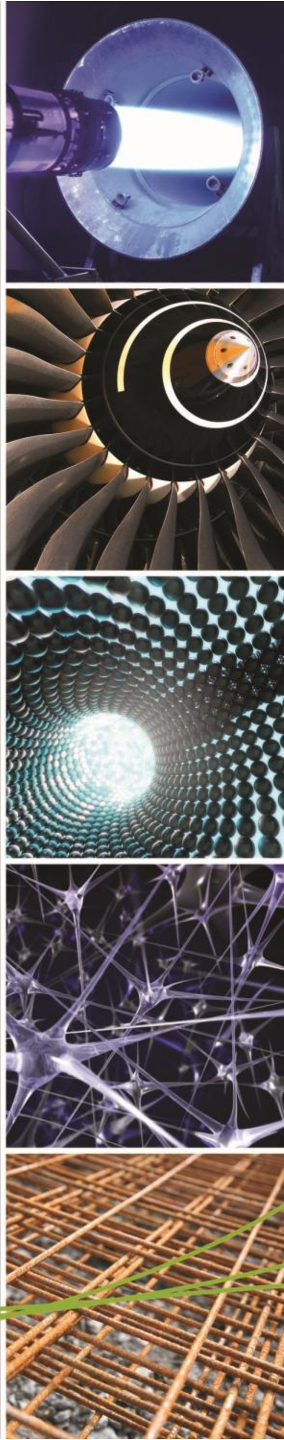
Swansea University
Prifysgol Abertawe

Advanced Structural Analysis
EGF316

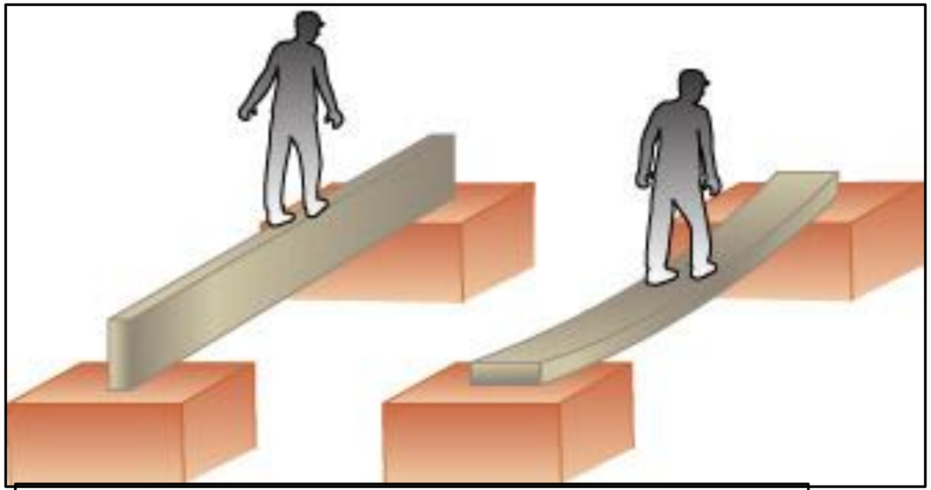
Bending and Section Properties



Lecture Content

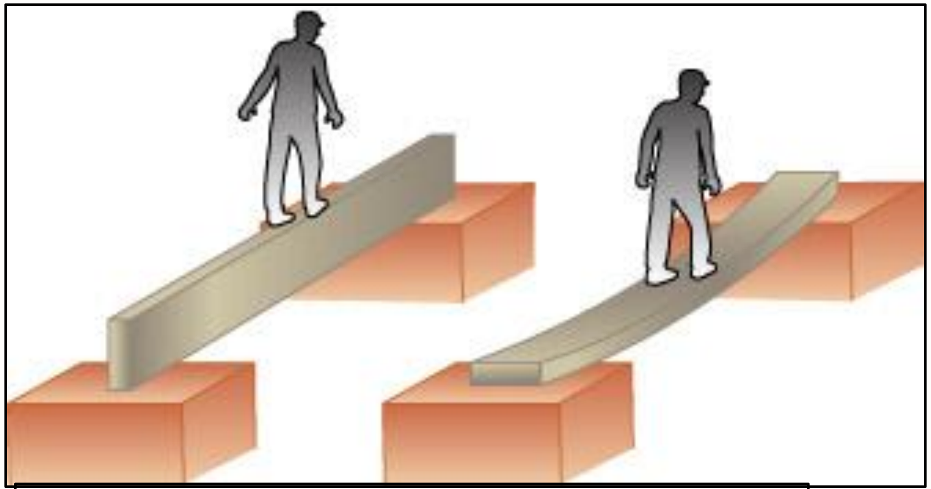
- Beam bending
- Stresses due to bending
- Symmetrical and Unsymmetrical (skew) bending
- Section properties
  - Centroid, First and Second moment of area
- Parallel axis theorem
- Principal axes
- Shear centre
- Torsion of circular sections

Beam bending



Credits: www.dlsweb.rmit.edu.au

Beam bending



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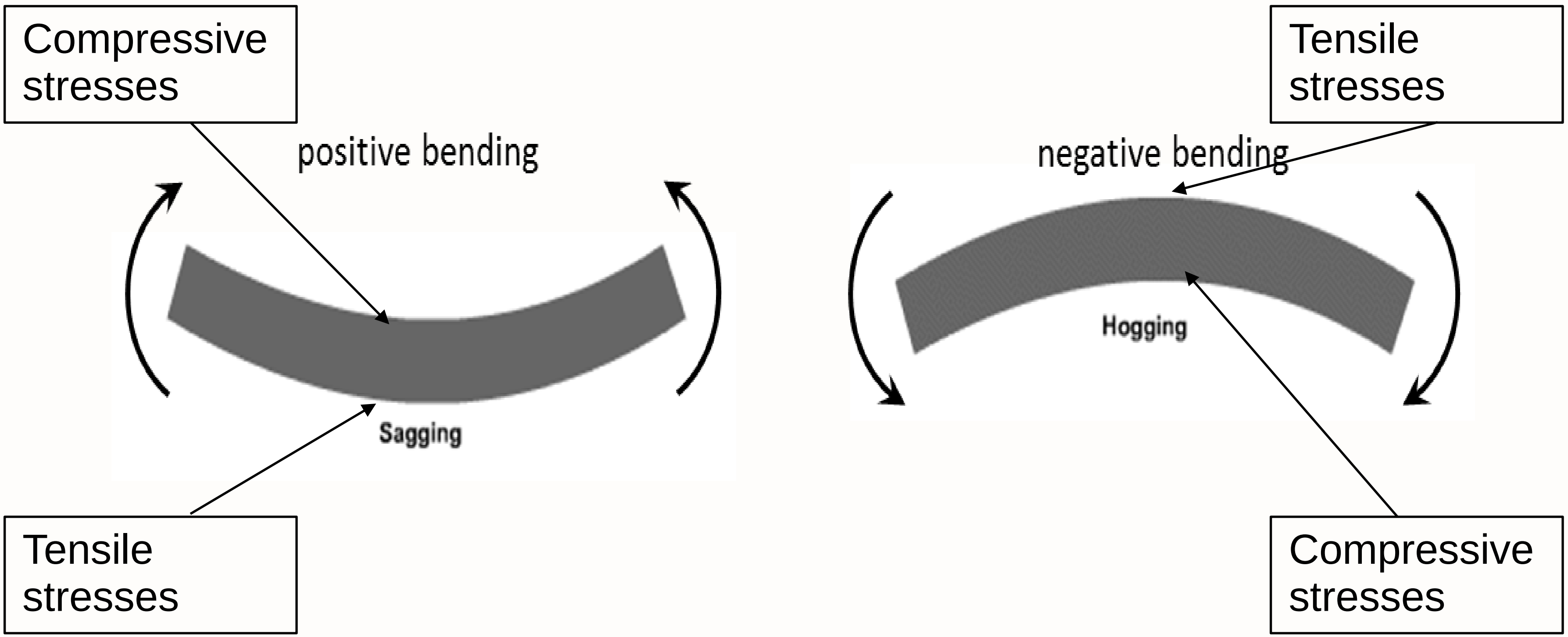
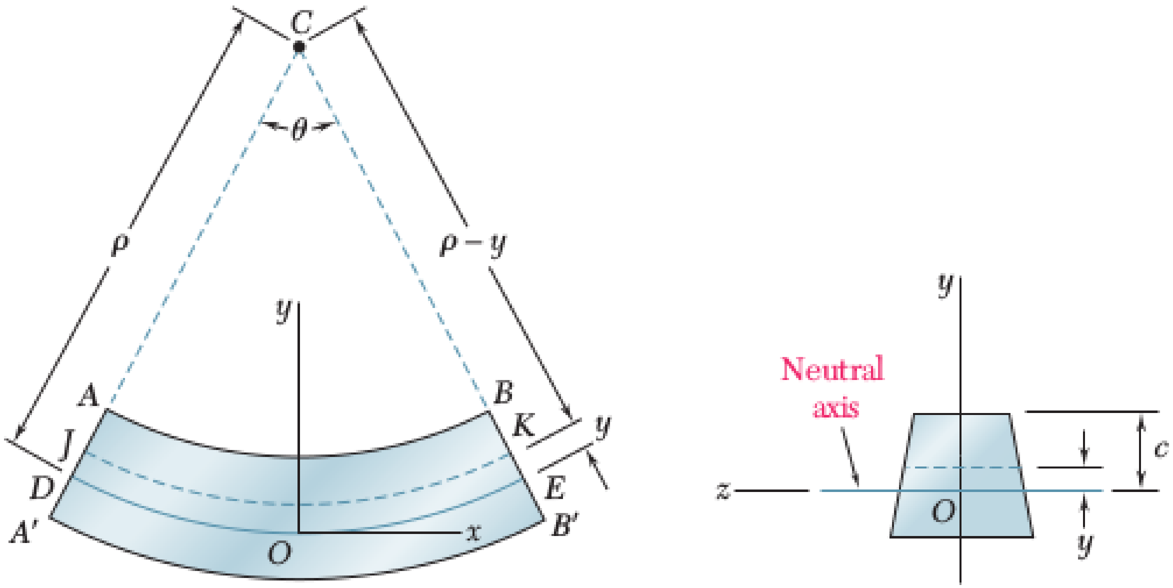
Important quantities:

1) Deflection/displacement (d)
2) Bending moment (M)
3) Shear force (S)
4) Bending stress (s)
5) Second moment of area (I)

Beam bending



Beam bending



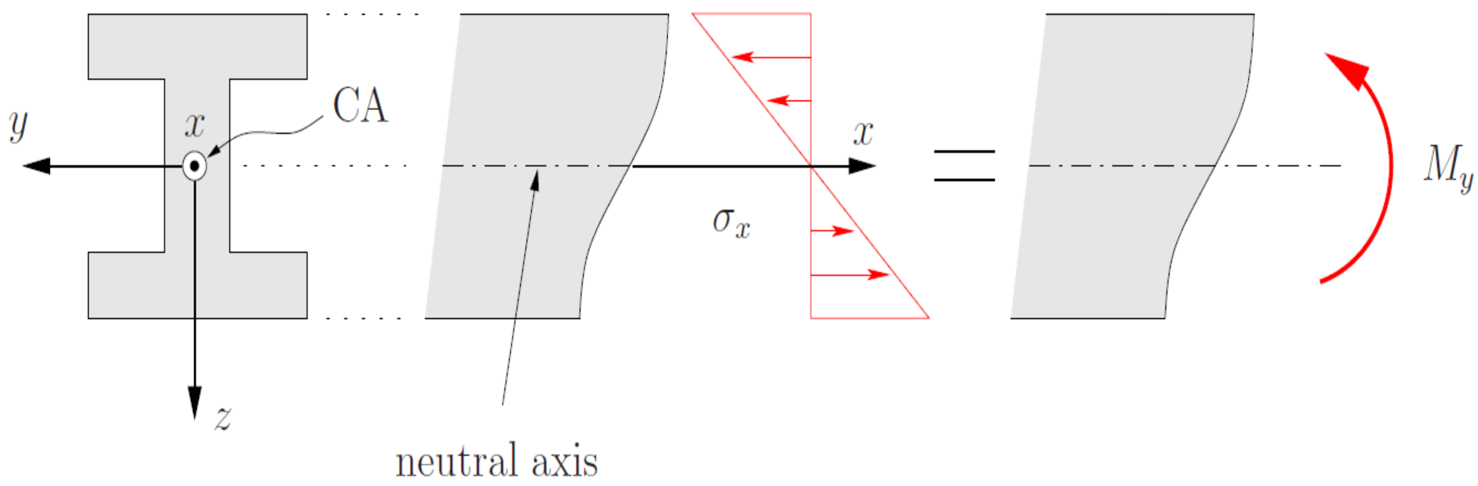
(a) Longitudinal, vertical section (plane of symmetry)

(b) Transverse section

$$EI\frac{d^2y}{dx^2} = M$$

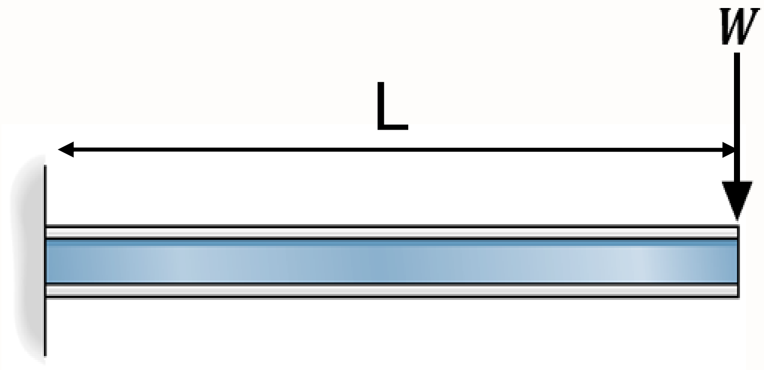
$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Stresses due to bending



$$\sigma = \frac{My}{I}; \ \sigma_{max} = \frac{My_{max}}{I}$$

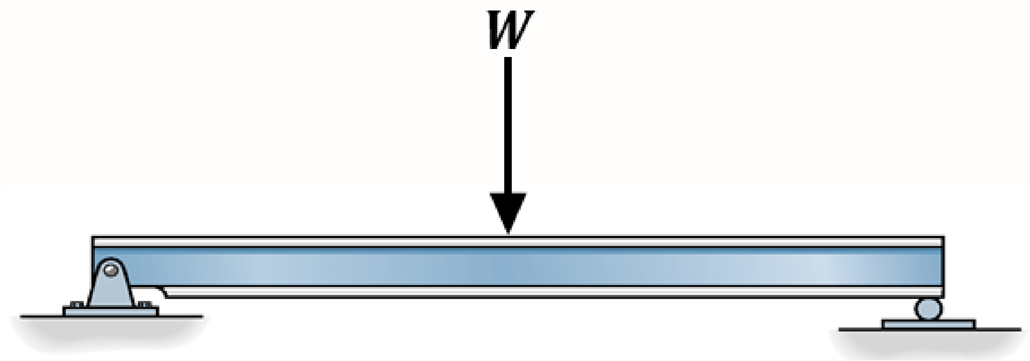
Cantilever Beam



$$M_{max} = WL$$

$$\delta_{max} = \frac{WL^3}{3EI}$$

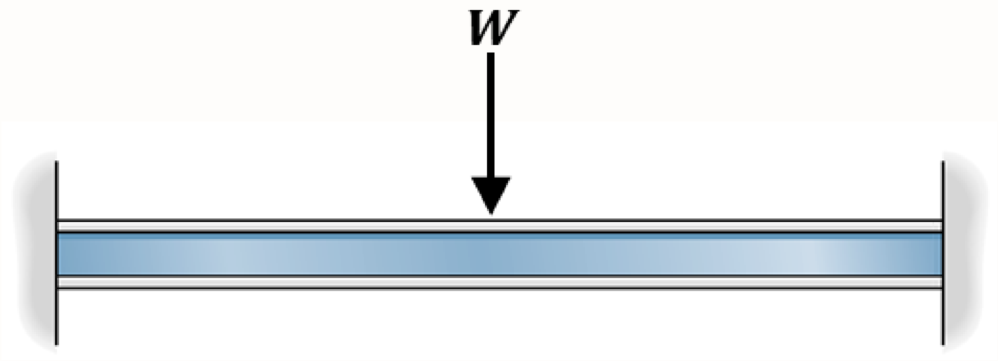
Simply Supported Beam



$$M_{max} = \frac{WL}{4}$$

$$\delta_{max} = \frac{WL^3}{48EI}$$

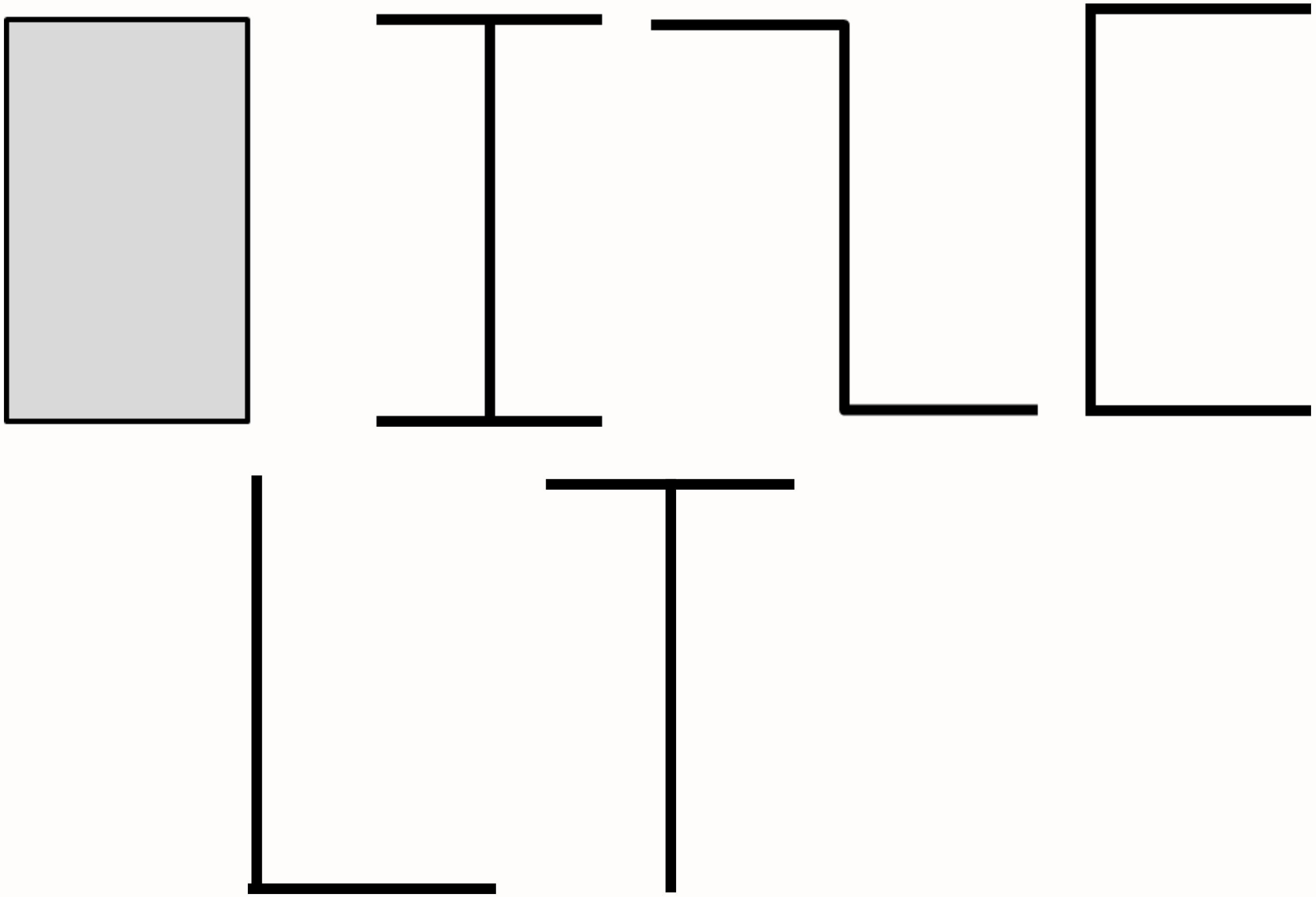
Built-in Beam



$$M_{max} = \frac{WL}{8}$$

$$\delta_{max} = \frac{WL^3}{192EI}$$

Section properties



First Moment of Area

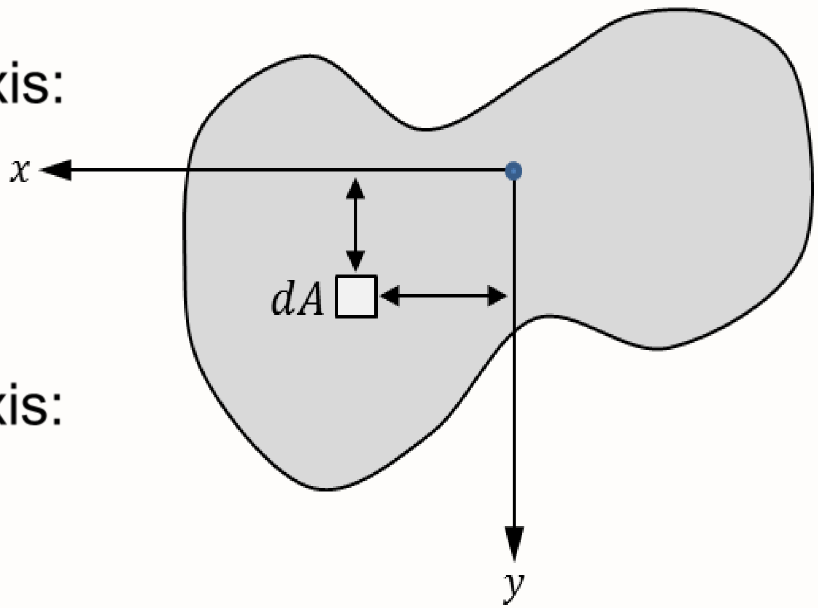
The *first moment of area* is a measure of the distribution of mass relative to an axis

First moment of area A about the x-axis:

$$m_x = \int y dA$$

First moment of area A about the y-axis:

$$m_y = \int x dA$$



The centroid is the point at which the **first moment of area goes to zero** for any orthogonal axis system

Locating the Centroid

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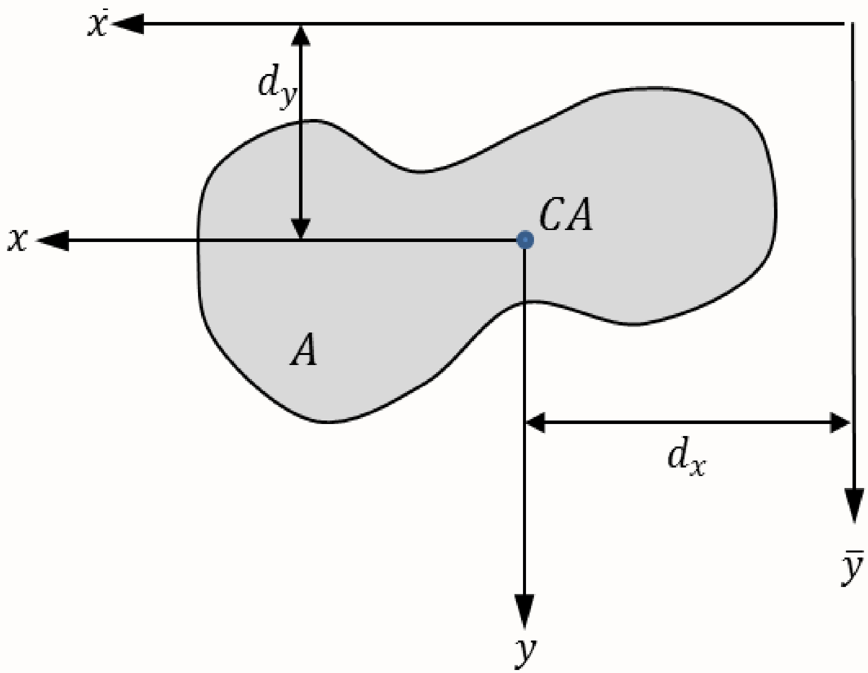
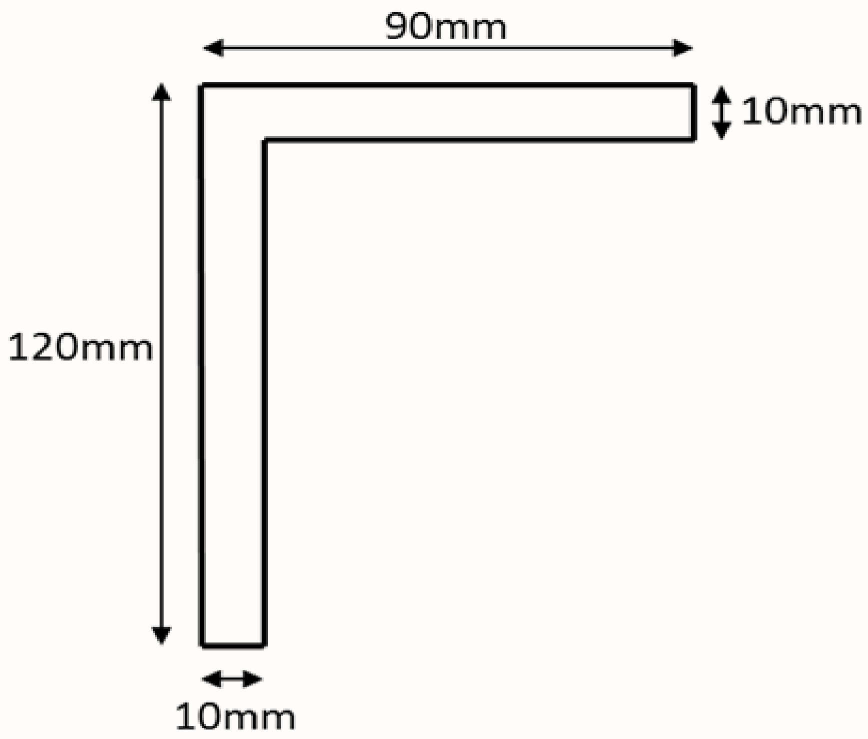
$$d_x = \frac{\int \bar{x}\,dA}{\int dA} = \frac{\sum_i \bar{x}_i A_i}{\sum_i A_i}$$

$$d_y = \frac{\int \bar{y}\,dA}{\int dA} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$$



Example 1

Locate the centroid for the below shape:



Second Moment of Area (moment of inertia)

Second Moment of Area

The *second moment of area* is a section property and is a measure of how far away the material is located from the neutral axis and therefore its resistance to bending.

The second moments of area must be taken relative to the centroid.

Second Moment of Area

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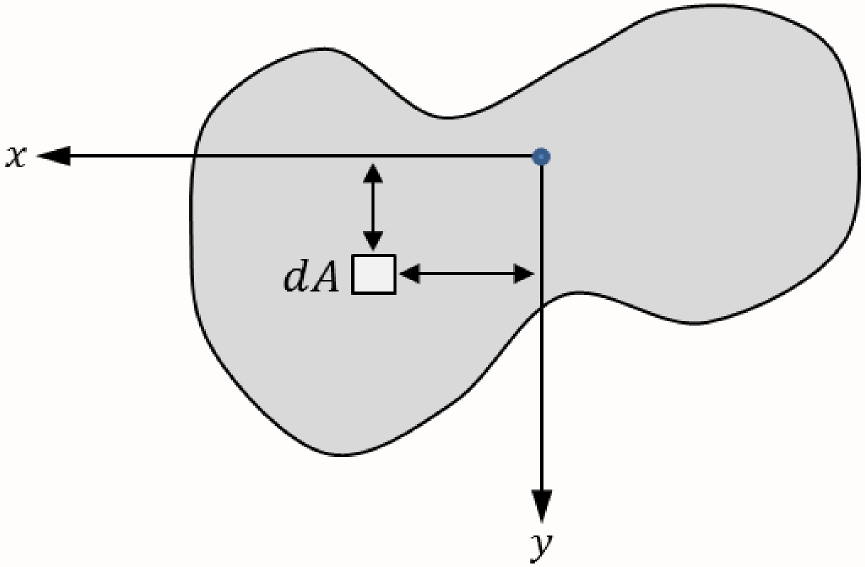
$$I_{xx} = \int y^2 dA = \sum y^2 A$$

$$I_{yy} = \int x^2 dA = \sum x^2 A$$

$$I_{xy} = \int xy dA = \sum xyA$$



I_{xx} and I_{yy} are always positive, but I_{xy} can be positive or negative.

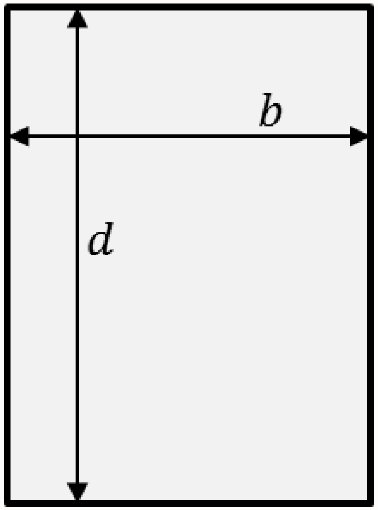
Some useful facts…

For a circular section of diameter D:

$$I = \frac{\pi D^4}{64} \qquad \text{and} \qquad J = \frac{\pi D^4}{32}$$

For a rectangular section:

$$I_{xx} = \frac{bd^3}{12}, \qquad I_{yy} = \frac{b^3 d}{12}, \qquad I_{xy} = 0$$



Parallel Axis Theorem

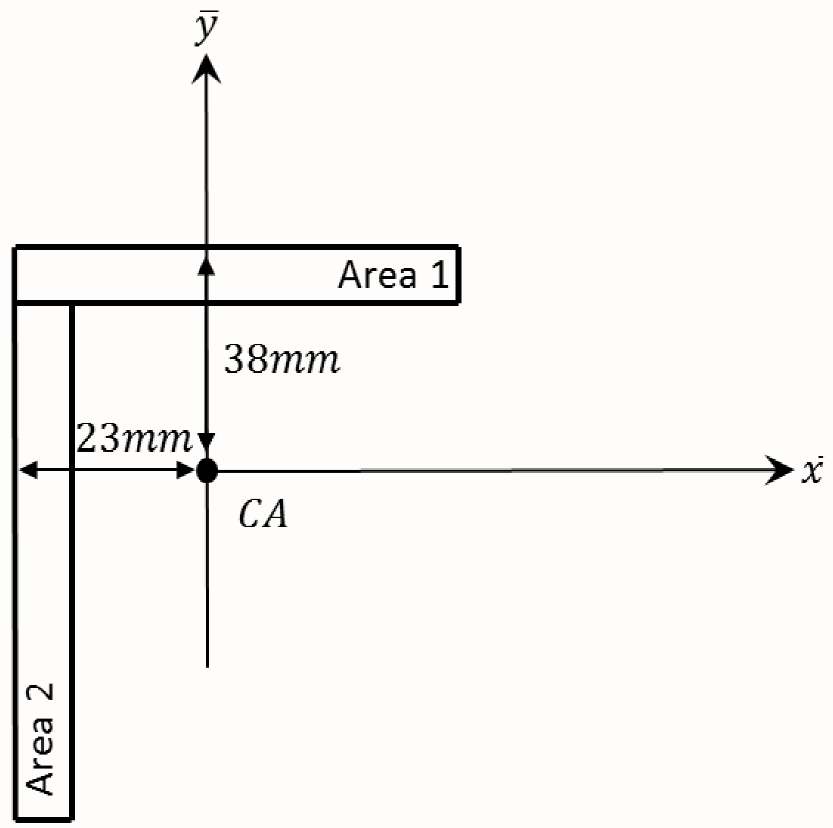
$$I_{xx} = \sum I_{xx_{ic}} + {y_i}^2 A_i$$

$$I_{yy} = \sum I_{y_{ic}} + {x_i}^2 A_i$$

$$I_{xy} = \sum I_{xy_{ic}} + x_i y_i A_i$$

Example (continued)

Calculate I_{xx}, I_{yy} and I_{xy}



Principal Axis

For the rectangle we saw that due to symmetry, the product second moment of area was zero.

There always exists an orientation of the coordinate system such that $I_{yz} = 0$. The associated coordinate axes are called the *principal directions* of the cross section.

A *principal axis* is one where bending about one axis does not result in any deflection (and hence stress/strain) perpendicular to that axis. There is no interaction between the two axes.

It follows that *every axis of symmetry is a principal axis*.

Principal Axis

It can be shown that:

$$\tan 2\theta = \frac{2I_{xy}}{(I_{yy} - I_{xx})}$$

θ is the angle of the principal axes relative to the x and y axes

Principal Axis

The second moments of area about principal axes, I_u and I_v:

$$I_{u,v} = \frac{1}{2}\left(I_{xx} + I_{yy}\right) \pm \frac{1}{2}\left(I_{xx} - I_{yy}\right)\sec 2\theta$$

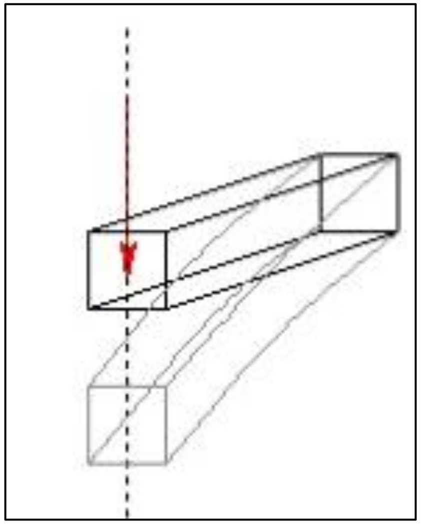
$$I_{u,v} = \frac{1}{2}\left(I_{xx} + I_{yy}\right) \pm \frac{1}{2}\sqrt{\left(I_{xx} - I_{yy}\right)^2 + 4I_{xy}^2}$$

Note that I_u and I_v are always positive

Example (continued)

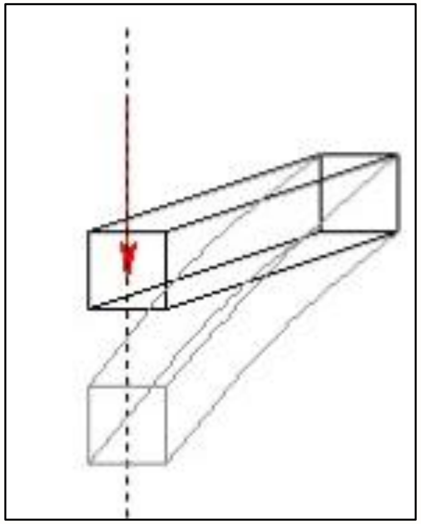
Calculate the second moments of area about principal axes, $I_{u,v}$ for the shape considered previously

Symmetrical Vs Unsymmetrical Bending

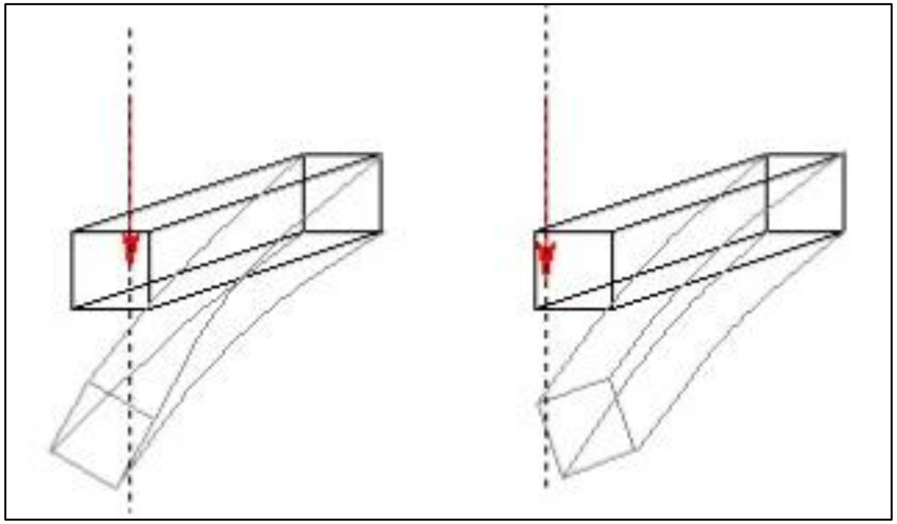


Symmetric

Symmetrical Vs Unsymmetrical Bending



Symmetric – no twist



Unsymmetric – twist

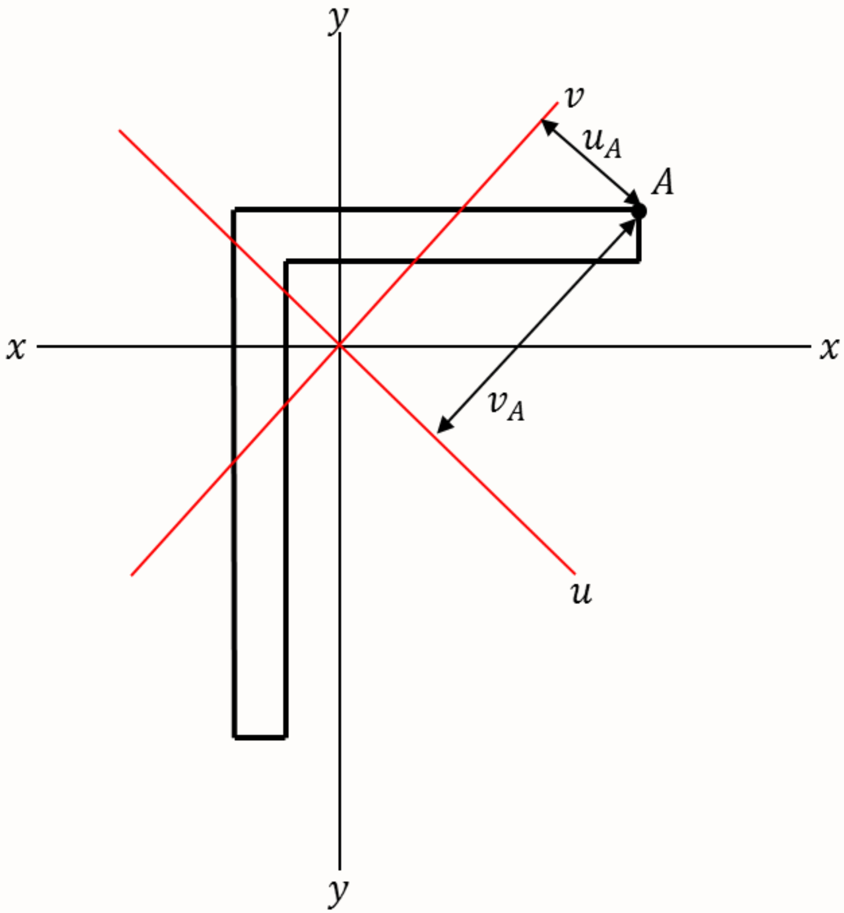
Unsymmetric Bending

$$\sigma_A = \frac{M_u}{I_u} v_A + \frac{M_v}{I_v} u_A$$

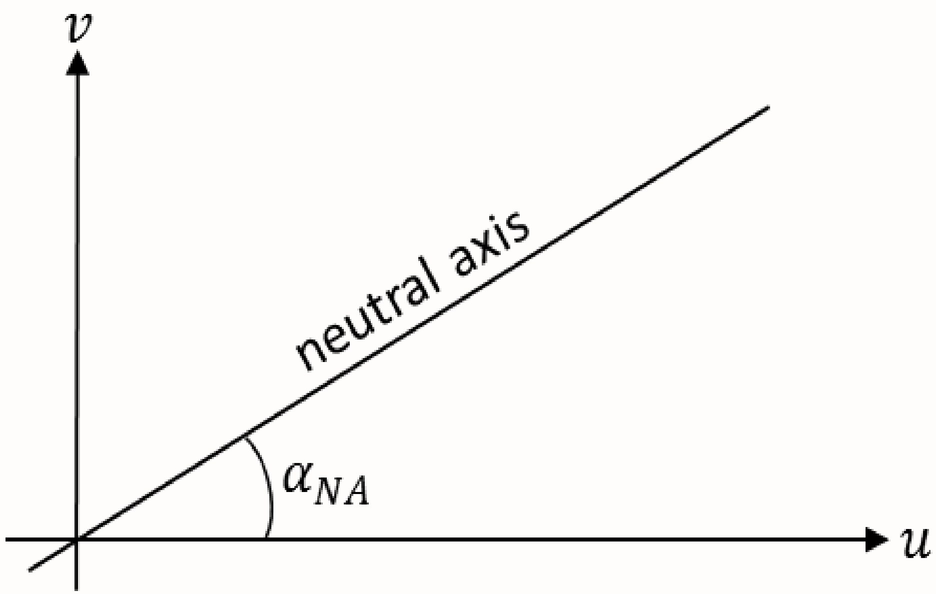
$$M_u = M_x \cos\theta,\ M_v = M_x \sin\theta$$

$$u_A = x_A \cos\theta + y_A \sin\theta$$

$$v_A = y_A \cos\theta - x_A \sin\theta$$



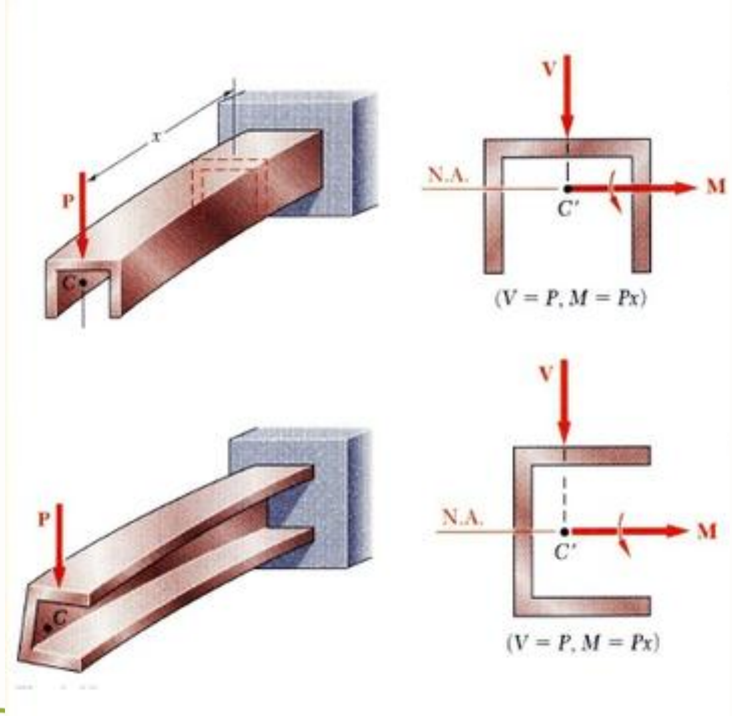
Unsymmetric Bending



$$\tan \alpha_{NA} = -\frac{M_v}{M_u}\frac{I_u}{I_v}$$

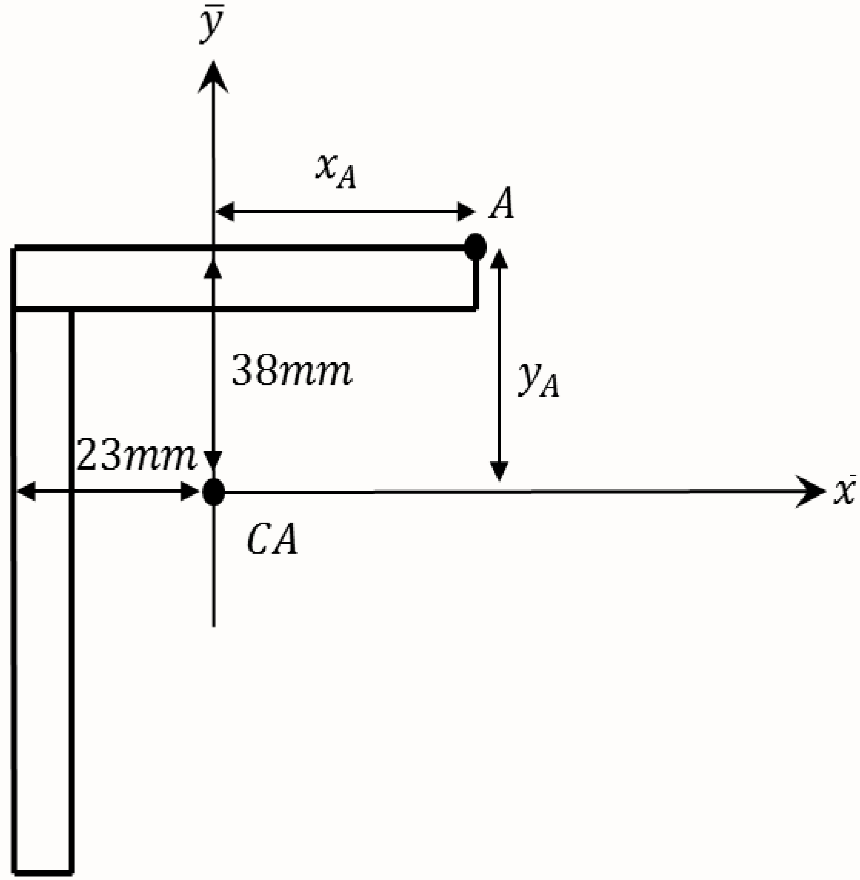
Shear center

Shear center is that point in the cross-section through which the applied loads produce no twisting.



Example 1 (continued)

Calculate the bending stress at Point A if $M_x = 1kNm$



Example 2

A 50mm by 50mm square section steel cantilever beam is 1m long and supports an end load of 100N.

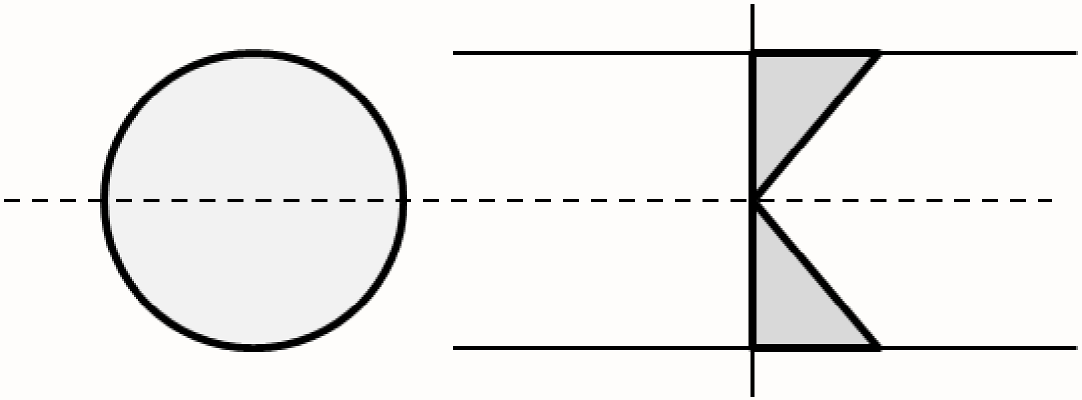
Calculate the maximum bending stress and the maximum deflection in the beam.

Assume Young's modulus to be 210GPa.

Torsion of Circular Sections

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

$$\tau = \frac{Tr}{J}$$



Example 3

A 2m length of 20mm diameter steel bar is subjected to a torque of 5kNm.

Calculate the maximum shear stress and the angle of twist.

Assume a Young's modulus, Poisson's ratio and yield stress of 210GPa, 0.3 and 300MPa respectively