Aeroelasticity & Experimental Aerodynamics

(AERO0032-1)

Lecture 1

Introduction – Equations of motion

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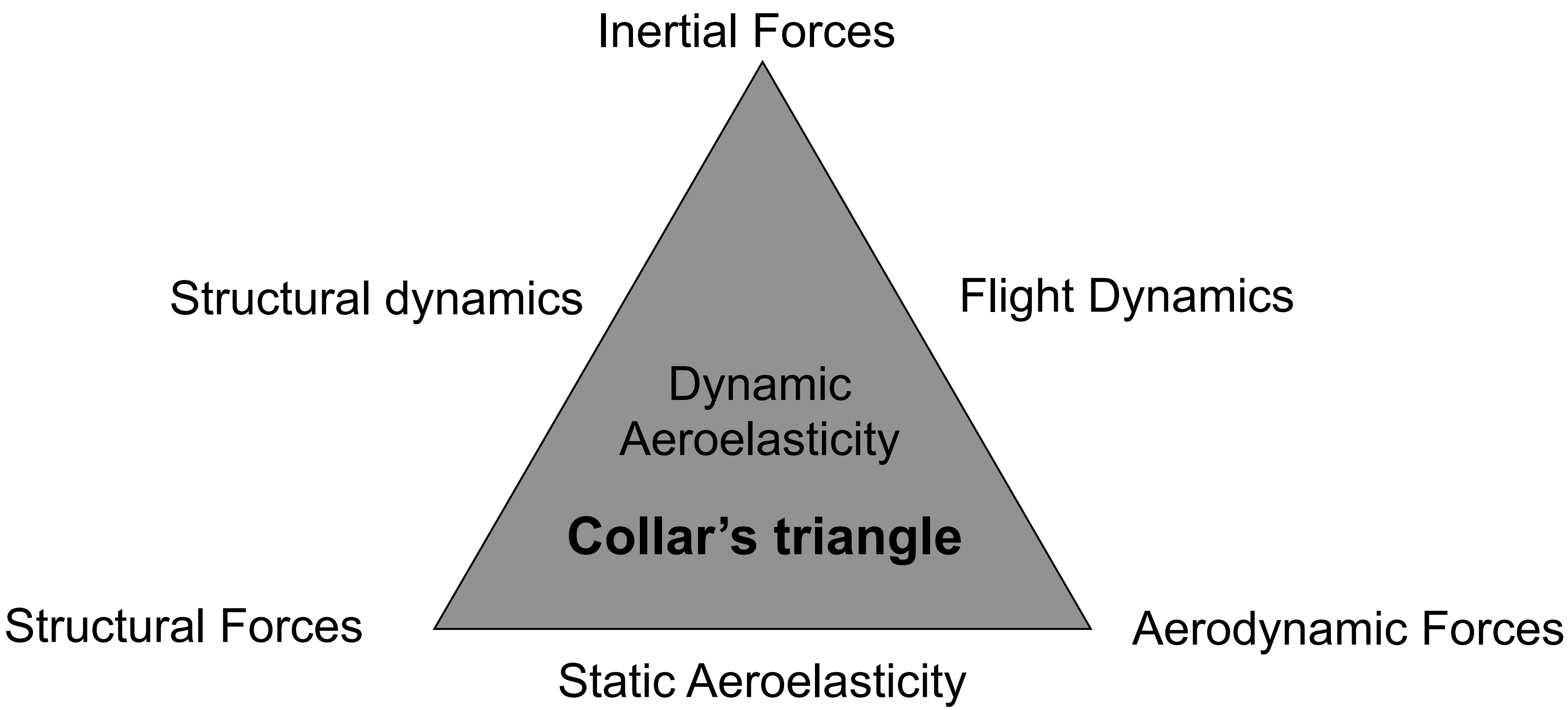
Introduction

- **Aeroelasticity** = study of the interaction of inertial, structural and aerodynamic forces
- Applications on aircrafts, buildings, surface vehicles etc.



$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f_a}(V_\infty, \dot{\mathbf{x}}(t), \mathbf{x}(t))$$

Why is it important?

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The interaction between these three forces can cause several undesirable phenomena:

- **Divergence** (static aeroelastic phenomenon)
- **Flutter** (dynamic aeroelastic phenomenon)
- **Vortex-induced vibration**, **buffeting** (unsteady aerodynamic phenomena)
- **Limit Cycle Oscillations** (nonlinear aeroelastic phenomenon)

Static Divergence

movie

NASA wind tunnel experiment on a forward swept wing

Flutter

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movie

Flutter experiment:
Winglet under fuselage of a F-16. Slow Mach number increase.

→ Prediction of the flutter Mach number from subcritical test data and to stop the test before flutter occurs.

Vortex-induced vibrations

movie

movie

Flow visualization of vortices shed behind a cylinder

Vortex-induced vibrations

(more details in Lecture 6)

Limit Cycle Oscillations

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movie

movie

Stall flutter of a wing at an angle of attack

Torsional flutter of a rectangle

Even more LCOs

movie

movie

Galloping of a bridge deck

(more details in Lecture 7)

Torsional flutter oscillations of a bridge deck

Many more LCOs

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Sub-critical LCO of a delta wing

In real applications

Out of the lab :

movie

movie

Tacoma Narrows Bridge Flutter

Glider Limit Cycle Oscillations

movie

Various phenomena

In real applications

Even on very expensive kits

movie

movie

Store-induced LCO on F-16

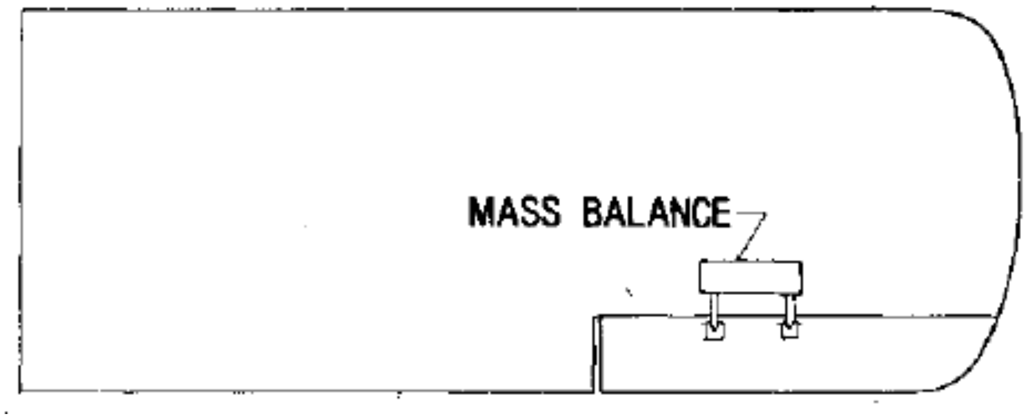
Fin buffeting on F-18

A bit of history

- The first ever flutter incident occurred on the Handley Page O/400 bomber in 1916 in the UK.
- A fuselage torsion mode coupled with an antisymmetric elevator mode (the elevators were independently actuated)
- The problem was solved by coupling the elevators

More history

- **Control surface flutter** became a frequent phenomenon during World War I and in the interwar period.
- It was solved in the mid-twenties by **mass balancing the control surface**.

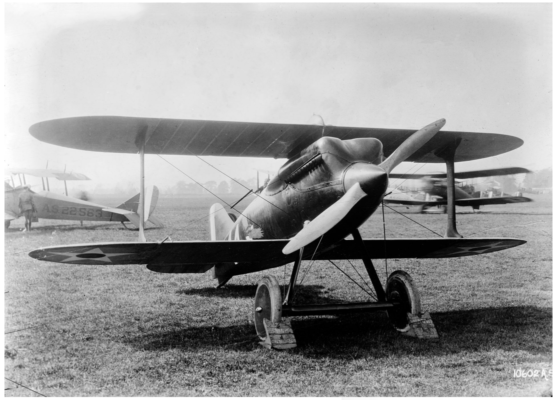


Attachment arm and balance weight on a Navion aileron. View is from the bottom looking toward the wing tip.

Increasing airspeed

- Aircraft flight speeds increased significantly during the 20s and 30s.
- A number of high-speed racing aircraft suffered from flutter problems

Supermarine S-4



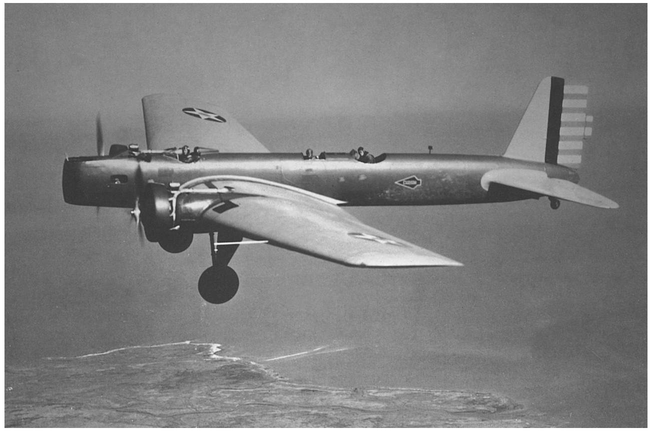
Curtiss R-6

Verville-Sperry R-3

Loening R-4

US flutter experiences in the 1930s

General Aviation YO-27: Wing-aileron and rudder-fuselage flutter



Fairchild F-24: Wing-aileron and tail flutter

Boeing YB-9A: Rudder-fuselage LCO

Curtiss YA-8: Rudder-fin flutter

Other historic examples

- Aircrafts that experienced aeroelastic phenomena
  - Handley Page O/400 (elevators-fuselage)
  - Junkers JU90 (fluttered during flight flutter test)
  - P80, F100, F14 (transonic aileron buzz)
  - T46A (servo tab flutter)
  - F16, F18 (external stores LCO, buffeting)
  - F111 (external stores LCO)
  - F117, E-6 (vertical fin flutter)
- Read ‘Historical Development of Aircraft Flutter’, I.E. Garrick, W.H. Reed III, Journal of Aircraft, 18(11), 897-912, 1981

F-117 crash

- Crash during an airshow in Maryland in 1997
- Four fasteners that connected the elevon actuator to the wing structure were missing.
- Reduction of the actuator-elevon stiffness, leading to elevon-wing flutter

movie

Aeroelastic investigations

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How to avoid these phenomena?

→**Wind tunnel testing**:

Aeroelastic scaling

→**Complete aircraft (prototype) testing:**

→ **Ground Vibration Testing :**

Complete modal analysis of aircraft structure

→ **Flight Flutter Testing** :

Demonstrate that flight envelope is flutter free

→**Aeroelastic Design**:

Computing critical airspeeds of Divergence, Flutter, Control Reversal

Wind Tunnel Testing

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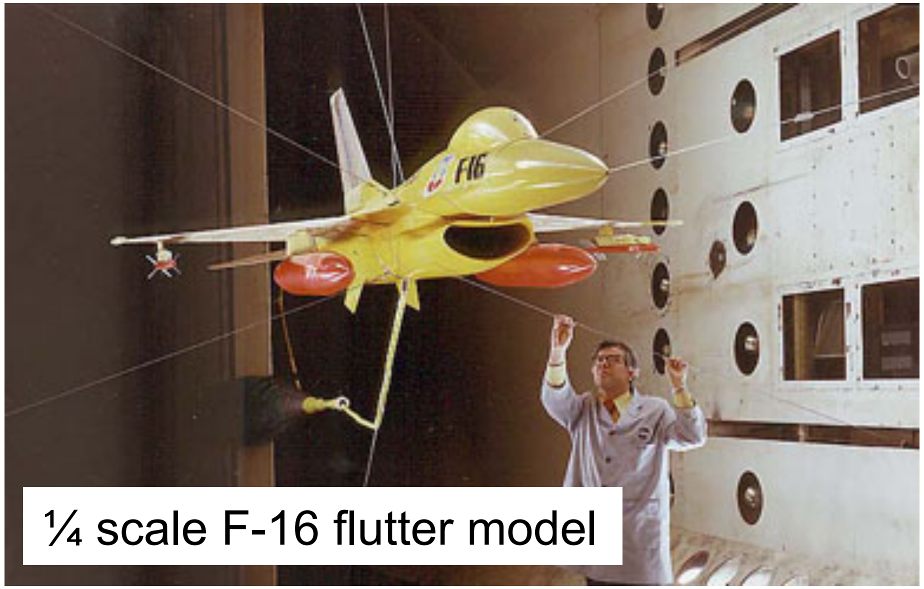
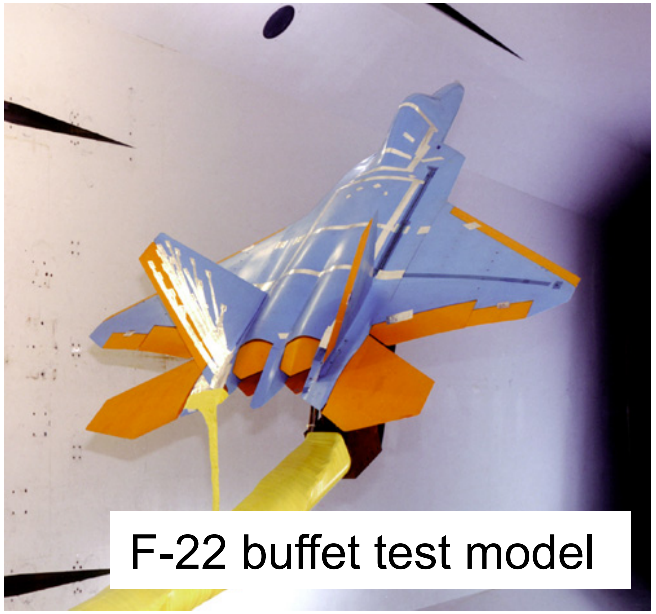
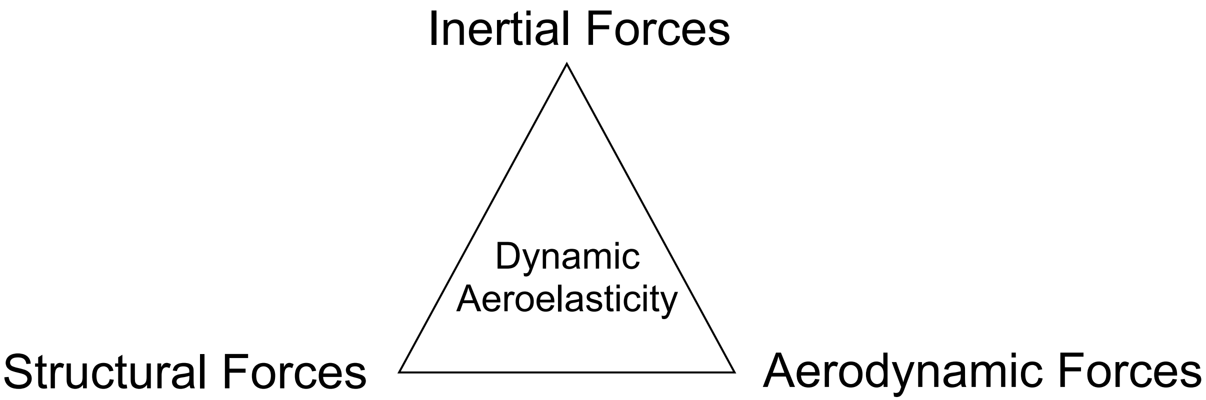


¼ scale F-16 flutter model



F-22 buffet test model

→ Scaling the **aeroelastic** behaviour of the aircraft



Wind Tunnel Testing

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Similarity laws:

→ Geometric similarity : Models dimensions
Turbulence scales
(CLA + topography)

→ Kinematic similarity : Power spectral density
Turbulence intensity
(Mean wind speed)

→ Dynamic similarity : Reynolds number
Reduced frequency fL/U

$$St\frac{\partial \bar{\mathbf{u}}}{\partial \bar{t}} + \bar{\mathbf{u}} \wedge \overrightarrow{rot}\bar{\mathbf{u}} = -\overrightarrow{grad}\bar{p} + \frac{1}{Re}\overrightarrow{rot}(\overrightarrow{rot}\bar{\mathbf{u}}) + \frac{1}{Fr^2}\bar{\mathbf{a}}$$

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}_{\mathbf{a}}(V_\infty, \dot{\mathbf{x}}(t), \mathbf{x}(t))$$

Damping ratio
Mass ratio

Un-respected similarity laws

→ Geometric discrepancies between model & prototype

→ Kinematic discrepancies between real incoming flow and wind tunnel flow

Wind Tunnel Testing

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Un-respected similarity laws

→ Dynamic discrepancies :

Reynolds number UL/v → Re effect for curved surfaces
→ Surface roughness / pressurized wind tunnel

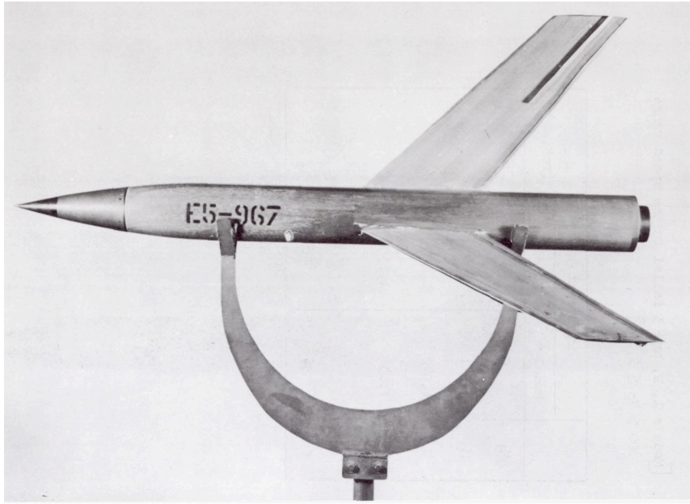
Reduced frequency fL/U → What happens in the lab is much faster than in reality
→ High frequency measurements

Damping ratio → Difficult measurement (prototype and model) and adjustment of damping (model)

Mass ratio → High density material needed

Wind Tunnel Testing

Limitations



→ Un-respected similarity laws

→ Support the model

→ Blockage effect

→ Wall constraint effects

→ Limited amount of measurements (low spatial resolution vs. high temporal resolution)

→ Cost

Wind Tunnel Testing

movie

Wind Tunnel Testing

movie

Wind Tunnel Testing

movie

Ground Vibration Testing

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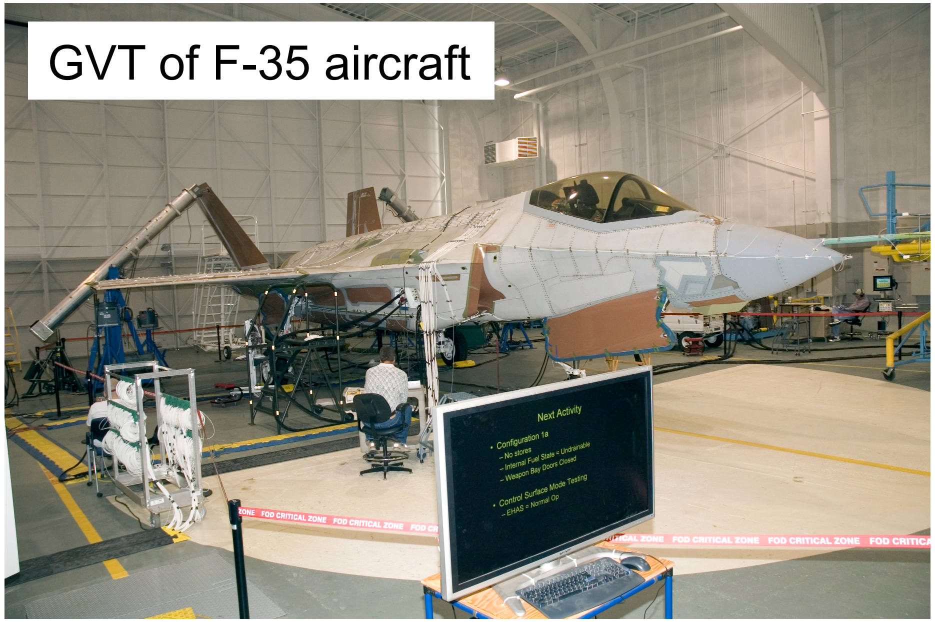
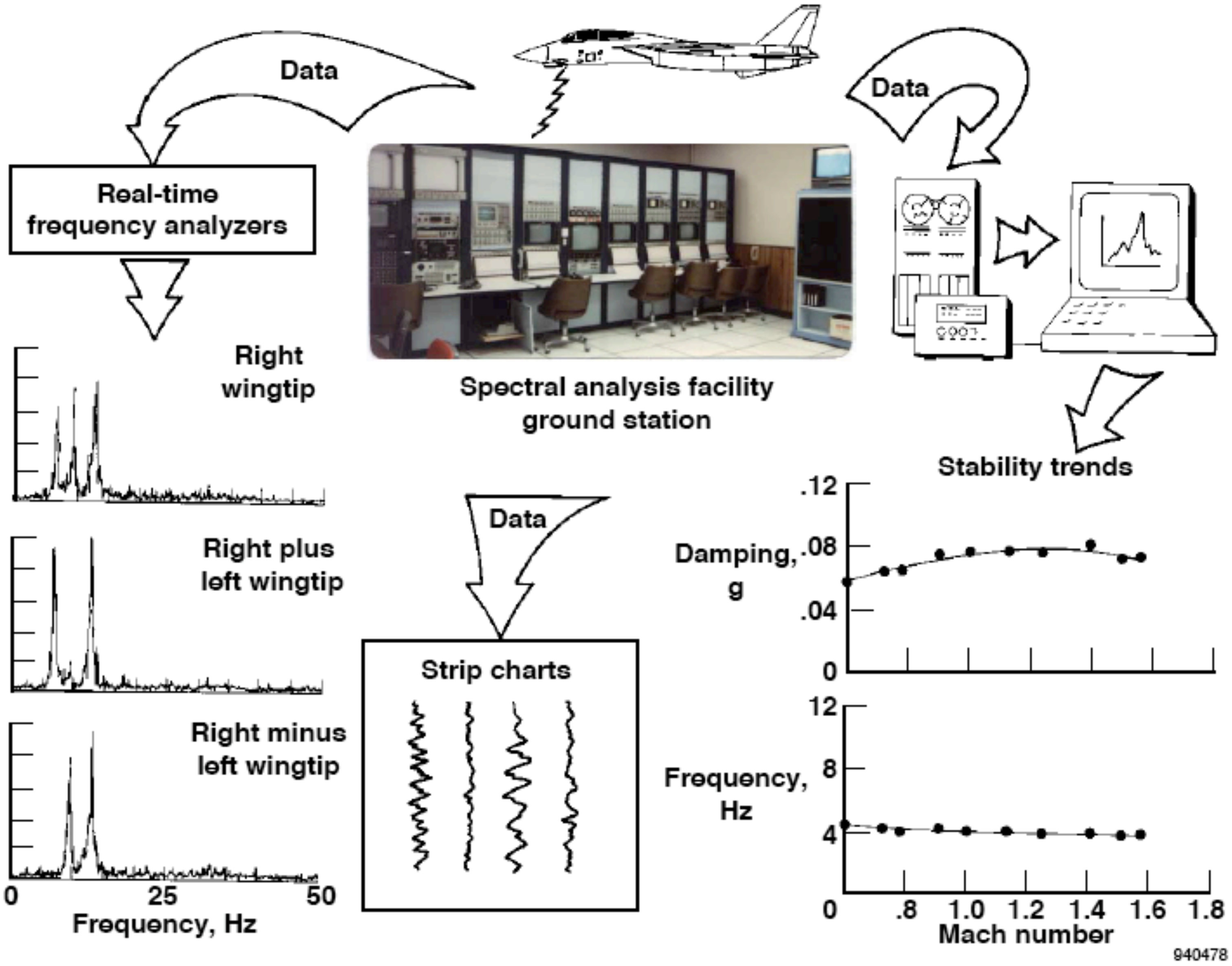


GVT of F-35 aircraft


GVT of A340


Space Shuttle GVT

Flight Flutter Testing



Flight Flutter Testing

movie

AIRBUS A350 Flight Flutter Testing

Aeroelastic Modeling

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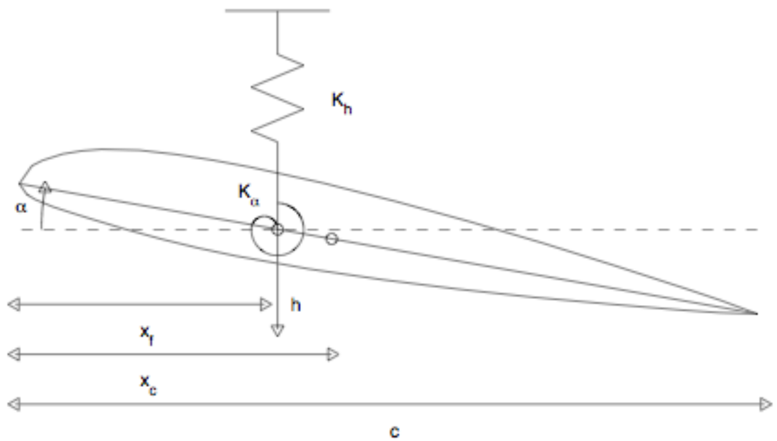
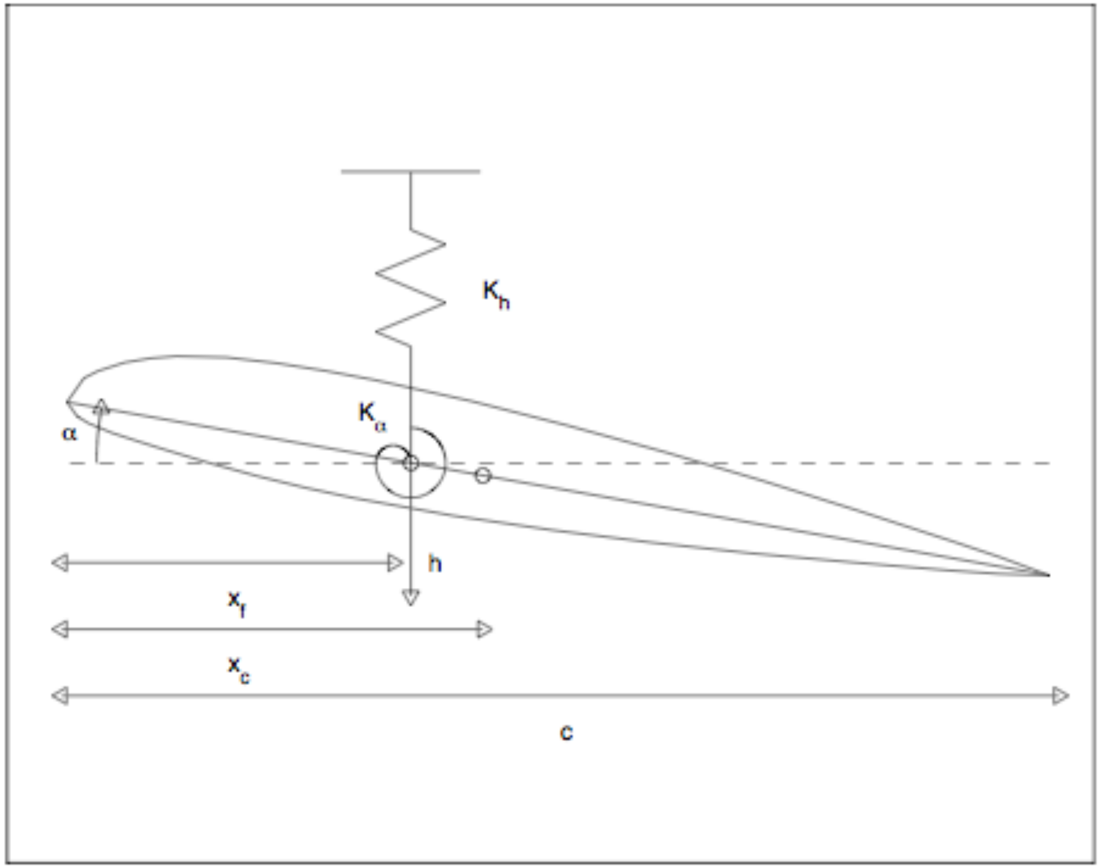
- Aircraft are very complex structures with many modes of vibration and can exhibit very complex fluid-structure interaction phenomena
- The exact modeling of the aeroelastic behaviour of an aircraft necessitates the coupled solution of:
  - The full compressible Navier Stokes equations
  - The full structural vibrations equations
- Start with something simpler: **pitch-plunge airfoil**



Pitch Plunge Airfoil



Two-dimensional, two degree-of freedom airfoil, quite capable of demonstrating most aeroelastic phenomena.

α = pitch degree of freedom
h = plunge degree of freedom
x_f = position of flexural axis (pivot)
x_c = position of centre of mass
K_h = plunge spring stiffness
K_α = pitch spring stiffness

In fact, we will simplify even further and consider a flat plate airfoil (no thickness, no camber)

Content of the course

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Introduction to aeroelastic modeling

Static aeroelastic phenomena : Divergence

Dynamic aeroelastic phenomena:

- Flutter
- Vortex-induced vibration
- Galloping

Practical aeroelasticity:

- Aeroelastic design
- Ground Vibration Testing, Flight Flutter Testing

7 lectures + 2 Matlab sessions + 3 Labs

Model

- Two aspects of each aeroelastic models:
  - A structural model
  - An aerodynamic model

- A control model can be added to represent the effects of actuators and other control elements

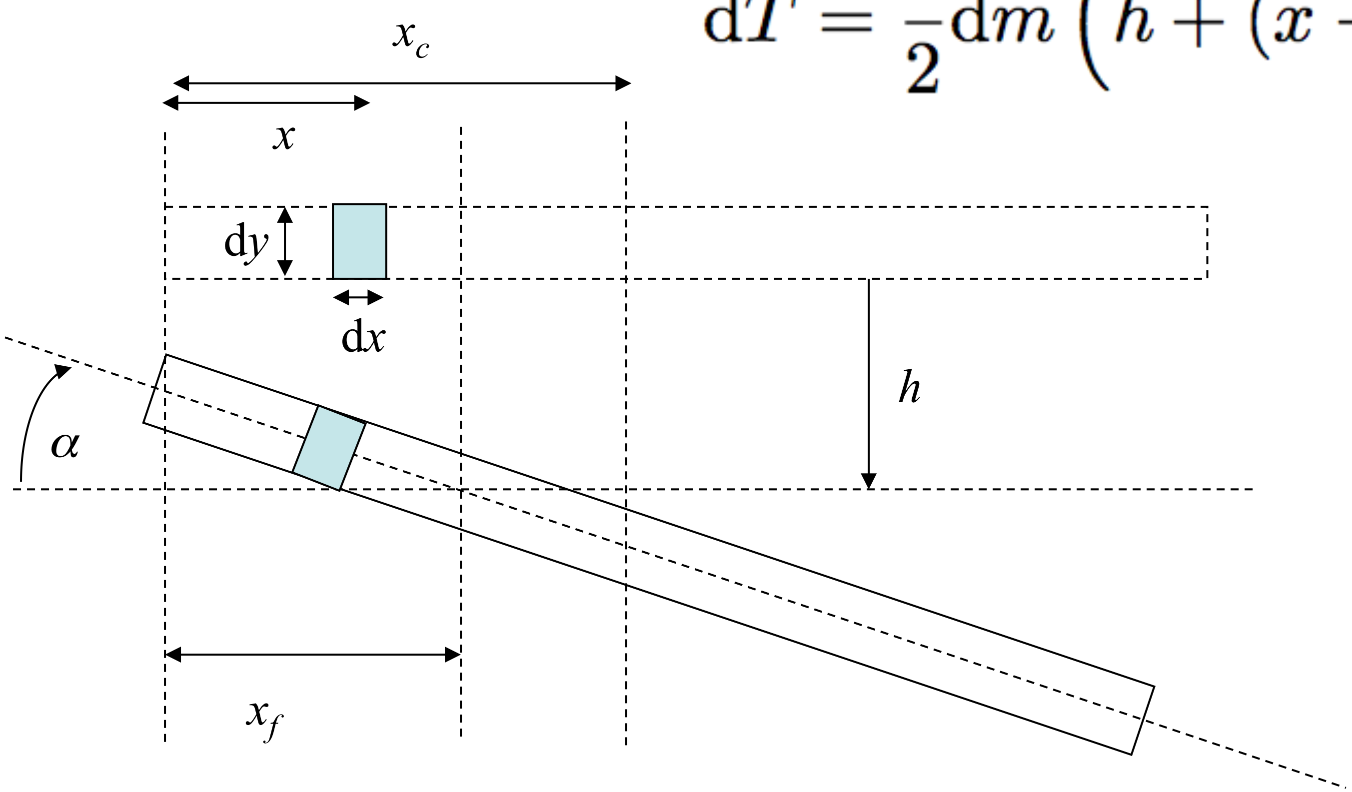
$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f_a}(V_\infty, \dot{\mathbf{x}}(t), \mathbf{x}(t)) + \mathbf{f_e}(t)$$

Structural Model

Use the **total energy conservation**

Kinetic energy of element dm :

$$\mathrm{d}T = \frac{1}{2}\mathrm{d}m\left(\dot{h} + (x - x_f)\dot{\alpha}\right)^2$$



Kinetic Energy

Integrating between 0 and c gives the total kinetic energy:

$$T = \frac{1}{2}m\dot{h}^2 + S\dot{h}\dot{\alpha} + \frac{1}{2}I_\alpha\dot{\alpha}^2$$

where
$$S = m(c/2 - x_f)$$
$$I_\alpha = \frac{1}{3}m\left(c^2 - 3cx_f + 3x_f^2\right)$$

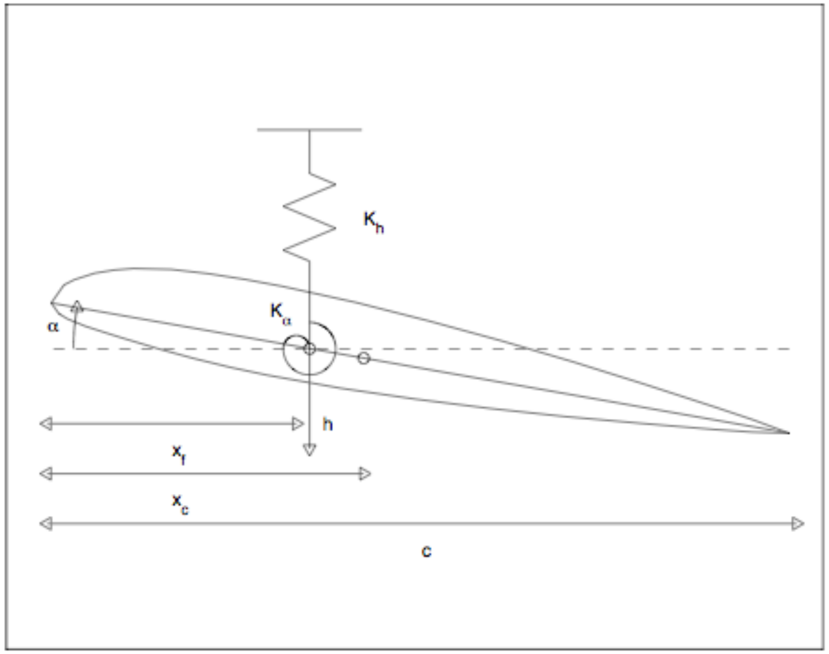
Note : we assume x_c = c /2 (center of gravity at mid-chord)

Potential Energy

Potential energy = Energy stored in the two springs:

$$V = \frac{1}{2}K_h h^2 + \frac{1}{2}K_\alpha \alpha^2$$

- Gravity can be conveniently ignored
- Total energy= kinetic energy + potential energy



Equations of motion

Equations of motion are obtained by inserting the expression for the total energy into Lagrange's equation

$$\frac{\partial}{\partial t}\left(\frac{\mathrm{d}T}{\mathrm{d}\dot{\mathbf{q}}}\right)+\frac{\mathrm{d}V}{\mathrm{d}\mathbf{q}}=\mathbf{0} \qquad \mathbf{q}=[h \quad \alpha]^T$$

→ Set of 2 equations of the form

$$\begin{pmatrix} m & S \\ S & I_\alpha \end{pmatrix}\begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix}+\begin{pmatrix} K_h & 0 \\ 0 & K_\alpha \end{pmatrix}\begin{Bmatrix} h \\ \alpha \end{Bmatrix}=\mathbf{0}$$

or, $\mathbf{M}\ddot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{Q}$

where $\mathbf{Q}$ is a vector of external forces

Note: no damping term is included at this stage

Aerodynamic model

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$$\begin{pmatrix} m & S \\ S & I_\alpha \end{pmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{pmatrix} K_h & 0 \\ 0 & K_\alpha \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -l(t) + p(t) \\ m_{xf}(t) + r(t) \end{Bmatrix}$$

- The possible aerodynamic models depends on flow regime and simplicity
- In general, only four flow regimes are considered by aeroelasticians:
  - Incompressible
  - Subsonic
  - Transonic
  - Supersonic

Lift and moment

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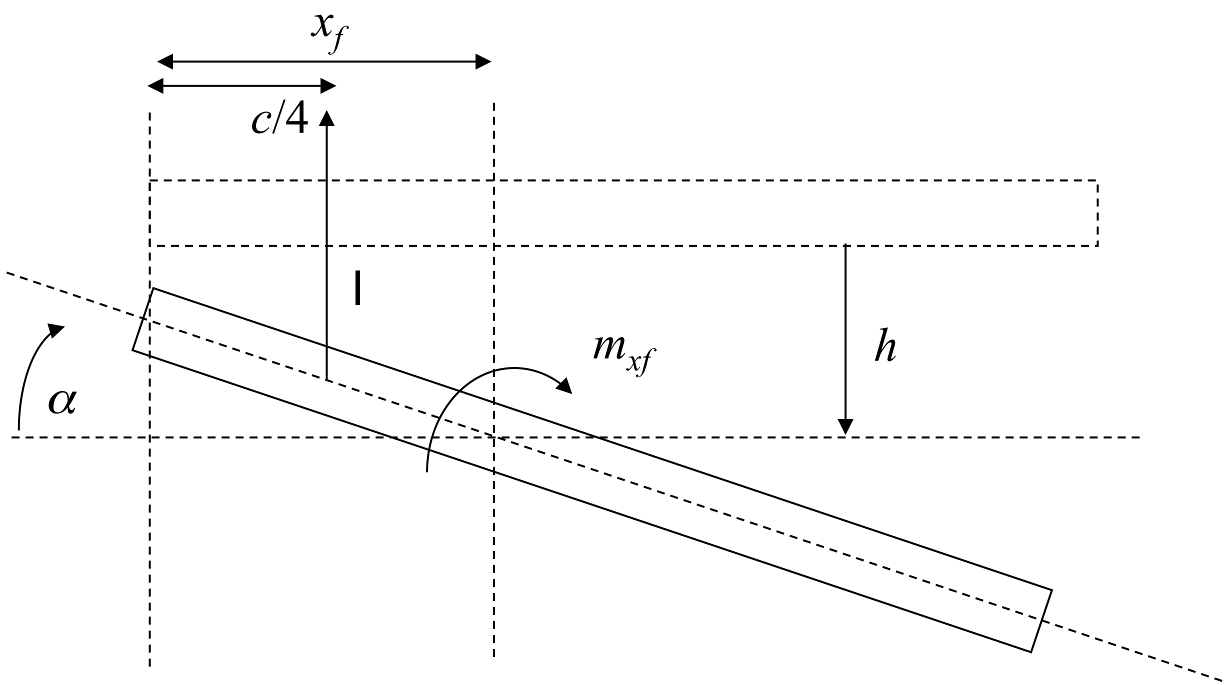
$$\begin{pmatrix} m & S \\ S & I_\alpha \end{pmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{pmatrix} K_h & 0 \\ 0 & K_\alpha \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -l(t) + p(t) \\ m_{xf}(t) + r(t) \end{Bmatrix}$$

- Lift at the quarter chord (aerodynamic center)
- Aerodynamic moment around the flexural axis

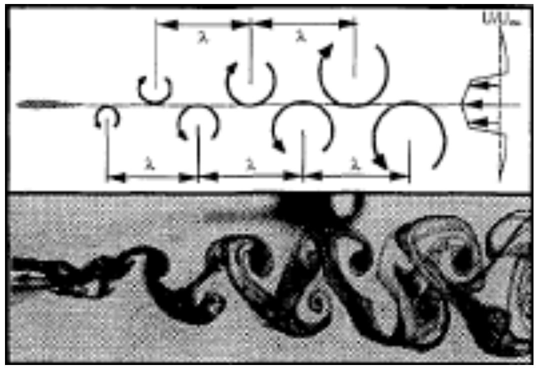


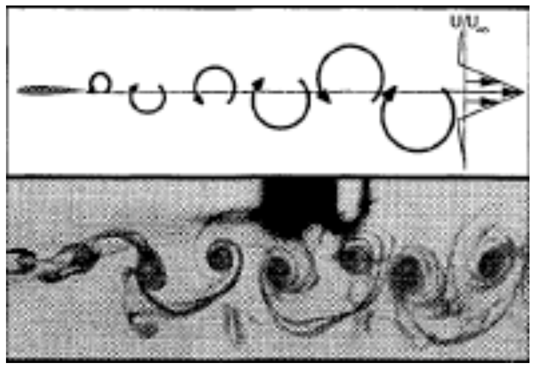
Incompressible, Unsteady Aerodynamics

Oscillating airfoils leave behind them a strong vortex street. The vorticity in the wake affects the flow over the airfoil:

The instantaneous aerodynamic forces depend not only on the instantaneous position of the airfoil but also on the position and strength of the wake vortices.

This means that instantaneous aerodynamic forces depend not only on the current motion of the airfoil but on all its motion history from the beginning of the motion.





Wake examples (Pitch)

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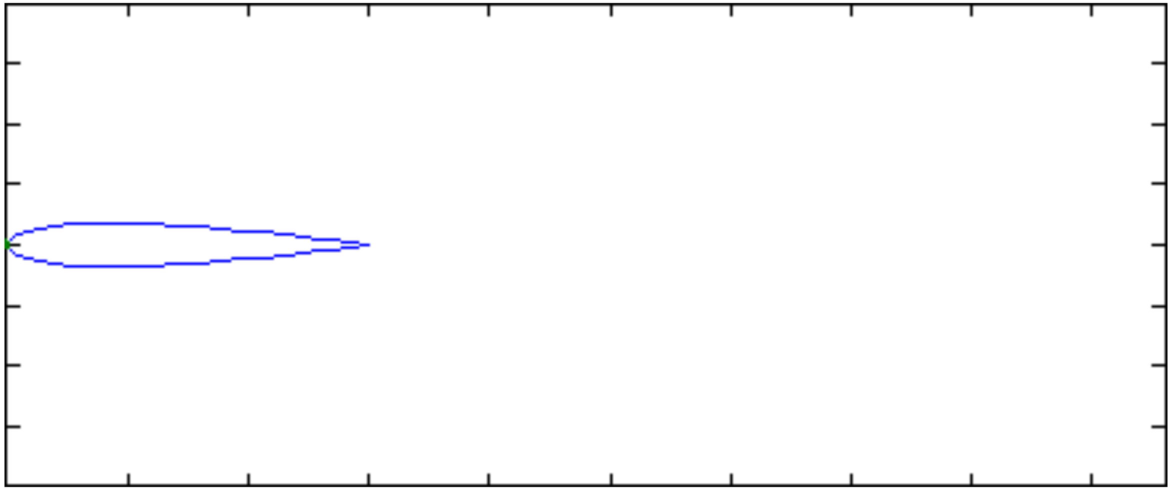
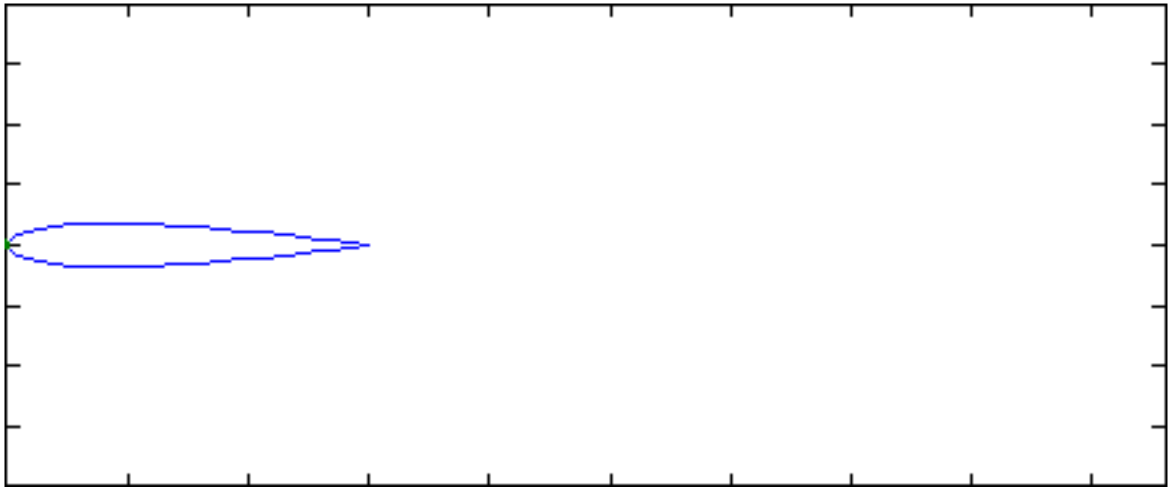
Pitching airfoil- Low frequency

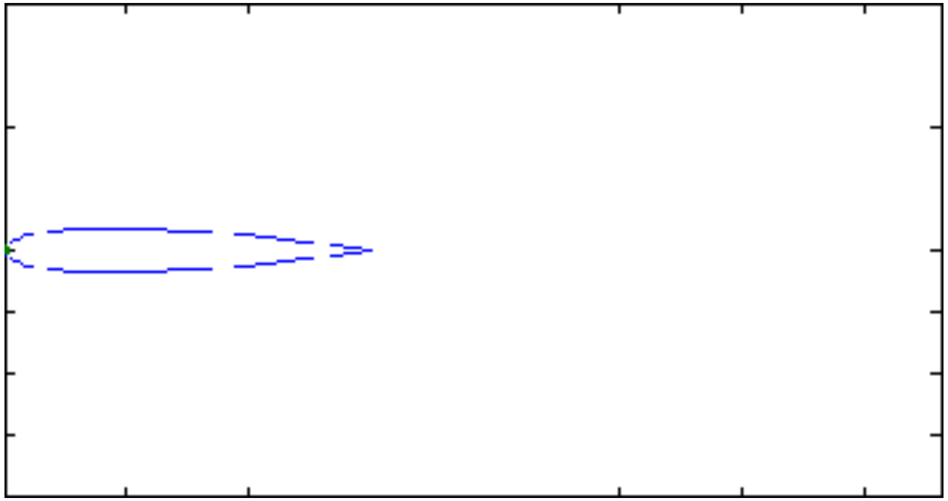


Pitching airfoil- High frequency

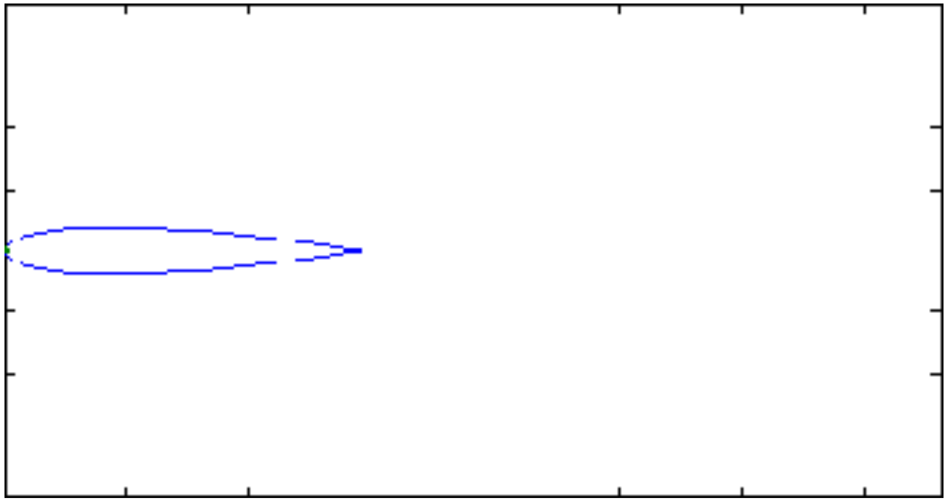


Wake examples (Plunge)

Plunging airfoil-
Low amplitude



Plunging airfoil-
High amplitude



Quasi-steady aerodynamics

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= Simplest possible modeling consists of **ignoring the effect of the wake**

- **Non-penetration** boundary condition (BC) :

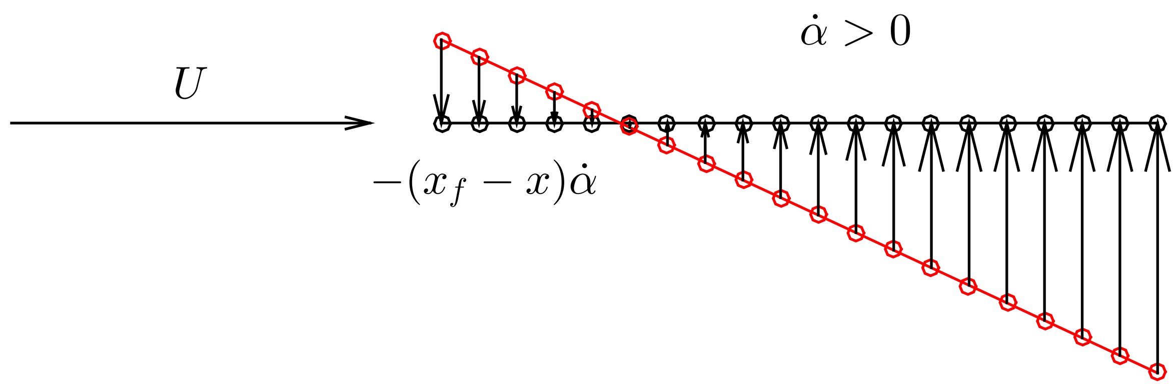
→ Total flow velocity at each point x of the airfoil must be parallel to the airfoil's camberline at that point

- Quasi-steady models assume that there are only **four contributions** to the aerodynamic forces:
  - Horizontal airspeed U, at angle $\alpha(t)$ to airfoil
  - Normal component of pitch speed, $-(x_f - x)\dot{\alpha}$
  - Airfoil plunge speed, $\dot{h}(t)$
  - Local velocity induced by the vorticity around the airfoil, $v_i(x,t)$

Quasi-steady aerodynamics

Effect of the pitch rate ?

Flow velocity seen by the flat plate at x: $-(x_f - x)\dot{\alpha}$



QS assumption: Flow velocity from the motion is much small than free stream velocity:

$$|(x_f - x)\dot{\alpha}| << U$$

Quasi-steady aerodynamics

→ **Effect of the pitch rate =** changing the flow angle at each chord-wise location:

– Free stream airspeed is still U

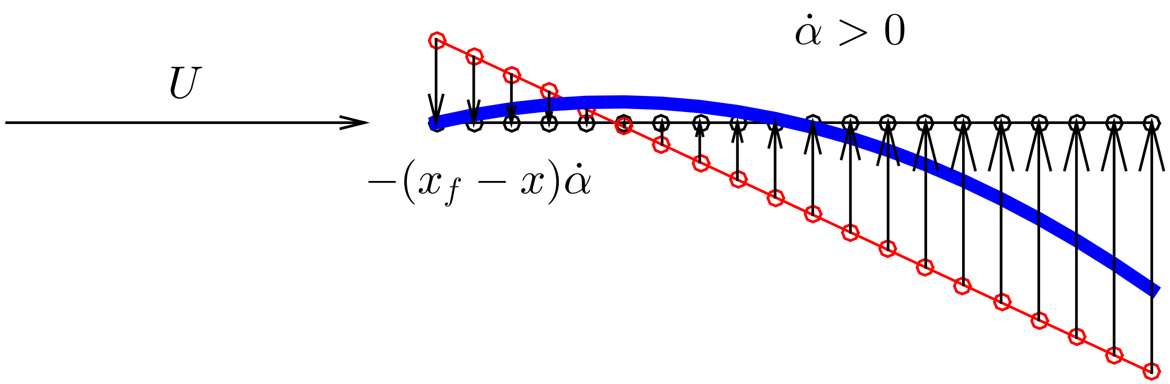
– Free stream angle is $-(x_f - x)\dot{\alpha}/U$

Negative angle of attack → Positive camber

→ Airfoil shape is **effectively cambered**

Camber slope = - effective aoa

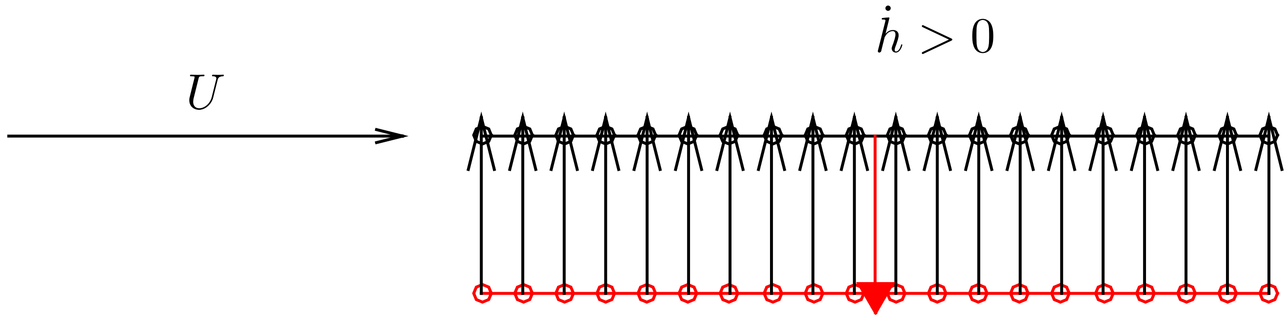
$$\mathrm{d}z/\mathrm{d}x = (x_f - x)\dot{\alpha}/U$$



Quasi-steady aerodynamics

Effect of the plunge rate ?

Also contribute to the relative airspeed, but constant over the entire airfoil



Assuming $\dot{h} << U$

→ Plunge rate change the total angle of attack:

$$\alpha_{\text{total}} = \alpha + \frac{\dot{h}}{U}$$

(*h* is defined positive downwards)

Lift coefficient

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From **thin airfoil theory**, $c_l = 2\pi\,(A_0 + A_1/2)$

where $A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{\mathrm{d}z}{\mathrm{d}x}\mathrm{d}\theta \qquad A_n = \frac{2}{\pi}\int_0^\pi \frac{\mathrm{d}z}{\mathrm{d}x}\cos n\theta \mathrm{d}\theta$

$$x = \frac{c}{2}(1-\cos\theta)$$

and effective camber from the pitching motion:

$$\mathrm{d}z/\mathrm{d}x = (x_f - x)\dot{\alpha}/U \qquad \alpha_{\text{total}} = \alpha + \frac{\dot{h}}{U}$$

$$\ldots \rightarrow \; c_l(t) = 2\pi\left(\alpha(t) + \frac{\dot{h}(t)}{U} + \left(\frac{3}{4}c - x_f\right)\frac{\dot{\alpha}(t)}{U}\right)$$

= total circulatory lift acting on the airfoil
(lift created by the vorticity distribution)

There is another type of lift acting on it (presented later)

Moment coefficient

- Moment coefficient around the leading edge (according to the thin airfoil theory) theory is given by $c_m = -c_l/4 - \pi\,(A_1 - A_2)/4$

- The moment coefficient around the flexural axis is given by $c_{mxf} = c_m + x_f c_l / c$

- Substituting and integrating yields

$$C_{m_{xf}}(t) = -\frac{c\pi}{8U}\dot{\alpha}(t) + \left(\frac{x_f}{c} - \frac{1}{4}\right) C_l(t)$$

Added Mass

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- Apart from the circulatory lift and moment, the air exerts another force on the airfoil.
- The wing is forcing a mass of air (fluid) around it to move. The air reacts and this force is known as the **added mass effect**.
- It can be seen as the effort required to move a cylinder of air with mass $\pi\rho b^2$ where $b=c/2$
- This force causes both lift and moment contributions

Full lift and moment

always important

$$l(t) = \rho\pi b^2\left(\ddot{h} - \left(x_f - \frac{c}{2}\right)\ddot{\alpha}\right) + \rho\pi b^2 U\dot{\alpha} + \rho U^2 c\pi\left(\alpha(t) + \frac{\dot{h}(t)}{U} + \left(\frac{3}{4}c - x_f\right)\frac{\dot{\alpha}(t)}{U}\right)$$

only important at high frequency

$$\begin{aligned} m_{xf}(t) \quad = \quad & \rho\pi b^2\left(x_f - \frac{c}{2}\right)\left(\ddot{h} - \left(x_f - \frac{c}{2}\right)\ddot{\alpha}\right) - \frac{\rho\pi b^4}{8}\ddot{\alpha} - \left(\frac{3}{4}c - x_f\right)\rho\pi b^2 U\dot{\alpha} + \\ & \rho U^2 e c^2\pi\left(\alpha(t) + \frac{\dot{h}(t)}{U} + \left(\frac{3}{4}c - x_f\right)\frac{\dot{\alpha}(t)}{U}\right) - \frac{1}{16}\rho U c^3\pi\dot{\alpha} \qquad (1) \end{aligned}$$

where $e = \left(\frac{x_f}{c} - \frac{1}{4}\right)$

These are substituted into the structural equations of motion:

$$\begin{pmatrix} m & S \\ S & I_\alpha \end{pmatrix}\begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{pmatrix} K_h & 0 \\ 0 & K_\alpha \end{pmatrix}\begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -l(t) + p(t) \\ m_{xf}(t) + r(t) \end{Bmatrix}$$

Full aeroelastic equations of motion

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$$\begin{pmatrix} m & S \\ S & I_\alpha \end{pmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{pmatrix} K_h & 0 \\ 0 & K_\alpha \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -l(t) + p(t) \\ m_{xf}(t) + r(t) \end{Bmatrix}$$

= Second order, linear, ordinary differential equations.

- Notice that the equations are of the form

$$\left(\mathbf{A} + \rho \mathbf{B}\right)\ddot{\mathbf{q}} + \left(\mathbf{C} + \rho U \mathbf{D}\right)\dot{\mathbf{q}} + \left(\mathbf{E} + \rho U^2 \mathbf{F}\right)\mathbf{q} = \mathbf{0}$$

- And that there are mass, damping and stiffness matrices **both aerodynamic and structural**

Pitch-plunge equations of motion

$$\begin{pmatrix} m & S \\ S & I_\alpha \end{pmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \rho\pi b^2 \begin{pmatrix} 1 & \left(\frac{c}{2} - x_f\right) \\ \left(\frac{c}{2} - x_f\right) & \left(\frac{c}{2} - x_f\right)^2 + \frac{b^2}{8} \end{pmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} +$$

$$\rho U c\pi \begin{pmatrix} 1 & \left(\frac{3}{4}c - x_f\right) + \frac{c}{4} \\ -ec & \left(\frac{c}{2} - x_f\right)^2 + \left(\frac{3}{4}c - x_f\right)\frac{c}{4} \end{pmatrix} \begin{Bmatrix} \dot{h} \\ \dot{\alpha} \end{Bmatrix} +$$

$$\begin{pmatrix} K_h & 0 \\ 0 & K_\alpha \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} + \rho U^2 c\pi \begin{pmatrix} 0 & 1 \\ 0 & -ec \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

= Full equations of motion for the pitch-plunge airfoil with quasi-steady aerodynamics.

Static Aeroelasticity

- Study of the static equilibrium of the system.
- Static means that **all velocities and accelerations are zero.**

The equations of motion become

$$\begin{pmatrix} K_h & \rho U^2 c\pi \\ 0 & K_\alpha - \rho U^2 e c^2 \pi \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Aerodynamic Coupling (1)

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- Apply an external moment M at the flexural axis
- The static equilibrium equations become

$$\begin{pmatrix} K_h & \rho U^2 c\pi \\ 0 & K_\alpha - \rho U^2 e c^2 \pi \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ M \end{Bmatrix}$$

- The second equation can only be satisfied if

$$\alpha = M/(K_\alpha - \rho U^2 e c^2 \pi)$$

- Then, the first equation can only be satisfied if

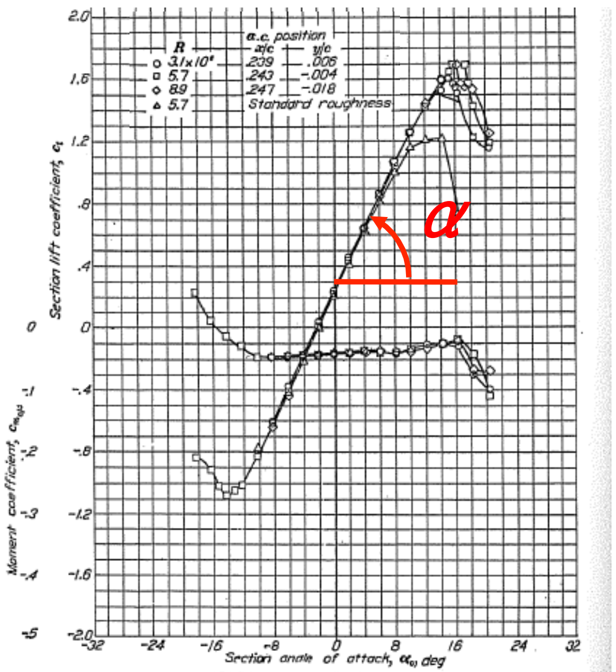
$$h = -\rho U^2 c\pi M/K_h(K_\alpha - \rho U^2 e c^2 \pi)$$

Aerodynamic Coupling (2)

- This phenomenon is called **aerodynamic coupling:**

Changing the pitch angle causes a change in the plunge.

- This is logical since increased pitch means increased lift.



NACA 2412 Wing Section

Aerodynamic Coupling (3)

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- If we apply a force F on the flexural axis
- The static EOM becomes

$$\begin{pmatrix} K_h & \rho U^2 c\pi \\ 0 & K_\alpha - \rho U^2 e c^2 \pi \end{pmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

- The second equation can only be satisfied if $\alpha = 0$
- The first equation then gives $h = F/K_h$

→No aerodynamic coupling:

Increasing the plunge does not affect the pitch.

- This is not the general case. The pitch-plunge model ignores 3D aerodynamic effects
- In real aircraft bending and torsion are both coupled.

Static Divergence (1)

- The second static equilibrium equation with an applied moment

$$\left(K_\alpha - \rho U^2 e c^2 \pi\right)\alpha = M$$

- If $K_\alpha > \rho U^2 e c^2 \pi$ → the spring and aerodynamic stiffness constitute a restoring force, which will balance the external moment

- If $K_\alpha < \rho U^2 e c^2 \pi$ → the spring and aerodynamic stiffness do not constitute a restoring force. Instead of balancing the external moment, they add to it.

  → The static equilibrium is unstable.

Static Divergence (2)

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- Static divergence in pitch occurs if the moment due to the lift around the flexural axis is higher than the structural restoring force and of opposite sign :

$$\rho U^2 e c^2 \pi \, \alpha > K_\alpha \alpha \quad \rightarrow \quad \rho U^2 e c^2 \pi > K_\alpha$$

- For every pitch stiffness there is an airspeed above which static divergence will occur

$$U = \sqrt{\frac{K_\alpha}{\rho e c^2 \pi}}$$

- This airspeed must be outside the flight envelope of the aircraft (with margin) to be safe.
- The pitch-plunge model does not allow for static divergence in plunge.
- Again, this is because it ignores 3D effects.

Static Divergence (3)

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movie

NASA wind tunnel experiment on a forward swept wing

Static Divergence (4)

Remember that $e = x_f / c - 1/4$

- If the flexural axis lies on the quarter-chord (aerodynamic center), → $e = 0$

→ $\rho U^2 e c^2 \pi \alpha = 0$ → no moment of the lift around the flexural axis

→ Static divergence is no longer possible

- If x_f is ahead of the aerodynamic centre, $e < 0$ and the static equilibrium equation becomes

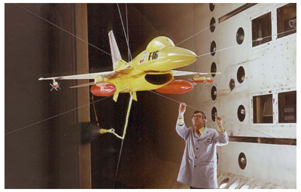
$$\left(K_\alpha + \rho U^2 |e| c^2 \pi\right) \alpha = M$$

→ Static divergence is no longer possible

Summary

- **Aeroelasticity** = study of the interaction of inertial, structural and aerodynamic forces
- **Instabilities**: divergence, flutter, VIV, galloping
- **Analysis techniques:**

→ Wind tunnel testing



Aeroelastic scaling (similarity laws)

→ Flight flutter testing

→ Aeroelastic design

2 dof's + QS aerodynamics

Divergence = static instability

$$\begin{pmatrix} m & S \\ S & I_\alpha \end{pmatrix}\begin{Bmatrix}\ddot h\\ \ddot\alpha\end{Bmatrix} + \rho\pi b^2 \begin{pmatrix} 1 & (\frac{c}{2} - x_f) \\ (\frac{c}{2} - x_f) & (\frac{c}{2} - x_f)^2+\frac{b^2}{8}\end{pmatrix}\begin{Bmatrix}\ddot h\\ \ddot\alpha\end{Bmatrix} + \rho U c\pi \begin{pmatrix} 1 & (\frac{3}{4} c - x_f)+\frac{c}{4} \\ -ec & (\frac{c}{2} - x_f)^2 + (\frac{3}{4} c - x_f)\frac{c}{4}\end{pmatrix}\begin{Bmatrix}\dot h\\ \dot\alpha\end{Bmatrix} + \begin{pmatrix}K_h & 0\\ 0 & K_\alpha\end{pmatrix}\begin{Bmatrix}h\\ \alpha\end{Bmatrix} + \rho U^2 c\pi\begin{pmatrix}0 & 1\\ 0 & -ec\end{pmatrix}\begin{Bmatrix}h\\ \alpha\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$