ADVANCED DESIGN OF STEEL AND COMPOSITE STRUCTURES



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European Erasmus Mundus Master Course

Sustainable Constructions
under Natural Hazards and Catastrophic Events

520121-1-2011-1-CZ-ERA MUNDUS-EMMC

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Module A – Design of non-uniform members

Introduction

Introduction

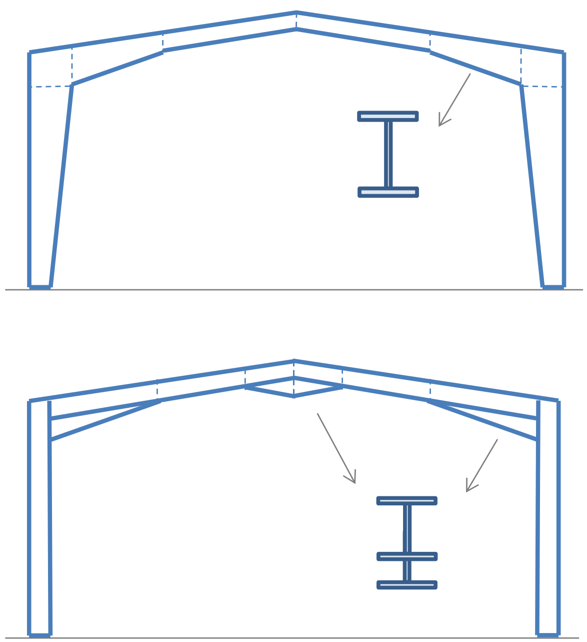
- Tapered steel members are used in steel structures
  - Structural efficiency → optimization of cross section capacity → saving of material
  - Aesthetical appearance

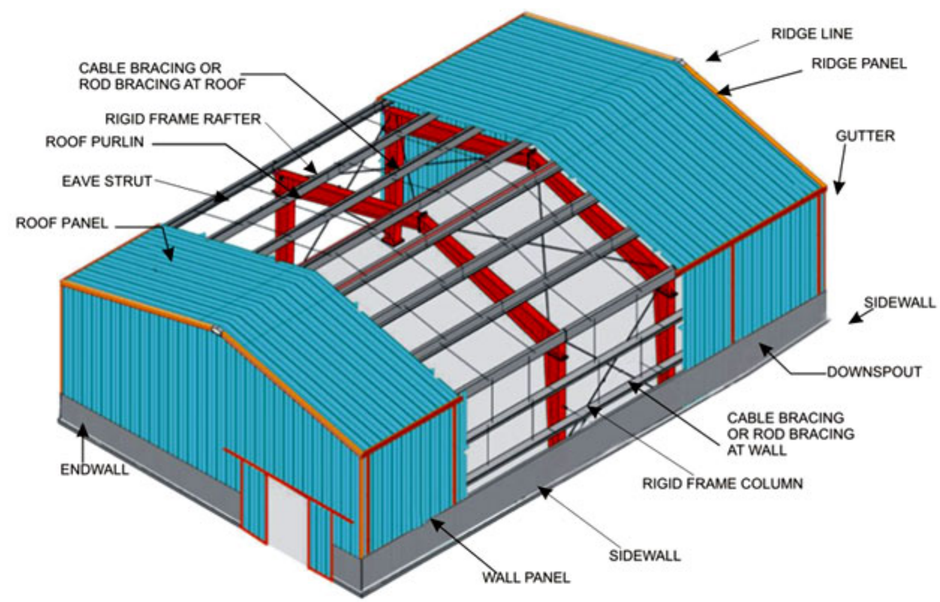
Multi-sport complex – Coimbra, Portugal

Construction site in front of the Central Station, Europaplatz, Graz, Austrial

Introduction

- Tapered members are commonly used in steel frames:
  - industrial halls, warehouses, exhibition centers, etc.





  - Adequate verification procedures are then required for these types of structures!

Introduction

- However, there are several difficulties in performing the stability verification of structures composed of non-uniform members;
  - Guidelines are inexistent or not clear for the designer
  - Due to this reason simplifications that are not mechanically consistent are adopted
    - These may be either too conservative or even
    - Unconservative!

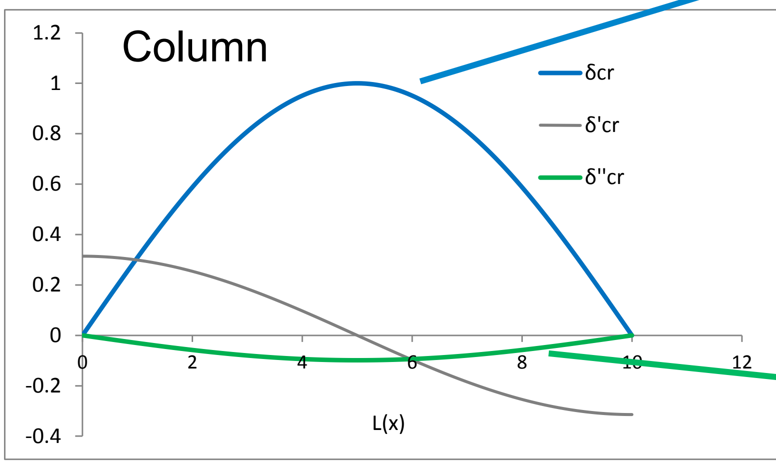
Non-uniform members

Approaches
and
Problems

Non-uniform members – approaches and problems

- Prismatic members – Clauses 6.3.1 to 6.3.3
  - Developed for prismatic members
  - Sinusoidal imperfections

$$\delta_0(x) = e_0 \sin\left(\frac{\pi x}{L}\right)$$



$$M^{II}(x) = EI\delta'' \propto \sin\left(\frac{\pi x}{L}\right)$$

  - Ayrton-Perry type equation:
    Is maximum at mid span:

$$\varepsilon(x) = \frac{N}{N_{Rk}} + \frac{M^{II}(x)}{M_{y,Rk}}$$

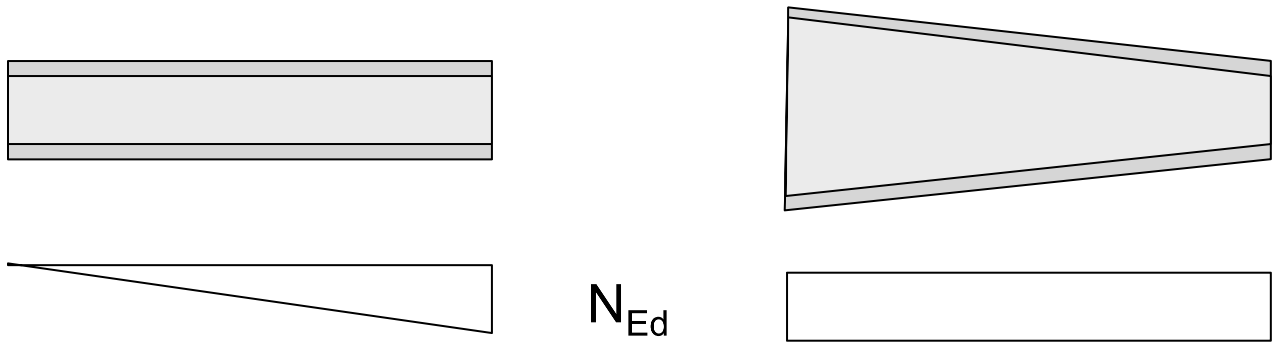
Constant

Sinusoidal

OK!

Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply ???



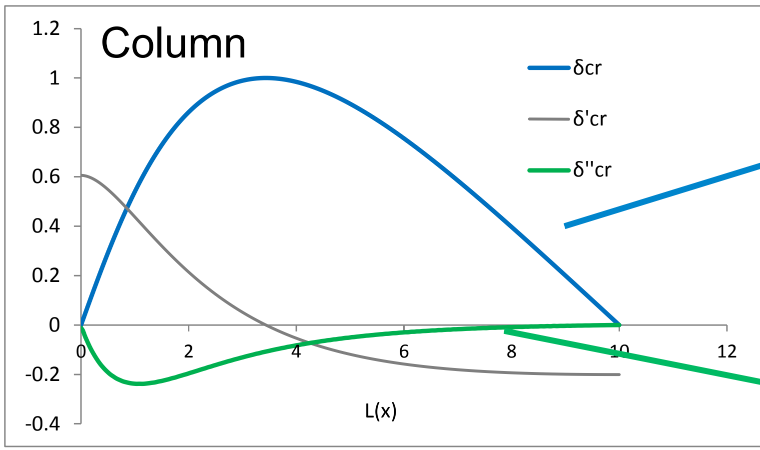
- Cross section utilization due to applied (first order) forces is not constant anymore.

$$\frac{N}{N_{Rk}}$$

Not Constant

Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply ???



$$\delta_0(x) = e_0 \sin\left(\frac{\pi x}{L}\right)$$

$$M^{II}(x) = EI\delta'' \propto \sin\left(\frac{\pi x}{L}\right)$$

- Ayrton-Perry type equation:
  Is it maximum at mid span ???

$$\varepsilon(x) = \frac{N}{N_{Rk}} + \frac{M^{II}(x)}{M_{y,Rk}}$$

KO!

Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply ???

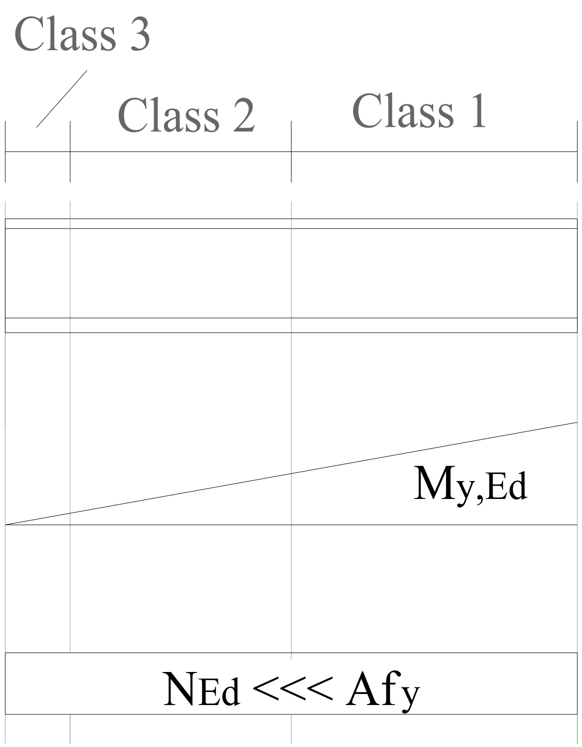
  - Position of the critical cross-section – not at mid span
    - Account for 2nd order effects; iterative procedure, not practical;

  - 1st order critical cross section is considered!

Non-uniform members – approaches and problems

- Non-uniform members – Clauses 6.3.1 to 6.3.3 apply ???

  - Variation of cross section class



- Definition of an equivalent class for the member

Non-uniform members – approaches and problems

- Non-uniform members – 2nd order analysis with imperfections

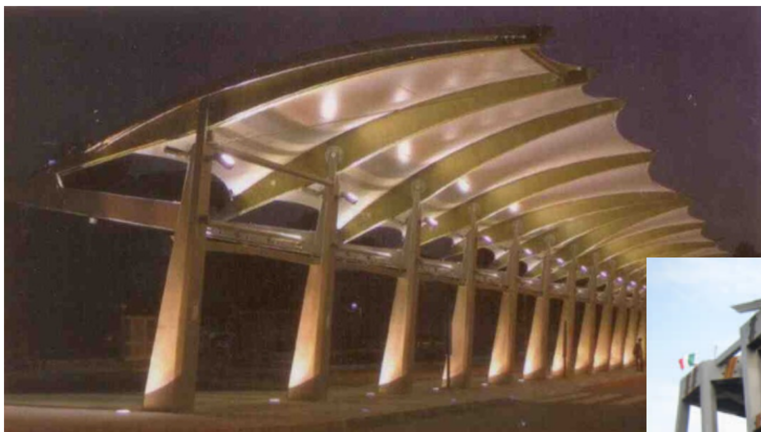
  - Definition of local imperfections:
    - Same problem: e_0/L calibrated for prismatic members with sinusoidal imperfections

Buckling curve acc. to EC3-1-1, Table 6.1	Elastic analysis	Plastic analysis
	e_0/L	e_0/L
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

Non-uniform members – approaches and problems

- Non-uniform members – 2nd order analysis with imperfections

  - Definition of local imperfections?



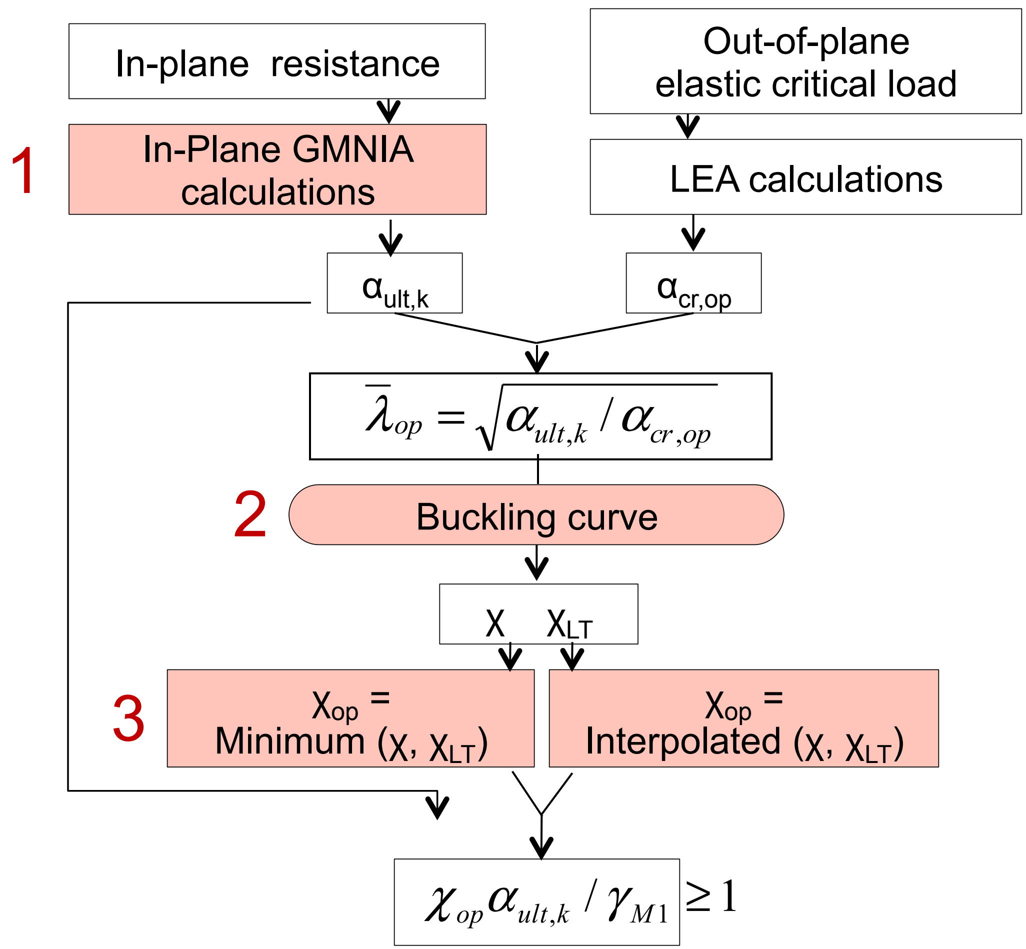
Auvent de la Gare Routière – Ermont

Italy pavilion, World Expo 2010 – Shanghai

Barajas Airport, Madrid, Spain

Non-uniform members – approaches and problems

- Non-uniform members – GENERAL METHOD (clause 6.3.4)

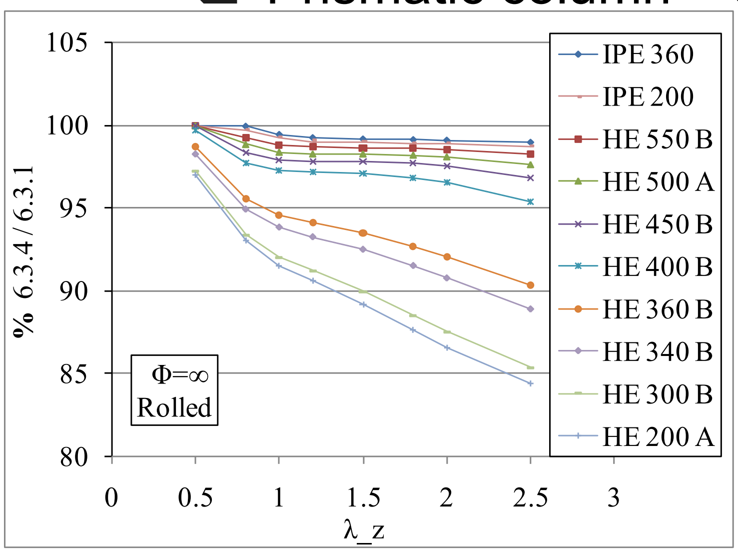


Non-uniform members – approaches and problems

- Non-uniform members – GENERAL METHOD (clause 6.3.4)

1 - $\alpha_{ult,k}$ should account for local second order effects?

  - If so: again → problem with definition of imperfections
  - Prismatic column – 6.3.4 vs. 6.3.1

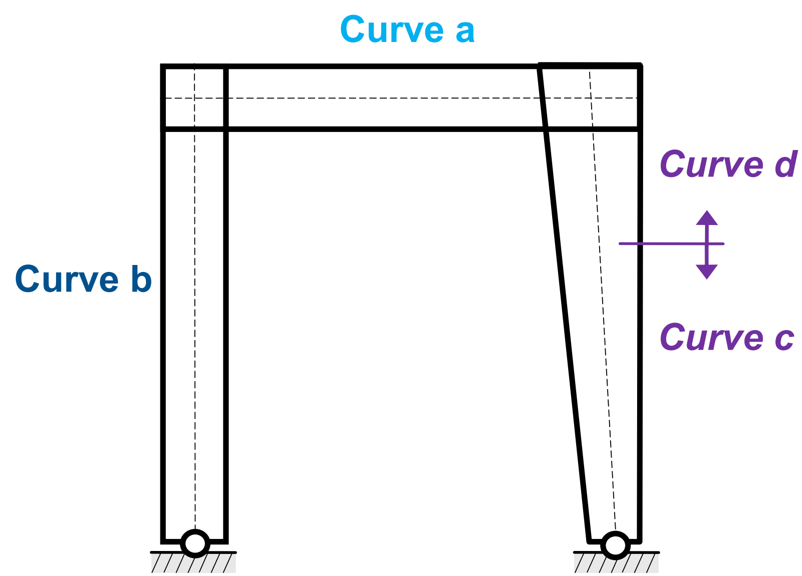


- Higher in-plane second order effects lead to a decrease in the out-of-plane reduction factor
- Even for the case of beam-columns this effect is not as restrictive.

Non-uniform members – approaches and problems

- Non-uniform members – GENERAL METHOD (clause 6.3.4)

2
- Definition of a proper buckling curve



- For some cases existing buckling curves may even be unconservative!

Non-uniform members – approaches and problems

- Non-uniform members – GENERAL METHOD (clause 6.3.4)

3
- Reduction factor – minimum or interpolation
  - Minimum
    - May be too conservative
    - Does not follow buckling mode correctly
  - Interpolation
    - Provides a transition between FB (M_y=0) and LTB (N=0)
    - What type of function for a correct mode transition?

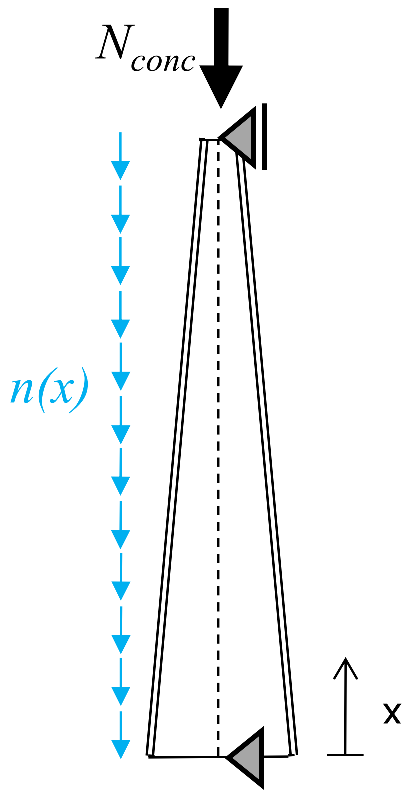
Non-uniform members – approaches and problems

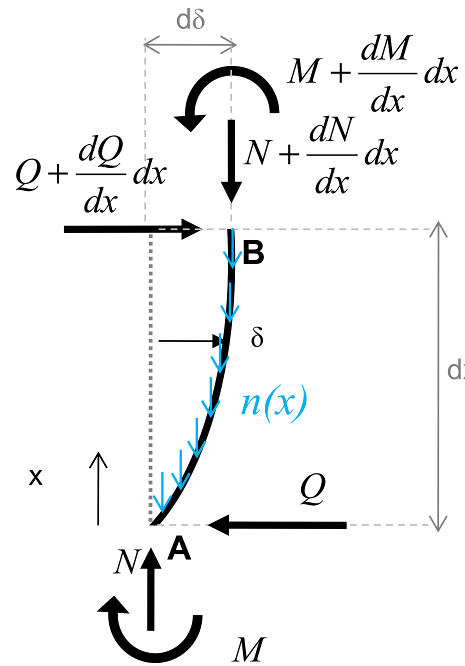
- Non-uniform members – FEM numerical analysis
  - Problem with definition of imperfections
  - Requires a high experience in FEM modeling from the user in order to achieve reliable results
  - Limited guidelines
- For the most simple cases it is preferable to provide simple rules which include as much as possible the real behavior of the member

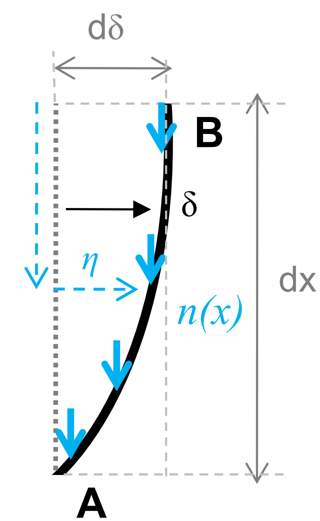
Tapered columns

Tapered columns
Differential equation

- Equilibrium







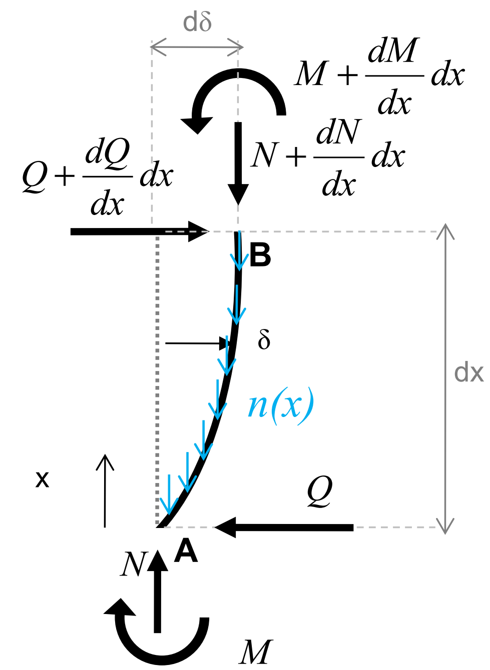
$$N(x)=N_{conc}+\int_x^L n(\xi)d\xi$$

Tapered columns
Differential equation

- Equilibrium

$$N(x) = N_{conc} + \int_x^L n(\xi) d\xi$$

+ neglect 2nd order terms



- Eq. Moments in B

$$N(x).dy + Qdx - \left(M + \frac{dM}{dx} dx \right) + M - \underbrace{\int_0^{dx} n(\xi) \eta d\xi}_{\approx 0} = 0$$

$$\rightarrow Q = \frac{dM}{dx} - N(x) \frac{dy}{dx}$$

- Eq. Horizontal

$$Q = Q + \frac{dQ}{dx} dx \quad \rightarrow \quad \frac{dQ}{dx} = 0$$

$$\frac{dQ}{dx} = 0 = \frac{d^2 M}{dx^2} - \frac{d}{dx}\left(N(x) \frac{d\delta}{dx} \right)$$

Tapered columns
Differential equation

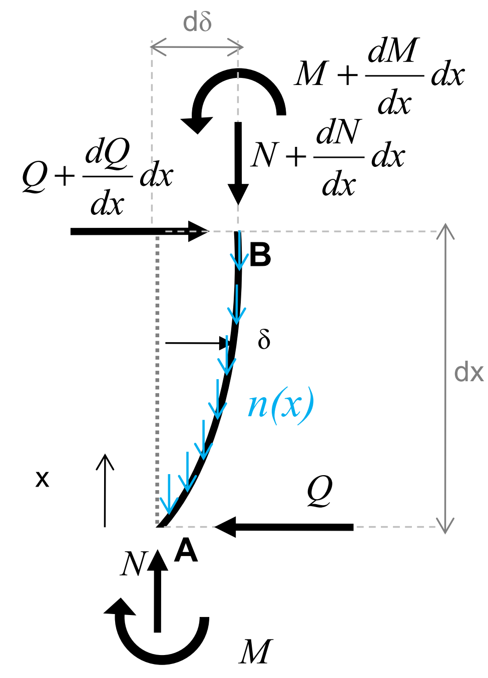
- Equilibrium

$$M(x) = -EI(x)\frac{d^2\delta}{dx^2} \quad + \quad 0 = \frac{d^2M}{dx^2} - \frac{d}{dx}\left(N(x)\frac{d\delta}{dx}\right)$$

$$E\frac{d^2}{dx^2}\left(I(x)\frac{d^2\delta}{dx^2}\right) + \frac{d}{dx}\left(N(x)\frac{d\delta}{dx}\right) = 0$$

$$E\left(I(x)\cdot\delta''\right)'' + \left(N(x)\cdot\delta'\right)' = 0$$

$$\begin{cases} N(x) = \alpha_{cr} N_{Ed}(x) \\ n(x) = \alpha_{cr} n_{Ed}(x) \\ \delta(x) = \delta_{cr}(x) \end{cases}$$



Tapered columns
Elastic critical load

- Differential equation

$$E\left(I(x)\cdot\delta''\right)'' + \left(N(x)\cdot\delta'\right)' = 0$$

- Proposal for determination of major axis critical load of tapered I columns (Marques et al, 2014)

$$N_{cr,Tap} = A\cdot N_{cr,min} \rightarrow A = \gamma_I^{\,0.56}\left(1 - 0.04\cdot tan^{-1}\left(\gamma_I - 1\right)\right)$$
$$\gamma_I = I_{y.max} / I_{y.min}$$

Tapered columns
Ayrton-Perry formulation

- Equilibrium

$$E\left(I(x)\cdot\delta''\right)'' + \left(N(x)\cdot\delta'\right)' = 0 \begin{cases} N(x) = \alpha_{cr} N_{Ed}(x) \\ n(x) = 0 \\ \delta(x) = \delta_{cr}(x) \end{cases}$$

$$\left(EI(x)\,\delta''\right)'' + \left(N(x)\,\delta' + N(x)\,\delta_0'\right)' = 0 \quad \begin{cases} N(x) = \alpha_b N_{Ed}(x) \\ \delta(x) = \dfrac{\alpha_b}{\alpha_{cr} - \alpha_b}\,\delta_0(x) \end{cases}$$

$$M(x) = -EI(x)\,\delta''(x) = -EI(x)\left[\frac{\alpha_b}{\alpha_{cr} - \alpha_b}\,\delta_0''(x)\right]$$

- First yield criterion

$$\varepsilon(x) = \frac{\alpha_b N_{Ed}(x)}{N_R(x)} + \frac{M(x)}{M_R(x)} = \frac{\alpha_b N_{Ed}(x)}{N_R(x)} + \frac{EI(x)\left[\dfrac{\alpha_b}{\alpha_{cr} - \alpha_b}\left(-\delta_0''(x)\right)\right]}{M_R(x)}$$

Tapered columns
Ayrton-Perry formulation

- Assumption for imperfection $\delta_0(x) = \delta_{cr}(x) e_0$

$$\varepsilon(x) = \frac{\alpha_b N_{Ed}(x)}{N_R(x)} + \frac{M(x)}{M_R(x)} = \frac{\alpha_b N_{Ed}(x)}{N_R(x)} + \frac{EI(x)\left[\frac{\alpha_b}{\alpha_{cr} - \alpha_b} \cdot \overbrace{(-1)\left(\delta''_{cr}(x) e_0\right)}^{-\delta''_0}\right]}{M_R(x)}$$

- Introducing slenderness and reduction factor definitions

$$\bar{\lambda}(x) = \sqrt{\frac{N_R(x)/N_{Ed}(x)}{\alpha_{cr}}} \qquad \chi(x) = \frac{\alpha_b}{N_R(x)/N_{Ed}(x)}$$

$$\varepsilon(x) = \chi(x) + \chi(x) \cdot \frac{1}{1 - \frac{\alpha_b}{\alpha_{cr}}} \left[e_0 \frac{N_R(x_c)}{M_R(x_c)}\right] \left[\frac{EI(x)\left(-\delta''_{cr}(x)\right)}{N_{Ed}(x)\alpha_{cr}}\right] \left[\frac{N_R(x)}{N_R(x_c)} \frac{M_R(x_c)}{M_R(x)}\right]$$

- At the critical location $\varepsilon(x_c^{II})=1$

$$1 = \chi(x_c^{II}) + \frac{\chi(x_c^{II})}{1 - \bar{\lambda}^2(x_c^{II})\chi(x_c^{II})} \left[e_0 \frac{N_R(x_c^{II})}{M_R(x_c^{II})}\right] \left[\frac{EI(x_c^{II}).(-\delta''_{cr}(x_c^{II}))}{\alpha_{cr}.N_{Ed}(x_c^{II})}\right]$$

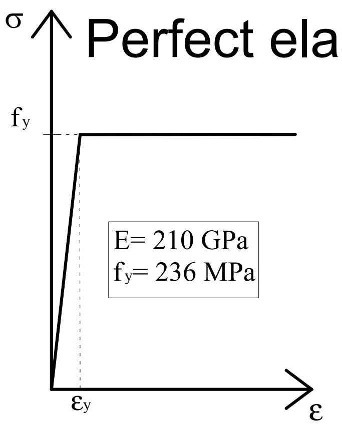
$$\alpha_{EC3}\left(\bar{\lambda}(x_c^{II}) - 0.2\right)$$

Design resistance of tapered columns and beams

Design resistance of tapered columns and beams

❑ Material:

Perfect elastic-plastic



❑ Boundary conditions:

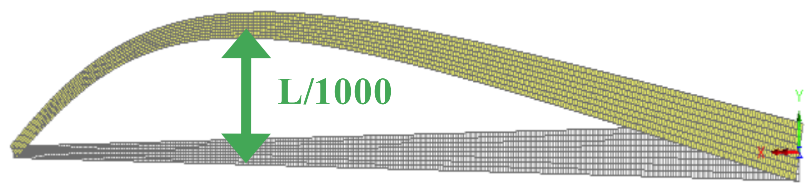
Fork supports

End cross sections remain straight

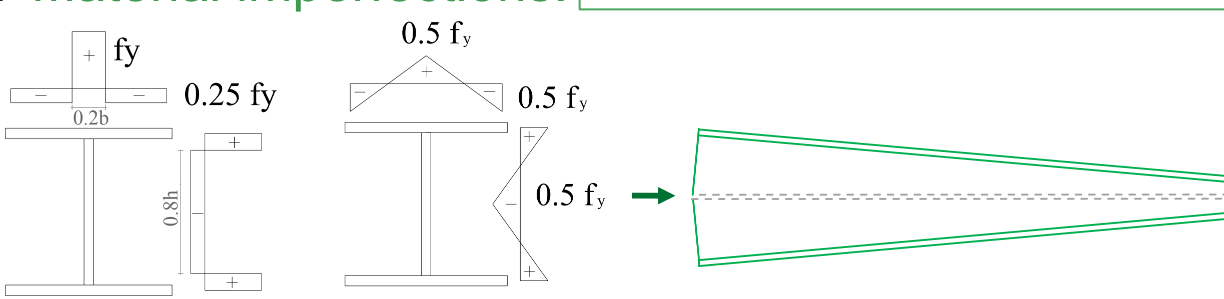
❑ Member imperfections:

With amplitude e_0=L/1000

Same shape as the buckling mode



❑ Material imperfections:



❑ Calculations:

LBA

GMNIA

Design resistance of tapered columns and beams

- Columns

$$\varepsilon(x) = \chi(x) \cdot \chi(x) \cdot \frac{1}{1-\frac{\alpha_b}{\alpha_{cr}}}\left[e_0 \frac{N_R(x_c)}{M_R(x_c)}\right]\left[\frac{EI(x)\left(-\delta''_{cr}(x)\right)}{N_{Ed}(x)\alpha_{cr}}\right]\left[\frac{N_R(x)}{N_R(x_c)}\frac{M_R(x_c)}{M_R(x)}\right]$$

$$\frac{N}{N_{Rk}}$$

$$\frac{M^{II}}{M_{Rk}} = \frac{-EI\delta''}{M_{Rk}}$$

- Beams

$$\varepsilon(x) = \chi_{LT}(x) \cdot \frac{\chi_{LT}(x)}{1-\bar{\lambda}_{LT}^2(x)\chi_{LT}(x)}\left[\bar{e}_0 \frac{A(x_c^{II})}{W_z(x_c^{II})}\right]\left[\frac{\bar{\lambda}_{LT}^2(x_c^{II})}{\bar{\lambda}_z^2(x_c^{II})}\right] \times \frac{\xi\left(-\delta''_{cr,h\min}(x)\right)EI_z(x)}{N_{cr,z,Tap}}\left[\frac{1+\frac{N_{cr,z,Tap}}{M_{cr,Tap}}\frac{h(x)}{2}}{1+\frac{M_{cr,Tap}}{N_{cr,z,Tap}}\frac{h_{min}}{2}}\right] \times \left[\frac{A(x)}{W_z(x)}\frac{W_z(x_c^{II})}{A(x_c^{II})}\right]\left[\frac{\bar{\lambda}_{LT}^2(x)}{\bar{\lambda}_z^2(x)}\frac{\bar{\lambda}_z^2(x_c^{II})}{\bar{\lambda}_{LT}^2(x_c^{II})}\right]$$

$$\frac{M_y}{M_{y,Rk}}$$

$$\frac{M_z^{II}}{M_{z,Rk}} = \frac{-EI_z v''}{M_{z,Rk}}$$

$$\frac{M_{\varpi}^{II}}{M_{\varpi,Rk}} = \frac{-EI_{\omega}\phi''}{M_{\varpi,Rk}}$$

Design resistance of tapered columns and beams

❑ Columns

$$\varepsilon(x) = \chi(x) + \chi(x)\cdot\frac{1}{1-\frac{\alpha_b}{\alpha_{cr}}}\left[e_0\frac{N_R(x_c)}{M_R(x_c)}\right]\left[\frac{EI(x)\left(-\delta''_{cr}(x)\right)}{N_{Ed}(x)\alpha_{cr}}\right]\left[\frac{N_R(x)}{N_R(x_c)}\frac{M_R(x_c)}{M_R(x)}\right]$$

$\alpha_{EC3}\left(\bar{\lambda}(x_c^{II})-0.2\right)$

$\eta_{uniform}(x_c^{II})$

$\eta_{non-uniform}(x_c^{II})$

❑ Beams

$$\varepsilon(x) = \chi_{LT}(x) + \frac{\chi_{LT}(x)}{1-\bar{\lambda}_{LT}^2(x)\chi_{LT}(x)}\left[\bar{e}_0\frac{A(x_c^{II})}{W_z(x_c^{II})}\right]\left[\frac{\bar{\lambda}_{LT}^2(x_c^{II})}{\bar{\lambda}_z^2(x_c^{II})}\right]\times\frac{\xi\left(-\delta''_{cr,h\min}(x)\right)EI_z(x)}{N_{cr,z,Tap}}\left[\frac{1+\frac{N_{cr,z,Tap}}{M_{cr,Tap}}\frac{h(x)}{2}}{1+\frac{M_{cr,Tap}}{N_{cr,z,Tap}}\frac{h_{min}}{2}}\right]\times\left[\frac{A(x)}{W_z(x)}\frac{W_z(x_c^{II})}{A(x_c^{II})}\right]\left[\frac{\bar{\lambda}_{LT}^2(x)}{\bar{\lambda}_z^2(x)}\frac{\bar{\lambda}_z^2(x_c^{II})}{\bar{\lambda}_{LT}^2(x_c^{II})}\right]$$

$\alpha_{LT}\left(\bar{\lambda}_z(x_c^{II})-0.2\right)$

Design resistance of tapered columns and beams

Columns

$$\varepsilon(x) = \chi(x) + \chi(x)\cdot\frac{1}{1-\frac{\alpha_b}{\alpha_{cr}}}\left[e_0\frac{N_R(x_c)}{M_R(x_c)}\right]\left[\frac{EI(x)\left(-\delta''_{cr}(x)\right)}{N_{Ed}(x)\alpha_{cr}}\right]\left[\frac{N_R(x)}{N_R(x_c)}\frac{M_R(x_c)}{M_R(x)}\right]$$

$$1=\chi(x_c^{II})+\chi(x_c^{II})\frac{1}{1-\bar{\lambda}^2(x_c^{II})\chi(x_c^{II})}\alpha_{EC3}(x_c^{II})\left(\bar{\lambda}(x_c^{II})-0.2\right)\times\beta(x_c^{II})$$

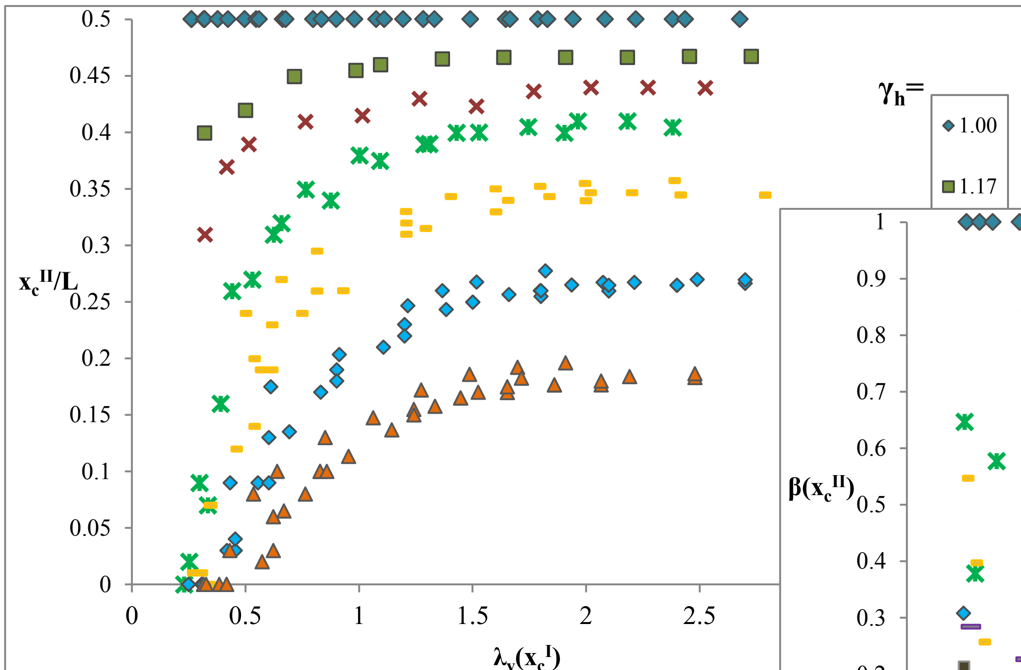
Beams

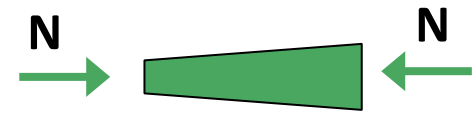
$$\varepsilon(x)=\chi_{LT}(x)+\frac{\chi_{LT}(x)}{1-\bar{\lambda}_{LT}^2(x)\chi_{LT}(x)}\left[\bar{e}_0\frac{A(x_c^{II})}{W_z(x_c^{II})}\right]\left[\frac{\bar{\lambda}_{LT}^2(x_c^{II})}{\bar{\lambda}_z^2(x_c^{II})}\right]\times\frac{\xi\left(-\delta''_{cr,h\min}(x)\right)EI_z(x)}{N_{cr,z,Tap}}\left[\frac{1+\frac{N_{cr,z,Tap}}{M_{cr,Tap}}\frac{h(x)}{2}}{1+\frac{M_{cr,Tap}}{N_{cr,z,Tap}}\frac{h_{min}}{2}}\right]\times\left[\frac{A(x)}{W_z(x)}\frac{W_z(x_c^{II})}{A(x_c^{II})}\right]\left[\frac{\bar{\lambda}_{LT}^2(x)}{\bar{\lambda}_z^2(x)}\frac{\bar{\lambda}_z^2(x_c^{II})}{\bar{\lambda}_{LT}^2(x_c^{II})}\right]$$

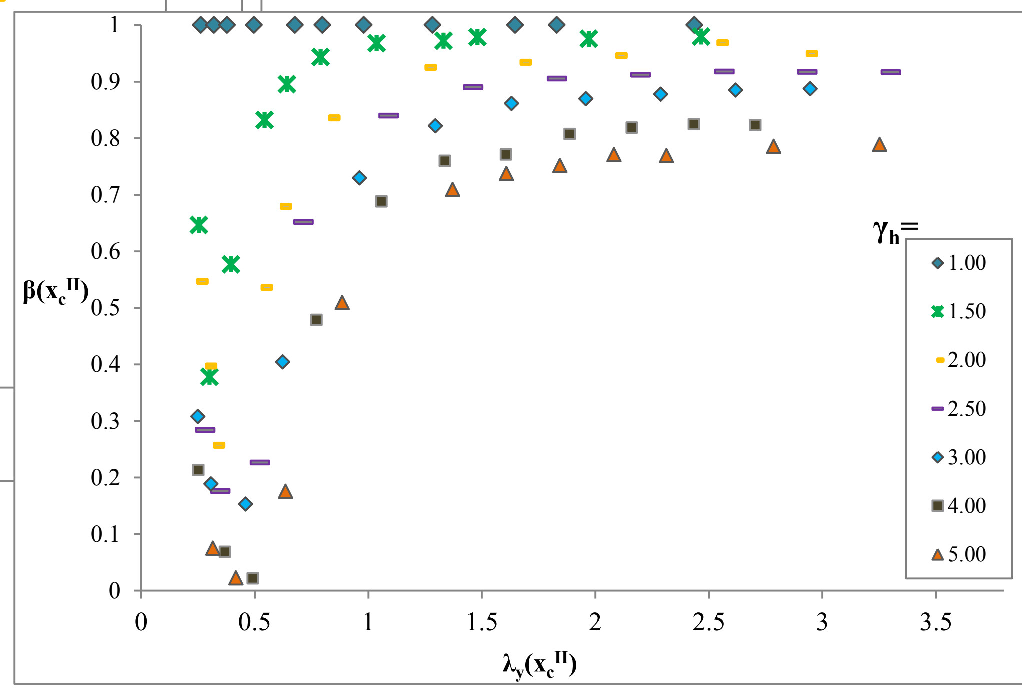
$$\varepsilon(x_c)=1 \quad\rightarrow\quad 1=\chi_{LT}(x_c^{II})+\frac{\chi_{LT}(x_c^{II})}{1-\bar{\lambda}_{LT}^2(x_c^{II})\chi_{LT}(x_c^{II})}\times\left(\alpha_{LT}\left(\bar{\lambda}_z(x_c^{II})-0.2\right)\right)\left[\frac{\bar{\lambda}_{LT}^2(x_c^{II})}{\bar{\lambda}_z^2(x_c^{II})}\right]\times\beta(x_c^{II})$$

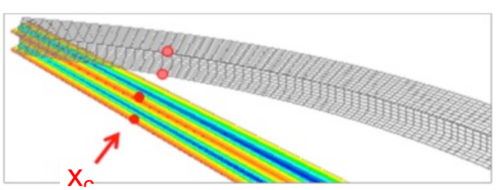
Design resistance of tapered columns and beams

- Interpretation of x_c^{II} and β – example (column)



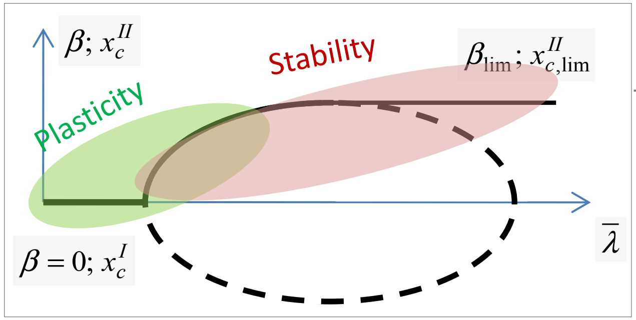


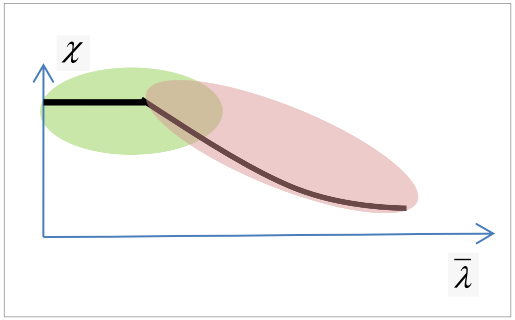




Design resistance of tapered columns and beams

1 - Simplification of the analytical models





- Columns

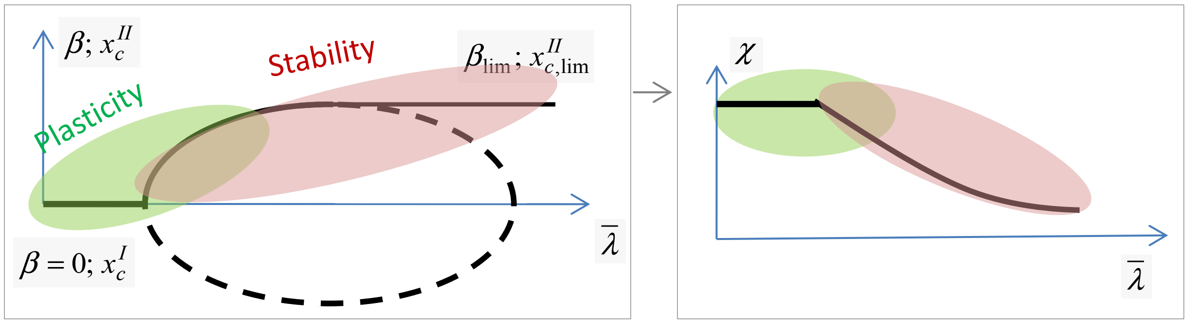
$$1 = \chi(x_{c,\lim}^{II}) + \chi(x_{c,\lim}^{II}) \frac{1}{1 - \bar{\lambda}^2(x_{c,\lim}^{II})\chi(x_{c,\lim}^{II})} \alpha\left(\bar{\lambda}(x_{c,\lim}^{II}) - 0.2\right) \times \underbrace{1}_{\beta_{\lim}}$$

- Beams

$$1 = \chi_{LT}(x_{c,\lim}^{II}) + \frac{\chi_{LT}(x_{c,\lim}^{II})}{1 - \bar{\lambda}_{LT}^2(x_c^{II})\chi_{LT}(x_c^{II})} \times \left(\alpha_{LT}\left(\bar{\lambda}_z(x_{c,\lim}^{II}) - 0.2\right)\right) \left[\frac{\bar{\lambda}_{LT}^2(x_{c,\lim}^{II})}{\bar{\lambda}_z^2(x_{c,\lim}^{II})}\right] \times \underbrace{1}_{\beta_{\lim}}$$

Design resistance of tapered columns and beams

1 - Simplification of the analytical models



- Columns

$$1=\chi(x^{II}_{c,lim})+\chi(x^{II}_{c,lim})\frac{1}{1-\bar{\lambda}^2(x^{II}_{c,lim})\chi(x^{II}_{c,lim})}\alpha\left(\bar{\lambda}(x^{II}_{c,lim})-0.2\right)\times\underbrace{1}_{\beta_{lim}}$$

- Beams

$$1=\chi_{LT}(x^{II}_{c,lim})+\frac{\chi_{LT}(x^{II}_{c,lim})}{1-\bar{\lambda}^2_{LT}(x^{II}_{c})\chi_{LT}(x^{II}_{c})}\times\left(\alpha_{LT}\left(\bar{\lambda}_z(x^{II}_{c,lim})-0.2\right)\right)\left[\frac{\bar{\lambda}^2_{LT}(x^{II}_{c,lim})}{\bar{\lambda}^2_{z}(x^{II}_{c,lim})}\right]\times\underbrace{1}_{\beta_{lim}}$$

2 - Transformation of variables

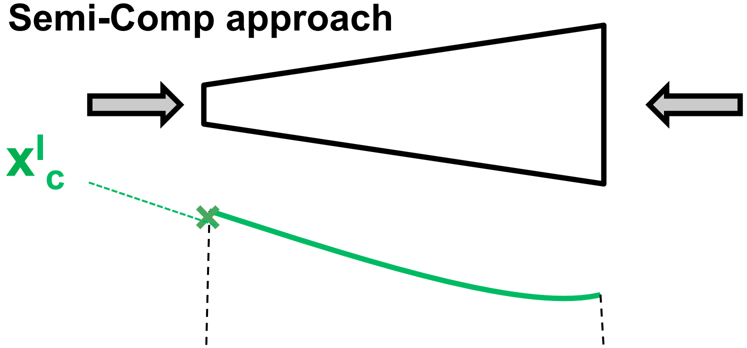
$$\varphi=\frac{\alpha_{ult,k}(x^{II}_{c,lim})}{\alpha_{ult,k}(x^{I}_{c})} \rightarrow \quad \bar{\lambda}\ (x^{II}_{c,lim})=\sqrt{\varphi}\times\bar{\lambda}(x^{I}_{c}) \qquad \chi(x^{II}_{c,lim})=\chi\ (x^{I}_{c})/\varphi$$

Design resistance of tapered columns and beams

COLUMNS – DESIGN METHODOLOGY

1. Required data

Calculate utilization $\varepsilon = N_{Ed}/N_{Rk}$ based on Semi-Comp approach



$$\alpha_{ilt,k}(x_c^I) = \frac{N_{Rk}(x_c^I)}{N_{Ed}(x_c^I)}$$

Calculate φ

(…)

Calculate α_{cr}

Design resistance of tapered columns and beams

COLUMNS – DESIGN METHODOLOGY

2. Application of the method

$$\eta = \alpha \times \left(\varphi \times \bar{\lambda}(x_c^I) - 0.2\right)$$

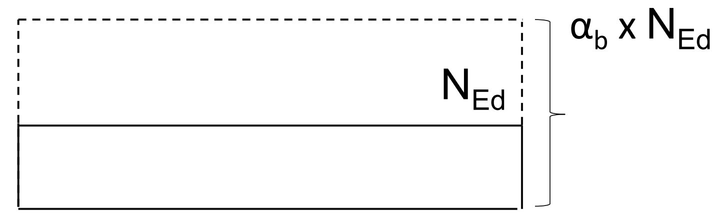
$$\bar{\lambda}(x_c^I) = \sqrt{\alpha_{ult,k}(x_c^I) / \alpha_{cr}}$$

$$\phi = 0.5 \times \left(1 + \varphi \times \eta \times \bar{\lambda}^2(x_c^I) + \varphi \times \bar{\lambda}^2(x_c^I)\right)$$

$$\chi(x_c^I) = \frac{\varphi}{\phi + \sqrt{\phi^2 - \varphi \times \bar{\lambda}^2(x_c^I)}} \leq 1$$

3. Verification

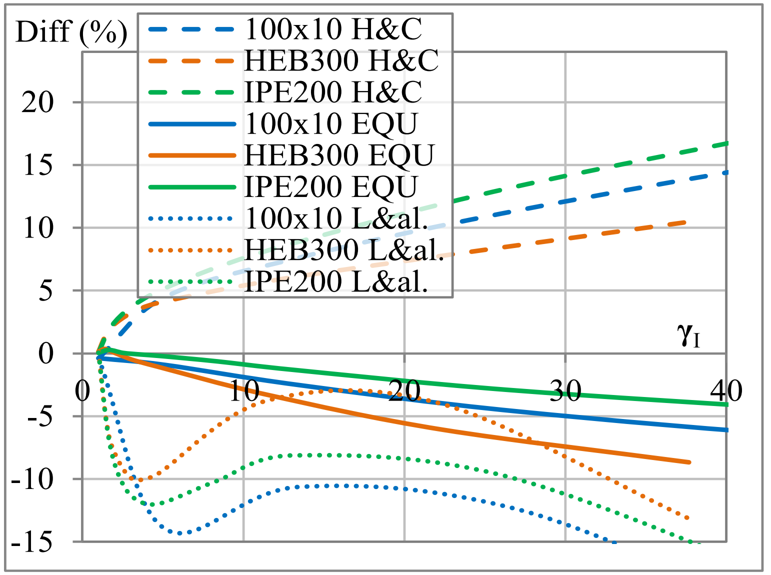
$$\chi(x_c^I) \times \alpha_{ult,k}(x_c^I) = \alpha_b \geq 1$$



Design resistance of tapered columns and beams

COLUMNS – DESIGN METHODOLOGY

- Necessary parameters
  - Critical load multiplier – α_{cr}
    - May be numerical
    - From the literature
    - For in-plane loading, a formula was developed considering Rayleigh-Ritz Method :



$$N_{cr,Tap} = A \cdot N_{cr,\min} \rightarrow A = \gamma_I^{\,0.56}\left(1 - 0.04 \cdot \tan^{-1}(\gamma_I - 1)\right)$$

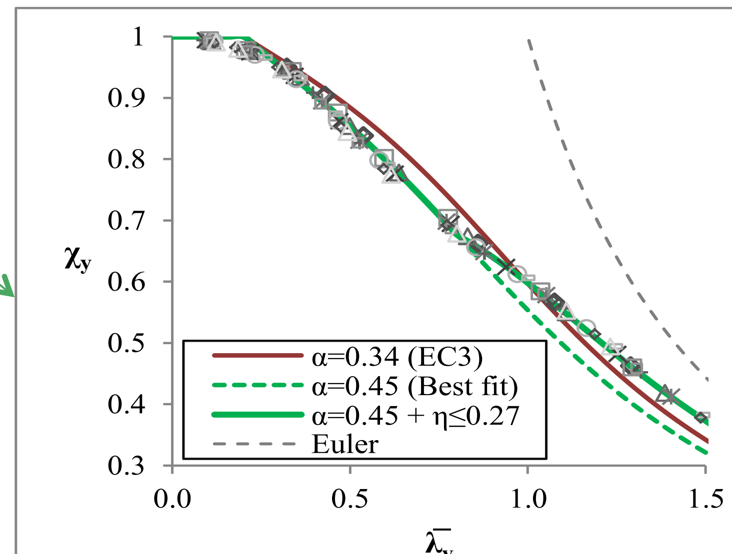
$$\gamma_I = I_{y.\max} / I_{y.\min}$$

Design resistance of tapered columns and beams

COLUMNS – DESIGN METHODOLOGY

- Necessary parameters
  - Imperfection factor α_y or α_z

	FB out-of-plane		FB in-plane	
	Hot-rolled:	Welded:	Hot-rolled:	Welded:
α	0.49	0.64	0.34	0.45
η	-	≤0.34	-	≤0.27



Design resistance of tapered columns and beams

COLUMNS – DESIGN METHODOLOGY

- Necessary parameters

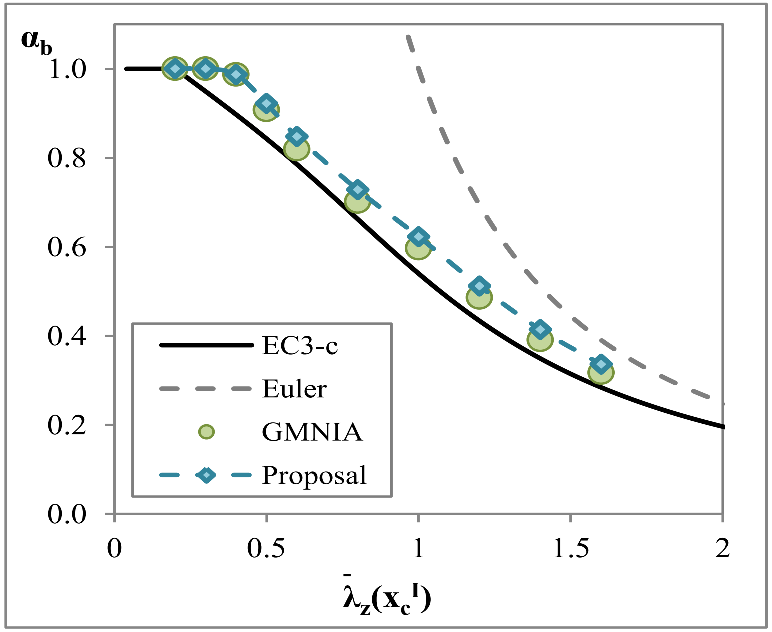
  - Overstrength factor φ_y or φ_z

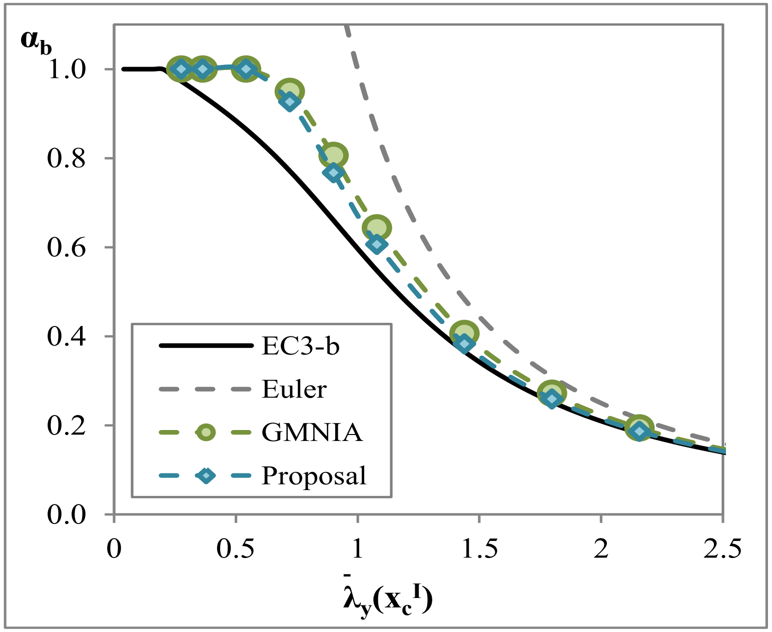
FB out-of-plane	FB in-plane
$1+\frac{ht_w}{A_{\min}}\left[\frac{(1+4\gamma_h)(\gamma_h-1)}{10\gamma_h}\right]$	$1+\frac{h_{\min}t_w}{A_{\min}}\frac{\gamma_h-1}{\gamma_h+1}$

Design resistance of tapered columns and beams

COLUMNS – DESIGN METHODOLOGY

- Results
  - Out-of-plane ($\gamma h = h_{max}/h_{min} = 1.8$)
  - In-plane ($\gamma h = h_{max}/h_{min} = 6$)

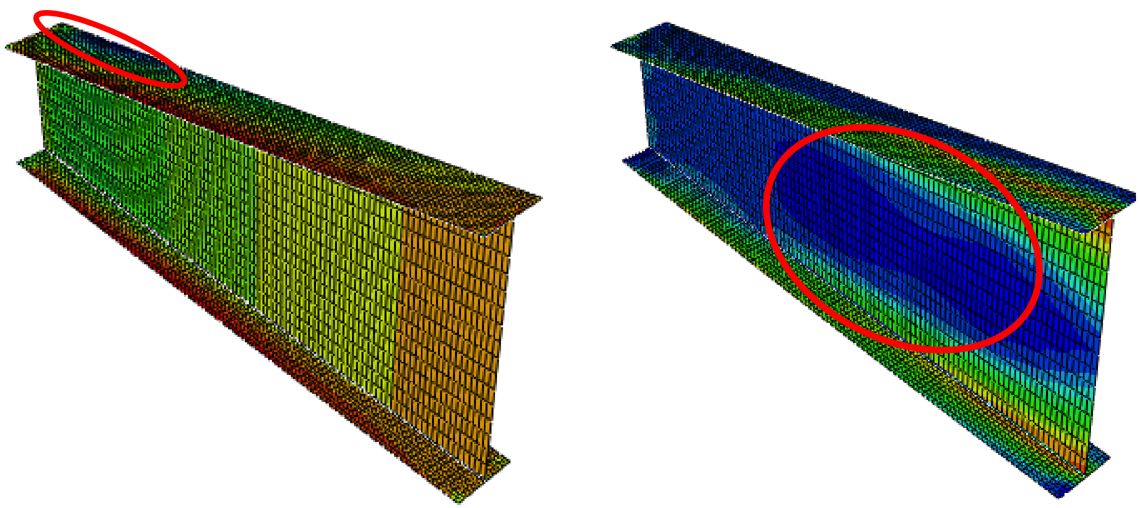




Design resistance of tapered columns and beams

COLUMNS – DESIGN METHODOLOGY

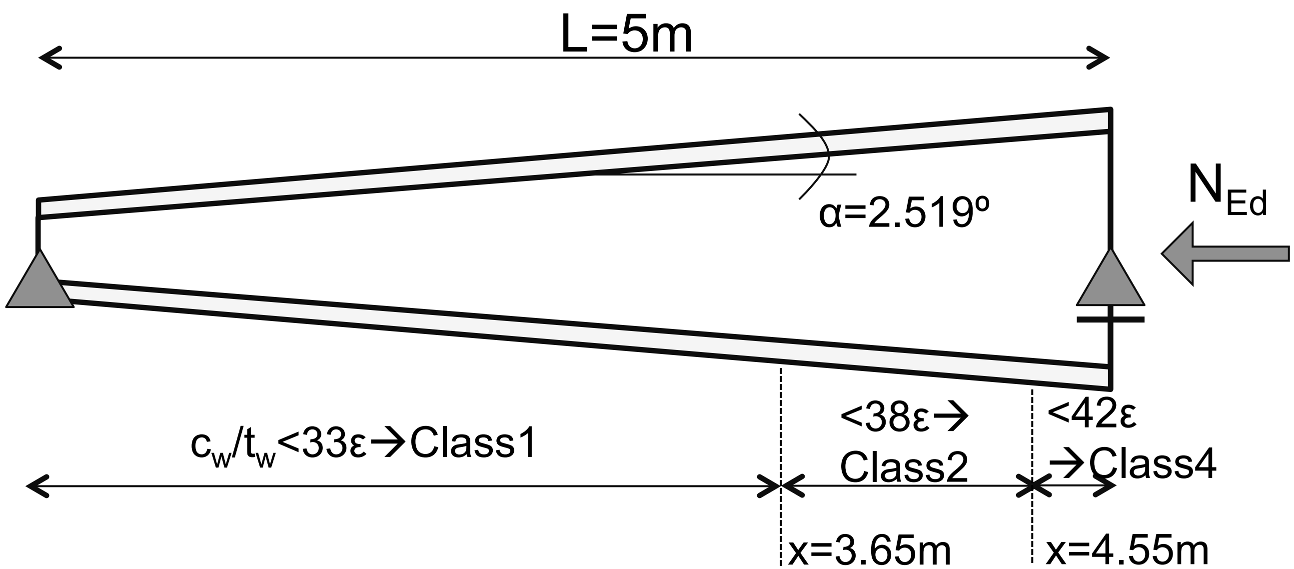
- Possible problems
  - Web buckling – critical location varies



- φ was calibrated considering critical location without local buckling effects!
- May lead to unsafe results!

Design resistance of tapered columns and beams

COLUMNS - EXAMPLE



$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
N_{Ed} = 1100 kN
L = 5 m

Flange class: c_f/t_f=(206/2-15/2)/25=3.82 <9ε → Class1

- Flange thickness in vertical plan

$$t_f' = t_f / \cos(\alpha) = 25.02mm$$

h_{min} = 220.05 mm
h_{max} = 660.05 mm

γ_h= 660.05/220.05=3

Design resistance of tapered columns and beams

COLUMNS - EXAMPLE

- Minor axis verification

- Calculation of slenderness at $x=x_c^I$

$$\bar{\lambda}(x_c^I)=\sqrt{\frac{N_{Rk}(x_c^I)/N_{Ed}}{\alpha_{cr}}}\approx\sqrt{\frac{N_{Rk}(x_c^I)/N_{Ed}}{N_{cr,h\min}/N_{Ed}}}=\sqrt{\frac{3022.1/1100}{3022.1/1100}}=1$$

$$N_{cr,h\min}=\frac{\pi^2 EI_{h\min}}{L^2}$$

- Overstrength-factor, φ

$$\varphi_z=1+\frac{h_{\min}t_w}{A_{\min}}\left[\frac{(1+4\gamma_h)(\gamma_h-1)}{10\gamma_h}\right]=1+\frac{220.05\times15}{12850}\left[\frac{(1+4\times3)(3-1)}{10\times3}\right]=1.222$$

- Determination of imperfection, η

$$\eta=\alpha\left(\sqrt{\varphi}\bar{\lambda}(x_c^I)-0.2\right)=0.64\left(\sqrt{1.222}\times1-0.2\right)=0.580>0.34\quad\rightarrow\quad\eta=0.34$$

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
N_{Ed} = 1100 kN
L = 5 m

Design resistance of tapered columns and beams

COLUMNS - EXAMPLE

- Minor axis verification

- Reduction factor

$$\phi = 0.5\left(1+\eta+\varphi\bar{\lambda}^2(x_c^I)\right) = 0.5\left(1+0.34+1.222\times1^2\right) = 1.281$$

$$\chi(x_c^I) = \frac{\varphi}{\phi+\sqrt{\phi^2-\varphi\times\bar{\lambda}^2(x_c^I)}} = \frac{1.222}{1.281+\sqrt{1.281^2-1.222\times1^2}} = 0.634 \leq 1$$

- Verification

$$N_{b,z,Rd} = \chi(x_c^I)N_{Pl}(x_c^I) = 0.634\times3022.1 = 1915.9\ kN > 1100\ kN$$

- Numerical analysis, GMNIA

$$N_{b,z,Rd} = 1904.68\ kN \rightarrow 0.5\%\ diff$$

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
N_{Ed} = 1100 kN
L = 5 m

Design resistance of tapered columns and beams

COLUMNS - EXAMPLE

- Major axis verification

- Critical load, $N_{cr,y,Tap}$

$$\gamma_I = \frac{I_{h\max}}{I_{h\min}} = \frac{132365 cm^2}{10471 cm^2} = 12.64$$

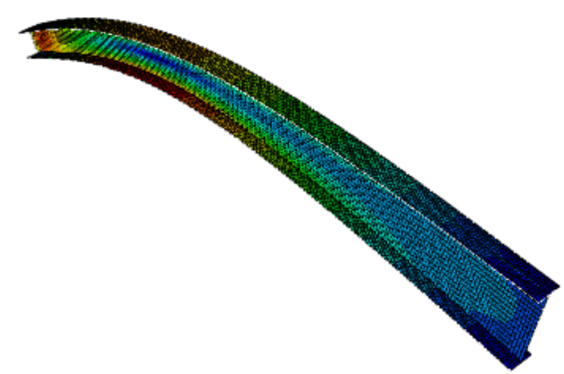
$$A = \gamma_I^{0.56}\left(1 - 0.04 \cdot \tan^{-1}(\gamma_I - 1)\right) = 3.894$$

$$N_{cr,y,Tap} = A \cdot N_{cr,\min} = 3.894 \times 8681 kN = 33804.56 kN$$

- Numerical buckling analysis, LBA:

$$N_{cr,y,Tap} = 32516 kN$$ → diff. 3.9%

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
N_{Ed} = 1100 kN
L = 5 m



Design resistance of tapered columns and beams

COLUMNS - EXAMPLE

- Major axis verification

- Calculation of slenderness at $x=x_c^I$

$$\bar{\lambda}(x_c^I) = \sqrt{\frac{N_{Rk}(x_c^I)/N_{Ed}}{\alpha_{cr}}} \approx \sqrt{\frac{N_{Rk}(x_c^I)/N_{Ed}}{N_{cr,y,Tap}/N_{Ed}}} = \sqrt{\frac{3022.1/1100}{33804.56/1100}} = 0.299$$

- Overstrength-factor, φ

$$\varphi_y = 1 + \frac{h_{\min} t_w}{A_{\min}} \frac{\gamma_h - 1}{\gamma_h + 1} = 1 + \frac{220.05mm \times 15mm}{12850mm^2} \frac{3-1}{3+1} = 1.128$$

- Determination of imperfection, η

$$\eta = \alpha\left(\sqrt{\varphi}\,\bar{\lambda}(x_c^I) - 0.2\right) = 0.45\left(\sqrt{1.128} \times 0.299 - 0.2\right) = 0.053 < 0.27 \quad \rightarrow \quad \eta = 0.053$$

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
N_{Ed} = 1100 kN
L = 5 m

Design resistance of tapered columns and beams

COLUMNS - EXAMPLE

- Major axis verification

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
N_{Ed} = 1100 kN
L = 5 m

- Reduction factor

$$\phi = 0.5\left(1+\eta+\varphi\bar{\lambda}^2(x_c^I)\right) = 0.5\left(1+0.053+1.128\times0.299^2\right) = 0.577$$

$$\chi(x_c^I) = \frac{\varphi}{\phi+\sqrt{\phi^2-\varphi\times\bar{\lambda}^2(x_c^I)}} = \frac{1.128}{0.577+\sqrt{0.577^2-1.128\times0.299^2}} = 1.07 > 1 \quad \chi(x_c^I) = 1$$

- Verification

$$N_{b,y,Rd} = \chi(x_c^I)N_{Pl}(x_c^I) = 1\times3022.1 = 3022.1\,kN > 1100\,kN$$

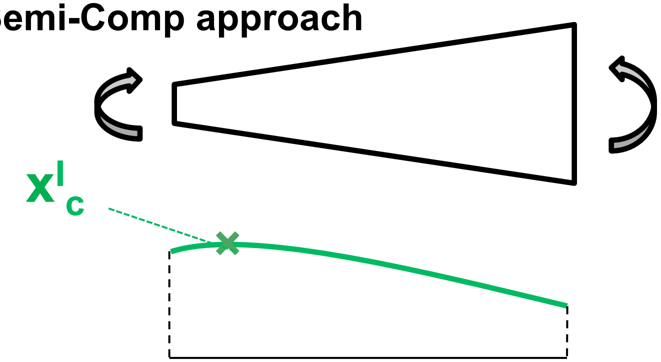
- Numerical analysis, GMNIA

$$N_{b,z,Rd} = 3022.1\,kN \rightarrow 0\%\ diff$$

BEAMS – DESIGN METHODOLOGY

1. Necessary data

Calculate utilization $\varepsilon = M_{Ed}/M_{Rk}$ based on Semi-Comp approach



$$\alpha_{ilt,k}(x_c^I) = \frac{M_{Rk}(x_c^I)}{M_{Ed}(x_c^I)}$$

Calculate φ

(...)

Calculate $x_{c,lim}^{II}$

(...)

Calculate α_{cr}

European Erasmus Mundus Master Course

Sustainable Constructions under Natural Hazards and Catastrophic Events

Design resistance of tapered columns and beams

BEAMS – DESIGN METHODOLOGY

2. Application of the method

$$W_{y,el}(x_c^{II})$$
$$W_{z,el}(x_c^{II})$$

$$N_{cr} \approx \frac{\pi^2 EI_z(x_c^{II})}{L^2}$$ $$N_{Rk}(x_c^{II})$$

$$\alpha_{LT} \quad (Taras)$$ $$\bar{\lambda}_z(x_c^{II})$$

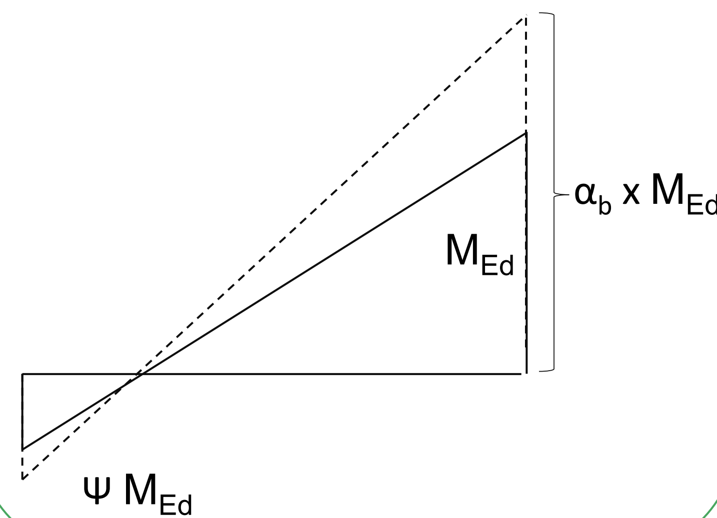
$$\eta = \alpha_{LT} \times \left(\bar{\lambda}_z(x_c^I) - 0.2\right)$$ $$\bar{\lambda}_{LT}(x_c^I) = \sqrt{\alpha_{ult,k}(x_c^I)/\alpha_{cr}}$$

$$\phi_{LT} = 0.5 \times \left(1 + \varphi \times \eta \times \frac{\bar{\lambda}^2_{LT}(x_c^I)}{\bar{\lambda}^2_z(x_c^{II})} + \varphi \times \bar{\lambda}^2_{LT}(x_c^I)\right)$$

$$\chi_{LT}(x_c^I) = \frac{\varphi}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \varphi \times \bar{\lambda}_{LT}^2(x_c^I)}} \leq 1$$

3. Verification

$$\chi_{LT}(x_c^I) \times \alpha_{ult,k}(x_c^I) = \alpha_b \geq 1$$



Design resistance of tapered columns and beams

BEAMS – DESIGN METHODOLOGY

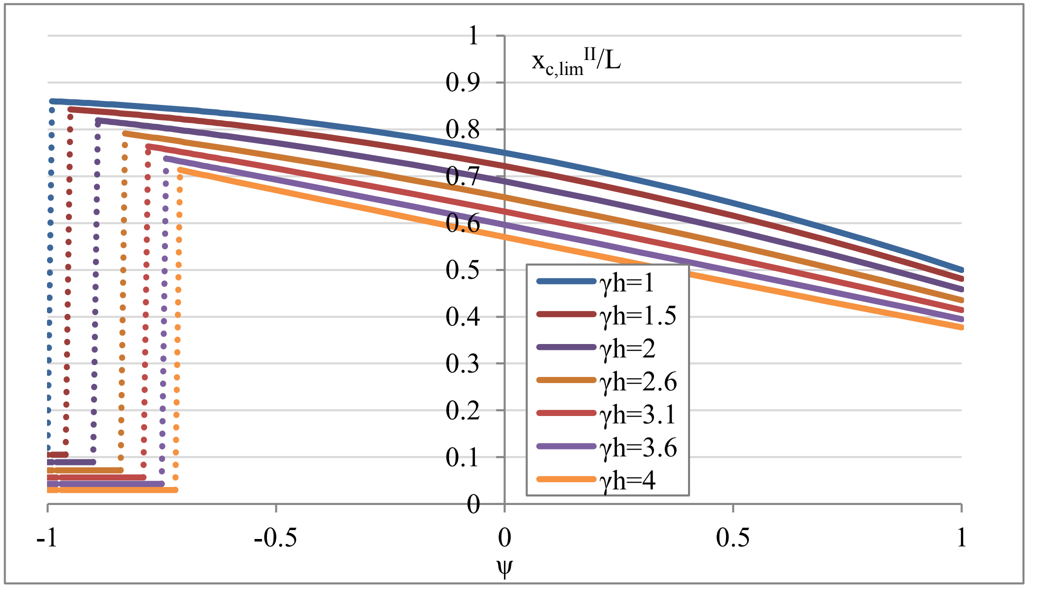
- Necessary parameters
  - Critical load multiplier – α_{cr}
    - May be numerical
    - From the literature
  - Imperfection factor α_{LT}
    - Model consistent with recently developed proposals for prismatic beams

	Hot-rolled:	Welded:
α	$0.16\sqrt{\frac{W_{y,el}(x^{II}_{c,\lim})}{W_{z,el}(x^{II}_{c,\lim})}} \leq 0.49$	$0.21\sqrt{\frac{W_{y,el}(x^{II}_{c,\lim})}{W_{z,el}(x^{II}_{c,\lim})}} \leq 0.64$
η	-	$\leq \sqrt{\frac{W_{y,el}(x^{II}_{c,\lim})}{W_{z,el}(x^{II}_{c,\lim})}}(0.12\psi^2 - 0.23\psi + 0.35)$

Design resistance of tapered columns and beams

BEAMS – DESIGN METHODOLOGY

- Necessary parameters
  - $x_{c,lim}^{II}$

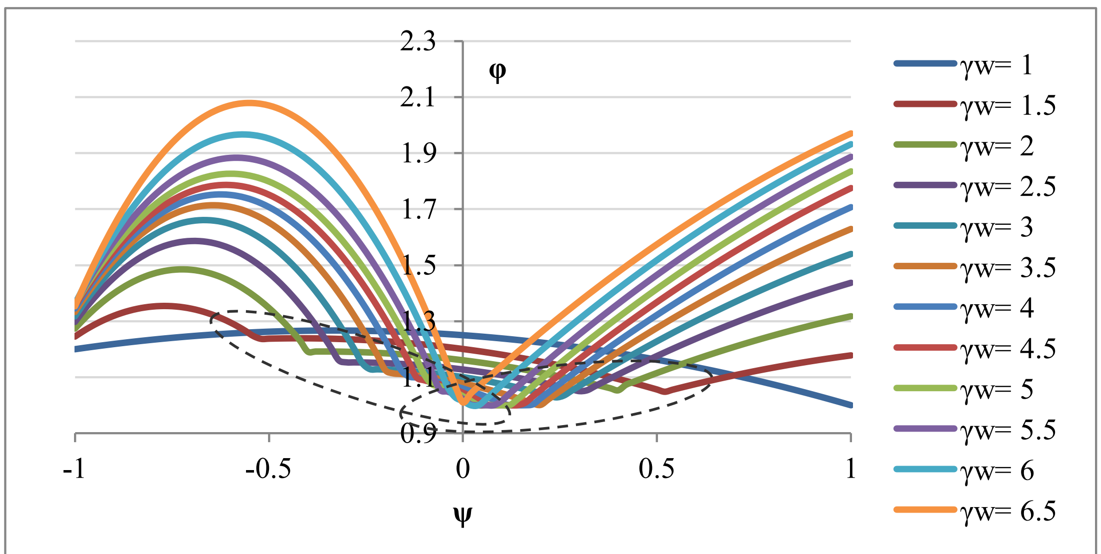


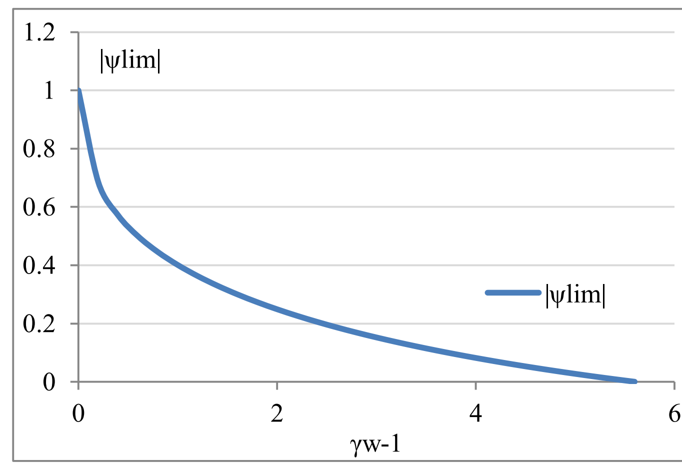
For ψ	$(0.75-0.18\psi-0.07\psi^2)+(0.025\psi^2-0.006\psi-0.06)(\gamma_h-1)\geq 0$ $If \quad \psi<0 \quad and \quad \lvert\psi\rvert\gamma_w \geq 1+1.214(\gamma_h-1), \quad x_{c,\lim}^{II}/L=0.12-0.03(\gamma_h-1)$
For UDL	$0.5+0.0035(\gamma_h-1)^2-0.03(\gamma_h-1)^2\leq 0.5$

Design resistance of tapered columns and beams

BEAMS – DESIGN METHODOLOGY

- Necessary parameters
  - Overstrength factor φ_{LT}





Design resistance of tapered columns and beams

BEAMS – DESIGN METHODOLOGY

- Necessary parameters
  - Overstrength factor φ_{LT}

UDL: $-0.0025 a_\gamma^2 + 0.015 a_\gamma + 1.05$

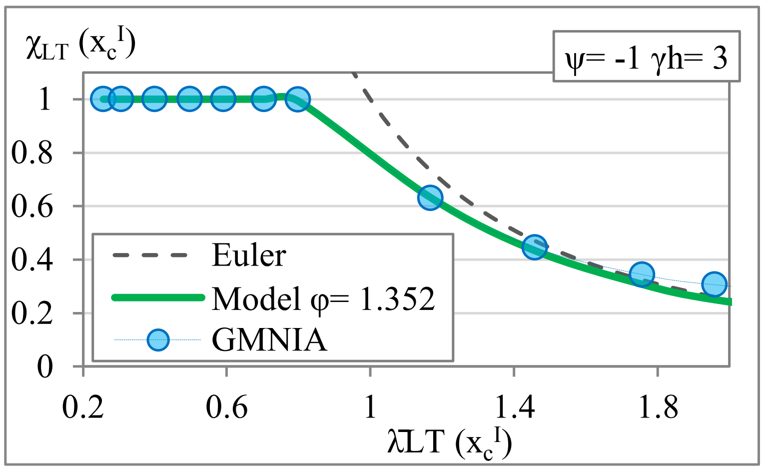
Ψ: $A \cdot \psi^2 + B \cdot \psi + C \quad \geq 1$

a_γ	$-0.0005 \cdot (\gamma_w - 1)^4 + 0.009 \cdot (\gamma_w - 1)^3 - 0.077 \cdot (\gamma_w - 1)^2 + 0.78 \cdot (\gamma_w - 1)$		
ψ_{lim}	$1 + 120 \cdot a_\gamma + 600 \cdot a_\gamma^2 - 210 \cdot a_\gamma^3 / 1 + 123 \cdot a_\gamma + 1140 \cdot a_\gamma^2 + 330 \cdot a_\gamma^3$		
φ_{LT}	$\psi < -\psi_{\lim}$	$-\psi_{\lim} \leq \psi \leq \psi_{\lim}$	$\psi > \psi_{\lim}$
A	$-0.0665 \cdot a_\gamma^6 + 0.718 \cdot a_\gamma^5 - 2.973 \cdot a_\gamma^4 + 5.36 \cdot a_\gamma^3 - 2.9 \cdot a_\gamma^2 - 2.1 \cdot a_\gamma - 1.09$	$\frac{-11.37 + 12090 \cdot a_\gamma - 8050 \cdot a_\gamma^2 + 1400 \cdot a_\gamma^3}{1 - 1058 \cdot a_\gamma + 705 \cdot a_\gamma^2 - 120 \cdot a_\gamma^3} + 11.22$	$0.008 \cdot a_\gamma^2 - 0.08 \cdot a_\gamma - 0.157$
B	$-0.1244 \cdot a_\gamma^6 + 1.3185 \cdot a_\gamma^5 - 5.287 \cdot a_\gamma^4 + 9.27 \cdot a_\gamma^3 - 5.24 \cdot a_\gamma^2 - 2.18 \cdot a_\gamma - 2$	$+0.02 \cdot a_\gamma^6 - 0.133 \cdot a_\gamma^5 + 0.425 \cdot a_\gamma^4 - 0.932 \cdot a_\gamma^3 + 1.05 \cdot a_\gamma^2 - 0.5 \cdot a_\gamma - 0.1$	$-0.033 \cdot a_\gamma^3 + 0.04 \cdot a_\gamma^2 + 0.48 \cdot a_\gamma + 0.37$
C	$-0.0579 \cdot a_\gamma^6 + 0.6003 \cdot a_\gamma^5 - 2.314 \cdot a_\gamma^4 + 3.911 \cdot a_\gamma^3 - 2.355 \cdot a_\gamma^2 + 0.02 \cdot a_\gamma + 0.3$	$0.02 \cdot a_\gamma^2 - 0.14 \cdot a_\gamma + 1.25$	$0.032 \cdot a_\gamma^3 - 0.092 \cdot a_\gamma^2 + 0.06 \cdot a_\gamma + 0.8$

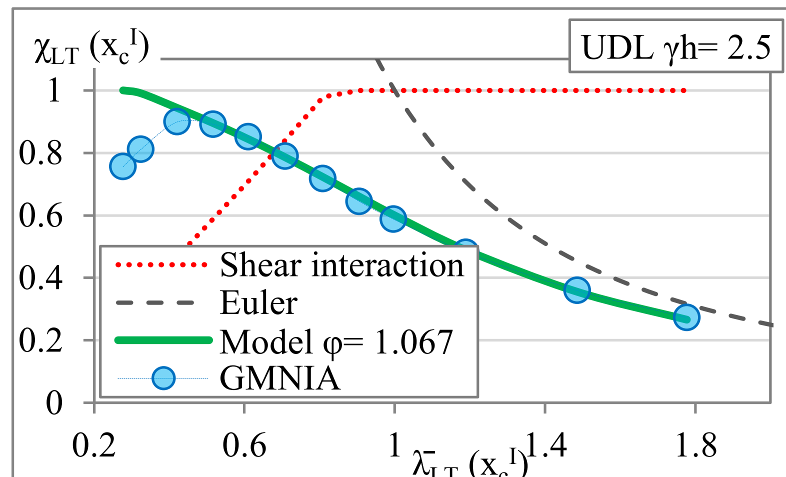
Design resistance of tapered columns and beams

BEAMS – DESIGN METHODOLOGY

- ❑ Results



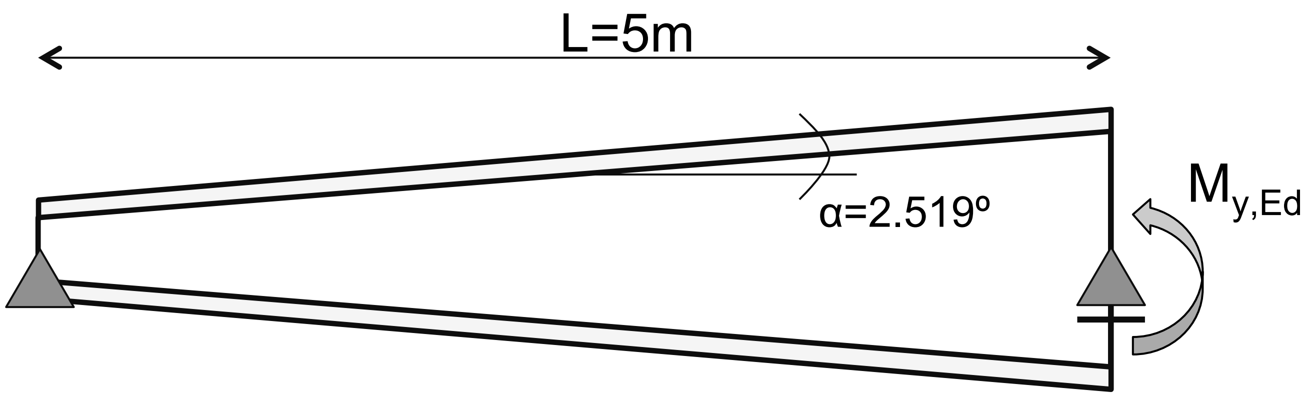
- ❑ Influence of shear!



- ❑ Possible problems:
  - ❑ Web buckling

Design resistance of tapered columns and beams

BEAMS - EXAMPLE



$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
$M_{y,Ed}$ = 300 kNm
L = 5 m

$c_w/t_w < 124\varepsilon \rightarrow$ Class1

Flange class: $c_f/t_f=(206/2-15/2)/25=3.82 < 9\varepsilon \rightarrow$ Class1

- Flange thickness in vertical plan

$$t_f' = t_f / \cos(\alpha) = 25.02mm$$

h_{min} = 220.05 mm
h_{max} = 660.05 mm

γ_h= 660.05/220.05=3

- V_{Ed}=300/5 kN = 60 kN $V_{pl,Rd,min}$=170x15x10^{-6} f_y=599.25 kN > 2V_{Ed}

$h_{w,max}/t_w=610/15=41<72\varepsilon$

Design resistance of tapered columns and beams

BEAMS - EXAMPLE

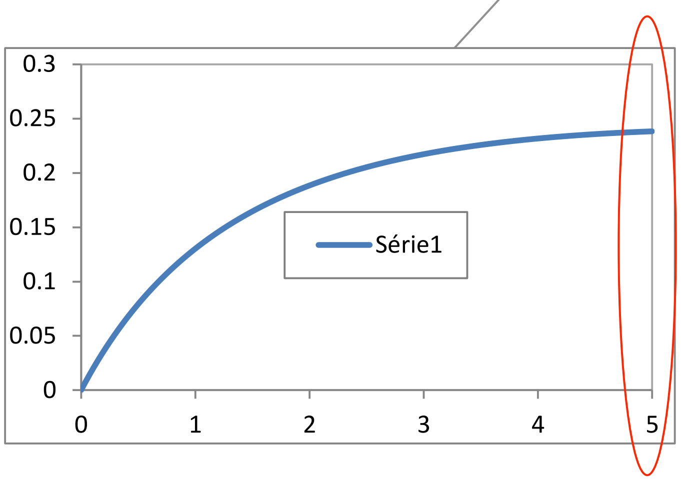
- Elastic critical moment, numerical analysis

$$M_{cr,Tap} = 2044.85\,kNm$$

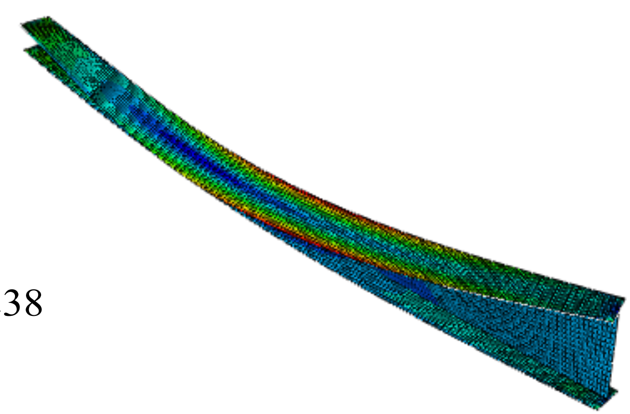
$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
$M_{y,Ed}$ = 300 kNm
L = 5 m

$$\varepsilon_M(x) = \frac{M_{y,Ed}(x)}{M_{y,Rd}(x)}$$

- $x = x_c^I = 5m$



$$\varepsilon_M(L) = 0.238$$



Design resistance of tapered columns and beams

BEAMS - EXAMPLE

- Calculation of slenderness at $x=x_c^I$

$$\bar{\lambda}(x_c^I) = \sqrt{\frac{\alpha_{ult,k}(x_c^I)}{\alpha_{cr}}} = \sqrt{\frac{1097.19/300}{2044.85/300}} = 0.733$$

- Second order failure location, $x=x_{c,lim}^{II}$

$$\gamma_w = \frac{W_{y,el,\max}}{W_{y,el,\min}} = \frac{4010.7mm^3}{951.7mm^3} = 4.214$$

$$\gamma_h = \ldots = 3$$

$$\psi = 0$$

$$x_{c,\lim}^{II}/L = \left(0.75 - 0.18\psi - 0.07\psi^2\right) + \left(0.025\psi^2 - 0.006\psi - 0.06\right)\left(\gamma_h - 1\right) = 0.63 \geq 0$$

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
$M_{y,Ed}$ = 300 kNm
L = 5 m

Design resistance of tapered columns and beams

BEAMS - EXAMPLE

- Over-strength factor, φ

$$\gamma_w = \frac{W_{y,el,\max}}{W_{y,el,\min}} = \frac{4010.7mm^3}{951.7mm^3} = 4.214$$

$$\gamma_h = \ldots = 3$$

$$\psi = 0$$

$$a_\gamma = -0.0005 \cdot (\gamma_w - 1)^4 + 0.009 \cdot (\gamma_w - 1)^3 - 0.077 \cdot (\gamma_w - 1)^2 + 0.78 \cdot (\gamma_w - 1) = 1.957$$

$$\psi = 0 \leq |\psi_{\lim}| \quad \forall \psi_{\lim} \rightarrow \begin{cases} A = \ldots = -1.097 \\ B = \ldots = -0.503 \\ C = \ldots = 1.053 \end{cases}$$

$$\longrightarrow \quad \varphi = A \cdot \psi^2 + B \cdot \psi + C \quad = 1.053 \geq 1$$

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
$M_{y,Ed}$ = 300 kNm
L = 5 m

- Determination of imperfection, η

$$\eta_{LT} = \alpha_{LT}\left(\bar{\lambda}_{LT}(x_c^{II}) - 0.2\right) \leq \sqrt{\frac{W_{y,el}(x_{c,\lim}^{II})}{W_{z,el}(x_{c,\lim}^{II})}}\left(0.12\psi^2 - 0.23\psi + 0.35\right)$$

$$\alpha_{LT} = 0.21\sqrt{\frac{W_{y,el}(x_{c,\lim}^{II})}{W_{z,el}(x_{c,\lim}^{II})}} \leq 0.64$$

$$\longrightarrow$$

$$\alpha_{LT} = 0.587$$

$$\bar{\lambda}_{LT}(x_c^{II}) = \sqrt{\frac{A(x_c^{II}) \cdot f_y}{\pi^2 EI(x_c^{II})/L^2}} = 1.150$$

$$\eta_{LT} = 0.557 < 0.978$$

Design resistance of tapered columns and beams

BEAMS - EXAMPLE

- Reduction factor

$$\phi_{LT} = 0.5 \times \left(1 + \varphi \times \eta \times \frac{\bar{\lambda}^2_{LT}(x_c^I)}{\bar{\lambda}^2_z(x^{II}_{c,\lim})} + \varphi \times \bar{\lambda}^2_{LT}(x_c^I) \right) = 0.901$$

$$\chi_{LT}(x_c^I) = \frac{\varphi}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \varphi \times \bar{\lambda}_{LT}^2(x_c^I)}} = 0.752 \leq 1$$

$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
$M_{y,Ed}$ = 300 kNm
L = 5 m

- Verification

$$M_{b,Rd} = \chi(x_c^I) M_{y,Rd}(x_c^I) = 0.751 \times 1097.19 = 825.45\ kN > M_{y,Ed}(x_c^I) = 300\ kNm$$

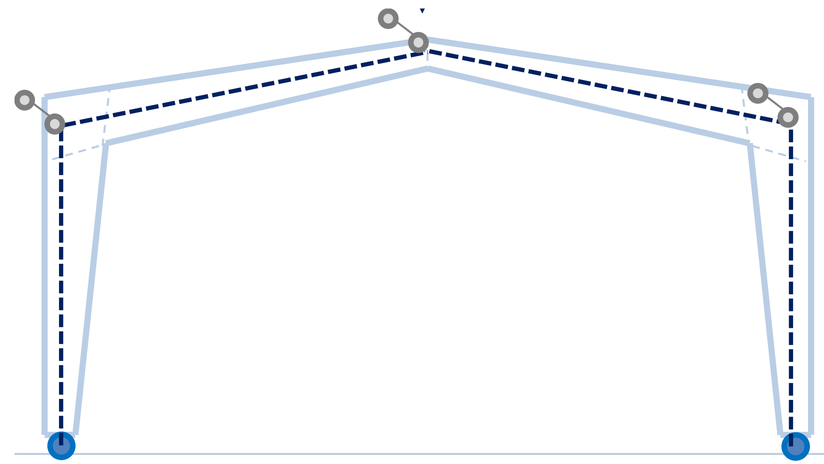
- Numerical analysis, GMNIA

$$M_{b,Rd} = 853.95\ kN \rightarrow 3.3\%\ diff$$

Beam-columns

Beam-columns

- How to solve the problem



	2nd order effects P-Δ + Global imperfections	2nd order effects P-δ + Local imperfections		Material nonlinearity	Check	Buckling length
	ϕ	$e_{0,y}$	$e_{0,z}$			
0	YES	YES	YES	YES	Max. Load factor (≡ GMMIA)	-
1	YES	YES	YES	NO	Cross section	-
2a	YES	YES	NO	NO	Out-of-plane Member stability procedures	L
2b	YES	NO	NO	NO	Member stability procedures	Global Lcr,z
3	NO	NO	NO	NO	Member stability procedures	Global Lcr,z Lcr,y

Beam-columns

❑ Difficulties for each approach

	2nd order effects P-Δ + Global imperfections	2nd order effects P-δ + Local imperfections		Material nonlinearity	Check	Buckling length
	ϕ	$e_{0,y}$	$e_{0,z}$			
0	YES	YES	YES	YES	Max. Load factor (≡ GMMIA)	-
1	YES	YES	YES	NO	Cross section	-
2a	YES	YES	NO	NO	Out-of-plane Member stability procedures	L
2b	YES	NO	NO	NO	Member stability procedures	L
3	NO	NO	NO	NO	Member stability procedures	Global Lcr,z Lcr,y

? ? ?

Buckling curve acc. to EC3-1-1, Table 6.1	Elastic analysis	Plastic analysis
	e_0/L	e_0/L
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

→ Need to calibrate e0/L acc. to new imperfection factors for welded members!

Beam-columns

- Difficulties for each approach

	2^{nd} order effects P-Δ + Global imperfections	2^{nd} order effects P-δ + Local imperfections		Material nonlinearity	Check	Buckling length
	ϕ	$e_{0,y}$	$e_{0,z}$			
0	YES	YES	YES	YES	Max. Load factor (≡ GMMIA)	-
1	YES	YES	YES	NO	Cross section	-
2a	YES	YES	NO	NO	Out-of-plane Member stability procedures	L
2b	YES	NO	NO	NO	Member stability procedures	L
3	NO	NO	NO	NO	Member stability procedures	Global Lcr,z Lcr,y

Calibrate e0/L for new imperfection factors

Need to develop
→In-plane
→Out-of-plane

Approaches:
→Interaction
→Generalized slenderness

Beam-columns

- Difficulties for each approach

	2^{nd} order effects P-Δ + Global imperfections	2^{nd} order effects P-δ + Local imperfections		Material nonlinearity	Check	Buckling length
	ϕ	$e_{0,y}$	$e_{0,z}$			
0	YES	YES	YES	YES	Max. Load factor (≡ GMMIA)	-
1	YES	YES	YES	NO	Cross section	-
2a	YES	YES	NO	NO	Out-of-plane Member stability procedures	L
2b	YES	NO	NO	NO	Member stability procedures	L
3	NO	NO	NO	NO	Member stability procedures	Global Lcr,z Lcr,y

- Focus on approach 2a:

→Develop out-of-plane verification procedure;

→ To account for LTB in the in-plane stability reduce Mpl by χ_{LT} in the cross section check

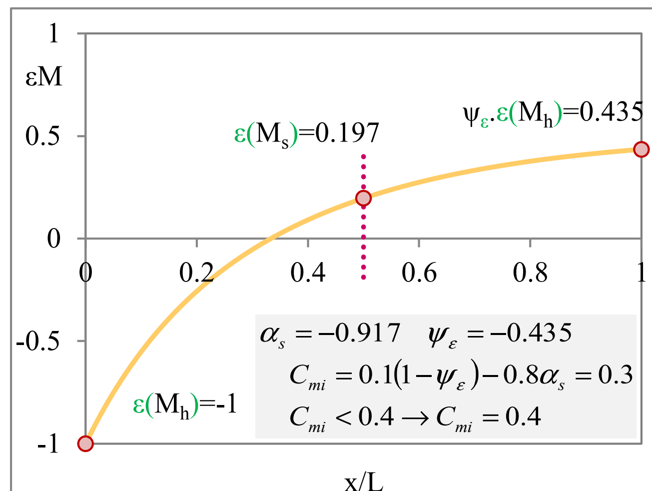
Beam-columns

- Adaptation of the Interaction formula

$$\frac{N_{Ed}(x^I_{c,N})}{\chi_z(x^I_{c,N})\,N_{Rk}(x^I_{c,N})/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}(x^I_{c,M})}{\chi_{LT}(x^I_{c,M})M_{y,Rk}(x^I_{c,M})/\gamma_{M1}} \leq 1.0$$

  - k_{zy} factor determined from Method 2 (Annex B)
    - Because yy and zz modes are clearly separated

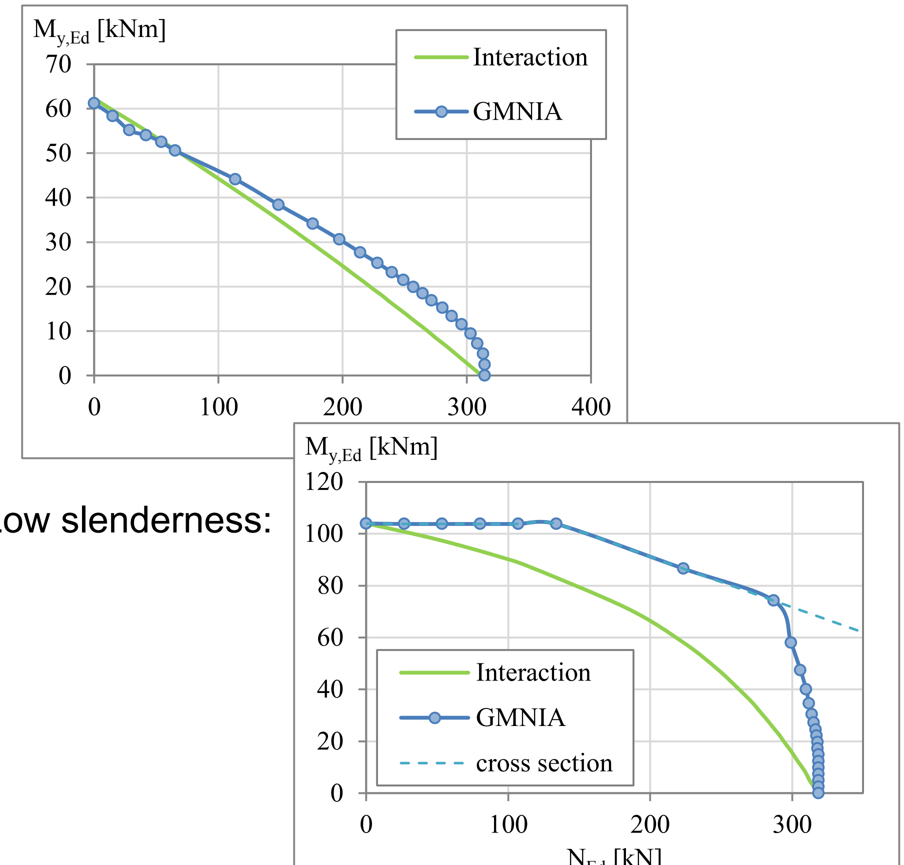
  - Cmy factors determined with
    MEMBER UTILIZATON
    $(M_y/M_{y,Rk})$

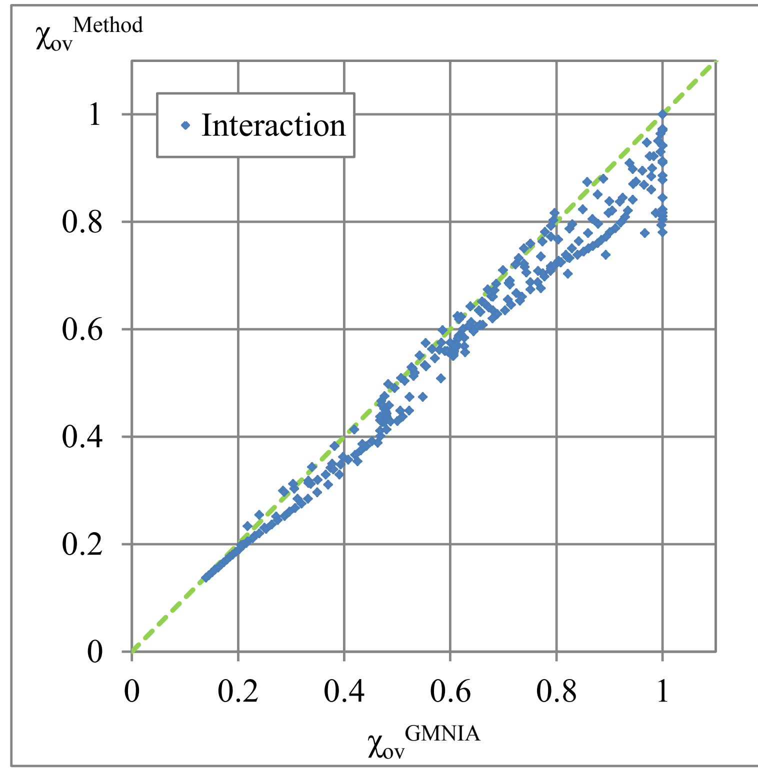


Beam-columns

- Adaptation of the Interaction formula
  - Results

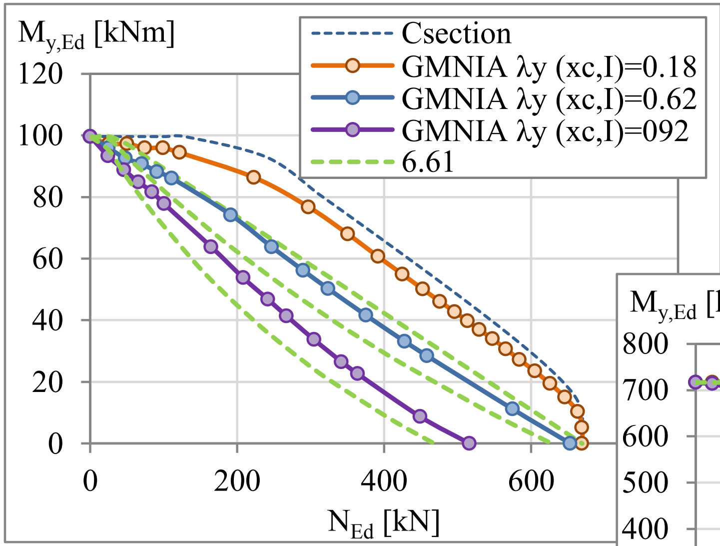
Low slenderness:

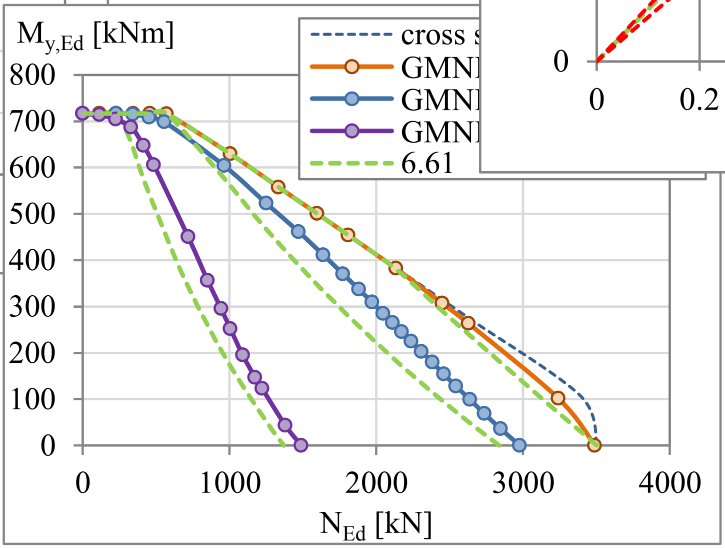


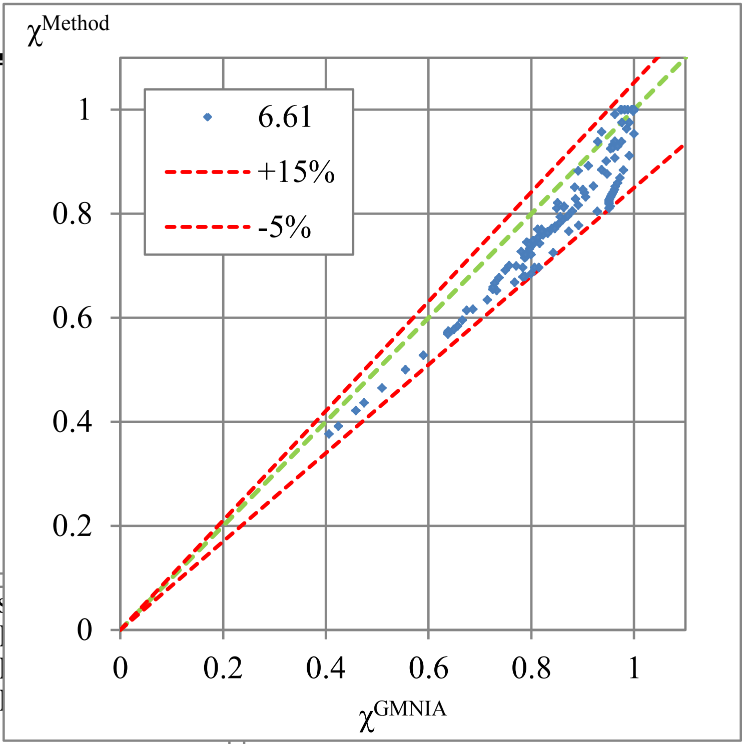


Beam-columns

- Adaptation of the Interaction formula
  - In-plane failure mode



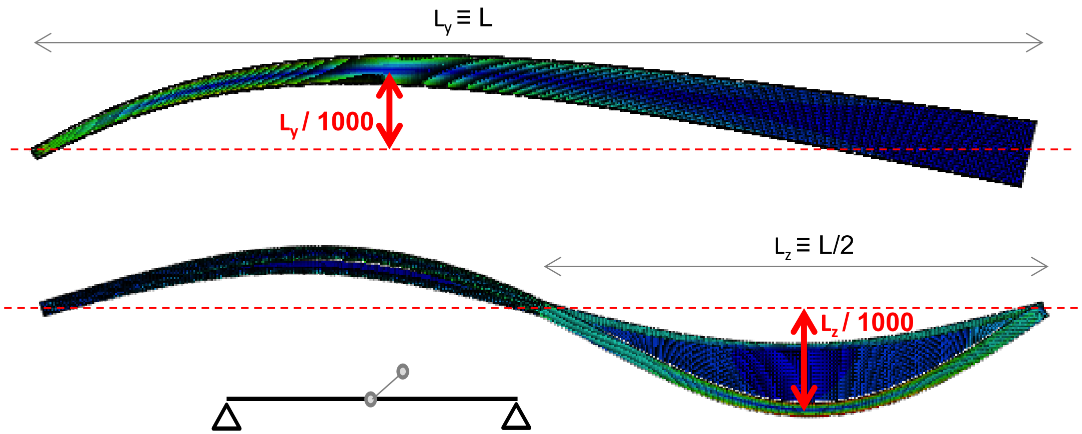


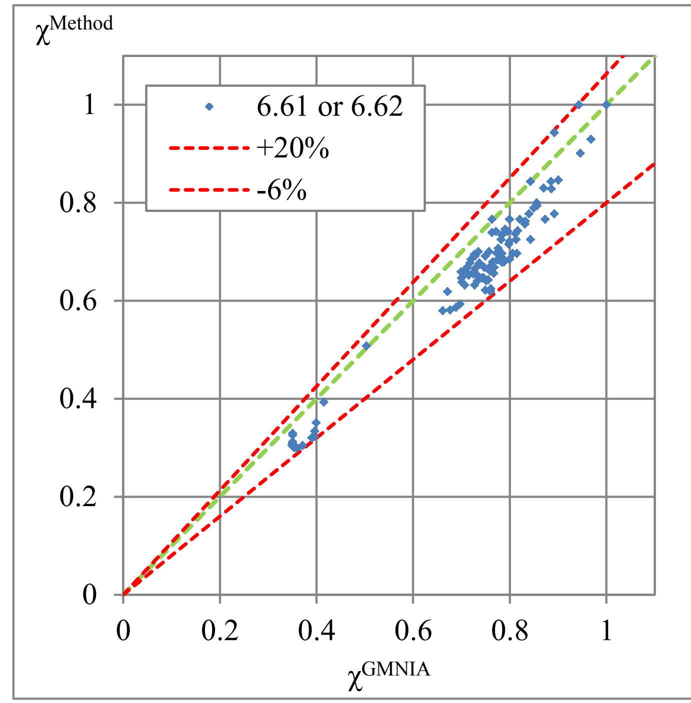


Also up to 20% conservative

Beam-columns

- ❑ Adaptation of the Interaction formula
  - ❑ In-plane and out-of-plane failure mode

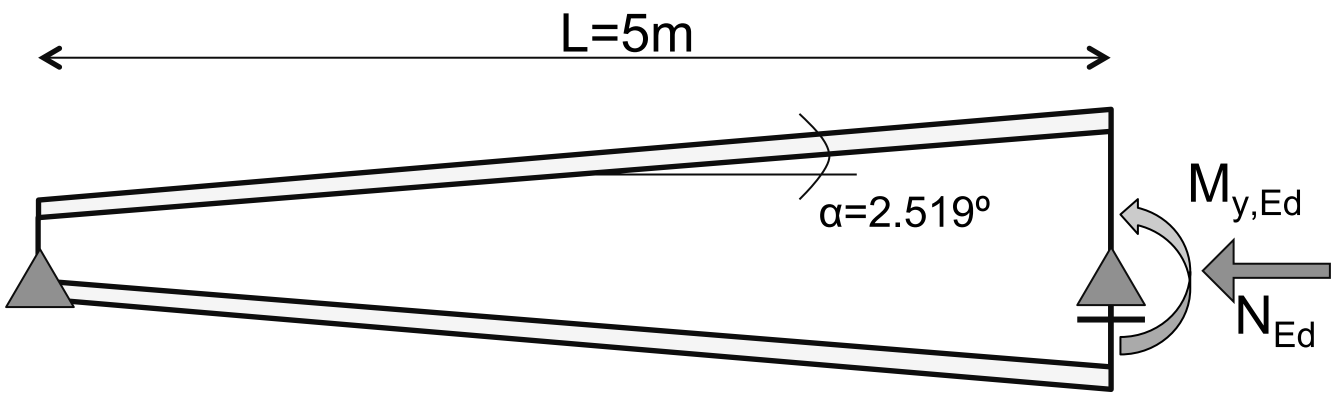




Also up to 20% conservative

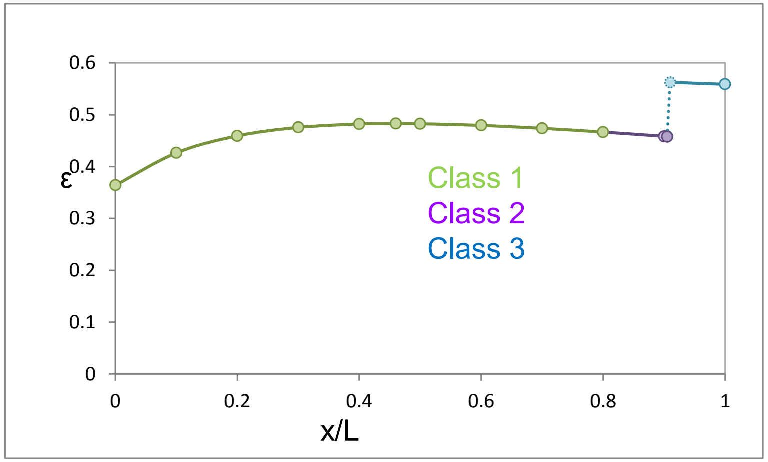
Beam-columns

BEAM-COLUMNS - EXAMPLE



$h_{w,min}$ = 170 mm
$h_{w,max}$ = 610 mm
b = 206 mm
tf = 25 mm
tw = 15 mm
S235
N_{Ed} = 1100 kN
$M_{y,Ed}$ = 300 kNm
L = 5 m

- Member class



Beam-columns

BEAM-COLUMNS - EXAMPLE

- Interaction formulae

$$\frac{N_{Ed}(x^I_{c,N})}{\chi_y(x^I_{c,N})N_{Rk}(x^I_{c,N})/\gamma_{M1}} + k_{yy}\frac{M_{y,Ed}(x^I_{c,M})}{\chi_{LT}(x^I_{c,M})M_{y,Rk}(x^I_{c,M})/\gamma_{M1}} \leq 1.0$$

$$\frac{N_{Ed}(x^I_{c,N})}{\chi_z(x^I_{c,N})N_{Rk}(x^I_{c,N})/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}(x^I_{c,M})}{\chi_{LT}(x^I_{c,M})M_{y,Rk}(x^I_{c,M})/\gamma_{M1}} \leq 1.0$$

- Data:

$$N_{Ed}(x^I_{c,N}) = 1100\ kN$$

$$M_{y,Ed}(x^I_{c,M}) = 300\ kNm$$

$$N_{Rk}(x^I_{c,N}) = N_{Rk}(0) = 3022.1\ kN$$

$$M_{y,Rk}(x^I_{c,M}) = M_{y,el}(5) = 942.53\ kNm$$

$$\gamma_{M1} = 1$$

$$\chi_z(x^I_{c,N}) = \chi_z(0) = 0.634$$

$$\chi_y(x^I_{c,N}) = \chi_y(0) = 1$$

$$\chi_{LT}(x^I_{c,M}) = \chi_y(5) = 0.752$$

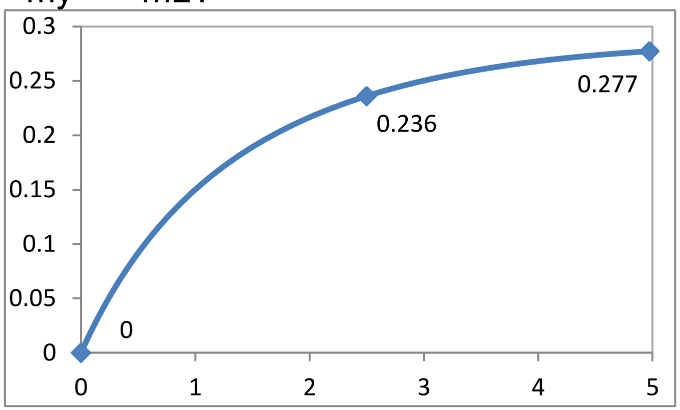
$$k_{yy} = ?$$

$$k_{zy} = ?$$

Beam-columns

BEAM-COLUMNS - EXAMPLE

- C_{my}, C_{mLT}



$$\psi_{el} = 0$$

$$\alpha_s = \varepsilon_{el}(M_s)/\varepsilon_{el}(M_h) = 0.236/0.277 = 0.8507$$

$$\Rightarrow C_{my} = C_{mLT} = 0.2 + 0.8 \times \alpha_s = 0.881$$

$$k_{yy} = C_{my} \times \left(1 + \underbrace{\left(0.6\sqrt{\varphi_y}\,\bar{\lambda}_y(x^I_{c,N})\right)}_{\le 0.6} \frac{N_{Ed}(x^I_{c,N})}{\chi_y(x^I_{c,N})\,N_{Rk}(x^I_{c,N})/\gamma_{M1}} \right) = 0.881 \times \left(1 + \underbrace{\left(0.6 \times \sqrt{1.128} \times 0.299\right)}_{\le 0.6} \frac{1100}{3022.1} \right) = 0.942$$

$$k_{zy} = 1 - \frac{\overbrace{0.05\sqrt{\varphi_z}\,\bar{\lambda}_z(x^I_{c,N})}^{\le 0.05}}{C_{m,LT} - 0.25} \frac{N_{Ed}(x^1_{c,N})}{\chi_z(x^1_{c,N})\,N_{Rk}(x^1_{c,N})/\gamma_{M1}} = 1 - \frac{\overbrace{0.05\sqrt{1.222} \times 1}^{\le 0.05}}{0.881 - 0.25} \frac{1100}{0.634 \times 3022.1} = 0.955$$

Beam-columns

BEAM-COLUMNS - EXAMPLE

- Verification

$$N_{Ed}(x^I_{c,N}) = 1100\ kN$$

$$M_{y,Ed}(x^I_{c,M}) = 300\ kNm$$

$$\chi_z(x^I_{c,N}) = \chi_z(0) = 0.634$$

$$\chi_y(x^I_{c,N}) = \chi_y(0) = 1$$

$$\chi_{LT}(x^I_{c,M}) = \chi_y(5) = 0.752$$

$$N_{Rk}(x^I_{c,N}) = N_{Rk}(0) = 3022.1\ kN$$

$$M_{y,Rk}(x^I_{c,M}) = M_{y,el}(5) = 942.53\ kNm$$

$$\gamma_{M1} = 1$$

$$k_{yy} = 0.942$$

$$k_{zy} = 0.955$$

→

$$\frac{N_{Ed}(x^I_{c,N})}{\chi_y(x^I_{c,N})\,N_{Rk}(x^I_{c,N})/\gamma_{M1}} + k_{yy}\frac{M_{y,Ed}(x^I_{c,M})}{\chi_{LT}(x^I_{c,M})M_{y,Rk}(x^I_{c,M})/\gamma_{M1}} \le 1.0$$

$$0.701 \le 1.0$$

$$\frac{N_{Ed}(x^I_{c,N})}{\chi_z(x^I_{c,N})\,N_{Rk}(x^I_{c,N})/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}(x^I_{c,M})}{\chi_{LT}(x^I_{c,M})M_{y,Rk}(x^I_{c,M})/\gamma_{M1}} \le 1.0$$

$$0.905 \le 1.0$$

→ LF ≈ 1/0.905=1.105

- Numerical analysis, GMNIA

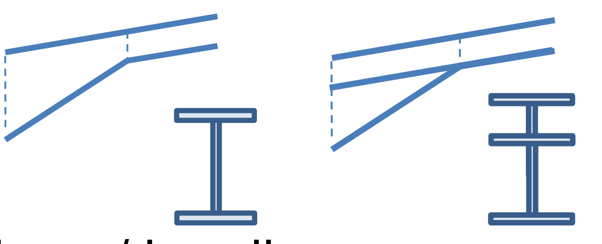
NEd,max=1337.75 kN
My,Ed,max= 364.84 kNm → LF = 1.216

$$\rightarrow 9.1\%\ diff$$

Research challenges

Research challenges

- Generalization of the over-strength factors to any shape of cross-section / loading
  - Utilization of the compressed flange
- Calibrate equivalent imperfections e_0/L for new imperfection factors and also for other types of cross section:



- Properly account for local buckling due to shear / bending
- Development of direct reduction factor approach for beam-columns
- Other boundary conditions
- Partial restraints
- …

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SOFTWARE

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