their wheel with a partner. Students earn a point for every correct number sentence and a bonus point for the number combination they had but their partner did not.
5. Play again with a different number. The person with the most points at the end is declared the winner.

Variations

- Students record subtraction sentences that equal the centre number. For example, if the number is 15 a student could make $40 - 20 - 5$.
- Increase or decrease the number of circles along the spokes.
- Students use a mixture of operations to reach the target number.

Reference: [Developing Efficient Numeracy Strategies 1](#) © State of New South Wales, Department of Education.

Target number'

Dan Finkel from 'Math for love' has given permission to use this resource to support teaching and learning in Mathematics.

This activity can be used with any number and students can use any operation. This can be in small groups or with partners or a whole-class exploration.

1. Display a "target" number on the board such as 37. Students try to record as many different equations (number sentences) that have that the target number as the total. Students share with a partner and discuss how the different number sentences link to the answer.

Teachers may like to increase the challenge by posing thoughtful limitations.

For example

- If we could only use 2 numbers, how many equations (number sentences) could we write?
 - If we could only use multiplication or division, how many equations (number sentences) could we write?
 - If you had to include fractions, how many equations (number sentences) could we write?
 - What if you had to use exactly 4 numbers, how many equations (number sentences) could we write?
 - What's the longest equation (number sentence) we could write?
 - What's the shortest equation (number sentence) we could write?
2. Alternatively, a range of numbers can be put on posters around the classroom. Students then add their ideas to the posters around the classroom.
 3. Students break into groups with one of the posters to see if the suggestions match the answer and possible suggestions to get the answer if the attempt was incorrect.
 4. Students might use [Think boards](#) (another example can be found in Appendix 2) to show how they are solving a problem.

Reference: [mathforlove.com – Target number lesson](#)

Finding numbers

This task can be completed by pairs or individuals. Students use their knowledge of additive strategies to develop alternative equations (number sentences).

1. Give students a copy of the grid in Appendix 3 (or similar). This grid has a variety of numbers which are all less than 100.
2. Students are asked: How many groups of two or three numbers which are side by side, across or down, that when added together make 100? How can you show these groups? Students share and identify some of the groups of numbers that added to 100 and the strategies they used. Students might share what they looked for to find a group of three?

Reference: [Developing Efficient Numeracy Strategies 1](#) © State of New South Wales, Department of Education.

Closest to 100

This task encourages students to use a range of additive thinking to reach the number 100. This can be extended beyond 100 so students gain experience with then applying these strategies to larger numbers, for example 789 or 1300. Students may work in groups of 3-4.

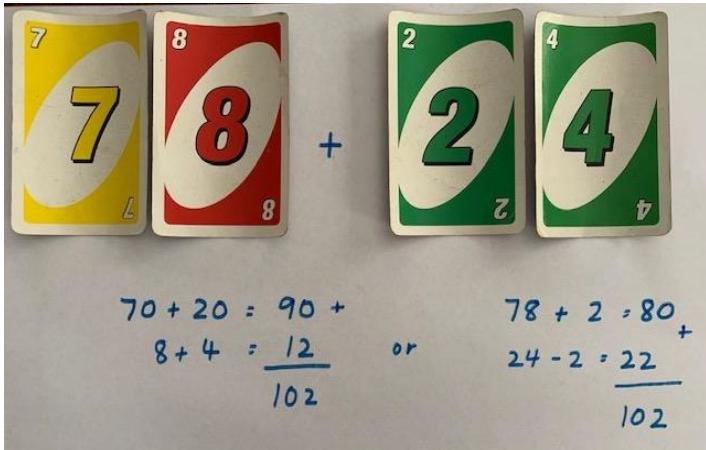
Watch the '[Closest to 100](#)' video to learn how to play.

You will need

- deck of cards
- whiteboards to show working

How to play

1. Players shuffle the cards and put them in a central pile. One person takes 6 cards and places them face up for everyone to see.
2. The goal is to use addition and subtraction to get as close to a total of 100 as possible.
3. Each card can only be used once. It can be used to form a 1- or 2-digit number.
4. Players score 0 points if they are able to reach exactly 100. Otherwise, they work out their points based on the difference between their total and 100. For example, if a team created a total of 98, they would score 2 points.
5. Keep a cumulative total of their difference to 100. The winner is the team to have the lowest points score at the end.
6. Students draw their thought process in solving the problem.
7. Share methods of getting to the target number.



Discuss

- Does your total need to be under 100?
- Can you win if your total is above 100?
- (Yes, as your total may be 105 and therefore your score is 5. The other team may have a score of 90 and their score will be 10. The smallest difference wins.)
- Can you use fewer cards to get to the target number? How did you do that?
- Was there a more efficient way of getting to the target number?

Variations

Extending

- Change the target number to a bigger number than 100 (for example, try 137 or 956)
- Change the target number to a fractional quantity, for example, 35-hundredths (0.35). In this case, the numerals can be worth tenths or hundredths.
- Use fewer cards to find a solution or change the rules so you use all the cards.
- Include multiplication and division in the challenge.

Supporting

- Change the target number to a smaller number (for example, 30)
- Provide more cards to find a solution
- Provide a calculator, bead string, number line or hundreds chart to empower students into the task
- Scaffold the task by playing a full example round as a class before students playing in pairs or teams of four.

Four strikes and you're out!

This game works well as a whole class versus the teacher and then played as a two-player game. This game supports students to think and reason numerically to solve a problem. The game is a mathematical version of 'Hangman'. Students use a variety of strategies to solve a number problem.

1. A problem is hidden from students, for example, $35 + 10 = 45$.
2. Display the 'problem frame' and the numbers 0-9.

$$\begin{array}{ccccccccc} \underline{\quad} & \underline{\quad} & + & \underline{\quad} & \underline{\quad} & = & \underline{\quad} & \underline{\quad} & & \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$

3. Students start by guessing a number 0-9. If the number is within the problem, it will be written in the corresponding space on the problem frame. If the number is not in the problem, it will be a 'strike'. To win, students need to determine all the numbers before they get four strikes.
4. As students progress, they may notice clues in the problem.

For example:

$$\underline{3} \quad \underline{5} \quad + \quad \underline{\quad} \quad \underline{\quad} \quad = \quad \underline{\quad} \quad \underline{5}$$

Students might notice that if there are two fives, this will mean in the second number, the digit in the units' place value must be a zero. They may also notice place value relationships which may help them to decide on what numbers to choose next. Ask students to explain what they notice and why this will help them to determine other numbers in the problem.

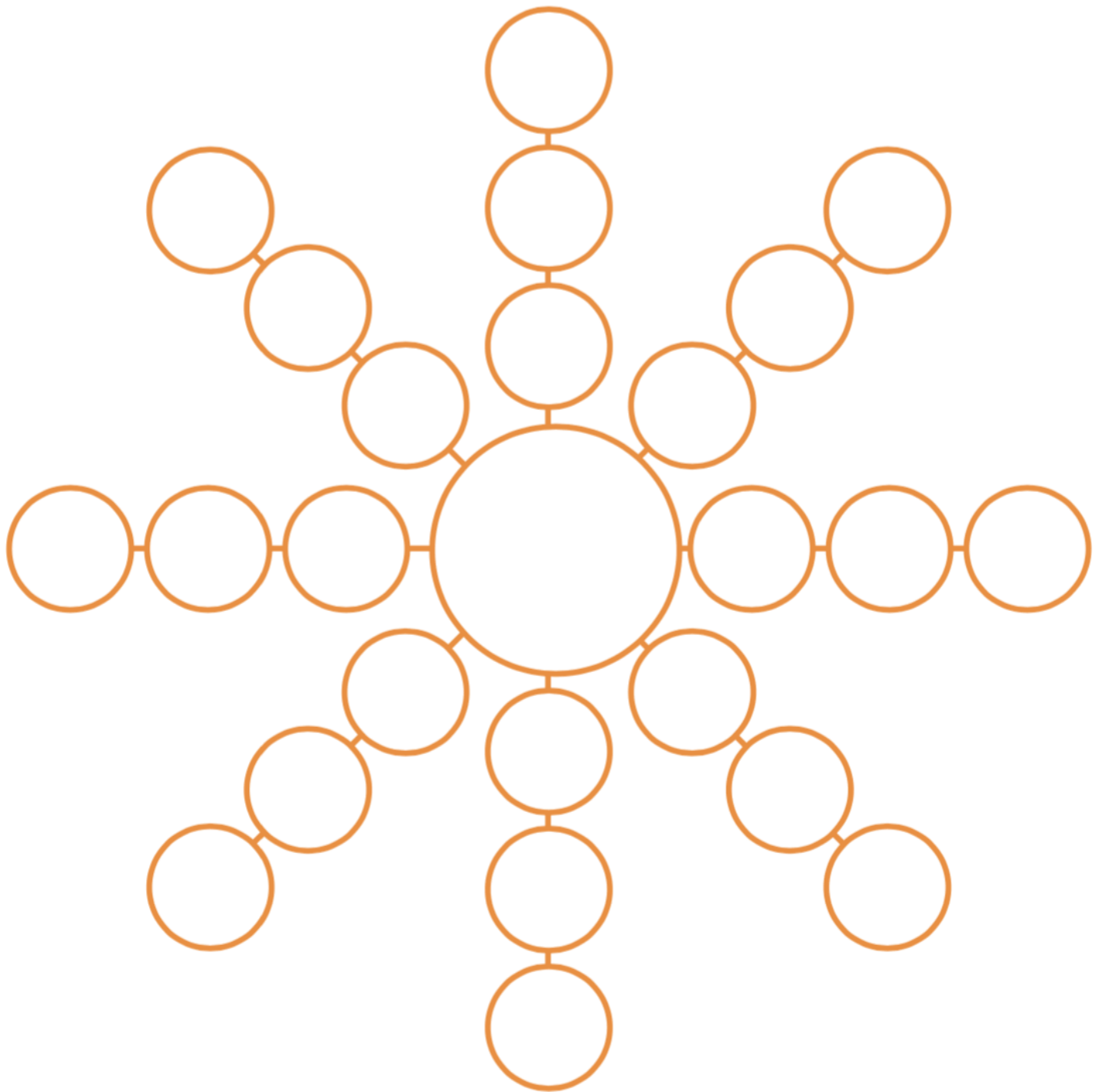
Variation

- This game can be played across the grades to support students with mental computation by adjusting the numbers, operations, and the number of blanks to reflect student needs.

Reference: From [Marilyn Burns – Four strikes and you're out](#). The game originally appeared in Tank, B., & Zolli, L. (2010). Teaching arithmetic: Lessons for addition and subtraction. Gate city books, North Carolina.

Appendix 1

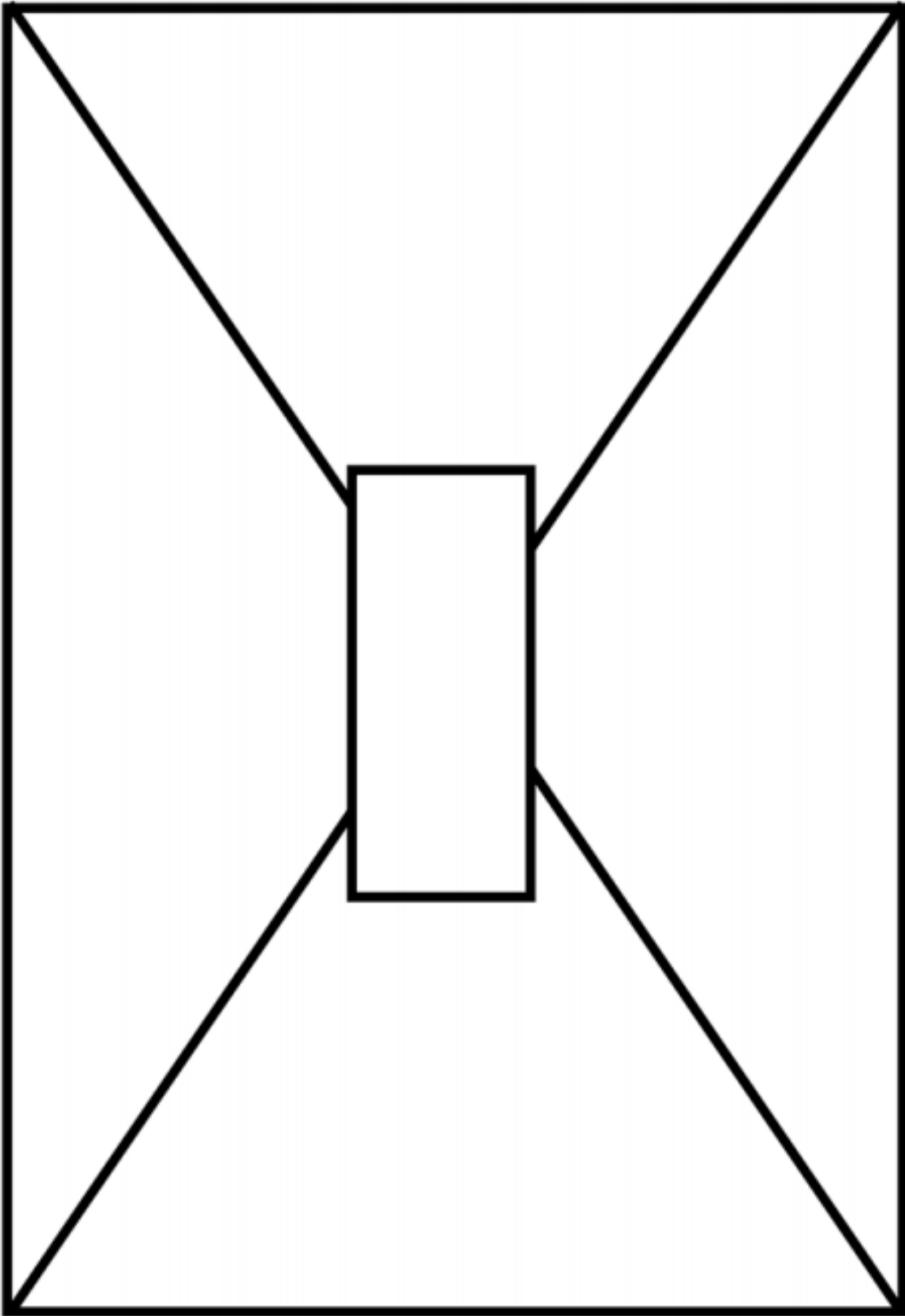
Addition wheel



Reference: [Developing Efficient Numeracy Strategies 1](#) © State of New South Wales, Department of Education.

Appendix 2

Think board



Appendix 3

Finding numbers: Number grid

| | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 10 | 79 | 30 | 48 | 22 | 70 | 30 | 47 |
| 90 | 21 | 55 | 9 | 8 | 92 | 33 | 70 |
| 78 | 22 | 45 | 1 | 80 | 20 | 33 | 66 |
| 25 | 25 | 50 | 90 | 91 | 18 | 34 | 10 |
| 10 | 21 | 8 | 71 | 9 | 52 | 64 | 36 |
| 55 | 43 | 45 | 44 | 11 | 30 | 36 | 18 |
| 35 | 57 | 93 | 4 | 31 | 40 | 29 | 71 |
| 17 | 25 | 36 | 39 | 58 | 60 | 31 | 29 |

Reference: [Developing Efficient Numeracy Strategies 1](#) © State of New South Wales, Department of Education.