ADVANCED ALGEBRA II

CHAPTER 6 – RATIONAL EXPONENTS AND RADICAL FUNCTIONS

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6.1 EVALUATE *n*th ROOTS AND USE RATIONAL EXPONENTS

WARM-UP / INTRODUCTION

CONCEPT DEVELOPMENT

REAL *n*th ROOTS OF *a*

Let *n* be an integer (n > 1) and let *a* be a real number.

***n* is an even integer**	***n* is an odd integer**

EXAMPLE 1: Find *n*th roots

A. n = 3, a = -125 B. n = 4, a = 81 C. n = 6, a = -64

RATIONAL EXPONENTS

RATIONAL EXPONENTS

Let $a^{1/n}$ be an *n*th root of *a*, and let *m* be a positive integer.

EXAMPLE 2: Evaluate expressions with rational exponents

Evaluate each expression.

A. $16^{3/2}$ B. $(-27)^{2/3}$ C. $32^{-2/5}$ D. $(-16)^{3/2}$ E. $-16^{3/2}$

F. $\left(\frac{27}{64}\right)^{2/3}$ G. $8^{3/2}$ H. $9^{-1/2}$ I. $\frac{1}{81^{-3/4}}$

EXAMPLE 3: Approximate roots with a calculator

A. $9^{1/5}$ B. $(\sqrt[4]{12})^3$ C. $\sqrt[6]{1234}$ D. $25^{-1/3}$

EXAMPLE 4: Solve equations using *n*th roots

Solve the equation.

A. $4x^5 = 128$ B. $(x - 3)^4 = 21$ C. $3x^{2/3} - 5 = 10$

EXAMPLE 5: Use *n*th roots in problem solving

SHOT PUT The shot used in men's shot put has a volume of about 905 cubic centimeters. Find the radius of the shot.

BOWLING A bowling ball has a surface area of about 232 square inches. Find the radius of the bowling ball.

HW 6.1A: p. 417 # 3 – 24

HW 6.1B: p. 417 # 25 – 32, 38 – 41, 50 – 58, 62, 64

MIXED REVIEW: p. 419 # 69 – 82

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6.2 APPLY PROPERTIES OF RATIONAL EXPONENTS

PROPERTIES OF RATIONAL EXPONENTS

Let *a* and *b* be real numbers and let *m* and *n* be rational numbers. The following properties have the same names as those listed earlier, but now apply to rational exponents as illustrated.

	Property	**Example**
1.		
2.		
3.		
4.		
5.		
6.		

EXAMPLE 1: Use properties of exponents

A. $5^{1/2} \cdot 5^{2/3}$

B. $\left(3^{1/2} \cdot 4^{2/3}\right)^2$

C. $\left(4^5 \cdot 2^5\right)^{-1/5}$

D. $\dfrac{6^{2/3}}{6^{1/5}}$

E. $\left(\dfrac{4^{2/3}}{7^{2/3}} \div \right)^2$

F. $\dfrac{3}{3^{1/4}}$

EXAMPLE 2: Apply properties of exponents

BIOLOGY A mammal's surface area *S* (in square centimeters) can be approximated by the model $S = km^{2/3}$ where *m* is the mass (in grams) of the mammal and *k* is a constant. Approximate the surface area of the rabbit that has a mass of 3.4×10^3 grams, if k = 9.75.

PROPERTIES OF RADICALS

Product Property of Radicals

Quotient Property of Radicals

EXAMPLE 3: Use properties of radicals

Use the properties of radicals to simplify the expression.

A. $\sqrt[3]{12} \times \sqrt[3]{18}$

B. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}}$

SIMPLEST FORM

EXAMPLE 4: Write radicals in simplest form

Write the expression in simplest form.

A.

B.

LIKE RADICALS

EXAMPLE 5: Add and subtract like radicals and roots

Simplify the expression.

A.

B.

C.

VARIABLE EXPRESSIONS

When *n* is odd

When *n* is even

EXAMPLE 6: Simplify expressions involving variables

A. B.

C. D.

EXAMPLE 7: Write variable expressions in simplest form

A. B.

C. D.

EXAMPLE 8: Add and subtract expressions involving variables

A. B.

C. D.

ADDITIONAL NOTES / ACTIVITIES:

HW 6.2A: p. 424 # 3 – 23

HW 6.2B: p. 424 # 24 – 50 even

HW 6.2C: p. 425 # 52 – 59, 60 – 80 even, 83, 84

MIXED REVIEW: p. 427 # 91 – 106

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6.3 PERFORM FUNCTION OPERATIONS AND COMPOSITION

OPERATIONS WITH FUNCTIONS

Let *f* and *g* be any two functions. A new function *h* can be defined by performing any of the four basic operations on *f* and *g*.

Operation	Definition	Example
Addition		
Subtraction		
Multiplication		
Division		

The domain of *h* consists of the x-values that are in the domains of both *f* and *g*. Additionally, the domain of the quotient does not include x-values for which g(x) = 0.

POWER FUNCTIONS

EXAMPLE 1: Add and subtract functions

EXAMPLE 2: Multiply and divide functions

EXAMPLE 3: Solve a multi-step problem

COMPOSITION OF FUNCTIONS

EXAMPLE 4: Find compositions of functions

EXAMPLE 5: Find compositions of functions

EXAMPLE 6: Solve a multi-step problem

HW 6.3A: p. 432 # 3 – 19 odd, 20, 22

HW 6.3B: p. 432 # 23, 25, 27, 28 – 35, 40 – 42, 43

MIXED REVIEW: p. 434 # 48 – 53, 58, 60, 62, 64, 66

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CHAPTER 6 – RATIONAL EXPONENTS AND RADICAL FUNCTIONS
6.4 USE INVERSE FUNCTIONS

DEFINITIONS / INTRODUCTION

EXAMPLE 1: Find an inverse relation

Find an equation for the inverse of the relation $y = -4x + 3$.

INVERSE FUNCTIONS

EXAMPLE 2: Verify that functions are inverses

Verify that and are inverses functions.

INVERSES OF NONLINEAR FUNCTIONS

EXAMPLE 4: Find the inverse of a power function

HORIZONTAL LINE TEST

EXAMPLE 5: Find the inverse of a cubic function

ADDITIONAL NOTES / ACTIVITIES

HW 6.4A: p. 442 # 3 – 11 odd, 15 – 20

HW 6.4B: p. 443 # 22 – 42 even, 48, 50

MIXED REVIEW: p. 445 # 52 – 66

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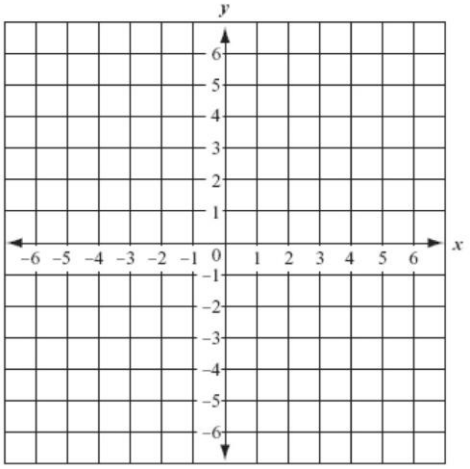
6.5 GRAPH SQUARE ROOT AND CUBE ROOT FUNCTIONS

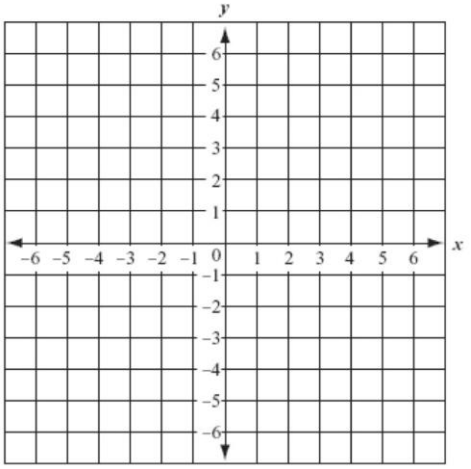
THE TWO IMPORTANT RADICAL FUNCTIONS

$f(x) = \sqrt{x} = x^{1/2}$



Domain: Range:

$f(x) = \sqrt[3]{x} = x^{1/3}$

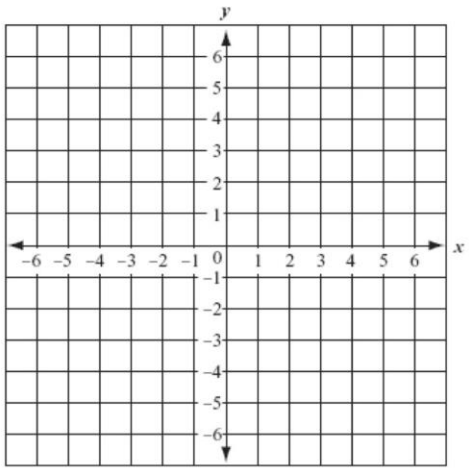


Domain: Range:

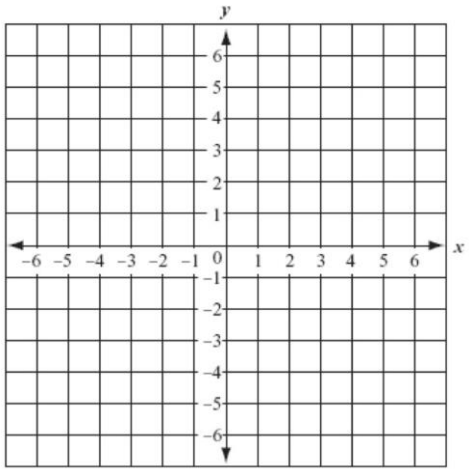
YOU WILL NEED TO MEMORIZE THESE GRAPHS!!!

Now, we will learn how to graph functions of the form $f(x) = a\sqrt{x - h} + k$ and $f(x) = a\sqrt[3]{x - h} + k$

EXAMPLE 1: Graph a square root function



EXAMPLE 2: Graph a cube root function



TRANSLATIONS OF RADICAL FUNCTIONS

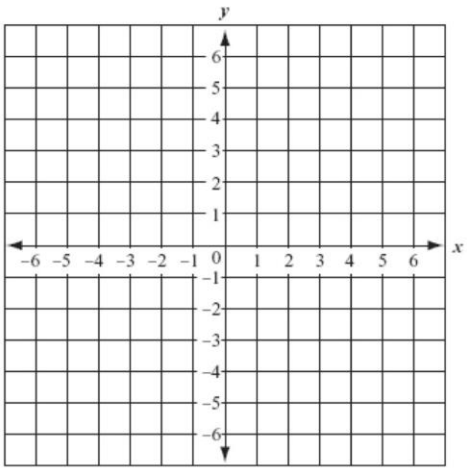
GRAPHS OF RADICAL FUNCTIONS

To graph $f(x)=a\sqrt{x-h}+k$ or $f(x)=a\sqrt[3]{x-h}+k$, follow these steps:

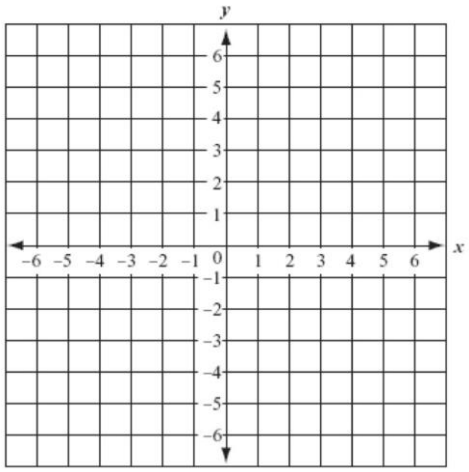
Step 1: **Sketch** the graph of $f(x)=a\sqrt{x}$ or $f(x)=\sqrt[3]{x}$

Step 2: **Translate** the graph horizontally *h* units and vertically *k* units

EXAMPLE 4: Graph a translated square root function



EXAMPLE 5: Graph a translated cube root function



HW 6.5A: p. 449 # 3 – 15 odd, 35

HW 6.5B: p. 449 # 16, 20, 22, 25, 28 – 33, 40 (Use Desmos)

MIXED REVIEW: p. 451 # 41 – 58

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6.6 SOLVE RADICAL EQUATIONS

RADICAL EQUATIONS

SOLVING RADICAL EQUATIONS

To solve a radical equation, follow these steps:

Step 1: **Isolate** the radical on one side of the equation, if necessary.

Step 2: **Raise** each side of the equation to the same power to eliminate the radical and obtain a Linear, quadratic, or other polynomial equation.

Step 3: **Solve** the polynomial equation using techniques that you learned in previous chapters. **Check your solution, strange things can happen!**

EXAMPLE 1: Solve a radical equation

GUIDED PRACTICE

A. B. C.

EXAMPLE 2: Solve an equation with a rational exponent

A. B.

GUIDED PRACTICE

Solve each equation. Check your solution.

A. B. C.

D. E. F.

EXTRANEOUS SOLUTIONS

EXAMPLE 3: Solve an equation with an extraneous solution

SQUARING TWICE

This is how extreme some of these problems can become, given the problem situation…

EXAMPLE 4: Solve an equation with two radicals

ADDITIONAL NOTES / ACTIVITIES

HW 6.6A: p. 456 # 3 – 33 odd

HW 6.6B: p. 457 # 34 – 42, 45, 47, 49, 51, 53, 56

MIXED REVIEW: p. 459 # 63 – 68, 75 – 80

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CHAPTER 6 REVIEW

In preparation for the Chapter 6 Examination, you **should** do the following things:

1. Study / Review Notes

2. Chapter 6 Review: p. 466 # 1 – 32

3. Chapter 6 Test: p. 469 # 1 – 38

4. Chapter 6 Standardized Test Preparation: p. 472 # 1 - 20

4. Chapter 6 Extra Practice: p. 1015 # 1 – 56