Advanced Algebra with Financial Applications
COURSE PROPOSAL

Advanced Algebra with Financial Applications
DESCRIPTION FOR HIGH SCHOOL COURSE CATALOG

Advanced Algebra with Financial Applications
Credits: One
Prerequisite: Algebra 1

Advanced Algebra with Financial Applications is a college-preparatory course that will use sophisticated mathematics to give you the tools to become a financially responsible young adult. The course employs algebra, precalculus, probability and statistics, calculus and geometry to solve financial problems that occur in everyday life. Real-world problems in investing, credit, banking, auto insurance, mortgages, employment, income taxes, budgeting and planning for retirement are solved by applying the relevant mathematics. Field projects, computer spreadsheets, and graphing calculators are key components of the course.

Advanced Algebra with Financial Applications
TEXTBOOK AND SUPPLEMENTAL MATERIALS

Gerver, R. & Sgroi, R. *Financial Algebra*. South-Western/Cengage Learning: Mason OH. Copyright 2014.

Supplemental Websites: Unit-specific websites are listed in the Course Outline section of this document, after the Mathematics Learning Goals.

Advanced Algebra with Financial Applications
BRIEF COURSE DESCRIPTION

Advanced Algebra with Financial Applications is a mathematical modeling course that is algebra-based, applications-oriented, and technology-dependent. The course addresses college preparatory mathematics topics from Advanced Algebra, Statistics, Probability, Precalculus, and Calculus under seven financial umbrellas: Banking, Investing, Credit, Employment and Income Taxes, Automobile Ownership, Independent Living, and Retirement Planning and Household Budgeting. The course allows students to experience the interrelatedness of mathematical topics, find patterns, make conjectures, and extrapolate from known situations to unknown situations. The mathematics topics contained in this course are introduced, developed, and applied in an as-needed format in the financial settings covered. Students are encouraged to use a variety of problem-solving skills and strategies in real-world contexts, and to question outcomes using mathematical analysis and data to support their findings. The course offers students multiple opportunities to use, construct, question, model, and interpret financial situations through symbolic algebraic representations, graphical representations, geometric representations, and verbal representations. It provides students a motivating, young-adult centered financial context for understanding and applying the mathematics they are guaranteed to use in the future, and is thusly aligned with the recommendations of the Common Core State Standards, as stated in this excerpt:

"...all students should be strongly encouraged to take math in all years of high school. ...An array of challenging options will keep math relevant for students, and give them a new set of tools for their futures..." From the Common Core State Standards

Advanced Algebra with Financial Applications
COURSE PURPOSE

Advanced Algebra with Financial Applications offers 11th and 12th grade students an opportunity to view the world of finance through a mathematical lens. The topics were developed using the Common Core State Standards in Mathematics, the California Mathematics Standards, and the NCTM Curriculum and Evaluation Standards. The mathematical formulas, functions, and pictorial representations used assist students in making sense of the financial world around them and equip them with the ability to make sound financial decisions.

The overarching purpose of the course is to develop the type of mathematically proficient students addressed in this excerpt from the Common Core State Standards for Mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Advanced Algebra with Financial Applications builds strength in reasoning and number sense, because the real-world applications demand that solutions make sense. Through contextual problem solving and the mathematical modeling of real situations, the course gives the students the motivation to persevere through routine and non-routine problems, and as a result, develop strength and confidence in their mathematics ability.

Advanced Algebra with Financial Applications
COURSE OUTLINE

Unit 1: Banking Services

In this unit, students use exponential functions to compute compound interest and compare it to simple interest. They derive formulas and use iteration to compute compound interest. They apply their findings to short-term, long-term, single deposit and periodic deposit accounts. The problems, activities and projects inherent in studying banking are a natural forum for all eight CCSS Mathematical Practice standards, but this unit highlights MP1, MP4, MP5, MP6, and MP8.

Common Core State Standards for Mathematical Content that are Addressed

A-CED4
A-SSE1a, A-SSE1b, A-SSE3
F-IF4, F-IF8b
F-BF1a
N-RN1, N-RN2

Mathematics Topics

- Derivation of the compound interest formula
- Exponential functions
- Computations based on iterative processes
- Limits of polynomial functions, rational functions, and sequences
- Natural logarithm as the inverse of the exponential function
- Exponential growth and decay
- Solving exponential equations
- Using inductive reasoning

Mathematics Learning Goals Cross-Referenced with Textbook and Key Assignments

- **Students will use the simple interest formula $I = PRT$ and using inverse operations to solve for missing variables in interest problems.** *Financial Algebra* section 3-3 teaches students how to compute I (interest), P (principal), R (rate), and T (time). They solve for one of the variables in terms of the others in numeric and literal equations. They use number sense to confirm the reasonableness of their answers.
- **Students will use iteration to show how compounding pays "interest on your interest."** *Financial Algebra* section 3-4 shows iterative processes that underscore the meaning of compound interest. Students use calendars to document the iteration and see where interest is added and then used as principle to compute subsequent interest. Key Assignment 1.1 requires students to guess future values based on the intuitive notion of compound interest.

- **Students will derive the compound interest formula $B = (1 + \frac{r}{n})^{nt}$ by using patterns and inductive reasoning.** *Financial Algebra* section 3-4 algebraically derives the formula for quarterly compounding, and then uses inductive reasoning to extend the formula into compounding over any period of time. Key Assignment 1.2 involves looking for patterns to make inductive inferences about a generalized compound interest formula.
- **Students will compute compound interest with and without the formula.** *Financial Algebra* sections 3-3 through 3-6 illustrates compounding with and without the formulas, and shows how daily compounding is adjusted for deposits and withdrawals.
- **Students will apply and interpret the limit notation $\lim_{x \to a} f(x) = b$.** *Financial Algebra* section 3-6 investigates limits since they will be used in the concept of continuous compounding. Students will compute limits of rational functions and other expressions. Key Assignment 1.3 will highlight a limit that is central to the continuous compounding formula.
- **Students will model an infinite series and finding a finite sum for an infinite series with common ratio ½.** *Financial Algebra* section 3-6 features the classic problem where someone walks across a room by first walking half the room, then half of the remaining half, etc. Students realize that they do actually cover the entire room, and that the sum is 1.
- **Students will compute limits of rational functions as $x \to \infty$. .** *Financial Algebra* section 3-6 has students examining numerators and denominators to intuitively determine function values as variable quantities approach infinity.
- **Students will approximate the natural base *e* by examining the sequence $\left\{\left(1 + \frac{1}{x}\right)^x\right\}$ for increasing values of x.** *Financial Algebra* section 3-6 discuss the "battle" between the decreasing base and the increasing exponent, which ends up with the natural base e. Key Assignment 1.3 uses calculator tables to make inferences about a rational function's pattern as a variable approaches infinity.
- **Students will inductively derive the natural base *e* using limits.** *Financial Algebra* section 3-6 has an activity that displays the pattern leading to the natural base e. This is a component of Key Assignment 1.3.
- **Students will apply the natural base *e* in the continuous compounding formula $B = Pe^{rt}$.** *Financial Algebra* section 3-6 uses the formula for continuous compounding to model the effect as compared to other types of compounding.
- **Students will be able to identify $y = ax^b$ as exponential decay when x < 1.** *Financial Algebra* section 5-6 models exponential decay by looking at depreciation of an automobile, which offers a contrast to growth via compound interest.
- **Students will be able to identify $y = ax^b$ as exponential growth when x > 1.** *Financial Algebra* section 8-5 uses exponential growth to model rent increases. The constant percent of annual increase is modeled by an exponential function.

- **Students will graph exponential functions of the type $y = ax^b$.** *Financial Algebra* section 5-6 has students using exponential regression on the calculator to model exponential functions. These functions are used to model automobile prices over the years.
- **Students will analyze rational function behavior and limits of the form $\lim_{x\to\infty} \frac{ax^n \pm b}{cx^m \pm d}$ where n = m, n >m, and n< m.** *Financial Algebra* section 3-6 has students examining numerators and denominators to compare their growth. The relative size of the numerators and denominators, and the powers of the highest exponents lead students to find rules for rational function limits.
- **Students will compute Annual Percentage Yield (APY) where $APY = \left(1 + \frac{r}{n}\right)^n - 1$, given the Annual Percentage Rate (APR).** *Financial Algebra* section 3-5 derives formulas used to compute and compare APY and APR.
- **Students will use the compound interest formula to derive the present value of a single deposit investment formula**

$$P = \frac{B}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Financial Algebra section 3-8 derives the present value formula in the context of compound interest.

- **Students will use the compound interest formula to derive the present value of a periodic deposit investment formula**

$$P = \frac{B\left(\frac{r}{n}\right)}{\left(1 + \frac{r}{n}\right)^{nt} - 1}$$

Financial Algebra section 3-8 adjusts the present value formula to include periodic deposits.

- **Students will use the future value of a periodic deposit investment formula**

$$B = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\left(\frac{r}{n}\right)}$$

Financial Algebra section 3-7 uses the formulas for the future value of a single-deposit investment and for a periodic deposit. Key Assignment 1.4 applies this formula to college costs, which is something many of the students are concerned with. They also have to imagine they are young parents and have to plan to save for college costs in the future, which they must compute using regression.

- **Students will adapt the algebra from banking formulas for input into a spreadsheet.** *Financial Algebra* sections 3-4 and 3-7 require students to adapt the structure of algebraic thinking to the language understood by the spreadsheet program.

This requires intense attention to precision since any deviation from the correct syntax will prevent the spreadsheet from computing correctly.

Internet Resources for Supporting Learning in the Banking Unit

Federal Deposit Insurance Corporation www.fdic.gov. Students can explore how they can maximize accounts that are covered by have different types of accounts in one bank.

Loan Payment Calculator www.bankrate.com. Students can easily compare many different types of loans by changing term, rate, and principle, and they can compare the software answers to answer they get using algebraic formulas.

Unit 2: Investing

Students are introduced to basic business organization terminology in order to read, interpret, chart and algebraically model stock ownership and transaction data. Statistical analysis plays a very important role in the modeling of a business. Using linear, quadratic, and regression equations in that process assists students in getting a complete picture of supply, demand, expense, revenue, and profit as they model the production of a new product. The problems, activities, and key assignments in this Investing Unit offer students opportunities to learn, explore, and use the CCSS Mathematical Practices MP1, MP2, MP3,MP4, MP5.

Common Core State Standards for Mathematical Content that are Addressed
A-CED1, A-CED2, A-CED3, A-CED4
A-REI2, A-REI3, A-REI4b, A-REI6, A-REI7, A-REI10, A-REI11,A-REI12
A-SSE1
F-IE4
F-IF1, F-IF4, F-IF5, F-IF7a, F-IF8, S-ID6
N-Q1, N-Q2, N-Q3
S-ID8, S-ID9

Mathematics Topics

- Algebraic ratios and proportions
- Algebraic representations of percent increase and decrease
- Pictorial representations of data
- Scatterplots
- Operations with functions
- Function domains
- Function evaluation
- Linear and quadratic functions to model situations

- Rational functions
- Systems of equations (linear/linear and linear/quadratic)
- Systems of inequalities
- Regression equations
- Extrapolation and interpolation
- Pearson Product-Moment Correlation Coefficient
- Axis of symmetry, roots, intercepts and concavity of parabolas
- Quadratic formula
- Absolute and relative extrema
- Explanatory, response, and lurking variables
- Causation vs. correlation for bivariate data
- Transitive Property of Dependence
- Zero Net Difference

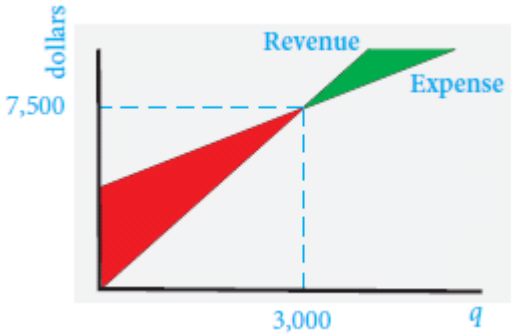
Mathematical Learning Goals Cross-Referenced with Textbook and Key Assignments

- **Students will construct, use, and interpret algebraic ratios and proportions.** *Financial Algebra* section 1-1 examines ratios as used to model business partnership shares. Here, given a set of n compound ratios and a total T, students write and solve equations in terms of x and T where the variable coefficients are the ratios and determine the amount associated with each ratio. *Financial Algebra* sections 1-2, 1-6, 1-8, and 1-9 explore the use of ratio and proportions in the context of stock market transactions, stock prices, and stock dividends. *Financial Algebra* sections 2-2, 2-5, and 2-6 offer students opportunities to model and interpret ratios and proportions within the context of a new business venture. In all of these sections, students use, interpret and evaluate rational expressions, algebraic fractions, ratios, and proportions
- **Students will determine, use, and interpret percent increase/decrease of monetary amounts.** *Financial Algebra* section 1-2 has students determine percent increase and decrease in the context of stock prices. Students then translate what they have learned to writing algebraic and spreadsheet formulas for percent net change.
- **Students will constructing and interpret pictorial representations of data.** *Financial Algebra* sections 1-3 and 1-4 teaches students how to chart and interpret stock market trends pictorially and graphically. *Financial Algebra* Chapter 2 focuses on the mathematical modeling of a business. Here, students use regression analysis, functions, and graphs to model how expense, revenue and profit functions work together to assist in setting the per unit price that will result in the maximum profit. Key Assignment 2.1 applies these skills in the context of charting corporate stocks using OHLC and Candlestick charts. Students also chart multiple time intervals of simple moving averages for closing prices in order to analyze crossover graphs. Based upon the interpretations of these graphs, students are asked to identify and explain stock trend reversal points.

- **Given a set of *n* data points, $p_1,\ p_2, p_3, ..., p_{n-1}, p_n$, students will calculate and interpret *d*-day simple moving averages by applying the Arithmetic Average Formula and the Subtraction/Addition Method.** *Financial Algebra* section 1-4 introduces students to the concept of data smoothing through the use of simple moving averages. Students calculate, chart, and interpret the simple moving average crossover graphs to make informed decisions about stock trend reversals. Key Assignment 2.1 uses real world data collected by students to reinforce the smoothing effects that simple moving averages have on that data.
- **In any *a-for-b* stock split, where *P* represent the pre-split price per share, students will calculate the post-split price per share using $\frac{b}{a} \times P$** *Financial Algebra* section 1-8 employs ratio and proportions to create pre- and post-split market price functions.
- **In any *a-for-b* stock split, where *D* represent the pre-split number of shares, students will calculate the post-split number of shares using $\frac{a}{b} \times D$** *Financial Algebra* section 1-8 employs ratios and proportions to create pre- and post-split outstanding shares functions.
- **Students will calculate the stock yield percentage using the formula $Yield = \frac{A}{C} \times 100$, where A represents the annual dividend per share and C represents the current price per share.** *Financial Algebra* section 1-9 defines yield as a ratio and offers students opportunities to calculate monetary yield amounts as well as write algebraic expressions that can be used to create yield formulas.
- **Students will construct and interpret scatterplots .** *Financial Algebra* sections 2-1 and 2-2 introduce students to scatter plots as an important part of regression analysis. Students will identify form, direction, and strength from a scatterplot. Section 2-8 has students construct and interpret scatterplots as part of the business model for determining optimal unit prices. Key Assignment 2.2 uses focus group data in the form (p,q) where p is a potential price of a new product and q is the quantity of the product that the focus group members would purchase if set at that price. Students construct and analyze a scatterplot of these ordered pairs.
- **Students will delineate causation vs. correlation for bivariate data.** *Financial Algebra* sections 2-1 and 2-2 introduce students to the concepts of causal and correlational relationships in the context of small business ventures.
- **Students will identify explanatory, response, and lurking variables.** *Financial Algebra* section 2-1 sets the stage for regression by examining business trend analysis variables.
- **Students will find, interpret, and graph linear regression equations and determine the correlation coefficient.** *Financial Algebra* sections 2-2 and 2-8 utilize linear regression analysis to model business situations. Students determine domains for which prediction using a regression line is considered extrapolating or interpolating. Students find and interpret the Pearson Product-Moment Coefficient of Correlation and use that value to make inferences about the line of best fit in a business model. Key Assignment 2.1 offers students an opportunity to use regression analysis in making stock price predictions using the closing price data they have collected over a 15 trading day

period. Key Assignment 2.2 has students determine a linear demand regression equation using focus group data in the form (p,q) where p is a potential price of a new product and q is the quantity of the product that the focus group members would purchase if set at that price. They must find the correlation coefficient and interpret the value as it pertains to the focus group data and the demand regression equation.

- **Students will create and evaluate functions, and use them to model situations.** *Financial Algebra* Chapters 1 and 2 have students creating, using, and graphing functions in the contexts of the stock market and business entrepreneurship.
- **Students will translate verbal situations into algebraic linear functions.** *Financial Algebra* sections 1-1, 1-2, 1-6, 1-7, 1-8, 1-9, and 1-10 offer students opportunities to model stock market situations using linear functions. Students also use linear functions in section 2-3 to model product demand and in section 2-4 to model fixed and variable expenses. Key Assignment 2.2 asks students to model expense and demand using linear functions.
- **Students will translate verbal situations into quadratic functions**. *Financial Algebra* sections 2-5 and 2-6 explore the use of quadratic functions to model revenue and profit. Key Assignment 2.2 has students model revenue and profit using quadratic functions.
- **Students will translate verbal situations into linear and quadratic inequalities.** *Financial Algebra* sections 2-6 through 2-8 use linear and quadratic inequalities to contextualize the relationships between and among expense, revenue, and profit functions.
- **Students will translate verbal situations into linear and quadratic inequalities.** *Financial Algebra* sections 2-6 through 2-8 use linear and quadratic inequalities to contextualize the relationships between and among expense, revenue, and profit functions.
- **Students will solve linear systems of equations and inequalities such as:**



- **and identify points of intersection and domains in the context of the problem situation.** *Financial Algebra* section 2-4 explores the use of linear systems to model revenue and expense situations. Students identify domains for which $f(x) > g(x)$, $f(x) = g(x)$, and $f(x) < g(x)$. Key Assignment 2.2 asks students to contextualize the domains of intersecting functions $f(x)$ and $g(x)$ where $f(x) > g(x)$, $f(x) = g(x)$, and $f(x) < g(x)$.
- **Students will solve linear-quadratic systems of equations and inequalities, and interpret the roots, intersection points, relative extrema, absolute extrema, and domains in the context of the problem situation.** *Financial Algebra* sections 2-6

through 2-8 models business contexts using linear-quadratic systems of equations. Key Assignment 2.2 uses linear-quadratic systems of equalities and inequalities to contextualize the modeling of product price setting for optimal profit.

- **Students will use the transitive property of dependence.** *Financial Algebra* section 2-8 sets the stage for the linear/quadratic business model by examining the concept of variable dependence (if x depends on y, and y depends on z, it follows that x depends on z.)
- **Students will determine the zero net difference.** *Financial Algebra* section 2-6 defines zero net difference functions as a function that is the difference of a quadratic revenue function and a linear expense function. The zero net difference points are the points of intersection of the linear/quadratic system.
- **Students will write algebraic formulas for use in spreadsheets.** *Financial Algebra* sections 1-2, 1-4, 1-8, 1-9, 2-2, and 2-6 ask students to create stock market and business model linear and quadratic formulas for use in spreadsheets.

Internet Resources for Supporting Learning in the Investing Unit
The New York Stock Exchange www.nyse.nyx.com This site offers students and educators detailed and current information about the New York Stock Exchange

Yahoo! Finance finance.yahoo.com Students can access historical and daily trading information on corporate stocks.

The Stock Market Game www.stockmarketgame.org This website offers students, teachers, and parents virtual investment opportunities in an educational setting.

Thirteen edonline "Could You Start A Business?"
http://www.thirteen.org/edonline/lessons/fe_start/index.html
Teachers and students explore the financial requirements needed to start a business.

Investopedia www.investopedia.com This is a comprehensive investment website for students and adults. It covers topics on investment, the stock markets, personal finance, trading, and much more. It is an excellent source of investing information for many of the student projects.

Unit 3: Employment and Income Taxes

Many Internal Revenue Service and Social Security Administration regulations can be modeled by using linear and polygonal functions that have different slopes over different domains. Line-by-line instructions for IRS forms can also be algebraically symbolized. The problems, activities and projects inherent in studying employment and income taxes are a natural forum for all eight CCSS Mathematical Practice standards, but this unit highlights MP1, MP4, MP5, MP6, and MP7.

Common Core State Standards for Mathematical Content that are Addressed
A-CED1, A-CED2, A-CED3, A-CED4
A-REI3
A-SSE1
F-BF1
F-IF1, F-IF2, F-IF4, F-IF7b, F-IF8
F-LE1

Mathematics Topics

- Point-slope form of linear equations
- Jump discontinuities
- Continuous functions with cusps
- Slope
- Compound inequality notation
- Piecewise functions
- Interval notation
- Percent increase and decrease
- Data analysis
- Algebraic modeling

Mathematics Learning Goals

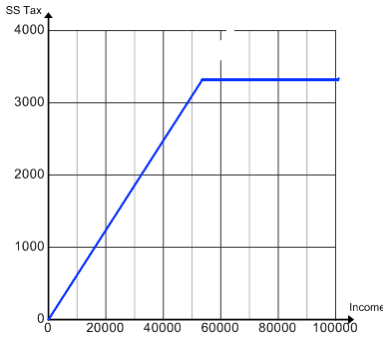
- **Students will determine and interpret domains of piecewise functions of the forms**

$$r(x) = \begin{cases} 29.95 \text{ if x is an integer and } x \le 2 \\ 29.95 + 14(x-2) \text{ if x is an integer and } x > 2 \end{cases}$$

$$c(x) = \begin{cases} 0.20x \text{ when } 0 \le x < 750 \\ 0.22x \text{ when } 750 \le x \le 1{,}000 \\ 0.25x \text{ when } x > 1{,}000 \end{cases}$$

Financial Algebra sections 7-1, 7-2, 5-1, 6-3, 9-3 and 10-2 require students to model real-life situations using piecewise functions. Key Assignment 3.2 involves modeling the FICA tax function, and interpreting its cusp and its slopes over different domains.

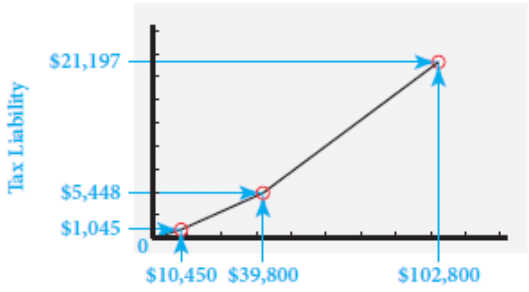
- **Students will graph piecewise functions with cusps such as**



- *Financial Algebra* section 6-5 and Key Assignment 3.2 require students to interpret the slope as a rate, and use the units of the rate to explain what the graph shows.

- **Students will compute measures of central tendency and rational functions such as** $a(x) = \dfrac{40r + 1.5tr}{t + r}$. *Financial Algebra* section 6-4 analyzes pension computations by expressing the mean as a rational function used to compute average salaries that determine pension amounts.
- **Students will use geometric sequences such as** $a_n = xr^n$ **with common ratio r.** *Financial Algebra* section 6-2 features the "penny doubled every day" problem to show the power of exponents. A pay scale that defies mathematical intuition is designed to show exponential growth with common ratio 2.
- **Students will express percent increases and decreases as rational functions.** *Financial Algebra* section 1-6 highlights classic percent increase and percent decrease using capital gains, and the students create rational functions to represent gains and losses.
- **Students will use the point-slope form** $y - y_1 = m(x - x_1)$ **and convert it to slope-intercept form** $y = mx + b$**.** *Financial Algebra* section 7-1 models the tax worksheet and derives the instructions on the tax worksheet using the piecewise function they create to model the tax schedules. Key Assignment 3.1 highlights the conversion of one form to the other, and shows how slope intercept form is used by the IRS on the tax worksheet. The ease of computation inherent in the slope-intercept form sacrifices information portrayed in the point-slope form about income thresholds and tax brackets.

- **Students will graph continuous polygonal functions with multiple slopes and cusps such as:**



Financial Algebra sections 7-1 and 7-2 uses continuous polygonal functions to model tax schedules. The cusp represents the points at which the tax brackets change. Key Assignment 3.2 features continuous functions with cusps derived from the FICA tax function. Students interpret the horizontal component, and its slope as a rate.

- **Students will translate verbal expressions into literal rational, exponential, and linear equations.** *Financial Algebra* sections 6-1 through 6-5 and 7-1 through 7-5, and in fact, all sections of the text, require students to frequently use equations to represent verbal situations. This is a necessary cornerstone of any math modeling course.
- **Students will express domains using compound inequality notation of the form** $t \geq t_1$ *and* $t < t_2$**. Students will express domains using compound inequality notation of the form** $t > t_1$ *and* $t \leq t_2$**, interval notation of the form** $t_1 < x \leq t_2$**, and tax schedule notation of the form "over** t_1 **but not over** t_2**."** *Financial Algebra* sections 7-1 and 7-2 use compound inequalities and interval notation to model the brackets on the income tax schedule. The tax brackets are defined by inequalities and students need to differentiate between "less than" and "less than or equal to" by reading carefully.
- **Students will model a tax bracket, given a compound inequality statement, and model a tax bracket to determine the tax using a linear equation of the form.** $y = a + p(x - t_1)$**where y is the tax, a is the base tax, p is the tax percentage expressed as a decimal, (**t_1 **is the lower boundary of the domain, and x is the taxable income).** *Financial Algebra* section 7-2 has students analyzing the IRS verbiage and compound inequalities, and translating from one to the other. *Financial Algebra* section 7-2 has the students computing taxes form the piecewise function, and from the linear function in slope-intercept form used on the IRS tax worksheet.
- **Students will interpret jump discontinuities**. *Financial Algebra* sections 6-5 and 7-2 use continuous functions with cusps to model the tax schedule. Students analyze this to explain the unfairness of a jump discontinuity when it is interpreted in the context of moving into another tax bracket..
- **Students will write equations in point-slope form.** *Financial Algebra* section 7-2 shows how the slope is the tax rate, and is the coefficient of the parenthetical expression that shows at what income that tax rate is effective.
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- **Students will create and interpret piecewise functions of the form**

$$f(x) = \begin{cases} 0.10x & 0 < x \le 16{,}050 \\ 1{,}605 + 0.15(x - 16{,}050) & 1{,}605 < x \le 65{,}100 \\ 8{,}962.50 + 0.25(x - 65{,}100) & 65{,}100 < x \le 131{,}450 \\ 25{,}550 + 0.28(x - 123{,}700) & 131{,}450 < x \le 200{,}300 \\ 44{,}828 + 0.33(x - 200{,}300) & 200{,}300 < x \le 357{,}700 \\ 96{,}770 + 0.35(x - 357{,}700) & x > 357{,}700 \end{cases}$$

  Financial Algebra section 7-2 shows how the tax bracket rates become slopes when equations are created and graphed. Where slopes change, the coordinates of the cusp show the income levels at which tax rates change.

  where f(x) represents the tax liability function for taxpayers using a given tax schedule with taxable incomes on a given domain.

- **Students will graph piecewise functions of the form and determine the cusps from the function notation such as:**

$$f(x) = \begin{cases} y = 0.10x & 0 < x \le 16{,}050 \\ y = 0.15x - 802.5 & 16{,}050 < x \le 65{,}400 \\ y = 0.25x - 7{,}312.5 & 65{,}100 < x \le 131{,}450 \end{cases}$$

  Financial Algebra section 7-2 shows how the tax bracket rates become slopes when equations are created and graphed. Where slopes change, the coordinates of the cusp show the income levels at which tax rates change.

- **Students will adapt all algebraic formulas in the unit for use in spreadsheets.** *Financial Algebra* sections 7-2 and 6-3 require students to adapt the structure of algebraic thinking to the language understood by the spreadsheet program. This requires intense attention to precision since any deviation from the correct syntax will prevent the spreadsheet from computing correctly. The spreadsheets will be computing payroll and computing taxes.

Internet Resources for Supporting Learning in the Employment/Income Taxes Unit

Internal Revenue Service www.irs.gov. Besides getting all the forms and instructions from this website, students can explore Publication 17: Your Federal Income Tax to find out all about tax deductions and tax credits.

Social Security Administration www.ssa.gov. Students can use this site to explore social security credits, benefits, and application procedures.

Department of Labor www.dol.gov. This site allows students to look up labor laws that affect all of them. Students should also go to their specific state's Department of labor website for state-specific regulations.

Unit 4: Automobile Ownership

Various functions, their graphs, and data analysis can be instrumental in the responsible purchase and operation of an automobile. In this unit, students will examine the mathematics of automobile advertising, sales and purchases, insurance, depreciation, safe driving, and accident reconstruction. The problems, activities, and key assignments in this Automobile Ownership Unit offer students opportunities to learn, explore, and use the CCSS Mathematical Practices MP1, MP3,MP4, MP5, MP6.

Common Core State Standards for Mathematical Content that are Addressed
A-CED2, A-CED3, A-CED4
A-REI2
A-SSE1b, A-SSE3
F-IF1, F-IF2, F-IF4, F-IF6, F-IF7a, F-IF7b, F-IF7e, F-IF8b, F-IF9
F-LE1b, F-LE1c, F-LE5
G-C5
S-ID1, S-ID2, S-ID3, S-ID4, S-ID6, S-ID7

Mathematics Topics

- Exponential/linear systems of equations
- Irrational functions
- Quadratic functions
- Arc length
- Piecewise functions
- Graphs of piecewise functions
- Systems of linear equations
- Frequency distributions
- Stem-and leaf plots
- Modified box-and-whisker plots
- Measures of dispersion
- Quartiles
- Interquartile range
- Outliers of a frequency distribution

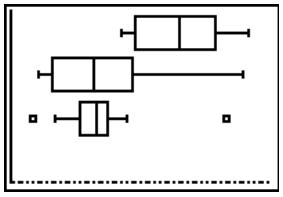
Mathematical Learning Goals Cross-Referenced with Textbook and Key Assignments

- **Students will transform raw data into a frequency distribution**. *Financial Algebra* section 5-3 shows students how to make sense out of large collections of data relating to automobile sales and purchases using frequency distributions. Students will compute measures of central tendency including the mean, median and mode, and explain appropriate uses of each. Key Assignment 4.1 asks students to chart internet automobile data and discuss the frequency distribute of that data.

- **Students will create and interpret stem and leaf plots and side-by-side steam plots such as**

9 8 1 1 1 1	87	1 2 2
3 2	88	2 4 6 7
7 5 4	89	1 3
7 6 6 6 6 5	90	2 7 7 7

  Financial Algebra sections 5-3 and 5-4 illustrate ways to chart data related to car sales, purchases and insurance. Stem and Leaf plots are offered as an alternative to frequency distribution lists. Students will compute measures of central tendency including the mean, median and mode, and explain appropriate uses of each. Key Assignment 4.1 has students construct stem and leaf plots to analyze automobile prices.
- **Students will create and interpret side-by-side, modified box and whisker plots as shown:**



  Financial Algebra sections 5-2 and 5-3 offer students contextual opportunities to compute measures of dispersion including the range, the quartiles, the interquartile range and the boundaries for outliers using the expressions $Q_1 - 1.5(IQR)$ and $Q_3 + 1.5(IQR)$. Key Assignment 4.1 has students construct box and whisker plots to analyze automobile prices.
- **Students will use linear and exponential regression analysis to model automobile related situations.** *Financial Algebra* section 5-6 offers students opportunities to investigate automobile depreciation using regression analysis. Key Assignment 4.1 has pair mileage and price data of similar make and model cars to create scatterplots and identify the regression line in order to make a presentation to the class about the predictive value of regression analysis as it pertains to used car purchases.
- **Students will create and interpret piecewise (split) functions of the form**

$$c(x) = \begin{cases} 38 \text{ when } x \le 4 \\ 38 + 6.25(x - 4) \text{ when } x > 4 \end{cases}$$

  Financial Algebra section 5-1 introduces students to continuous piecewise functions based on classified ad costs and commission payment schedules. Domains and cusps are identified and interpreted. Students will graph piecewise functions using mutually exclusive domains.
- **Students will model automobile depreciation using linear function models.** *Financial Algebra* section 5-5 employs the Internal Revenue Service linear depreciation method to model car depreciation over time. Students contextualize slope, intercepts and function values. Key Assignment 4.2 asks students to use linear and exponential graphing techniques in the context of automobile depreciation.
- **Students will create, use, and interpret linear systems of equations.** *Financial Algebra* section 5-5 addresses the interrelatedness of the automobile linear expense and

linear depreciation functions. Students contextualize system domains and the intersection point. Key Assignment 4.2 asks students to use linear and exponential graphing techniques in the context of automobile depreciation.

- **Students will model exponential depreciation as $y = Px^b$ where P is the purchase price and $x < 1$, and compare the depreciation to an increasing linear expense function.** *Financial Algebra* section 5-6 develops the algebraic representation of a depreciation percentage and uses it in the exponential depreciation equation. Students set up linear/exponential systems and contextualize the domains and point of intersection. Key Assignment 4.2 asks students to use linear and exponential graphing techniques in the context of automobile depreciation.
- **Students will set up and use algebraic ratios.** *Financial Algebra* section 5-7 requires students to analyze driving data by setting up rational functions.
- **Students will create and use the total stopping distance function which is created by adding a linear reaction distance function $RD = 0.75\left(\frac{5280s}{60^2}\right)$ to a quadratic braking distance function $BD = 5(.1s)^2$.** *Financial Algebra* section 5-8 models driving safety data by setting up and using linear and quadratic functions as components of the stopping distance formula. Key Assignment 4.3 examines the physics of driving as it relates to the stopping distance function.
- **Students will use and interpret a square root function.** *Financial Algebra* section 5-9 offers students the skid speed square root function. Data from accidents are analyzed using this function. Students will use multi-variable square root functions such as the skid length $S = \sqrt{30Dfn}$. Key Assignment 4.3 examines the physics of driving as it relates to the square root skid speed function.
- **Students will use geometry theorems involving chords intersecting in a circle and radii perpendicular to chords to determine yaw mark arc length.** *Financial Algebra* section 5-9 expands the use of the square root function to introduce the formula for determine the speed of a vehicle based on yaw marks. The yaw mark skid speed function is a composite function f(g(x)) where f(x) is a square root function and g(x) is a quadratic function. Students find the radius of the yaw and compute the arc lengths. Key Assignment 4.3 examines the physics of driving as it relates to the yaw speed function.
- **Students will adapt all algebraic formulas from the unit for use in spreadsheets**. *Financial Algebra* sections 5-7 and 5-9 ask students to create automobile linear and quadratic, exponential and square root formulas for use in spreadsheets.

Internet Resources for Supporting Learning in the Automobile Ownership Unit

Kelly Blue Book www.kbb.com This website gives students access to real-time data for new and used car sales.

Gas Buddy www.gasbuddy.com Here, students are able to get up to date information about gas prices throughout the country.

Teen Driver Source www.teendriversource.org/teen This website is sponsored by the Children's Hospital of Philadelphia. It offers a great deal of research and statistical information that can assist teens to become better drivers.

Motor Vehicle Crash Investigation and Reconstruction
www.Crashforensics.com/papers.cfm?PaperID=37
This website is used as a student reading assignment in the section on accident reconstruction.

Insurance Institute for Highway Safety www.iihs.org
Here, students are offered information and statistics on a variety of highway safety topics.

Unit 5: Consumer Credit

Becoming familiar with credit terminology and regulations is critical in making wise credit decisions. Credit comes at a price and in this unit students learn how to use mathematics to make wise credit choices that fit their needs, current financial situation, and future goals. The problems, activities, and key assignments in this Consumer Credit Unit offer students opportunities to learn, explore, and use the CCSS Mathematical Practices 1, 2, 4, 5, 6, 7.

Common Core State Standards for Mathematical Content that are Addressed
A-CED3
A-SSE1, A-SSE2, A-SSE3
F-IF8b
F-BF1a
F-LE5
N-Q1, N-Q2
S-ID6a

Mathematics Topics

- Algebraic proportions
- Linear, quadratic, cubic, and exponential equations
- Exponential growth and decay
- Regression equations
- Inverse function of an exponential equation
- Logarithms
- Summation notation

Mathematics Learning Goals Cross-Referenced with Textbook and Key Assignments

- **Students will create, evaluate, interpret and solve algebraic proportions**. *Financial Algebra* sections 4-1 and 4-2 utilize ratios and proportions in the context of short term and long term credit situations.
- **Students will model situations using linear, quadratic, cubic, and exponential equations.** *Financial Algebra* sections 4-2 and 4-3 ask students to explore loan information and model that data using regression analysis to find the line or curve of best fit.
- **Students will create, use, and interpret exponential growth and decay equations that model given situations.** *Financial Algebra* sections 4-2 and 4-3 examine exponential equations in the context of loans. Students apply an exponential equation in the form of the monthly payment formula $M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t}-1}$ where the exponent is present in both the numerator and the denominator. Key Assignment 5.1 has students using exponential functions to analyze a consumer credit situation.
- **Students will use the slope-intercept form y=Mx+b where M is the exponential monthly payment equation** $M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t}-1}$**.**
  Financial Algebra sections 4-2 and 4-3 offer students opportunities to explore the monthly payment function. Key Assignment 5.1 has students use the monthly payment formula as a means to analyze a consumer credit situation.
- **Students will use model and calculate the finance charge using the exponential monthly payment formula and the retail price.** *Financial Algebra* sections 4-3 through 4-6 examine loan and credit card transactions and statements using the monthly payment formula. Key Assignment 5.2 asks students to calculate a finance charge by using exponential functions as they deconstruct a credit card statement.
- **Students will use inverse functions to create the natural logarithm function.** *Financial Algebra* section 4-3 introduces students to logarithms as a means of solving for the time, t, in the monthly payment formula to determine the length of a loan. Key Assignment 5.1 has students using logarithmic functions to analyze a consumer credit situation.
- **Students will interpret and use summation notation to model the average daily balance.** $\sum_{i=1}^{n} \frac{d_n}{n}$
  Financial Algebra section 4-6 uses summation to deconstruct the entries on a monthly credit card statement. Students will calculate the average daily balance and finance charge using the summation notation formula. Key Assignment 5.2 asks students to use summation in order to determine the finance charge.

- **Students will create and use algebraic formulas and apply them for use in spreadsheets.** *Financial Algebra* sections 4-1, 4-2, and 4-5 ask students to create linear, exponential and logarithmic formulas in the context of consumer credit for use in spreadsheets. Key Assignment 5.2 asks students to create a spreadsheet that models a credit card statement situation.

Internet Resources For Supporting Learning in the Consumer Credit Unit

The Federal Reserve http://www.federalreserve.gov/consumerinfo/fivetips_creditcard.htm
This easy to read website offers students suggestions for managing a credit card account.

Index Credit Cards http://www.indexcreditcards.com/creditcardlist.html
This website offers students detailed information about all credit cards that are available. Here they can compare the terms of the credit card agreements and find information specific to different types of credit cards.

Teens' Guide To Spending www.teensguidetomoney.com/spending/
The "Teens Guide To Money" website has 7 links to spending and credit topics of interest to teenagers.

MSN Money
http://money.msn.com/credit-rating/article.aspx?post=45196da9-83b9-4229-8435-14415c64fe16
This is a teen's guide to understanding credit scores.

The College Board
https://bigfuture.collegeboard.org/pay-for-college/loans/8-tips-for-taking-out-student-loans
The College Board offers students tips for taking out student loans and paying for college.

Unit 6: Independent Living

In this unit, students work their way through the mathematics that models moving, renting, and purchasing a place to live. They also explore the geometric demands of floor plans and design, and discover the relationship between area and probability. The problems, activities, and key assignments in this Independent Living Unit offer students opportunities to learn, explore, and use the CCSS Mathematical Practices 1, 4, 5.

Common Core State Standards for Mathematical Content that are Addressed
A-APR6
A-CED2, A-CED3
A-LE1
A-REI6
A-SSE1
F-BF1

G-C5
G-MG3
S-ID6a, S-ID6c, S-ID8

Mathematics Topics

- The apothem of a regular polygon
- Area of a regular polygon
- Areas of shaded regions
- Rational functions
- The Monte Carlo Method
- Exponential functions
- Dilations and scale

Mathematics Learning Goals Cross-Referenced with Textbook and Key Assignments

- **Students will use systems of linear equations to model a given situation.** *Financial Algebra* section 8-1 asks students model a home move situation using a system of linear equations.
- **Students will use linear regression to model a given situation and make predictions using the regression equation.** *Financial Algebra* section 8-1 offers students an opportunity to determine a regression equation that best fits data representing the square footage and monthly rent of similar apartments.
- **Students will use rational functions with multiple independent variables**. *Financial Algebra* section *8-3* asks students to set up and interpret rational functions representing back-end $b = \frac{m + p/12 + h/3 + c + d}{a/12}$, front-end $f = \frac{m + p/12 + h/12}{x/12}$, and debt-to-income ratios for mortgage applications. In sections 8-3 and 8-4 students use the rational function representing the monthly loan payment to investigate home loan costs. Key Assignment 6.4 asks students to set up rational functions in order to determine the reduction in interest from extra mortgage payments.
- **Students will use the apothem to derive the formula for the area of a regular polygon, and apply the formula.** *Financial Algebra* section 8-2 focuses on floor plans. Students learn how to use the apothem to calculate the square footage of a regular polygonal plot ($A = \frac{1}{2}ap$**)**. Key Assignment 6.2 has students work with floor plan blueprints to determine areas of plane figures that have sections removed from them. Key Assignment 6.3 offers students an opportunity to determine areas by first finding the apothem.

- **Students will use probability to find the area of irregular plane region (The Monte Carlo Method)**

$$\frac{\text{number of points inside region}}{\text{number of random points generated}} = \frac{\text{area of irregular region}}{\text{area of framing rectangle}}$$

Financial Algebra section 8-2 discusses the use of the Monte Carlo method to determine the area of an irregular shaped plot of land. Key Assignment 6.1 uses the Monte Carlo method to find the area of an irregular landscape design.
- **Students will use factors of dilations to draw to scale**. *Financial Algebra* section 8-2 has students drawing and interpreting house plans to scale. Students will solve scale drawing problems using proportions.
- **Students will compute areas of irregular and shaded regions.** *Financial Algebra* section 8-2 reviews area formulas and area processes for regular and irregular regions.
- **Students will use multi-variable rational functions.** *Financial Algebra* section 8-2 uses multi-variable rational functions to compute air conditioner BTU needs.
- **Students will create and use exponential equations.** *Financial Algebra* section 8-5 uses exponential equations and exponential regression equations to model percentage rent increases.
- **Students will adapt all algebraic formulas for use in spreadsheets.** *Financial Algebra* sections 8-3, 8-4, and 8-5 offer students opportunities to model home ownership situations using spreadsheets.

Internet Resources For Supporting Learning in the Independent Living Unit

Khan Academy
http://www.khanacademy.org/economics-finance-domain/core-finance/housing/renting-v-buying/v/renting-vs--buying-a-home
This educational video walks students through the mathematics of comparing home rental vs a home purchase.

New York Times
http://www.nytimes.com/interactive/business/buy-rent-calculator.html
This is an interactive rent vs buy calculator that gives monetary information in a variety of formats.

Mortgage Calculator http://www.mortgagecalculator.org
This is one of the many mortgage calculator sites available online.

Mortgage Points Calculator
http://www.bankrate.com/calculators/mortgages/mortgage-loan-points-calculator.aspx
This is one of the many mortgage point calculator sites available online.

Unit 7: Retirement Planning and Budgeting

The focus of this unit is on the mathematics of fiscal plans that workers can make years ahead of their retirement date. This involves a detailed study of retirement savings plans, both personal and federal, employee pension programs, and life insurance. Additionally, students are asked to call upon the knowledge acquired in all of the preceding units in order to create and chart a responsible personal budget plan, to mathematically analyze cash flow, and to determine net worth. The problems, activities and projects inherent in studying budgeting and retirement planning are a natural forum for all eight CCSS Mathematical Practice standards, but this unit highlights MP1, MP2, MP4, MP5, MP6, and MP8.

Common Core State Standards for Mathematical Content that are Addressed
A-CED3
A-REI10
A-SSE1
F-BF1
F-IF4, F-IF5, F-IF7a, F-IF7b, F-IF8b
N-Q1, N-Q2
N-VM6
S-MD1, S-MD2, S-MD4, S-MD5

Mathematics Topics

- Expected value of a probability distribution
- Greatest Integer function
- Sectors and central angles
- Exponential Equations
- Rational expressions as combinations of rational and polynomial expressions
- Piecewise Greatest Integer Function
- Systems of linear and piecewise functions
- Domains, constants, coefficients, dependent and independent variable

Mathematics Learning Goals Cross-Referenced with Textbook and Key Assignments

- **Students will use the future value of a periodic investment formula of the form**

$$B = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}}$$

  to predict balances after t years when given a periodic deposit amount, an investment return rate, and compounding information. *Financial Algebra* section 3-7 teaches students how to compute the future value of an account in which the same periodic investment is made over a long period of time.

- **Students will use the present value of a periodic investment formula of the form**

$$P = \frac{B\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$$

to determine the principal when given a future value, a time in years, an investment return rate, and compounding information. *Financial Algebra* section 3-8 shows students how much they have to contribute over a certain length of time to reach a future financial goal. This is part of Key Assignment 7.2, in which students act as financial planners.

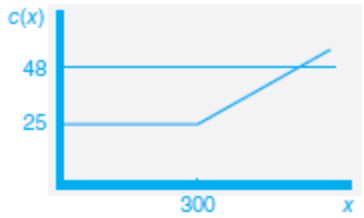
- **Students will write rational expressions as a combination of rational and polynomial expressions.** *Financial Algebra* section 9-1 expresses income brackets as compound inequalities which serve as the domain for tax functions. Students will use inequalities to define domains when creating algebraic expressions. *Financial Algebra* section 9-1 uses multivariable rational functions to compute requirements to meet specific financial goals. This is also a component of Key Assignment 7.2.
- **Students will use inequalities to define domains when creating algebraic expressions**. *Financial Algebra* section 9-1 expresses income brackets as compound inequalities that serve as the domain for tax functions.
- **Students will create and analyze discrete probability distributions.** *Financial Algebra* section 9-4 requires students to set up a table representing a discrete probability distribution by filling in the values of a random variable and the associated probabilities. Key Assignment 7.1 uses the mortality tables to compute life insurance benefits at different periods in the policy of a term insurance policy.
- **Students will write rational expressions to represent increase over time.** *Financial Algebra* section 9-4 requires students to express percent increases in life insurance premiums as rational functions.
- **Students will create, interpret, and graph the greatest integer function of the form** $[x]$. *Algebra* section 10-2 incorporates the greatest integer function to model billing that is dependent on entries being integers.
- **Students will determine and interpret the expected value of a probability distribution where the expected value is of the form** $\sum_{i=1}^{n} x_i f(x_i)$. *Financial Algebra* section 9-4 develops the concept of expected value, in the context of life insurance. The random variable is the life insurance benefit. Mortality tables supply the probabilities associated with each value of the random variable.
- **Students will incorporate the greatest integer function into a piecewise function of the form**

$$c(x) = \begin{cases} a \text{ when } x \le b \\ a + c(x-d) \text{ when } x > b \text{ and x is an integer} \\ a + c([x-d]+1) \text{ when } x > b \text{ and x is not an integer} \end{cases}$$

Financial Algebra section 10-2 combines piecewise functions with the greatest integer

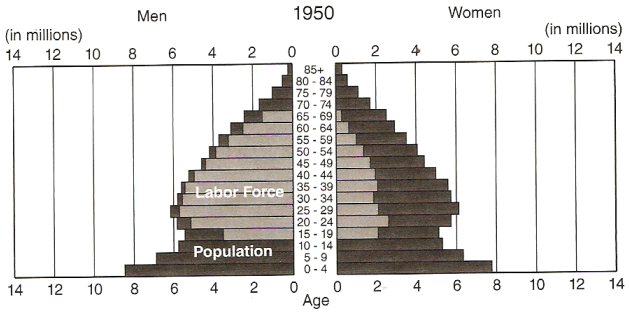
function when the criteria involved in the splitting of the function depends on whether the input variable is an integer or not.

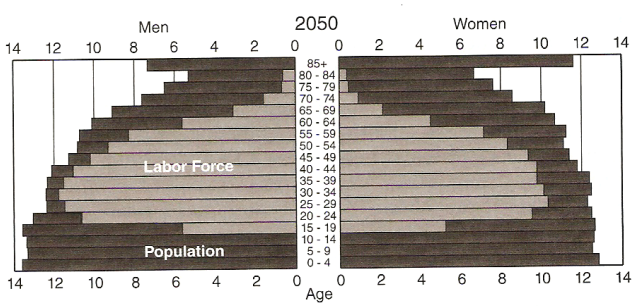
- **Students will create, interpret, and graph a system of a linear and a piecewise function and determining the point of intersection as shown in the following graph:**



Financial Algebra section 10-2 compares two optional billing plans offered by a utility to determine at what usage the costs are equivalent. This would be one facet of the many variable inherent in the completion of Key Assignment 7.4.

- **Students will use sectors and central angles of a circle to depict proportional categories on a pie chart when given categorical information.** *Financial Algebra* section 10-3 requires the students to proportionally divide a circle into sectors that represent given percentages, to display budget allocation.
- **Students will create and interpret budget line equations of the type** $C_x x + C_y y = B$ **where** C_x **represents the cost of the first of two items and** C_y **represents the cost of the second of two items, x and y represent quantities under consideration and B represents an amount budgeted.** *Financial Algebra* section 10-3 features these graphs. Key Assignment 7.4 investigates the combinations of possible purchases within a constrained budget.
- **Students will use multiple representations to chart data such as**





Bureau of Labor Statistics

Financial Algebra sections 10-3 and 10-4 uses a matrix to chart budgeting for a household over a year. Key Assignment 7.3 uses spreadsheets to calculate cash flow, net worth, and debt reduction.

Internet Resources for Supporting Learning in the Budgeting/Retirement Planning Unit

Mapping Your Future www.mappingyourfuture.org. Students can get budget tips and then plan a budget by entering their expenses into a software template.

Practical Money Skills www.practicalmoneyskills.com. This site gives students access to a budget worksheet and budget calculators.

Budget Calculator www.bankrate.com. This site offers a comprehensive budget calculator students can use to examine increases and decreases in expenses and how they affect a budget. The speed of the software's recalculation allows students to repeatedly explore the affect of changing expense

Advanced Algebra with Financial Applications
KEY ASSIGNMENTS

The Key Assignments presented in this section are well-aligned with the CCSS Standards for Mathematical Practice. The assignments are all verbal problem solving activities that relate to the unit being studied. Students must represent the verbal situation symbolically, manipulate those symbols to arrive at an answer, and then interpret that answer in the context of the problem. This offers students opportunities to make sense of quantities and their relationships within those problem-solving settings through multiple representations. Students can approach, access, and deconstruct the necessary mathematics using handheld graphing utilities, manipulatives, spreadsheets, and/or software. The assignments throughout this course require students to attend to precision in their responses both in the computational and algebraic fluency required to arrive at those answers and in the units used to contextualize the answers.

The prevalence of mathematical modeling assignments allows students to practice seeking out mathematical structure in what may seem to them to be an unstructured situation. Identifying and exploiting the structure leads students to a richer understanding of the themes and regularities that are present in the real world. Students make tables, find patterns, and offer conjectures based on the patterns. This form of inductive reasoning is a cornerstone of mathematical thinking. The assignments and other course-related activities optimize students' exposure to extrapolating what they have learned to routine and non-routine mathematically-dependent situations they encounter in their futures.

Most assignments require the student to prepare a presentation on their finished work. This can be a PowerPoint show, a webinar, a poster presentation, or a presentation using transparencies. The student audience gets to critique the presentation, ask questions, and make comments, in a firmly established, constructive, positive "safe" zone. The presentation is graded, and the quality of student critiques and comments can also be graded.

Unit 1: Banking

Key Assignment 1.1: How Interest Method Affects Monetary Growth

Mathematics: Simple interest, compound interest
Mathematics Learning Goals: To determine how increased compounding affects growth.

Students are first introduced to the meaning of compounding numerically via mathematical iteration. Before embarking on a rigorous study of limits and compound interest algebraic formulas, students are asked "How much would $1,000 grow to, in one year, at 100% interest compounded continuously?" The 100% interest and continuous compounding often leads them to guess much higher than the actual amount. Their guesses are recorded, and a statistical analysis of their guesses is made. Outliers are carefully noted. The findings of this activity are scrutinized after students complete Key Assignment 3.

Key Assignment 1.2: Deriving the Compound Interest Formula

Mathematics: Inductive reasoning, exponential functions, rational functions
Mathematics Learning Goals: To use patterns and induction to generate for selected forms of compounding and adapt them to monthly, weekly, daily, and hourly compounding.

Students will compute interest for each interest period over a semi-annual and quarterly compounded account for a given balance and interest rate. They will derive the general algebraic formulas for these two types of compounding. They will then look for patterns in the semi-annual and quarterly compound interest formulas to inductively conjecture about the general formula for compounding. They will then find formulas for monthly, weekly, daily and hourly compounding, and compute and compare the interest earned over one year for these accounts.

Key Assignment 1.3: Using Limits to Derive the Natural Base e

Mathematics: Rational functions, exponential functions
Mathematics Learning Goals: To use substitution and patterns to generate a series that approaches e as x approaches infinity.

Students will be introduced to the notion of limits and limit notation and apply it to the compound interest formulas previously derived. They will increase the number of compoundings by first computing interest when the compounding period is every minute, and then every second, for a given balance and interest rate. They will then let the number of compoundings 'n' approach infinity to see what happens to the annual interest as the number of compoundings approaches infinity. They will analyze the compound interest formula without the balance, and explain the "battle" between the base and the exponent of the expression $\left(1+\frac{1}{n}\right)^n$ as $n \to \infty$.

Key Assignment 1.4: Future Value and College Costs

Mathematics: Rational functions, regression
Mathematics Learning Goals: To estimate the cost of a college education in 18 years and determine how much needs to be saved each month to have the costs covered by the 18^{th} year.

Students pick a college and find out the cost of tuition, room and board (if necessary) and fees over the past ten years. They set up a regression line or curve of best fit. They then predict the cost of a college education in 18 years (as if they just had a child and were trying to save for college). They then use the prevailing interest rate and the future value formula to determine the monthly periodic deposit that would be necessary to have the full college cost saved by the child's 18^{th} birthday. They then do the problem with interest rates slightly higher than the prevailing rate.

Unit 2: Investing

Key Assignment 2.1:Charting a Corporate Stock

Mathematics: Data Analysis, regression, prediction, modeling, graphical interpretation
Mathematics Learning Goals: The goal of this assignment is to have students use mathematical modeling to chart and interpret stock market trends over a 15-day period. They will make trend predictions based on simple moving average crossover analysis as well as regression models.

Each student selects a corporation traded on the New York Stock Exchange. They produce a background paper, PowerPoint presentation or poster board display on that corporation. Students chart the open, close, high, low and volume data for 15 consecutive trading days. They graph the data using two different formats and then discuss trends that the data shows. They will also calculate three different cluster-lengths of moving averages and, using those clusters, they will create superimposed line graphs. Students discuss trading implications based upon stated domains of graph pairs before and after any intersection points. Finally, they determine the closing price curve of best fit using regression analysis. They must state the regression equation and support why their stated curve best fits the data of closing prices. Students will then use the curve of best fit to predict a closing price on the 16th trading day. They compare that predicted price with the actual closing price on the 16th day and find a percent error.

Key Assignment 2.2: Mathematically Modeling A Business

Mathematics: Linear and quadratic functions, linear/linear Systems, linear/quadratic systems, regression analysis
Mathematics Learning Goals: To have students create linear and quadratic models for a start-up business. They will graph and interpret systems of these regression and modeling equations in order to explore the relationship between and among expense, demand, price, revenue and profit.

Students are given a market research scenario for a new product, attained from a focus group questionnaire. The research contains a list of ordered pairs in the form (p,q) where p is a potential price and q is the quantity of the product that the focus group member would purchase if it was set at that price. Using these ordered pairs, students construct a scatterplot, determine the correlation coefficient, and identify a linear regression equation in which q is the independent variable and p is the dependent variable. Then, given information about expenses, they are to set up a linear expense function in terms of the quantity demanded. The quadratic revenue and profit equations are determined and graphed on the same axes with the expense function. Students identify and interpret the breakeven points, the coordinates of the maximum

point on the revenue graph, the coordinates of the maximum point on the profit graph, and the price at which the product should be sold in order to maximize profit. Finally, students are told the initial price per share for the company's stock and asked to determine the number of shares that must be sold in order to have enough money to start this business.

Unit 3: Employment and Income Taxes

Key Assignment 3.1: Creating the Tax Worksheet

Mathematics: Domains, piecewise functions, linear functions and graphs, point-slope form, slope-intercept form, graphs with cusps.
Mathematics Learning Goals: To derive the slope-intercept form used on the IRS tax worksheet by translating tax tables into piecewise functions.

The tax tables give taxpayers a function in which the independent variable is the taxable income and the dependent variable is the tax. It is convoluted and has confused taxpayers for years. Within the last decade, the IRS created a worksheet that uses the slope-intercept form of the equations of a line to simplify calculations for the taxpayer. In this Key Assignment, students interpret the IRS Schedule, express the domains using compound inequality notation, and create the piecewise function that models the IRS intentions. They then covert this function, which is a translated version of point-slope form, into the slope-intercept form to create the tax worksheet.

Key Assignment 3.2: Graphing the FICA Tax Function

Mathematics: Piecewise functions, slope, cusps, linear equations
Mathematics Learning Goals: To use graphs to compare the FICA tax longitudinally over a prescribed number of years.

Students look up the FICA tax percents, and maximum taxable incomes to create piecewise functions for each of the last six years. They compute the maximum FICA tax, and graph all six years on the same axes, and use the graph to write a paragraph on what has happened to FICA taxes over those years. They discuss the significance of the coordinates of the cusp. They do the same for the tax years 1981-86, and compare the last six years to the years 1981-1986. The assignment is replicated using the Medicare tax percent.

Unit 4: Automobile Ownership

Key Assignment 4.1: Using Statistics to Negotiate Auto Transactions

Mathematics: Bivariate data, **c**orrelation, regression, mean, median, mode, quartiles, interquartile range, outliers, modified box-and-whisker plots, stem-and-leaf plots, frequency distributions, scatterplots.
Mathematics Learning Goals: To use measures of central tendency and measures of dispersion to mathematically negotiate the buying and/or selling of an automobile.

Students choose a make, model and year for an automobile. They use the Internet and newspaper classified ads to find 10-20 of those cars for sale. They get the price of the car and the mileage it has. They construct modified box-and-whisker plots and describe the frequency distribution. They pair each car's price with its mileage to create a scatterplot. They classify the association as positive or negative. They find the regression line and correlation coefficient and interpret the relationship as strong, moderate or weak, and discuss its linearity. Their results are presented to the class via PowerPoint presentation or poster presentation.

Key Assignment 4.2: Automobile Cost and Depreciation

Mathematics: Exponential regression, **g**raphing linear and exponential functions, rational functions, linear/exponential systems, systems of linear equations, slope-intercept form.
Mathematics Learning Goals: To use graphing techniques to compare the value of a car to the expense of purchasing it throughout its lifetime.

Using the monthly payment rational function, students graph the cost C of purchasing a new car, using the down payment as the y-intercept, and the monthly payment as the slope. They then investigate three types of depreciation: straight-line, exponential, and historical bath tub graphs. They graph the cost and depreciation functions on the same set of axes to find the month at which the total cost C of owning the car surpasses its value V as it depreciates. They identify and interpret the domains on which $C > V$ and $C < V$.

Key Assignment 4.3: The Physics of Driving

Mathematics: Quadratic equations, radical functions, arc length, geometry of the circle.
Mathematics Learning Goals: To use the mathematics listed to determine braking distances and to gather data from accidents scenes.

Students use formulas to determine reaction distance, braking distance, and figure out the speed a car was going based on its skid marks. The braking-distance formula is a quadratic function, with speed as the independent variable. The skid speed formula is an irrational function that has three independent variables. Students also use the geometry of the circle to compute the radius of a given yaw mark, which is a curved skid mark, and use the radius and friction factor to find the speed the car was going when it began to skid. The students then prepare a PowerPoint or poster presentation for the driver's education class in their school.

Unit 5: Consumer Credit

Key Assignment 5.1: Can I Afford This Loan?

Mathematics: Exponential functions, logarithmic functions, system of exponential and linear functions, modeling, graphical interpretation
Mathematics Learning Goals: To use three modalities to determine the affordability of a loan: exponential formula evaluation, logarithmic formula evaluation, and interpreting an exponential/linear system. To use technology (graphing utility and/or spreadsheet) to make the determinations required and justify their responses.

Students are given a scenario in which a family must make a decision about the affordability of a loan based on the principal, the loan-length, the APR and the maximum affordable monthly payment the family is able to make towards loan debt reduction. Students determine the affordability of the loan in three different ways: using the monthly payment function, interpreting the graphs of the system of equations defined by the exponential monthly payment function and the linear maximum affordable monthly payment, and using the logarithmic loan length function. They are then asked to construct two spreadsheets: a monthly payment spreadsheet that charts the monthly payment as loan length time varies from 1 to 20 years, and a loan length spreadsheet that charts time as monthly payments vary from $100 to $1000. Finally, students must write up a summary analysis for this situation explaining how the algebraic modeling by the spreadsheet formulas supports their prior work.

Key Assignment 5.2: Mathematically Modeling a Credit Card Statement

Mathematics: Algebraic modeling and spreadsheet formula creation
Mathematics Learning Goals: To algebraically model a month of activity on a person's credit card.

Students create a 21-day credit calendar that depicts algebraic representations of daily balances based upon an opening balance of Y dollars, an X-dollar purchased on the 8^{th} day, a Z dollar payment on the 13^{th} day, and a W-dollar purchased on the 20^{th} day. Using these representations from the calendar, they write algebraic expressions for the sum of the daily balances, the average daily balance, and the finance charge for this 21-day period given that the APR on this credit card is P%. Students then create a spreadsheet that models the situation described above and test their spreadsheet for a given data set.

Unit 6: Independent Living

Key Assignment 6.1: Areas of Irregular Plane Figures

Mathematics: Probability, ratios, random integers, graphing, random number table
Mathematics Learning Goals: To use the Monte Carlo method to find the area of any regular or irregular plane figure.

Students superimpose a grid on an irregular plane figure that is part of a landscape design. They outline the irregular figure with a rectangle and use a random number generator from a calculator, or a random number table, to generate 500 points, which they plot on their rectangular grid. As they plot each point, they note if it is inside or outside of the irregular region. They find the percent of random points that landed in the irregular region and take that percent of the area of the enclosing rectangle to approximate the area of the irregular region.

Key Assignment 6.2: Areas of Shaded Regions

Mathematics: Area formulas

Mathematics Learning Goals: To determine areas of plane figures that have sections removed from them.

As part of a unit on floor plans and interior design, students compute areas of floors to find the cost of new flooring. They also compute the cost of paint by taking the areas of the walls and subtracting window and door areas. They employ the area of a circle, square, triangle, rectangle, trapezoid, and parallelogram, and create a poster display on what a specific room cost to redo.

Key Assignment 6.3: The Apothem and the Area of a Regular Polygon

Mathematics: Inscribed circles, area of a triangle, perimeter, congruence.
Mathematics Learning Goals: To derive a formula for the area of any regular polygon.

Students use the area of a triangle to find the area of a regular polygon. They divide a regular polygon into triangles, by connecting the center to each vertex. They draw in the altitude, which is renamed the apothem, and find the area of the triangle. They discuss the congruence of the n triangles formed in the regular n-gon, and multiply to find the area of the polygon. They then model this algebraically, and use the commutative property of multiplication to derive the formula that the area is half the product of the apothem and the perimeter of the regular polygon.

Key Assignment 6.4: How Increased Payments Affect Mortgages

Mathematics: Rational functions
Mathematics Learning Goals: To determine the reduction in interest that extra mortgage payments result in.

Students use the monthly payment formula to compute the monthly payment for a hypothetical mortgage amount over 15 and 30 years. They compute the total payments, based on 12 monthly payments each year, and the total interest for the entire loan. They then use a mortgage calculator to assume an extra, 13th payment is made each year, so payments are made once every 4 weeks instead of once each month. They compute the interest and new total repayment period and compare the total interest to the original conventional mortgage to see the savings in total years and interest.

Unit 7: Retirement and Budgeting

Key Assignment 7.1: How Do Life Insurance Companies Earn a Profit?

Mathematics: Expected value, random variables, probability distributions
Mathematics Learning Goals: To use probability distributions and mortality tables to compute the profit earned on a five-year term life insurance policy.

Students use the probability inherent in mortality tables and life insurance annual premiums to compute the expected profit for a life insurance company's term policy. They create

probability distributions for the random variable profit and compute expected profit by summing the products of the individual profits and probabilities for each year of the policy. They compute the minimum annual premium the company must charge to earn a profit.

Key Assignment 7.2: Planning For Retirement

Mathematics: Exponential equations, expected value, data analysis, modeling and predicting
Mathematics Learning Goals: To apply prior knowledge from the banking unit to make decisions about the feasibility of a retirement plan.

Students are given financial information about a prospective retiree and asked to act as a financial retirement planner. The prospective retiree has also supplied the planner with desired monetary goals in retirement. Based upon information about savings plans, social security benefits, pensions, and life insurance policies, and using formulas learned in this unit, the planner is to write up a financial plan for the prospective retiree that includes at least two ways of meeting the goals and has mathematical justification for the recommendations made.

Key Assignment 7.3: Cash Flow, Net Worth and Debt Reduction

Mathematics: Algebraic ratios, modeling, linear equations
Mathematics Learning Goals: To create a spreadsheet that calculates cash flow, net worth, and debt to income ratio.

Students are given a budget spreadsheet that contains the headings of income, fixed expenses, variable expenses, and non-monthly expenses. There are sub-headings under each of these listing specific categories relating to the heading. Students are given a full accounting of a person's financial status and asked to build a spreadsheet that calculates that person's cash flow. In addition, the students are given information about the person's assets and liabilities and are asked to add it to the spreadsheet and determine the net worth. Finally, based upon the calculation of the debt-to-income ratio, students are asked to develop a debt reduction plan for the individual if necessary.

Key Assignment 7.4 Budget Line Equations

Mathematics: Linear equations, domain, range, constraints, modeling,
Mathematics Learning Goals: To construct and interpret a graphical representation of a particular aspect of a budget.

A budget line graph allows the user to interpret many combinations of product usage based upon given constraints. The interpretation of the combinations allows the user to make decisions about affordability. Students are given information about a particular aspect of a personal budget. This data contains prices and budgeting constraints. Students are asked to construct a budget line equation. They then examine the regions above, on, and below the budget line to identify points representing affordability data. Students make recommendations for this budget item based upon the interpretation of the budget line graph.

Advanced Algebra with Financial Applications
INSTRUCTIONAL METHODS AND STRATEGIES

The instructional strategies used throughout this course are varied, targeted, and rooted in the CCSS Standards for Mathematical Practice. Just as the Standards are interrelated, the methods used in this course are. Together, the practices referenced in this section serve to build mathematical confidence, interest and strength.

The Advanced Algebra with Financial Applications program's instructional strategies cover these basic umbrellas:

- Motivational Unit Openers
- Essential Questions
- Reading
- Discussion/interaction
- Presentation of model problems
- Extensions and problem solving
- Differentiation of instruction
- Experiential learning
- Use of technology

The **motivational unit openers** are real-life problems that need to be solved mathematically. Students realize that they "need to know" this material, as they will be encountering financial matters every days of their adult lives. Financial situations are inherently natural motivators. Since all of the problems in the course are real-world applications, lessons must integrate **reading and discussion** on a daily basis. An **essential question**, written on the board each day, serves as a focal point as algebraic symbols are used to represent the situation. These applications are all embedded in prose, so every new topic begins with a reading passage that acts as a springboard to a full-class discussion. This lively interactive feature of every lesson sets a constructive, motivating stage for the mathematics that follows.

The direct instruction/lecture component is highlighted by the investigation of **model problems** on each skill covered. After each model problem, students look for structure and regularity and try to apply it in a situation rooted in the model problem just completed. This gives the students a chance to see if they understood the new concept before moving on to a deeper problem for which the previous problem was an entry condition. Students are then asked to extend their understanding by looking for patterns and **extending** previously-used strategies. The applications at the end of each section give students a chance to practice as part of their classwork and homework. The program spirals previously-learned material on a daily basis. The sequential nature of the introduction of each new skill, followed by immediate practice, allows students to monitor their progress often. Class notes include vocabulary and financial explanatory material as well as mathematical procedures.

The model problems and applications generally graduate in difficulty level, allowing the teacher to **differentiate instruction**. Since abstract reasoning can be difficult for many students, the instructions are graduated so students can grasp the higher level skills by meeting them step-by-step. This strategy allows student and teacher to identify the exact juncture at which the student is having difficulty. This makes diagnostics and intervention more pointed.

There is much opportunity for **experiential learning**. Projects require the students to get out in the field and meet with brokers, bankers, local businesses, etc. Guest speakers at several junctures bring the outside world right into the classroom. Students act as moderators and compile questions for the guest speaker. For some projects, data is gathered and statistically analyzed. Students present their work to the class, and they field questions and comments from their classmates.

Technology plays a key role in the development of Advanced Algebra with Financial Applications topics. The graphing calculator is a daily tool, and its algebraic and graphing features are extensively used. Spreadsheets appear in every unit so students can model situations using algebra and technology.

Advanced Algebra with Financial Applications
ASSESSMENT METHODS

A variety of formative and summative assessment methods are used throughout Advanced Algebra with Financial Applications in order to assess student learning. The assessments are aligned with the course purpose and the instructional strategies used, and with the Common Core Standards for the development of mathematically proficient students. In the activities listed below, students are offered assessment opportunities to address mathematics as a sense-making tool, problem solve, reason, construct arguments, offer mathematics-justified critiques of arguments, , model, use appropriate tools, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning. The assessment grading percentages contributing to the student's quarter course grade are offered in parentheses next to the assessment name.

FORMATIVE ASSESSMENTS (30%)

CLASS PARTICIPATION (15%)

- **Do Now Activities** are assessments that can be used as a vehicle for the teacher to determine whether students have acquired skills, strategies, and content necessary for subsequent work in a topic. This diagnostic feature allows the teacher to adjust the lesson accordingly, if entry conditions are not fully met.

- **Check Your Understanding** problems are offered to students immediately after the teacher has introduced a new concept or procedure. These problems offer students and teacher alike an immediate assessment opportunity that is confined to the single new skill just addressed. The teacher can adjust the lesson to follow based upon review of these problems.

- **Extend Your Understanding** problems are more advanced problems that use the concepts and procedures just learned and take them to another level. These can be offered to all students or differentiated for selected students depending on the nature of the problems.

- **Ticket to Leave Activities** are ungraded activities that offer the teacher an opportunity to determine the level of understanding students acquired on the skills, strategies, and content of the day's lesson. These activities can be used by the teacher to adjust the following day's lesson.

- **Direct and Indirect Teacher Questions** are immediate formative methods of assessing students' understanding. In-class discussion is a critical part of Advanced Algebra with Financial Applications. The teacher should initiate discussion through focused questioning.

- Through the **Exploration of Essential Questions** (one per lesson), the teacher assesses student understanding both pre-instruction and post-instruction. The essential question is offered to the students at the beginning of the first lesson on the topic and a discussion ensures. That same essential question is revisited during the instruction and/or post-instruction to assess student growth and learning.

- Reading and writing are an essential part of Advanced Algebra with Financial Applications. Teachers will use **written and oral response to reading** (from the textbook, newspapers, magazines, Internet, brochures, laws, etc.) as a way of assessing understanding. Some writing activities will offer students an opportunity to interpret data that is displayed in a pictorial representation. Based upon the data, they are asked to write a short, newspaper-type story centered on the graph. There is one such activity for each chapter.

HOMEWORK (15%)

- **Homework Assignments** are a daily evaluation and reflection device for both student and teacher. The level of proficiency with the homework questions should allow the teacher the opportunity to adjust the lesson as needed. The homework acts as a barometer for students, so they can formulate questions, and attempt problems on their own.

SUMMATIVE ASSESSMENTS (70%)

- **Lesson-Opener Quizzes** are short, graded, cumulative assessments that can test for prerequisite skills and/or mastery of recently taught material. These assessments are averaged and count as one full-period exam grade.

- **Full-Period Exams** are graded summative assessments that test student acquisition of skills, strategies, and content.

- **Experiential Learning** activities are project-based assessment tools that are offered to students as long-term assignments. Students are asked to do research in a variety of forms and formats in order to accomplish a task that is related to the skills, strategies, and content covered in the chapter. Their projects can be submitted in print, electronic, or presentation format. Precision and accuracy will be scrutinized during their presentations as well as the ability to use mathematical tools appropriately and strategically. Each project is valued as a single full-period exam grade.