Advanced Statistical Techniques for Economic Forecasting

Seminar for the

National Association of Business Economics

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Course Outline

- Forecasting
- Considerations Basic to Successful Forecasting
- Evaluating and Combining Forecasts
- Nature of Economic Modeling
- Diagnostics (Serial Correlation, Collinearity, and Influence)
- Distributed Lag Models
- Multi-Equation Models (Simultaneous, Seemingly Unrelated, Recursive)

- Time-Series Models

  Auto Regressive Integrated Moving Average Models (ARIMA) (Univariate) (Box-Jenkins)

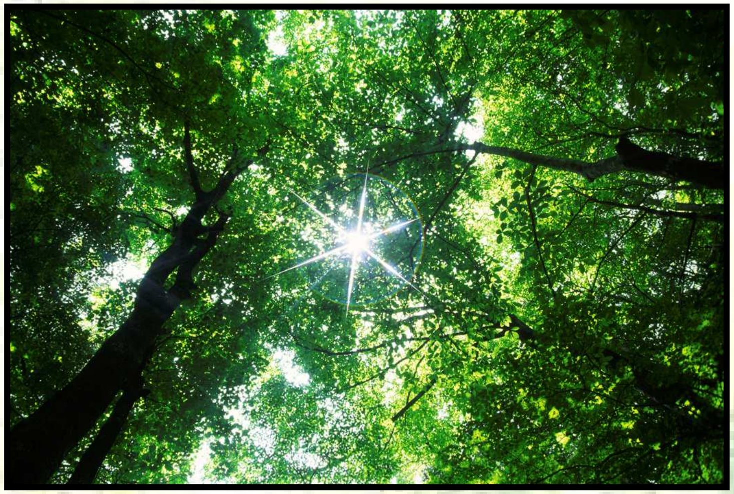
  Vector Autoregression Models (VAR) (Multivariate)

    Impulse Response Functions

    Granger Causality

  Cointegration

  Error Correction Models

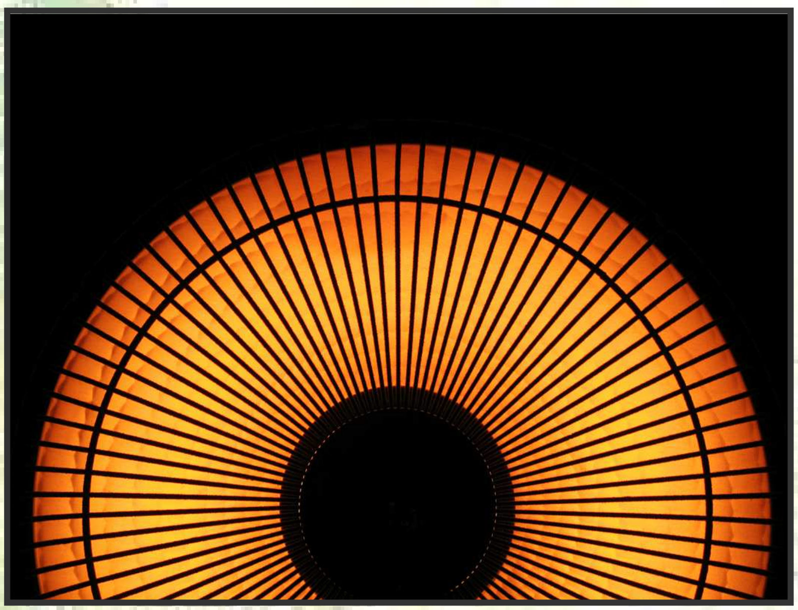


Econometric results -- indeed a term that for many economists conjures up horrifying visions of well-meaning but perhaps marginally skilled, likely nocturnal, individuals sorting through endless piles of computer print-outs. One "final"print-out is then chosen for seemingly mysterious reasons and the rest discarded to be recycled through a local paper processor and another computer printer for other "econometricians" to to repeat the process ad infinitum. Besides supplying a lucrative business for the paper recyclers, what useful output, if any, results from such a process? This question lies at the heart of the so called "science" of econometrics as currently applied, a practice which has been called "data mining," "number crunching," "model sifting, " "data grubbing," "fishing," "data massaging," and even "alchemy," among other less palatable terms. All of these euphemisms describe basically the same process: choosing an econometric model based on repeated experimentation with available sample data.

Zimmerman, Rod F., " Reporting Econometric Results: Believe It or Not?" *Land Economics*, 60 (February 1984):122.

Section 1:

NABE ASTEF

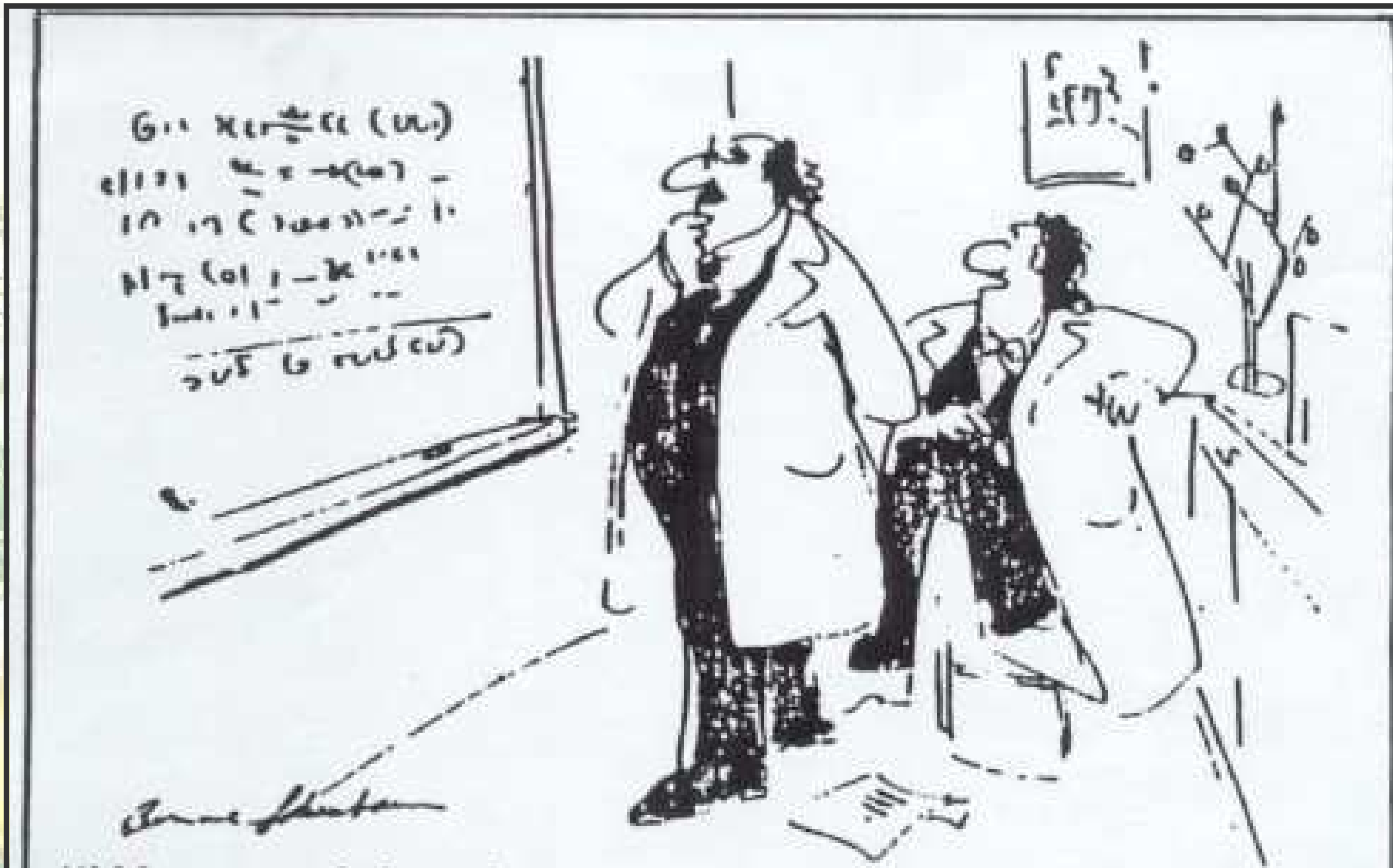


Tea-Leaf Reading, Soothsaying, or Quantitative Business Analysis?

Business analysts often need to be in position to:

1. Interpret the economic and financial landscape.
2. Forecast various economic and financial activities.

How does one achieve these objectives?

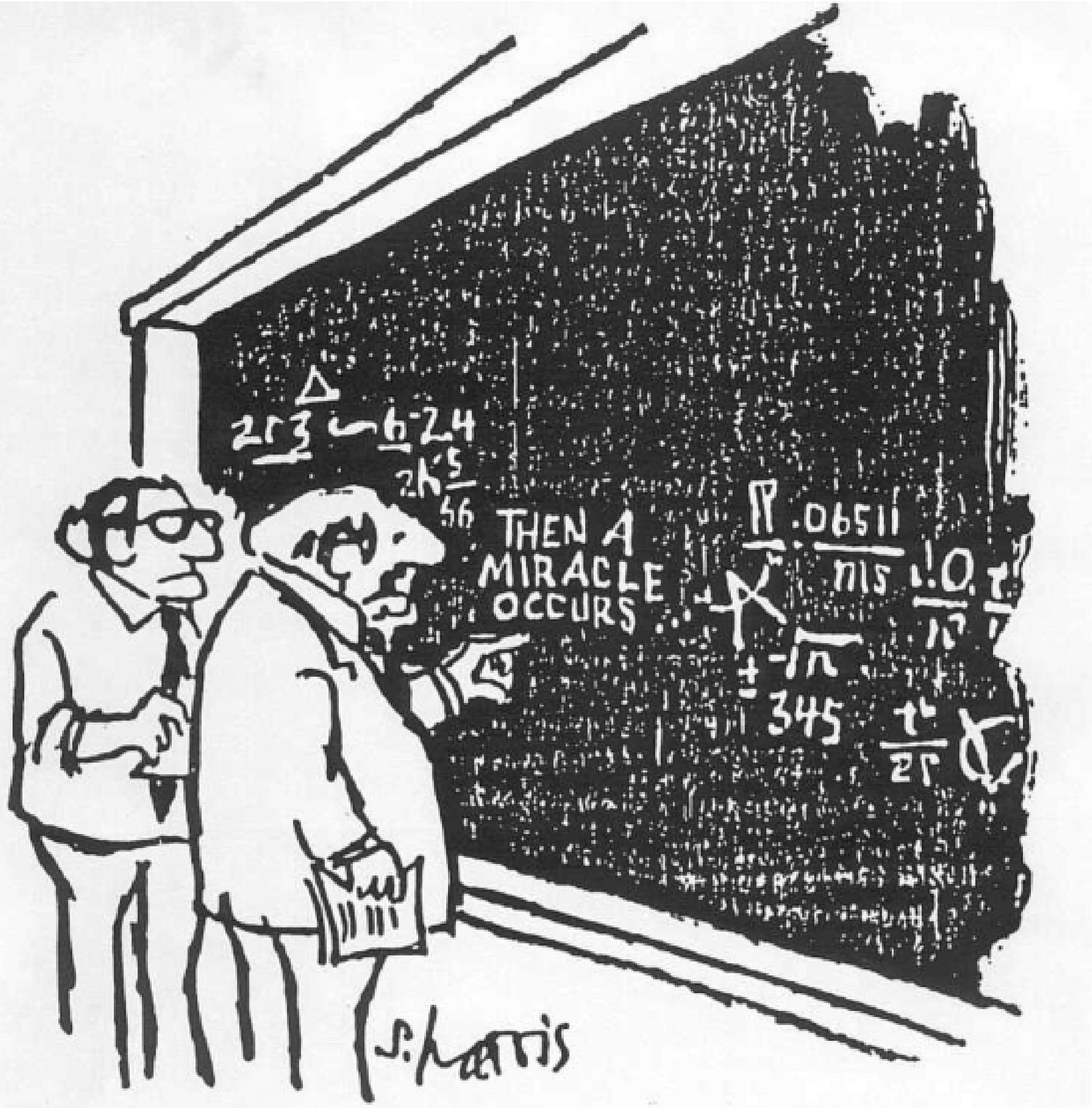


"We could take an uneducated guess."

Course of Action:

Development of

Formal Quantitative Models



"I think you should be more explicit here in step two."

Modeling Approaches

Econometric (Structural) Models

- Single-Equation
- Multi-Equation
  - Simultaneous Models
  - Seemingly Unrelated Regression Models
  - Recursive Models

Time-Series Models

- Univariate
  - Box-Jenkins ARIMA Models
- Multivariate
  - Vector Autoregression (VAR) Models
  - Error Correction Models

Model specification driven by:

- Data (# of observations)
- Data (# of variables)
- Theoretical Considerations
- Atheoretical Considerations
- Dynamics
- Experience
- Question(s) Addressed

Critical ingredient in all models:

- **"Sufficiently large" amount of historical data**

- **"Ask not what you can do to the data but rather what the data can do for you."**

- **Data Types**

  Time-Series

  Cross-Sectional

  Combination

Quote from Lord Kelvin:

"I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind."

Mathematical Considerations:

- Functional form
- Derivatives to capture marginal effects (changes)

Statistical Considerations:

- Estimation Procedures
  - Ordinary Least Squares
  - Maximum Likelihood
- Tests of Hypotheses
- p-values and Levels of Significance

Model Selection Criteria:

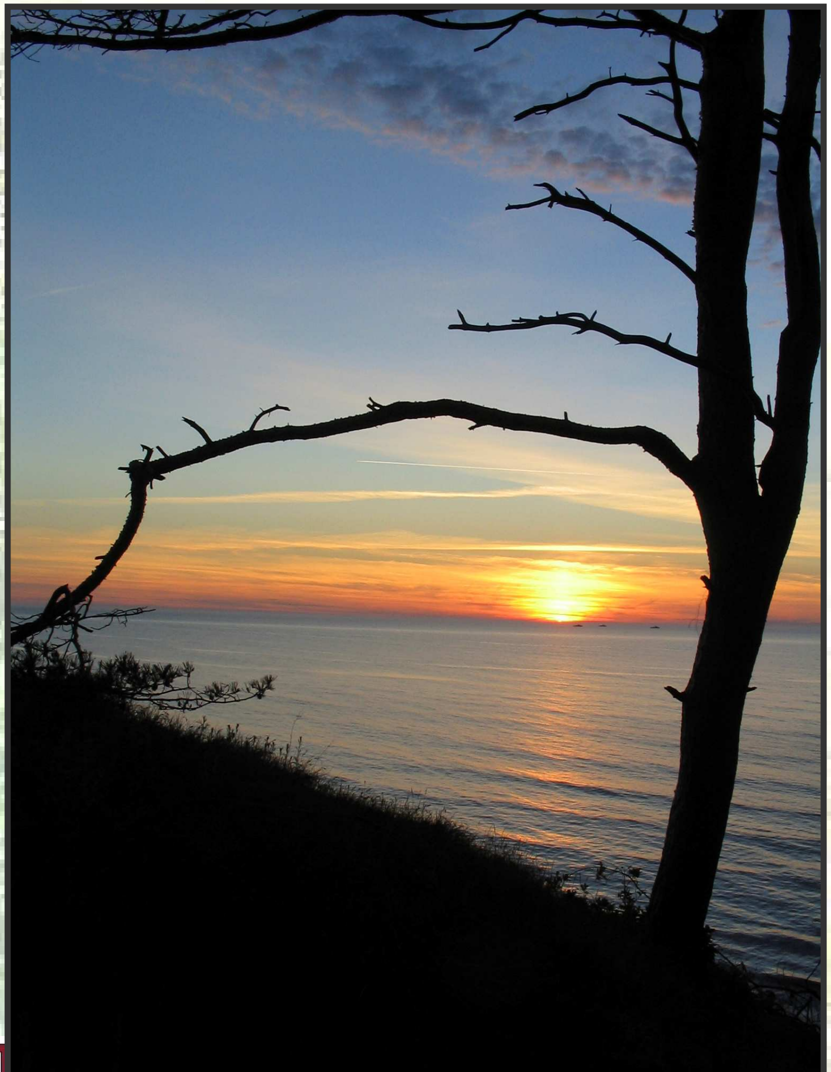
R^2

$\bar{R}^2$

Akaike Information Criterion (AIC)

Schwarz or Bayesian Information Criterion (SIC) or (BIC)

Key Diagnostics



- Serial Correlation

  (Autocorrelation)

- Heteroscedasticity

- Collinearity Diagnostics

- Influence Diagnostics

- Structural Change

Example of Structural Model

The effect of unionization on earnings hypothesis:

Unionization increases real wages

$$\ln w = 0.28 \ln Q + 0.77Z + 0.84 \ln P + 0.228U$$
$$\quad (0.056) \quad (0.28) \quad (0.32) \quad (0.094)$$
$$- 0.16UZ - 1.94U \ln P + CONSTANT$$
$$\quad (1.10) \quad (0.99)$$

$R^2 = 0.891$

w = relative wages (average hourly compensation in unionized industries relative to compensation in nonunionized industries)

Q = relative value of output

Z = unemployment rate

P = measure of expected inflation

U = measure of the extent of union membership

UZ = interaction variable

UlnP = interaction variable

Distributed Lag Model

Example:

$$\text{Sales}_t = \beta_0 + \beta_1 \text{PRICE}_t + \beta_2 \text{INCOME}_t + w_0 \text{ADV}_t + w_1 \text{ADV}_{t-1} + \ldots + w_m \text{ADV}_{t-m} + \varepsilon_t$$

$$\text{SR } \textit{effect} \rightarrow w_0$$

$$\text{LR } \textit{effect} \rightarrow \sum_{i=0}^{m} w_i$$

$$\text{MEAN } \textit{lag} \rightarrow \frac{\sum_{i=0}^{m} i w_i}{LR_{effect}}$$

Models wherein the dependent variables correspond to choices

Probit Models

Logit Models

Censored Response Models (Dependent variables are discontinuous)

Tobit Models

Heckman Sample Selection Procedure

Count Data Models (Dependent Variables are integers)

ARCH/GARCH Models (Variance of Stochastic Disturbance term not constant)

Multi-equation Models

Simultaneous equations model

$$Y_1 = \tau_{10} + B_{12}Y_2 + \ldots + B_{1g}Y_g + \tau_{11}x_1 + \tau_{12}x_2 + \ldots + \tau_{1K}x_K + \in_1$$

$$Y_2 = \tau_{20} + B_{21}Y_1 + \ldots + B_{2g}Y_g + \tau_{21}x_1 + \tau_{22}x_2 + \ldots + \tau_{2K}x_K + \in_2$$

$$Y_g = \tau_{g0} + B_{g1}Y_1 + \ldots + B_{gg}Y_g + \tau_{g1}x_1 + \tau_{g2}x_2 + \ldots + \tau_{gK}x_K + \in_g$$

Analytically Derived Reduced Forms

Impact, Interim, Total Multipliers

Seemingly Unrelated Regression Models

$$Y_{1t} = \beta_{11} X_{1t,1} + \beta_{12} X_{1t,2} + \ldots + \beta_{1k1} X_{1t,k1} + \varepsilon_{1t}$$

$$Y_{2t} = \beta_{21} X_{2t,1} + \beta_{22} X_{2t,2} + \ldots + \beta_{2k2} X_{2t,k2} + \varepsilon_{2t}$$

$$\vdots$$

$$Y_{mt} = \beta_{m1} X_{mt,1} + \beta_{M2} X_{mt,2} + \ldots + \beta_{mkm} X_{mt,km} + \varepsilon_{mt}$$

Recursive Model

Example (Lee and Lloyd) Model for the Oil Industry

$$
\begin{aligned}
R_{1t} &= \alpha_1 + \tau_1 M_t + u_{1t} \\
R_{2t} &= \alpha_2 + \beta_{21} R_{1t} + \tau_2 M_t + u_{2t} \\
R_{3t} &= \alpha_3 + \beta_{31} R_{1t} + \beta_{32} R_{2t} + \tau_3 M_t + u_{3t} \\
R_{4t} &= \alpha_4 + \beta_{41} R_{1t} + \beta_{42} R_{2t} + \beta_{43} R_{3t} + \tau_4 M_t + u_{4t} \\
R_{5t} &= \alpha_5 + \beta_{51} R_{1t} + \beta_{52} R_{2t} + \beta_{53} R_{3t} + \beta_{54} R_{4t} + \tau_5 M_t + u_{5t} \\
R_{6t} &= \alpha_6 + \beta_{61} R_{1t} + \beta_{62} R_{2t} + \beta_{63} R_{3t} + \beta_{64} R_{4t} + \beta_{65} R_{5t} + \tau_6 M_t + u_{6t} \\
R_{7t} &= \alpha_7 + \beta_{71} R_{1t} + \beta_{72} R_{2t} + \beta_{73} R_{3t} + \beta_{74} R_{4t} + \beta_{75} R_{5t} + \beta_{76} R_{6t} + \tau_7 M_t + u_{7t}
\end{aligned}
$$

Where R_1 = Rate of Return on Security 1 (* Imperial Oil)
Where R_2 = Rate of Return on Security 2 (* Sun Oil)
Where R_7 = Rate of Return on Security 7 (* Standard Oil of Indiana)
Where M_t = Rate of Return on the Market Index
Where u_{it} = Disturbances (i = 1, 2, …, 7)

- Box-Jenkins Modeling (ARIMA Models)
    - AR, MA, ARMA, ARIMA models
    - Model specification, estimation, diagnostic checking
    - Forecasting
- Unit Roots and Unit Root Tests (Stationarity)
- Vector Autoregressions (VARs)
    - Specification and Estimation of VARs
    - Causality (Granger Causality)
- Cointegration
- Error Correction Models

The Box-Jenkins Approach

One of the most widely used methodologies for the analysis of time-series data.

Basic Steps in the Box-Jenkins Methodology

(1) differencing the series so as to achieve stationarity

(2) identification of preliminary model(s)

(3) estimation of the model(s)

(4) diagnostic checking

(5) using the model for forecasting

- Purely Random Process (White Noise)

A sequence of mutually independent identically distributed random variables

constant mean, constant variance

- Random Walk Process

$$Y_t = Y_{t-1} + \varepsilon_t$$

- Moving-Average Process MA(q)

$$Y_t = \varepsilon_t + B_1 \varepsilon_{t-1} + \ldots + B_q \varepsilon_{t-q}$$

- Autoregressive Process AR(p)

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \ldots + \alpha_p Y_{t-p} + \varepsilon_t$$

- Autoregressive Moving-Average Process ARMA (p,q)

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \ldots + \alpha_p Y_{t-p} + \varepsilon_t + B_1 \varepsilon_{t-1} + \ldots + B_q \varepsilon_{t-q}$$

Vector Autoregression

When we have several time series, we need to take into account the interdependence between them.

VAR Approach

A multiple time-series generalization of the autoregressive (AR) model.

$$y_{1t} = \alpha_{11} y_{1,t-1} + \alpha_{12} y_{1,t-2} + \ldots + \alpha_{1m} y_{1,t-m} + \alpha_{21} y_{2,t-1} + \alpha_{22} y_{2,t-2} + \ldots + \alpha_{2n} y_{2,t-n} + \epsilon_{1t}$$

$$y_{2t} = \beta_{11} y_{1,t-1} + \beta_{12} y_{1,t-2} + \ldots + \beta_{1p} y_{1,t-p} + \beta_{21} y_{2,t-1} + \beta_{22} y_{2,t-2} + \ldots + \beta_{2s} y_{2,t-s} + \epsilon_{2t}$$

Error Correction Model (ECM)

If x_t and y_t are cointegrated, there is a long-run relationship between them. Furthermore, the short-run dynamics can be described by the ECM.

If $x_t \sim I(1)$, $y_t \sim I(1)$ and $z_t = y_t - \beta x_t \sim I(o)$ and x_t and y_t are cointegrated. The Granger representation theorem says that, therefore, x_t and y_t may be considered to be generated by ECMs of the form.

$$\Delta x_t = \rho_1 z_{t-1} + \text{lagged}\left(\Delta x_t, \Delta y_t\right) + \in_{1t}$$

$$\Delta y_t = \rho_1 z_{t-1} + \text{lagged}\left(\Delta x_t, \Delta y_t\right) + \in_{2t}$$

At least one of the ρ_1 and ρ_2 are non-zero, and $\varepsilon_{1t}, \varepsilon_{2t}$ are white-noise errors. Note that z_{t-1} is the one-period lag of the residuals from the cointegrated relationship between x_t and y_t.

Software Packages for Econometrics/Time Series

Without Regard to Order:

- SAS/ETS
- SPSS
- RATS
- SHAZAM
- LIMDEP
- FORECAST PRO
- EVIEWS
- GAUSS
- STATA
- TSP

No Single Package is Optimal for Every Situation

Forecasting

and

Forecast Evaluation

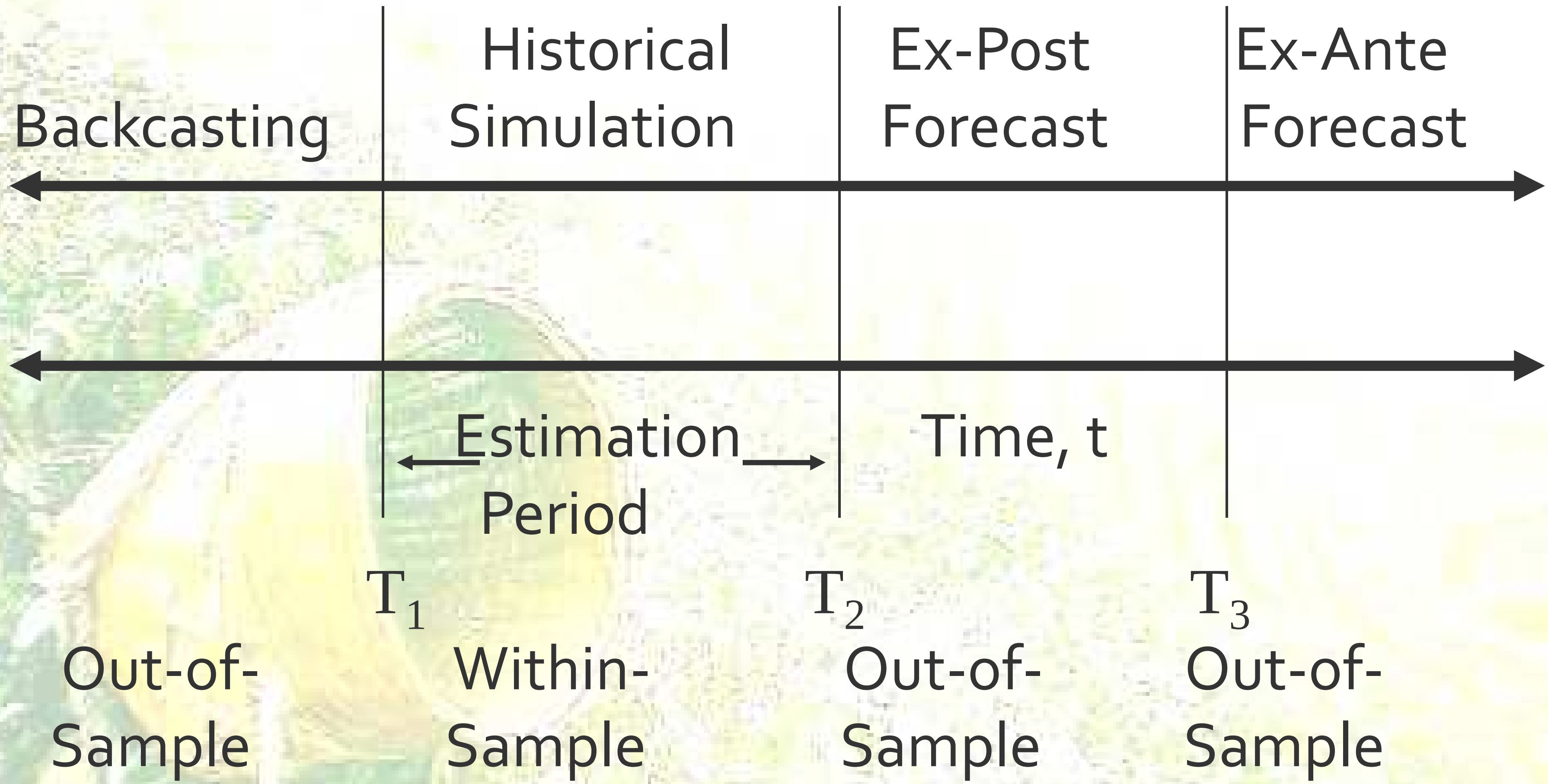
Forecasting

Self-styled "prophets" who mislead us should be reminded that among the ancient Scythians, when prophets predicted things that failed to come true, they were laid, shackled hand and foot, on a little cart filled with heather and drawn by oxen, on which they were burned to death.

"Those who live by the crystal ball should learn to eat ground glass."

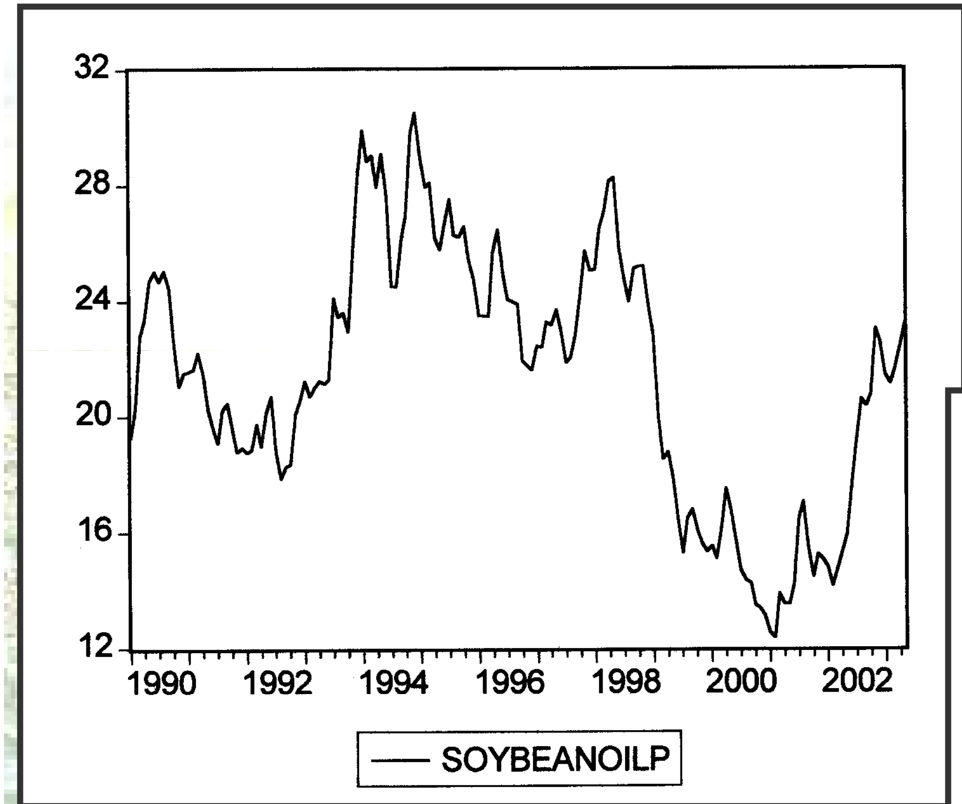
Ted Moskovitz

Forecasting



Forecast Methods

- **Qualitative**
- **Quantitative**
  - **Formal**
    - **Econometric Analysis**
    - **Time Series Analysis**
      (ARIMA, VAR, Error Correction Models)
  - **Intuitive (Naive)**
    - **Seasonal**
    - **Trend Extrapolation**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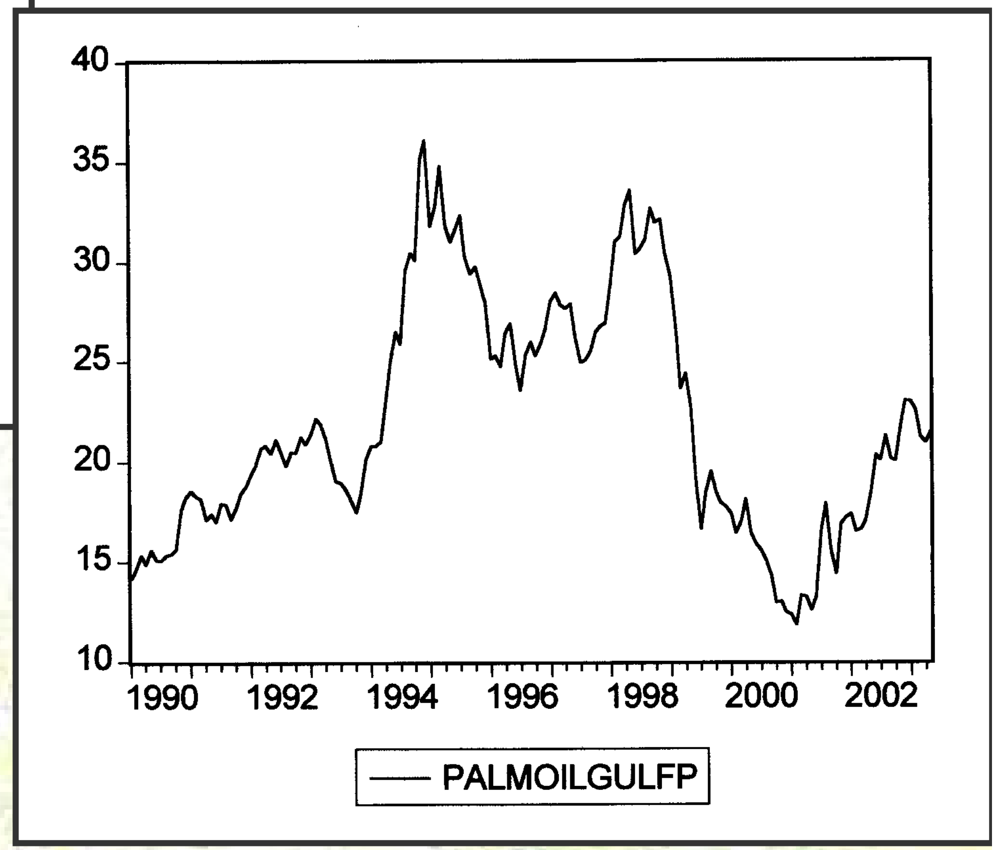
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1998
2000
2002
PALMOILGULFP

Six Considerations Basic to Successful Forecasting

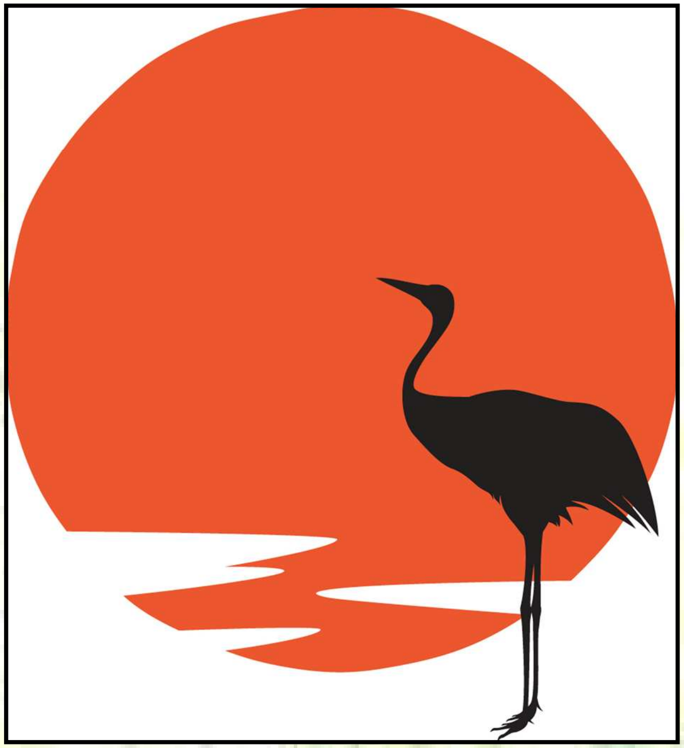
- Decision Environment and Loss Function
- Forecast Object
- Forecast Statement
- Forecast Horizon
- Information Set
- Methods

Forecast Error

Let Y Denote a Series and Let $\hat{Y}$ Its Forecast

Forecast Error (e)

$$\mathbf{e} = Y - \hat{Y}$$



Loss Functions

A Representation of the Loss Associated with a Forecast, Dependent on the Size of the Forecast Error

L(e)

Requirements of a Loss Function:

(1) L(0)=0

(2) L(e) is Continuous

(3) L(e) is Increasing in the Absolute Value of e

Quadratic Loss or Squared-Error Loss

$$L(e)=e^2$$

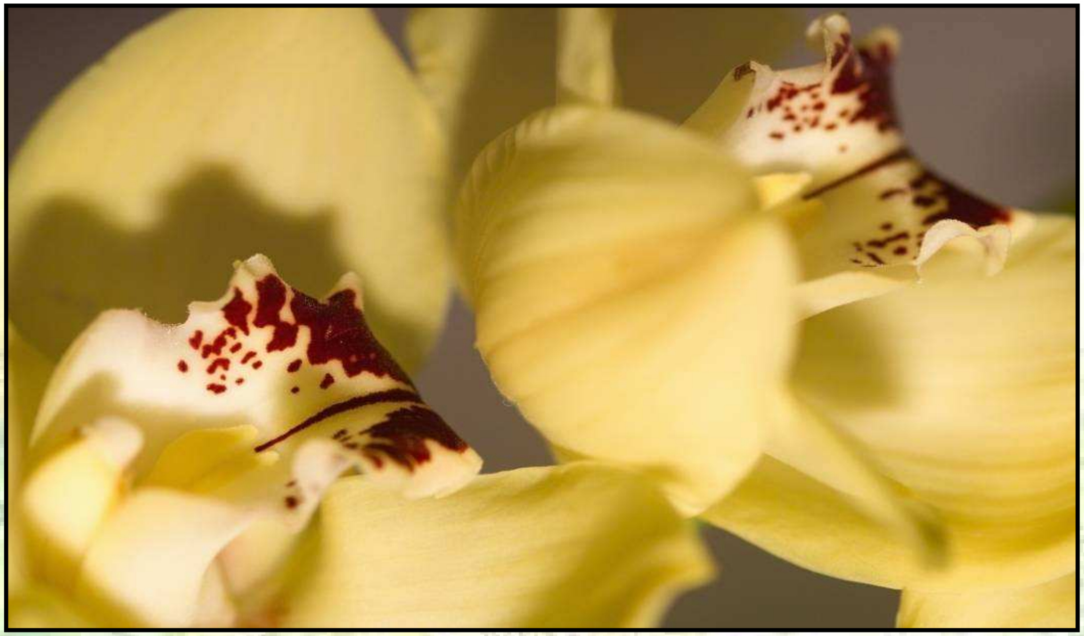
Absolute Loss or Absolute-Error Loss

$$L(e) = |e|$$

Symmetric Loss Functions

Direction-Of-Change Forecast Takes on Two Values

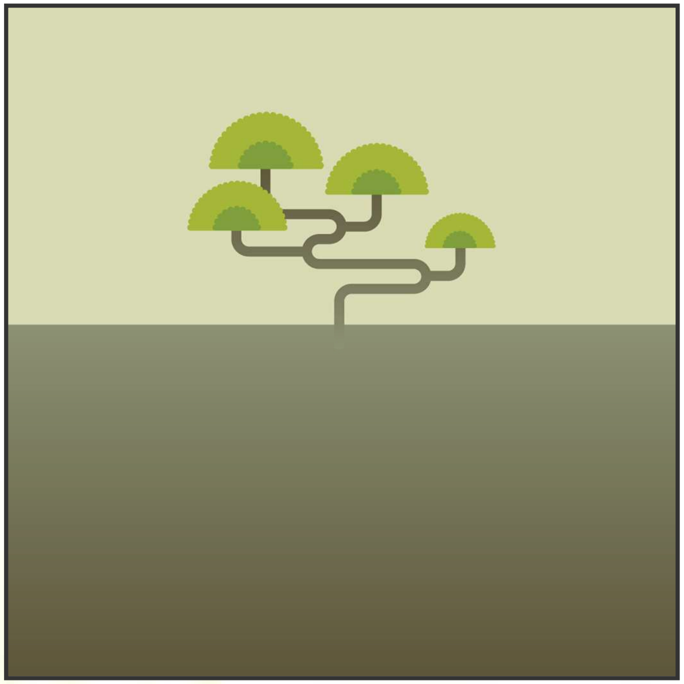
$$L\left(Y,\hat{Y}\right) = \begin{cases} 0 \text{ if } \operatorname{sign}(\Delta Y) = \operatorname{sign}\left(\Delta \hat{Y}\right) \\ 1 \text{ if } \operatorname{sign}(\Delta Y) \neq \operatorname{sign}\left(\Delta \hat{Y}\right) \end{cases}$$



Point Forecast

A single number

FV_{t+h}

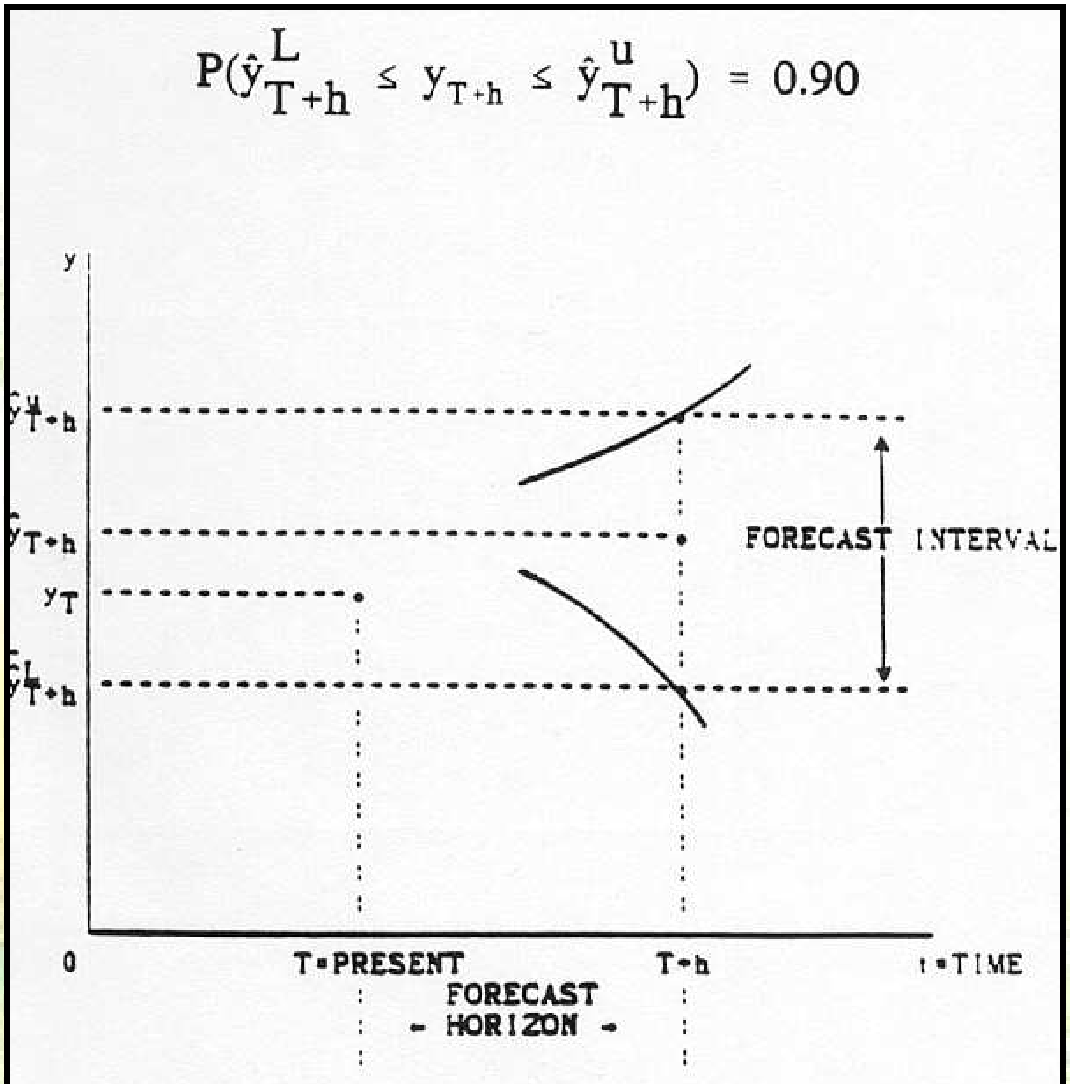


Interval Forecast -

Places Confidence or Tolerance Bands on the Forecast

$$FV_{t+h} - kSE_{(FV_{t+h})} < RV_{t+h} < FV_{t+h} + kSE_{(FV_{t+h})}$$

Forecast Interval



THE POINT FORECAST AND THE INTERVAL FORECAST

Parsimony Principle

Ceteris Paribus, Simple Models are Preferable to Complex Models

Shrinkage Principle

Imposing Restrictions on Models Often Improves Marginal Effects Analysis and Forecast Performance

Kiss Principle — Keep it Sophisticatedly Simple

There is no efficacious substitute for economic analysis in business forecasting. Some maverick may hit a homerun on occasion; but over the long season, batting averages tend to settle down to a sorry level when the more esoteric methods of soothsaying are relied upon.

Better to be wrong in good company than run the risk of being wrong all alone.

Often a forecaster is forced to give a single point estimate because his boss or others cannot handle a more complicated concept. Then he must figure out for himself which point estimate will do them the most good, or the least harm.

Evaluating and Combining Forecasts

Measures of Forecast Accuracy

- Mean Error
- Mean Absolute Error
- MSE
- Decomposition of MSE
- RMSE
- MAPE
- WAPE
- Theil U Statistic

Combining Forecasts (Composite Forecasts)

- Minimum Forecast Error Variance
- Arithmetic Average
- Use of Regression to Find Optimal Weights

Economic Forecasting & Science

Paul A. Samuelson --

If prediction is the ultimate aim of all science, then we forecasters ought to award ourselves the palm for accomplishment, bravery or rashness.

... we (economists) are better than anything else in heaven and earth at forecasting aggregate business trends -- better than gypsy tea-leaf readers, Wall Street soothsayers and chartist technicians, hunch-playing heads of mail-order chains, or all-powerful heads of state.

Forecasting models have their limitations because they deal with human behavior and ever-changing institutions.

Insight on the Forecasting Process

Forecasting is an important part of the economic policy-making process.

Forecasting Terminology

Forecast — a quantitative estimate (or set of estimates) about the likelihood of future events based on past and current information

Point forecasts — those which yield a single number

Interval forecasts — indicate in each period the interval in which it is hoped the actual value will lie

"If you twist my arm, you can make me give a single number as a guess about next year's GNP. But you will have to twist hard. My scientific conscience would feel more comfortable giving you my subjective probability distribution for all the values of GNP."

— Samuelson

Forecast Process

(1) Design Phase

(2) Specification Phase

(3) Evaluation Phase

Clearly & Levenbach

The Professional Forecaster

A forecast is a systematic process of decisions and actions performed in an effort to predict the future. A forecast is not an end product, but rather an input to the decision-making process.

Algebraic Measures of Forecast Accuracy

"In science and in real economic life,

it is terribly important

not to be wrong much."

— Samuelson

Forecast Evaluation

(1) MSE = Mean Squared Error $\frac{1}{M}\sum_{t=1}^{M}\left(F_t - A_t\right)^2$

(2) RMSE = Root Mean Squared Error $\left(MSE\right)^{1/2}$

(3) MAE = Mean Absolute Error $\frac{1}{M}\sum_{t=1}^{M}\left|F_t - A_t\right|$

(4) MAPE = Mean Absolute Percent Error $\frac{1}{M}\sum_{t=1}^{M}\left|\frac{F_t - A_t}{A_t}\right| \times 100$

(5) WAPE = Weighted Absolute Percent Error $\sum_{t=1}^{M} w_t\left|\frac{F_t - A_t}{A_t}\right| \times 100$, where $w_t = \frac{A_t}{\sum_{t=1}^{M} A_t}$

Decomposition of MSE

$$U_m = \frac{(\bar{F} - \bar{A})^2}{MSE} \qquad U_S = \frac{(S_F - S_A)^2}{MSE} \qquad U_C = \frac{2(1 - r)\, S_F S_A}{MSE}$$

↑ mean (or bias) proportion

↑ variance proportion

↑ covariance proportion

$$U_m + U_S + U_C = 1$$

With this decomposition,

U_M Measure of bias - unequal central tendencies of the actual and forecasted values

$$(\bar{F} - \bar{A})^2$$

U_S Measure of unequal variation - squared difference between standard deviations, both actual and forecasted

$$(S_F - S_A)^2$$

U_C Measure of incomplete covariation - correlation coefficient r between actual and forecasted values

$$2(1 - r)S_F S_A$$

$$U_M = (\bar{F} - \bar{A})^2 / MSE; U_S = (S_F - S_A)^2 / MSE;$$

$$U_C = 2(1 - r)S_F S_A / MSE$$

$$U_M + U_S + U_C = 1$$

U_C - nonsystematic random error, cannot be avoided

U_M, U_S - represent systematic errors that should be avoided

$U_M \rightarrow 0$ as $\overline{F} = \overline{A}$ if U_M large, then average predicted value deviates substantially from average realized value

$U_S \rightarrow 0$ as $S_F = S_A U_S$ indicates ability of the model to replicate the degree of variability; if U_S large, then the actual series fluctuated considerably but the simulated series shows little fluctuation or vice versa

$U_C \rightarrow 0$ as $r = 1$; can never hope that forecasters will be able to predict so that all points are located on the straight line of perfect forecasts

$U_C \rightarrow$ remaining error after deviations from average values and deviations in variabilities have been accounted for

Another Decomposition

$$MSE = (\overline{F} - \overline{A})^2 + (S_F - rS_A)^2 + (1 - r^2)S_A^2$$

$$U_M = (\overline{F} - \overline{A})^2 / MSE$$
$$U_R = (S_F - rS_A)^2 / MSE$$
$$U_D = (1 - r^2)S_A^2 / MSE$$

U_M, U_R should not differ much from zero

U_D should be close to unity

$$U_M + U_R + U_D = 1$$

$U_R \rightarrow$ Regression component

$U_D \rightarrow$ Disturbance component

Consider this decomposition in relation to the regression $A_t = F_t + e_t$

If residuals have zero mean, then $\bar{A} = \bar{F} \rightarrow U_M = 0.$

If regression coefficient truly 1, then $U_R = 0.$

MSE then will consist of only one term, the disturbance proportion.

EXAMPLE OF OUT-OF-SAMPLE EVALUATION

ACTUAL	ARIMA	FORECAST ERROR	ABSOLUTE FORECAST ERROR	SQUARED ERROR	ABSOLUTE PERCENT ERROR	WEIGHT	WEIGHTED ABSOLUTE PERCENT ERROR
1293594	1392156	-98562	98562	9714467844	7.62	0.0700542	0.53
1275381	1294103	-18722	18722	350513284	1.47	0.0690679	0.10
1559312	1549253	10059	10059	101183481	0.65	0.0844440	0.05
1789155	1860777	-71622	71622	5129710884	4.00	0.0968911	0.39
2139428	2153039	-13611	13611	185259321	0.64	0.1158600	0.07
1458652	1686154	-227502	227502	51757160004	15.60	0.0789928	1.23
1517930	1810875	-292945	292945	85816773025	19.30	0.0822030	1.59
1541403	1351399	190004	190004	36101520016	12.33	0.0834742	1.03
2107742	2015857	91885	91885	8442853225	4.36	0.1141441	0.50
1664925	1730865	-65940	65940	4348083600	3.96	0.0901635	0.36
2118102	1649788	468314	468314	219318002596	22.11	0.1147051	2.54
18465624			140833.27 (MAE)	38296866116 (MSE)	8.37 (MAPE)	1	8.39 (WAPE)
				195695.85 (RMSE)			

THEIL U2 0.11 IN LEVELS

THEIL U2 0.53 W/ CHANGES

ACTUAL	ACTUAL SQ	ACTUAL CHANGE	ACTUAL CHANGE SQUARED
1293594	1673385436836	NA	
1275381	1626596695161	-18213	331713369
1559312	2431453913344	283931	80616812761
1789155	3201075614025	229843	52827804649
2139428	4577152167184	350273	122691174529
1458652	2127665657104	-680776	463455962176
1517930	2304111484900	59278	3513881284
1541403	2375923208409	23473	550981729
2107742	4442576338564	566339	320739862921
1664925	2771975255625	-442817	196086895489
2118102	4486356082404	453177	205369393329
			1446184482236
			1202574.11

DECOMPOSITION OF MSE

UM 0.0002	ARIMA BAR 1681296.91	ACTUAL BAR 1678693.09
US 0.0505	ARIMA STD DEV 260548.00	ACTUAL STD DEV 304534.88
UC 0.9493	CORR(ARIMA,ACTUAL) 0.77	
1		

UR 0.0174 UD 0.9825 UM 0.0002 1

ACTUAL=ARIMA + ERROR

Regression Output:

Constant		163752.57807
Std Err of Y Est		214445.08728
R Squared		0.5942972579
No. of Observations		11
Degrees of Freedom		9
X Coefficient(s)	0.9010547183	
Std Err of Coef.	0.2481601362	

Composite Forecasting — often alternative forecasts of the same data are available, each of which contains information independent of others.

- Bates and Granger (1969)
- Granger and Newbold (1977)
- Just and Rausser (1981)
- Bessler and Brandt (1981)

Bates and Granger suggest that if the objective is to make as good a forecast as possible, the analyst should attempt to combine the forecasts.

Composite forecasting can provide forecasts which are preferred to the individual forecasts used to generate the composite.

$$CF = w_1 FM_1 + w_2 FM_2$$

Building composite forecasts requires that the analyst select weights to assign the individual forecasts. How to weigh?

(1) Minimum variance weighting minimize the variance of the forecast errors over the forecast period.

$$w_1 = \frac{\sigma_2^2 - \rho \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2 \rho \sigma_1 \sigma_2}$$

$$w_2 = 1 - w_1$$

σ^2_i → the forecast error variance associated with method i; ρ is the correlation coefficient between the errors of forecasts i and j.

(2) Simple averaging

$$\mathbf{CF} = \frac{\mathbf{FM1 + FM2}}{\mathbf{2}}$$

More generally,

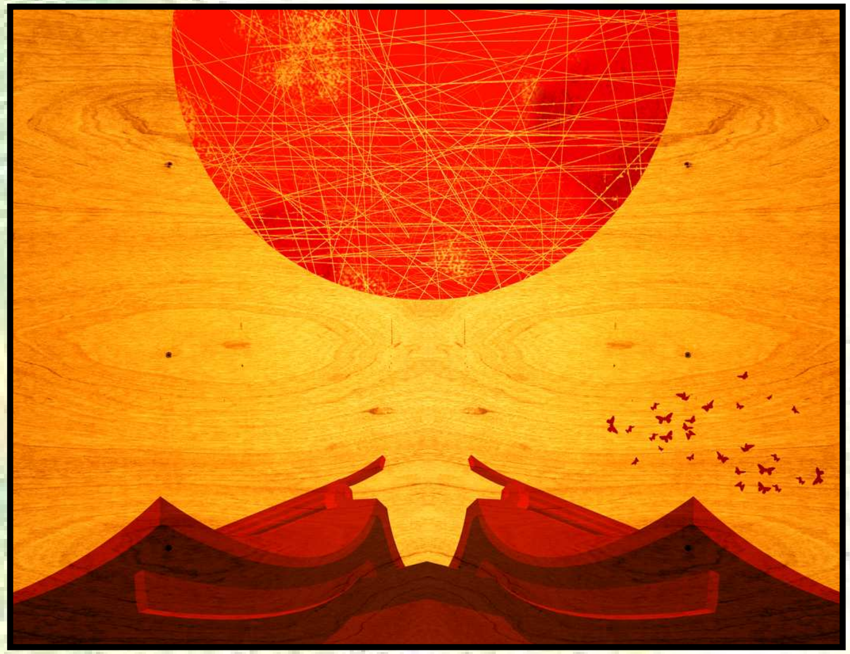
$$\frac{\sum_{i=1}^{r} \mathbf{FM_i}}{\mathbf{r}}$$

Weights for Composite Forecasting

$$\textbf{Run} \quad \textbf{Actual}_t = W_1 FM1_t + W_2 FM2_t + \epsilon_t$$

$$\textbf{Restrict} \quad W_1 + W_2 = 1$$

Use same procedure for more than two forecast methods.



Example:

Ex-Post Forecast Evaluation of Germany Sales

ARIMA Model

Winters Model

ACTUAL	ARIMA	WINTERS
1293594	1392156	1571411
1275381	1294103	1401020
1559312	1549253	1664467
1789155	1860777	1936854
2139428	2153039	1936245
1458652	1686154	1821541
1517930	1810875	1640003
1541403	1351399	1613390
2107742	2015857	1944321
1664925	1730865	1792553
2118102	1649788	1754221

EXAMPLE OF COMPOSITE FORECASTING

ACTUAL	ARIMA	HOLT WINTERS	COMPOSITE AVERAGE	COMPOSITE WEIGHTS ARIMA=.5913128 HOLT-WINTERS=.4086872
1293594	1392156	1571411	1481784	1465415
1275381	1294103	1401020	1347562	1337799
1559312	1549253	1664467	1606860	1596339
1789155	1860777	1936854	1898816	1891869
2139428	2153039	1936245	2044642	2064438
1458652	1686154	1821541	1753848	1741485
1517930	1810875	1640003	1725439	1741042
1541403	1351399	1613390	1482395	1458471
2107742	2015857	1944321	1980089	1986621
1664925	1730865	1792553	1761709	1756076
2118102	1649788	1754221	1702005	1692468

ACTUAL-HW	ARIMA-HW
-277817	-179255
-125639	-106917
-105155	-115214
-147699	-76077
203183	216794
-362889	-135387
-122073	170872
-71987	-261991
163421	71536
-127628	-61688
363881	-104433

Regression Output:

Constant		-24218.17
Std Err of Y Est		206909.84
R Squared		0.1631598
No. of Observations		11
Degrees of Freedom		9
X Coefficient(s)	0.5913128	0.4086872
Std Err of Coef.	0.4463861	

	ARIMA APE	HW APE	CA APE	CW APE
	7.62	21.48	14.55	13.28
	1.47	9.85	5.66	4.89
	0.65	6.74	3.05	2.37
	4.00	8.26	6.13	5.74
	0.64	9.50	4.43	3.51
	15.60	24.88	20.24	19.39
	19.30	8.04	13.67	14.70
	12.33	4.67	3.83	5.38
	4.36	7.75	6.06	5.75
	3.96	7.67	5.81	5.47
	22.11	17.18	19.64	20.10
MAPE	8.37	11.46	9.37	9.14