CONTEMPORARY MATHEMATICS

735

# Advances in Complex Geometry

JHU-UMD Complex Geometry Seminar
2015–2018
Johns Hopkins University, Baltimore, Maryland
and
University of Maryland, College Park, Maryland

Yanir A. Rubinstein
Bernard Shiffman
Editors

AMS American Mathematical Society

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# Preface

There is a rich history of research in complex geometry and complex analysis of one and several variables at Johns Hopkins University (JHU; Baltimore, Maryland) and at the University of Maryland (UMD; College Park, Maryland). W.-L. Chow came to Hopkins in 1948, remaining there throughout his career and bringing K. Kodaira to Hopkins from 1962 to 1965. At College Park, an international conference in several complex variables convened in 1970, the proceedings of which are published in a two-part volume[1] edited by J. Horváth. Then in 1973, the University of Maryland held a Special Year in Complex Function Theory; some of the talks during that year are published in a volume[2] edited by W. E. Kirwan and L. Zalcman. As the editors of that volume wrote, the common denominator for the contributions was "a certain emphasis, in point of view or in method, on problems having concrete geometric content". Subsequently, C. Berenstein and the second editor of the present proceedings organized a Hopkins–Maryland joint seminar in complex analysis, which ran through 1989. Seminars on complex geometry then continued at Hopkins through 2012, organized by the second editor of this proceedings together with V. V. Shokurov and S. Zucker during 1991–2004 and with R. Wentworth and S. Zelditch during 2004–10. The complex geometry seminars at Hopkins were also co-organized by J. Noguchi in 1998 and by the first editor of the present proceedings in 2008–09. Additional activity was provided by conferences involving complex geometry sponsored by the Japan–U.S. Mathematics Institute (JAMI)[3] at Johns Hopkins in 1991, 1998, 2004, and 2019.

In 2012, the editors of this proceedings revived the Hopkins–Maryland joint seminar by establishing the JHU-UMD Complex Geometry Seminar, with the assistance of R. Wentworth, S. Wolpert and Y. Yuan. Topics presented at the seminar include geometric flows, canonical metrics, geometric stability, pluripotential theory, the Monge–Ampère equation, zeros of random holomorphic sections, $L^2$ extension, deformation theory, Bergman kernel, special Lagrangians, and Gromov–Witten theory, as well as topics on the interface of complex geometry with convex geometry, symplectic geometry, algebraic geometry, and topology.

This volume contains contributions from speakers at the seminar during three academic years from 2015 to 2018. The volume begins with a survey by T. Darvas of recent developments in pluripotential theory and its application to Kähler–Einstein metrics. The next article is a survey by S. Dinew of recent advances in the theory of local regularity of plurisubharmonic functions and the complex Monge–Ampère

[1] *Several Complex Variables I, II*, Lecture Notes in Mathematics, Vols. 155, 185, Springer, 1970, 1971.

[2] *Advances in Complex Function Theory*, Lecture Notes in Mathematics, Vol. 505, Springer, 1976.

[3] https://mathematics.jhu.edu/events/jami/.

equation. The article by F. R. Harvey and H. B. Lawson concerns properties of pluriharmonic functions arising in the generalized potential theories associated to subequations. Next, C. Li and G. Tian discuss orbifold regularity for weak Kähler–Einstein metrics of certain singular Fano varieties via resolution of singularities. The article by Z. Lu and H. Xu concerns the spectrum and self-adjointness properties of the Laplacian on the moduli space of polarized Calabi-Yau manifolds. L. Ni and F. Zheng review recent progress in the study of compact Kähler manifolds with positive orthogonal Ricci curvature. D. H. Phong, S. Picard, and X. Zhang describe the long-time behavior of the Anomaly flow on unimodular Lie groups. The volume concludes with an article by Z. Slodkowski on the pseudoconcave decomposition of the core of a complex manifold.

We would like to thank T. Darvas, H. J. Hein, J. Martinez-Garcia, V. P. Pingali, R. Wentworth, S. Wolpert, and H. Xu for their assistance in the organization of the seminar during 2015–2018. We are also grateful for the financial and logistical support of the mathematics departments at Johns Hopkins University and the University of Maryland. Finally, we thank the speakers for their inspiring contributions to the mathematical activities of our two institutions.

Yanir A. Rubinstein
Bernard Shiffman

## List of seminars

**2015–2016**

Gábor Székelyhidi (University of Notre Dame)
*The J-flow on toric manifolds*

Ruadhaí Dervan (University of Cambridge)
*K-stability of finite covers*

Gang Tian (Princeton University)
*K-stability implies CM-stability*

Blaine Lawson (Stony Brook University)
*Differential inequalities and generalized pluripotential theories*

Zhiqin Lu (University of California at Irvine)
*On the $L^2$ estimates on moduli space of Calabi-Yau manifolds*

Tamás Darvas (University of Maryland)
*Infinite-dimensional geometry on the space of Kähler metrics and applications to canonical Kähler metrics*

Joaquim Ortega-Cerdà (Universitat de Barcelona)
*Sampling polynomials in algebraic varieties*

Dror Varolin (Stony Brook University)
*Berndtsson's Convexity Theorem and the $L^2$ Extension Theorem*

**2016–2017**

Ben Weinkove (Northwestern University)
*Monge-Ampère equations on complex and almost complex manifolds*

Hao Xu (University of Pittsburgh)
*Asymptotic expansion of Bergman and heat kernels*

Mu-Tao Wang (Columbia University)
*Lagrangian curvature flows in cotangent bundles of spheres*

Jake Solomon (Hebrew University of Jerusalem)
*Point-like bounding chains in open Gromov-Witten theory*

Duong Phong (Columbia University)
*Supersymmetric vacua of superstrings and geometric flows*

Xiaofeng Sun (Lehigh University)
*Deformation of Fano manifolds*

Xiaojun Huang (Rutgers University)
*Bergman-Einstein metrics on strongly pseudoconvex domains of* $\mathbb{C}^n$.

Mattias Jonsson (University of Michigan)
*A variational approach to the Yau-Tian-Donaldson conjecture*

**2017–2018**

Zbigniew Slodkowski (University of Illinois at Chicago)
*Pseudoconcave decompositions in complex manifolds*

Sebastien Picard (Columbia University)
*The anomaly flow and the Hull-Strominger system*

Sławomir Dinew (Jagiellonian University, Kraków)
*Singular sets of plurisubharmonic functions*

Thomas Bloom (University of Toronto)
*Universality for zeros of random polynomials*

Lei Ni (University of California at San Diego)
*Metric characterizations of the projectivity*

Jeff Streets (University of California at Irvine)
*Generalized Kähler-Ricci flow in the commuting case*

735 **Yanir A. Rubinstein and Bernard Shiffman, Editors,** Advances in Complex Geometry, 2019

731 **Robert G. Niemeyer, Erin P. J. Pearse, John A. Rock, and Tony Samuel, Editors,** Horizons of Fractal Geometry and Complex Dimensions, 2019

730 **Alberto Facchini, Lorna Gregory, Sonia L'Innocente, and Marcus Tressl, Editors,** Model Theory of Modules, Algebras and Categories, 2019

729 **Daniel G. Davis, Hans-Werner Henn, J. F. Jardine, Mark W. Johnson, and Charles Rezk, Editors,** Homotopy Theory: Tools and Applications, 2019

728 **Nicolás Andruskiewitsch and Dmitri Nikshych, Editors,** Tensor Categories and Hopf Algebras, 2019

727 **André Leroy, Christian Lomp, Sergio López-Permouth, and Frédérique Oggier, Editors,** Rings, Modules and Codes, 2019

726 **Eugene Plotkin, Editor,** Groups, Algebras and Identities, 2019

725 **Shijun Zheng, Marius Beceanu, Jerry Bona, Geng Chen, Tuoc Van Phan, and Avy Soffer, Editors,** Nonlinear Dispersive Waves and Fluids, 2019

724 **Lubjana Beshaj and Tony Shaska, Editors,** Algebraic Curves and Their Applications, 2019

723 **Donatella Danielli, Arshak Petrosyan, and Camelia A. Pop, Editors,** New Developments in the Analysis of Nonlocal Operators, 2019

722 **Yves Aubry, Everett W. Howe, and Christophe Ritzenthaler, Editors,** Arithmetic Geometry: Computation and Applications, 2019

721 **Petr Vojtěchovský, Murray R. Bremner, J. Scott Carter, Anthony B. Evans, John Huerta, Michael K. Kinyon, G. Eric Moorhouse, and Jonathan D. H. Smith, Editors,** Nonassociative Mathematics and its Applications, 2019

720 **Alexandre Girouard, Editor,** Spectral Theory and Applications, 2018

719 **Florian Sobieczky, Editor,** Unimodularity in Randomly Generated Graphs, 2018

718 **David Ayala, Daniel S. Freed, and Ryan E. Grady, Editors,** Topology and Quantum Theory in Interaction, 2018

717 **Federico Bonetto, David Borthwick, Evans Harrell, and Michael Loss, Editors,** Mathematical Problems in Quantum Physics, 2018

716 **Alex Martsinkovsky, Kiyoshi Igusa, and Gordana Todorov, Editors,** Surveys in Representation Theory of Algebras, 2018

715 **Sergio R. López-Permouth, Jae Keol Park, S. Tariq Rizvi, and Cosmin S. Roman, Editors,** Advances in Rings and Modules, 2018

714 **Jens Gerlach Christensen, Susanna Dann, and Matthew Dawson, Editors,** Representation Theory and Harmonic Analysis on Symmetric Spaces, 2018

713 **Naihuan Jing and Kailash C. Misra, Editors,** Representations of Lie Algebras, Quantum Groups and Related Topics, 2018

712 **Nero Budur, Tommaso de Fernex, Roi Docampo, and Kevin Tucker, Editors,** Local and Global Methods in Algebraic Geometry, 2018

711 **Thomas Creutzig and Andrew R. Linshaw, Editors,** Vertex Algebras and Geometry, 2018

710 **Raphaël Danchin, Reinhard Farwig, Jiří Neustupa, and Patrick Penel, Editors,** Mathematical Analysis in Fluid Mechanics, 2018

709 **Fernando Galaz-García, Juan Carlos Pardo Millán, and Pedro Solórzano, Editors,** Contributions of Mexican Mathematicians Abroad in Pure and Applied Mathematics, 2018

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This volume contains contributions from speakers at the 2015–2018 joint Johns Hopkins University and University of Maryland Complex Geometry Seminar. It begins with a survey article on recent developments in pluripotential theory and its applications to Kähler–Einstein metrics and continues with articles devoted to various aspects of the theory of complex manifolds and functions on such manifolds.

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