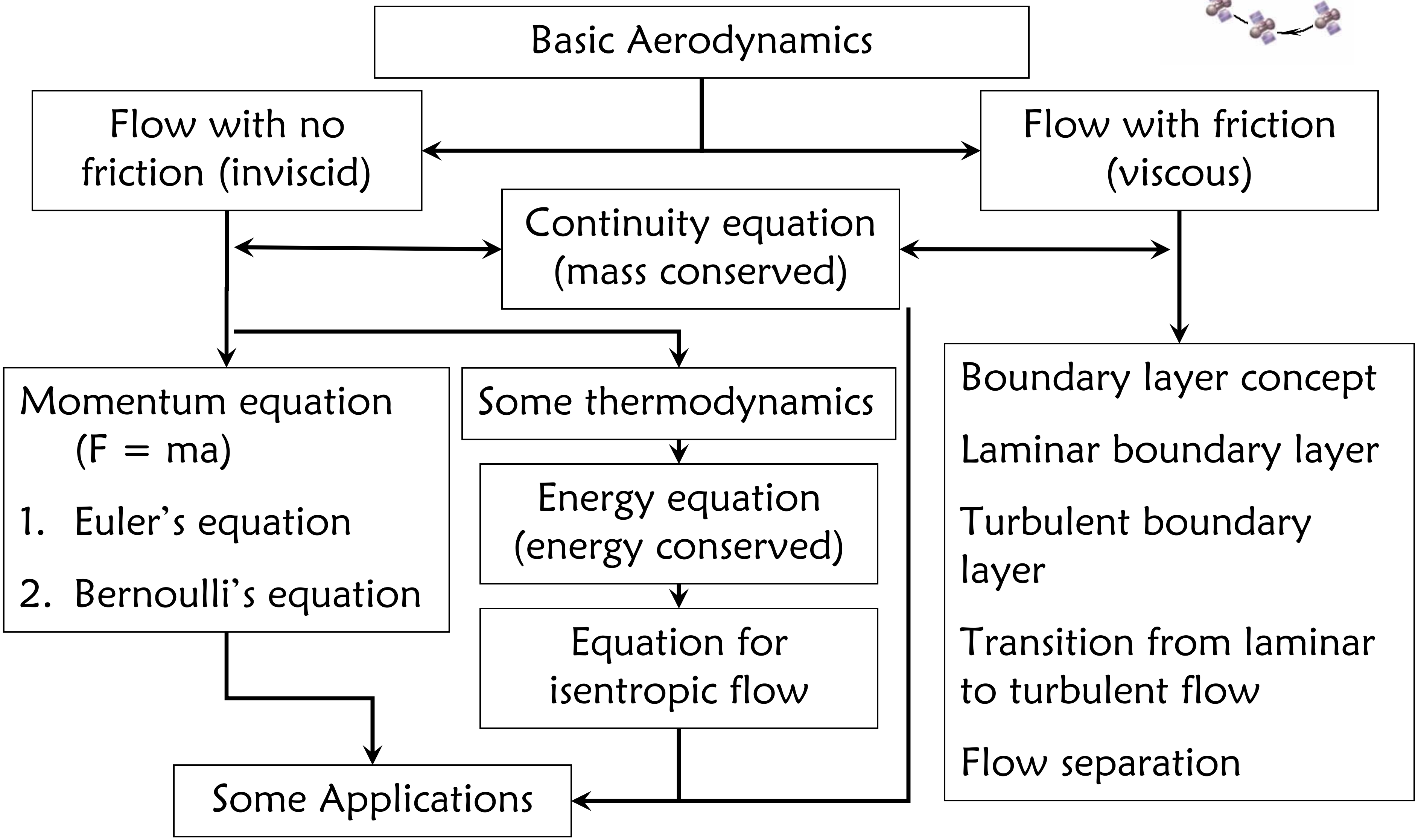
Virginia Tech

# Aerodynamics

AOE



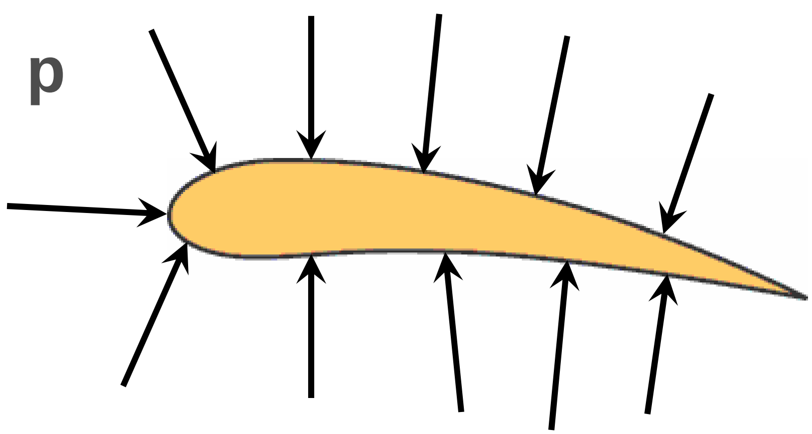

Reading: Chapter 4

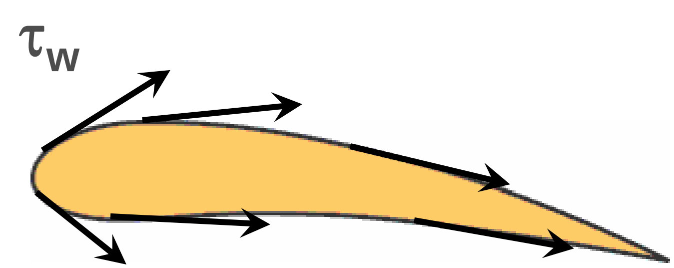
# Recall: Aerodynamic Forces

- "Theoretical and experimental aerodynamicists labor to calculate and measure flow fields of many types."
- … because "the aerodynamic force exerted by the airflow on the surface of an airplane, missile, *etc.*, stems from only two simple natural sources:

  Pressure distribution on the surface (normal to surface)

  Shear stress (friction) on the surface (tangential to surface)



# Fundamental Principles

- Conservation of mass
  - ⇒ Continuity equation (§§ 4.1-4.2)
- Newton's second law (F = ma)
  - ⇒ Euler's equation & Bernoulli's equation (§§ 4.3-4.4)
- Conservation of energy
  - ⇒ Energy equation (§§ 4.5-4.7)

# First: Buoyancy

- One way to get lift is through Archimedes' principle of buoyancy
- The buoyancy force acting on an object in a fluid is equal to the weight of the volume of fluid displaced by the object
- Requires integral

(assume $\rho_0$ is constant)

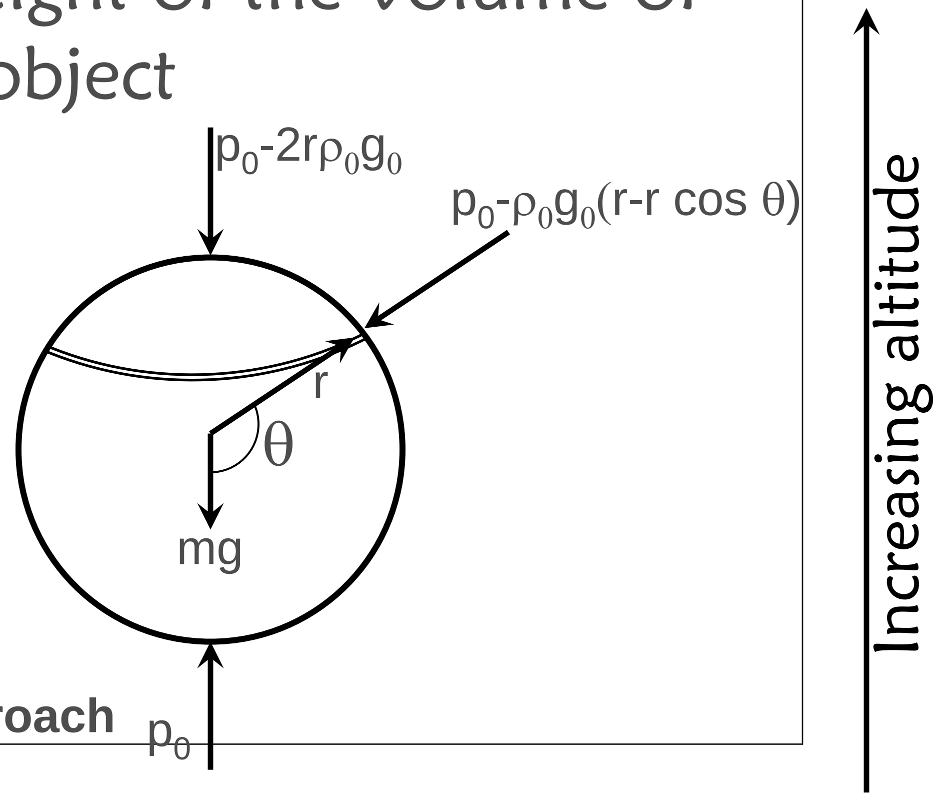
$p = p_0 - \rho_0 g_0 (r - r\cos\theta)$

Force is

$p\,dA = [p_0 - \rho_0 g_0 (r - r\cos\theta)]\,dA$

$dA = 2\pi r^2 \sin\theta\, d\theta$

**Integrate using "shell element" approach**



# Buoyancy: Integration Over Surface of Sphere

- Each shell element is a ring with radius $r \sin \theta$, and width $r\,d\theta$

  Thus the differential area of an element is

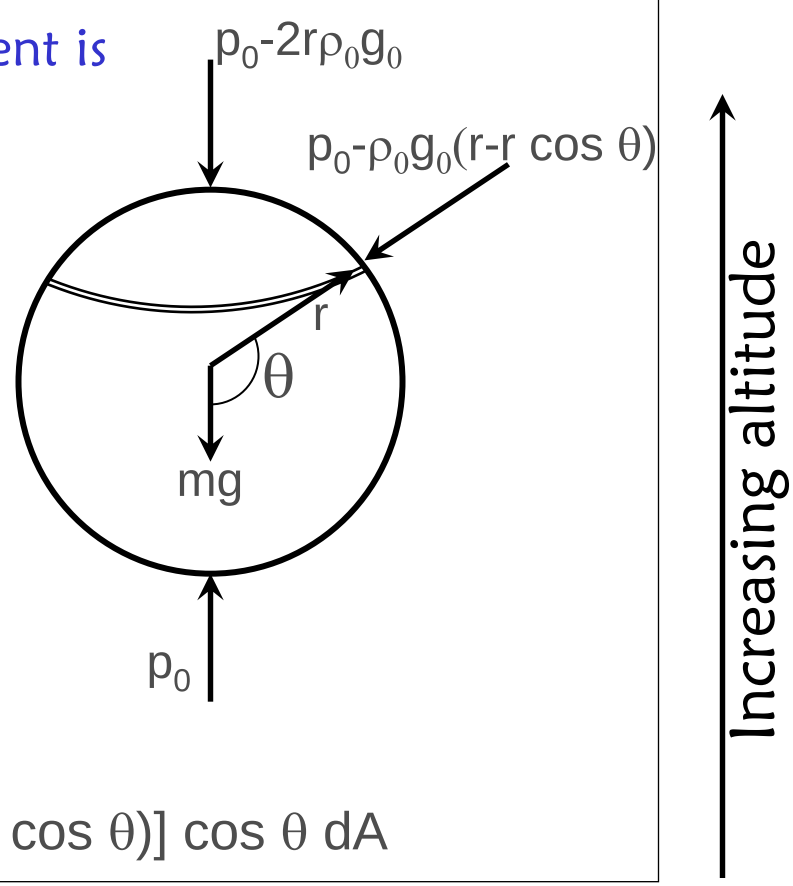
  $dA = 2\pi r^2 \sin\theta\, d\theta$

- Pressure at each point on an element is

  $p = p_0 - \rho_0 g_0 (r - r\cos\theta)$

- Force is pressure times area

  $dF = p\,dA = [p_0 - \rho_0 g_0 (r - r\cos\theta)]\,dA$




- Vertical pressure force is

  $dF\cos\theta = p\,dA\cos\theta = [p_0 - \rho_0 g_0 (r - r\cos\theta)]\cos\theta\,dA$

# Buoyancy: Integration Over Surface of Sphere (continued)

- Total vertical pressure force is found by integrating from $\theta = 0$ to $\theta = \pi$ :

$$F_{vp} = 2\pi r^2 \int [p_0 - \rho_0 g_0 (r - r\cos\theta)] \cos\theta \sin\theta \, d\theta$$

- Some useful identities:

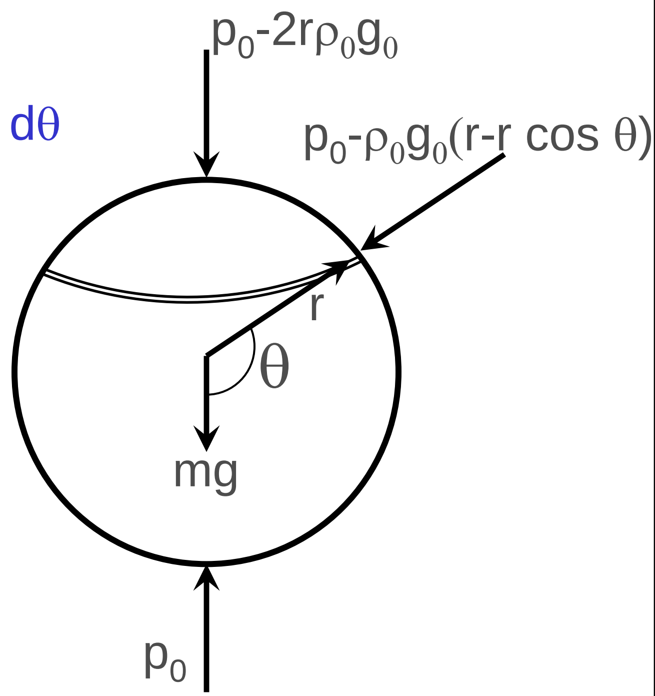
$$\int \cos\theta \sin\theta \, d\theta = \tfrac{1}{2}\sin^2\theta$$

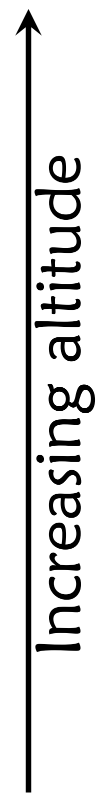
$$\int \cos^2\theta \sin\theta \, d\theta = -\tfrac{1}{3}\cos^3\theta$$

- Put them together to get

$$F_{vp} = \tfrac{4}{3}\pi r^3 \bullet \rho_0 \bullet g_0$$

- The first bit is the volume of the sphere; multiplying by density gives mass of fluid displaced; multiplying by gravity gives weight of fluid displaced



# Buoyancy: Forces on a Sphere (continued)

- Total vertical pressure force is

  $F_{vp} = {}^{4}/_{3}\pi r^{3} \bullet \rho_0 \bullet g_0$

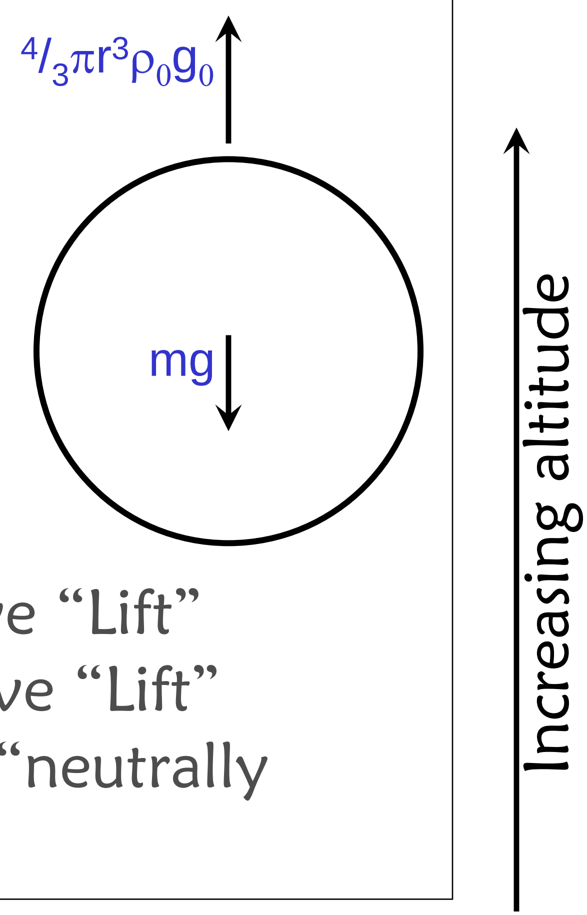
  or

  $F_{vp} = W_v$ (weight of volume of fluid)

- Thus the total vertical force on the sphere is

  $F_v = W_v - W_s$

  where $W_s = mg$ is the weight of the sphere




- If $W_v > W_s$, then the net force is a positive "Lift"
- If $W_v < W_s$, then the net force is a negative "Lift"
- If $W_v = W_s$, then the sphere is said to be "neutrally buoyant"

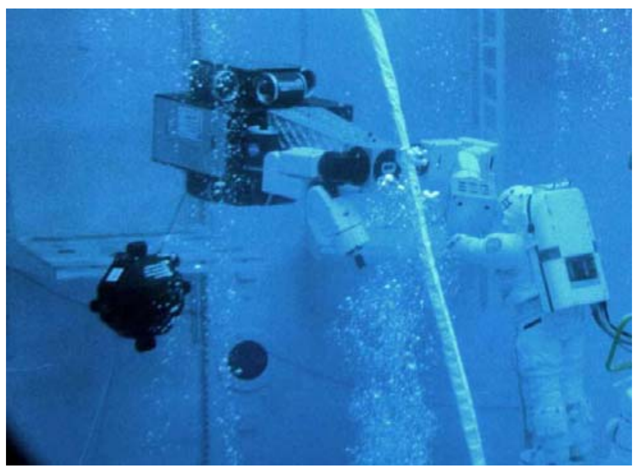
# Neutral Buoyancy Tanks

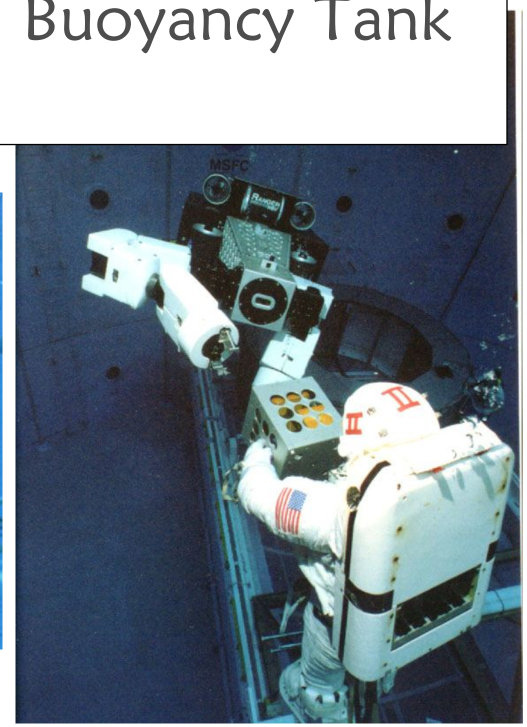
- Neutral buoyancy is useful for simulating the freefall environment experienced by astronauts
- NASA's Marshall Space Flight Center has a Neutral Buoyancy Simulator

  http://www1.msfc.nasa.gov/NEWSROOM/background/facts/nbs.htm
- University of Maryland has a Neutral Buoyancy Tank

  http://www.ssl.umd.edu/facilities/facilities.html

The deck area of the neutral buoyancy tank. In the water is a mockup of an International Space Station truss.



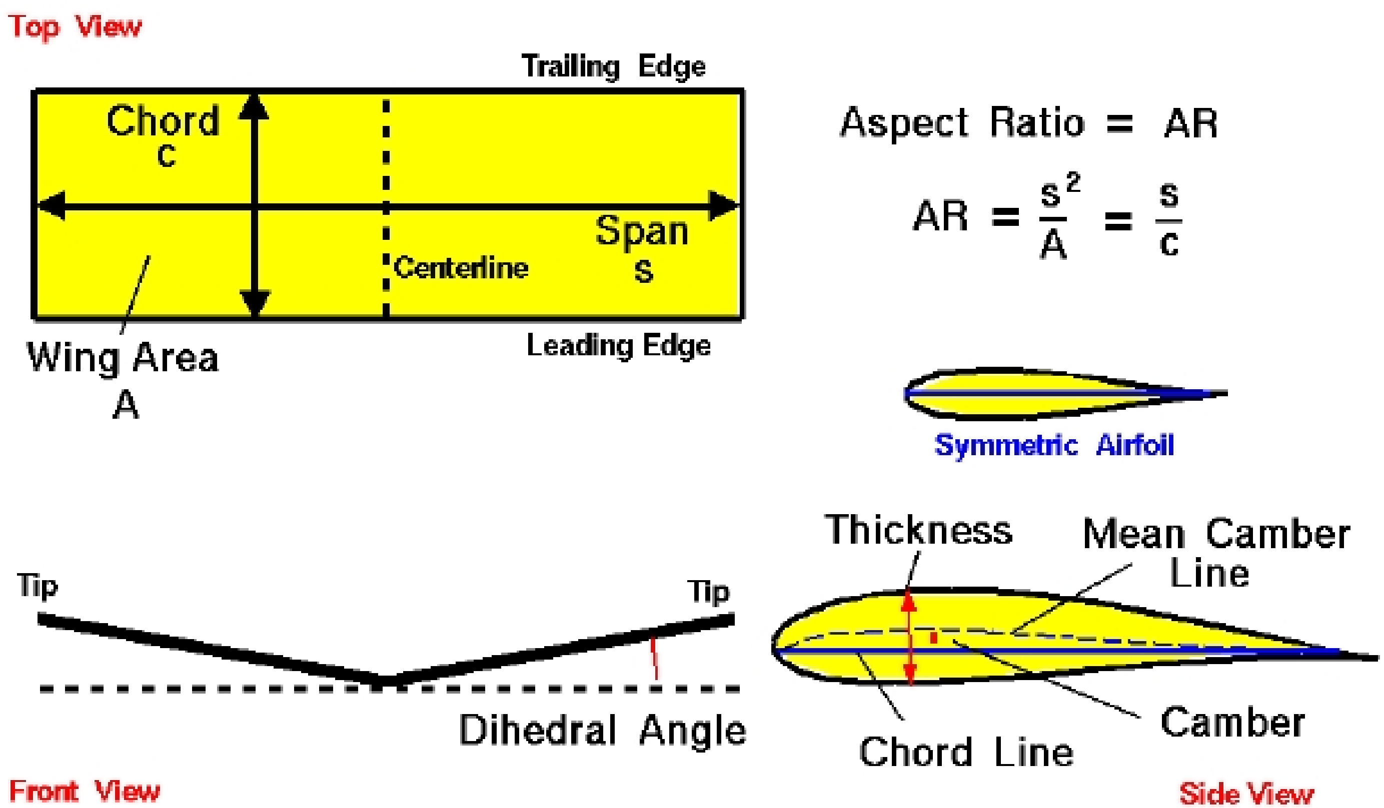


# What's In Our Toolbox So Far?

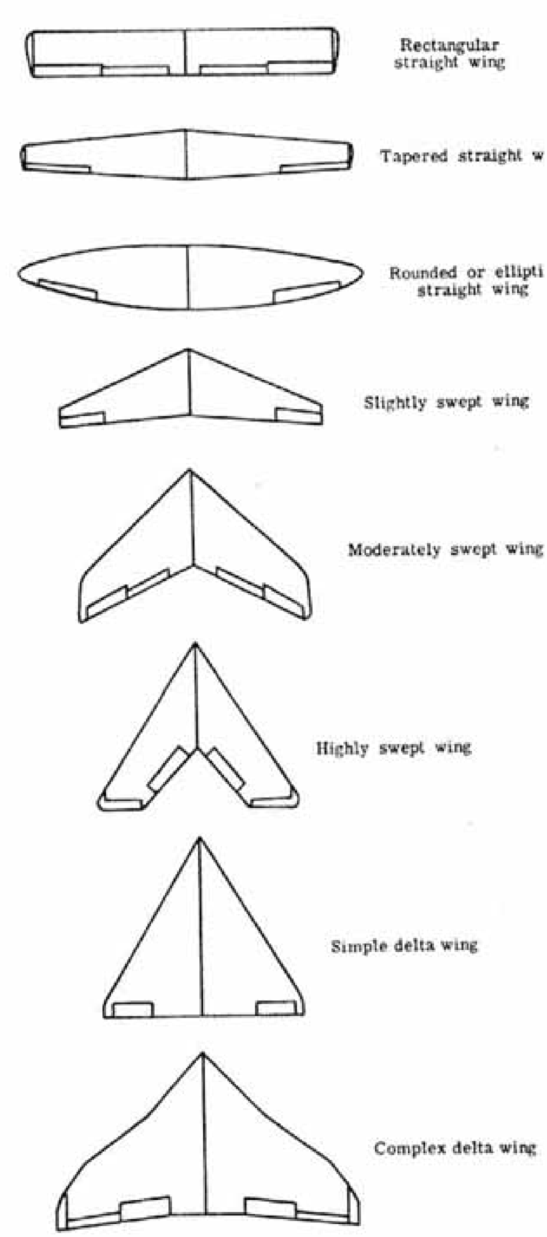
- Four aerodynamic quantities, flow field
- Steady vs unsteady flow
- Streamlines
- Two sources of all aerodynamic forces
- Equation of state for perfect gas
- Standard atmosphere: six different altitudes
- Hydrostatic equation
- Linear interpolation, local approximation
- Lift due to buoyancy
- Viscous vs inviscous flow

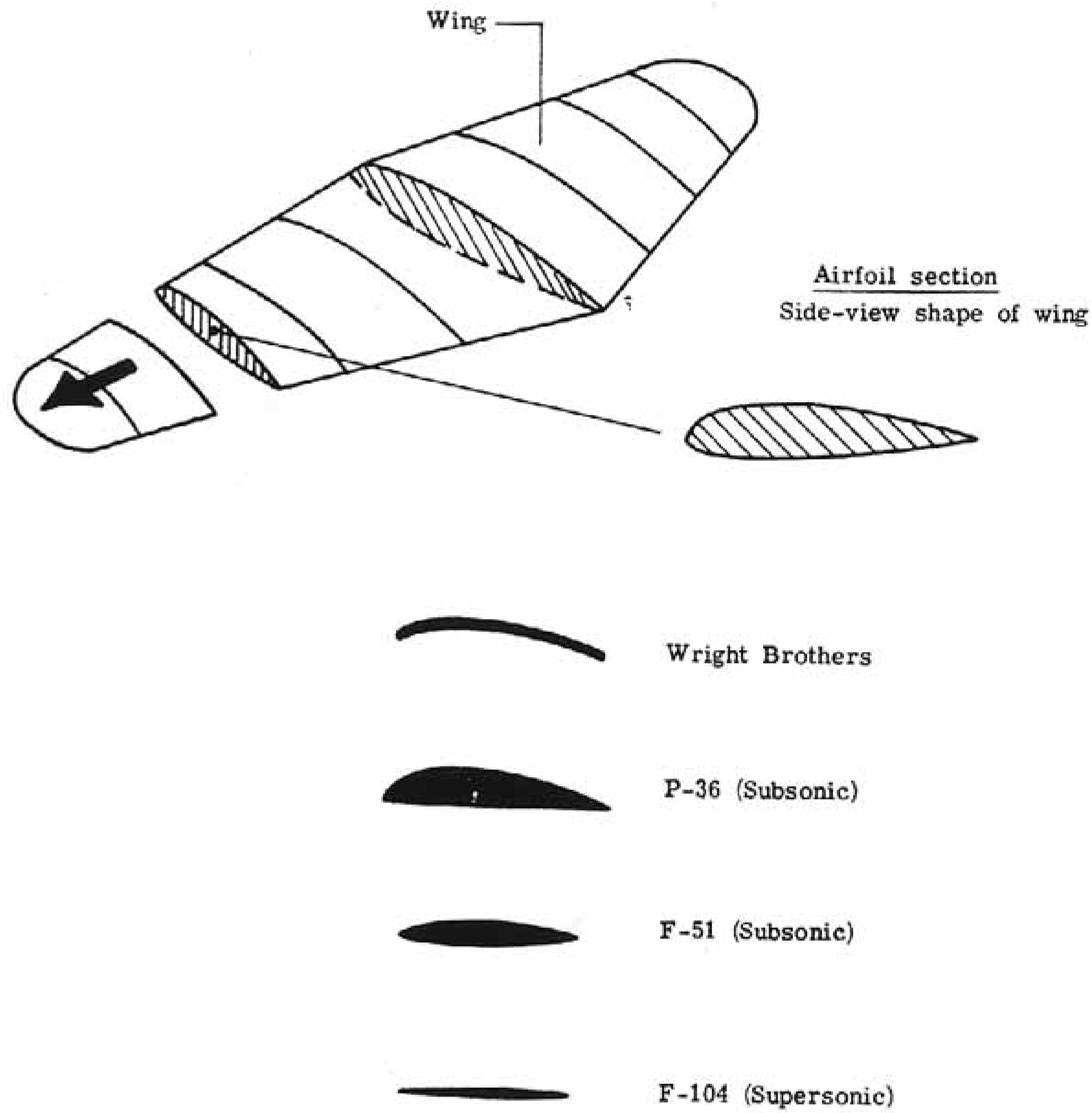
# Lift from Fluid Motion

- **First:** Airplane wing geometry
- Span, Chord, Area, Planform, Aspect Ratio, Camber, Leading and Trailing Edges



# Some Wing Shapes



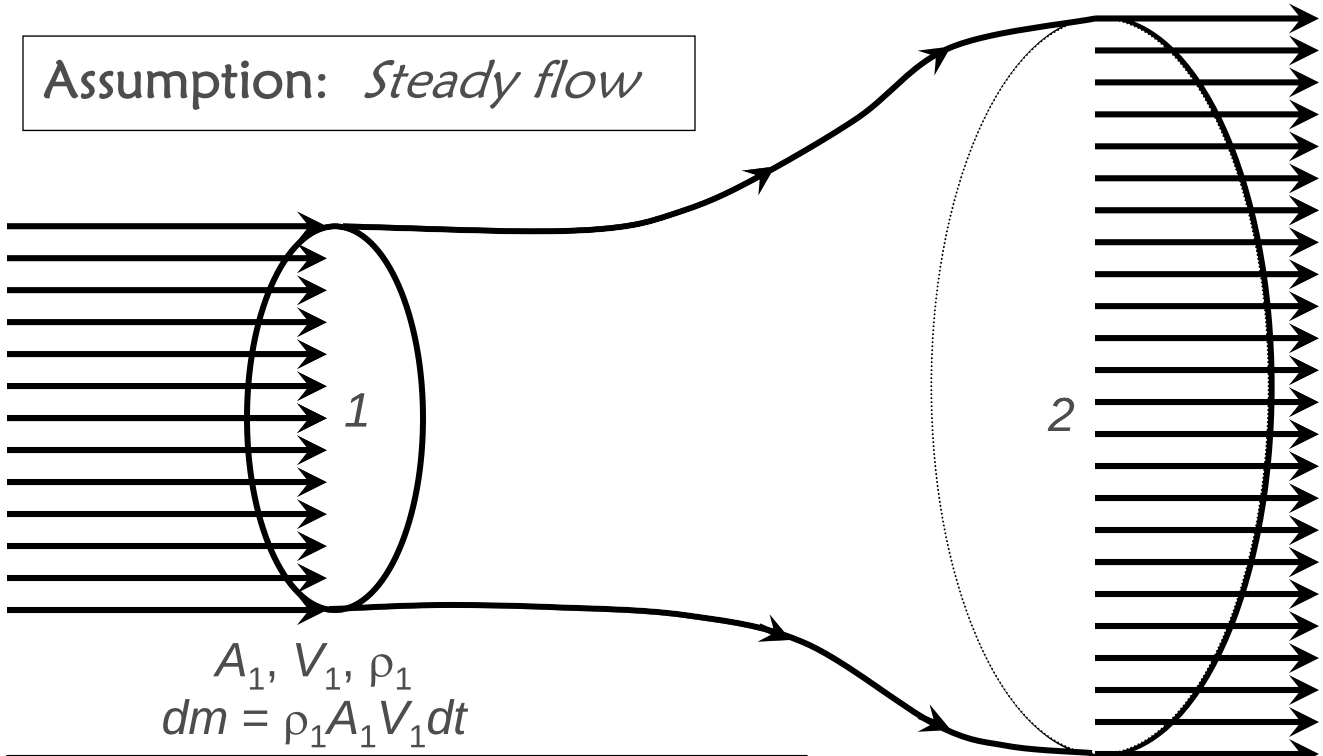
Virginia Tech

AOE

# Continuity

**Physical principle:** Mass can be neither created nor destroyed.

**Assumption:** *Steady flow*




$A_1, V_1, \rho_1$
$dm = \rho_1 A_1 V_1 dt$

$A_2, V_2, \rho_2$
$dm = \rho_2 A_2 V_2 dt$

Volume bounded by streamlines is called a *stream tube*

At entry point (1):

$dm/dt = \rho_1 A_1 V_1$

At exit point (2):

$dm/dt = \rho_2 A_2 V_2$

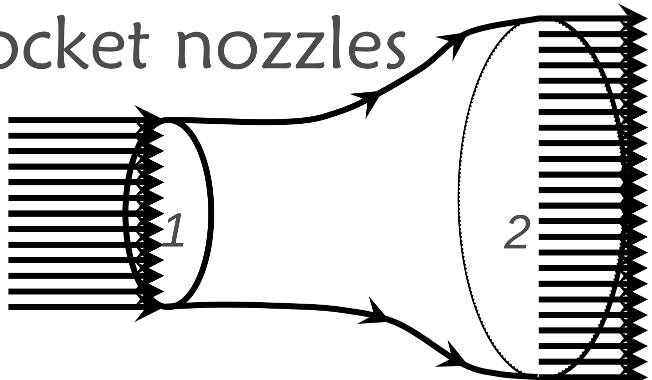
Since mass is conserved, these two expressions must be equal; hence

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

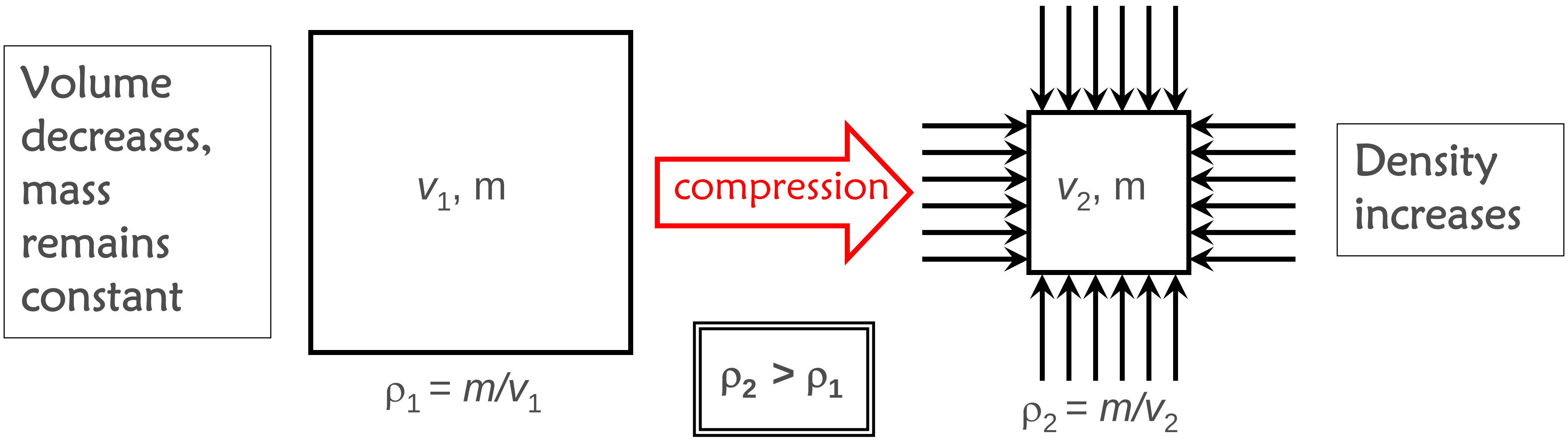
This is the *continuity equation for steady flow*

# Remarks on Continuity

- In the stream tube figure, the velocities and densities at points 1 and 2 are **assumed** to be uniform across the cross-sectional areas
- In reality, V and ρ do vary across the area and the values represent mean values
- The continuity equation is used for flow calculations in many applications such as wind tunnels and rocket nozzles
- Stream tubes do not have to represent physical flow boundaries



# Compressible *vs* Incompressible




- **Compressible flow:** flow in which the density of the fluid changes from point to point
  - In reality, all flows are compressible, but $\Delta\rho$ may be negligible
- **Incompressible flow:** flow in which the density of the fluid is constant
  - Continuity equation becomes $A_1V_1 = A_2V_2$

# Compressible *vs* Incompressible

- Incompressible flow does not exist in reality
- However, many flows are "incompressible enough" so that the assumption is useful
- Incompressibility is an excellent model for
  - Flow of liquids such as water and oil
  - Low-speed aerodynamics (<100 m/s or <225 mph)
- For incompressible flow, the continuity equation can be written as $V_2 = A_1 V_1 / A_2$
- Thus if $A_1 > A_2$ then $V_1 < V_2$

# Example 4.1

Consider a convergent duct with an inlet area $A_1$ = 5 m$^2$. Air enters this duct with velocity $V_1$ = 10 m/s and leaves the duct exit with a velocity $V_2$ = 30 m/s. What is the area of the duct exit?

First, check that the velocities involved are < 100 m/s, which implies *incompressible flow.*

Then use

$$A_2 = A_1 V_1 / V_2 = (5\ \text{m}^2)(10)/(30) = 1.67\ \text{m}^2$$

# Example 4.2

Consider a convergent duct with an inlet area $A_1 = 3\ \text{ft}^2$ and an exit area $A_2 = 2.57\ \text{ft}^2$. Air enters this duct with velocity $V_1 = 700\ \text{ft/s}$ and a density $\rho_1 = 0.002\ \text{slug/ft}^3$, and leaves the duct exit with a velocity $V_2 = 1070\ \text{ft/s}$. What is the density of the air at the duct exit?

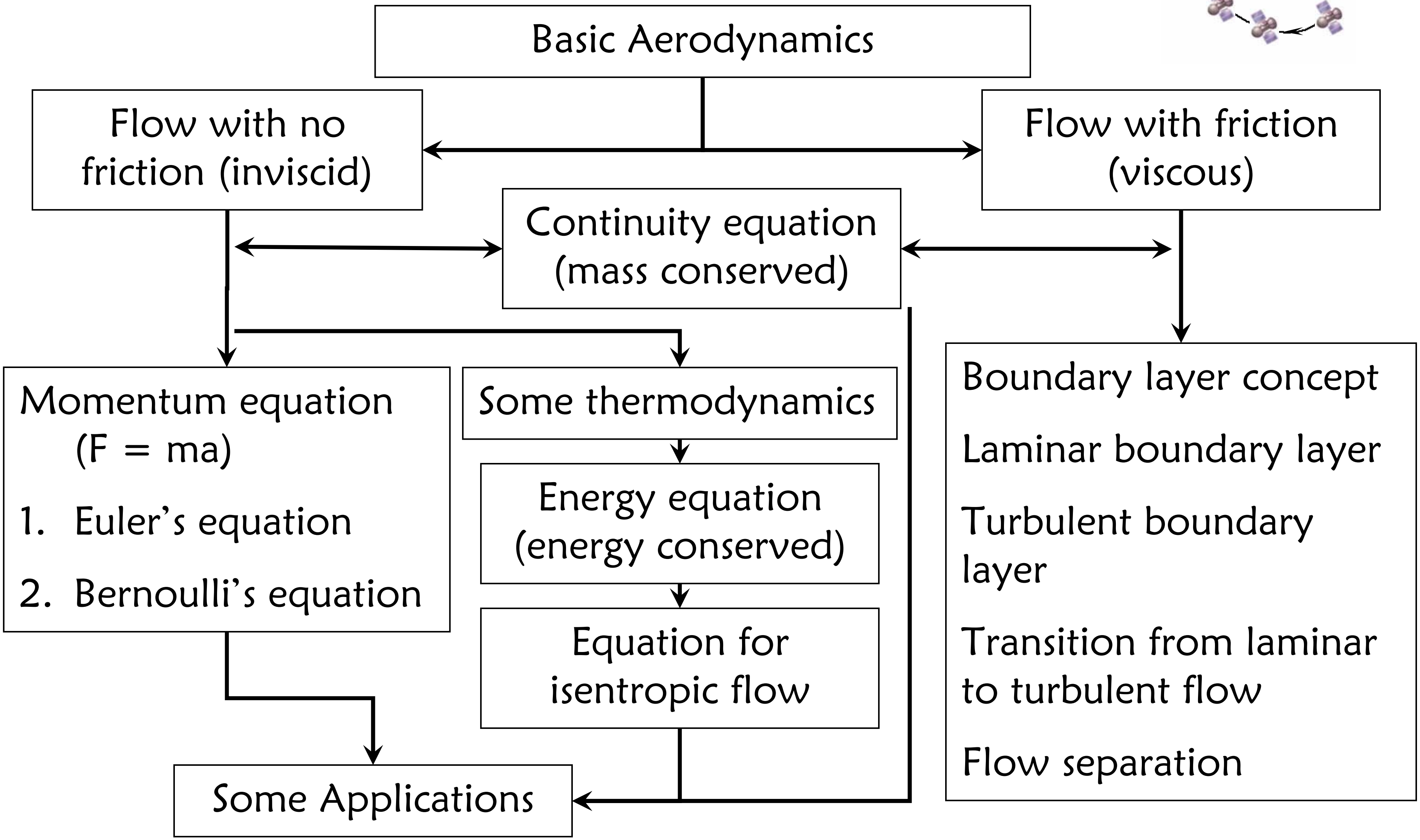
First, check that the velocities involved are $> 300\ \text{ft/s}$, which implies *compressible flow.*

Then use

$$\rho_2 = \rho_1 A_1 V_1/(A_2 V_2) = 0.00153\ \text{slug/ft}^3$$

# Aerodynamics






Reading: Chapter 4

# Momentum Equation

- Continuity equation does not involve pressure
- Pressure ⇒ Force ⇒ Change in momentum ⇒ Change in velocity

  Force = d(momentum)/d*t*  What Newton said

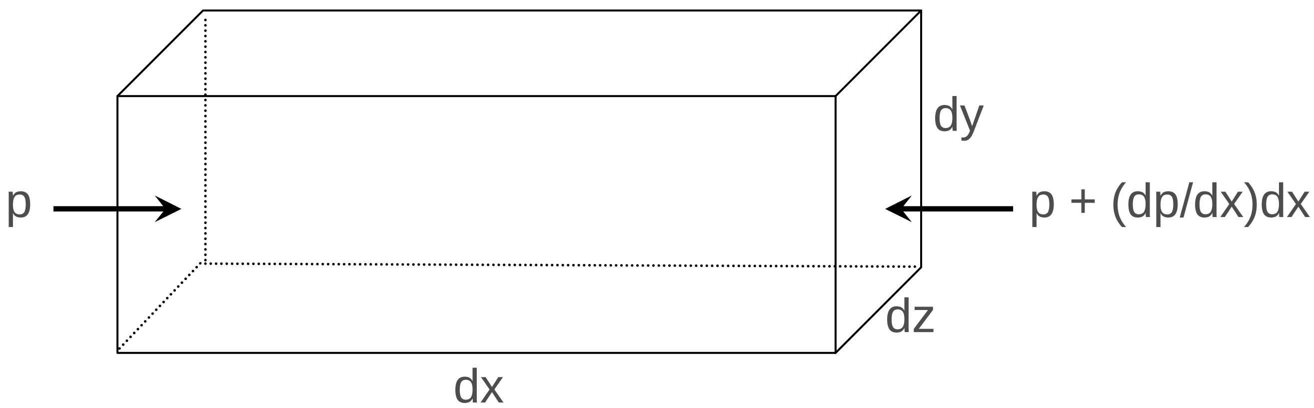
  Force = d(*mv*)/dt  but only applies if  m=const

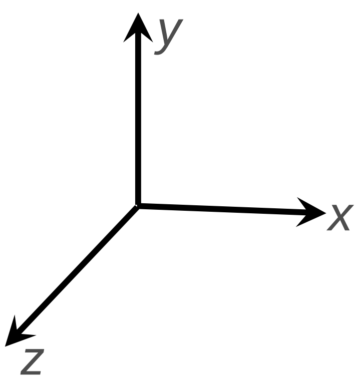
  $F = m\, dv/dt$

  $F = ma$

- We apply $F = ma$ to the fluid by summing the forces acting on a single infinitesimally small particle of fluid

# Free-Body Diagram




- Assume element is moving in x direction
- Force on element has three sources:
  - Normal pressure distribution: p
  - Shear stress distribution: $\tau_w$
  - Gravity: $\rho$ dx dy dz g
- Ignore gravity, smaller than other forces
- Consider force balance in x direction
- Force = Pressure × Area

# Force Balance

- Force on left face: $F_L = p\ dy\ dz$
- Force on right face: $F_R = (p+[dp/dx]dx)\ dy\ dz$

  $F = F_L - F_R = p\ dy\ dz - (p+[dp/dx]dx)\ dy\ dz$

  $F = -(dp/dx)\ dx\ dy\ dz$
- Mass of the fluid element is

  $m = \rho\ dx\ dy\ dz$
- Acceleration of the fluid element

  $a = dV/dt = (dV/dx)(dx/dt) = (dV/dx)V$
- Newton's second law

  $F = ma \Rightarrow \boxed{dp = -\rho\ V\ dV}$ **Euler's Equation**
- Also referred to as the Momentum Equation
  - Keep in mind that we assumed steady flow and ignored gravity and friction, thus this is the momentum equation for *steady, inviscid flow*
  - However, Euler's equation applies to compressible and incompressible flows

# Incompressible Flow

- If the flow is incompressible, then $\rho$ is constant
- The momentum equation can be written as

  $dp + \rho V\,dV = 0$

- Integrating along a streamline between two points 1 and 2 gives

  $p_2 - p_1 + \rho\,(V_2^2 - V_1^2)/2 = 0$

- Which can be rewritten as

  $p_2 + \rho\,V_2^2/2 = p_1 + \rho V_1^2/2$

  Or

  $p + \rho\,V^2/2 =$ constant along a streamline

- This equation is known as Bernoulli's equation

# Euler's and Bernoulli's Equations

- Bernoulli's equation

  $p_2 + \rho V_2^2/2 = p_1 + \rho V_1^2/2$

  – Holds for inviscid, incompressible flow

  – Relates properties of different points along a streamline

- Euler's equation

  $dp = -\rho V dV$

  – Holds for inviscid flow, compressible or incompressible

- These equations represent Newton's Second Law applied to fluid flow, and relate pressure, density, and velocity

# Euler's and Bernoulli's Equations

- Bernoulli's equation

  $p_2 + \rho V_2^2/2 = p_1 + \rho V_1^2/2$

  - Holds for inviscid, incompressible flow
  - Relates properties of different points along a streamline
- Euler's equation

  $dp = -\rho V dV$

  - Holds for inviscid flow, compressible or incompressible
- These equations represent Newton's Second Law applied to fluid flow, and relate pressure, density, and velocity

# Example 4.3

Consider an airfoil in a flow of air, where far ahead (upstream) of the airfoil, the pressure, velocity, and density are 2116 lb/ft$^2$, 100 mi/h, and 0.002377 slug/ft$^3$, respectively. At a given point A on the airfoil, the pressure is 2070 lb/ft$^2$. What is the velocity at point A?

First, we must use consistent units. Using the fact that 60 mi/h ≈ 88 ft/s, we find that V = 100 mi/h = 146.7 ft/s. This flow is slow enough that we can assume it is incompressible, so we can use Bernoulli's equation:

$$p_1 + \rho\, V_1^2/2 = p_A + \rho V_A^2/2$$

Where "1" is the far upstream condition, and "A" is the point on the airfoil. Solving for velocity at A gives

$$V_A = 245.4 \text{ ft/s}$$

# Example 4.4

Consider a convergent duct with an inlet area $A_1 = 5\ m^2$. Air enters this duct with velocity $V_1 = 10$ m/s and leaves the duct exit with a velocity $V_2 = 30$ m/s. If the air pressure and temperature at the inlet are $p_1 = 1.2 \times 10^5$ N/m$^2$ and $T_1 = 330$K, respectively, calculate the pressure at the exit.

First, compute density at inlet using equation of state:

$$\rho_1 = p_1/(R\ T_1) = 1.27\ kg/m^3$$

Assuming compressible flow, use Bernoulli's equation to solve for $p_2$:

$$p_2 = p_1 + \rho(V_1^2 - V_2^2)/2 = 1.195 \times 10^5\ N/m^2$$

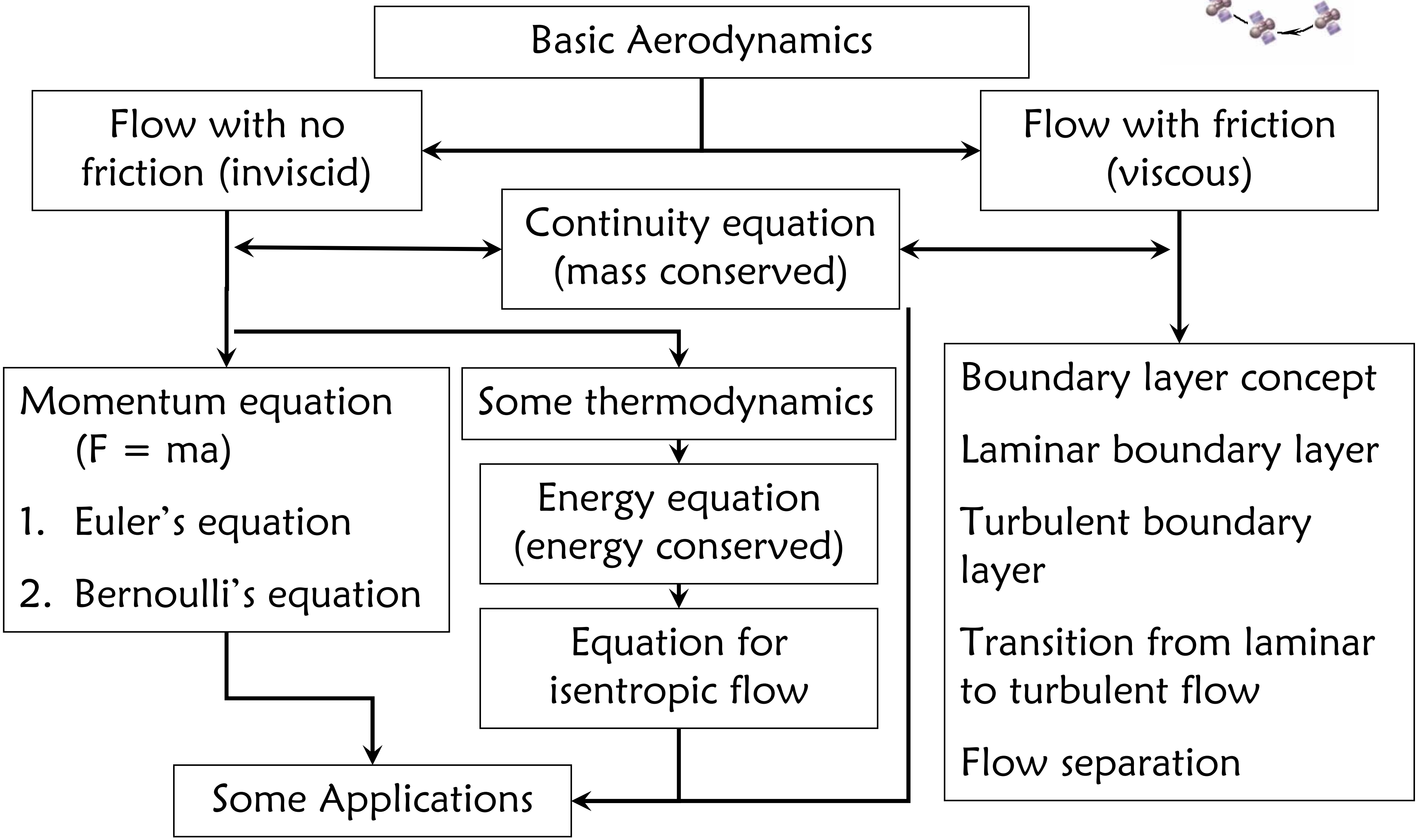
# Example 4.5

Consider a long dowel with semicircular cross section

…

See pages 135-141 in text

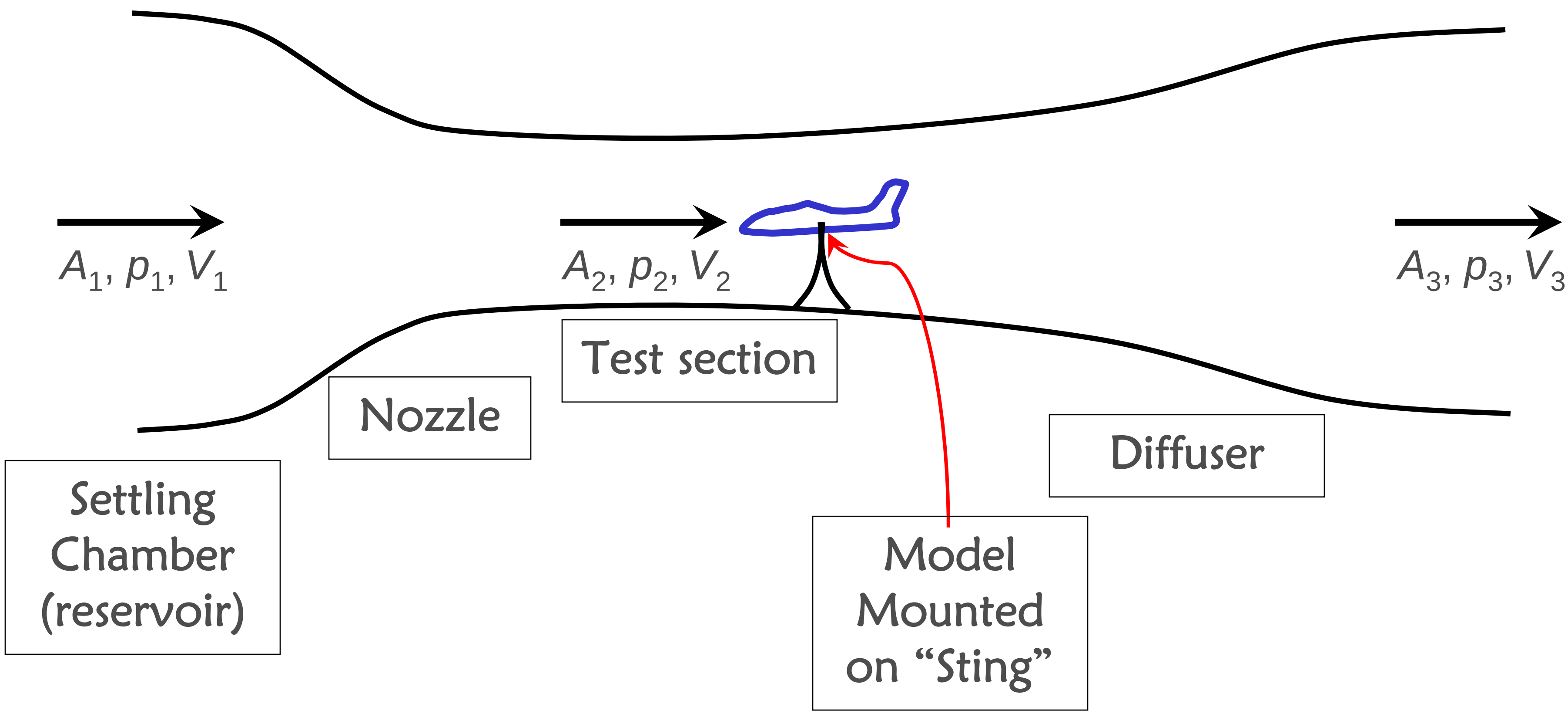
# Aerodynamics




Reading: Chapter 4

# §4.10: Low-Speed Subsonic Wind Tunnels

Assumption: *Steady incompressible flow*




Continuity and Bernoulli's Equation apply

# Wind Tunnel Calculations

Continuity $\Rightarrow V_1 = (A_2/A_1)V_2$

Bernoulli $\Rightarrow V_2^2 = 2(p_1-p_2)/\rho + V_1^2$

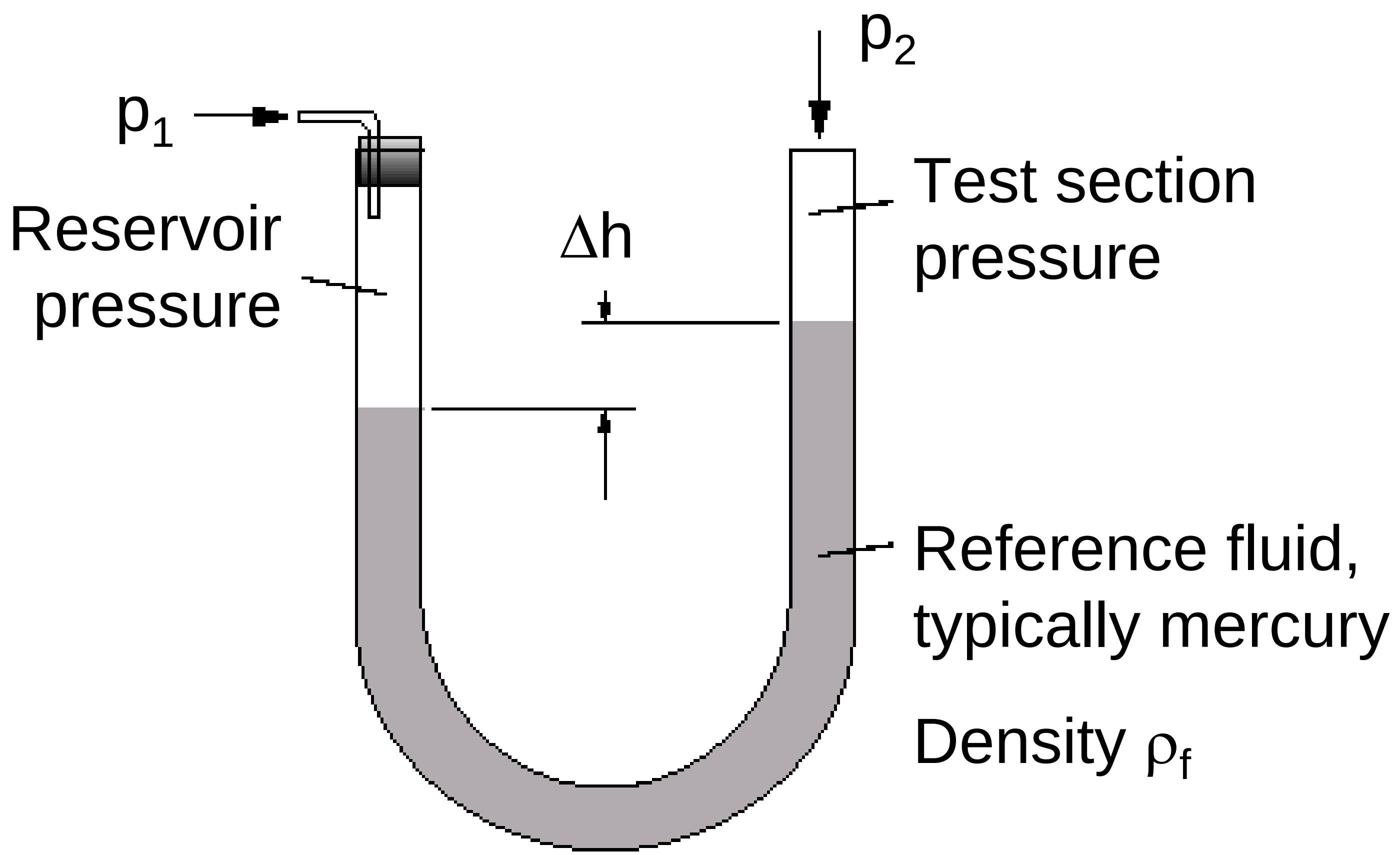
Combine to get

$$V_2 = \{ 2(p_1-p_2) / [\rho(1- (A_2/A_1)^2)] \}^{1/2}$$

The ratio $A_2/A_1$ is fixed for a given wind tunnel, and the density $\rho$ is constant for low-speed tunnels, so the "control" is $p_1-p_2$

How to determine $p_1-p_2$?

# Manometer




$$p_1 A = p_2 A + w A \Delta h, \quad w = \rho_f g$$

$$p_1 - p_2 = A + w \Delta h, \text{ So } \quad \Delta h \Rightarrow V_2$$

# Example 4.13

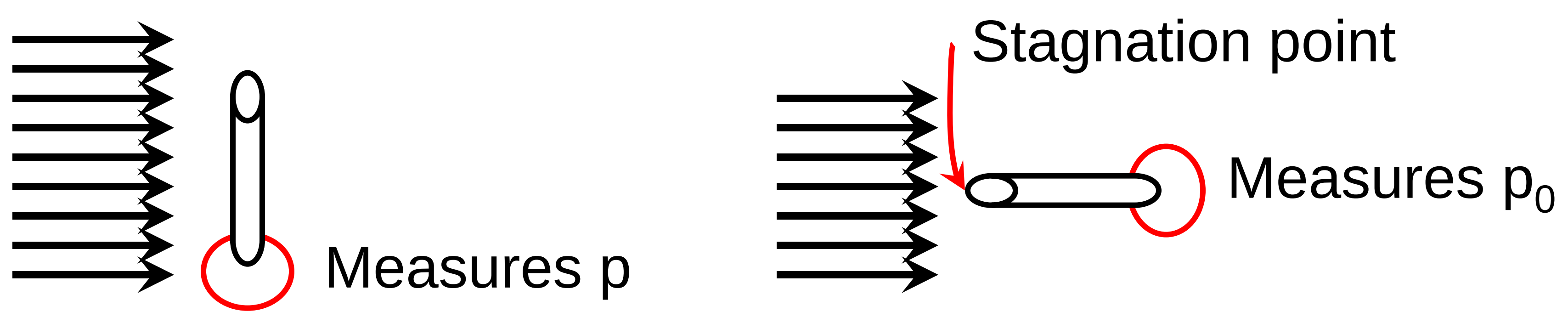
In a low-speed subsonic wind tunnel, one side of a mercury manometer is connected to the reservoir and the other side is connected to the test section. The contraction ratio of the nozzle $A_2/A_1 = 1/15$. The reservoir pressure and temperature are $p_1 = 1.1$ atm and $T_1 = 300$ K. When the tunnel is running the height difference between the two columns of mercury is 10 cm. The density of liquid mercury is $1.36 \times 10^4$ kg/m$^3$. Calculate the airflow velocity $V_2$ in the test section.

# §4.11: Measurement of Airspeed

- Total pressure *vs* static pressure
- Static pressure is the pressure we've been using all along, and is the pressure you'd feel if you were moving along with the fluid
- Total pressure includes the static pressure, but also includes the "pressure" due to the fluid's velocity, the so-called dynamic pressure
- Imagine a hollow tube with an opening at one end and a pressure sensor at the other, and imagine inserting it into a flow in two different ways

# Pitot Tube

- This device is called a Pitot Tube (after Henri Pitot, who invented it in 1732; see §4. 23)
- The orientation on the left measures the static pressure (the pressure in all our calculations so far)
- The orientation on the right measures the total pressure, or the pressure if the flow is reduced to zero velocity



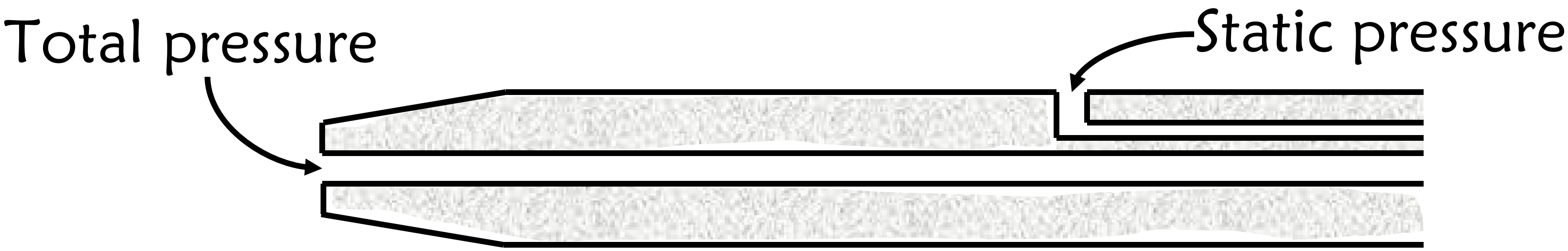
# Pitot Tube for Incompressible Flow

- The two tube orientations are used together
- One measures static pressure p, and the other measures total pressure $p_0$
- Since the total pressure is measured by removing all the velocity, and we're assuming incompressible flow, we can apply Bernoulli's equation to see that

  $p + \rho V^2/2 = p_0$

  Static pressure + Dynamic Pressure = Total Pressure
- Dynamic pressure, the $\rho V^2/2$ term, is frequently denoted by $q = \rho V^2/2$

# Using the Pitot-static Probe




- The two pressures are measured by a pressure transducer
- Bernoulli's equation (incompressible flow only!) can be written as

  $p_0 = p + q \qquad (q = \rho V^2/2)$

- Solve for velocity

  $V = [2(p_0 - p)/\rho]^{1/2}$

- A Pitot-static tube provides an airspeed measurement

# Example 4.16

The altimeter on a low-speed Cessna 150 reads 5000 ft. The outside temperature is T = 505°R. If a Pitot tube on the wingtip measures p = 1818 lb/ft$^2$, what is the true velocity of the airplane? What is the equivalent airspeed?

# Overview of the “Rest” of Aerodynamics

- We will not cover the remainder of Ch. 4, but here are some highlights
- First Law of Thermodynamics leads to relationships between **energy**, **temperature**, **heat, enthalpy**, and **specific heat**
- **Energy** has units of Joules
- **Enthalpy** has units of Joules but also accounts for temperature
- **Adiabatic** $\Rightarrow$ no heat is added or removed
- **Reversible** $\Rightarrow$ no frictional losses
- **Isentropic** $\Rightarrow$ adiabatic and reversible