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Advanced Income Capitalization

Course Handbook

PC510CH--I

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Lecture 4.
DCF and Yield Capitalization Using an Overall Yield Rate

I. Concept of yield capitalization

A. Conversion of future benefits into present value by applying appropriate yield rate (See Session 1 for complete definition.)

1. Typical investor's anticipated yields reflected in yield rates for market value

2. Individual investor's requirements reflected in yield rate for investment value

B. General steps

1. Selection of holding period

2. Forecast of all future cash flows or cash flow patterns

3. Selection of appropriate yield or discount rate

4. Conversion of future benefits into present value

a. Discounting each annual future benefit

b. Developing overall rate that reflects income pattern, value change, and yield rate

C. Cash flow and income

1. In DCF analysis, cash flow (CF) refers to *the periodic income attributable to the interests in real property*. (*The Appraisal of Real Estate*, p. 520)

2. Each cash flow discounted to present value; total of all present values equals the total value of income to the real property interest being appraised.

3. Expected resale proceeds (reversion) forecast and discounted at end of projection period.

D. Discounted cash flows may be net operating income *(I_O)* to entire property or cash flows to specific interests:

1. Pre-tax cash flow *(PTCF)* to equity interest (equity dividend)
2. After-tax cash flow *(ATCF)* to equity interest
3. Debt service for mortgage interest

II. General DCF formula

A. Expression of yield capitalization

$$PV = \frac{CF_1}{(1+Y)^1} + \frac{CF_2}{(1+Y)^2} + \frac{CF_3}{(1+Y)^3} + \ldots + \frac{CF_n}{(1+Y)^n}$$

1. PV = present value
2. CF = cash flow for period specified
3. Y = appropriate periodic yield, or discount rate
4. n = number of periods in the projection

B. Separate discounting of each payment and adding of all present values to obtain the present value.

C. Use of formula to estimate

1. Total property value (V_O)
2. Loan value (V_M)
3. Equity value (V_E)
4. Leased fee value (V_{LF})
5. Leasehold value (V_{LH})
6. Other interests

(See *The Appraisal of Real Estate*, pp. 520–23.)

D. Valuation of any series of periodic incomes, with or without a reversion, with the basic formula.

E. DCF synonymous with yield capitalization

1. Terminology. DCF is sometimes used when the cash flows are explicitly projected for each year of the investment holding period and the cash flows (including reversion) are discounted to estimate value.

2. Techniques. Any of the yield capitalization techniques discussed in Session 4 can be thought of as a DCF analysis; all involve estimating a value that represents the discounted present value of estimated future cash flows.

3. In some techniques, assumption of a pattern of change in income and property value over the holding period is applied rather than an explicit projection of cash flow for each year.

F. Estimating property value with DCF analysis

Example 4.1. Estimating property value with DCF

Assume that I_O is level. The resale of the property is estimated to be $2,300,000 at the end of a five-year holding period.

Year	1	2	3	4	5
PGI	$300,000	$300,000	$300,000	$300,000	$300,000
Less vacancy & collection loss	18,000	18,000	18,000	18,000	18,000
EGI	$282,000	$282,000	$282,000	$282,000	$282,000
Less operating expenses	82,000	82,000	82,000	82,000	82,000
I_O	$200,000	$200,000	$200,000	$200,000	$200,000
Net resale proceeds (5 years)					$2,300,000

Estimate the present value (PV) of the property using a property yield rate (Y_O) of 12% to discount the cash flows.

Suggested Solution 4.1. Estimating property value with DCF

$$PV = \frac{CF_1}{(1+Y)^1} + \frac{CF_2}{(1+Y)^2} + \frac{CF_3}{(1+Y)^3} + \ldots + \frac{CF_n}{(1+Y)^n}$$

$$= \frac{\$200{,}000}{1.12} + \frac{\$200{,}000}{(1.12)^2} + \frac{\$200{,}000}{(1.12)^3} + \frac{\$200{,}000}{(1.12)^4}$$

$$+ \frac{\$200{,}000}{(1.12)^5} + \frac{\$2{,}300{,}000}{(1.12)^5}$$

$$= \$2{,}026{,}037$$

Alternatively, the same answer could be obtained by using the present value factors from the tables.

Year	Cash flow		*PV* factor @ 12%		Present Value
1	$ 200,000	×	0.892857	=	$ 178,571
2	$ 200,000	×	0.797194	=	159,439
3	$ 200,000	×	0.711780	=	142,356
4	$ 200,000	×	0.635518	=	127,104
5	$ 200,000	×	0.567427	=	113,485
5*	$2,300,000	×	0.567427	=	1,305,082
Total present value					$2,026,037

*Resale

Shortcut Method

$ 200,000 (cash flow) × 3.604776 (*PV* annuity at 12%) = $ 720,955
$2,300,000 (reversion) × 0.567427 (*PV* factor at 12%) = 1,305,082
$2,026,037

G. DCFs with uneven cash flows

Example 4.2. DCF with uneven cash flows

Assume that incomes and expenses are increasing over time as shown in the following projection of I_O. The resale price is estimated to be $2,300,000. *What is the present value using a property yield rate (Y_O) of 12%?*

Year	**1**	**2**	**3**	**4**	**5**
PGI	$300,000	$312,000	$324,480	$337,459	$350,958
Less vacancy & collection loss	18,000	18,720	19,469	20,248	21,057
EGI	$282,000	$293,280	$305,011	$317,211	$329,901
Less operating expenses	82,000	85,024	88,183	94,580	98,021
I_O	$200,000	$208,256	$216,828	$222,631	$231,880
Resale price (5 years)					$2,300,000

Suggested Solution 4.2. DCF with uneven cash flows

Year	**Cash Flow**		***PV* Factor @ 12%**		**Present Value**
1	$ 200,000	×	0.892857	=	$ 178,571
2	$ 208,256	×	0.797194	=	166,020
3	$ 216,828	×	0.711780	=	154,334
4	$ 222,631	×	0.635518	=	141,486
5	$ 231,880	×	0.567427	=	131,575
5*	$2,300,000	×	0.567427	=	1,305,082
Total present value					$2,077,068

* Resale

The present value estimate is $2,077,068, which implies an overall capitalization rate of 9.63% ($200,000/$2,077,068).

H. Terminal capitalization rate (R_N)—*The rate used to convert income, e.g., NOI, cash flow, into an indication of the anticipated value of the subject real property at the end of an actual or anticipated holding period. The terminal capitalization rate is used to estimate the resale value of the property. Also called* reversionary capitalization rate *or* going-out capitalization rate.

1. Considerations and uses

    a. Overall capitalization rate a buyer might use to value property when it is sold at end of current owner's holding period

    b. I_O is for first year of ownership of *next* owner (one year *after* the end of the current owner's holding period)

c. Overall capitalization rate (R_O) sometimes referred to as *going-in capitalization rate* to distinguish it from terminal capitalization rate

d. Income capitalized with terminal capitalization rate reflects

   1) Expectation of changes in market rates over first owner's holding period

   2) Number of original leases still in effect

e. Estimated resale price unaffected by any below- or above-market rent because, ideally, all leases renewed at estimated market rental rate before end of holding period

Example 4.3. Resale using R_N, implied R_O

Given the following cash flow summary including information on resale and terminal capitalization rate, what is the present value and implied R_O?

Estimated Year 6 I_O (I_N) = $240,000 (based on separate analysis)

Gross resale	= I_O (I_N) (Year 6)/R_N
	= $240,000/0.10
	= $2,400,000
Net resale	= gross resale – costs of sale @ 3%
	= $2,400,000 – $72,000
	= $2,328,000

Year	Cash Flow		*PV* Factor @ 12%		Present Value
1	$ 200,000	×	0.892857	=	$ 178,571
2	$ 208,256	×	0.797194	=	166,020
3	$ 216,828	×	0.711780	=	154,334
4	$ 222,631	×	0.635518	=	141,486
5	$ 231,880	×	0.567427	=	131,575
5*	$2,328,000	×	0.567427	=	1,320,970
Total present value					$2,092,956

* Resale price

Suggested Solution 4.3. Resale using R_N, implied R_O

The present value of the total property is $2,092,956, which implies an R_O of 9.56% ($200,000/$2,092,956).

2. Relationship between going-in and terminal capitalization rate

   a. Comparison of implied overall capitalization rate (9.56% in Example 4.3) with terminal capitalization rate (10%) to determine whether proper relationship is implied.

   b. Higher rate due to risk

      1) Difficulty in accurately forecasting income for year (n + 1)

      2) Older, more depreciated buildings

      3) Higher if less growth in I_O

      4) Lower if more growth in I_O

Example 4.4. Implied change in value

Refer to Example 4.3. *Calculate the implied change in value (Δ) over the holding period and compare it with the change implied in the* I_O.

Suggested Solution 4.4. Implied change in value

The I_O increased from $200,000 to $240,000, which is a total of 20%, over the five years from Year 1 to Year 6. Over the same five years value would increase from $2,092,956 to $2,400,000, which is a total of about 14.7%. The appraiser should ask whether a 20% increase in net operating income and a 14.7% increase in property value would reflect the expectations of a typical investor. If not, the appraiser may need to alter the input assumptions.

I. Percentage change in value

1. Sometimes useful to solve for value, assuming resale will change by a certain percentage (annual or total) over holding period

2. Dollar amount of resale price not known until value is found

   a. The resale price depends on the present value that is being calculated.

   b. At the same time, present value depends on the resale price! Attempt to solve for present value presents a mathematical problem.

   c. Solutions

      1) Algebra

      2) Yield capitalization formulas—Developed to eliminate the algebra by solving for an overall capitalization rate that gives the same answer as the algebra.

      3) Computer—Can be programmed to solve for the present value while allowing the resale price to depend on the present value. Electronic spreadsheets also can be used to solve the problem.

Example 4.5. Solving present value by algebra

Assumptions:

I_O (level)	\$200,000
Holding period	5 years
Δ_O	15%
Y_O	12%

Calculate the PV. From the PV calculate the implied overall capitalization rate. Prove that the answer is correct by calculating an IRR for the implied cash flows.

Suggested Solution 4.5. Solving present value by algebra

Calculation of value and reversion value

A property discount rate (Y_O) of 12% results in the following present value estimate (*PV*).

Year	Cash Flow		*PV* Factor @ 12%		Present Value
1	$200,000	×	0.892857	=	$178,571
2	200,000	×	0.797194	=	159,439
3	200,000	×	0.711780	=	142,356
4	200,000	×	0.635518	=	127,104
5	200,000	×	0.567427	=	113,485
5*	($V + 0.15V$)	×	0.567427	=	$\underline{0.652541V}$
Total present value					$720,955 + $0.652541V$

* Resale

$$
\begin{aligned}
V &= \$720{,}955 + 0.652541\,V \\
0.347459\,V &= \$720{,}955 \\
PV &= \$2{,}074{,}936
\end{aligned}
$$

Calculation of overall capitalization rate

$$
\begin{aligned}
R_O &= I_O/V_O \\
&= \$200{,}000/\$2{,}074{,}936 \\
&= 0.0964
\end{aligned}
$$

Proof

That the internal rate of return for the implied cash flows is equal to the property discount rate (Y_O) proves that the answer is correct. The implied cash flows are shown below.

The implied resale price is estimated as follows:

$$\begin{aligned}\text{Resale price} &= V_O \times (1 + \Delta_O)\\ &= (\$2{,}074{,}936 \times 1.15)\\ &= \$2{,}386{,}176\end{aligned}$$

The resale price and the assumptions about the I_O provide

Proof

Year	**Cash Flow**
0	($2,074,936)
1	200,000
2	200,000
3	200,000
4	200,000
5	200,000
5*	2,386,176

IRR = 12.00%

* Resale

Alternatively, the cash flows for Years 1 to 5 (including the reversion) could have been discounted at a yield rate of 12% to prove algebraically that the present value is $2,074,936.

III. Yield capitalization formulas

A. Considerations and uses

1. Developed to solve DCF problem

2. Developed to reflect specific assumptions on how property value and income are expected to change over the investment period

3. Used to calculate an overall capitalization rate

4. This capitalization rate then divided into first year I_O to solve for value

B. Differences between direct capitalization and yield capitalization

1. In direct capitalization, R_O is derived directly from market data without considering the expected rate of return on capital or the means of recapture.

2. In yield capitalization, R_O must be determined by taking into account the required rate of return on capital and the timing of recapture as reflected in the projected income pattern.

3. Yield capitalization is still market-oriented because yield rates should be based on what typical investors require in the market.

4. Overall capitalization rate (R_O) differs from the overall yield rate (Y_O)

   a. Overall rate does not explicitly account for the impact of changes in income and property value on the yield rate earned over the entire investment-holding period.

   b. Overall yield rates can be higher or lower than overall capitalization rates because of the change in income and property value.

C. General yield capitalization formula

$$\boldsymbol{R_O = Y_O - A}$$

1. Yield capitalization formulas express the overall capitalization rate in terms of the overall yield rate (Y_O) and an adjustment factor (A) that accounts for the way that changes in income and property value cause the overall yield rate to differ from the overall capitalization rate.

2. If the adjustment for increase is treated as a positive change and the adjustment for decrease is treated as a negative change, the overall capitalization rate (R_O) can be expressed in terms of the overall yield rate (Y_O) and an adjustment rate (A) with a simple formula.

3. Increasing income or property value

   a. Overall yield rate higher than overall capitalization rate

   b. Capitalization rate equals total yield rate minus an adjustment for expected growth.

4. Decreasing income or property value

   a. Overall yield rate lower than overall capitalization rate

   b. Overall capitalization rate equals yield rate plus adjustment for expected loss

5. Adjustment (*A*)

   a. The adjustment (*A*) is often expressed as the *rate* of change in the overall property value. The symbol Δ_O is used to represent the *total* percentage change in the overall property value.

      Note. The Greek letter delta (Δ) is used to represent total change and the subscript (*O*) indicates that the change is for the entire, or overall, property.

   b. To calculate *A*, multiply Δ_O by an annualizer to convert the total change in value into an annual rate of change.

6. With the conversion factor *a*, the general formula for R_O is expressed

   $$R_O = Y_O - \Delta_O a$$

   where

   R_O is the overall capitalization rate

   Y_O is the overall yield rate

   Δ_O is the total percentage change in the overall value during the holding period

   a is the annualizer.

7. When the appropriate capitalization rate has been determined, obtain an indication of property value V_O by dividing the first year income (I_O) by the overall capitalization rate (R_O).

   $\boldsymbol{V_O = I_O/R_O}$

8. Advantages of yield capitalization formulas

   a. Provides convenient way of solving for value when using DCF problems that require algebra

   b. Includes specific assumptions about projected change in income and property value

   c. Clearly shows conceptual relationship between overall capitalization rates and overall yield rates

      1) If direct capitalization is used to obtain overall capitalization rates from comparable sales, we should understand what this rate implies about the expectations for changes in property value that would result in an overall yield rate that makes sense.

      2) Overall capitalization rates can differ significantly for properties that have different potential for changes in income and/or property value even if investors require the same overall yield rate for each property.

9. Obtaining estimates of change

   a. Analyzing sales using rate extraction techniques

   b. Confirmation of sales

   c. Interviews with actual and potential buyers

IV. Yield capitalization formula for level income and a percentage change in value

A. Level-income formula

1. Assumes level income but property value will change by a specified overall change (Δ_O)

2. Conversion factor *a* is a sinking fund factor ($1/S_{\overline{n}|}$). Thus the yield capitalization formula becomes

$$\boldsymbol{R_O = Y_O - \Delta_O\, 1/S_{\overline{n}|}}$$

where

$1/S_{\overline{n}|}$ is calculated using the overall yield rate (Y_O) as the interest rate and investment holding period (n) as the number of periods. The sinking fund factor can also be found in the financial tables.

Example 4.6. Yield capitalization formula

Refer to Example 4.5. The property was assumed to generate a stable I_O of $200,000 per year for the next five years. Expected total property appreciation is 15% during this five-year period. The appraiser is asked to value the property to yield 12%. *Find the same answer using the yield capitalization formula.*

Suggested Solution 4.6. Yield capitalization formula

To solve this problem, use the formula $R_O = Y_O - \Delta_O 1/S_{\overline{n}|}$.

$1/S_{\overline{n}|}$ is the sinking fund factor for 12.00%, five years. According to the tables or a financial calculator, the sinking fund factor is 0.157410, so R_O is calculated

$$\begin{aligned} R_O &= Y_O - \Delta_O\, 1/S_{\overline{n}|} \\ &= 0.1200 - (0.15)(0.157410) \\ &= 0.0964 \end{aligned}$$

$$\begin{aligned} \text{Value} &= I_O/R_O \\ &= \$200{,}000/0.0964 \\ &= \$2{,}074{,}947 \end{aligned}$$

Proof

Year	Cash Flow
0	($2,074,947)
1	200,000
2	200,000
3	200,000
4	200,000
5	200,000
5*	$2,386,189

* Resale

The *IRR* for the above cash flows is 12.00%, which is the assumed yield rate. Thus the value estimate is correct.

Note. R_O is less than Y_O because return on capital will come from the increase in property value at resale.

B. Comparison of techniques

Example 4.7. Comparison of techniques—Algebra and yield capitalization formula solutions

A property is projected to have level I_O of $10,000 per year for five years. Property values are expected to increase 20% total over the five years. *Using a discount rate of 10%, what is the value?*

Suggested Solution 4.7. Comparison of techniques—Algebra and yield capitalization formula solutions

Solution using algebra with present value factors

$$\begin{aligned} V &= (I_O \times A_{\overline{n}|}) + [(1 + \Delta_O) \times V \times 1/S^n] \\ &= (10{,}000 \times 3.7908) + (1.2 \times V \times 0.620921) \\ &= \$148{,}720 \end{aligned}$$

Solution using yield capitalization formula

$$R_O = Y_O - \Delta_O\ 1/S_{\overline{n}|}$$

Y_O = 10%, Δ_O = 20%, and $1/S_{\overline{n}|}$ is 0.163797

$$\begin{aligned} R_O &= 0.10 - (0.20 \times 0.163797) \\ &= 0.067241 \end{aligned}$$

$$\begin{aligned} V_O &= I_O / R_O \\ &= \$10{,}000/0.067241 \\ &= \$148{,}720 \end{aligned}$$

V. Perpetuity—Income stream that is expected to be level forever (see *The Appraisal of Real Estate*, pp. 529–30).

A. Considerations and uses

1. Equivalent mathematically and conceptually to assumption that income and value remain constant over a finite holding period

2. According to the general formula

$R_O = Y_O - \Delta_O a$, the overall capitalization rate (R_O) becomes the overall yield rate (Y_O) when there is no change in value because Δ_O equals zero.

B. No change in value in perpetuity

Example 4.8. No change in value

Given:

I_O (level)	\$200,000
Holding period	5 years
Y_O	12.00%

Calculate the property value.

Suggested Solution 4.8. No change in value

$$R_O = Y_O$$
$$= 0.1200$$
$$V_O = I_O/R_O$$
$$= \$200{,}000/0.12$$
$$= \$1{,}666{,}667$$

Proof

Year	Cash Flow
0	(\$1,666,667)
1	200,000
2	200,000
3	200,000
4	200,000
5 (including reversion)	\$1,866,667

IRR = 12.00%

VI. Yield capitalization formula for changing income, straight-line premise—Assumes I_O changes by same dollar amount each year (See *The Appraisal of Real Estate*, p. 526.)

A. The formula is

$$R_O = Y_O - \Delta_O \, 1/n$$

where

n is the holding period

Δ_O is total change in value over holding period

$1/n$ is the appropriate annualizer (a)

$\Delta_O \, 1/n$ gives the percentage change in value for the first year.

The changes in income and value are related but not equal.

B. Straight-line premise

Example 4.9. Changing income–Straight line

Assume that the first year I_O is \$16,000 and that it changes on a straight-line basis. The property value is estimated to decrease a total of 20% over a five-year holding period.

What is the value using a 12% discount rate?

Suggested Solution 4.9. Changing income—Straight line

$$
\begin{aligned}
R_O &= Y_O - \Delta_O \, 1/n \\
\Delta_O \, 1/n &= -0.20/5 = -0.04 \\
R_O &= 0.12 - (-0.04) = 0.1600 \\
V_O &= \$16{,}000/0.1600 = \$100{,}000
\end{aligned}
$$

Example 4.10. Calculating dollar amount of income change

Refer to Example 4.9. *What is the implied change in I_O in Example 4.9?*

Suggested Solution 4.10. Calculating dollar amount of income change

The change in I_O must be found by the following formula:

Periodic change in income ($\$\Delta_I$) = $V_O \times \Delta_O\ 1/n \times Y_O$

$$\begin{aligned} \$\Delta_I &= \$100{,}000 \times (-0.04) \times 0.12 \\ &= \$480 \end{aligned}$$

This means that I_O is assumed to decrease by $480 per year.

Example 4.11. Calculating cash flows implied by yield capitalization formula

Refer to Examples 4.9 and 4.10. *What are the implied cash flows each year?*

Suggested Solution 4.11. Calculating cash flows implied by yield capitalization formula

Because it was assumed that value would decrease by 20%, the implied resale price after five years is $100,000 × (1 – 0.20) = $80,000.

Note. This could not be determined until *after* the value of $100,000 is found. Using the above I_O and estimated resale, the cash flows for 5 years are as follows:

Year	**1**	**2**	**3**	**4**	**5**
Cash flow	$16,000	$15,520	$15,040	$14,560	$94,080

Example 4.12. Proof of estimate from yield capitalization formula

Prove your answer in Example 4.11 using DCF analysis.

Suggested Solution 4.12. Proof of estimate from yield capitalization formula

The present value of these cash flows at 12% is $100,000, which proves the answer checks out using DCF analysis.

VII. Yield capitalization formula for changing income, exponential curve (constant ratio)—Assumes that I_O and property value are changing by a constant ratio; i.e., the same annual compounded rate. (See *The Appraisal of Real Estate*, pp. 526, 530.)

A. Considerations and uses

1. This results in an exponential growth pattern for the I_O.

2. When both income and value change at a constant ratio, the capitalization rate can be determined without tables using the general formula

$$R_O = Y_O - a$$

3. Where *a* can be replaced with the periodic compound rate of change (*CR*), the formula then becomes

$$R_O = Y_O - CR$$

where Y_O is the yield rate per period and *CR* is the rate of change per period.

Note. The annualizer (*a*) was not necessary because *CR* is already the annual change in value. In effect, "$\Delta_O\ a$" is replaced by *CR*.

4. Implicitly the going-in rate and the terminal capitalization rate are the same.

B. Changing income

Example 4.13. Changing income–Exponential curve

Assume that the initial I_O is $200,000. The I_O and property value are expected to increase by 3% per year. *What is the value of the property using a 12% discount rate? Prove your answer using DCF analysis.*

Suggested Solution 4.13. Changing income—Exponential curve

$$\begin{aligned} R_O &= Y_O - CR \\ &= 0.1200 - 0.03 \\ &= 0.0900 \end{aligned}$$

$$\begin{aligned} V_O &= I_O/R_O \\ &= \$200{,}000/0.0900 \\ &= \$2{,}222{,}222 \end{aligned}$$

Proof

Year	Cash Flow
0	($2,222,222)
1	200,000
2	206,000
3	212,180
4	218,545
5	$2,801,266

IRR = 12.00%
* $I_{O_5} + [V_O \times (1 + CR)^N]$
= [$225,102 + ($2,222,222)(1.03)^5]
= $2,801,266

Example 4.14. Terminal capitalization rate (R_N) with changing income

Refer to Example 4.13. *Calculate the R_N based on the Year 6 I_O.*

Suggested Solution 4.14. Terminal capitalization rate (R_N) with changing income

Note. If the I_O continues to increase 3% per year, it will be $231,855 in Year 6. Using the Year 6 I_O the terminal capitalization rate (R_N) can be calculated as follows:

$$\begin{aligned} R_N &= I_{O_6}/[V_O \times (1 + CR)^{N}] \\ &= \$231{,}855/\$2{,}576{,}164 \\ &= 9.00\% \end{aligned}$$

This is exactly the same as the going-in capitalization rate that was calculated in Example 4.13.

Note. This is *not* a coincidence. This yield capitalization formula implicitly assumes that the capitalization rate will be constant through time; therefore, the terminal capitalization rate will always equal the going-in capitalization rate.

Example 4.15. Changing income—Exponential

Assume an income-producing property is expected to produce a I_O of \$50,000 for the first year. Then both I_O and value are expected to grow at a constant ratio of 2% per year. *What is the value of the property using an 11% property yield rate?*

Suggested Solution 4.15. Changing income—Exponential

$$
\begin{aligned}
R_O &= Y_O - CR \\
&= 0.1100 - 0.02 \\
&= 0.0900
\end{aligned}
$$

$$
\begin{aligned}
V_O &= I_O/R_O \\
&= \$50{,}000/0.0900 \\
&= \$555{,}556
\end{aligned}
$$

Example 4.16. Changing income—Exponential

Assume that the initial I_O is $10,000. The property value is expected to increase 4% per year over a five-year holding period. Income will also increase 4% per year.

Example 4.16a.

What is the value assuming a discount rate of 14%?

Suggested Solution 4.16a.

R_O = $Y_O - CR$
= 0.1400 – 0.04 = 0.1000

V_O = I_O/R_O
= $10,000/0.1000
= $100,000

Proof

I_O is as follows:

Year	1	2	3	4	5
I_O	$10,000	$10,400	$10,816	$11,249	$11,699

The resale price is

$V_O \times (1 + CR)^N$
$= \$100,000 \times (1.04)^5$
$= \$121,665$

The present value of income and resale discounted at 14% is $100,000.

Example 4.16b.

What is the terminal capitalization rate?

Suggested Solution 4.16b

The I_O in Year 6 would be $12,166.50

The terminal capitalization rate is 10.00%, the same as the going-in capitalization rate.

VIII. Assumptions about recapture of capital

A. The yield capitalization formulas (based on compound interest concepts) automatically provide for the appropriate allocation of I_O to return on and return of capital.

B. Appraisers normally are not concerned with what investors do with the return *of* capital portion of the I_O versus the return *on* capital portion.

1. Useful to examine assumptions about reinvestment of return of capital each year

2. Alternative assumption about reinvestment rate of return of capital

C. Specific assumptions about reinvestment of return of capital are made when yield capitalization formulas are applied.

D. Inwood premise

1. Applies to stream of income that is a level ordinary annuity.

2. Present value of a stream of income is based on a single discount rate.

3. Assumes that income will be sufficient to return all investment capital and pay specified return.

4. Inwood capitalization rate

a. Reciprocal of ordinary level annuity (present value of $1 per period) factor found in financial tables

b. Constructed by adding interest rate to a sinking fund factor based on same interest rate and duration of income stream

5. Premise consistent with use of compound interest tables to calculate the present value of the income stream

6. Present value of expected reversion or any other benefit not included in income stream must be added to obtain total present value of the investment (See *The Appraisal of Real Estate*, pp. 526–28).

Example 4.17. Inwood premise

Assume that I_O is \$10,000 per year for five years. *What is the value assuming an overall yield rate (Y_O) of 10% under the Inwood Premise?*

Suggested Solution 4.17. Inwood premise

Financial tables

Use the present value of \$1 per period (ordinary annuity) factor

$\$10,000 \times 3.79079 = \$37,908$

Yield capitalization formula

The property model is used. Since income is level the formula is

$$R_O = Y_O - \Delta_O \, 1/S_{\overline{n}|}$$

Because there is no reversion, the property will lose 100% of its value. This means that Δ_O is –1.0. Thus the yield capitalization formula becomes

$$R_O = Y_O + 1/S_{\overline{n}|}$$

which is the Inwood premise.

Substituting the assumptions from the Example 4.17:

$$\begin{aligned} R_O &= 0.10 + 0.163797 \\ &= 0.263797 \end{aligned}$$

Using this overall rate, the value is estimated as follows:

$$\begin{aligned} V_O &= \$10,000/0.263797 \\ &= \$37,908 \end{aligned}$$

Note. The sinking fund factor ($1/S_{\overline{n}|}$) is based on a 10% discount rate. This implies that a portion of the I_O could be reinvested at 10% to replace the investment. Therefore, Y_O represents return on capital and $1/S_{\overline{n}|}$ represents return of capital.

Example 4.18. Inwood premise—Return on capital and return of capital

Refer to Example 4.17. *Show that the Inwood premise is equivalent to the following*

4.18a. *A constant return on capital each year*

4.18b. *Return of capital reinvested in a sinking fund at the yield rate (Y_O) which can be used to replace the asset at the end of its economic life*

Suggested Solution 4.18. Inwood Premise—Return on capital and return of capital

I_O	\$10,000.00
Less return on capital (10% of \$37,908)	3,790.80
Return of capital	\$ 6,209.20

If the return of capital (\$6,209.20) is placed in a sinking fund earning 10%, the fund will accumulate to \$37,908 after five years, which is the exact amount necessary to replace the investment.

E. Hoskold premise

1. Employs two separate interest rates

    a. Speculative rate, representing a fair rate of return on capital commensurate with risks

    b. Safe rate for a sinking fund designed to return all invested capital in lump sum at termination of investment

2. Stream of income of limited duration and is sufficient to

    a. Pay a fair return on capital at speculative rate

    b. Contribute necessary installments to a conservative, minimum-risk sinking fund (See *The Appraisal of Real Estate*, pp. 691–92.)

3. Assumes that the portion of I_O necessary to replace investment (capital recovery or return of capital) is reinvested at a "safe rate" lower than the yield rate (Y_O) used to value the rest of the I_O.

4. Designed for situations where the property value would decrease to zero over a holding period (as does the Inwood premise)

Example 4.19. Hoskold premise

Refer to Example 4.17. Assume that a portion of the I_O has to be set aside at a 5% safe rate to replace the investment every five years. All other assumptions remain the same. *What is the value of the property?*

Suggested Solution 4.19. Hoskold premise

Use the yield capitalization formula from the Inwood premise except the sinking fund factor should be based on the safe rate of 5% rather than the yield rate of 10%. Calculate the overall rate as follows:

$$
\begin{aligned}
R_O &= Y_O + 1/S_{\overline{n}|} \\
&= 0.10 + 0.180975 \\
&= 0.280975
\end{aligned}
$$

Because the sinking fund factor ($1/S_{\overline{n}|}$) is calculated at a 5% rate rather than the 10% rate used in Example 4.17, the capitalization rate is higher and the value is lower. The value is now

$$
\begin{aligned}
V_O &= \$10{,}000/0.280975 \\
&= \$35{,}590
\end{aligned}
$$

The value is lower because a portion of I_O had to be set aside at 5% to accumulate $35,590 after five years to replace the investment.

I_O	$10,000
Less return on capital (10% of $35,590)	3,559
Return of capital	$ 6,441

The future value of $6,441 at 5% for five years is $35,590 (rounded), which replaces the investment.

IX. Land and building residual techniques with yield capitalization

A. Other uses of yield capitalization formula

1. Land capitalization rate

$\boldsymbol{R_L = Y_L - \Delta_L\, a}$

2. Building capitalization rate

$\boldsymbol{R_B = Y_B - \Delta_B\, a}$

Notes. In this case the change in value (Δ) is for the interest being valued, e.g., the change in land value Δ_L or the change in building value Δ_B.

The annualizer, *a*, is the sinking fund factor for the holding period at the Y_O, if income is level.

3. Advantage of formula: allows for different assumptions about changes in land value versus changes in building value

4. Yield rate (*Y*)

a. Typically assumed to be same for each.

b. Differences in risk might warrant a different yield rate for the land versus that of the building.

B. Application of other income patterns to development of land and building capitalization rates

1. Level income

2. Straight-line premise

3. Exponential growth in income and value

Example 4.20. Building residual with yield capitalization formula

Assume that I_O is level at \$200,000, the land value is constant, and the building becomes worthless at the end of its economic life.

V_L	=	\$450,000
Y_O	=	9.5%
R_L	=	9.5%*
Building life	=	25 Years
$1/S_{\overline{n}\rceil}$	=	0.010959**

* Same as Y_O because the change in land value is assumed to be zero (like a perpetuity).

** Sinking fund factor for 25 years at a 9.5% internal rate.

$$\boldsymbol{R_B = Y_O + \text{Recapture rate } (1/S_{n\rceil}) = 0.095 + 0.010959 = 0.1060}$$

Because the holding period is the entire economic life of the property and building value is zero at the end of the holding period, the change in building value, Δ_B, is – 1.

Therefore,

$$R_B = Y_O + 1/S_{n\rceil}$$

What is the total property value using the building residual technique?

Suggested Solution 4.20. Building residual with yield capitalization formula

I_O	200,000
Less I_L (\$450,000 × 0.095)	42,750
I_B	\$ 157,250

$$V_B = I_B/R_B$$
$$= \$157{,}250/0.1060$$
$$= \$1{,}484{,}051$$

V_B	\$1,484,051
Plus V_L	450,000
Total V_O	\$1,934,051

The assumptions are consistent with the classic building residual technique.

Other assumptions about income pattern and change in land and building value also could be made to develop land and building capitalization rates, e.g., assumption that building income was falling with building value even though land income and land value remained constant.

Example 4.21. Building capitalization rate and building income with straight-line formula

Refer to Example 4.20. *Use the straight-line capitalization formula to develop a building capitalization rate. What does this imply about the change in building income?*

Suggested Solution 4.21. Building capitalization rate and building income with straight-line formula

$$
\begin{aligned}
R_B &= Y_B + 1/n \\
&= 0.095 + 1/25 \\
&= 0.095 + 0.04 \\
&= 0.1350
\end{aligned}
$$

The value of the building would now be

$157,250/0.1350 = $1,164,815.

Note. The building value is now lower than when the assumption was made that the building income was level. This occurs because the assumption now is that building income is declining.

The implied decline in building income is found as follows:

$$
\begin{aligned}
\$\Delta_I &= V_B \times \Delta_B\ 1/n \times Y_{\boldsymbol{O}} \\
&= \$1{,}164{,}815 \times -1.0\ (1/25) \times 0.095 \\
&= -\$4{,}426 \text{ per year.}
\end{aligned}
$$

Comment. These techniques should be used only when they represent a reasonable approximation of the expected pattern of I_O. If it appears unlikely that the building income (I_B) will decrease $4,426 per year, the straight-line capitalization formula should not be used.