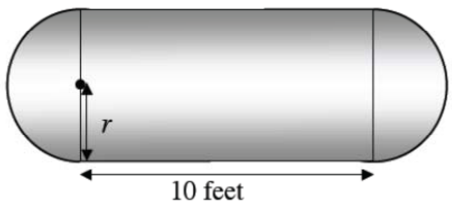
Opening Task

Propane Tanks

People who live in isolated or rural areas have their own tanks of natural gas to run appliances like stoves, washers, and water heaters.

These tanks are made in the shape of a cylinder with hemispheres on the ends.



The Insane Propane Tank *C*ompany makes tanks with this shape, in different sizes.

The cylinder part of every tank is exactly 10 feet long, but the radius of the hemispheres, r , will be different depending on the size of the tank.

The company want to double the capacity of their standard tank, which is 6 feet in diameter.

What should the radius of the new tank be? ______________________

Explain your thinking and show your calculations.

____

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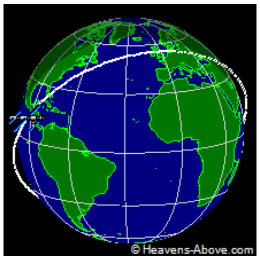
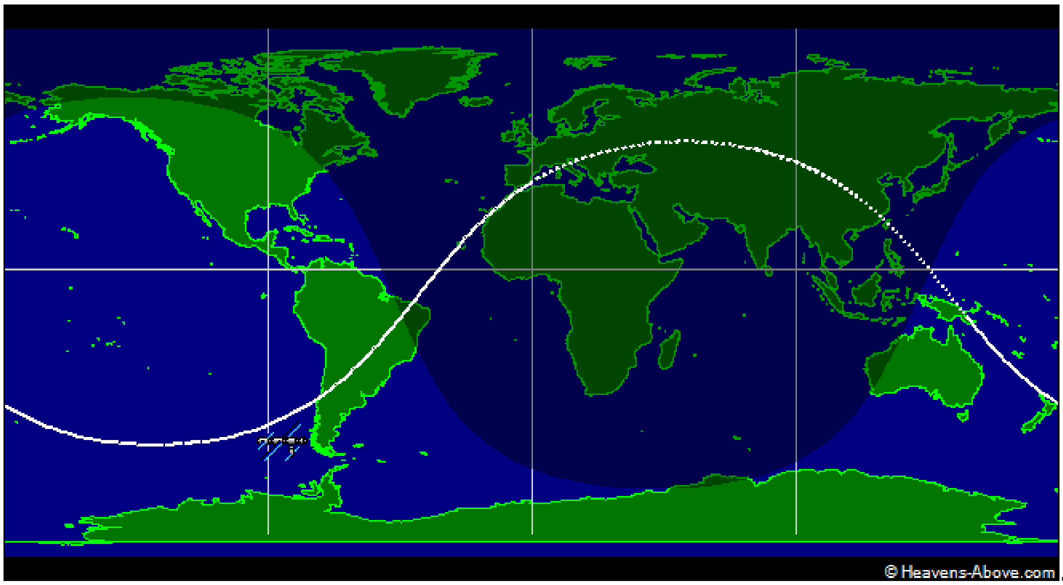
Focus Task

The space station orbits the Earth at a 51.6453° inclination every 92.413 minutes. If you take the orbital path 

and project it onto a flat map



the resulting ground track appears sinusoidal that alternates between 51.6453° N and 51.6453° S.

On a given day the space station crossed the equator going North at 5:15pm CDT. Write a periodic function that will model the ground track of the space station. When will it cross the equator next? When was the first time it crossed after 12:00pm CDT on that day?

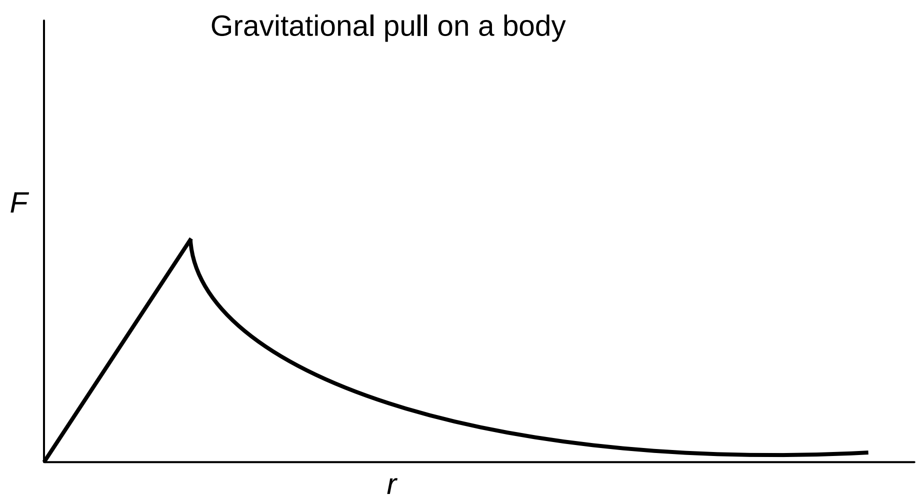
Coherence Task

In the graph below the left-hand piece is defined by the equation: $F = G(\frac{4}{3}\pi r \rho M_{you})$. The right-hand piece is defined by the equation: $= \frac{GM_{Earth}M_{you}}{r^2}$. Determine the transition point.

$M_{Earth} = 5.97\ \times 10^{24}\ Kg$

$G = 6.673 \times 10^{-11}\ N\ {m^2}/{kg^2}$

$\rho = 2.7\ {g}/{cm^3}$



<u>Overview of High School Math</u>

CCGPS Coordinate Algebra

The fundamental purpose of Coordinate Algebra is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Coordinate Algebra uses algebra to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CCGPS Analytic Geometry:

The focus of Analytic Geometry on the coordinate plane is organized into 6 critical areas. Transformations on the coordinate plane provide opportunities for the formal study of congruence and similarity. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. The study of circles uses similarity and congruence to develop basic theorems relating circles and lines. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. Quadratic expressions, equations, and functions are developed; comparing their characteristics and behavior to those of linear and exponential relationships from Coordinate Algebra. Circles return with their quadratic algebraic representations on the coordinate plane. The link between probability and data is explored through conditional probability. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CCGPS Advanced Algebra:

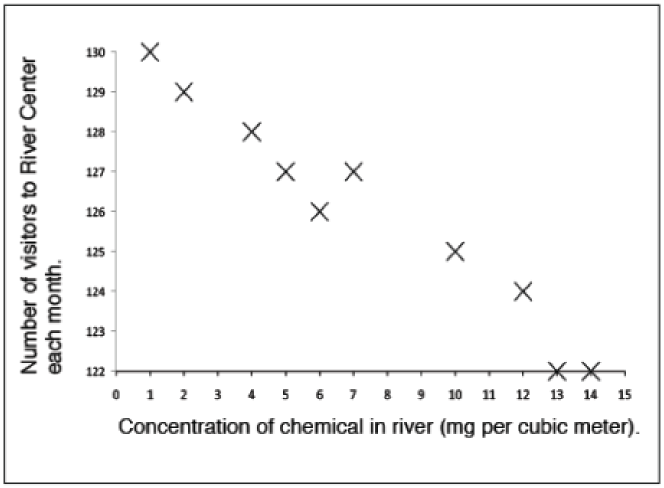
It is in Advanced Algebra that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into six critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to model periodic phenomena. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Deep Understanding Task

Muddying The Waters

The manager of the Riverside Center is concerned about visitor numbers.
He is certain the Center's popularity has been badly affected by an increase in river pollution.
He feels the local Environmental Agency should do something about it.

To support his argument he measured the chemical concentration in the river each month.
He also counted the number of people visiting the Center over several months.
He used the results to draw this chart.

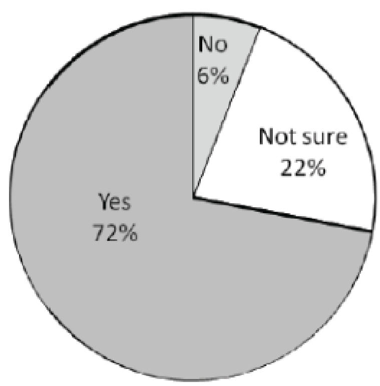


Scatter chart: Chemical concentration and number of visitors

At the same time the manager asked 18 visitors this question:
"The odor you can smell originates from the pollution in the river.
Is it spoiling your enjoyment of the Center?"

He displayed the results as a pie chart.

Pie chart showing the percentage of visitors whose enjoyment was spoiled.



The centre manager writes to the Environmental Officer to try to get something done about the river pollution.

> Dear Environmental Officer,
>
> Please find enclosed two charts.
>
> The scatter plot clearly shows that the increase in the concentration of the chemical in the river has caused a real drop-off in visitor numbers to the Center over the last year.
>
> The pie chart proves that people (not surprisingly) don't like the acrid smell of pollution wafting up from the river.
>
> The river needs to be cleaned up; it's not good for the environment and it's certainly not good for my business. Please let me know what action you intend to take.
>
> Yours faithfully,
>
> Manager, Riverside Center

Tasks

1. Describe in detail what you think the two charts show.

2. Do you think the Riverside Center Manager's argument is fair?
Explain your reasoning.

<u>Application Task</u> Illustrative Mathematics

You have been hired for a summer internship at a marine life aquarium. Your job requires that you dilute brine for the saltwater fish tanks. The brine is water and 15.8% sea salt (by weight). Thus, the salt concentration of the brine is 15.8%.

a. The supervisor has asked you to add fresh water in half-liter amounts to one liter of the brine. Let S be the function that assigns to each half-liter amount of fresh water added, x, the salt concentration of the resulting mixture. Write an expression for $S(x)$.

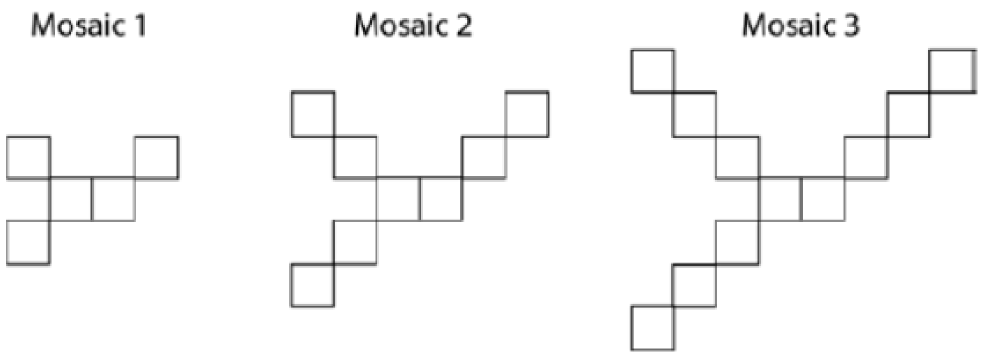
b. Describe how the graph of S is related to the graph of $y=1/x$.

c. Sketch the graph of S.

d. How much fresh water should you add to get a mixture which is 4% sea salt, approximately the salt concentration of the ocean?

Balanced Approach Task

Mosaics



Reuben learned in art class that a mosaic is made by arranging small pieces of colored material (such as glass or tile) to create a design. Reuben created a mosaic using tiles, then decided on a growing pattern and created a second and third mosaic. Reuben continued his pattern by building additional mosaics. He counted the number of tiles in each mosaic and then represented the data in multiple ways. He thinks he sees a relationship between the mosaic number and the total number of tiles in the mosaic.

1. Represent Reuben's data from the mosaics problem in at least three ways, including a general function rule, to determine the number of tiles in any mosaic.

2. Write a description of how your rule is related to the mosaic picture. Include a description of what is constant and what is changing as tiles are added.

3. How many tiles would be in the tenth mosaic? Use two different representations to show how you determined your answer.

4. Would there be a mosaic in Reuben's set that uses exactly 57 tiles? Explain your reasoning using at least one representation.

5. In Reuben's mosaic, there are 2 tiles in the center. How would the function rule change if the center of the mosaic contained 4 tiles instead? Explain your reasoning using two different representations.

Advanced Algebra

What's in Analytic Geometry B / Advanced Algebra

Unit 1	Unit 2
Extending the Number System	**Quadratic Functions**
Extend the properties of exponents to rational exponents. **MCC9-12.N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. **MCC9-12.N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **Use properties of rational and irrational numbers.** **MCC9-12.N.RN.3** Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **Perform arithmetic operations with complex numbers.** **MCC9-12.N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. **MCC9-12.N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **MCC9-12.N.CN.3 (+)** Find the conjugate of a complex number; use conjugates to find ~~moduli and~~ quotients of complex numbers. **Perform arithmetic operations on polynomials** **MCC9-12.A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *(Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.)*	**Use complex numbers in polynomial identities and equations.** **MCC9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions. **Interpret the structure of expressions** **MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **Write expressions in equivalent forms to solve problems** **MCC9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.★ **MCC9-12.A.SSE.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.★ **Create equations that describe numbers or relationships** **MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and~~ quadratic functions, ~~and simple rational and exponential functions~~.★ **MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **Solve equations and inequalities in one variable** **MCC9-12.A.REI.4** Solve quadratic equations in one variable. **MCC9-12.A.REI.4a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **MCC9-12.A.REI.4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. **Solve systems of equations** **MCC9-12.A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. **Interpret functions that arise in applications in terms of the context** **MCC9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.★

	MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Analyze functions using different representations</u>** **MCC9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.IF.7a** Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima.★ **MCC9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.IF.8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **MCC9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Build a function that models a relationship between two quantities</u>** **MCC9-12.F.BF.1** Write a function that describes a relationship between two quantities.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.BF.1a** Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.BF.1b** Combine standard function types using arithmetic operations. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Build new functions from existing functions</u>** **MCC9-12.F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Construct and compare linear, quadratic, and exponential models and solve problems</u>** **MCC9-12.F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★ **<u>Summarize, represent, and interpret data on two categorical and quantitative variables</u>** **MCC9-12.S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ **MCC9-12.S.ID.6a** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize ~~linear~~, quadratic, ~~and exponential~~ models.★

Advanced Algebra

Unit 3	Unit 4
Modeling Geometry	**Applications of Probability**
<u>Translate between the geometric description and the equation for a conic section</u> **MCC9-12.G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **MCC9-12.G.GPE.2** Derive the equation of a parabola given a focus and directrix. **<u>Use coordinates to prove simple geometric theorems algebraically</u>** **MCC9-12.G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *(Restrict to context of circles and parabolas)*	**<u>Understand independence and conditional probability and use them to interpret data</u>** **MCC9-12.S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★ **MCC9-12.S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★ **MCC9-12.S.CP.3** Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ **MCC9-12.S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.★ **MCC9-12.S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.★ **<u>Use the rules of probability to compute probabilities of compound events in a uniform probability model</u>** **MCC9-12.S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ **MCC9-12.S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★