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Advanced Methods for Security Constrained Financial Transmission Rights (FTR)

Stephen Elbert, Steve.Elbert@PNNL.gov

Karan Kalsi, Kurt Glaesemann, Mark Rice, Maria Vlachopoulou, Ning Zhou

Pacific Northwest National Laboratory, Richland, WA

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Motivation - Challenges with FTR Calculations

- Financial Transmission Rights (FTR) improve power market operation efficiency by providing financial tool to hedge price risk associated with congestion
- Mitigate incentives for inefficient transmission investment
- FTR auction is formulated as a linear programming optimization problem
- FTR calculations are computationally expensive because
  - Large number of security constraints (**N-1 contingency analysis**)
  - Many FTR variables (**obligatory and optional FTR bids**)
  - Multiple time periods (**security constraints coupled** & no. of constraints increase **exponentially** with no. of categories)
- FTR computation must be finished in time to improve market efficiency

Objectives

- Develop innovative mathematical reformulation of the FTR problem
- Compare multiple solvers for FTR computations
- Developed approaches will be able to
  - Support N-1 Simultaneous Feasibility Test (SFT) e.g. DC contingency analysis
  - Support both optional and obligatory FTR bids
  - Support multi-period FTR calculation (e.g. winter, summer and annual)
- Algorithms designed to solve FTR problem should be parallelizable to support large-scale implementation in a cloud environment

Problem Formulation

- **Power flow constraints**
  - B is (singular) admittance matrix
  - θ_i are the bus voltage angles
  - A is FTR location matrix

$$\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n} \\ B_{21} & B_{22} & \cdots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & \cdots & \cdots & B_{nn} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & \cdots & \cdots & A_{nm} \end{bmatrix} \begin{bmatrix} FTR_1 \\ FTR_2 \\ \vdots \\ FTR_m \end{bmatrix}$$

- **Thermal constraints**
  - C converts voltage angles to line flows
  - L_i are transmission line limits

$$-\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_b \end{bmatrix} \le \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{b1} & \cdots & \cdots & C_{bn} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \le \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_b \end{bmatrix}$$

- **Bid-in constraints**

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \le \begin{bmatrix} FTR_1 \\ FTR_2 \\ \vdots \\ FTR_m \end{bmatrix} \le \begin{bmatrix} FH_1 \\ FH_2 \\ \vdots \\ FH_m \end{bmatrix}$$

- **Combine**

$$-\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_b \end{bmatrix} \le \underbrace{\begin{bmatrix} C_{12} & C_{13} & \cdots & C_{1n} \\ C_{22} & C_{23} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{b2} & \cdots & \cdots & C_{bn} \end{bmatrix} \begin{bmatrix} B_{22} & B_{23} & \cdots & B_{2n} \\ B_{32} & B_{33} & \cdots & B_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n2} & \cdots & \cdots & B_{nn} \end{bmatrix}^{-1} \begin{bmatrix} A_{21} & A_{22} & \cdots & A_{2m} \\ A_{31} & A_{32} & \cdots & A_{3m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & \cdots & \cdots & A_{nm} \end{bmatrix}}_{\boldsymbol{A}\text{ to solve}} \underbrace{\begin{bmatrix} FTR_1 \\ FTR_2 \\ \vdots \\ FTR_m \end{bmatrix}}_{\boldsymbol{x}} \le \underbrace{\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_b \end{bmatrix}}_{\boldsymbol{b}}$$

- ***A*** **dimension is**
  (constraints x bids)

Standard FTR Solvers

- CPLEX (industry standard)
  - Primal simplex; most basic LP solver method
    - Updates tableau containing objective function and constraint information at every iteration
    - Consistently slower on FTR than dual simplex
  - Dual simplex; fastest of the CPLEX methods
    - Similar to primal simplex method, but uses dual formulation of the LP to improve convergence time of optimization
    - Core computation is a linear solve; scales as cube of size
  - Barrier; an interior point method (best for large sparse problems)
    - A primal-dual logarithmic barrier algorithm that generates a sequence of strictly positive primal and dual solutions
    - Fewest iterations but each is more computationally intense

PNNL FTR solver –
Parallel Adaptive Non-linear Dynamical System (NDS)

- Transform LP into coupled set of non-linear dynamical equations
- Dynamical system converges to stable states which are solutions of primal and dual LP problems respectively

Non-linear Dynamical System

Primal

maximize $c^T x$
subject to $Ax \leq b$ and $x \geq 0$

Dual

minimize $b^T y$
subject to $A^T y \geq c$ and $y \geq 0$

$$\frac{dx}{dt} = k_1 \left(c - A^T \left(y + k \frac{dy}{dt} \right) \right)$$

$$\frac{dy}{dt} = k_2 \left(-b + A \left(x + k \frac{dx}{dt} \right) \right)$$

$$k_1 = \frac{K}{i} \qquad i = 1, 2, \ldots M \qquad k_2 = \frac{1}{k_1}$$

- Kernel is a pair of easily parallelized matrix-vector operations: scale as square of problem size (constraints x variables)

Implementation notes

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- Obligatory and Optional bids sorted into separate blocks
  - Obligatory bids may use dense matrix arithmetic
  - Optional bids use sparse matrix arithmetic (up to 50% sparse)
- A matrix segments communicated once at beginning
- A and A^T stored separately to maximize unit stride access
- Global sums for product vectors and distribution of x and y vectors are the only communication after initialization
- Symmetry of Obligatory bids reduces computation by two

Validation test cases

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Cases	Constraints	Bids
1a. WECC 230 single period	5,362	5,790
2a. WECC 230 single period & many bids	5,362	100,000
3a. WECC 230 multi-period	10,724	17,370
4a. WECC 230 multi-period & many bids	10,724	300,000
1b. WECC 100 single period	19,094	22,455
2b. WECC 100 single period & many bids	19,094	100,000
3b. WECC 100 multi-period	38,188	67,365
4b. WECC 100 multi-period & many bids	38,188	300,000

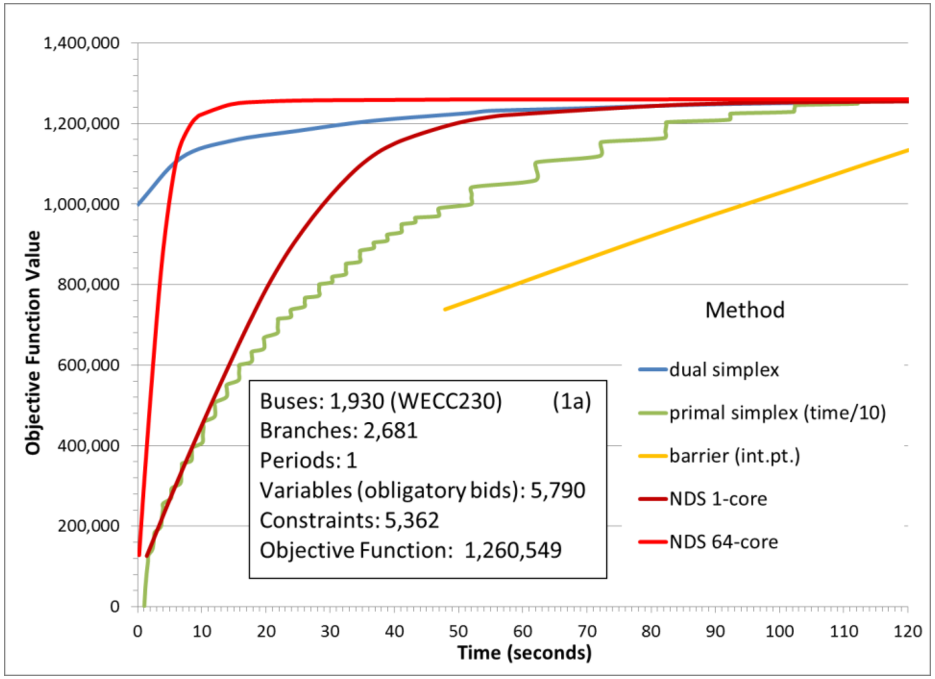
- WECC 230 & 100 model power flow on transmission lines operating at min of
  - 230 kV (1,930 buses and 2,681 branches)
  - 100 kV (7,485 buses and 9,547 branches)

- Multi-period problems have independent periods plus a coupling block

FTR bids	PF (winter)	PF (summer)
Bids (winter)	■	
Bids (summer)		■
Bids (annual,)	■	■

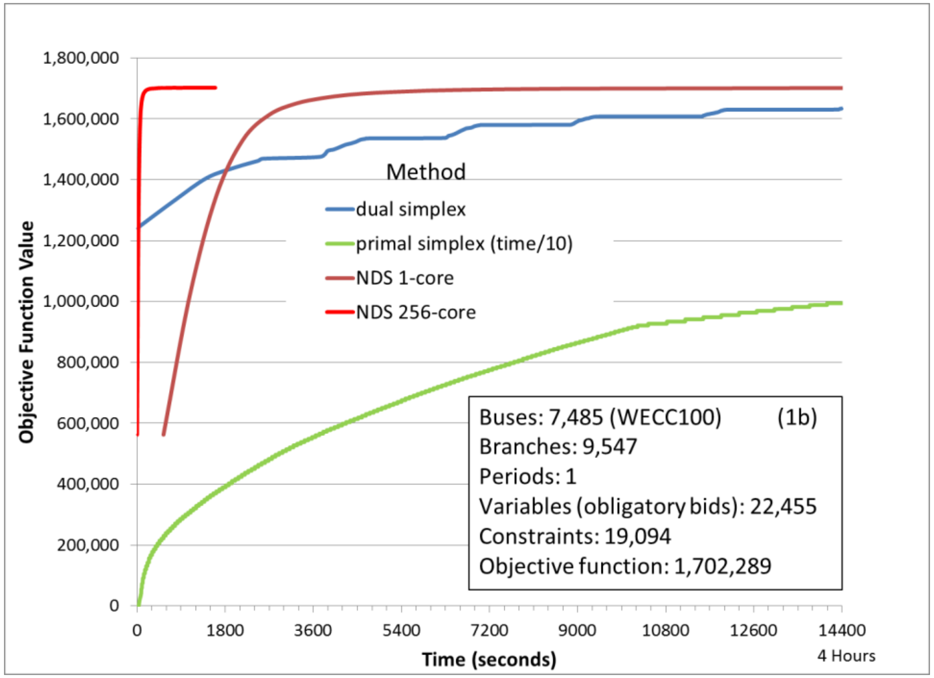
Results – single period cases

- WECC 230



- Primal simplex takes 20 min, dual simplex and serial NDS (1 core) takes 2 minutes
- Parallel NDS is six times faster than dual simplex (CPLEX)

- WECC 100



- At 4 hours, dual simplex not yet converged
- Parallel NDS 46X faster than dual simplex

Results – single period & many bids (100,000)

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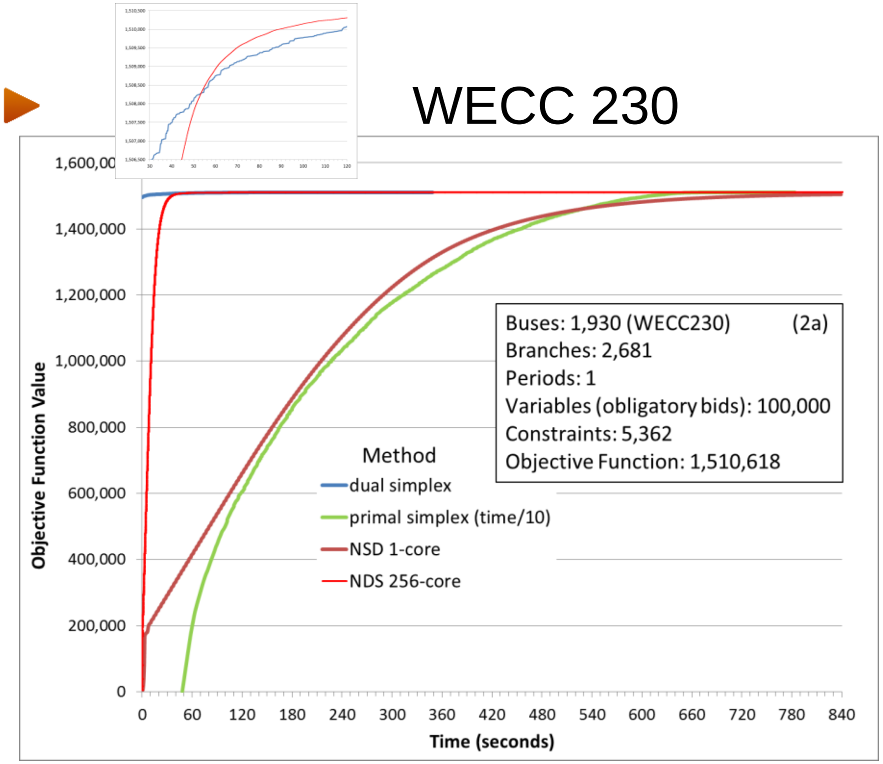
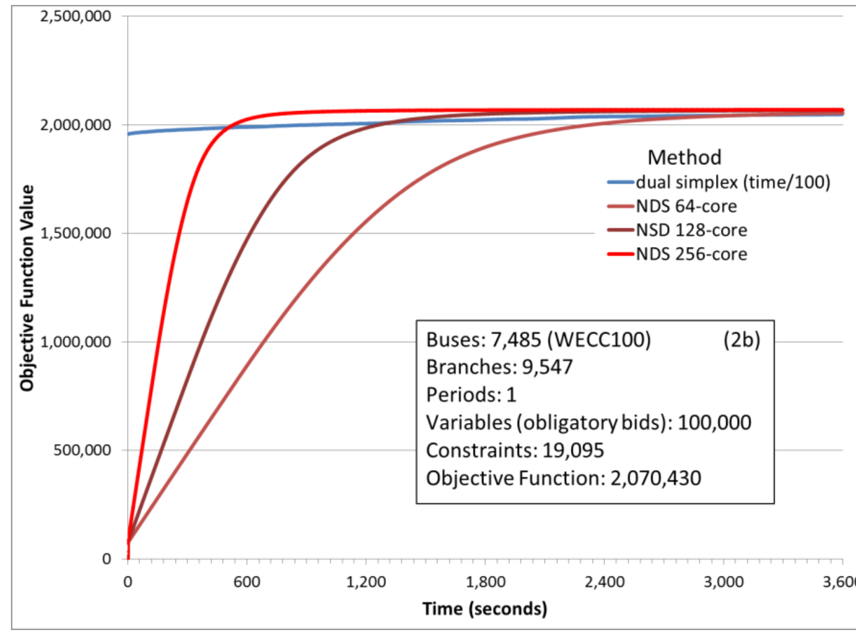
- WECC 230



- NDS 256-core and CPLEX comparable results (cross at 53 seconds)

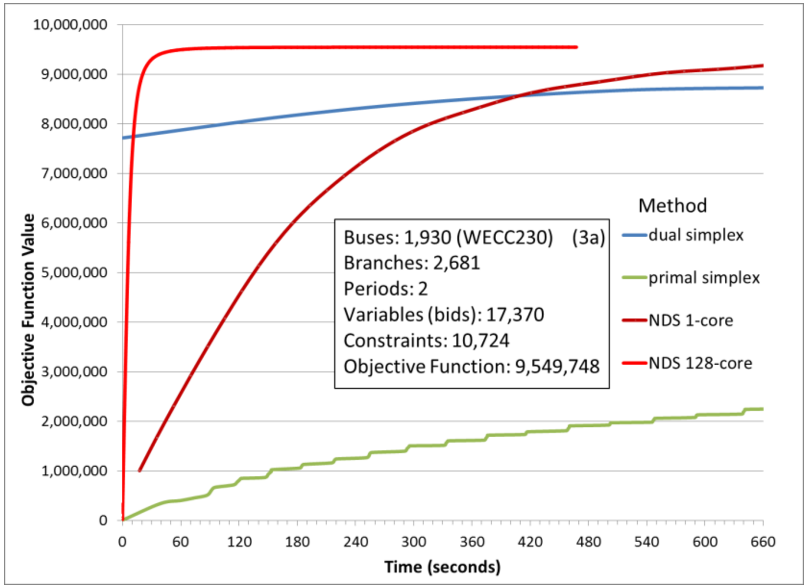
- WECC 100



- NDS 100 times faster when crossing CPLEX curve
- NDS scaling well

Results – two period (summer/winter) cases

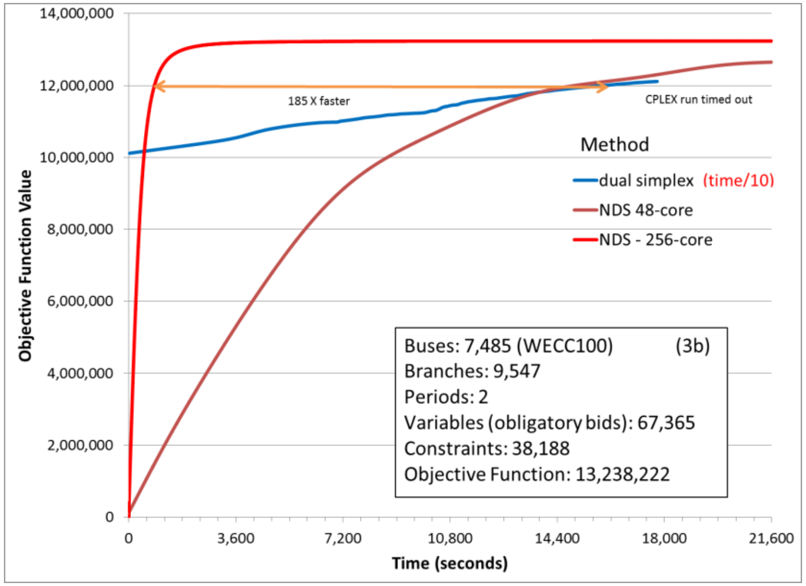
- WECC 230



- Serial NDS is faster than CPLEX
- NDS 128-core is 17 times faster

The bigger the problem,
The faster the relative performance

- WECC 100



- CPLEX no longer practical—time is divided by 10 and not converged
- NDS 256-core is 185 times faster
- 1.7 billion non-zero matrix elem.

Results – two periods & many bids (300,000)

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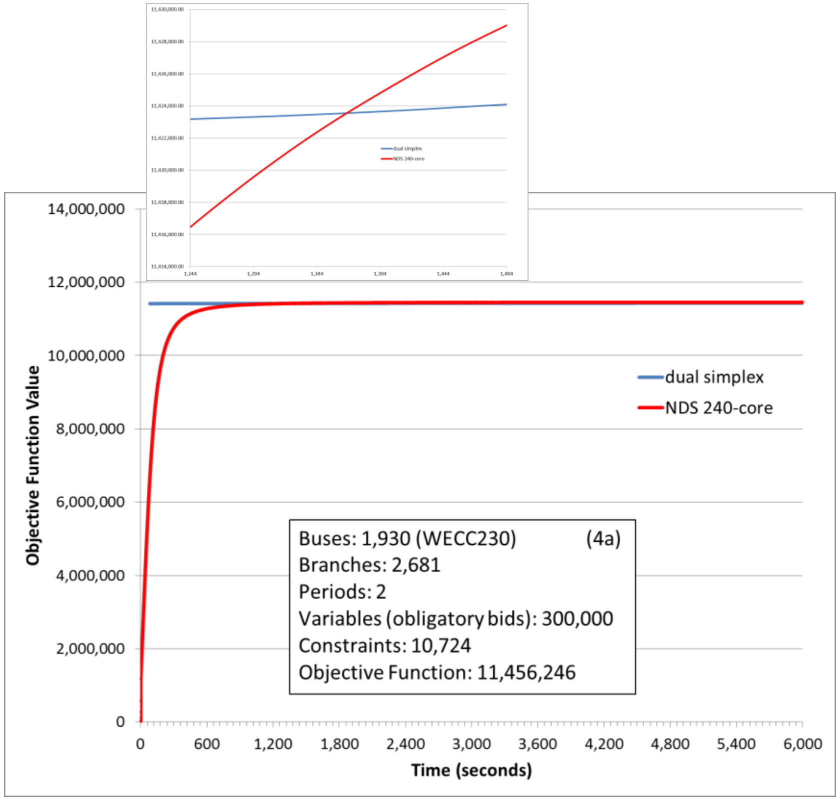
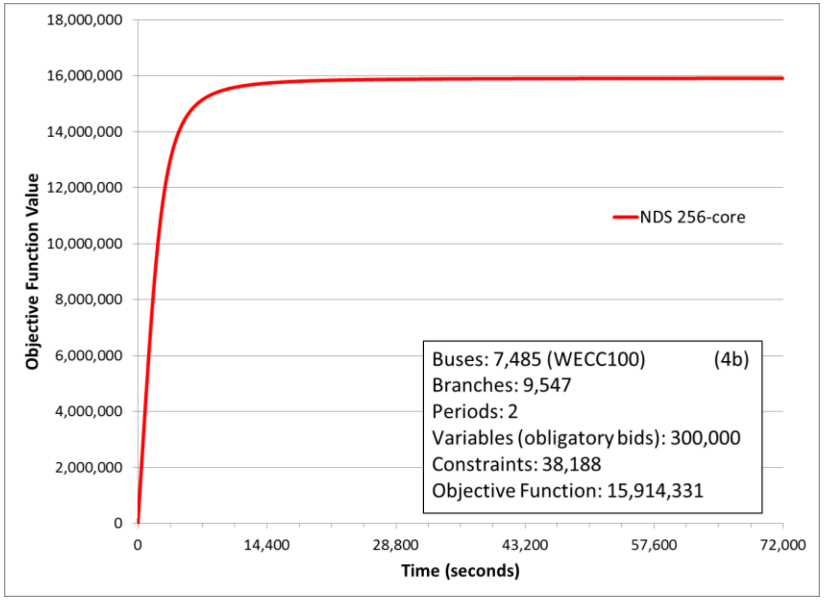
WECC 230



WECC 100



- Additional cores and code improvements → solution in under 4 hours
- 15.3 billion non-zero matrix elem.

Real-world data 1

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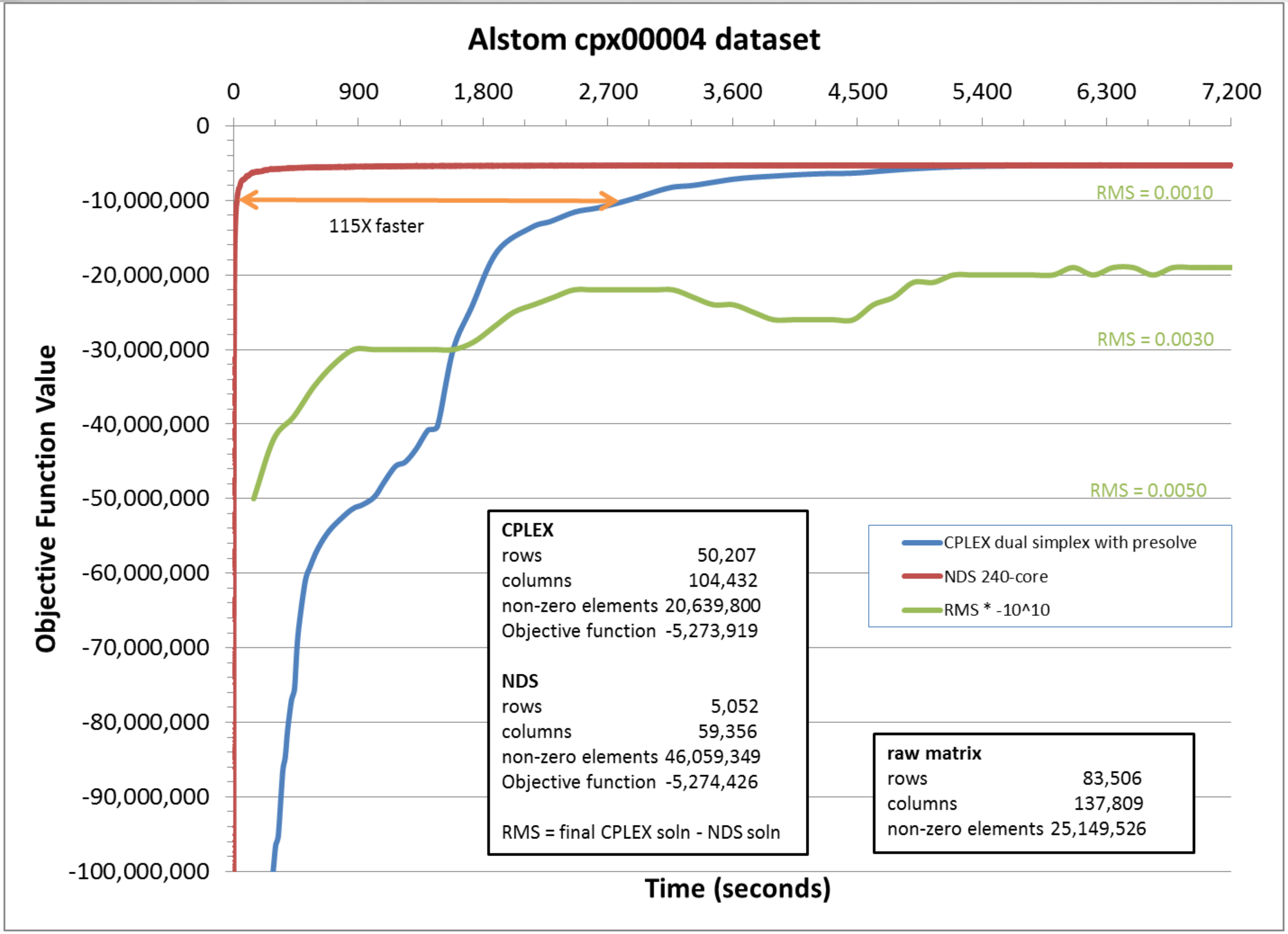
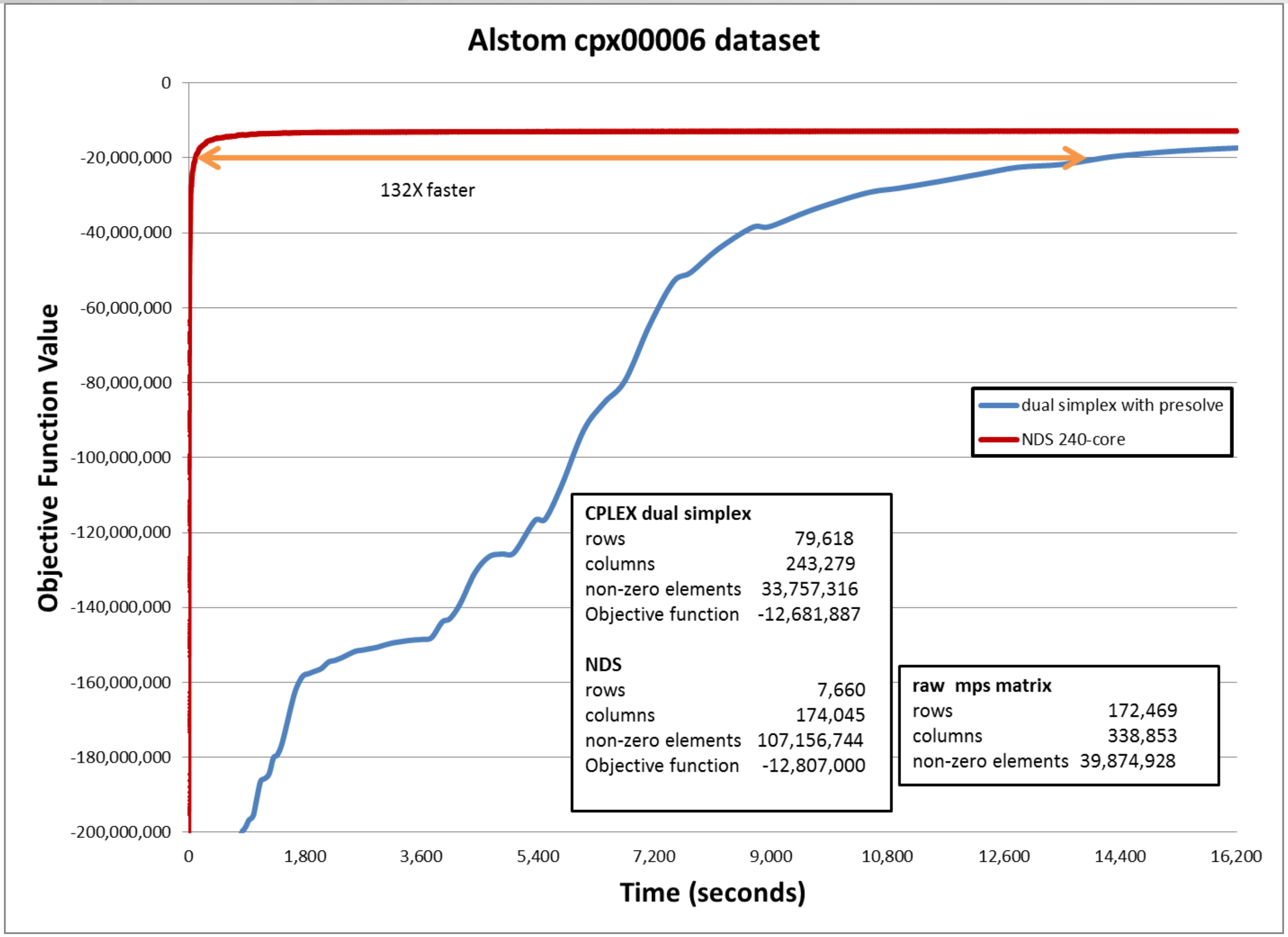


CPLEX time per iteration slows by 85x from beginning to end due to backfill

Real World Data 2



CPLEX time per iteration slows by 269x from beginning to end due to backfill

Summary

- Developed novel non-linear dynamical system based FTR solver
- Easily parallelized to solve large linear programming (LP) problems for FTR application within few hours (cloud compatible)
- Parallel NDS more computationally efficient than CPLEX for LP
  - Computational kernel of CPLEX is linear solver that scales as cube of problem size
  - NDS kernel is matrix-vector multiplication that scales as square
  - NDS avoids backfill (filing in zeros) of coupled blocks
  - Maintains numerical stability through using only original matrix
  - Uses dense algorithm for obligatory bids, sparse (50%) for optional bids
  - Half the arithmetic for obligatory bids (two inner products differ only in sign)
  - Data loaded efficiently in parallel
- Further enhancements
  - Further improve parallelization (asynchronous communication, sparse ops)
  - Refine adaptive time stepping and explore ode time stepping for faster convergence

Future

- Develop quadratic programming capability
  - Improved FTR constraints
- Explore other application needing LP and/or QP capability
  - Transmission planning
  - Locational Marginal Pricing (LMP)
  - Optimal Power Flow (OPF)
- Explore using method with discrete problems
  - Mixed Integer Programming (MIP)
  - Resource Scheduling and Commitment (RSC) (aka Unit Commitment)
  - Stochastic RSC

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References

- Kalsi K, ST Elbert, M Vlachopoulou, N Zhou, and Z Huang. 2012. "Advanced Computational Methods for Security Constrained Financial Transmission Rights." to be presented at the *IEEE Power and Energy Society General Meeting 2012. 22-27 July 2012, San Diego.*

- Elbert ST, K Kalsi, M Vlachopoulou, MJ Rice, KR Glaesemann, and N Zhou. 2012. "Advanced Computational Methods for Security Constrained Financial Transmission Rights: Structure and Parallelism." to be presented at the IFAC (International Federation of Automatic Control) *8th PP&PSC (Power Plant and Power Systems Control) Symposium*, 2-5 Sept. 2012, Toulouse, France.