African Fractals

MODERN COMPUTING AND INDIGENOUS DESIGN

RON EGLASH

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PART I
Introduction

CHAPTER

I

Introduction to fractal geometry

Fractal geometry has emerged as one of the most exciting frontiers in the fusion between mathematics and information technology. Fractals can be seen in many of the swirling patterns produced by computer graphics, and they have become an important new tool for modeling in biology, geology, and other natural sciences. While fractal geometry can indeed take us into the far reaches of high-tech science, its patterns are surprisingly common in traditional African designs, and some of its basic concepts are fundamental to African knowledge systems. This book will provide an easy introduction to fractal geometry for people without any mathematics background, and it will show how these same categories of geometric pattern, calculation, and theory are expressed in African cultures.

Mathematics and culture

For many years anthropologists have observed that the patterns produced in different cultures can be characterized by specific design themes. In Europe and America, for example, we often see cities laid out in a grid pattern of straight streets and right-angle corners. Another grid, the Cartesian coordinate system, has long been a foundation for the mathematics used in these societies. In many works of Chinese art we find hexagons used with extraordinary geometric precision—a choice that might seem arbitrary were it not for the importance of the number six in the hexagrams of their fortunetelling system (the *I Ching*), in the anatomy charts for acupuncture (*liù-qì* or "six spirits"), and even in Chinese architecture.[1] Shape and number are not only the universal rules of measurement and logic; they are also cultural tools that can be used for expressing particular social ideas and linking different areas of life. They are, as Claude Lévi-Strauss would put it, "good to think with."

Design themes are like threads running through the social fabric; they are less a commanding force than something we command, weaving these strands into many different patterns of meaning. The ancient Chinese empires, for example, used a base-10 counting system, and they even began the first universal metric system.[2] So the frequent use of the number 60 in Chinese knowledge systems can be linked to the combination of this official base 10 notation with their sacred number six. In some American cities we find that the streets are numbered like Cartesian coordinates, but in others they are named after historical figures, and still others combine the two. These city differences typically correspond to different social meanings—an emphasis on history versus efficiency, for example.

Suppose that visitors from another world were to view the grid of an American city. For a city with numbered streets, the visitors (assuming they could read our numbers) could safely conclude that Americans made use of a coordinate structure. But do these Americans actually understand coordinate mathematics? Can they use a coordinate grid to graph equations? Just how sophisticated is their mathematical understanding? In the following chapter, we will find ourselves in a similar position, for African settlement architecture is filled with remarkable examples of fractal structure. Did precolonial Africans actually understand and apply fractal geometry?

As I will explain in this chapter, fractals are characterized by the repetition of similar patterns at ever-diminishing scales. Traditional African settlements typically show this "self-similar" characteristic: circles of circles of circular dwellings, rectangular walls enclosing ever-smaller rectangles, and streets in which broad avenues branch down to tiny footpaths with striking geometric repetition. The fractal structure will be easily identified when we compare aerial views of these African villages and cities with corresponding fractal graphics simulations.

What are we to make of this comparison? Let's put ourselves back in the shoes of the visitors from another planet. Having beamed down to an American settlement named "Corvallis, Oregon," they discover that the streets are not numbered, but rather titled with what appear to be arbitrary names: Washington, Jefferson, Adams, and so on. At first they might conclude that there is nothing mathematical about it. By understanding a bit more about the cultural meaning, however, a mathematical pattern does emerge: these are names in historical succession. It might be only ordering in terms of position in a series (an "ordinal" number), but there is some kind of coordinate system at work after all. African designs have to be approached in the same way. We cannot just assume that African fractals show an understanding of fractal geometry, nor can we dismiss that possibility. We need to listen to what the designers and users of these structures have to say about it. What appears to be an unconscious or accidental pattern might actually have an intentional mathematical component.

Overall, the presence of mathematics in culture can be thought of in terms of a spectrum from unintentional to self-conscious. At one extreme is the emergence of completely unconscious structures. Termite mounds, for example, are excellent fractals (they have chambers within chambers within chambers) but no one would claim that termites understand mathematics. In the same way, patterns appear in the group dynamics of large human populations, but these are generally not patterns of which any individual is aware. Unconscious structures do not count as mathematical knowledge, even though we can use mathematics to describe them.

Moving along this spectrum toward the more intentional, we next find examples of decorative designs which, although consciously created, have no explicit knowledge attached to them. It is possible, for example, that an artist who does not know what the word "hexagon" means could still draw one with great precision. This would be a conscious design, but the knowledge is strictly implicit.[3] In the next step along our spectrum, people make these components explicit—they have names for the patterns they observe in shapes and numbers. Taking the intention spectrum one more step, we have rules for how these patterns can be combined. Here we can find "applied mathematics." Of course there is a world of difference between the applied math of a modern engineer and the applied math of a shopkeeper—whether or not something is intentional tells us nothing about its complexity.

Finally we move to "pure mathematics," as found in the abstract theories of modern academic mathematicians. Pure math can also be very simple—for example, the distinction between ordinal numbers (first, second, third) and cardinal numbers (one, two, three) is an example of pure math. But it would not be enough for people in a society simply to use examples of both types; they would have to have words for these two categories and explicitly reflect on a comparison of their properties before we would say that they have a theory of the distinction between ordinal and cardinal numbers. While applied mathematics makes use of rules, pure math tells us why they work—and how to find new ones.

This book begins by moving along the spectrum just described. We will start by showing that African fractals are not simply due to unconscious activity. We will then look at examples where they are conscious but implicit designs, followed by examples in which Africans have devised explicit rules for generating these patterns, and finally to examples of abstract theory in these indigenous knowledge systems. The reason for taking such a cautious route can be expressed in terms of what philosopher Karl Popper called "falsifiability." Popper pointed out that everyone has the urge to confirm their favorite theories; and so we have to take precautions not to limit our attention to success—a theory is only good if you try to test it for failure. If we only use examples where African knowledge systems successfully matched fractal geometry, we would not know its limitations. There are indeed gaps where the family of theories and practices centered around fractal geometry in high-tech mathematics has no counterpart in traditional Africa. Although such gaps are significant, they do not invalidate the comparison, but rather provide the necessary qualifications to accurately characterize the indigenous fractal geometry of Africa.

Overview of the text

Following the introduction to fractal geometry in the next section, in chapter 2 we will explore fractals in African settlements. It will become clear that the explanation of unconscious group activity does not fit this case. When we talk to the indigenous architects, they are quite explicit about those same fractal features we observe, and use several of the basic concepts of fractal geometry in discussing their material designs and associated knowledge systems. Termites may make fractal architectures, but they do not paint abstract models of the structure on its walls or create symbols for its geometric properties. While these introductory examples won't settle all the questions, we will at least have established that these architectural designs should be explained by something more than unintentional social dynamics.

In chapter 3 we will examine another explanation: perhaps fractal settlement patterns are not unique to Africa, and we have simply observed a common characteristic of all non-Western architectures. Here the concept of design themes become important. Anthropologists have found that the design themes found in each culture are fairly distinct—that is, despite the artistic diversity within each society, most of the culture's patterns can be characterized by specific geometric practices. We will see that although fractal designs do occur outside of Africa (Celtic knots, Ukrainian eggs, and Maori rafters have some excellent examples), they are not everywhere. Their strong prevalence in Africa (and in African-influenced southern India) is quite specific.

Chapter 4 returns to this exploration with fractals in African esthetic design. These examples are important for two reasons. First, they remind us that we cannot assume explicit, formal knowledge simply on the basis of a pattern. In contrast to the fractal patterns of African settlement architecture, these aesthetic fractals, according to the artisans, were made "just because it looks pretty that way." They are neither formal systems (no rules to the game) nor do the artisans' report explicit thinking ("I don't know how or why, it just came to me"). Second, they provide one possible route by which a particular set of mathematical concepts came to be spread over an enormous continent. Trade networks could have diffused the fractal aesthetic across Africa, reinforcing a design theme that may have been scattered about in other areas of life. Of course, such origin stories are never certain, and all too easy to invent.

Part II of this book, starting with chapter 5, presents the explicit design methods and symbolic systems that demonstrate fractal geometry as an African knowledge system. As in the introduction to fractals in the first chapter, I will assume the reader has no mathematics background and provide an introduction to any new concepts along with the African versions. We will see that not only in architecture, but in traditional hairstyling, textiles, and sculpture, in painting, carving, and metalwork, in religion, games, and practical craft, in quantitative techniques and symbolic systems, Africans have used the patterns and abstract concepts of fractal geometry.

Chapter 10, the last in part II, is the result of my collaboration with an African physicist, Professor Christian Sina Diatta. A sponsor for the Fulbright fellowship that enabled my fieldwork in west and central Africa, Dr. Diatta took the idea of indigenous fractals and ran with it, moving us in directions that I would never have considered on my own, and still have yet to explore fully.

In the third and final part of this book we will examine the consequences of African fractal geometry: given that it does exist, what are its social implications? Chapter 11 will briefly review previous studies of African knowledge systems. We will see that although several researchers have proposed ideas related to the fractal concept—Henry Louis Gates's "repetition with revision," Léopold Senghor's "dynamic symmetry," and William Fagg's "exponential morphology" are all good examples—there have been specific obstacles that prevented anthropologists and others from taking up these concepts in terms of African mathematics.

Chapter 12 covers the political consequences of African fractals. On the one hand, we will find there is no evidence that geometric form has any *inherent* social meaning. In settlement design, for example, people report both oppressive and liberatory social experiences with fractal architectures. Fractal versus nonfractal ("Euclidean") geometry does not imply good versus bad. On the other hand, people do invest abstract forms with particular local meanings. To take a controversial example, recent U.S. supreme court decisions declared that voting districts cannot have "bizarre" or "highly irregular" shapes, and several of these fractal contours have been replaced by the straight lines of Euclidean form. If fractal settlement patterns are traditional for people of African descent, and Euclidean settlement patterns for Europeans, is it ethnocentric to insist on only Euclidean voting district lines?

Chapter 13 will examine the cultural history of fractal geometry and its mathematical precursors in Europe. We will see that the gaps are not one-sided: just as Africans were missing certain mathematical ideas in their version of fractal geometry, Europeans were equally affected by their own cultural views and have been slow to adopt some of the mathematical concepts that were long championed by Africans. Indeed, there is striking evidence that some of the sources of mathematical inspiration for European fractals were of African origin. The final chapter will move forward in time, highlighting the contemporary versions of fractal design that have been proposed by African architects in Senegal, Mali, and Zambia, and other illustrations of possible fractal futures.

But to understand all this, we must first visit the fractal past.

A historical introduction to fractal geometry

The work of Georg Cantor (1845–1918), which produced the first fractal, the Cantor set (fig. 1.1), proved to be the beginning of a new outlook on infinity. Infinity had long been considered suspect by mathematicians. How can we claim to be using only exact, explicit rules if we have a symbol that vaguely means "the number you would get if you counted forever"? So many mathematicians, starting with Aristotle, had just banned it outright. Cantor showed that it was possible to keep track of the number of elements in an infinite set, and did so in a deceptively simple fashion. Starting with a single straight line, Cantor erased the middle third, leaving two lines. He then carried out the same operation on those two lines, erasing their middles and leaving four lines. In other words, he used a sort of feedback loop, with the end result of one stage brought back as the starting point for the next. This technique is called "recursion." Cantor showed that if this recursive construction was continued forever, it would create an infinite number of lines, and yet would have zero length.

Not only did Cantor reintroduce infinity as a proper object of mathematical study, but his recursive construction could be used as a model for other "pathological curves," such as that created by Helge von Koch in 1904 (figs. 1.2, 1.3). The mathematical properties of these figures were equally perplexing. Small portions looked just like the whole, and these reflections were repeated down to infinitesimal scales. How could we measure the length of the Koch curve? If

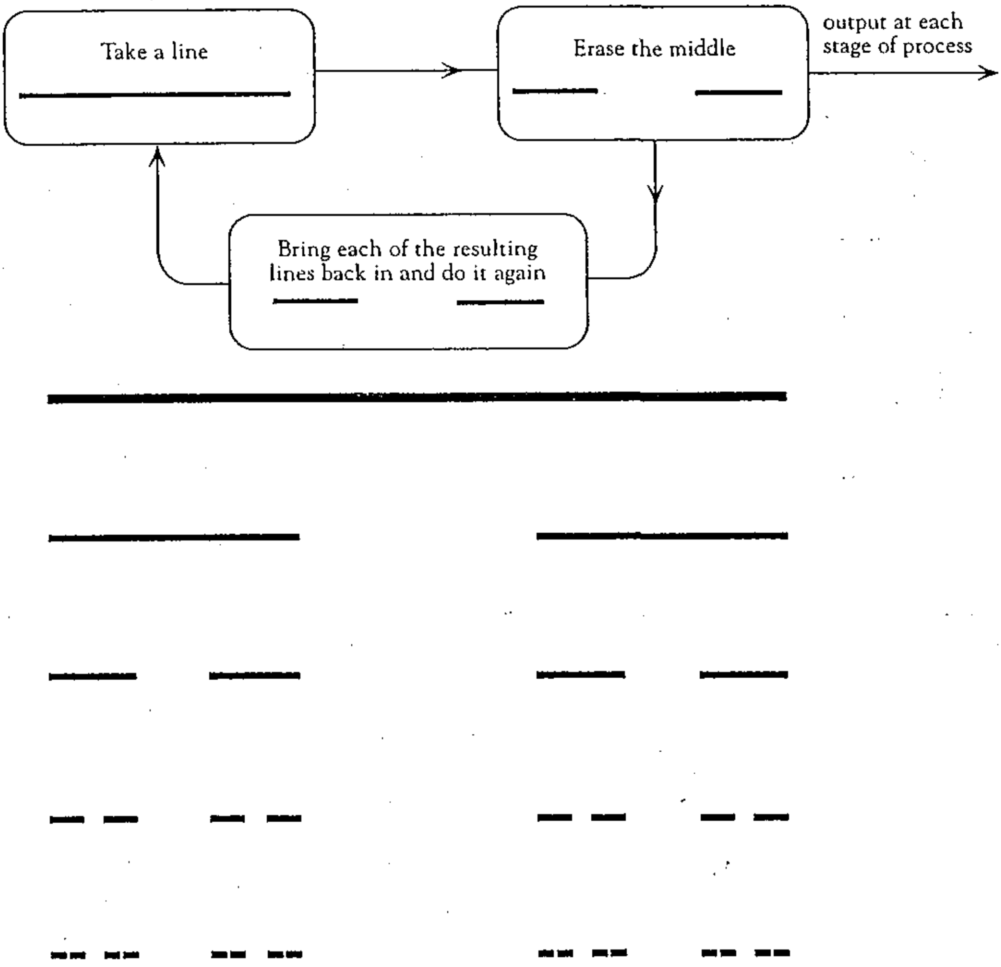


FIGURE 1.1
The Cantor set
In 1877 Georg Cantor came up with the idea of repeatedly subdividing a line to illustrate the concept of an infinite set. This looping technique is called recursion. By specifying that the recursion continues forever, Cantor was able to define an infinite set.

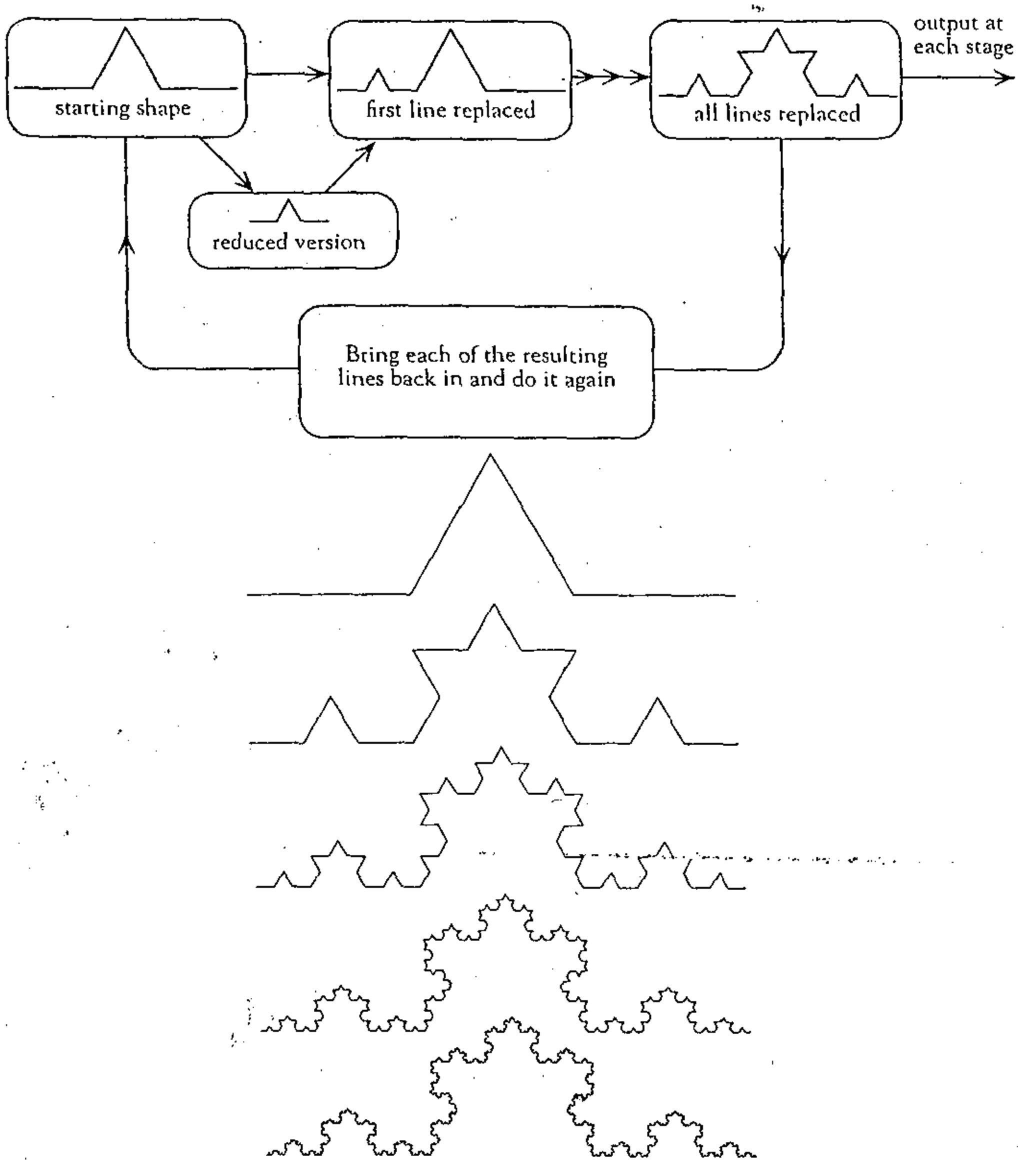


FIGURE 1.2

The Koch curve

Helge von Koch used the same kind of recursive loop as Cantor, but he added lines instead of erasing them. He began with a triangular shape made of four lines, the "seed." He then replaced each of the lines with a reduced version of the original seed shape.

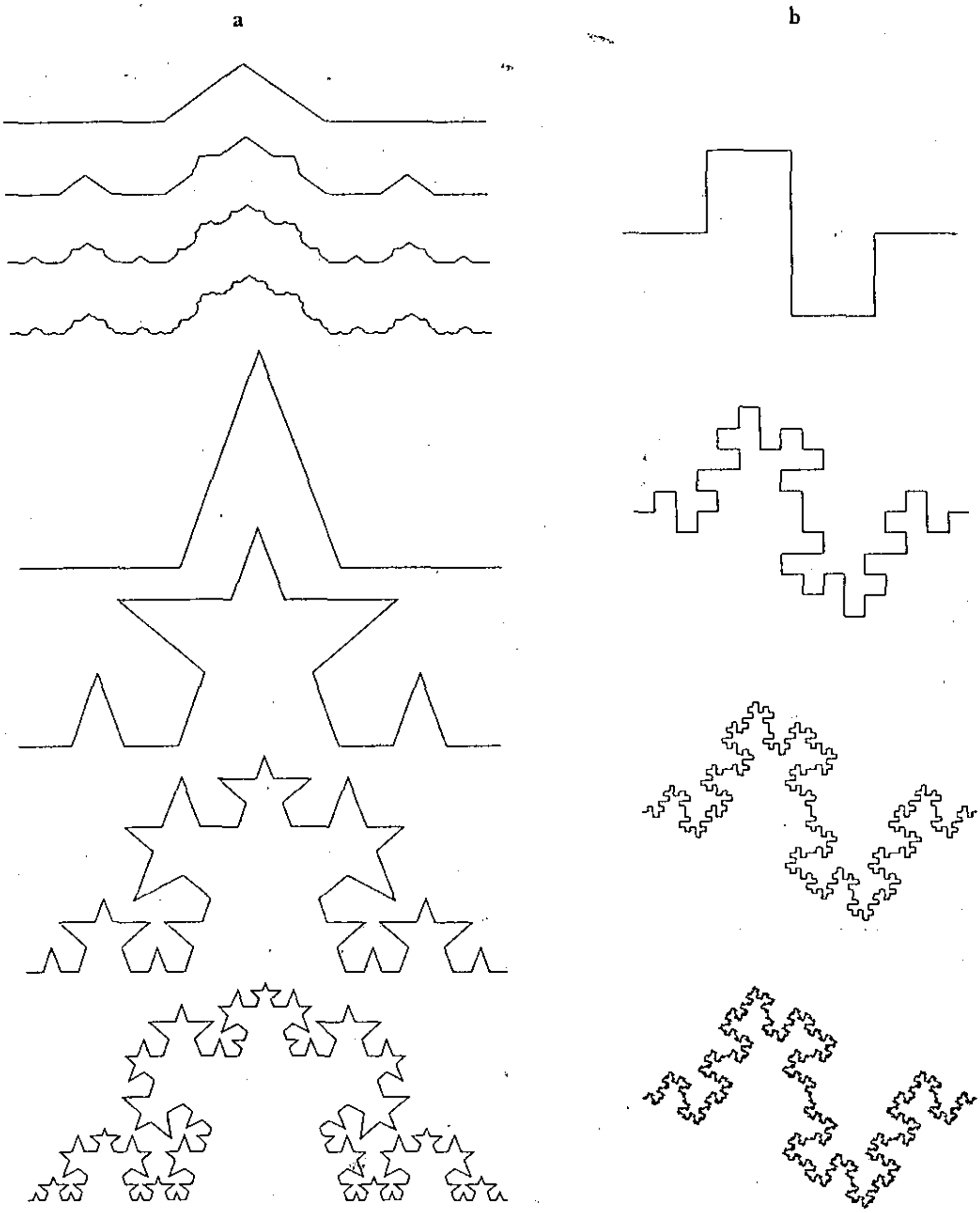


FIGURE 1.3

Koch curve variations

There is nothing special about the particular shape Koch first used. For example, we can make similar shapes that are more flat or more spiked using variations on the seed shape (a). Nor is there anything special about triangles—any shape can undergo this recursive replacement process. Mathematician *Giuseppe Peano, for example, experimented with rectangular seed shapes such as* those in (b).

we hold up a six-inch ruler to the curve (fig. 1.4) we get six inches, but of course that misses all the nooks and crannies. If we use a smaller ruler, we get greater length, and with a smaller one even greater length, and so on to infinity. Obviously this is not a very useful way to characterize one of these curves. A new way of thinking about measurement was needed. The answer was to plot these different measures of ruler size versus length, and see how fast we gain length as we shrink the ruler (fig. 1.5). This rate (the slope) tells us just how crinkled or tortuous the curve is. For extremely crinkled curves, the plot will show that we rapidly gain length as we shrink the ruler, so it will have a steep slope. For relatively smooth curves, you don't gain much length as you shrink the ruler size, so the plot has a *shallow slope*.

To mathematicians this slope was more than just a practical way to characterize crinkles. Recall that when we first tried to measure the length of the Koch curve, we found that its length was potentially infinite. Yet this infinite length fits into a bounded space. Mathematician Felix Hausdorff (1868–1942) found that this paradox could be resolved if we thought of the pathological curves as somehow taking up more than one dimension, as all normal lines do, but less than two dimensions, as flat shapes like squares and circles do. In Hausdorff's view, the Koch curve has a fractional dimension, approximately 1.3, which is the slope of our ruler-versus-length plot. Being pure mathematicians, they were fascinated with this idea of a fractional dimension and never thought about putting it to practical use.

The conceptual leap to practical application was created by Benoit Mandelbrot (b. 1924), who happened upon a study of long-term river fluctuations by British civil servant H. E. Hurst. Hurst had found that the yearly floods of rivers did not have any one average, but rather varied over many different scales—there were flood years, flood decades, even flood centuries. He concluded that the only way to characterize this temporal wiggliness was to plot the amount of fluctuation at each scale and use the slope of this line. Mandelbrot realized that this was equivalent to the kind of scaling measure that had been used for Cantor's pathological curves. As he began to apply computer graphics (figs. 1.6, 1.7), he found that these shapes were not pathological at all, but rather very common throughout the natural world. Mountain ranges had peaks within peaks, trees had branches made of branches, clouds were puffs within puffs—even his own body was full of recursive scaling structures.

The fractal simulations for natural objects in figure 1.7 were created just like the Cantor set, Koch curve, and other examples we have already seen, with a seed shape that undergoes recursive replacement. The only difference is that some of these simulations require that certain lines in the seed shape do not get

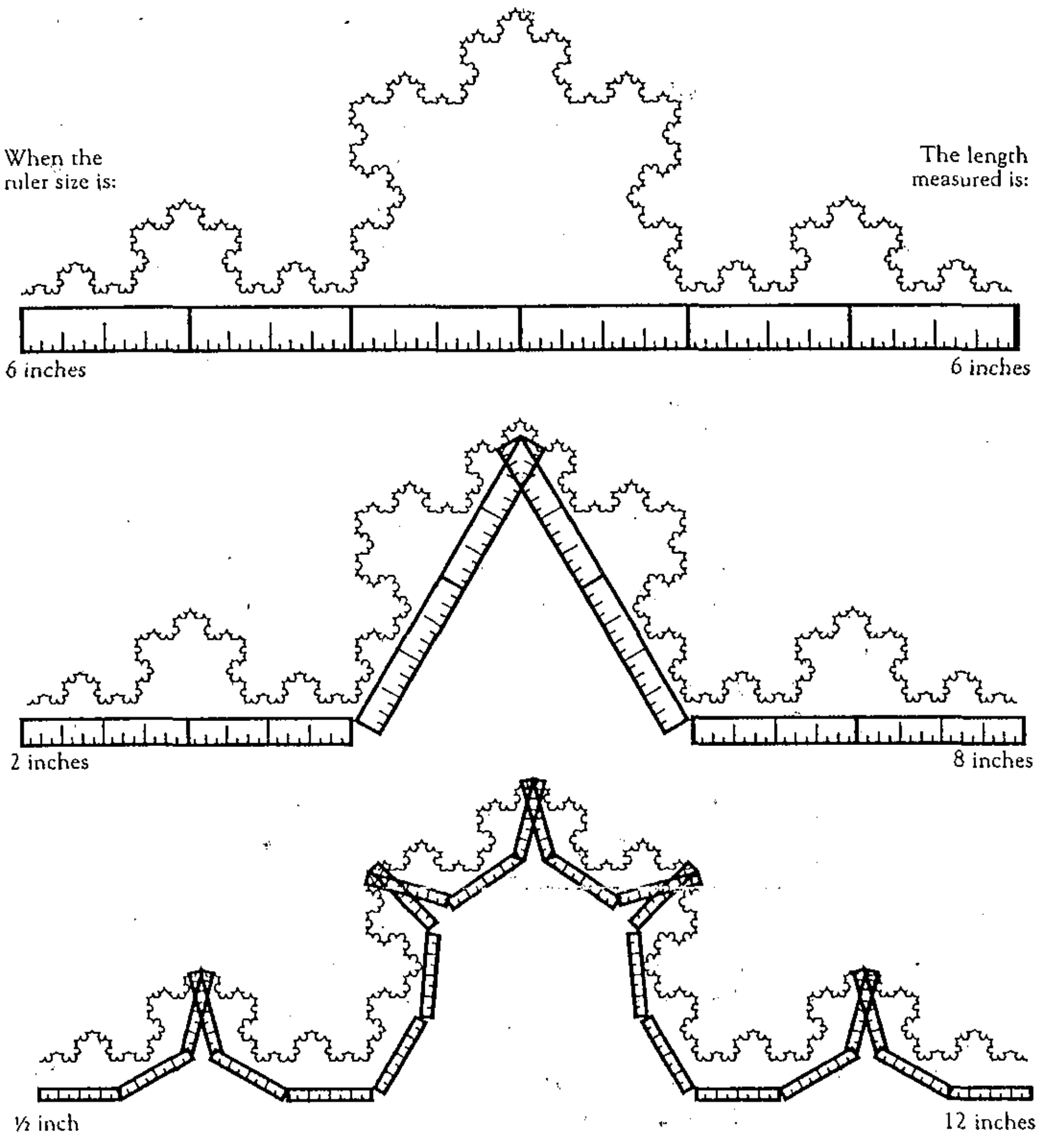


FIGURE I.4

Measuring the length of fractal curves

The new curves of Cantor, Koch, and others represented a problem in measurement theory. The length of the curve depends on the size of the ruler. As we shrink the ruler down, the length approaches infinity.

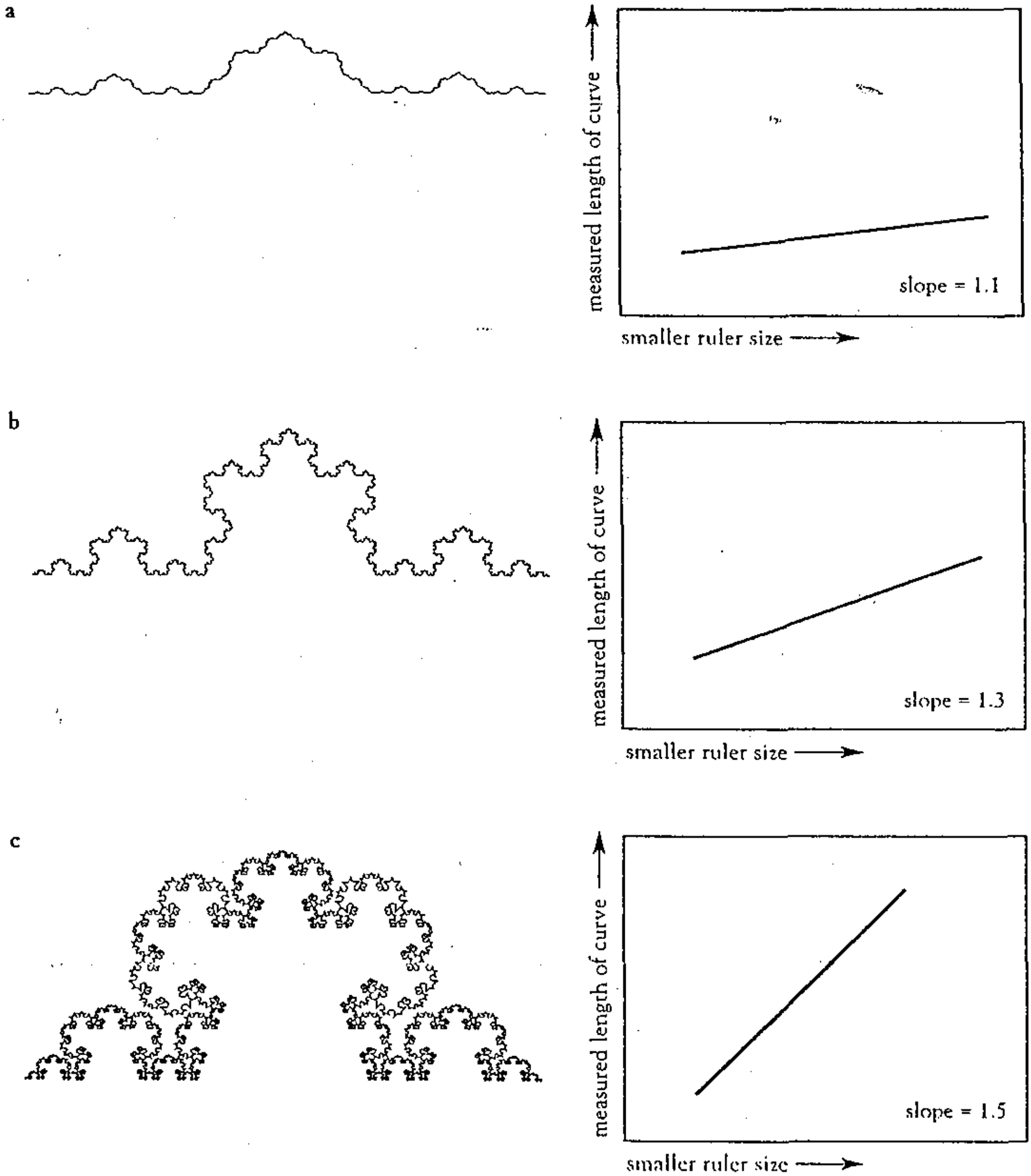


FIGURE 1.5

A better way to measure fractal curves

Our experiment in shrinking rulers wasn't a total waste. In fact, it turns out that if you keep track of how the measured length changes with ruler size, you get a very good way of characterizing the curve. A relatively smooth fractal won't increase length very quickly with shrinking ruler size, but very crinkled fractals will. (a) This smooth Koch curve doesn't add much length with shrinking ruler size, so the plot shows only a small rise. (b) Since a small ruler can get into all the nooks and crannies, this more crinkled Koch curve shows a steeper rise in measured length with a shrinking ruler. (c) An extremely tortuous Koch curve has a very steep slope for its plot.
Note for math sticklers: These figures are plotted on a logarithmic graph.

replaced. This is illustrated for the lung model at the bottom of figure 1.7. The lines that get replaced in each iteration are called "active lines." Those that do not get replaced are called "passive lines." We will be using the distinction between active and passive lines in simulations for African designs as well.

Mandelbrot coined the term "fractal" for this new geometry, and it is now used in every scientific discipline from astrophysics to zoology. It is one of the most powerful tools for the creation of new technologies as well as a revolutionary approach to the analysis of the natural world. In medicine, for example, fractal

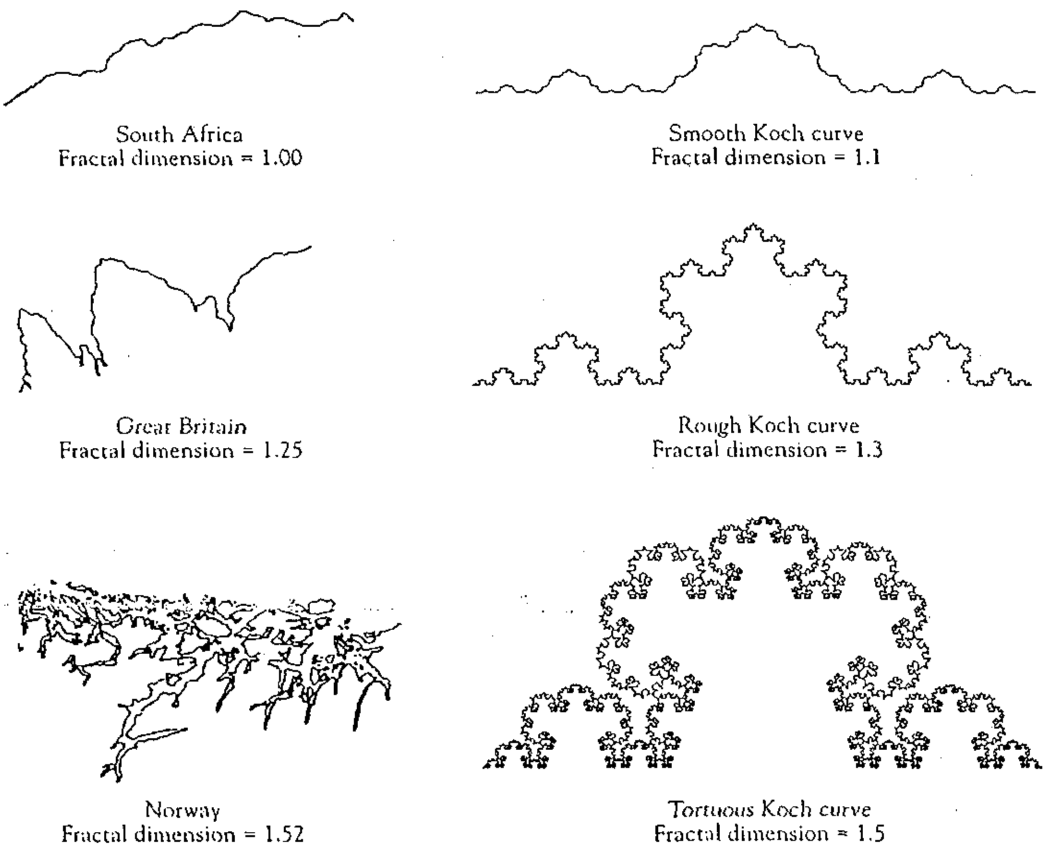
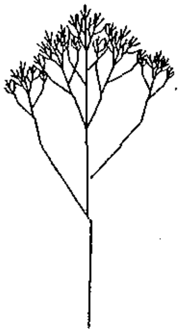


FIGURE 1.6

Measuring nature with fractal geometry

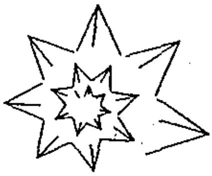
Although the curves of Cantor and others were introduced as abstractions without physical meaning, Benoit Mandelbrot realized that their scaling measure, which he called "fractal dimension," could be put to practical use in characterizing irregular shapes in nature. The classic example is the measurement of coastlines. Even though it is a very crude model, we can see how the variations of the roughness in the Koch curve are similar to the variations in these coasts. Note that the fractal dimension is our plot slope from figure 1.5; the coastlines were measured in the same way.

a

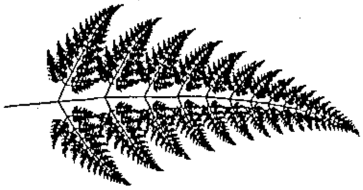


acacia tree

clouds



shell

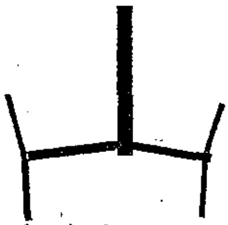


fern

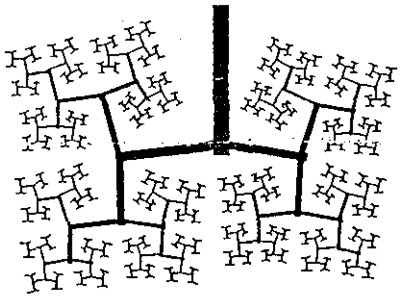
b

This vertical line is passive.

These two horizontal lines (gray) are the active lines that will be replaced by a reduced version of this seed shape.



After the first iteration we see that only the active lines were replaced; the passive line remains the same. Now there are three passive lines (center) and four active lines (the ends).



By the eighth iteration we can see the similarity to the scaling structure of the human lungs.

FIGURE 1.7

Simulating nature with fractal geometry

In his experiments with computer graphics, Mandelbrot found that fractal shapes abound in nature, where continual processes such as biological growth, geological change, and atmospheric turbulence result in a wide variety of recursive scaling structures (a). The recursive construction of these natural shapes is basically the same as that of the other fractal shapes we have seen so far. In some examples, like the lung model (b), certain lines of the original seed shape do not participate in the replacement step; they are called "passive lines." The ones which do go through replacement are called "active lines." Each step is referred to as an "iteration."

dimension can be used as a diagnostic tool. A healthy lung has a high fractal dimension, but when black lung disease begins it loses some of the fine branching—a condition that can be detected by measuring the fractal dimension of the X ray. For this reason, Benoit Mandelbrot was recently named an honorary member of the French Coal Miners Union.

Of course, no revolution is without its counterrevolutionaries. It was not long before some scientists started objecting that Mandelbrot was ignoring the presence of the natural objects that could be described by Euclidean geometry, such as crystals or eggs. It's true that not all of nature is fractal—and this will be an important point for us to keep in mind. Some writers have mistakenly attempted to portray Africans as "more natural"—a dangerous and misleading claim, even when made by well-meaning romantics. Since fractals are associated with nature, a book about "African fractals" could be misinterpreted as support for such romantic organicists. Pointing out that some Euclidean shapes exist in the realm of nature makes it easier to understand that African fractals are from the artificial realm of culture. Before moving on to these African designs, let's review the basic characteristics of fractal geometry.

Five essential components of fractal geometry

RECURSION

We have seen that fractals are generated by a circular process, a loop in which the output at one stage becomes the input for the next. Results are repeatedly returned, so that the same operation can be carried out again. This is often referred to as "recursion," a very powerful concept. Later we will distinguish between three different types of recursion, but for now just think of it in terms of this iterative feedback loop. We've already seen how iteration works to create the Cantor set and the Koch curve. Although we can create a mathematical abstraction in which the recursion continues forever, there are also cases where the recursion will "bottom out." In our generation of the Koch curve, for example, we quit once the lines get too small to print. In fact, any physically existing object will only be fractal within a particular range of scales.

SCALING

If you look at the coastline of a continent—take the Pacific side of North America for instance—you will see a jagged shape, and if you look at a small piece of that coastline—say, California—we continue to see similar jaggedness. In fact, a similar jagged curve can be seen standing on a cliff overlooking a rocky California shore, or even standing on that shore looking at one rock. Of course, that's only roughly similar, and it's only good for a certain range of scales, but it is astonishing to realize how well this works for many natural features. It is this "scaling" property of nature that allows fractal geometry to be so effective for modeling. To have a "scaling shape" means that there are similar patterns at different scales within the range under consideration. Enlarging a tiny section will produce a pattern that looks similar to the whole picture, and shrinking down the whole will give us something that looks like a tiny part.

SELF-SIMILARITY

Just how similar do these patterns have to be to qualify as a fractal? Mathematicians distinguish between statistical self-similarity, as in the case of the coastline, and exact self-similarity, as in the case of the Koch curve. In exact self-similarity we need to be able to show a precise replica of the whole in at least some of its parts. In the Koch curve a precise replica of the whole could be found within any section of the fractal ("strictly self-similar"), but this isn't true for all fractals. The branching fractals used to model the lungs and acacia tree (fig. 1.7), for example, have parts (e.g., the stem) that do not contain a tiny image of the whole. Unlike the Koch curve, they were not generated by replacing every line in the seed shape with a miniature version of the seed; instead, we used some passive lines that were just carried though the iterations without change, in addition to active lines that created a growing tip by the usual recursive replacement.

INFINITY

Since fractals can be limited to a finite range of scales, it may seem like infinity is just a historical artifact, at best a Holy Grail whose quest allowed mathematicians serendipitously to stumble across fractals. It is this kind of omission that has made many pure mathematicians rather nonplussed about the whole fractal affair, and in some cases downright hostile (cf. Krantz 1989). There is no way to connect fractals to the idea of dimension without using infinity, and for many mathematicians that is their crucial role.

FRACTIONAL DIMENSION

How can it be that the Koch curve, or any member of its fractal family, has infinite length in a finite boundary? We are used to thinking of dimension as only whole numbers—the one-dimensional line, the two-dimensional plane—but the theory of measurement that governs fractals allows dimensions to be fractions. Consider, for example, the increasing dimension of the Koch curves in figure 1.6. Above the top, we could go as close as we like to a one-dimensional line. Below the bottom, we could make the curve so jagged that it starts to fill in two-dimensional areas of the plane. In between, we need an in-between dimension.

Looking for fractals in African culture

As we examine African designs and knowledge systems, these five essential components will be a useful way to keep track of what does or does not match fractal geometry. Since scaling and self-similarity are descriptive characteristics, our first step will be to look for these properties in African designs. Once we establish that theme, we can ask whether or not these concepts have been intentionally applied, and start to look for the other three essential components. We will now turn to African architecture, where we find some of the clearest illustrations of indigenous self-similar designs.

CHAPTER 2

Fractals in African settlement architecture

Architecture often provides excellent examples of cultural design themes, because anything that is going to be so much a part of our lives—a structure that makes up our built environment, one in which we will live, work or play—is likely to have its design informed by our social concepts. Take religious architecture for example. Several churches have been built using a triangular floor plan to symbolize the Christian trinity; others have used a cross shape. The Roman Pantheon was divided into three vertical levels: the bottom with seven niches representing the heavenly bodies, the middle with the 12 zodiac signs, and on top a hemisphere symbolizing the order of the cosmos as a whole.[1] But we don't need to look to grandiose monuments; even the most mundane shack will involve geometric decisions—should it be square or oblong? pitched roof or flat? face north or west?—and so culture will play a role here as well.

At first glance African architecture might seem so varied that one would conclude its structures have nothing in common. Although there is great diversity among the many cultures of Africa, examples of fractal architecture can be found in every corner of the African continent. Not all architecture in Africa is fractal—fractal geometry is not the only mathematics used in Africa—but its repeated presence among such a wide variety of shapes is quite striking.

In each case presented here we will compare the aerial photo or architectural diagram of a settlement to a computer-generated fractal model. The fractal simulation will make the self-similar aspects of the physical structure more evident, and in some cases it will even help us understand the local cultural meaning of the architecture. Since the African designers used techniques like iteration in building these structures, our virtual construction through fractal graphics will give us a chance to see how the patterns emerge through this process.

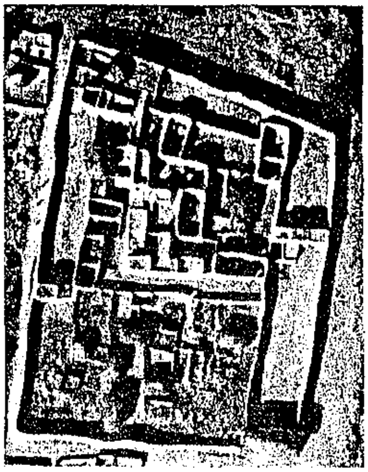
Rectangular fractals in settlement architecture

If you fly over the northern part of Cameroon, heading toward Lake Chad along the Logone River, you will see something like figure 2.1a. This aerial photo shows the city of Logone-Birni in Cameroon. The Kotoko people, who founded this city centuries ago, use the local clay to create huge rectangular building complexes. The largest of these buildings, in the upper center of the photo, is the palace of the chief, or "Miarre" (fig. 2.1b). Each complex is created by a process often called "architecture by accretion," in this case adding rectangular enclosures to preexisting rectangles. Since new enclosures often incorporate the walls of two or more of the old ones, enclosures tend to get larger and larger as you go outward from the center. The end result is the complex of rectangles within rectangles within rectangles that we see in the photo.

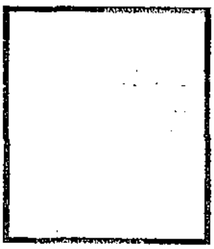
Since this architecture can be described in terms of self-similar scaling—it makes use of the same pattern at several different scales—it is easy to simulate using a computer-generated fractal, as we see in figures 2.1c–e. The seed shape of the model is a rectangle, but each side is made up of both active lines (gray) and passive lines (black). After the first iteration we see how a small version of the original rectangle is reproduced by each of the active lines. One more iteration gives a range of scales that is about the same as that of the palace; this is enlarged in figure 2.1e.

During my visit to Logone-Birni in the summer of 1993, the Miarre kindly allowed me to climb onto the palace roof and take the photo shown in figure 2.1f. I asked several of the Kotoko men about the variation in scale of their architecture. They explained it in terms of a combination of patrilocal household expansion, and the historic need for defense. "A man would like his sons to live next to him," they said, "and so we build by adding walls to the father's house." In the past, invasions by northern marauders were common, and so a larger defensive wall was also needed. Sometimes the assembly of families would outgrow this defensive enclosure, and so they would turn that wall into housing, and build an even larger enclosure around it. These scaling additions created the tradition of self-similar shapes we still see today, although the population is far below the

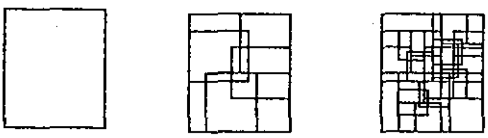
a. An aerial view of the city of Logone-Birni in Cameroon. The largest building complex, in the center, is the palace of the chief.
Photo courtesy Musée de l'Homme, Paris.



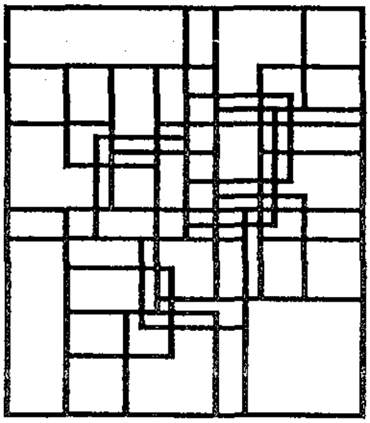
b. A closer view of the palace. The smallest rectangles, in the center, are the royal chambers.



c. Seed shape for the fractal simulation of the palace. The active lines, in gray, will be replaced by a scaled-down replica of the entire seed.



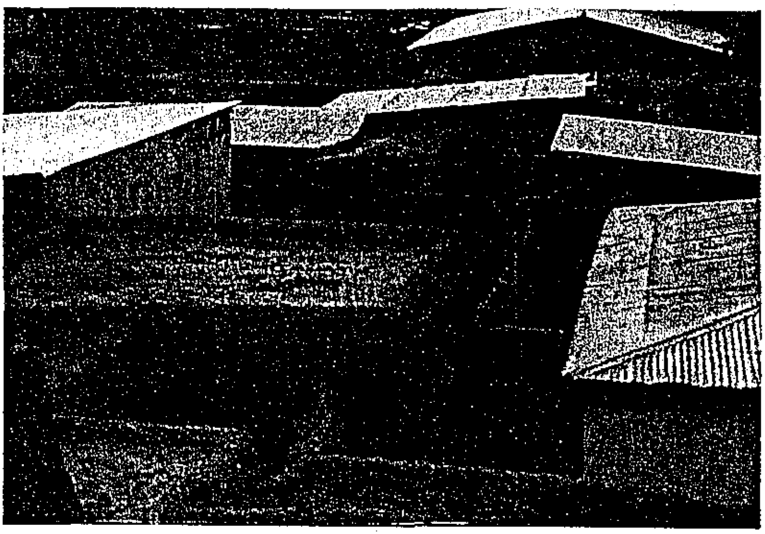
d. First three iterations of the fractal simulation.



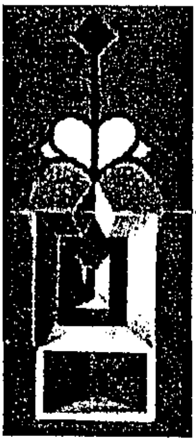
e. Enlargment of the third iteration.

FIGURE 2.1
Logone-Birni

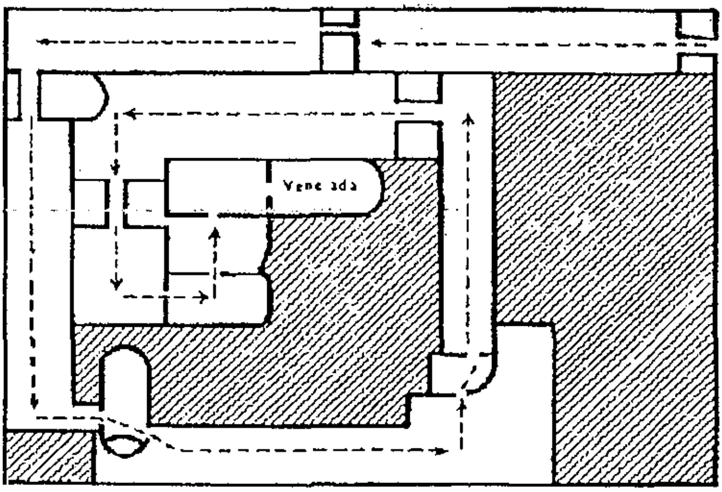
(figure continues)



f. Photo by the author taken from the roof of the palace.



g. The guti, the royal insignia, painted on the palace walls. *By permission of Lebeuf 1969.*



Le chemin de la lumière

h. The spiral path taken by visitors to the throne. *By permission of Lebeuf 1969.*

FIGURE 2.1 *(continued)*
Inside Logone-Birni

original 180,000 estimated for Logone-Birni's peak in the nineteenth century. At that time there was a gigantic wall, about 10 feet thick, that enclosed the perimeter of the entire settlement.

The women I spoke with were much less interested in either patrilineage or military history; their responses concerning architectural scaling were primarily about the contrast between the raw exterior walls and the stunning waterproof finish they created for courtyards and interior rooms. This began by smoothing wet walls flat with special stones, applying a resin created from a plant extract, and then adding beautifully austere decorative lines.

The most important of these decorative drawings is the *guti*, a royal insignia (fig. 2.1g). The central motif of the guti shows a rectangle inside a rectangle inside a rectangle; it is a kind of abstract model that the Kotoko themselves have created. The reason for choosing scaling rectangles as a symbol of royalty becomes clear when we look at the passage that one must take to visit the Miarre (fig. 2.1h). The passage as a whole is a rectangular spiral. Each time you enter a smaller scale, you are required to behave more politely. By the time you arrive at the throne you are shoeless and speak with a very cultured formality.[2] Thus the fractal scaling of the architecture is not simply the result of unconscious social dynamics; it is a subject of abstract representation, and even a practical technique applied to social ranking.

To the west near the Nigerian border the landscape of Cameroon becomes much greener; this is the fertile high grasslands region of the Bamileke. They too have a fractal settlement architecture based on rectangles (fig. 2.2a), but it has no cultural relation to that of the Kotoko. Rather than the thick clay of Logone-Birni, these houses and the attached enclosures are built from bamboo, which is very strong and widely available. And there was no mention of kinship, defense, or politics when I asked about the architecture; here I was told it is patterns of agricultural production that underlie the scaling. The grassland soil and climate are excellent for farming, and the gardens near the Bamileke houses typically grow a dozen different plants all in a single space, with each taking its characteristic vertical place. But this is labor intensive, and so more dispersed plantings—rows of corn and ground-nut—are used in the wider spaces farther from the house. Since the same bamboo mesh construction is used for houses, house enclosures, and enclosures of enclosures, the result is a self-similar architecture. Unlike the defensive labyrinth of Kotoko architecture, where there were only a few well-protected entryways, the farming activities require a lot of movement between enclosures, so at all scales we see good-sized openings. The fractal simulation in figures 2.2b,c shows how this scaling structure can be modeled using an open square as the seed shape.

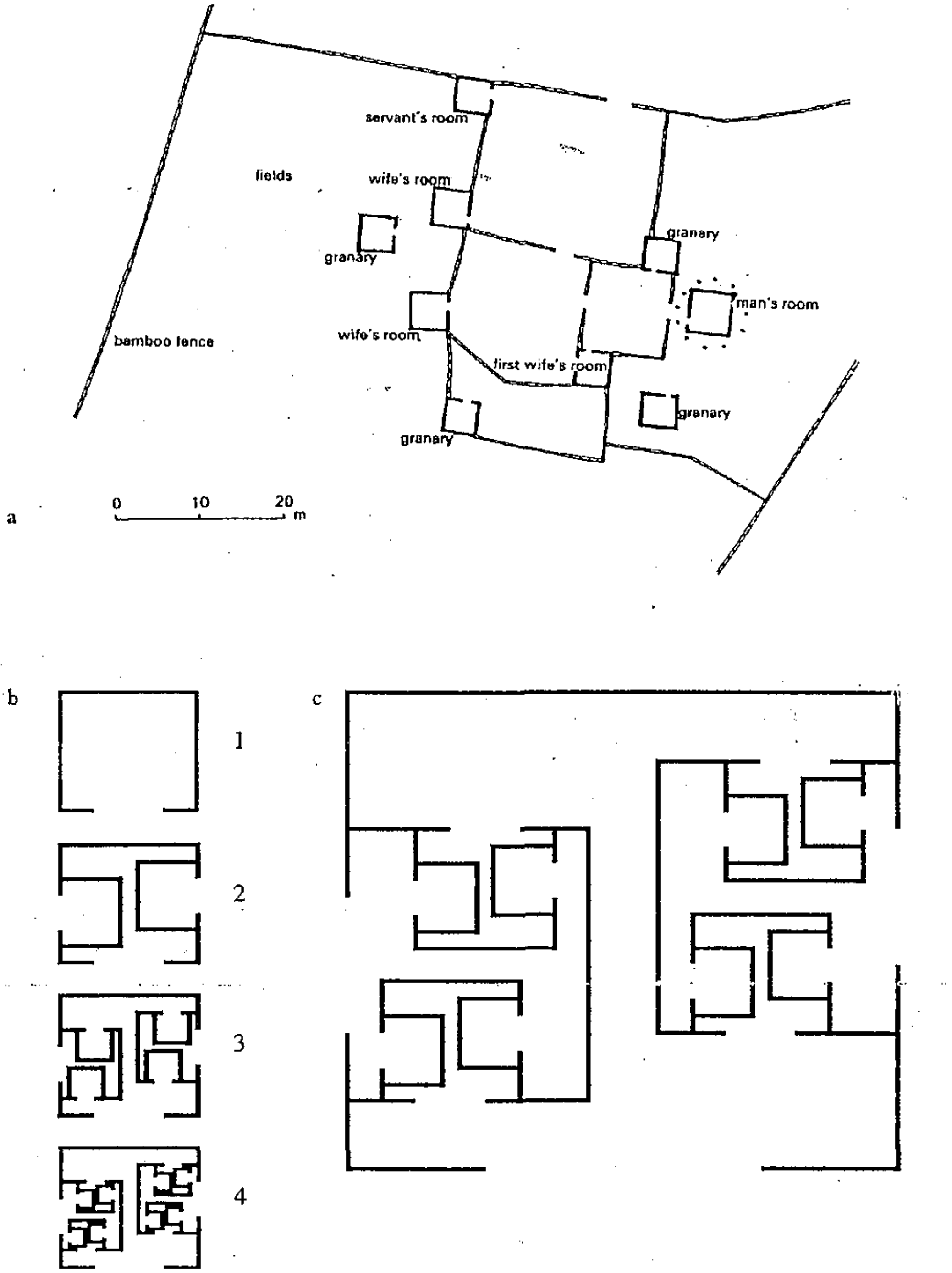


FIGURE 2.2

Bamileke settlement

(a) Plan of Bamileke settlement from about 1960. (b) Fractal simulation of Bamileke architecture. In the first iteration ("seed shape"), the two active lines are shown in gray. (c) Enlarged view of fourth iteration.

(a, *Beguin 1952; reprinted with permission from* ORSTOM).

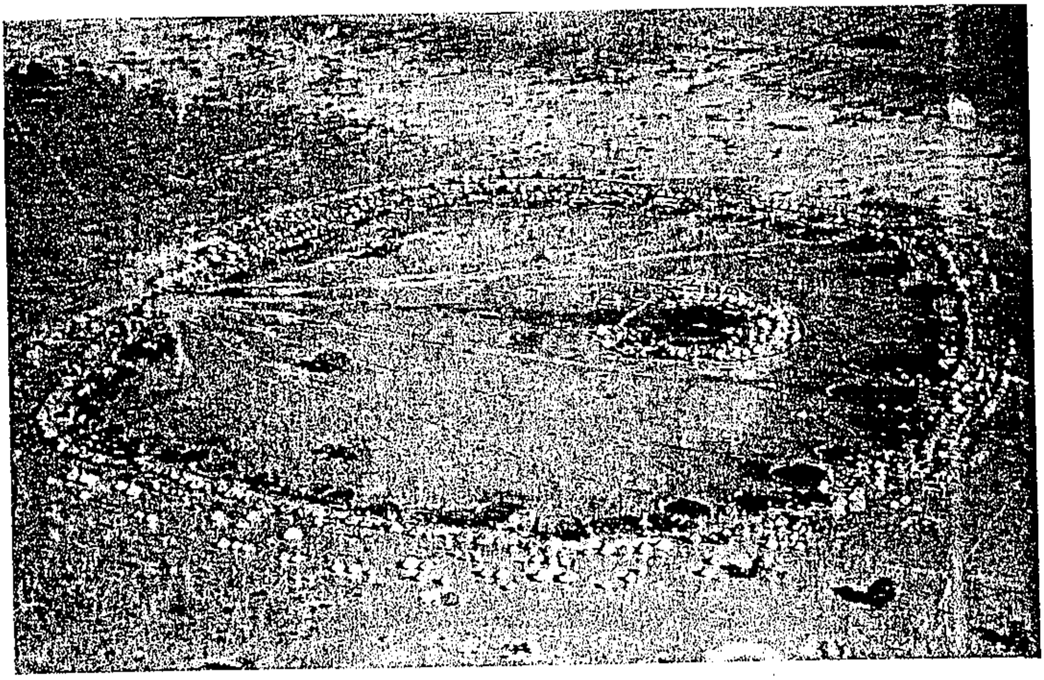
Circular fractals in settlement architecture

Much of southern Africa is made up of arid plains where herds of cattle and other livestock are raised. Ring-shaped livestock pens, one for each extended family,[3] can be seen in the aerial photo in figure 2.3a, a Ba-ila settlement in southern Zambia. A diagram of another Ba-ila settlement (fig. 2.3d) makes these livestock enclosures ("kraals") more clear. Toward the back of each pen we find the family living quarters, and toward the front is the gated entrance for letting livestock in and out. For this reason the front entrance is associated with low status (unclean, animals), and the back end with high status (clean, people).[4] This gradient of status is reflected by the size gradient in the architecture: the front is only fencing, as we go toward the back smaller buildings (for storage) appear, and toward the very back end are the larger houses. The two geometric elements of this structure—a ring shape overall, and a status gradient increasing with size from front to back—echoes throughout every scale of the Ba-ila settlement.

The settlement as a whole has the same shape: it is a ring of rings. The settlement, like the livestock pen, has a front/back social distinction: the entrance is low status, and the back end is high status. At the settlement entrance there are no family enclosures at all for the first 20 yards or so, but the farther back we go, the larger the family enclosures become.

At the back end of the interior of the settlement, we see a smaller detached ring of houses, like a settlement within the settlement. This is the chief's extended family. At the back of the interior of the chief's extended family ring, the chief has his own house. And if we were to view a single house from above, we would see that it is a ring with a special place at the back of the interior: the household altar.

Since we have a similar structure at all scales, this architecture is easy to model with fractals. Figure 2.3b shows the first three iterations. We begin with a seed shape that could be the overhead view of a single house. This is created by active lines that make up the ring-shaped walls, as well as an active line at the position of the altar at the back of the interior. The only passive lines are those adjacent to the entrance. In the next iteration, we have a shape that could be the overhead view of a family enclosure. At the entrance to the family enclosure we have only fencing, but as we go toward the back we have buildings of increasing size. Since the seed shape used only passive lines near the entrance and increasingly larger lines toward the back, this iteration of our simulation has the same size gradient that the real family enclosure shows. Finally, the third iteration provides a structure that could be the overhead view of the whole settlement. At the entrance to the settlement we have only fencing, but as we go toward



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b

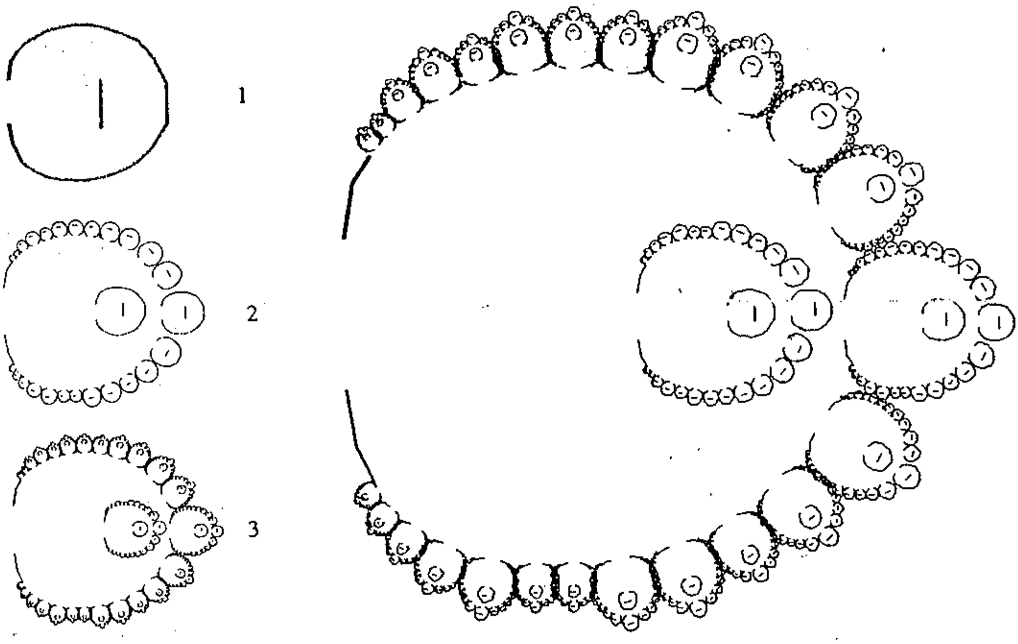


FIGURE 2.3

Ba-ila

(a) Aerial photo of Ba-ila settlement before 1944. (b) Fractal generation of Ba-ila simulation. Note that the seed shape has only active lines (gray) except for those near the opening (black). (a, *American Geographic Institute.*)

the back we have enclosures of increasing size. Again, by having the seed shape use only passive lines near the entrance and increasingly larger lines toward the back, this iteration of our simulation has the same size gradient that the real settlement shows.

I never visited the Ba-ila myself; most of my information comes from the classic ethnography by Edwin Smith and Andrew Dale, published in 1920. While their colonial and missionary motivations do not inspire much trust, they often showed a strong commitment toward understanding the Ba-ila point of view for social structure. Their analysis of Ba-ila settlement architecture points out fractal attributes. They too noted the scaling of house size, from those less than 12 feet wide near the entrance, to houses more than 40 feet wide at the back, and explained it as a social status gradient; "there being a world of difference between the small hovel of a careless nobody and the spacious dwelling of a chief" (Smith and Dale 1968, 114).

It is in Smith's discussion of religious beliefs, however, that the most striking feature of the Ba-ila's fractal architecture is illuminated. Unlike most missionaries of his time, Smith was a strong proponent of respect for local religions. He was no relativist—understanding and respect were strategies for conversion—but his delight in the indigenous spiritual strength comes across clearly in his writings and provided him with insight into the subtle relation of the social, sacred, and physical structure of the Ba-ila architectural plan.

> In this village there are about 250 huts, built mostly on the edge of a circle four hundred yards in diameter. Inside this circle there is a subsidiary one occupied by the chief, his family, and cattle. It is a village in itself, and the form of it in the plan is the form of the greater number of Ba-ila villages, which do not attain to the dimensions of Shaloba's capital. The open space in the center of the village is also broken by a second subsidiary village, in which reside important members of the chief's family, and also by three or four miniature huts surrounded by a fence: these are the *manda a mizhimo* ("the manes' huts") where offerings are made to the ancestral spirits. Thus early do we see traces of the all-pervading religious consciousness of the Ba-ila. (Smith and Dale 1968, 113)

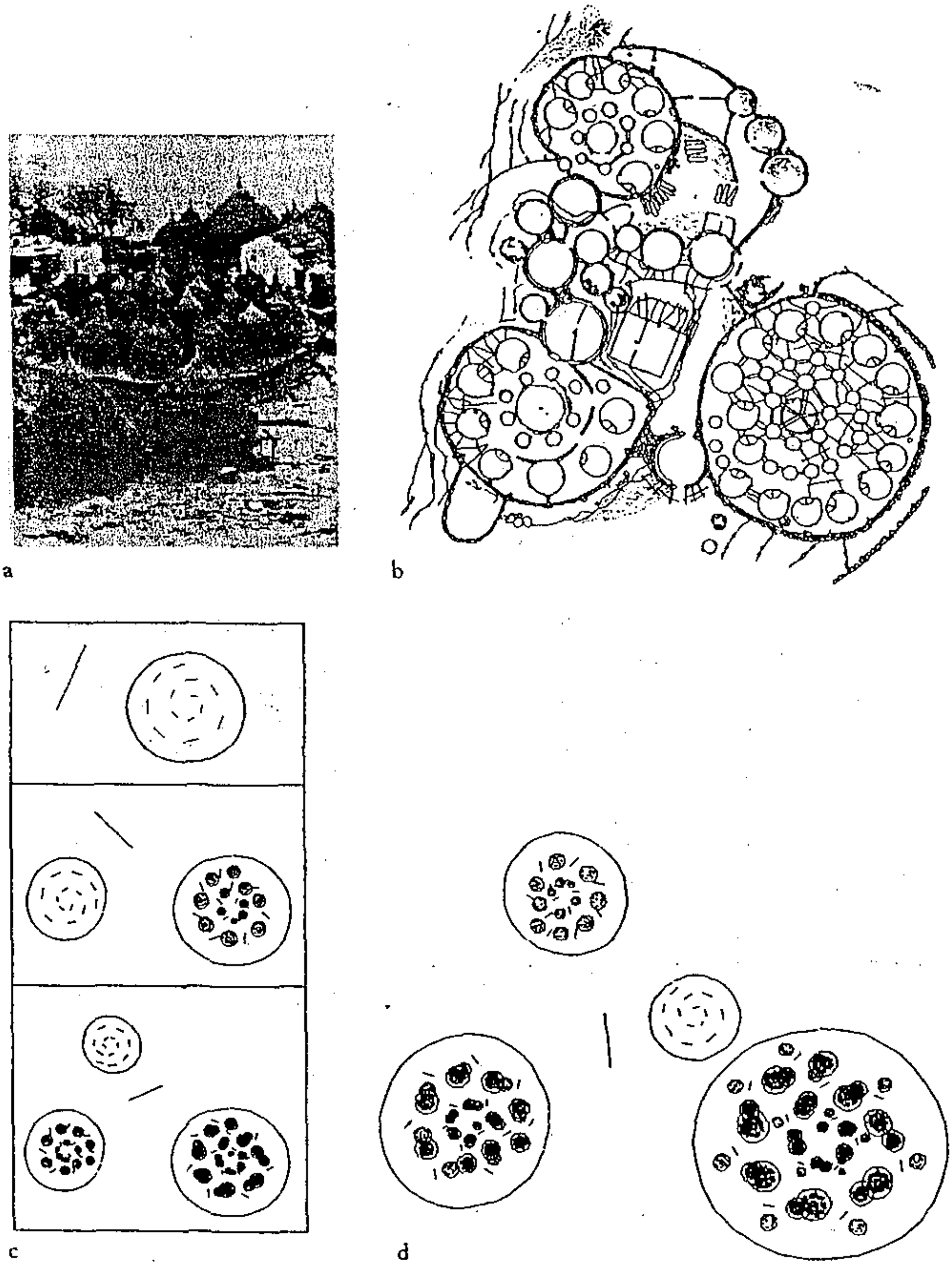
In the first iteration of the computer-generated model there is a detached active line inside the ring, at the end opposite the entrance. This was motivated by the ring comprising the chief's family, but it also describes the location of the sacred altar within each house. As a logician would put it, the chief's family ring is to the whole settlement as the altar is to the house. It is not a status gradient, as we saw with the front-back axis, but rather a recurring functional role between different scales: "The word applied to the chief's relation to his people is *kulela*: in the extracts given above we translate it 'to rule,' but it has this only as a secondary meaning. Kulela is primarily to nurse, to cherish; it is the word applied to a woman caring for her child. The chief is the father of the community; they are his children, and what he does is *lela* them" (Smith and Dale 1968, 307).

This relationship is echoed throughout family and spiritual ties at all scales, and is structurally mapped through the self-similar architecture. The nesting of circular shapes—ancestral miniatures to chief's house ring to chief's extended family ring to the great outer ring—was not a status gradient, as we saw for the enclosure variation from front to back, but successive iterations of *lela*.

A very different circular fractal architecture can be seen in the famous stone buildings in the Mandara Mountains of Cameroon. The various ethnic groups of this area have their own separate names, but collectively are often referred to as Kirdi, the Fulani word for "pagan," because of their strong resistance against conversion to Islam. Their buildings are created from the stone rubble that commonly covers the Mandara mountain terrain. Much of the stone has natural fracture lines that tend to split into thick flat sheets, so these ready-made bricks—along with defensive needs—helped to inspire the construction of their huge castlelike complexes. But rather than being the Euclidean shapes of European castles, this African architecture is fractal.

Figure 2.4a shows the building complex of the chief of Mokoulek, one of the Mofou settlements. An architectural diagram of Mokoulek, drawn by French researchers from the ORSTOM science institute, shows its overall structure (fig. 2.4b). It is primarily composed of three stone enclosures (the large circles), each of which surrounds tightly spiraled granaries (small circles). The seed shape for the simulation requires a circle, made of passive lines, and two different sets of active lines (fig. 2.4c). Inside the circle is a scaling sequence of small active lines; these will become the granaries. Outside the circle there is a large active line; this will replicate the enclosure filled with granaries. By the fourth iteration we have created three enclosures filled with spiral clusters of granaries, plus one unfilled. The real diagram of Mokoulek shows several unfilled circles—evidence that not everything in the architectural structure can be accounted for by fractals. Nevertheless, an important feature is suggested by the simulation.

In the first iteration we see that the large external active line is to the left of the circle. But since it is at an angle, the next iteration finds this active line above and to the right. If we follow the iterations, we can see that the *dynamic construction* of the complex has a spiral pattern; the replications whorl about a central location. This spiral dynamic can be missed with just a static view—I certainly didn't see it before I tried the simulation—but our participation in the virtual construction makes the spiral quite evident.[5] The similarity between the small spirals of granaries inside the enclosures and this large-scale spiral shape of the



a b c d

FIGURE 2.4
Mokoulek

(a) Mokoulek, Cameroon. The small buildings inside the stone wall are granaries. The rectangular building (top right) holds the sacred altar. (b) Architectural diagram of Mokoulek. (c) First three iterations of the Mokoulek simulation. The seed shape is composed of a circle drawn with passive lines (black) and with gray active lines both inside and outside the circle. (d) Fourth iteration of the Mokoulek simulation.
(a *and* b, *by permission from Seignobos 1982.*)

complex as a whole indicates that the fractal appearance of the architecture is not merely due to a random accumulation of various-sized circular forms. The idea of circles of increasing size, spiraling from a central point, has been applied at two different scales, and this structural coherence is confirmed by the architects' own concepts.

In our simulation the active line became located toward the center of the spiral. The Mofou also think of their architecture as spiraling from this central location, which holds their sacred altar. The altar is a kind of conceptual "active line" in their schema; it is responsible for the iterations of life. Seignobos (1982) notes that this area of the complex is the site of both religious and political authority; it is the location for rituals that generate cycles of agricultural fertility and ancestral succession. This geometric mapping between the scaling circles of the architecture and the spiritual cycles of life is represented in their divination ("fortunetelling") ritual, in which the priest creates concentric circles of stones and places himself at the center. As in the *guti* painting in Logone-Birni, in which the Kotoko had modeled their scaling rectangles, the Mofou have also created their own scaling simulation.

By the time I arrived at Mokoulek in 1994 the chief had died, and the ownership of this complex had been passed on to his widows. The new chief told me that the design of this architecture, including that of his new complex, began with a precise knowledge of the agricultural yield. This volume measure was then converted to a number of granaries, and these were arranged in spirals. The design is thus not simply a matter of adding on granaries as they are needed; in fact, it has a much more quantitative basis than my computer model, which I simply did by eyeball.

Not all circular architectures in Africa have the kind of centralized location that we saw in Mokoulek. The Songhai village of Labbezanga in Mali (fig. 2.5a), for example, shows circular swirls of circular houses without any single focus. But comparing this to the fractal image of figure 2.5b, we see that a lack of central focus does not mean a lack of self-similarity. It is important to remember that while "symmetry" in Euclidean geometry means similarity within one scale (e.g., similarity between opposite sides in bilateral symmetry), fractal geometry is based on symmetry between different scales. Even these decentralized swirls of circular buildings show a scaling symmetry.

Paul Stoller, an accomplished ethnographer of the Songhai, tells me that the rectangular buildings that can be seen in figure 2.5a are due to Islamic influence, and that the original structure would have been completely circular. Thanks to Peter Broadwell, a computer programmer from Silicon Graphics Inc., we were able to run a quantitative test of the photo that confirmed what our eyes

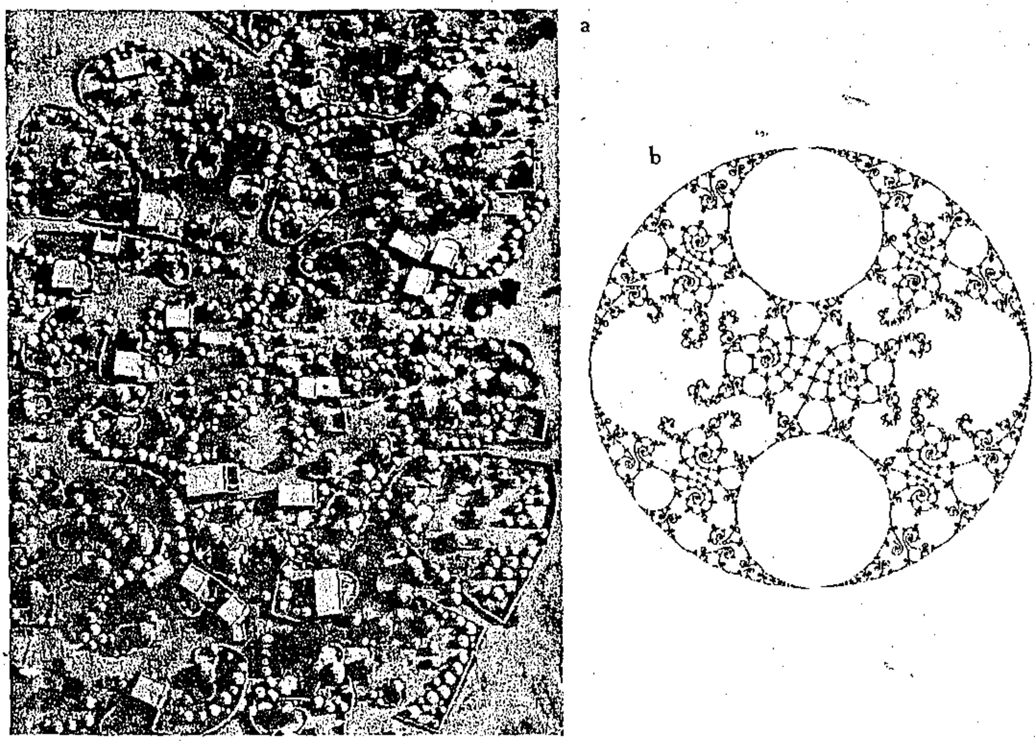


FIGURE 2.5
Labbezanga
(a) Aerial view of the village of Labbezanga in Mali. (b) Fractal graphic.
(a, *photo by Georg Gerster;* b, *by permission of Benoit Mandelbrot.*)

were telling us: the Songhai architecture can be characterized by a fractal dimension similar to that of the computer-generated fractal of figure 2.5b.[6]

This kind of dense circular arrangement of circles, while occurring in all sorts of variations, is common throughout inland west Africa. Bourdier and Trinh (1985), for example, describe a similar circular architecture in Burkina Faso. The scaling of individual buildings is beautifully diagrammed in their cover illustration (fig. 2.6a), a portion of one of the large building complexes created by the Nankani society. As for the Songhai, foreign cultural influences have now introduced rectangular buildings as well. In the Nankani complex the outermost enclosure (the perimeter of the complex) is socially coded as male. As we move in, the successive enclosures become more female associated, down to the circular woman's *dégo* (fig. 2.6b), the circular fireplace, and finally the scaling stacks of pots (fig. 2.6c).

Using a technique quite close to that of the Kotoko women, the women of Nankani also waterproof and decorate these walls. The recurrent image of a triangle in these decorations (see walls of dégo) represents the *zalanga*, a nested stack of calabashes (circular bowls carved from gourds) that each woman keeps in her kitchen (fig. 2.6d). Since these calabashes are stacked from large to small, they (and the rope that holds them) form a triangle—thus the triangular decorations also represent scaling circles, just in a more abstract way. The smallest container in a woman's zalanga is the *kumpio*, which is a shrine for her soul. When she dies, the zalanga, along with her pots, is smashed, and her soul is released to eternity. The eternity concept, associated with well-being, is symbolically

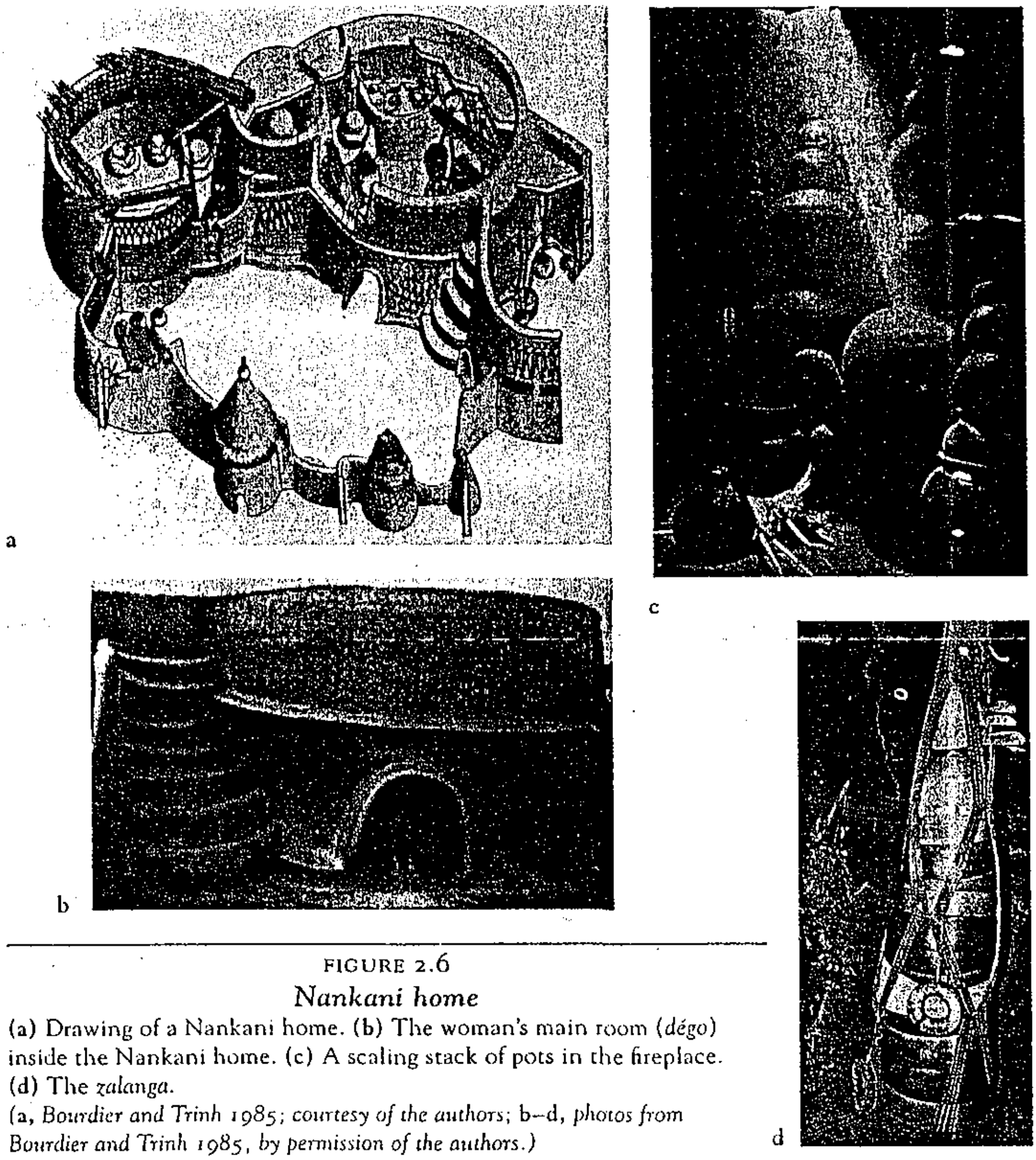


FIGURE 2.6
Nankani home
(a) Drawing of a Nankani home. (b) The woman's main room (*dégo*) inside the Nankani home. (c) A scaling stack of pots in the fireplace. (d) The *zalanga*.
(a, *Bourdier and Trinh* 1985; *courtesy of the authors*; b–d, *photos from* Bourdier and Trinh 1985, *by permission of the authors.*)

represented by the coils of a serpent of infinite length, sculpted into the walls of these homes.

From the 20-meter diameter of the building complex to the 0.2-meter kumpio—and not simply at one or two levels in between, but with dozens of self-similar scalings—the Nankani fractal spans three orders of magnitude, which is comparable to the resolution of most computer screens. Moreover, these scaling circles are far from unconscious accident: as in several other architectures we have examined, they have made conscious use of the scaling in their social symbolism. In this case, the most prominent symbolism is that of birthing. When a child is born, for example, it must remain in the innermost enclosure of the women's dégo until it can crawl out by itself. Each successive entrance is—spatially as well as socially—a rite of passage, starting with the biological entrance of the child from the womb. It leaves each of these nested chambers as the next iteration in life's stages is born. The zalanga models the entire structure in miniature, and its destruction in the event of death maps the journey in reverse: from the circles of the largest calabash to the tiny kumpio holding the soul—from mature adult to the eternal realm of ancestors who dwell in "the earth's womb." There is a conscious scheme to the scaling circles of the Nankani: it is a recursion which bottoms-out at infinity.

Branching fractals

While African circular buildings are typically arranged in circular clusters, the paths that lead through these settlements are typically not circular. Like the bronchial passages that oxygenate the round alveoli of the lungs, the routes that nourish circular settlements often take a branching form (e.g., figure 2.7). But despite my unavoidably organicist metaphor, these cannot be simply reduced to unconscious traces of minimum effort. For one thing, conscious design criteria are evident in communities in which there is an architectural transition from circular to rectangular buildings, since they can choose to either maintain or erase the branching forms.

Discussion concerning such decisions are apparent in the settlement of Banyo, Cameroon, where the transition has a long history (Hurault 1975). I found that few circular buildings were left, but those that were still intact served as an embodiment of cultural memory. This role was honored in the case of the chief's complex and exploited in the case of a blacksmith's shop, which was the site of occasional tourist visits. After passing approval by the government officials and the sultan, I was greeted by the official city surveyor, who—considering the fact that his raison d'être was Euclideanizing the streets—showed surprising

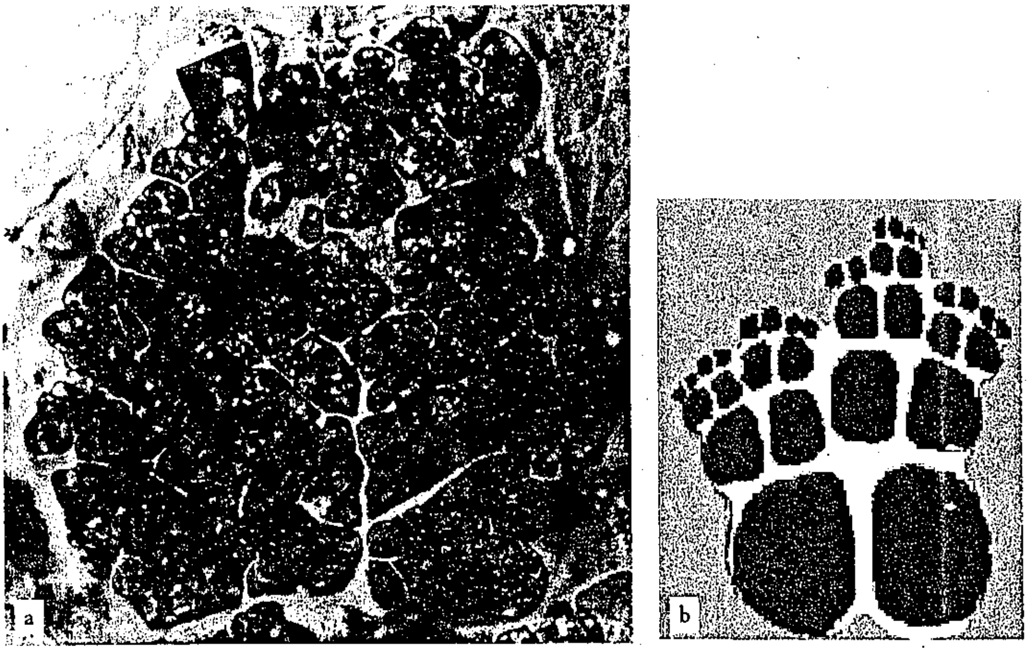


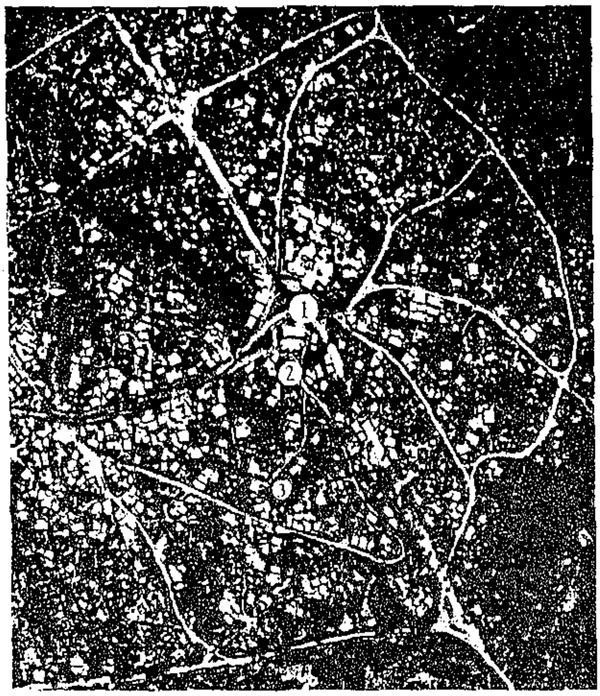
FIGURE 2.7

Branching paths in a Senegalese settlement

(a) Aerial photo of a traditional settlement in northeast Senegal. The space between enclosure walls, serving as roads and footpaths, creates a branching pattern. (b) A branching fractal can be created by the background of a scaling set of circular shapes.
(a, courtesy Institut Geographique du Senegal.)

appreciation for my project and helped me locate the most fractal area of the city (fig. 2.8a). At the upper left of the photo we see a portion of the Euclidean grid that covers the rest of the city, but most of this area is still fractal. The location of this carefully maintained branching—fanning out from a large plaza that is bordered by the palace of the sultan and the grand mosque—is no coincidence. By marking my position on the aerial photo as I traveled through (fig. 2.8b), I was later able to create a map by digitally altering the photo image (fig. 2.8c). This provides a stark outline—looking much like the veins in a leaf—of the fractal structure of this transportation network. I may have plunged through a wall or two in creating this map, but it certainly underestimates the fine branching of the footpaths, as I did not attempt to include their extensions into private housing enclosures.

How does the creation of these scaling branches interact with the kinds of iterative construction and social meaning we have seen associated with other examples of fractal architecture? A good illustration can be found in the



a

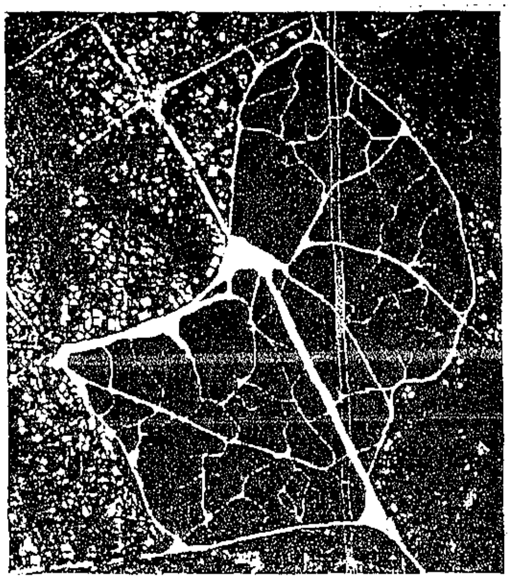
b

Position 1—outside palace   Position 2—road below mosque   Position 3—narrow walkway

FIGURE 2.8

Branching paths in Banyo

(a) Aerial photo of the city of Banyo, Cameroon. (b) Successive views of the branching paths, as marked on the photo above. The clay walls require their own roof, which comes in both thatched and metal versions along the walkway in the last photo. (c) Aerial photo of Banyo with only public paths showing. (a, *courtesy National Institute of Cartography, Cameroon.*)



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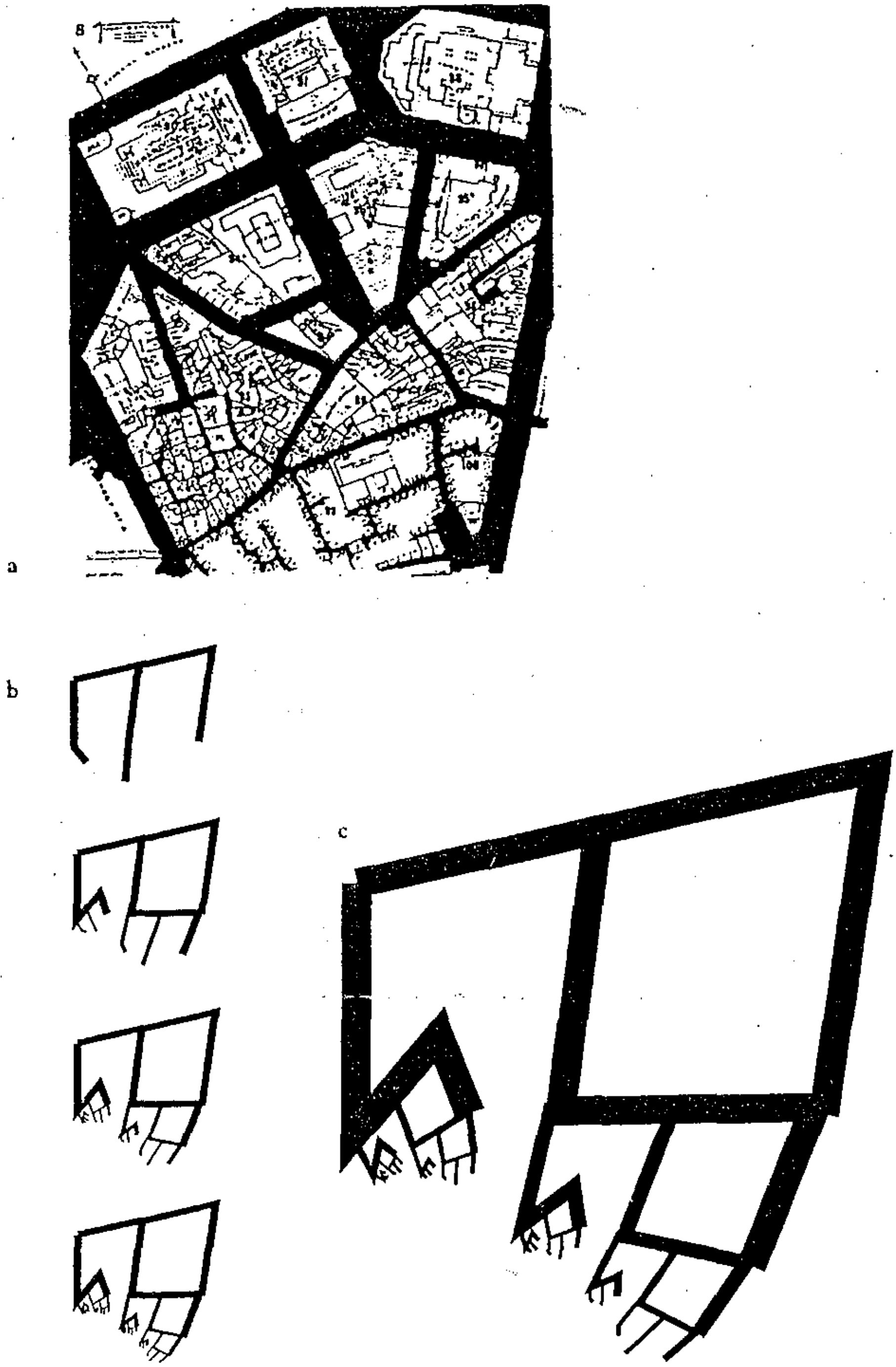


FIGURE 2.9

Streets of Cairo

(a) Map of streets of Cairo, 1898. (b) Fractal simulation for Cairo streets. (c) Enlarged view of fourth iteration.

branching streets of North African cities. Figure 2.9a shows a map of Cairo, Egypt, in 1898. The map was created by an insurance company, and I have colored the streets black to make the scaling branches more apparent. Figure 2.9b shows its computer simulation. Delaval (1974) has described the morphogenesis of Saharan cities in terms of successive additions similar to the line replacement in the fractal algorithms we have used here. The first "seed shape" consists of a mosque connected by a wide avenue to the marketplace, and successive iterations of construction add successive contractions of this form.

Since these fractal Saharan settlement architectures predate Islam (see Devisse 1983), it would be misleading to see them as an entirely Muslim invention; but given the previous observations about the introduction of Islamic architecture as an interruption of circular fractals in sub-Saharan Africa, it is important to note that Islamic cultural influences have made strong contributions to African fractals as well. Heaver (1987) describes the "arabesque" artistic form in North African architecture and design in terms that recall several fractal concepts (e.g., "cyclical rhythms" producing an "indefinitely expandable" structure). He discussed these patterns as visual analogues to certain Islamic social concepts, and we will examine his ideas in greater detail in chapter 12 of this book.

Conclusion

Throughout this chapter, we have seen that a wide variety of African settlement architectures can be characterized as fractals. Their physical construction makes use of scaling and iteration, and their self-similarity is clearly evident from comparison to fractal-graphic simulations. Chapter 3 will show that fractal architecture is not simply a typical characteristic of non-Western settlements. This alone does not allow us to conclude an indigenous African knowledge of fractal geometry; in fact, I will argue in chapter 4 that certain fractal patterns in African decorative arts are merely the result of an intuitive esthetic. But as we have already seen, the fractals in African architecture are much more than that. Their design is linked to conscious knowledge systems that suggest some of the basic concepts of fractal geometry, and in later chapters we will find more explicit expressions of this indigenous mathematics in astonishing variety and form.

CHAPTER 3

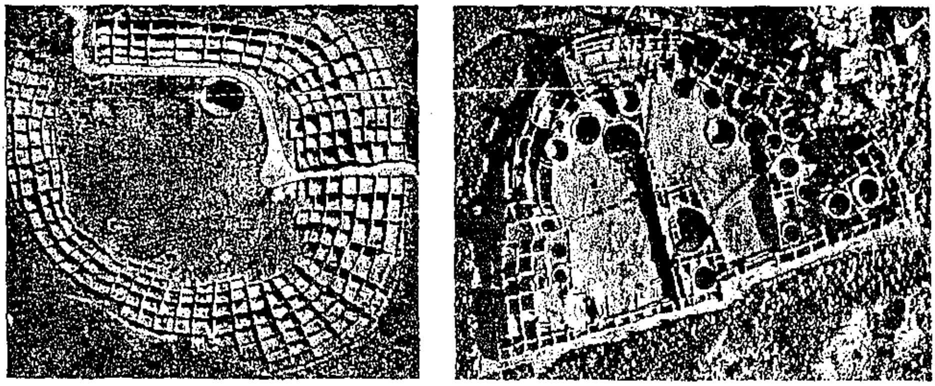
Fractals in cross-cultural comparison

The fractal settlement patterns of Africa stand in sharp contrast to the Cartesian grids of Euro-American settlements. Why the difference? One explanation could be the difference in social structure. Euro-American cultures are organized by what anthropologists would call a "state society." This includes not just the modern nation-state, but refers more generally to any society with a large political hierarchy, labor specialization, and cohesive, formal controls—what is sometimes called "top-down" organization. Precolonial African cultures included many state societies, as well as an enormous number of smaller, decentralized social groups, with little political hierarchy—that is, societies that are organized "bottom-up" rather than "top-down."[1] But if fractal architecture is simply the automatic result of a nonstate social organization, then we should see fractal settlement patterns in the indigenous societies of many parts of the world. In this chapter we will examine the settlement patterns found in the indigenous societies of the Americas and the South Pacific, but our search will turn up very few fractals. Rather than dividing the world between a Euclidean West and fractal non-West, we will find that each society makes use of its particular design themes in organizing its built environment. African architecture tends to be fractal because that is a prominent design theme in African culture. In fact, this cultural specificity of design themes is true not only for architecture, but for many other types of material design and cultural practices as well. We will begin our survey with a brief look at the design themes in Native American societies, which included both hierarchical state empires as well as smaller, decentralized tribal cultures.

Native American design

The Ancestral Pueblo society dwelled in what is now the southwestern United States around 1100 C.E. Aerial photos of these sites (fig. 3.1) are some of the most famous examples of Native American settlements. But as we can see from this vantage point, the architecture is primarily characterized by an enormous *circular* form created from smaller *rectangular* components—certainly not the same shape at two different scales. This juxtaposition of the circle and the quadrilateral (rectangle or cross-shaped) form is not a coincidence; the two forms are the most important design themes in the material culture of many Native American societies, including both North and South continents.

As far as architecture is concerned, there are no examples of the nonlinear scaling we saw in Africa. The only Native American architectures that come close are a few instances of linear concentric figures (fig. 3.2a). Shapes approximating concentric circles can be seen in the Poverty Point complex in north-



a b

FIGURE 3.1
Euclidean geometry in Native American architecture
(a) Aerial photo of Bandelier, one of the Ancestral Pueblo settlements (starting around 1100 C.E.) in norhtwestern New Mexico. (b) Aerial photo of Pueblo Bonito, another Ancestral Pueblo settlement (starting around 950 C.E.). Note that they are mostly rectangular at the smallest scale and circular at the largest scale.
(a, *photo by Tom Baker*; b, *photo by Georg Gerster*.)

ern Louisiana, for example, and there were concentric circles of tepees in the Cheyenne camps. The step-pyramids of Mesoamerica look like concentric squares when viewed from above. But linear concentric figures are not fractals. First, these are linear layers: the distance between lines is always the same, and thus the number of concentric circles within the largest circle is finite. The non-linear scaling of fractals requires an ever-changing distance between lines,

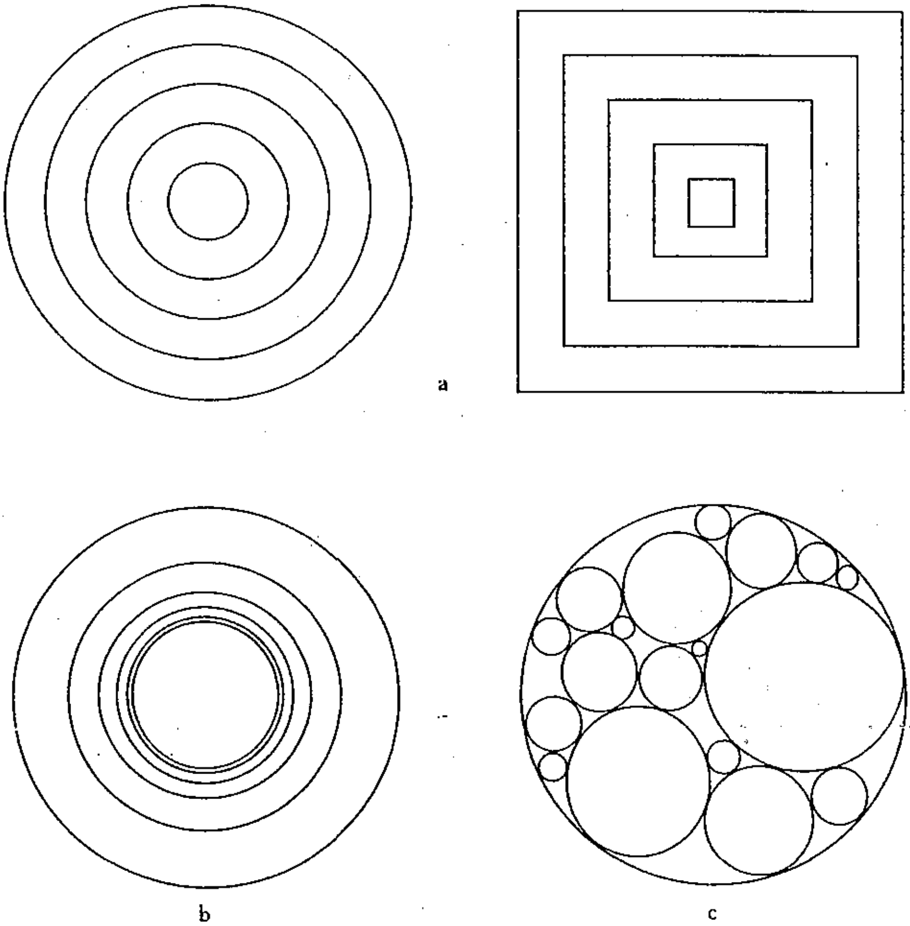


FIGURE 3.2

Linear concentric forms in Native American architecture

(a) Native American architecture is typically based on quadrilateral grids or a combination of circular and grid forms. The only examples of scaling shapes are these linear concentric forms. In the Poverty Point complex, for example, concentric circles were used, and concentric squares can be seen if we look at the Mexican step pyramids from above. These forms are better characterized as Euclidean than fractal for two reasons: (b) First, they are linear. Here is an example of a nonlinear concentric circle. While the linear version must have a finite number of circles, this figure could have an infinite number and still fit in the same boundary. (c) Second, they only scale with respect to one point (the center). Here is an example of circles with more global scaling symmetry.

which means there can be an infinite number in a finite space (fig. 3.2b). Second, even nonlinear concentric circles are only self-similar with respect to a single locus (the center point), rather than having the global self-similarity of fractals (fig. 3.2c).

The importance of the circle is detailed in a famous passage by Black Elk (1961), in which he explains that "everything an Indian does is in a circle, and that is because the Power of the World always works in circles, and everything tries to be round." But he goes on to note that his people thought of their world as "the circle of the four quarters." A similar combination of the circle and quadrilateral form can be seen many Native American myths and artifacts; it is not their only design theme, but it can be found in a surprising number of different societies. Burland (1965), for example, shows a ceremonial rattle consisting of a wooden hoop with a cross inside from southern Alaska, a Navajo sand painting showing four equidistant stalks of corn growing from a circular lake, and a Pawnee buffalo-hide drum with four arrows emanating from its circular center. Nabokov and Easton (1989) describe the cultural symbolism of the tepee in terms of its combination of circular hide exterior and the four main struts of the interior wood supports. Waters (1963) provides an extensive illustration of the cultural significance of combining the circular and cross form in his commentary on the Hopi creation myth.

The fourfold symmetry of the quadrilateral form has lead to some sophisticated conceptual structures in Native American knowledge systems. In Navajo sand painting, for example, the cruciform shape represents the "four directions" concept, similar to the Cartesian coordinate system. While orderly and consistent, it is by no means simple (see Witherspoon and Peterson 1995). The four Navajo directions are also associated with corresponding sun positions (dawn, day, evening, night), yearly seasons (spring, summer, fall, winter), principal colors (white, blue, yellow, black), and other quadrilateral divisions (botanical categories, partitions of the life cycle, etc.). These are further broken into intersecting bipolarities (e.g., the east/west sun path is broken by the north/south directions). Combined with circular curves (usually representing organic shapes and processes), the resulting schema are rich cultural resources for indigenous mathematics (see Moore 1994). But, except for minor repetitions (like the small circular kivas in the Chaco canyon site of fig. 3.1) there is nothing particularly fractal about these quadrilateral designs.

Many Mesoamerican cities, such as the Mayans' Teotihuacán, the Aztec's Tenochtitlán, and the Toltec's Tula, embedded a wealth of astronomical knowledge in their rectangular layouts, aligning their streets and buildings with heavenly objects and events (Aveni 1980). J. Thompson (1970) and Klein (1982) describe the quadrilateral figure as an underlying theme in Mesoamerican geometric thinking, from small-scale material construction techniques such as weaving, to the heavenly cosmology of the four serpents. Rogelio Díaz, of the Mathematics Museum at the University of Querétaro, points out that the skin patterns of the diamondback rattlesnake were used by the Mayans to symbolize this concept (fig. 3.3a).

Comparing the Mayan snake pattern with an African weaving based on the cobra skin pattern (fig. 3.3b), we can see how geometric modeling of similar natural phenomena in these two cultures results in very different representations. The Native American example emphasizes the Euclidean symmetry *within one* size frame ("size frame" because the term "scale" is confusing in the context of snake skin). This Mayan pattern is composed of four shapes of the same size, a fourfold symmetry. But the African example emphasizes fractal symmetry, which is not about similarity between right/left or up/down, but rather similarity *between different* size frames. The African snake pattern shows diamonds within diamonds within diamonds. Neither design is necessarily more accurate: cobra skin does indeed exhibit a fractal pattern—the snake's "hood," its twin white patches, and the individual scales themselves are all diamond shaped—and yet snake skin patterns (thanks to the arrangement of the scales) are also characteristically in diagonal rows, so they are accurately modeled as Euclidean structures as well. Each culture chooses to emphasize the characteristics that best fit its design theme.

There are a few cases in which Native Americans have used scaling geometries in artistic designs. Several of these were not, however, part of the traditional repertoire.[2] Navajo blankets, for example, were originally quite linear; it was only on examining Persian rugs that Navajo weavers began to use more scaling styles of design (and even then the designs were much more Euclidean than the Persian originals; see Kent 1985). The Pueblo "storyteller" figures have some scaling properties, but they are of recent (1960s) origin. Pottery and calabash (carved gourd) artisans in Africa often create scaling by allowing the design adaptively to change proportion according to the three-dimensional form on which it is inscribed (see "adaptive scaling" in chapter 6), but this was quite rare in Native American pottery until the 1960s.

Finally, there are three Native American designs that are both indigenous and fractal. The best case is the abstract figurative art of the Haida, Kwakiutl, Tlingit, and others in the Pacific Northwest (Holm 1965). The figures, primarily carvings, have the kind of global, nonlinear self-similarity necessary to qualify as fractals and clearly exhibit recursive scaling of up to three or four iterations. They also make use of adaptive scaling, as illustrated by the shrinking series of

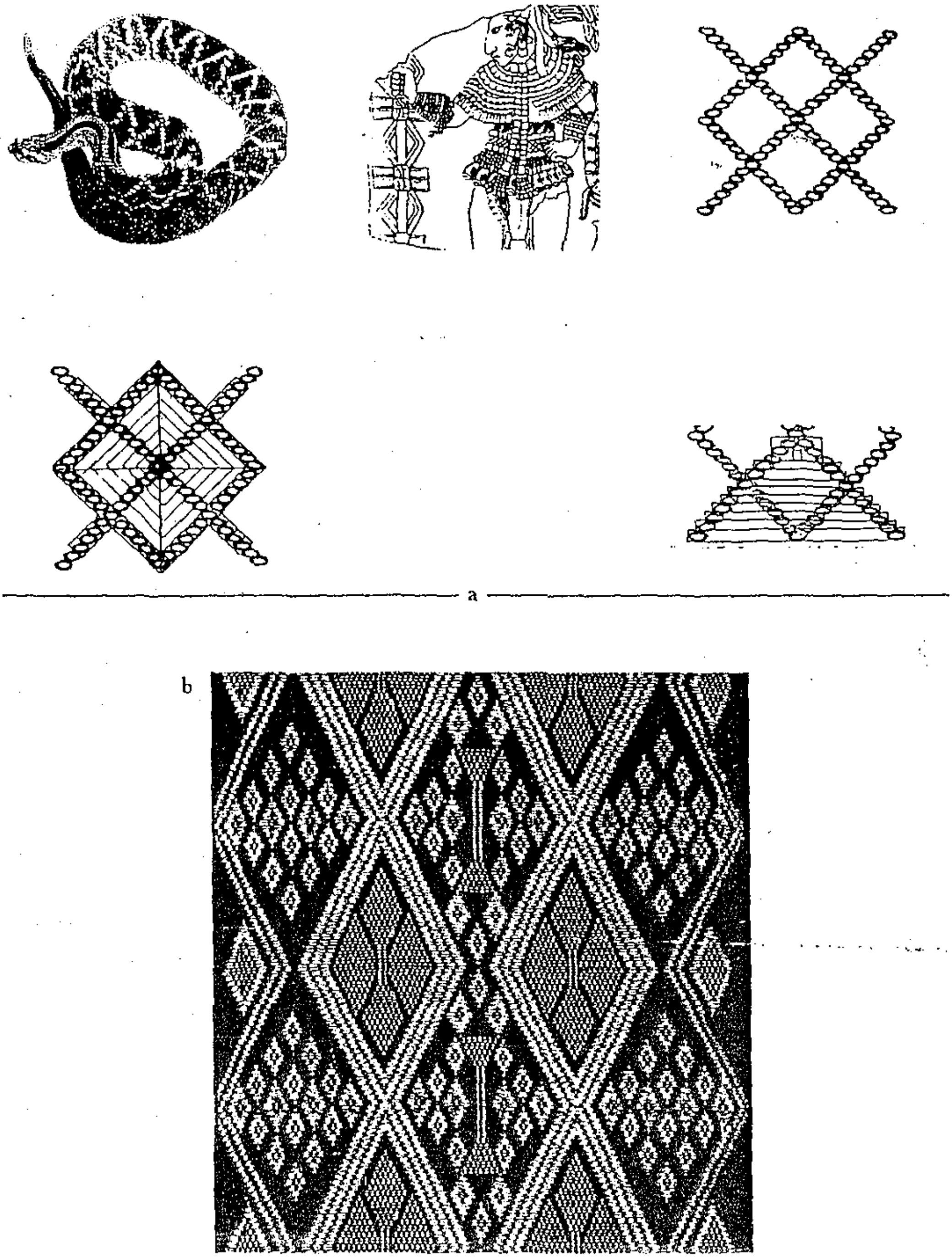


FIGURE 3.3

Snakeskin models in Native American and African cultures

(a) Rogelio Díaz of the Mathematics Museum at the University of Querétaro shows how the skin patterns of the diamondback rattlesnake were used by the Mayans to symbolize a cosmology based on quadrilateral structure. (b) The Mandiack weavers of *Guinea-Bissau* have also created an abstract design based on a snakeskin pattern, but chose to emphasize the fractal characteristics.

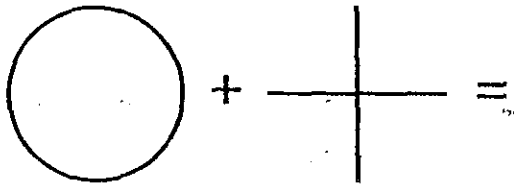
figures on the diminishing handles of soup ladles. Researchers since Adams (1936) have pointed to the similarity with early Chinese art, which also has some beautiful examples of scaling form, and its style of curvature and bilateral symmetry could indeed be culturally tied to these New World designs through an ancient common origin. However, I doubt that is the case for the scaling characteristics. The Pacific Northwest art appears to have developed its scaling structure as the result of competition between artisans for increasingly elaborate carvings (Faris 1983). Although some researchers have attributed the competition pressure to European trading influences, the development of the scaling designs was clearly an internal invention.

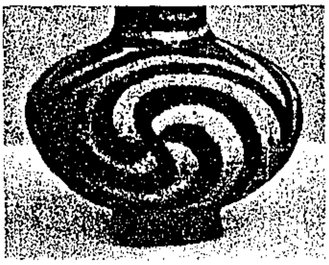
The other two traditional Native American designs do not qualify as fractals quite as well. One involves the saw-tooth pattern found in several basket and weaving designs. When two saw-tooth rows intersect at an angle, they create a triangle made from triangular edges. But these typically have only two iterations of scale, and there is no indication in the ethnographic literature that they are semantically tied to ideas of recursion or scaling (see Thomas and Slockish 1982, 18). The other is an arrangement of spiral arms often found on coiled baskets. Again, this is self-similar only with respect to the center point, but there are some nonlinear scaling versions (that is, designs that rapidly get smaller as you move from basket edge to center). However, these designs generally appear to be a fusion between the circular form of the basket and the cruciform shape of the arms: again more a combination of two Euclidean shapes than a fractal.

One of the most common examples of this fusion between the circle and the cross is the "bifold rotation" pattern in which the arms curve in opposite directions, as shown in figure 3.4a. Figure 3.4b shows the figure of a bat from Mimbres pottery with a more complex version of the bifold rotation. Euclidean symmetry has been emphasized in this figure; for example, the ears and mouth of the bat have been made to look similar to increase the bilateral symmetry, and the belly is drawn as a rectangle. Figure 3.4c shows the figure of a bat from an African design; it is a zigzag shape that expands in width from top to bottom, representing the wing of the bat. Here we see neglect of the bilateral symmetry of the bat, and an emphasis on the scaling folds of a single wing. Again, the Native American representation makes use of its quadrilateral/circular design theme, just as the African representation of the bat emphasizes scaling design.

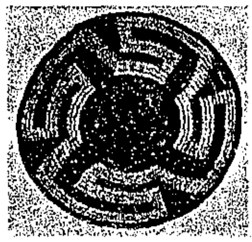
There is plenty of complexity and sophistication in the indigenous geometry and numeric systems of the Americas (see Ascher 1991, 87–94; Closs 1986; Eglash 1998b), but with the impressive exception of the Pacific Northwest carvings, fractals are almost entirely absent in Native American designs.

a

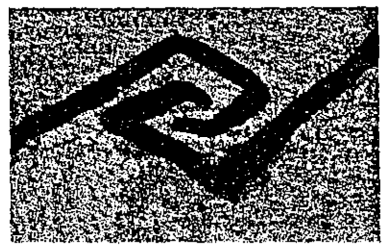




Arkansas pottery



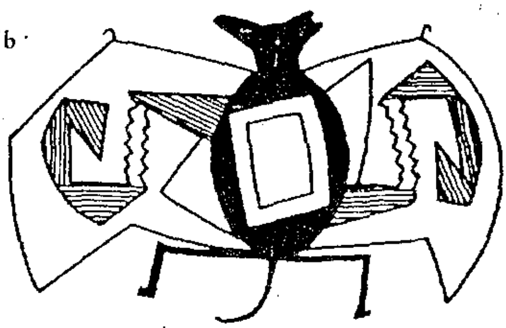
Pima basket



Southwestern pottery motif

(a) The circular and quadrilateral forms were often combined in Native American designs as a fourfold or bifold rotation.

b



c

(b) This image of a bat, from a Mimbres pottery in Southwestern Native American tradition, shows an emphasis on circular and quadrilateral form. The ear and the mouth, for example, are made to look similar to emphasize bilateral symmetry, and the belly is drawn as a rectangle. It also shows the wing bones as a bifold rotation pattern.

(c) This African sculpture of a bat, from the Lega society of Zaire, pays little attention to the bilateral symmetry of the bat's body but gives an emphasis on the scaling symmetry of the wing folds, shown as an expanding zigzag pattern.

FIGURE 3.4

The bifold rotation in Native American design

(a: *Left, from* Miles 1963. Center, *from* Southwest Indian Craft Arts *by Clara Lee Tanner. Copyright 1968 by the Arizona Board of Regents. Reprinted by permission of the University of Arizona Press. Right, courtesy Don Crowe.* b, *from Zaslow 1977, courtesy of the author.* c, *courtesy of Daniel Biebuyck.)*

Designs of Asia and the South Pacific

Several of the South Pacific cultures share a tradition of decorative curved and spiral forms, which in certain Maori versions—particularly their rafter and tattoo patterns—would certainly count as fractal (see Hamilton 1977). These are strongly suggestive of the curvature of waves and swirling water. Classic Japanese paintings of water waves were also presented as fractal patterns in Mandelbrot's (1982) seminal text (plate C16). These may have some historic relation to scaling patterns in Chinese art (see Washburn and Crowe 1988, fig. 6.9), which are based on swirling forms of water and clouds, abstracted as spiral scaling structures. While both the Japanese and Chinese patterns are explicitly associated with an effort to imitate nature, these Maori designs are reported to be more about culture—in particular, they emphasize mirror-image symmetries, which are associated with their cultural themes of complimentarity in social relations (Hanson 1983).

In almost all other indigenous examples, however, the Pacific Islander patterns are quite Euclidean. Settlement layout, for instance, is typically in one or two rows of rectangular buildings near the coasts, with circular arrangements of rectangles also occurring inland (see Fraser 1968). The building construction is generally based on a combination of rectangular grids with triangular or curved arch roofs. Occasionally these triangular faces are decorated with triangles, but otherwise nonscaling designs dominate both structural and decorative patterns.[3]

Again, it is important to note that this lack of fractals does not imply a lack of sophistication in their mathematical thinking. For example, Ascher (1991) has analyzed some of the algorithmic properties of Warlpiri (Pacific Islander) sand drawings. Similar structures are also found in Africa, where they are called *lusona*. But while the lusona tend to use similar patterns at different scales (as we will see in chapter 5), the Warlpiri drawings tend to use different patterns at different scales. Ascher concludes that the Warlpiri method of combining different graph movements is analogous to algebraic combinations, but the African lusona are best described as fractals.

Complicating my characterization of the South Pacific as dominated by Euclidean patterns is the extensive influence of India. It is perhaps no coincidence that the triangle of triangles mentioned above is most common in Indonesia. In architecture, a famous exception to the generally Euclidean form is that of Borobudur, a temple of Indian religious origin in Java. Although northern India tends toward Euclidean architecture, explicit recursive design is seen in several temples in southern India—the Kandarya Mahadeo in Khajuraho is one of the clearest examples—and is related to recursive concepts in religious cosmology. These same areas in southern India also have a version of the lusona drawings, and many other examples of fractal design. Interestingly, these examples from southern India are the products of Dravidian culture, which is suspected to have significant historical roots in Africa.

European designs

Most traditional European fractal designs, like those of Japan and China, are due to imitation of nature—a topic we will take up in the following chapter. There are at least two stellar exceptions, however, that are worth noting. One is the scaling iterations of triangles in the floor tiles of the Church of Santa Maria in Cosmedin Rome (see plate 5.7 in Washburn and Crowe 1988). I have not been able to determine anything about their cultural origins, but they are clearly artistic invention rather than imitation of some natural form. The other can be found in certain varieties of Celtic interlace designs. Nordenfalk (1977) suggests that these are historically related to the spiral designs of pre-Christian Celtic religion, where they trace the flow of a vital life force. They are geometrically classified as an Eulerian path, which is closely associated with mathematical knot theory (cf. Jones 1990, 99).

Conclusion

Fractal structure is by no means universal in the material patterns of indigenous societies. In Native American designs, only the Pacific Northwest patterns show a strong fractal characteristic; Euclidean shapes otherwise dominate the art and architecture. Except for the Maori spiral designs, fractal geometry does not appear to be an important aspect of indigenous South Pacific patterns either. That is not to say that fractal designs appear nowhere but Africa—southern India is full of fractals, and Chinese fluid swirl designs and Celtic knot patterns are almost certainly of independent origin.[4] The important point here is that the fractal designs of Africa should not be mistaken for a universal or pancultural phenomenon; they are culturally specific. The next chapter will examine the question of their mathematical specificity.

CHAPTER

4

Intention and invention in design

Before we can discuss the fractal shapes in African settlement architectures as geometric knowledge, we need to think carefully about the relation between material designs and mathematical understanding. Designs are best seen as positioned on a range or spectrum of intention. At the bottom of the range are unintentional patterns, created accidentally as the by-product of some other activity. In the middle of the range are designs that are intentional but purely intuitive, with no rules or guidelines to explain its creation. At the upper end of the range, we have the intentional application of explicit rules that we are accustomed to associating with mathematics. The following sections will examine the fractal designs that occur in various positions along this intentionality spectrum.

Fractals from unconscious activity

An excellent example of unintentional fractals can be found in the work of Michael Batty and Paul Longley (1989), who examined the shape of large-scale urban sprawl surrounding European and American cities (fig. 4.1). While the blocks of these cities are typically laid out in rectangular grids, at very large scales—around 100 square miles—we can see that the process of population growth has created an irregular pattern. This type of fractal, a "diffusion limited aggregation," also

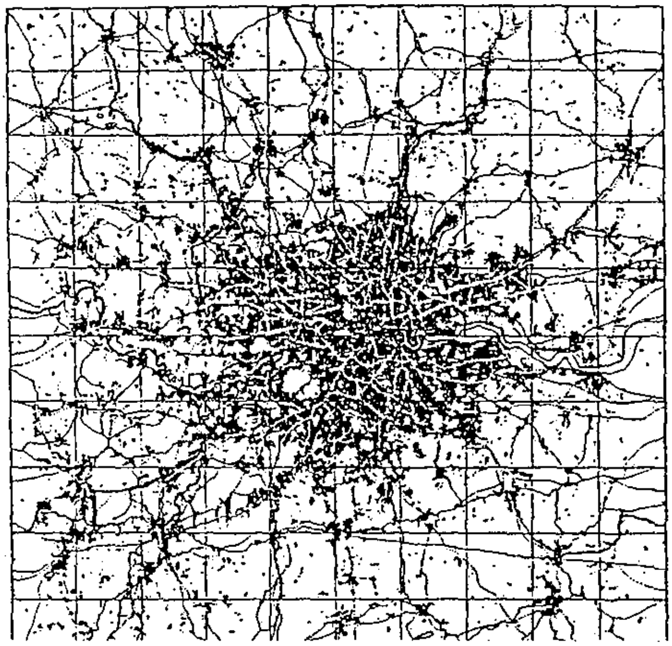


FIGURE 4.1
Urban sprawl in London
Large-scale urban sprawl generally has a fractal structure. The urban sprawl fractals only exist at very large scales—about 100 sq. miles—and result from the unconscious accumulation of urban population dynamics. At levels of conscious intent (e.g., the grid of city blocks), European cities are typically Euclidean. Area is 10 × 10 kilometers.
(Reprinted with permission from Batty et al. 1989.)

occurs in chemical systems when particles in a solution are attracted to an electrode. Fractal urban sprawl is clearly the result of unconscious social dynamics, not conscious design. At the smaller scales in which there is conscious planning, European and American settlement architectures are typically Euclidean.

Fractals from nature: mimesis versus modeling

It might be tempting to think that the contrast between the Euclidean designs of Europe and the fractal designs of Africa can be explained by the important role of the natural environment in African societies. But this assumption turns out to be wrong; if anything, there is a tendency for indigenous societies to favor Euclidean shapes. Physicist Kh. S. Mamedov observed such a contrast in his reflections on his youth in a nomadic culture:

> My parents and countrymen . . . up to the second world war had been nomads. . . . Outside our nomad tents we were living in a wonderful kingdom of various curved lines and forms. So why were the aesthetic signs not formed from them, having instead preserved geometric patterns . . . ? [I]n the cities where the straight-line geometry was predominant the aesthetic signs were formed . . . with nature playing the dominating role. . . . [T]he nomad did not need the "portrait" of an oak to be carried with him elsewhere because he could view all sorts of oaks every day and every hour . . . while for the townsfolk their inclination to nature was more a result of nostalgia. (Mamedov 1986, 512–513)

Contrary to romantic portraits of the "noble savage" living as one with nature, most indigenous societies seem quite interested in differentiating themselves from their surroundings. It is the inhabitants of large state societies, such as those of modern Europe, who yearn to mimic the natural. When European designs are fractal, it is usually due to an effort to mimic nature. African fractals based on mimicry of natural form are relatively rare; their inspiration tends to come from the realm of culture.

How should we place such nature-based designs in our intentionality spectrum? That depends on the difference between mimesis and modeling. Mimesis is an attempt to mirror the image of a particular object, a goal explicitly stated by Plato and Aristotle as the essence of art, one that was subsequently followed in Europe for many centuries (see Auerbach 1953). A photograph is a good example of mimesis. A photo might capture the fractal image of a tree, but it would be foolish to conclude that the photographer knows fractal geometry. If artisans are simply trying to copy a particular natural object, then the scaling is an unintended by-product.

The most important attributes that separate mimesis from modeling are abstraction and generalization. Abstraction is an attempt to leave out many of the concrete details of the subject by creating a simpler figure whose structure is still roughly analogous to the original—often called a "stylized" representation in the arts. Generalization means selecting an analogous structure that is common to all examples of the subject; what is often referred to as an "underlying" form or law.[1] For example, Mandelbrot (1981) points to the European Beaux Arts style as an attempt not merely to imitate nature, but to "guess its laws." He notes that the interior of the Paris opera house makes use of scaling arches-within-arches, a pattern that generalizes some of the scaling characteristics of nature, but is not a copy of any one particular natural object.

Since the ultimate generalization is a mathematical model, why didn't design practices such as the Beaux Arts style result in an early development of fractal geometry? For Europeans, such lush ornamentation was presented—and generally accepted—as embodying the *opposite* of mathematics; it was an effort to create designs that could only be understood in irrational, emotional, or intuitive terms. Even some movements against this attempt, such as the use of distinctly Euclidean forms in the high modern arts style, simply reinforced the association because it only offered a reversal, suggesting that "mathematical" shapes (cubes, spheres, etc.) could be esthetically appreciated. With rare exceptions (e.g., Thompson 1917), mimesis of nature in pre-WW II European art traditions merely furthered the assumption that Euclidean geometry was the only true geometry.[2]

The difference between mimesis and modeling provides two different positions along the intentionality spectrum. The least intentional would be merely holding a mirror to nature—for example, if someone was just shooting a camera or painting a realistic picture outdoors and happened to include a fractal object (cloud, tree, etc.). This mimesis does not count as mathematical thinking. More intentional is a stylized representation of nature. If the artist has reduced the natural image to a structurally analogous collection of more simple elements, she has created an abstract model. Whether or not such abstractions move toward more mathematical models is a matter of local preference.

The two examples of African representations of nature we observed in the previous chapter certainly show that the artisans have gone beyond mere mimesis. The Mandiack cobra pattern we saw in figure 3.2. shows a strictly systematic scaling pattern. This textile design conveys the scaling property of the natural cobra skin pattern—diamonds at many scales—in a stylized or abstract way. We can take this idea a step further by examining another Bwami bat sculpture (fig. 4.2). This spiral pattern is also a stylized representation of the natural scaling of the bat's wing, but it is a different geometric design than the expanding zigzag pattern we saw in figure 3.4c. It is more styl-

FIGURE 4.2
Stylized sculpture of a bat
Another Lega bat sculpture, but unlike the zigzag design we saw in figure 3.4c, here the scaling of the wing folds is represented by a spiral.
(By permission of the Museum of African Art, N.Y.)

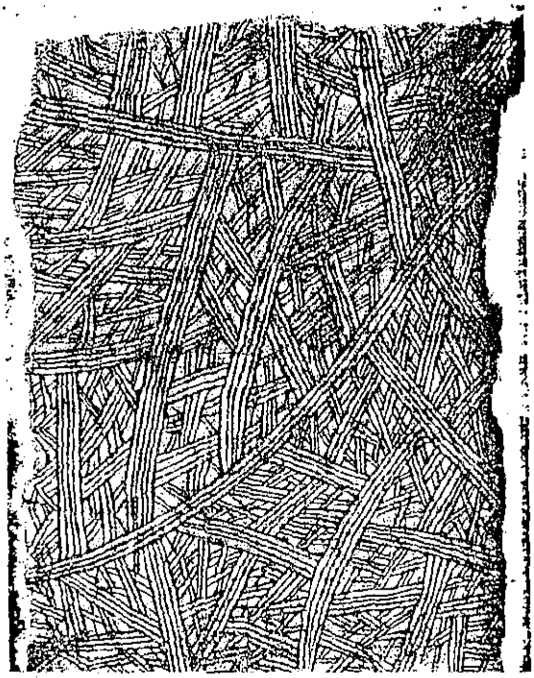
ized in the sense of being further abstracted from the original natural bat's wing. This provides further evidence that the sculptors were focused on the scaling properties—the generalized underlying feature—and not particular concrete details.

The greatest danger of this book is that readers might misinterpret its meaning in terms of primitivism. The fact that African fractals are rarely the result of imitating natural forms helps remind us that they are *not* due to "primitives living close to nature." But even for those rare cases in which African fractals are representations of nature, it is clearly a self-conscious abstraction, not a mimetic reflection. The geometric thinking that goes into these examples may be simple, but it is quite intentional.

The fractal esthetic

Just as we saw how designs based on nature range from unconscious to intentional, artificial designs also vary along a range of intention, with some simply the result of an intuitive inspiration, and others a more self-conscious application of rules or principles. The examples of African fractals in figure 4.3 did not appear to be related to anything other than the artisan's esthetic intuition or sense of beauty. As far as I could determine from descriptions in the literature and my own fieldwork, there were no explicit rules about how to construct these designs, and no meaning was attached to the particular geometric structure of the figures other than looking good. In particular, I spent several weeks in Dakar wandering the streets asking about certain fractal fabric patterns and jewelry designs, and the insistence that these patterns were "just for looks" was so adamant that if someone finally had offered an explanation, I would have viewed it with suspicion.

Since some professional mathematicians report that their ideas were pure intuition—a sudden flash of insight, or "Aha!" as Martin Gardner puts it—we shouldn't discount the geometric thinking of an artisan who reports "I can't tell you how I created that, it just came to me." Esthetic patterns clearly qualify as intentional designs. On the other hand, there isn't much we can say about the mathematical ideas behind these patterns; they will have to remain a mystery unless something more is revealed about their meaning or the artisan's construction techniques. It is worth noting, however, that they do contribute to the fractal design theme in Africa. Esthetic patterns help inspire practical designs, and vice versa. Since ancient trade networks were well established, the diffusion of esthetic patterns is probably one part of the explanation for how fractals came to be so widespread across the African continent.



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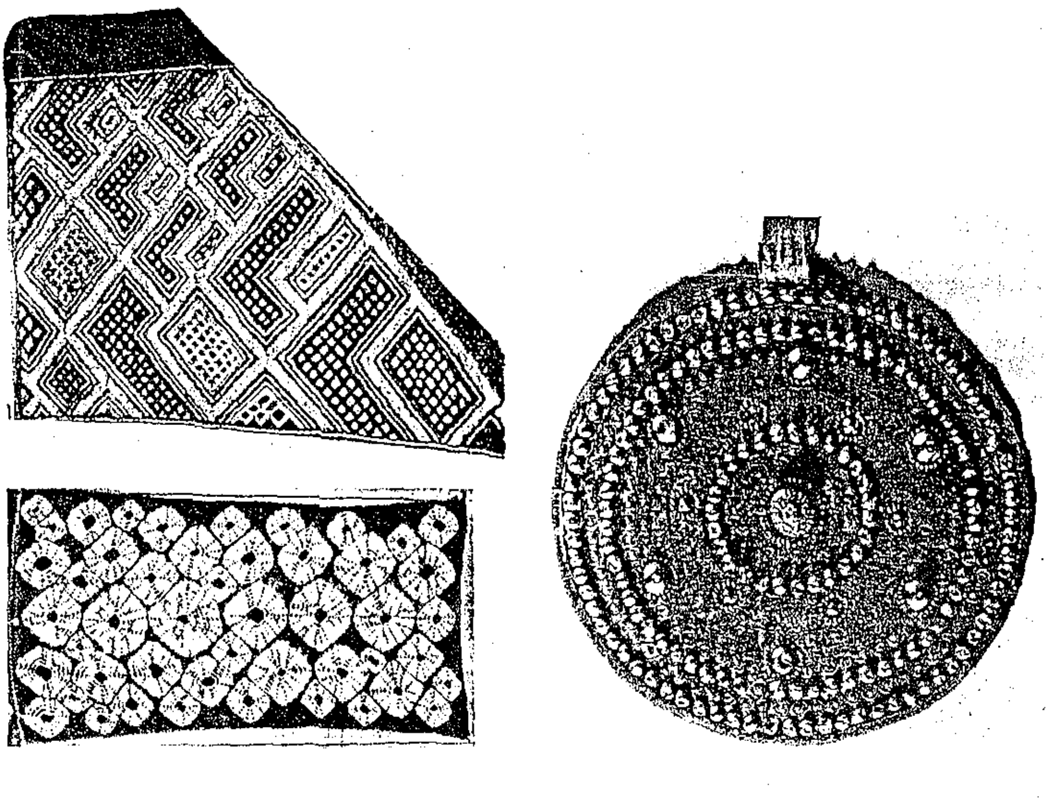


FIGURE 4.3

Esthetic fractals

(a) Meurant (quoted in Reif 1996) reports that the Mbuti women who created this fractal design, a bark-cloth painting, told him the design was not "telling stories," nor was it "representing any particular object." (b) Scaling patterns can be found in many African decorative designs that are reported to be "just for beauty." *Upper left*, Shoowa Raffia cloth; *lower left*, Senegalese tie dye; *right*, Senegalese pendant.

(a, *courtesy Georges Meruant.*b: *Upper left, British Museum; lower left, from Musée Royal de l'Afrique Central, Belgium; right, photo courtesy IFAN, Dakar.)*

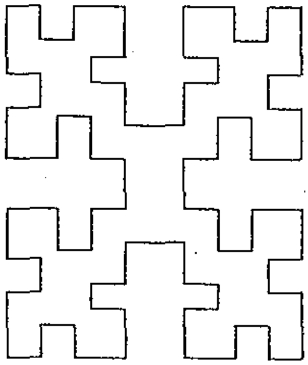


FIGURE 4.4
The quincunx fractal
A customer in Touba, Senegal, selects a fractal quincunx pattern for his leather neck bag. The quincunx is historically important because of its use by early African American "man of science" Benjamin Banneker.

Of course, there are plenty of African designs that are strictly Euclidean, but even these can occur in "fractalized" versions. One particularly interesting example is the *quincunx* (fig. 4.4). The basic quincunx is a pattern of five squares, with one at the center and one at each corner. The design is common in Senegal, where it is said to represent the "light of Allah." The quincunx is historically important because the image was recorded as being of religious significance to the early African American "man of science" Benjamin Banneker. Since evidence shows that Banneker's grandfather (Bannaka) came from Senegal, the quincunx is a fascinating possibility for geometry in the African diaspora (see Eglash 1997c for details). Because of the fractal esthetic, this religious symbol is often arranged in a recursive pattern—five squares of five squares—as shown in figure 4.4 in the design for a leather neck bag.

Finally, there are also examples of the fractal esthetic in common household furnishings. Euro-American furniture is differentiated by form and function—stools are structured differently from chairs, which are structured differently from couches. But in African homes one often sees different sizes of the same shape (fig. 4.5). A similar difference has been noted in cross-cultural comparisons of housing. Whereas Euro-Americans would never think to have a governer's mansion shaped like a peasant's shack (or vice versa), precolonial African architecture typically used the same form at different sizes (as we saw for the status distinctions in the Ba-ila settlement in chapter 2). It is unfortunate that this African structural characteristic is typically described in terms of a lack—as the absence of shape distinctions rather than as the presence of a scaling design theme.

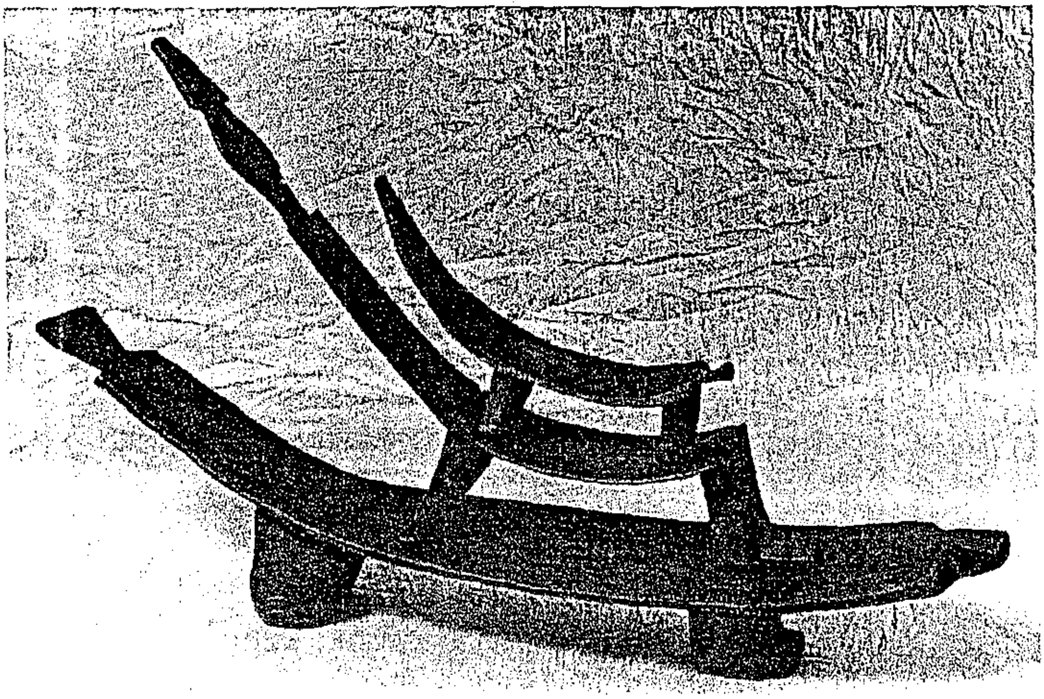


FIGURE 4.5

The fractal esthetic in household objects

African stools, chairs, and benches are often created in a scaling series.
(*Photo courtesy of Africa Place, Inc.*)

Conclusion

We now have some guidelines to help determine which fractal designs should count as mathematics, which should not, and which are in between. Figure 4.6 summarizes this spectrum. Fractals produced by unconscious activity, or as the unintentional by-product from some other purpose, cannot be attributed to indigenous concepts. But some artistic activities, such as the creation of stylized represen-

Unintentional	*Intentional but implicit*	*Intentional and explicit*
Unconscious activity • urban sprawl	*Conscious use of natural scaling* • stylistic abstraction of natural scaling	*Construction techniques*
Accidental fractals • "mirror" portrait of nature (*mimesis*; e.g., photography)	*Esthetic design* • intuitive fractal design theme	*Knowledge systems*

FIGURE 4.6

From unconscious accident to explicit design tations of nature or purely esthetic designs, do show intentional activity focused on fractals. Such examples may be restricted in terms of geometric thinking—the artisans may only report that the design suddenly came to them in a flash of intuition—but these are clearly distinguished from those which are unconscious or accidental. The following chapters will consider examples that are not only intentional, but also include enough explicit information about design techniques and knowledge systems to be easily identifiable as mathematical practice and ideas.

PART

II

African fractal mathematics

CHAPTER 5

Geometric algorithms

The word "algorithm" derives from the name of an Arab mathematician, Al-Khwarizmi (c. 780–850 C.E.), whose book *Hisab al-jabr w'al-muqabala* (Calculation by Restoration and Reduction) also gave us the word "algebra." Although Al-Khwarizmi focused on numeric procedures for solving equations, the modern term "algorithm" applies to any formally specified procedure. A geometric algorithm gives explicit instructions for generating a particular set of spatial patterns. We have already seen how iterations of such pattern-generating procedures can produce fractals on a computer screen; in this chapter we will examine two indigenous algorithms that also use iteration to produce scaling designs: the 45-degree-angle constructions of the Mangbetu, and the lusona drawings of the Chokwe.

Geometry in Mangbetu design

The Mangbetu occupy the Uele River area in the northeastern part of the Democratic Republic of Congo (formally Zaire). Archaeological evidence shows iron smelting in the area since 2300 B.C.E., but the Mangbetu, coming from drier lands around present-day Uganda, did not arrive until about 1000 C.E. Through both conflict and cooperation, they exchanged cultural traditions with other societies of the area: Bantu-speaking peoples such as the Buda, Bua and Lese, and Ubangian-speaking peoples such as the Azande, Bangba, and Barambo. Around 1800 a number of small chiefdoms were consolidated into the first Mangbetu kingdom. Although it lasted only two generations, a tradition of courtly prestige continued even in small villages and spread to many of the Mangbetu's trading partners. This combination of cultural diversity, exchange, and prestige resulted in a thriving artistic tradition.

A detailed account of Mangbetu history and traditions can be found in *African Reflections: Art from Northeastern Zaire*. Schildkrout and Keim (1990) begin their analysis by showing that the most famous aspect of Mangbetu art, the "naturalistic look," was actually quite rare in the traditional Mangbetu society of the nineteenth century. During a research expedition to the Congo in 1914 (the origin of the photos used here), mammalogist Herbert Lang became fascinated with lifelike carvings of human figures, and as word spread that he was paying high prices for them, more of these carvings were produced. Other collectors came to buy these pieces, and eventually the economic rewards for producing naturalistic Mangbetu art became so strong that it replaced other styles.

Schildkrout and Keim show that originally the most important esthetic was not naturalism, but abstract geometric design. The indigenous fascination with artifice and abstraction was ignored by colonizers, and their preconceptions of Africans as nature-loving "children of the forest" became a self-fulfilling expectation. But the artifacts and photographic records from the 1914 expedition provide us with excellent examples of traditional Mangbetu patterns, as well as an opportunity to infer some of their techniques.

Figure 5.1 shows the decorative end of an ivory hatpin. Like the architecture and esthetic patterns we have seen, this is clearly a scaling design, but the precision of the pattern suggests that there may be a more

FIGURE 5.1
Mangebetu ivory sculpture
(Transparency no. 3935, photograph by Lynton Gardiner, courtesy American Museum of Natural History.)

formal geometric process at work. Similar design can be seen at work in the Mangbetu's geometric style of personal adornment. Figure 5.2a shows a Mangbetu hairstyle, popular during the time that this carving was created (about 1914), which featured a disk angled to the vertical at 45 degrees. Men often wore a hat with the top flattened, forming the same angle, as seen in figure 5.2b. Just as a plane cuts diagonally through the top of the heads in the ivory sculpture of figure 5.1, real Mangbetu headdresses also terminated in a 45-degree angle.

This was only one part of an elaborate geometric esthetic based on multiples of the 45-degree angle. Figure 5.2b shows an ivory hatpin, ending in a disk perpendicular to it, inserted perpendicular to the hat. To its right, a small ivory arrow pinned to the hat points horizontally, thus forming an angle of 135 degrees with the hatpin. Each part of the ensemble was aligned by a multiple of the 45-degree angle. This adornment style included artificial elongation of the head, which is clearly visible in the photograph in figure 5.2b. Elongation was accomplished by wrapping a cloth band around the head of infants; the woman in figure 5.2a is weaving one of these bands. Head elongation resulted in an angle of 135 degrees between the back of the head and the neck.

a

b

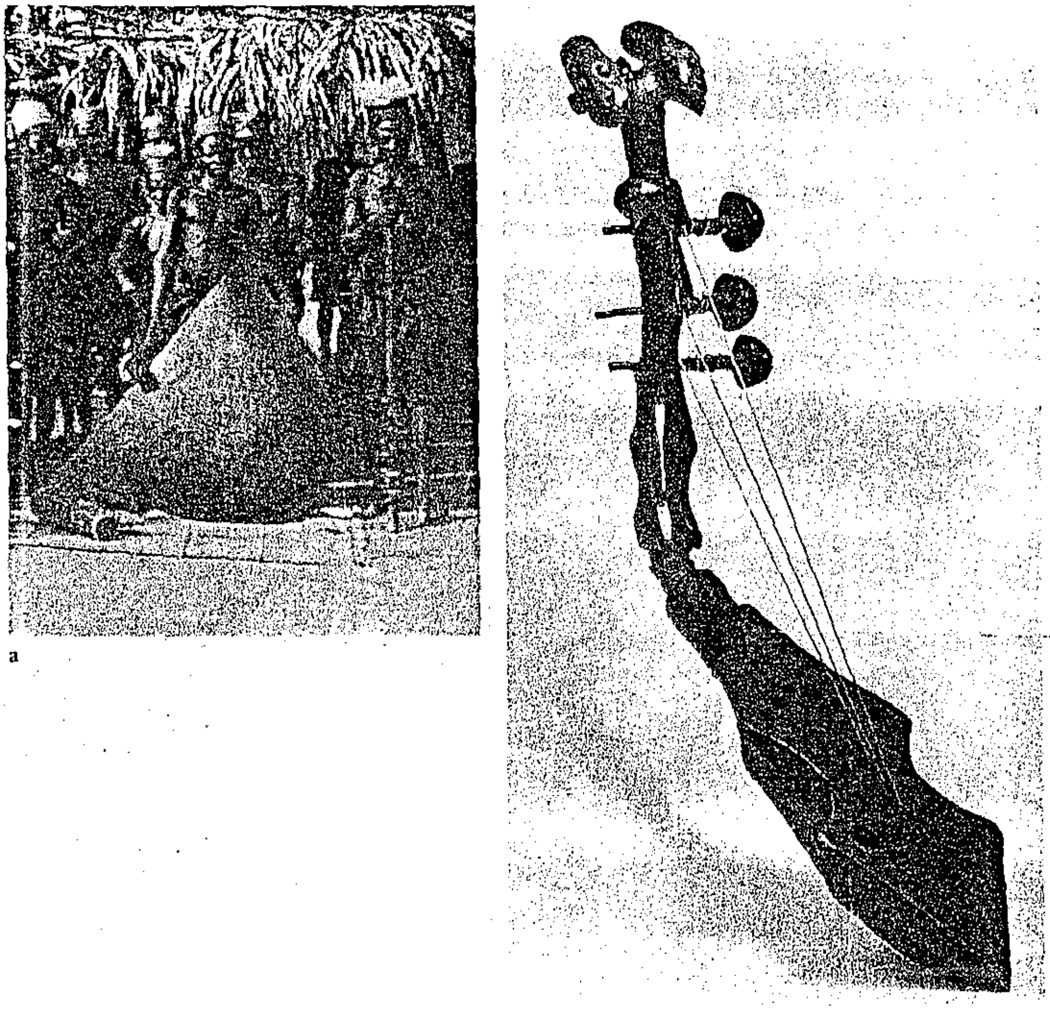
FIGURE 5.2

Geometric design in Mangbetu personal adornment

(a) Mangbetu woman weaving headband. (b) Mangbetu chief.

(a, negative no. 111919, photograph by H. Lang, courtesy American Museum of Natural History; b, negative no. 224105, photograph by H. Lang, courtesy American Museum of Natural History.)

While the Mangbetu geometric conception of the body may have inspired the 45-degree-angle design theme, those designs were certainly not limited to simple mimicry of anatomy. We can clearly see this in their musical instruments. The drum in figure 5.3a, for example, has its upper surface cut at a 45-degree angle to the vertical. The stringed instrument shown in figure 5.3b has a resonator that meets the vertical tuning stem at a 135-degree angle. Even in the case of anthropomorphic designs, the artisans elaborated on the human form in ways that show



a

b

FIGURE 5.3
Geometric design in Mangbetu musical instruments
(a) Drum. (b) Harp.
(*a, negative no. 111896, photograph by H. Lang, courtesy American Museum of Natural History; b, courtesy Rietberg Museum Zurich, photograph by Wettstein and Kauf.*)

creative—and not merely imitative—applications of geometrical thinking. For example, there is an anthropomorphic decorative motif at the end of the tuning stem shown in figure 5.3b, but these human heads are not simply mimicking human form. In figure 5.2b we saw that the Mangbetu had a 135-degree angle between the back of the head and the neck. The carved heads in figure 5.3b have a 90-degree angle between the back of the head and the neck. Such distortions indicate active geometric thinking rather than passive reflection of natural anatomical angles (which, recalling the artificial head elongation, were not so natural to begin with).

There are also purely abstract designs that make use of multiples of 45 degrees, as we see in figure 5.4. Modern Mangbetu report that the creation of a design reflected the artisan's desire to "make it beautiful and show the intelligence of the creator" (Schildkrout and Keim 1990, 100). This suggests another reason for artisans to adhere to angles that are multiples of 45 degrees: if there were no rules to follow, then it would have been difficult to compare designs and demonstrate one's ingenuity. By restricting the permissible angles to a small set, they were better able to display their geometric accomplishments.

Combining this 45-degree-angle construction technique with the scaling properties of the ivory carving in figure 5.1 can reveal its underlying structure. The carving has three interesting geometric features: First, each head is larger than the one above it and faces in the opposite direction. Second, each head is framed by two lines, one formed by the jaw and one formed by the hair; these lines intersect at approximately 90 degrees. Third, there is an asymmetry: the left side shows a distinct angle about 20 degrees from the vertical.

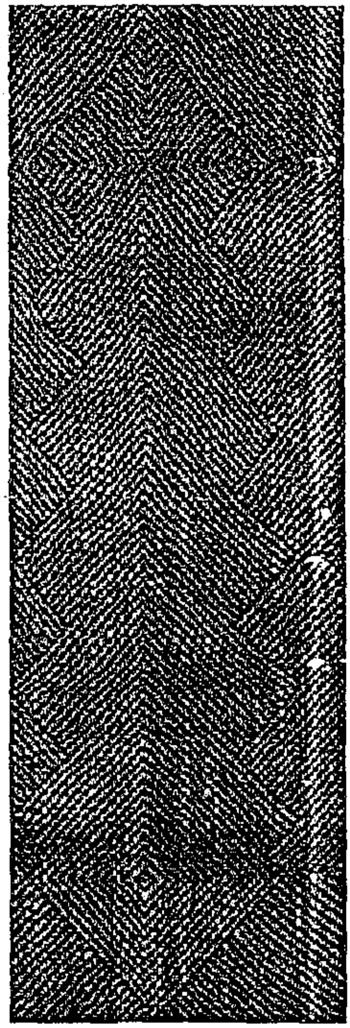


FIGURE 5.4
Mangebetu ivory sculpture
(Transparency no. 3929, photograph by Lynton Gardiner, courtesy American Museum of Natural History.)

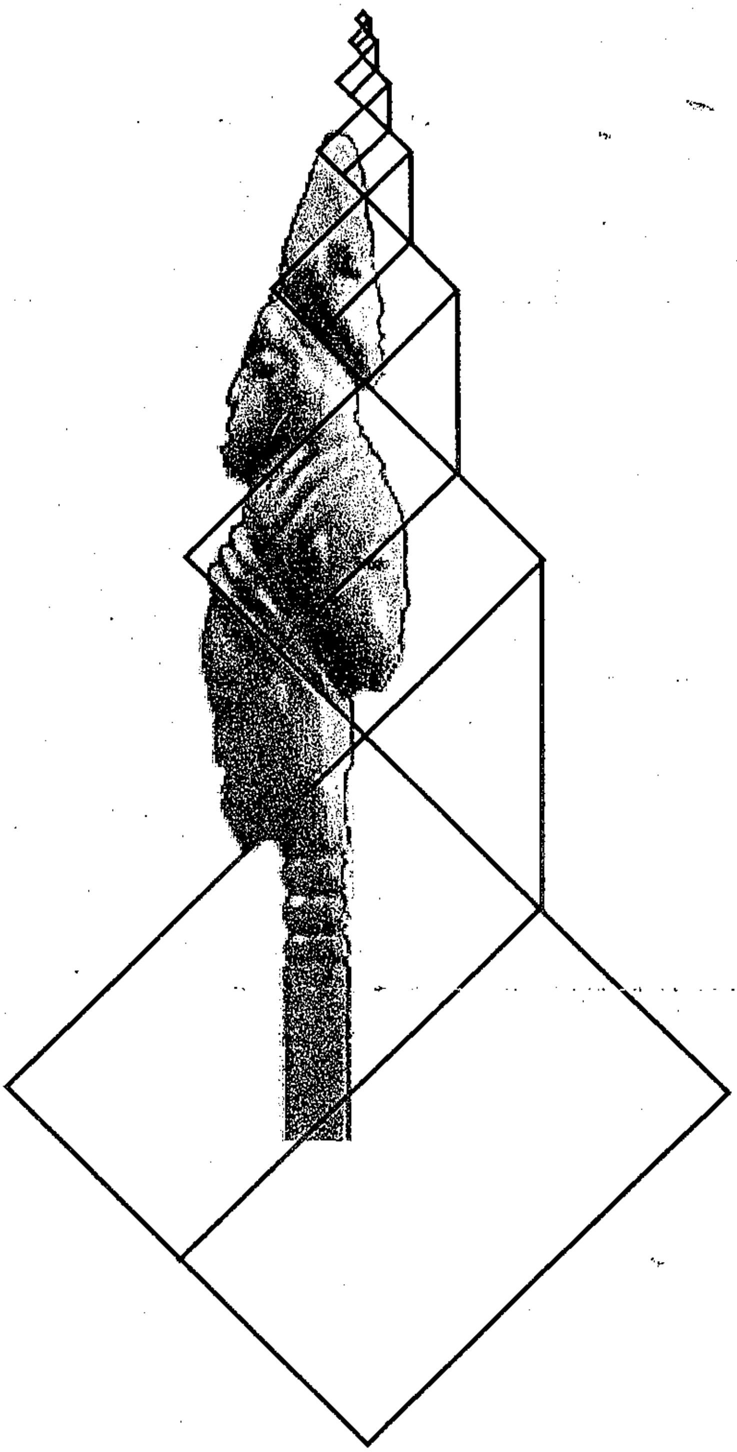


FIGURE 5.5
Geometric analysis of an ivory sculpture

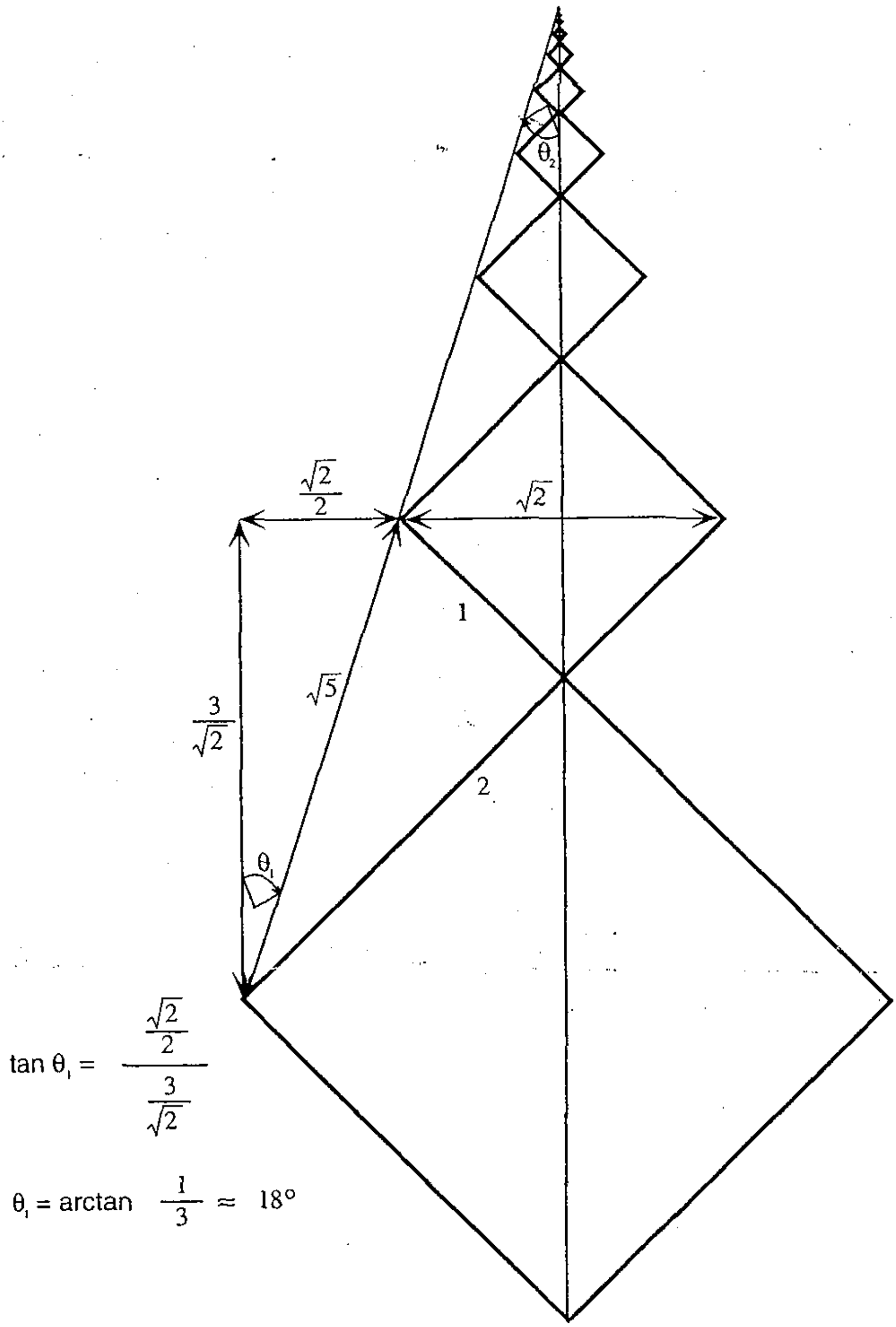


FIGURE 5.6

Geometric relations in the Mangbetu iterative squares structure

Since θ_1 and θ_2 are the alternate interior angles of a transversal intersecting two parallel lines, $\theta_1 = \theta_2$.

All of these features can be accounted for by the structure shown in figure 5.5. This sequence of shrinking squares can be constructed by an iterative process, bisecting one square to create the length of the side for the next square, as indicated in the diagram. We will never know for certain if this iterative-squares construction was the concept underlying the sculpture's design, but it does match the features identified above. In the ivory sculpture, the left side is about 20 degrees from the vertical. In the iterative-squares structure, the left side is about 18 degrees from the vertical, as shown in figure 5.6. Here we see that the construction algorithm can be continued indefinitely, and the resulting structure can be applied to a wide variety of math teaching applications, from simple procedural construction to trigonometry (Eglash 1998a).

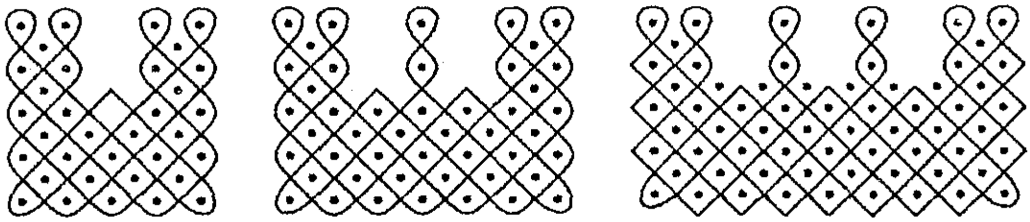
Lusona

The Chokwe people of Angola had a tradition of creating patterns by drawing lines called "lusona" in the sand. Gerdes (1991) notes that the lusona sand drawings show the constraints necessary to define what mathematicians call an "Eulerian path": the stylus never leaves the surface and no line is retraced. The lusona also tend to use the same pattern at different scales, that is, successive iterations of a single geometric algorithm. Figure 5.7 shows the first three iterations of one of the dozens of lusona that were recorded by missionaries during the nineteenth century, when the lusona tradition was still intact.

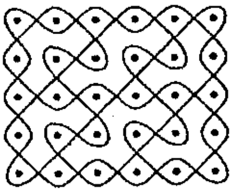
As in the case of the Mangbetu 45-degree constructions, the restriction to an Eulerian path provides the Chokwe with a means to compare designs within a single framework, and to show how increasing complexity can be achieved within these constraints of space and logic. But unlike the competitive basis for comparison that the Mangbetu describe, the Chokwe made use of these figures to create group identity. The reports indicate that the lusona were used in an age-grade initiation system; rituals that allowed each member to achieve the status of reaching the next, more senior level of identity. By using more complex lusona, the iterations of social knowledge passed on in the initiation become visualized by the geometric iterations. In chapter 8 we will see other examples of iterative scaling patterns in initiation rituals. This tradition of group identity through knowledge of the lusona was also deployed by the Chokwe as a way to deflate the ego of overconfident European visitors, who found themselves unable to replicate the lusona of many children.

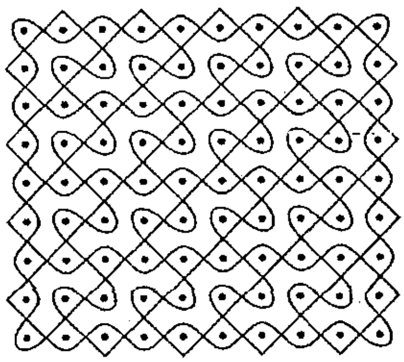
Conclusion

These two examples, the Mangbetu ivory carving and the lusona drawings, help us see that African fractals are not just the result of spontaneous intuition; in some cases they are created under rule-bound techniques equivalent to Western mathematics. And their cultural significance makes it clear that all mathematical activity—*no matter in which society it is found*—is produced through an interaction between the freedom of local human invention and the universal constraints we discover in space and logic.

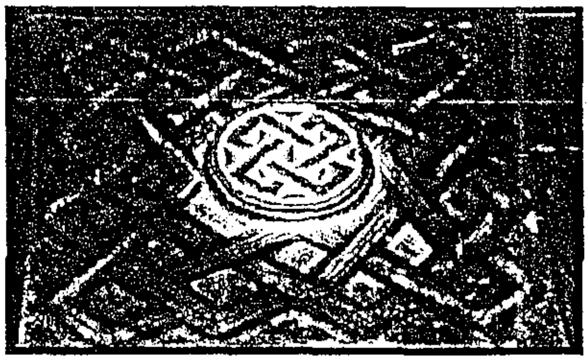


"Myombo"—trees of the ancestors.





"Path of a hunted bird."



a b

FIGURE 5.7
Lusona

(a) These figures, "lusona," were traditionally drawn in sand by the Chokwe people of Angola. Successive iterations of the same algorithm were sometimes used to produce similar patterns of increasing size. (b) The first and third iterations of another lusona algorithm carved into a wooden box lid.

(a, *based on drawings in Gerdes 1995.*)

Recall that in both examples the role of "constraint" was crucial to the development of their scaling geometry. For the Mangbetu's design it was the constraints of straight-edge construction with angles at multiples of 45 degrees. For the Chokwe's lusona it was the constraints of an Eulerian path. But in each case the choice of particular objective constraints—deciding which of the infinite laws of space and logic we are concerned with—was established by and for the social relations of the community. In the case of the Mangbetu it was artistic competition, and in the case of the Chokwe it was age-grade identity. In other words, the invention and discovery components of mathematics are inextricably linked through social expression.

Philosophic perspectives on the relation of culture and mathematics will be further discussed in part II, but to do so we need a fuller portrait of African fractal geometry. The next chapter will examine African conceptions of the most fundamental characteristic of fractals: nonlinear scaling.

CHAPTER

6

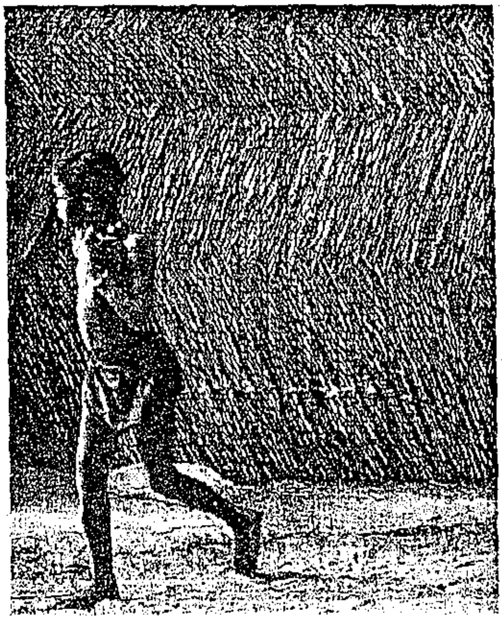
Scaling

We have already seen many examples of scaling in African designs. In the settlement architecture of chapter 2, for example, the computer simulations clearly show that we can think about these patterns in terms of fractal geometry. How do the African artisans think about scaling? Is it just intuition, or do they use explicit mathematical practices in thinking about similarity at different sizes? By examining varieties of designs with different scaling properties, and comparing these with the artisans' discussions of the patterns, we can gain some insight into scaling as a mathematical concept in African cultures.

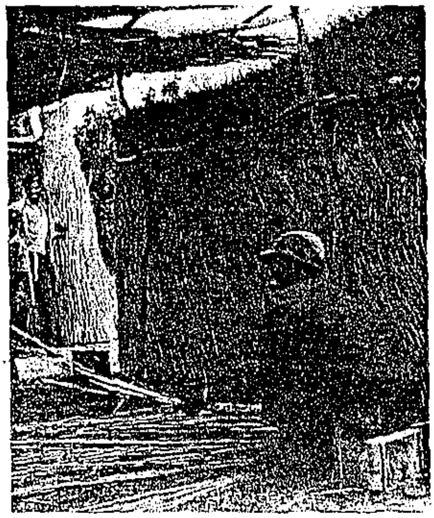
Power law scaling in windscreens from the Sahel

The Sahel is a broad band of arid land between the Sahara Desert and the rest of sub-Saharan Africa. Since there are few trees and a great deal of millet cultivation, it is not surprising that artisans use millet stalks to weave fences, walls, and other constructions. But the consistent use of a nonlinear scaling pattern in these straw screens (fig. 6.1a) is a bit odd. Rather than uniform lengths, the rows of millet straw get shorter and shorter as they go up. In the United States we are used to the image of "the white picket fence" as a symbol of unchanging, linear repetition, yet here the fences are distinctly nonlinear. While I was in Mali on

a



The straw windscreen in Niger.



b Windscreen under construction in Mali.

c

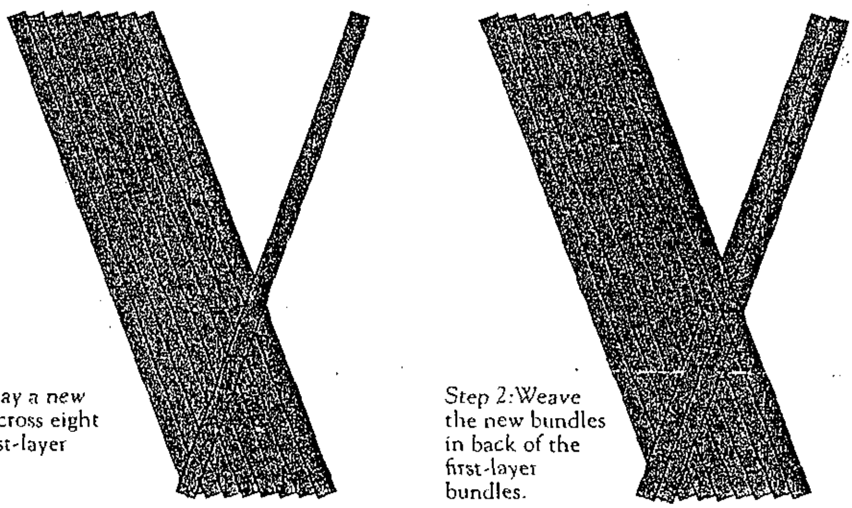


FIGURE 6.1

An African windscreen

(a) The diagonal lengths of these rows from bottom to top: L = 16 12 8 6 5 5 3 3 2 2

This pattern is quantitatively determined by the African artisans. Here we see how the bundles of straw are first laid in long diagonal rows, then a row at the opposite angle is interlaced in back of it. The length of each diagonal row—how high up you go before doing the interlace step—is determined by *counting a certain number of diagonals to be crossed. In the first layer (c) we go* over eight, then six, then four, then three.

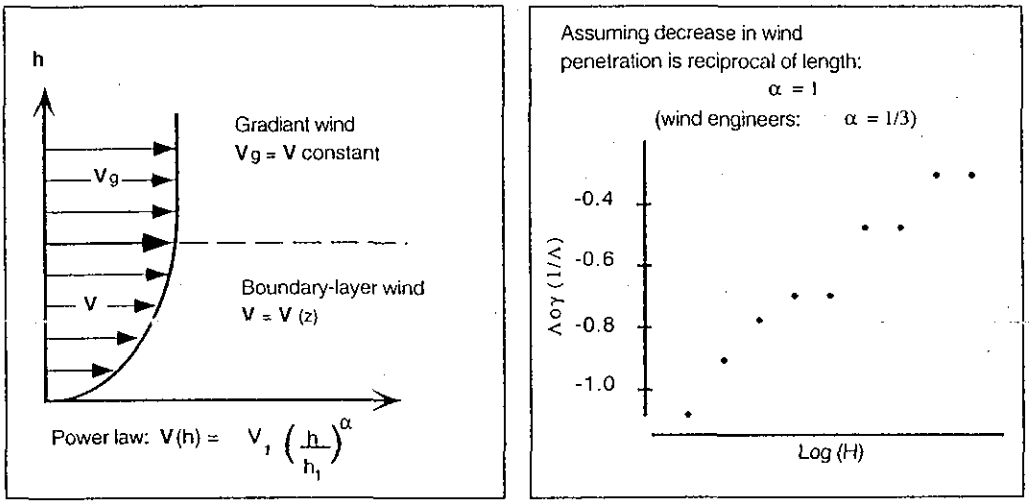
Each bundle is about 2 inches across the diagonal, which is why the lengths go as double the number of crossings. The odd numbered lengths are created by splitting the bundles in two.

Why do the lengths repeat in pairs as we go toward the top? There is a discrete approximation to the continuous nonlinear scale that the African artisans follow.

(a, *photo by permission of Gardi 1973.*) *(figure continues)* the outskirts of the capital city of Bamako, I had the opportunity to interview some of the artisans who create these screens and was provided with a striking example of indigenous application of the scaling concept.

The artisans began by explaining that in "fertile areas" such as the forests of the south, the screens are not made with scaling rows but rather with rows of long, uniform length. This is because the long rows use less straw and take less time to make. But here in the Sahel, they said, we have strong winds and dust. The shortest rows are the ones that keep out dust the best, because they are the tightest weave. But they also take more materials and effort. "We know that the wind blows stronger as you go up from the ground, so we make the windscreen to match—that way we only use the straw needed at each level."

The reasoning the artisans reported is equivalent to what an engineer would call a "cost-benefit" analysis; developing the maximum in function (keeping out dust) for a minimum of cost (effort and materials). My primary interest here is in showing that the scaling concept in Africa can be much more sophisticated than just an observation, "the same thing in different sizes." The creation



d e

FIGURE 6.1 *(continued)*

(d) The relation between wind speed and vertical height as shown in the *Wind Engineering Handbook*. (e) The African windscreen makers say that they have scaled the rows of straw to match the change of wind speed with height. If we assume, just for simplicity, that the decrease in wind penetration is the reciprocal of the length, then we can get the African estimate for α by measuring the slope of row length versus height on a log-log graph. This gives $\alpha = 1$, whereas the engineers use $\alpha = 1/3$—not bad for a ballpark estimate.

Note that the graph is in a very straight line, except where the discrete nature of the screen (the screen makers must count in whole-number units due to the straw bundles) forces an approximation by repeating the same length twice.

of the windscreen as an optimal design required matching the scaling variation of wind speed versus height to a scaling variation in lengths of straw. By transferring this concept between two completely different domains, the artisans have demonstrated that they understand scaling in the abstract; indeed, the design essentially plots the relation of wind speed to height on a straw graph.

Although I was concerned only with the overall relation of scaling and reasoning, I measured the rows just to see how close they came to what a Western engineer would develop for an optimal match with wind speed. If the straw screen had linear scaling, then each row would decrease in length by the same amount (e.g., 12 inches, 10 inches, 8 inches, etc.). But the rows decrease less and less with height; it turns out that the screen design shows a close fit to what is called a "power law"—that is, it scales according to an exponent (fig. 6.1c). Figure 6.1b, reprinted from the *Wind Engineering Handbook*, shows the equation of wind speed with height most commonly used by engineers—also a power law. So the Sahel windscreen is not only a practical application of the abstract scaling concept, it is also a fairly accurate one. Of course, one might object that the indigenous engineers did not actually set up the algebra and perform the optimizing calculation. But I asked three American mathematicians how they would set up these equations to determine the optimal design, and all three said the same thing: "I wouldn't solve it analytically, I'd just graph the equations on the computer and see where the functions peaked." Whether we make our graphs on a computer screen or a straw screen doesn't matter, as long as we get the right answer.

Stretching space in kente cloth

If someone in America were asked to think of an African textile, *kente* cloth would be the most likely image. Its combination of strong colors, bold designs, and associations with ancient kingdoms of West Africa has made it a favorite for imports. But most of the imported kente cloth is created by automated machine, and while I would fiercely defend it as "authentic," the need for pattern repetition in automation has eliminated a wonderful scaling transformation that can be seen in the older patterns created on hand looms (fig. 6.2a). The scaling change is not just small and large versions of the same thing; rather, it is as if the design was drawn on a rubber sheet, which was half stretched and half contracted. In Ghana I traveled to the village of Bonwire, where hand-loom weaving is still practiced, and asked the artisans there why this scaling transformation was created.

The weavers replied that they think of the compressed version as the original pattern, and said they call it "spreading" when they create the stretched version. The reason they gave for the spreading pattern can best be understood with

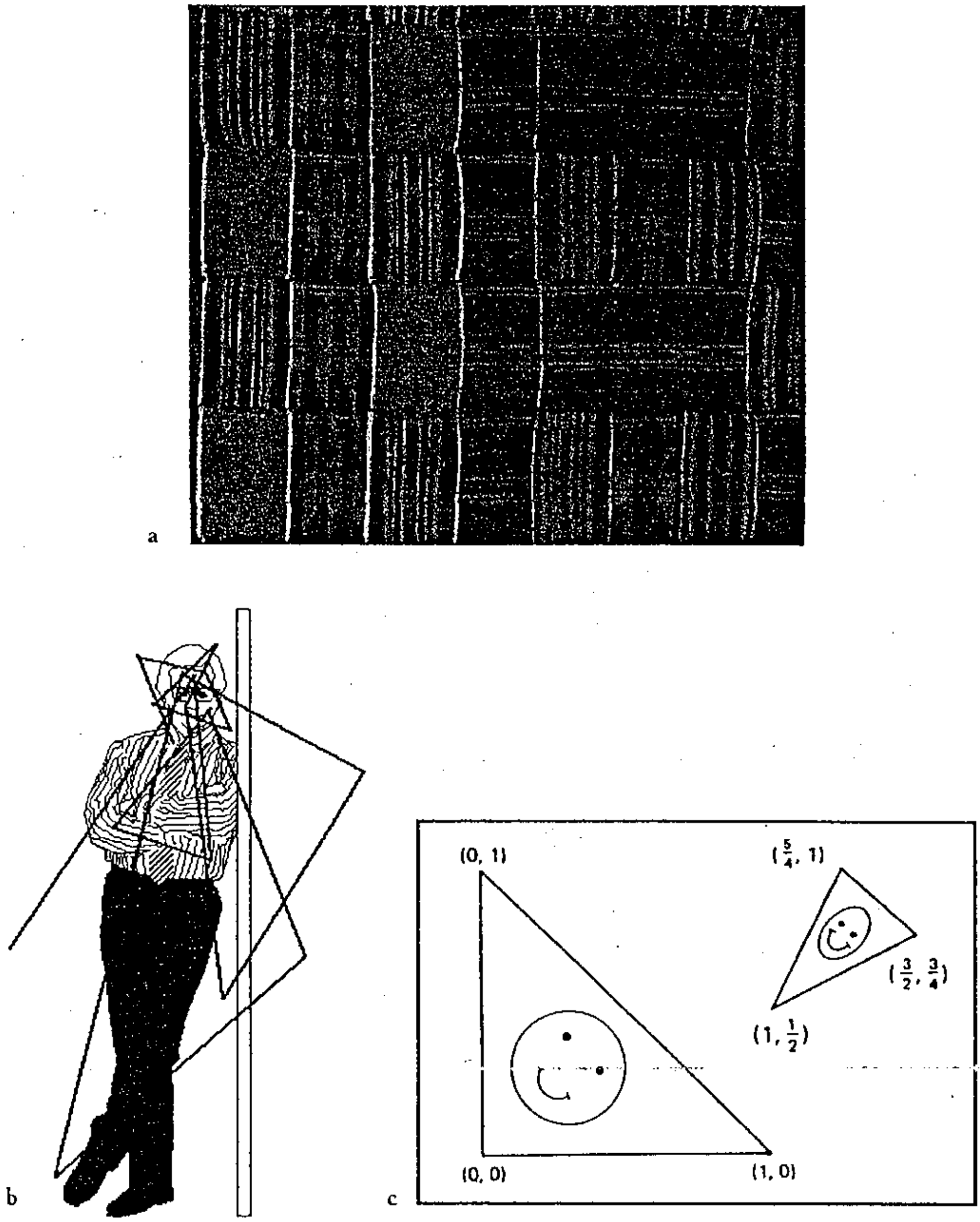


FIGURE 6.2

Kente cloth

(a) In this traditional kente cloth design, stretched and compressed versions of the same pattern appear. The weavers call this "spreading" the pattern. (b) Why are weavers spreading the pattern? They say that our eyes give "heavy looks" to the face, and only "light looks" to the rest of the body. This is what neurobiologists call "saccadic" eye movements. Unlike "tracking" eye movements, which are continuous, saccadic movements are discrete and tend to leap about. Since kente cloth was traditionally worn as a toga over the shoulder, the part near the face was given a compressed pattern, and the part along the body a stretched pattern, to match the scaling of the saccadic eye movements. (c) The compression of space is used in mathematics to model scaling patterns, like that of the saccadic eye movements. Mathematicians call this a "contractive affine transformation."

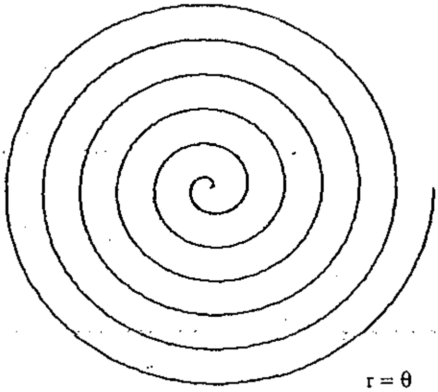
the following experiment. Hold your finger in front of your face, and without moving your head, track the finger with your eyes as you move it slowly across the visual field. Now try the same thing again, smoothly tracking the visual field, but without the finger to guide your eyes. You'll find that it can't be done! Your eye moves involuntarily in little jumps, called "saccadic" movements. When a person comes into your visual field, those same saccadic movements densely cover the face, and then make a few glances at the body (fig. 6.2b). The weavers in Bonwire reported the same idea: "When you see a person you give heavy looks to the face, and light looks to the body." They explained that the purpose of the scaling change is to match this visual scaling: the compressed part of the pattern is the cloth worn over the shoulder, and the stretched part is worn down the length of the body.

The mathematical term for this operation is "contractive affine transformation" (fig. 6.2c), which can be used for creating fractals through a method called "iterated function systems" (see Wahl 1995, 156–157). In kente cloth there is no iteration—the operation is done only once—but it does show active thinking about a scaling transformation. As in the case of the windscreen, the weavers are taking a rather abstract observation about a time-varying quantity and mapping this model into a material design.

Logarithmic spirals

In chapter 3 (fig. 3.2) we examined the contrast between nonlinear concentric circles and linear concentric circles. In the same way, nonlinear spirals are easy to understand if we contrast them with linear spirals (fig. 6.3a). The linear spiral, also called an Archemedean spiral in honor of the Greek mathematician who favored it, is in the shape of a coiled rope or watch spring. Each revolution brings you out by the same distance (just as each layer in the linear concentric circle was the same thickness). For that reason, a linear spiral of a finite diameter can have only a finite number of turns. A nonlinear spiral of finite diameter can have an infinite number of turns, because even though there is less and less space remaining as one goes toward the center, the distance between each revolution can get smaller and smaller.

A good example of this nonlinear scaling can be seen in the logarithmic spiral (fig. 6.3b). Logarithmic spirals are typical structures in two different categories of natural phenomena. On the one hand, they are found in astonishing varieties of organic growth. Theodore Cook's *The Curve of Life* (1914), for example, shows dozens of logarithmic spirals from every branch of the evolutionary tree: snail and nautilus shells; the horns of rams and antelope; algae, pinecones,



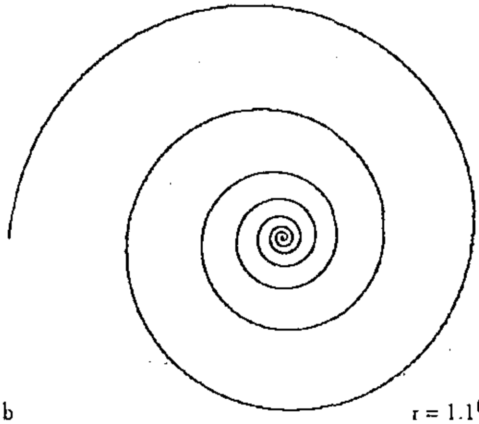
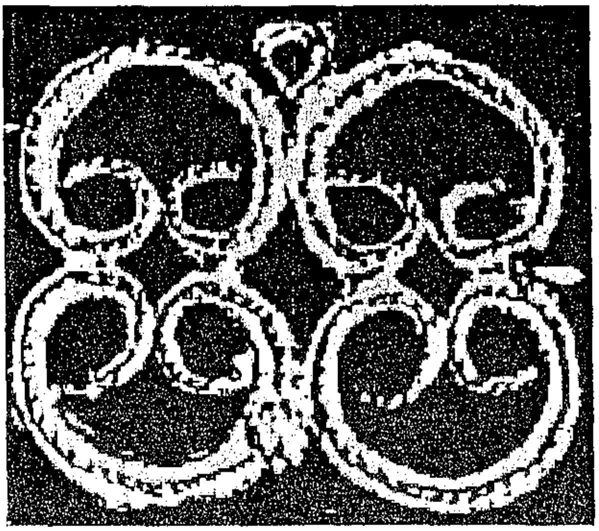


FIGURE 6.3
Spirals
(a) In the linear spiral of Archimedes, there is a constant distance between each revolution. Only a finite number of turns can fit in this finite space. (b) In the logarithmic spiral, there is an increasing distance between each revolution. An infinite number of turns can fit in this finite space.

and sunflowers; and even anatomical parts of the human ear and heart. Many researchers have speculated on why this is so; their answer is typically that living systems need to keep the same proportions as they grow, and so a scaling curve allows the same form to be maintained. I prefer to think of it as recursion: if we look at the chambered nautilus, for example, we can think of each new chamber as the next iteration through the same scaling algorithm.

On the other hand, logarithmic spirals are also found in fluid turbulence. We become aware of this when we watch a hurricane from space, or simply admire the swirls of water along a riverbank. Explanations for these fluid curves are much less speculative, since we can write equations for turbulence and show them producing logarithmic spirals in computer simulations (as we will see in chapter 7). But the Euro-American tradition is not the only one interested in simulacra. The artists of what is now Ghana—particularly those of the Akan society—long ago abstracted the logarithmic spiral for precisely these two categories. Their symbols for the life force (fig. 6.4a) are clearly related to the "curves of life," and icons for Tanu, the river god (fig. 6.4b), show the logarithmic swirls of turbulence.



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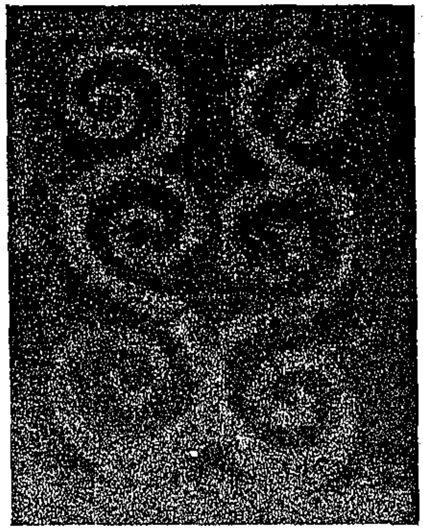


b

FIGURE 6.4
Logarithmic spirals

(a) Several Ghanaian iconic figures, such as this goldweight, link a spiritual force with the structure of living systems through logarithmic spirals. This example is particularly striking since it shows how spirals can be combined with bilateral symmetry to create other self-similar shapes (the large diamond shape created by the meeting of the large spiral arms is repeated on either side by the small diamond at the meeting of the small spiral arms). (b) This figure, again based on logarithmic spirals, appears on the temples of Tanu, the river god, and links this spiritual force to the geometric structure of fluid turbulence.
(a, *photo courtesy Doran Ross.*)

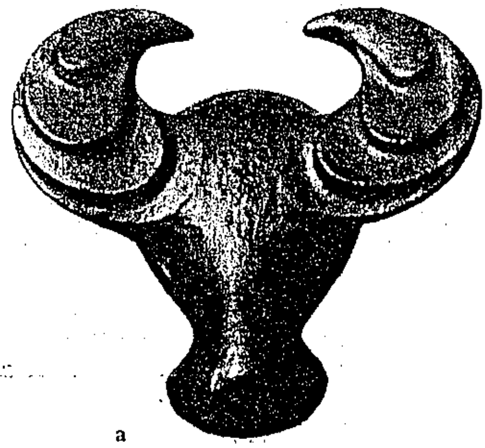
Again, we need to avoid the assumption that the Ghanaian log spirals are simply mimetic "reflections" of nature, and examine how they are used and designed. The Akan and other societies of Ghana created a collection of specific icons that several researchers have compared to a written language. But rather than composed of the vast number of symbols we call "words," the Ghanaian symbolic vocabulary is much smaller, and each symbol refers not to a single word but an entire social, religious, or philosophical concept. Moreover, in many cases the structure of the symbol is not arbitrary (as Gregory Bateson said, "There is nothing 'sevenish' about the numeral 7"), but rather is shaped so that each icon's geometric structure recalls the concept it represents. In other words, they are not only abstractions in the sense of being stylized, but also generalizations in the sense of the designers' intent to find an underlying structure that all examples have in common. For this reason we can accurately describe the Ghanaian log spiral icons as geometric models for the phenomena of organic growth and fluid turbulence.

Some aspects of these designs illustrate a conscious reflection on their geometric properties. Figure 6.4a, for example, not only displays the log spiral's Euclidean symmetry—for we can see how clockwise and counterclockwise spirals compare—but also experiments with other kinds of scaling symmetry: note that the large diamond shape created by the meeting of the large spiral arms is repeated on both sides by the small diamond at the meeting of the small spiral arms. Can this scaling be continued in further iterations? I will leave that question as an exercise for the readers.

There are hints that the precolonial Ghanaian designers were headed toward a quantitative approach in their log spiral designs. Figure 6.5a shows the sculpture of a water buffalo in which they have inscribed uniform discrete steps. I don't think this was motivated by numeric measures, but rather the reverse. By cutting these steps we can clearly gauge the nonlinear nature of the spiral—the way steps of a constant increment show an increasing amount of curve generated—and this practice could have led to quantitative measures. Another move in that direction would generalize such discretized logarithmic

FIGURE 6.5
Logarithmic scaling
in Ghanaian designs

(a) Logarithmic scaling can be demonstrated in a three-dimensional curve by showing how discrete steps of the same vertical increment lead to rapidly increasing area. (b) Overhead view of pyramid-shaped goldweight. (c) Logarithmic plot of goldweight triangle lengths.
(a, *photo from the Metropolitan Museum of Art.* b, *photo courtesy George Arthur, Marshall University.*)



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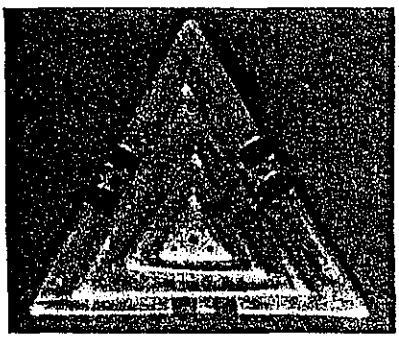

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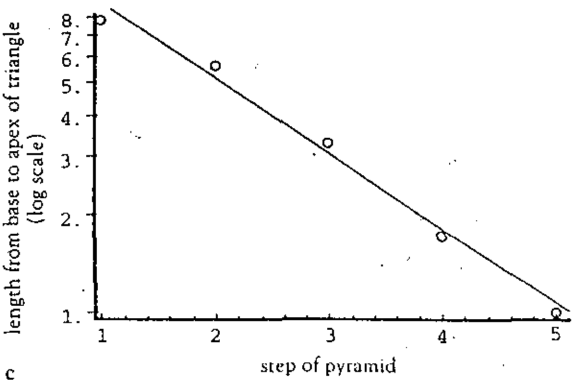

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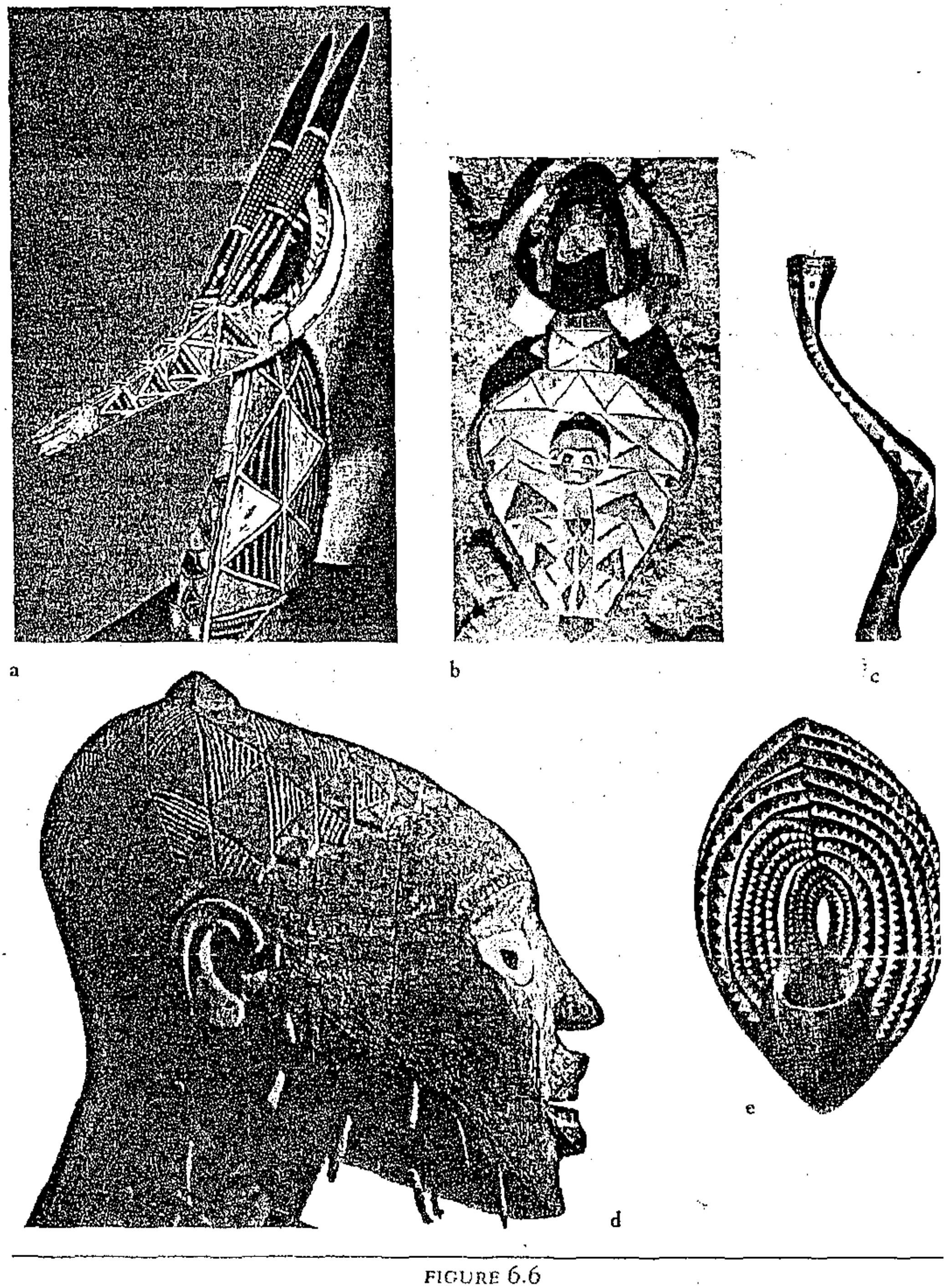


a b c d e

FIGURE 6.6

Adaptive scaling with triangles

(a) Antelope headdress created by the Krumba of Burkina Faso. (b) Mask sold in Accra, Ghana, based on design used in the Sakara-Bounou religious dances. (c) Representation of the water spirit created by the Baga of Guinea. (d) Sculpture from the Congo. (e) A Kikuyu wooden shield. The wood has a nonlinear curve toward the center, and the triangles are scaled to match.
(a, *courtesy Musée de l'Homme*. c, *Metropolitan Museum of Art; photo by Eliot Elisofon*. d, *Detroit Museum of Art*. e, *British Museum; from Zaslavsky 1973*.)

scaling to forms other than spirals, and that did indeed occur, as we can see in figure 6.5b, one of the Akan gold weights. A plot of the length of these triangles (fig. 6.5c) indicates that reasonable accuracy was achieved in this indigenous logarithmic scaling practice.

Adaptive scaling

So far this chapter has focused on questions of intentionality, precision, and mathematical reasoning in African scaling designs. Adaptive scaling has little mathematical sophistication, but it too is an important part of the African fractal design theme. By adapting the scale of a pattern to fit various forms, a number of esthetic and practical effects can be achieved. These examples fall into two categories. In conformal mapping, the pattern simply fits along the contours of a concrete, preexisting structure. In global mapping, the pattern is distorted by compression or expansion—as we saw happen along one dimension in kente cloth—according to a more universal, abstract transformation.

Figure 6.6 shows several examples of conformal mapping on triangles. My search of the facial markings of antelope of the western Sudan did not turn up anything like the scaling pattern of figure 6.6a; these triangles are decorative additions, sized to fit into the shape of the sculpture. Other examples (fig. 6.6b–e) show a series of triangles conforming to the scaling contours of a mask, a sinuous curve, a carved human figure, and a shield. Figure 6.7a shows conformal mapping in the hairstyle Americans call "corn-rowing"; its simulation is shown in figure 6.7b. The Yoruba name for this style is *ipako elede*, which means the nape of the neck of a boar—because the boar's bristles show a similar nonlinear scaling. Figure 6.7c shows a hairstyle that combines conformal mapping with iteration. Adaptive scaling of circles can be seen in the Senegalese textile in figure 6.7d.

A practical application of conformal mapping appears in figure 6.7e, an aerial photo of the Nkong-mondo quarter in the city of Edéa in southern Cameroon, where we see a scaling series of houses. As explained to me by one member of the neighborhood, Mr. Sosso, the houses were constructed along a narrowing ridge, and the scaling was simply conforming to the natural landscape. However, the oldest inhabitant of this Bassa neighborhood, Mr. Bellmbock, told me that the pattern was created because people wanted neighbors of a similar economic class next door, so that the range in house size reflected an economic gradient, from poorest to wealthiest. Mr. Bellmbock lived in the smallest house, and Mr. Sosso in the largest, so I would not discount the possibility that there was an economic scaling as well.

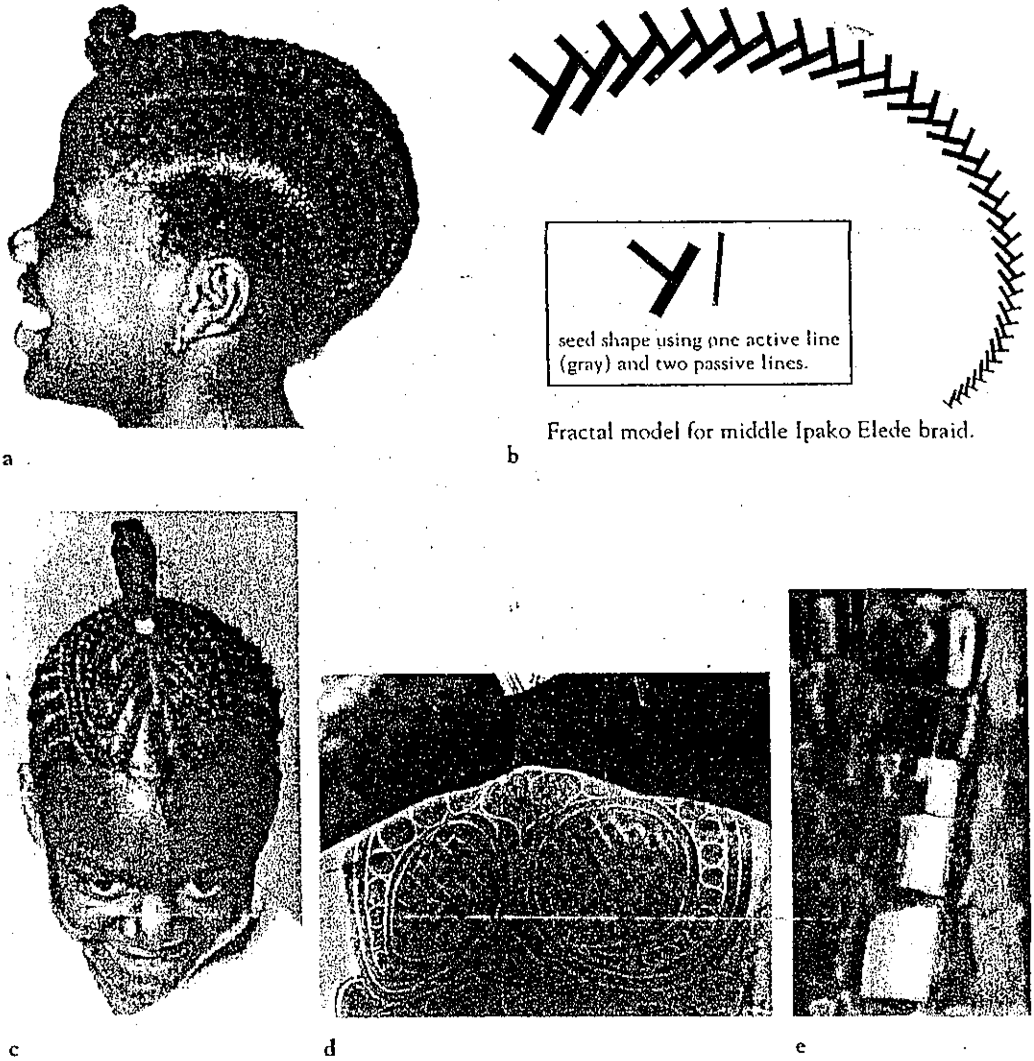


FIGURE 6.7

Adaptive scaling based on various shapes

(a, b) A Yoruba hairstyle, Ipako Elede, adapts the scaling of the braids to the nonlinear contours of the head. (c) This hairstyle begins by braiding a small horseshoe shape in the top center, and then tracing the contour in increasing perimeters—a combination of adaptive scaling and iteration. (d) Fitting circles between intersecting curves creates a scaling series in this textile design from Guinea. (e) An aerial photo of the Nkong-mondo quarter in the city of Edéa in southern Cameroon, where we see a scaling series of houses.

(a, *from Sagay 1983*. c, *from Sagay 1983*. d, *photo courtesy IFAN, Dakar.)*

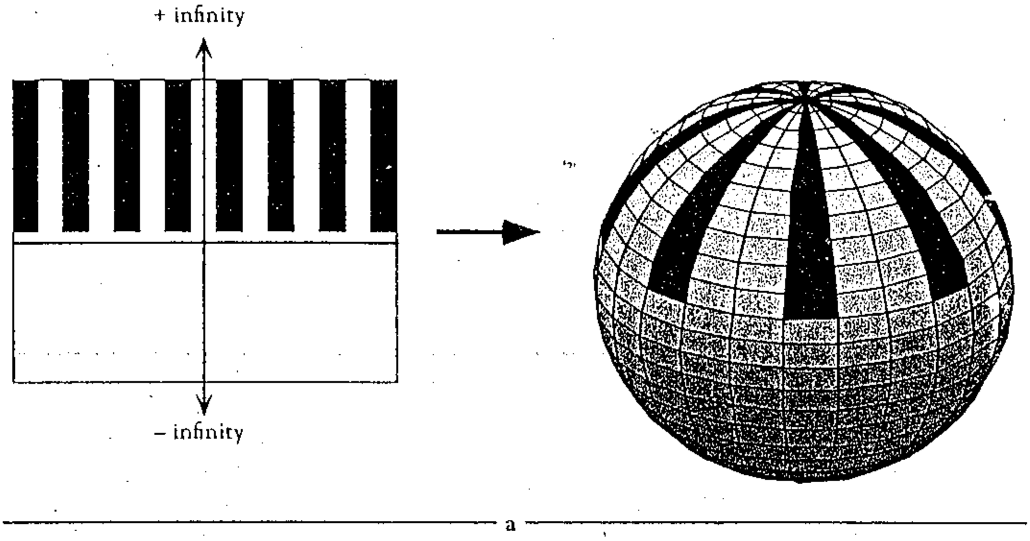
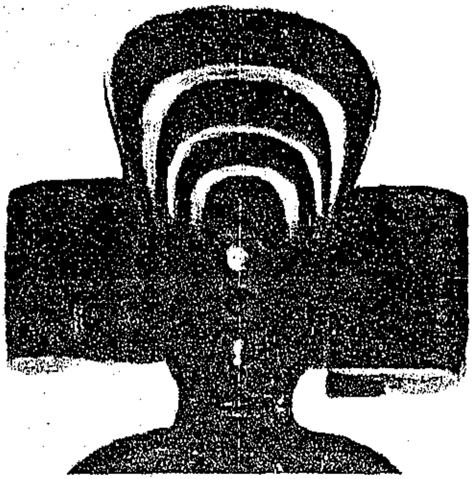


FIGURE 6.8
Mapping from the plane to a spherical surface
(a) Mapping bars of infinite length from the plane to a sphere.
(b) A Yoruba hairstyle, Koroba ("bucket").
(b, *from Sagay 1983.*)

It is possible to misread these examples of conformal mapping as being the product of artisans who are strongly guided by concrete forms rather than abstract thought. But adaptive scaling can also be seen in more abstract examples: global transformations in which space itself is distorted. This is a common operation in Western geometry, the most frequent example being a mapping between the plane and a sphere (fig. 6.8a). Figure 6.8b shows a hairstyle that appears to have a planar design mapped onto a spherical surface. Figure 6.9 provides an even more abstract illustration, the inverse of the previous mapping—now going from spherical to rectangular—and utilizing three dimensions instead of two. In this Chokwe sculpture, the entire human figure is distorted as if its spherical volume had been mapped to a cubic volume; the resulting nonlinear scaling is dramatically illustrated by the discrete steps in the headdress. Art historian William Fagg (1955) made a similar suggestion about other African designs, which he compared to the drawings of natural growth by biologist D'Arcy Thompson: "I believe that the morphology of African sculpture may be usefully studied . . . by reference to mathematics. . . . For example in certain masks



a

b

FIGURE 6.9
Mapping from a spherical volume to a rectangular volume
(a) Bastin (1992, 68) shows that this Chokwe crown, the Cipenya-Mutwe, is made up of linear bands in real life. The nonlinear scaling we see in this sculpture can be explained as the inverse of the transformation we saw in figure 6.8a. Rather than shrinking as we move from the center to the margins, the inverse mapping causes expansion from center to margins. This is not only the inverse of the previous mapping, but also operates on three-dimensional volume rather than surface. Similar transformations are used in neuroscience to model the ways that tactile receptors are mapped from body to brain, since there is a much greater density of sensory neurons at the extremes. (b) The reason for this transformation is to invoke the impression of power and stability (Chanda 1993). The meaning has nothing in particular to do with geometric mapping, other than achieving the desired effect, but it is interesting to note that the transformation is uniformly applied to all external areas, even to the extent of deforming the forehead. (*a, courtesy Jacques Kerchache and Museum of Mankind, London. b, courtesy Museum of the Philadelphia Civic Center.*)

for the Gelde society the natural . . . physiognomy is 'blown up,' so to speak, in a way which could be plotted on a set of flaring exponential coordinates." (1917, 43).

Conclusion

The examples of scaling designs in this chapter vary greatly in purpose, pattern, and method. While it is not difficult to invent explanations based on unconscious social forces—for example, the flexibility in conforming designs to material surfaces as expressions of social flexibility—I do not think that any such explanation can account for this diversity. From optimization engineering, to modeling organic life, to mapping between different spatial structures, African artisans have developed a wide range of tools, techniques, and design practices based on the conscious application of scaling geometry. In the next chapter, we will see that African numeric systems also share many fractal characteristics.

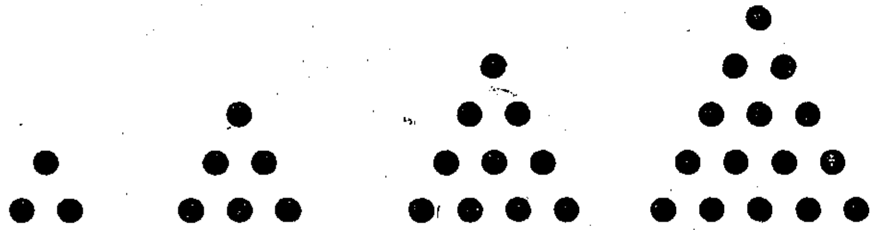
CHAPTER 7

Numeric systems

So far we have focused on geometric structures rather than numeric systems. The only exception was in the windscreen, where the nonlinear scaling was created by counting a specific sequence of diagonal straw rows. But there are many other instances in which the African approach to fractal geometry makes use of numbers.

Nonlinear additive series in Africa

The counting numbers (1,2,3 . . .) can be thought of as a kind of iteration, but only in the most trivial way.[1] It is true that we could produce the counting numbers from a recursive loop, that is, a function in which the output at one stage becomes the input for the next: $X_{n+1} = X_n + 1$. But this is a strictly linear series, increasing by the same amount each time—the numeric equivalent of what we saw in the linear concentric circle and linear spiral. Addition can, however, produce nonlinear series,[2] and there are at least two examples of nonlinear additive series in African cultures. The triangular numbers (1,3,6,10,15 . . .) are used in a game called "tarumbeta" in east Africa (Zaslavsky 1973, 111). Figure 7.1 shows how these numbers are derived from the shape of triangles of increasing size, and how the numeric series can be created by a recursive loop. As in the case of cer-



number of stones:	3	6	10	15
number of iterations:	2	3	4	5

A game called "tarumbeta" in East Africa makes use of the triangular numbers, starting with 3 (3, 6, 10, 15 . . .). In this game, one player calls out a count as he removes stones consecutively, left to right and bottom to top, while the other player, with his back turned, must signal whenever the first stone in a row has been removed.

The stones in each triangular array can be built up in an iterative fashion, that is, the next triangle can be created by adding another layer to any side of the previous triangle. The number to be added in each additional layer is simply the number of iterations. For each iteration i, and total number of stones N, we have:

$N_{i+1} = N_i + i$ (starting with $N_0 = 0$)

$1 = 0 + 1$ (a trivial array, not used in the game)

$3 = 1 + 2$

$6 = 3 + 3$

$10 = 6 + 4$

$15 = 10 + 5$

.

.

.

In other words, the next number will be given by the last number plus the iteration count:

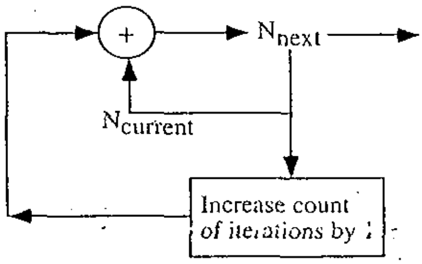


FIGURE 7.1
The triangular numbers in an East African game

tain formal age-grade initiation practices (see chapters 5 and 8), the simple versions are used by smaller children, and the higher iterations are picked up with increasing age. While there is no indication of a formal relationship in this instance, there is still an underlying parallel between the iterative concept of aging common to many African cultures—each individual passing through multiple turns of the "life cycle"—and the iterative nature of the triangular number series.

Another nonlinear additive series was found in archaeological evidence from North Africa. Badawy (1965) noted what appears to be use of the Fibonacci series in the layout of the temples of ancient Egypt. Using a slightly different approach, The Fibonacci series (1, 1, 2, 3, 5, 8, 13 . . .) was found by Badawy (1965) in his study of the layout of the temples of Egypt. His analysis was quite complex, but it is not difficult to create a simple visualization. Here we see the series in the successive chambers of the temple of Karnak.

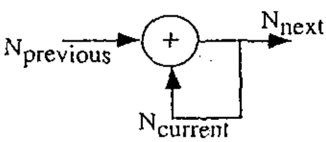
The Fibonacci series is produced by adding the previous number to the current number to get the next number, starting with 1 + 1 = 2. For each iteration i, the number N in the series *is given by:*

$$N_{i+1} = N_i + N_{i-1}$$

that is,

$$N_{next} = N_{current} + N_{previous}$$

$$\begin{aligned} 1 + 1 &= 2 \\ 1 + 2 &= 3 \\ 2 + 3 &= 5 \\ 3 + 5 &= 8 \\ 5 + 8 &= 13 \end{aligned}$$



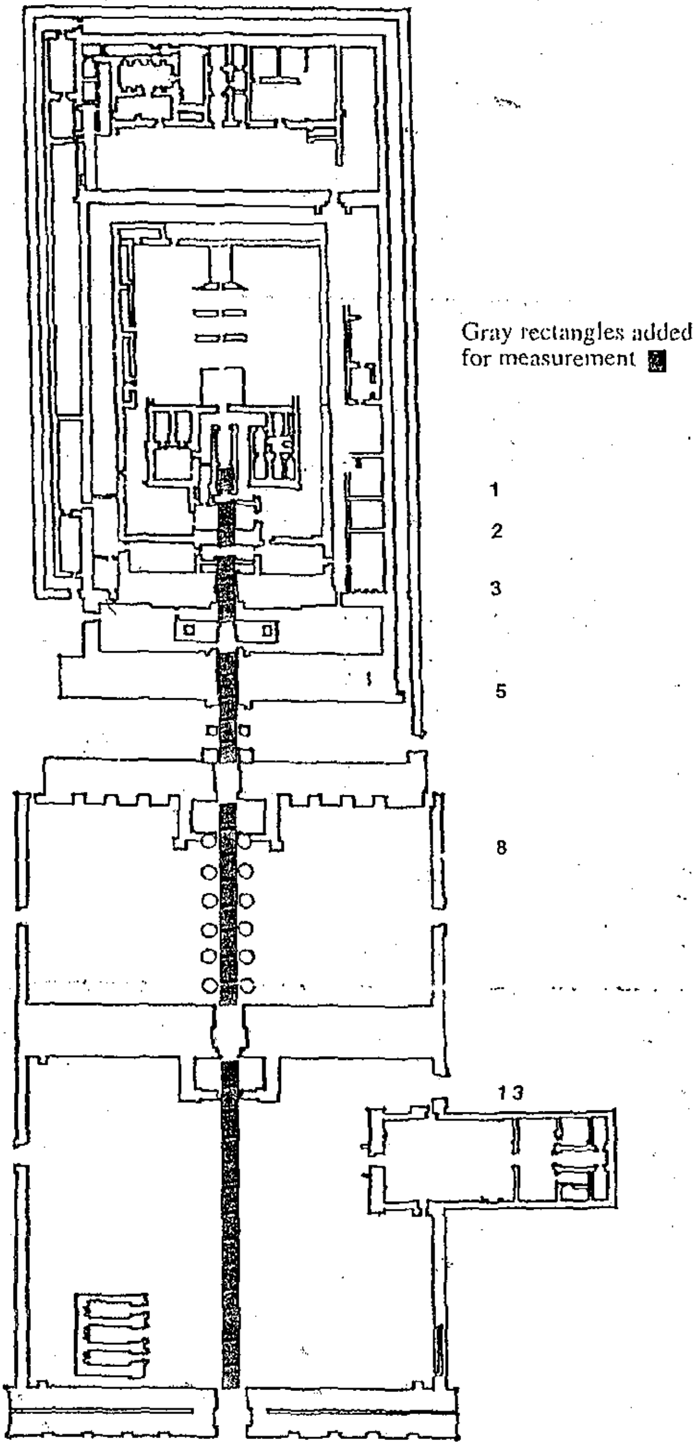


FIGURE 7.2

The Fibonacci series in ancient Egypt

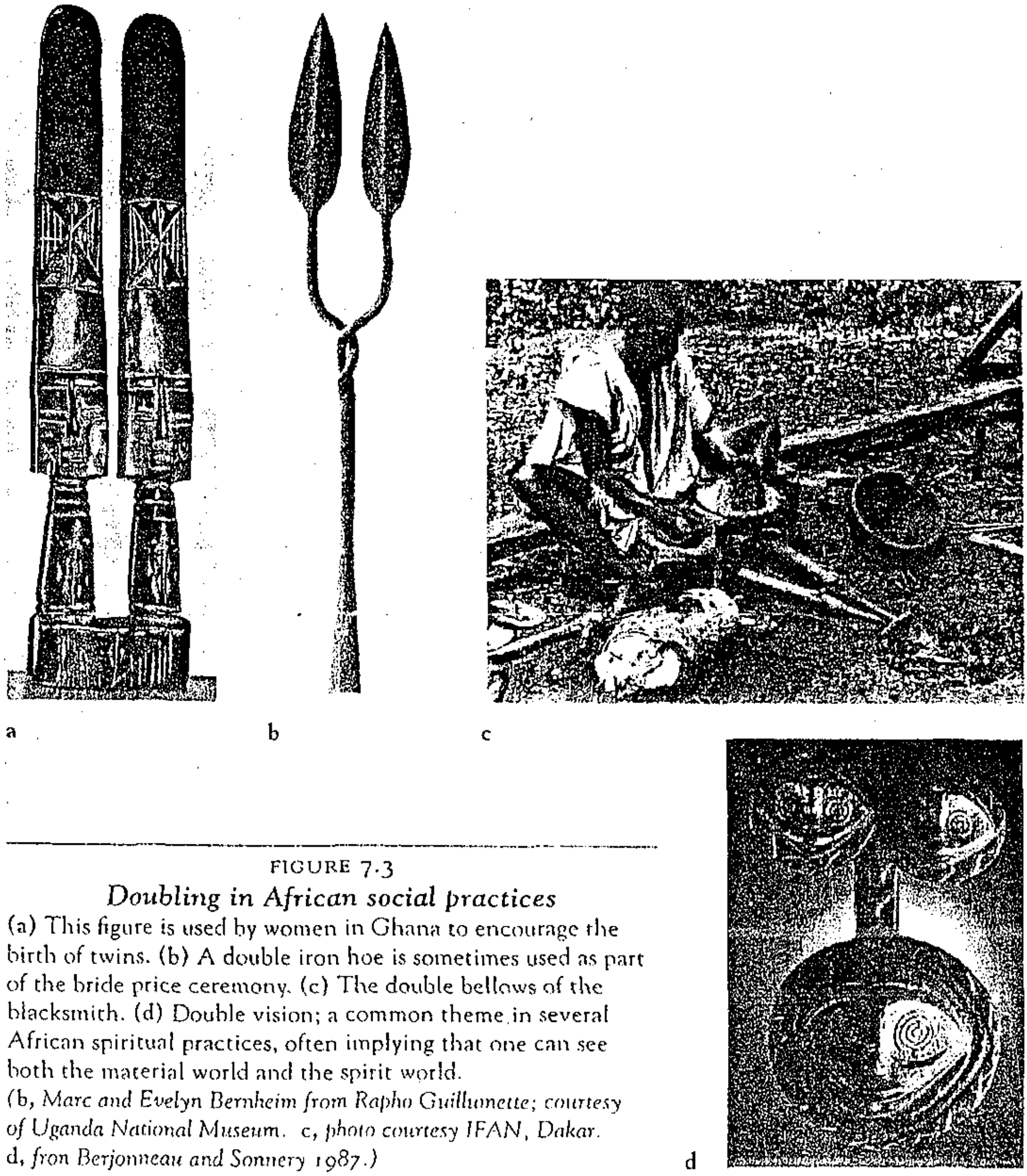
I found a visually distinct example of this series in the successive chambers of the temple of Karnak, as shown in figure 7.2a. Figure 7.2b shows how these numbers can be generated using a recursive loop. This formal scaling plan may have been derived from the nonnumeric versions of scaling architecture we see throughout Africa. An ancient set of balance weights, apparently used in Egypt, Syria, and Palestine circa 1200 B.C.E., also appear to employ a Fibonacci sequence (Petruso 1985). This is a particularly interesting use, since one of the striking mathematical properties of the series is that one can create any positive integer through addition of selected members—a property that makes it ideal for application to balance measurements (Hoggatt 1969, 76). There is no evidence that ancient Greek mathematicians knew of the Fibonacci series. There was use of the Fibonacci series in Minoan design, but Preziosi (1968) cites evidence indicating that it could have been brought from Egypt by Minoan architectural workers employed at Kahun.

Doubling series in Africa

Some accounts report that Africans use a "primitive" number system in which they count by multiples of two. It is true that many cases of African arithmetic are based on multiples of two, but as we will see, base-2 systems are not crude artifacts from a forgotten past. They have surprising mathematical significance, not only in relation to African fractals, but to the Western history of mathematics and computing as well.

The presence of doubling as a cultural theme occurs in many different African societies and in many different social domains, connecting the sacredness of twins, spirit doubles, and double vision with material objects, such as the blacksmith's twin bellows and the double iron hoe given in bridewealth (fig. 7.3). Figure 7.4a shows the Ishango bone, which is around 8,000 years old and appears to show a doubling sequence. Doubling is fundamental to many of the counting systems of Africa in modern times as well. It is common, for example, to have the word for an even number 2N mean "N plus N" (e.g., the number 8 in the Shambaa language of Tanzania is "ne na ne," literally "four and four"). A similar doubling takes place for the precisely articulated system of number hand gestures (fig. 7.4b), for example, "four" represented by two groups of two fingers, and "eight" by two groups of four. Petitto (1982) found that doubling was used in multiplication and division techniques in West Africa (fig. 7.4c). Gillings (1972) details the persistent use of powers of two in ancient Egyptian mathematics as well, and Zaslavsky (1973) shows archaeological evidence suggesting that ancient Egypt's use of base-2 calculations derived from the use of base-2 in sub-Saharan Africa.

Doubling practices were also used by African descendants in the Americas. Benjamin Banneker, for example, made unusual use of doubling in his calculations, which may have derived from the teachings of his African father and grandfather (Eglash 1997c). Gates (1988) examined the cultural significance of doubling in West African religions such as vodun and its transfer to "voodoo" in the Americas. In the religion of Shango, for example, the vodun god of thunder and lightning is represented by a double-bladed axe (fig. 7.5a), used by Shango devotees in the new world as well (R. Thompson 1983). Figure 7.5b shows



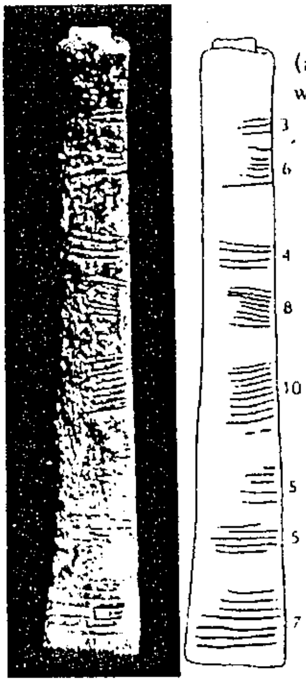
a b c d

FIGURE 7.3

Doubling in African social practices

(a) This figure is used by women in Ghana to encourage the birth of twins. (b) A double iron hoe is sometimes used as part of the bride price ceremony. (c) The double bellows of the blacksmith. (d) Double vision; a common theme in several African spiritual practices, often implying that one can see both the material world and the spirit world.

(b, Marc and Evelyn Bernheim from Rapho Guillumette; courtesy of Uganda National Museum. c, photo courtesy IFAN, Dakar. d, fron Berjonneau and Sonnery 1987.)



a.

(a) The Ishango bone, estimated to be over 8,000 years old, shows what appears to be use of doubling: 3 + 3 = 6, 4 + 4 = 8, 10 = 5 + 5.

b

(b) Even numbers are typically represented by doubling in the precisely articulated system of African hand gestures.

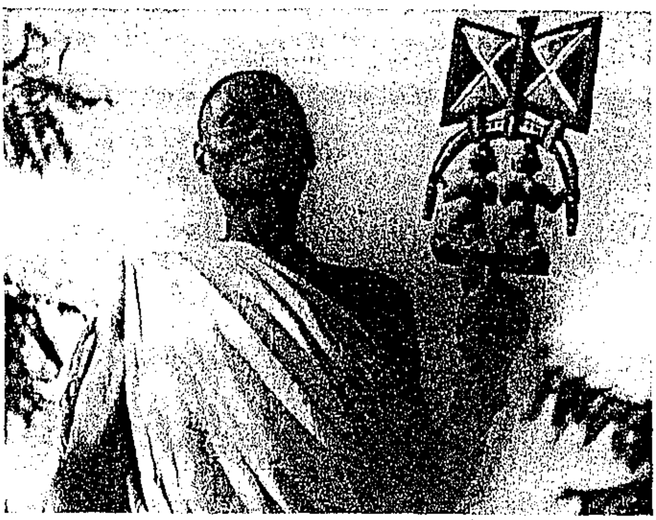
(c) Doubling was traditionally used by tailors in West Africa when doing large mental multiplications; it is essentially based on what we would call factoring.

For example, 3 × 273 ("3 taken 273 times") would be calculated by successively doubling 3 (6, 12, 24 . . .) while keeping track of the counterpart in powers of two (2, 4, 8 . . .). When the next power of two would overshoot 273, he then has to memorize the number reached so far through doublings of 3 (768), while subtracting the power of two that was reached (273 – 256 = 17). Then he starts again, doubling 3, and keeping track of the powers of two. When the next power of two would overshoot 17, he again memorizes the number reached through doublings of 3 (48) and subtracts the power of two (17 – 16 = 1). Since one is left over, he just needs to add an additional 3. The answer is then given by the sum of the underlined terms: 768 + 48 + 3 = 819.

Despite the complexity of the method, the tailors were quite fast at performing these silent mental operations.

FIGURE 7.4
Doubling in African arithmetic
(*a and b, from From Zaslavsky 1973.*)

the use of a doubling sequence in the structure of a Shango temple and in religious ceremonies (ritual choreography aligning two priests, four children, eight legs). A curator at the Musée Ethnographique in Porto Novo, Benin, who specialized in Shango explained to me that these doubling structures were used because the god of lightning required a portrait of the forked structure of a lightning bolt. The model is particularly interesting in that the lengths of each iteration are shortened, so that one could have infinite doublings in a finite



(a) Shango, the god of lightning, is part of the vodun religion of Benin and was one of the important components in the creation of the voodoo religion in the New World. Here we see the double-bladed "thunder axe," with another double blade within each side.

a

b

(b) Shango temple and initiation. Here we see the doubling sequence carried out further, using the bilateral symmetry of the human body itself in the last iteration. This is used to symbolize the bifurcating pattern of the lightning bolt.

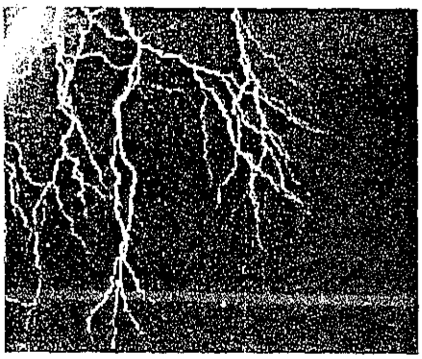


FIGURE 7.5

Doubling in the religion of Shango

(a, courtesy IFAN, Dakar. b: both center photos, courtesy IFAN, Dakar; lower right, courtesy Dave Crowley, www.stormguy.com.)

space—a true fractal. The self-similar structure of lightning has been a favorite example for fractal geometry texts (see Mandelbrot 1977). The doubling sequence used to model the fractal structure of lightning in Shango would not give an accurate value for the empirical fractal dimension—real lightning tends to branch much more than doubling allows for—but it's enough to know that the vodun representation offers a testable quantitative model.

The most mathematically significant aspect of doubling in African religion occurs in the divination ("fortunetelling") techniques of vodun and its religious relatives (Eglash 1997b). The famous Ifa divination system (fig. 7.6) is based on tossing pairs of flat shells or seeds split in two. Each lands open-side or closed-side (like "heads or tails" in a coin toss). They are connected by a doubled chain to make four pairs. Each group of four pairs gives one of the 16 divination symbols, which tell the future of the diviner's client. The Ifa system is what a mathematician would call "stochastic," that is, it operates by pure chance. But a closely related divination system, Cedena, has a nonstochastic element—it is closer to what mathematicians call "deterministic chaos."

My introduction to *cedena*, or sand divination, took place in Dakar, Senegal, where the local Islamic culture credits the Bamana (also known as Bambara) with a potent pagan mysticism. Almost all diviners had some kind of physical deformity—"the price paid for their power."[3] One diviner seemed quite willing to teach me about the system, suggesting that it "would be just like school." The first few sessions went smoothly, with the diviner showing me a symbolic code in which each symbol, represented by a set of four vertical dashed lines drawn in the sand, stood for some archetypical concept (travel, desire, health, etc.) with which he assembled narratives about the future. But when I finally asked how he derived the symbols—in particular, the meaning of some of the patterns drawn prior to the symbol writing—they all laughed at me and shook their heads. "That's the secret!" My offers of increasingly high payments were met with disinterest. Finally, I tried to explain the social significance of cross-cultural mathematics. I happened to have a copy of Linda Garcia's *Fractal Explorer* with me and began by showing a graph of the Cantor set, explaining its recursive construction. The head diviner, with an expression of excitement, suddenly stopped me, snapped the book shut, and said "show him what he wants!"

As it turns out, the recursive construction of the Cantor set was just the right thing to show, because the Bamana divination is also based on recursion (fig. 7.7). The divination begins with four horizontal dashed lines, drawn rapidly, so that there is some random variation in the number of dashes in each. The dashes are then connected in pairs, such that each of the four lines is left with either one single dash (in the case of an odd number) or no dashes (all pairs, the case

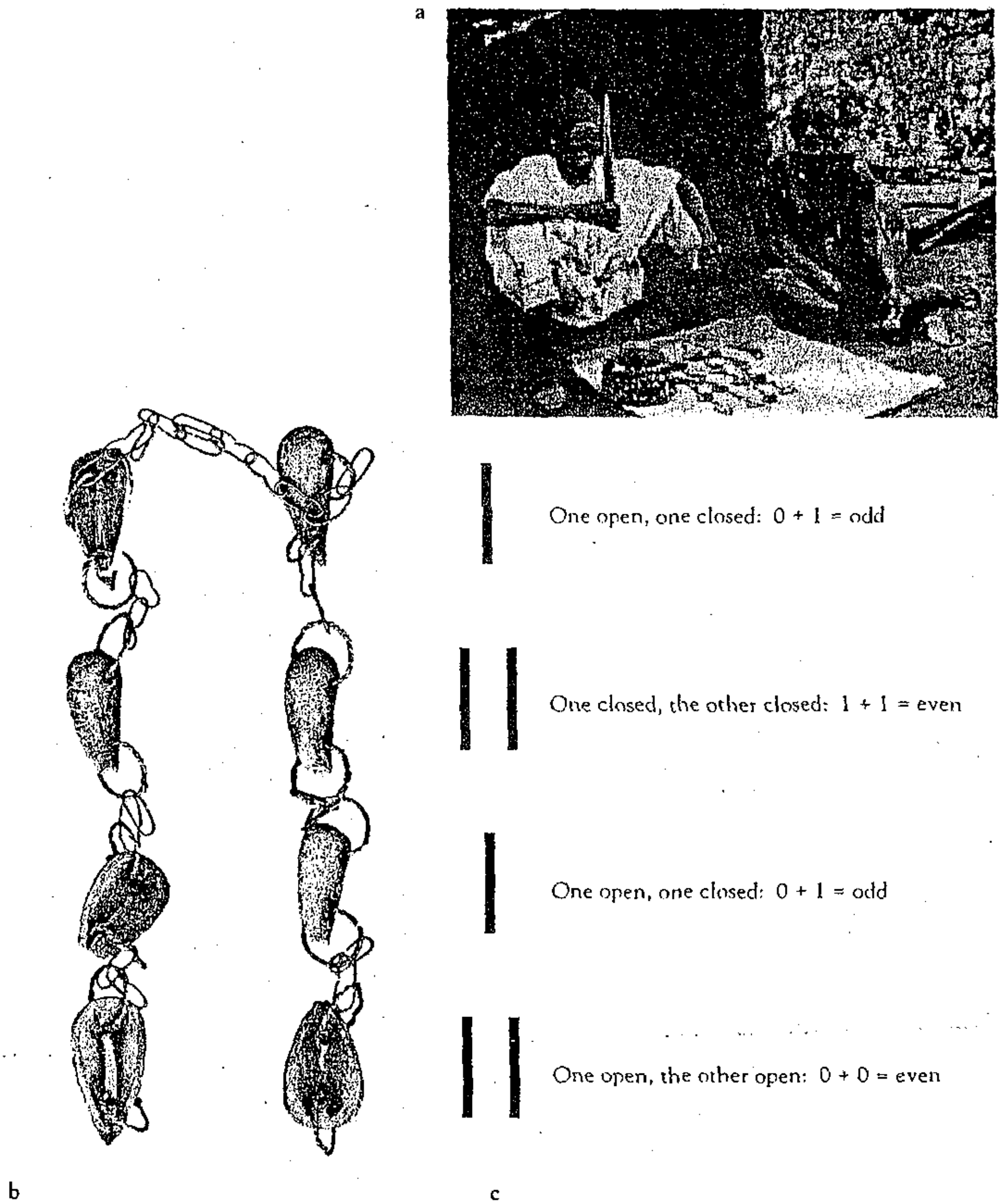


FIGURE 7.6

Binary codes in divination

(a) This Nigerian priest is telling the future by Ifa divination, in which pairs of flat shells or seeds split in two are tossed with each landing open-side or closed-side. They are connected by a doubled chain to make four pairs, giving a total of 16 divination symbols. In this version of Ifa (used in the Abigba region of Nigeria) they use two doubled chains and consider the cast more accurate if there is a correlation between the two sets. (b) Here we see a chain using split seeds. Each half lands either "closed" (meaning we see the rounded outside) or "open" (meaning we see the interior). By using open to represent 0 (double lines), and closed to represent 1 (single line), we can see how the divination symbol is obtained. (c) The divination chain is interpreted as pairs summing to odd (one stroke) or even (two strokes).

(a, photo by E. M. McClelland, courtesy Royal Anthropological Institute.)

of an even number). The narrative symbol is then constructed as a column of four vertical marks, with double vertical lines representing an even number of dashes and single lines representing an odd number. At this point the system is similar to the famous Ifa divination: there are two possible marks in four positions, so 16 possible symbols. Unlike Ifa, however, the random symbol production is repeated four times rather than two. The difference is quite significant. Each of the Ifa symbol pairs are interpreted as one of 256 possible Odu, or verses. The Ifa diviner must memorize the Odu; hence, four symbols would be too cumbersome (65,536 possible verses). But the Bamana divination does not require any verse memorization; as we will see, its use of recursion allows for verse self-assembly.

As in the additive sequences we examined, the divination code is generated by an iterative loop in which the output of the operation is used as the input for the next stage. In this case, the operation is addition modulo 2 ("mod 2" for short), which simply gives the remainder after division by two. This is the same even/odd distinction used in the parity bit operation that checks for errors on contemporary computer systems. There is nothing particularly complex about mod 2; in fact, I was quite disappointed at first because its reapplication destroyed the potential for a binary placeholder representation in the Bamana divination. Rather than interpret each position in the column as having some meaning (*as would our binary number 1011, which means one 1, one 2, zero 4s,* and one 8), the diviners reapplied mod 2 to each row of the first two symbols and to each row of the last two. The results were then assembled into two new symbols, and mod 2 was applied again to generate a third symbol. Another four symbols were created by reading the rows of the original four as columns, and mod 2 was again recursively applied to generate another three symbols.

The use of an iterative loop, passing outputs of an operation back as inputs for the next stage, was a shock to me; I was at least as taken aback by the sand symbols as the diviners had been by the Cantor set. *It would be naive to* claim that this was somehow a leap outside of our cultural barriers and power differences—in fact, that's just the sort of pretension that the last two decades of reflexive anthropology has been dedicated against—but it would also be ethnocentric to rule out those aspects that would be attributed to mathematical collaboration elsewhere in the world: the mutual delight in two recursion fanatics discovering each other. And the appearance of the symbols laid out in two groups of seven—the Rosicrucian's mystic number—added some numerological icing on the cake.

The following day I found that the presentation had not been complete: an additional two symbols were left out. These were also generated by mod 2 recursion using the two bottom symbols to create a fifteenth, and using that last symbol with the first symbol to create a sixteenth (bringing the total depth of recursion to five iterations). The fifteenth symbol is called "this world," and the sixteenth is "the next world," so there was good reason to separate them from the others. The final part of the system—creating a narrative from the symbols—was still unclear, but I was assured that it could be learned if I carefully followed their instructions. I was to give seven coins to seven lepers, place a kola nut on

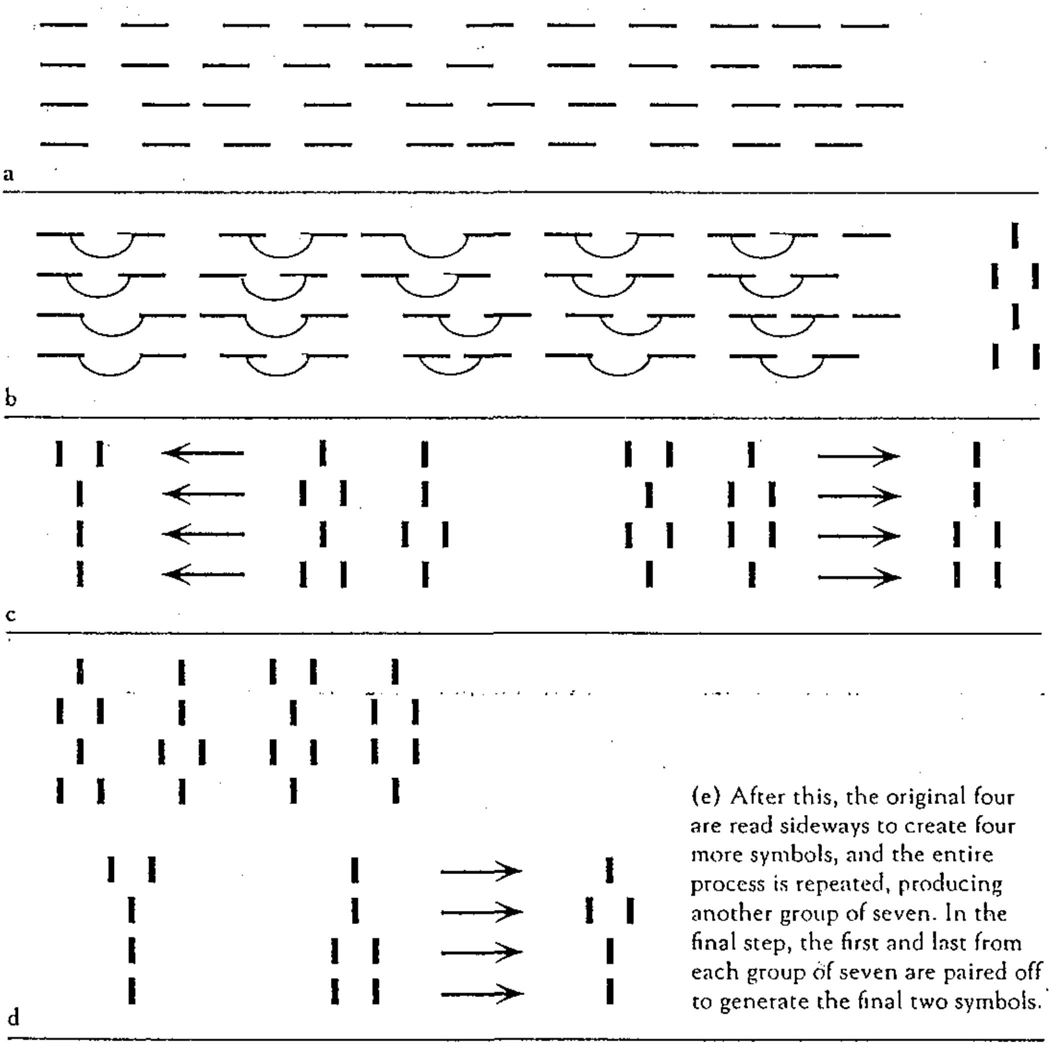


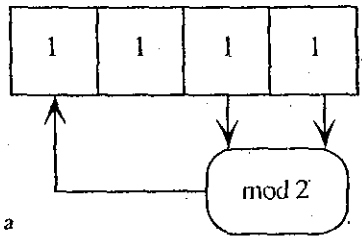
FIGURE 7.7
Bamana sand divination
(a) Four sets of random dashes are drawn. (b) Each of the dashes is paired, and the odd/even results are recorded. (c) The process is repeated four times, resulting in four symbols. Each row of the first two symbols and the last two symbols are paired off to generate two new symbols. (d) The two newly generated symbols, now placed below the original four, are again paired off to generate a seventh symbol.

a pile of sand next to my bed at night, and in the morning bring a white cock, which would have to be sacrificed to compensate for the harmful energy released in the telling of the secret. I followed all the instructions, and the next morning bought a large white cock at the market. They held the chicken over the divination sand, and I was told to eat the bitter kola nut as they marked divination symbols on its feet with an ink pen. A little sand was thrown in its mouth, and then I was told to hold it down as prayers were chanted. There was no action on the part of the diviner; the chicken simply died in my hands.

While still a bit shaken by the chicken's demise (as well as experiencing a respectable buzz from the kola nut), I was told the remaining mystery. Each symbol has a "house" in which it belongs—for example, the position of the sixteenth symbol is "the next world"—but in any given divination most symbols will not be located in their own house. Thus the sixteenth symbol generated might be "desire," so we would have desire in the house of the next world, and so on. Obviously this still leaves room for creative narration on the part of the diviner, but the beauty of the system is that no verses need to be memorized or books consulted; *the system creates its own complex variety.*

The most elegant part of the method is that it requires only four random drawings; after that the entire symbolic array is quickly self-generated. Self-generated variety is important in modern computing, where it is called "pseudorandom number generation" (fig. 7.8). These algorithms take little memory, but can generate very long lists of what appear to be random numbers, although the list will eventually start over again (this length is called the "period" of the algorithm). Although the Bamana only require an additional 12 symbols to be generated in this fashion, a maximum-length pseudorandom number generator using their initial four symbols will produce 65,535 symbols before it begins to repeat.

A similar system for self-generated variety was developed as a model for the "chaos" of nonlinear dynamics by Marston Morse (1892–1977). Before the 1970s, mathematicians had assumed that, besides a few esoteric exceptions (the algorithms for producing irrational numbers such as $\sqrt{2}$), the output of an equation would eventually start repeating. That assumption was partly based on European cultural ideas about free will: complex behavior could not be the result of predetermined systems (see Porter 1986). It was not until the 1960s–'70s that mathematicians realized that even simple, common equations describing things like population growth or fluid flow could result in what they called "deterministic chaos"—an output that never repeats, giving the appearance of random numbers from a nonrandom (deterministic) equation. Morse developed the minimal case for such behavior.



a

1111
0111
0011
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0100
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1100
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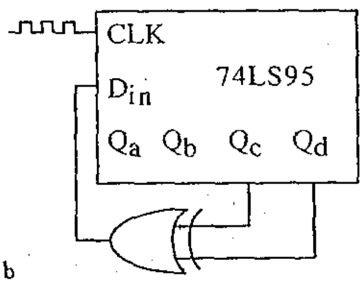


b

FIGURE 7.8

Pseudorandom number generation from shift register circuits

(a) If we think of the two-strokes as zero and single stroke as one, the Bamana divination system is almost identical to the process of pseudorandom number generation used by digital circuits called "shift registers." Here the circuit takes mod 2 of the last two bits in the register and places the result in the first position. The other bits are shifted to the right, with the last discarded.

This four-bit shift register will only produce 15 binary words before the cycle starts over, but the period of the cycle increases with more bits ($2^n - 1$). For the entire 16 bits (four symbols of four bits each) that begin the Bamana divination, 65,535 binary words can be produced before repeating the cycle.

(b) Electrical circuit representation of a four-bit shift register combined with exclusive-or to perform the mod 2 operation. While schoolteachers are making increasing use of African culture in the mathematics classroom, few have explored the potential applications to technology education.

The construction of the Morse sequence begins by counting from zero in binary notation: 000, 001, 010, 011 . . . . It then takes the sum of the digits in each number—0 + 0 + 0 = 0, 0 + 0 + 1 = 1, etc.—and finally mod 2 of each sum. The result is a sequence with many recursive properties,[4] but of endless variety. Morse did the same "misreading" of the binary number as did the Bamana—although he did not have an anthropologist scowling at him for ignoring place value—and he did it for the same reason: combined with the mod 2 operation, it maximizes variety.

In my reading of divination literature I eventually came across the duplicate of the Bamana technique 5,000 miles to the east in Malagasy *sikidy* (Sussman and Sussman 1977), which inspired a study of the history of its diffusion. The strong similarity of both symbolic technique and semantic categories to what Europeans termed "geomancy" was first noted by Flacourt (1661), but it was not until Trautmann (1939) that a serious claim was made for a common source for this Arabic, European, West African, and East African divination technique. The commonality was confirmed in a detailed formal analysis by Jaulin (1966). But where did it originate?

Skinner (1980) provides a well-documented history of the diffusion evidence, from the first specific written record—a ninth-century Jewish commentary by Aran ben Joseph—to its modern use in Aleister Crowley's *Liber 777*. The oldest Arabic documents (those of az-Zanti in the thirteenth century) claim the origin of geomancy (*ilm al-raml*, "the science of sand") through the Egyptian god Idris (Hermes Trismegistus); while we need not take that as anything more than a claim to antiquity, a Nilotic influence is not unreasonable. Budge (1961) attempts to connect the use of sand in ancient Egyptian rituals to African geomancy, but it is hard to see this as unique. Mathematically, however, geomancy is strikingly out of place in non-African systems.

Like other linguistic codes, number bases tend to have an extremely long historical persistence. Even under Platonic rationalism, the ancient Greeks held 10 to be the most sacred of all numbers; the Kabbalah's Ayin Sof emanates by 10 Sefirot, and the Christian West counts on its "Hindu-Arabic" decimal notation. In Africa, on the other hand, base-2 calculation was ubiquitous, even for multiplication and division. And it is here that we find the cultural connotations of doubling that ground the divination practice in its religious significance.

The implications of this trajectory—from sub-Saharan Africa to North Africa to Europe—are quite significant for the history of mathematics. Following the introduction of geomancy to Europe by Hugo of Santalla in twelfth-century Spain, it was taken up with great interest by the pre-science mystics of those times—alchemists, hermeticists, and Rosicrucians (fig. 7.9). But these European geomancers—Raymond Lull, Robert Fludd, de Peruchio, Henry de Pisis, and others—persistently replaced the deterministic aspects of the system with chance. By mounting the 16 figures on a wheel and spinning it, they maintained their society's exclusion of any connections between determinism and unpredictability. The Africans, on the other hand, seem to have emphasized such connections. In chapter 10 we will explore one source of this difference: the African concept of a "trickster" god, one who is both deterministic and unpredictable.

On a video recording I made of the Bamana divination, I noticed that the practitioners had used a shortcut method in some demonstrations (this may have been a parting gift, as the video was shot on my last day). As they first taught me, when they count off the pairs of random dashes, they link them by drawing short curves. The shortcut method then links those curves with larger curves, and those below with even larger curves. This upside-down Cantor set shows that they are not simply applying mod 2 again and again in a mindless fashion. The self-similar physical structure of the shortcut method vividly illustrates a recursive process, and as a nontraditional invention (there is no record of its use elsewhere) it shows active mathematical practice. Other African divination practices

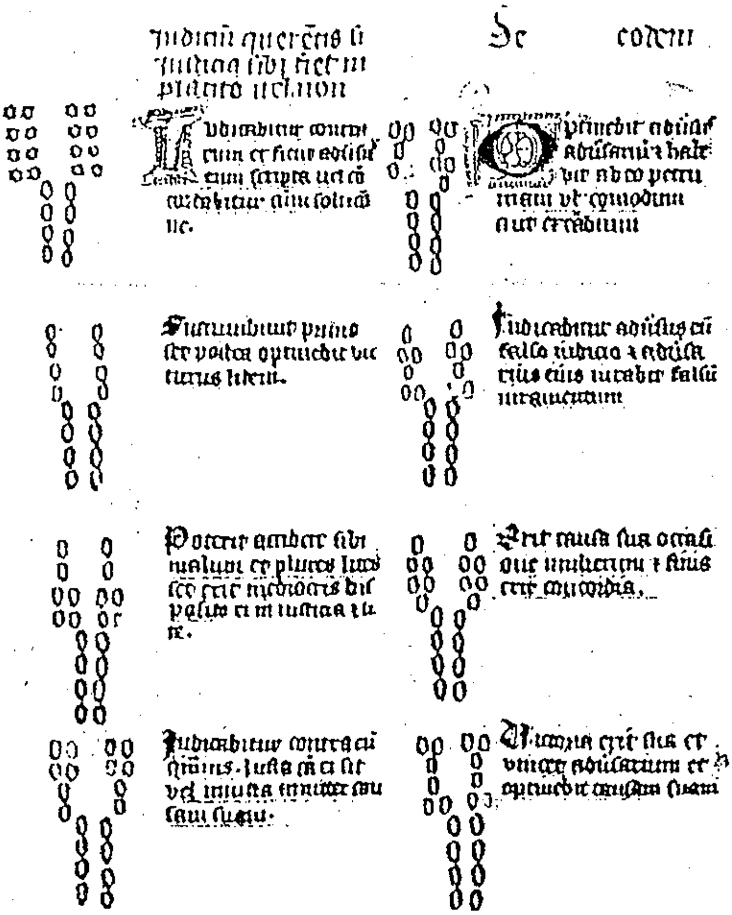


FIGURE 7.9
Geomancy
African divination was taken up under the name "geomancy" by European mystics. This chart was drawn for King Richard II in 1391.
(From Skinner 1980.)

can be linked to recursion as well; for example Devisch (1991) describes the Yaka diviners' "self-generative" initiation and uterine symbolism.

Before leaving divination, there is one more important connection to mathematical history. While Raymond Lull, like other European alchemists, created wheels with sixteen divination figures, his primary interest was in the combinatorial possibilities offered by base-2 divisions. Lull's work was closely examined by German mathematician Gottfried Leibniz, whose *Dissertatio de arte combinatoria*, published in 1666 when he was twenty, acknowledges Lull's work as a precursor. Further exploration led Leibniz to introduce a base-2 counting system, creating what we now call the binary code. While there were many other influences in the lives of Lull and Leibniz, it is not far-fetched to see a historical path for base-2 calculation that begins with African divination, runs through the geomancy of European alchemists, and is finally translated into binary calculation, where it is now applied in every digital circuit from alarm clocks to supercomputers.

In a 1995 interview in *Wired* magazine, techno-pop musician Brian Eno claimed that the problem with computers is that "they don't have enough African in them." Eno was, no doubt, trying to be complimentary, saying that there is some intuitive quality that is a valuable attribute of African culture. But in doing so he obscured the cultural origins of digital computing and did an injustice to the very concept he was trying to convey.

Discrete self-organization in Owari

Figure 7.10a shows a board game that is played throughout Africa in many different versions variously termed *ayo*, *bao*, *giuthi*, *lela*, *mancala*, *omweso*, *owari*, *tei*, and *songo* (among many other names). Boards that were cut into stones, some of extreme antiquity, have been found from Zimbabwe to Ethiopia (see Zaslavsky 1973, fig. 11-6). The game is played by scooping pebble or seed counters from one cup, and placing one of those counters into each cup, starting with the cup to the right of the scoop. The goal is to have the last counter land in a cup that has only one or two counters already in it, which allows the player to capture these counters. In the Ghanaian game of owari, players are known for utilizing a series of moves they call a "marching group." They note that if the number of counters in a series of cups each decreases by one (e.g., 4-3-2-1), the entire pattern can be replicated with a right-shift by scooping from the largest cup, and that if the pattern is left uninterrupted it can propagate in this way as far as needed for a winning move (fig. 7.10b). As simple as it seems, this concept of a self-replicating pattern is at the heart of some sophisticated mathematical concepts.

John von Neumann, who played a pivotal role in the development of the modern digital computer, was also a founder of the mathematical theory of self-organizing systems. Initially, von Neumann's theory was to be based on self-reproducing physical robots. Why work on a theory of self-reproducing machines? I believe the answer can be found in von Neumann's social outlook. Heims's (1984) biography emphasizes how the disorder of von Neumann's precarious youth as a Hungarian Jew was reflected in his adult efforts to impose a strict mathematical order on various aspects of the world. In von Neumann's application of game theory to social science, for example, Heims writes that his "Hobbesian" assumptions were "conditioned by the harsh political realities of

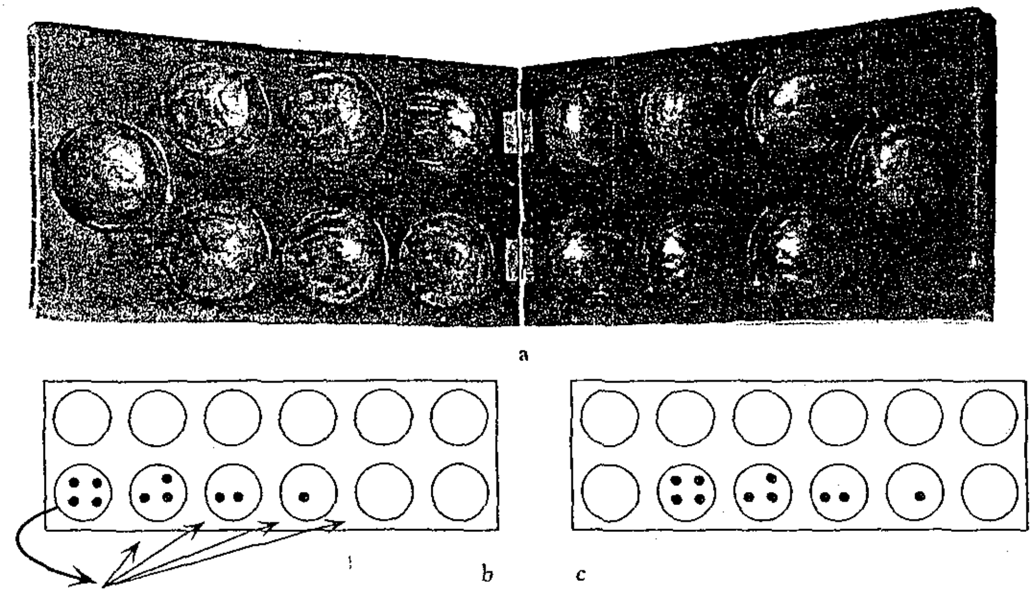


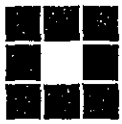
FIGURE 7.10
Owari

(a) The owari board has 12 cups, plus one cup on each side for captured counters. This board is hinged in the center, with a beautifully carved cover (see fig. 7.14). (b) Scoop from the first cup, and plant one counter in each succeeding cup. (c) The Marching Group is replicated with a right-shift. Repeated application will allow it to propagate around the board.

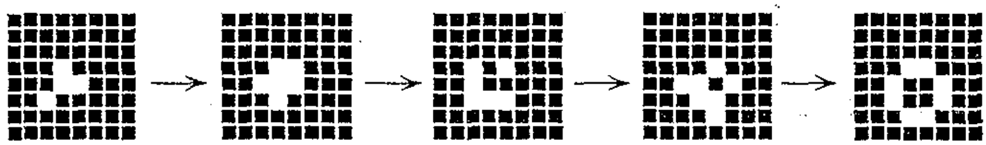
his Hungarian existence." His enthusiasm for the use of nuclear weapons against the Soviet Union is also attributed to this experience.

During the Hixon Symposium (von Neumann 1951) he was asked if computing machines could be built such that they could repair themselves if "damaged in air raids," and he replied that "there is no doubt that one can design machines which, under suitable circumstances, will repair themselves." His work on nuclear radiation tolerance for the Atomic Energy Commission in 1954–1955 included biological effects as well as machine operation. Putting these facts together, I cannot escape the creepy conclusion that von Neumann's interest in self-reproducing automata originated in fantasies about having a more perfect mechanical progeny survive the nuclear purging of organic life on this planet.

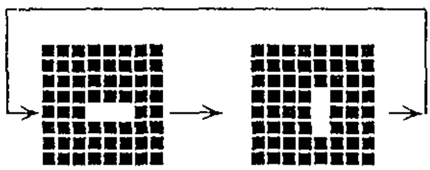
Models for physical robots turned out to be too complex, and at the suggestion of his colleague Stanislaw Ulam, von Neumann settled for a graphic abstraction: "cellular automata," as they came to be called. In this model (fig. 7.11a), each square in a grid is said to be either alive or dead (that is, in one of two possible states). The iterative rules for changing the state of any one square are based



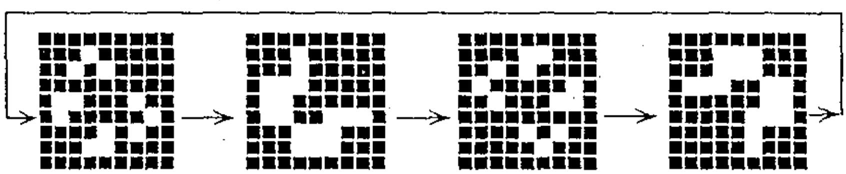
In the cellular automaton called "the game of life," each cell in the grid is in one of two states: live or dead. Here we see a live cell in the center, surrounded by dead cells in its eight nearest neighbors. The state of each cell in the next iteration is determined by a set of rules. In "classic" life (the rules first proposed by John Horton Conway), a dead cell becomes a live cell if it has three live nearest neighbors, and a cell dies unless it has two or three live neighbors.



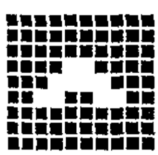
This initial condition produces a fixed pattern after four iterations. The patterns occurring before it settles down to stability are called the "transient."



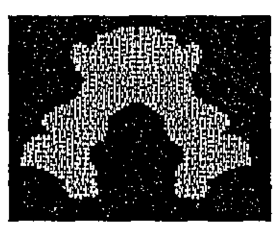
This stable pattern flips back and forth between these two states. This is called a "period-2" pattern.



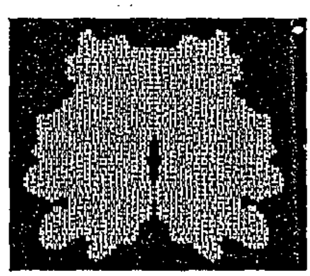
A period-4 pattern. Periods of any length can be produced, as we saw in the previous examples of pseudorandom number generation. Deterministic chaos, in which the pattern never repeats (i.e., a period-infinity pattern, like the Morse sequence), is also possible.



Iteration 49



Iteration 133



Iteration 182

A constant-growth pattern, shown in high resolution, looks similar to the cross-section of an internal organ. The rules: a dead cell becomes a live cell if it has three live nearest neighbors, and a cell dies only if it has seven or eight live neighbors.

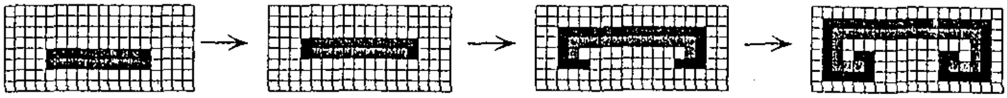
FIGURE 7.11

Cellular automata

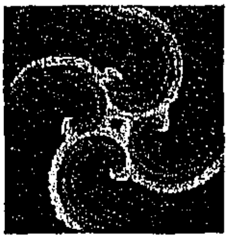
on the eight nearest neighbors (e.g., if three or more nearest-neighbors are full, the cell becomes full in the next iteration). At first, researchers carried out on these cellular automata experiments on checkered tablecloths with poker chips and dozens of human helpers (Mayer-Kress, pers. comm.), but by 1970 it had been developed into a simple computer program (Conway's "game of life"), which was described by Martin Gardner in his famous "Mathematical Games" column in *Scientific American*. The "game of life" story was an instant hit, and computer screens all over the world began to pulsate with a bizarre array of patterns (fig. 7.11b). As these activities drew increasing professional attention, a wide range of mathematically oriented scientists began to realize that the spontaneous emergence of self-sustaining patterns created in certain cellular automata were excellent models for the kinds of self-organizing patterns that had been so elusive in studies of fluid flow and biological growth.

Since scaling structures are one of the hallmarks of both fluid turbulence and biological growth, the occurrence of fractal patterns in cellular automata attracted a great deal of interest. But a more simple scaling structure, the logarithmic spiral (fig. 7.12), has garnered much of the attention. Even back in the 1950s mathematician Alan Turing, whose theory of computation provided von Neumann with the inspiration for the first digital computer, began his research on "biological morphogenesis" with an analysis of logarithmic spirals in growth patterns. Markus (1991) notes that the application areas for cellular automata models of spiral waves include nerve axons, the retina, the surface of fertilized eggs, the cerebral cortex, heart tissue, and aggregating slime molds. In the text for CALAB, the first comprehensive software for experimenting with cellular automata, mathematician Rudy Rucker (1989, 168) refers to systems that produce paired log spirals as "Zhabotinsky CAs," after the chemist who first observed such self-organizing patterns in artificial media: "When you look at Zhabotinsky CAs, you are seeing very striking three dimensional structures; things like paired vortex sheets in the surface of a river below a dam, the scroll pair stretching all the way down to the river bottom. . . . In three dimensions, a Zhabotinsky reaction would be like two paired nautilus shells, facing each other with their lips blending. The successive layers of such a growing pattern would build up very like a fetus!"

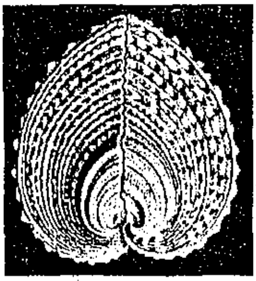
Figure 7.13 shows how the owari marching-group system can be used as a one-dimensional cellular automaton to demonstrate many of the dynamic phenomena produced on two-dimensional systems.[5] Earlier we noted that the Akan and other Ghanaian societies had a remarkable precolonial use of logarithmic spirals in iconic representations for living systems. The Ghanaian fourfold spiral (fig. 6.4a) and the four-armed computer graphic in figure 7.12b are



(a) Paired spirals emerge from a three-state cellular automation. Black cells are live, white cells are dead, and gray cells are in a refractory or "ghost" state. The rules: Any dead nearest neighbors of a live cell become live in the next iteration, and any live cell goes into the ghost state in the next iteration. The refractory layer acts as a memory, providing the directed growth (i.e., the breaking of symmetry) needed to create a spiral pattern.

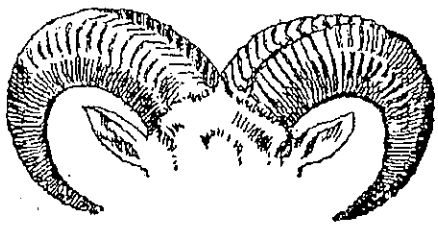


(b) This four-armed logarithmic spiral from Markus (1991) was produced by a six-state cellular automaton in which a sequence of ghost states corresponds to increasingly dark shades of gray. The system makes use of a very high-resolution grid as well as some random noise to prevent the tendency for the patterns to follow the grid shape (as in the square contours of the spiral above). Compare with the Ghanaian fourfold spiral in figure 6.4a.



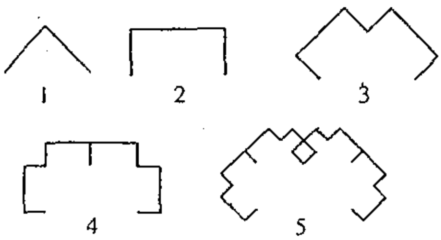
Bivalve shell.
(From Haeckel 1904.)

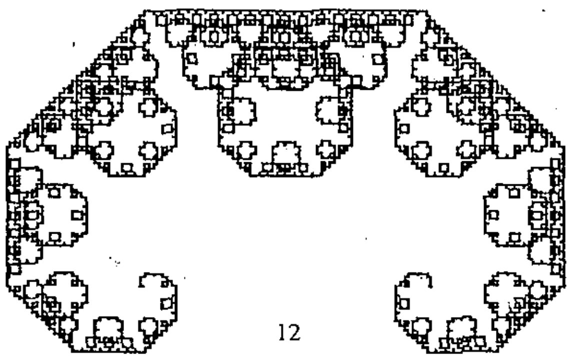
Mushroom cut in half.



North African sheep.
(From Cook 1914.)

(c) Paired logarithmic spirals often occur in natural growth forms.



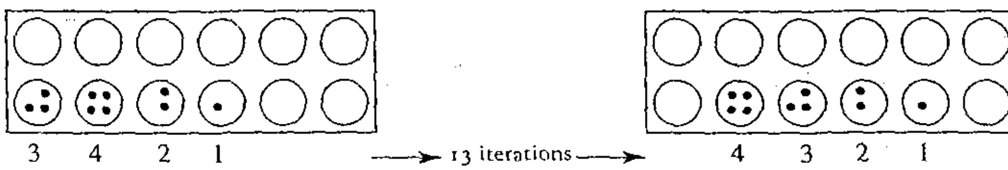


(d) Recursive line replacement, as we saw for other fractal generations, can also produce such paired spirals.

FIGURE 7.12
Spirals in cellular automata

We can view the owari board as a one-dimensional cellular automaton. One dimension is not necessarily a disadvantage; in fact, most of the professional mathematics on cellular automata (see Wolfram 1984, 1986) have been done on one-dimensional versions, because it is easier to keep track of the results. They can show all the dynamics of two dimensions.

The patterns noted by traditional owari players offer a great deal of insight into self-organizing behavior. Their observation of a class of self-propagating patterns, the "marching group," provides an excellent starting point.



3421→532→43111→4222→3331→442→5311→42211→3322→433→4411→4552→33211→4321

The marching group is an example of a constant pattern. Here we see counters in the *initial sequence* 3421 converge on their marching formation simply by repeating the "scoop from the left cup" rule through 13 iterations.

Just as we saw in two-dimensional cellular automata, transients of many different lengths can be produced. Transients of maximum length are used as an endgame tactic by indigenous Ghanaian players, who call it "slow motion"—accumulating pieces on your side to prevent your opponent from capturing them. In nonlinear dynamics, the constant pattern is called a "point attractor," and the transients would be said to lie in the "basin of attraction."

The marching group rule can also produce periodic behavior (a "limit cycle" or "periodic attractor" in nonlinear dynamics terms). Here is a period-3 system using only four counters:

211→22→31→211

Which leads to marching groups, and which ones lead to periodic cycles?

The numbers which lead to marching groups—1, 3, 6, 10, 15 . . . —should look familiar to readers: it's the triangular numbers we saw in tarumbeta!

The period of cycles in between each marching group is given by one plus the iteration level of the previous triangular number reached.

(*Note:* Some sequences will be truncated for 13, 14, and 15 since there are more counters than holes.)

Total number of counters	Behavior (after transients)
1	Marching
2	Period 2
3	Marching
4	Period 3
5	Period 3
6	Marching
7	Period 4
8	Period 4
9	Period 4
10	Marching
11	Period 5
12	Period 5
13	Period 5
14	Period 5
15	Marching

FIGURE 7.13
Owari as one-dimensional cellular automaton

quite distant in terms of the technologies that produced them, but there may well be some subtle connections between the two. Since cellular automata model the emergence of such patterns in modern scientific studies of living systems, and certain Ghanaian log spiral icons were also intended as generalized models for organic growth, it is not unreasonable to consider the possibility that the self-organizing dynamics observable in owari were also linked to concepts of biological morphogenesis in traditional Ghanaian knowledge systems.

Rattray's classic volume on the Asante culture of Ghana includes a chapter on owari, but unfortunately it only covers the rules and strategies of the game. Recently Kofi Agudoawu (1991) of Ghana has written a booklet on owari "dedicated to Africans who are engaged in the formidable task of reclaiming their heritage," and he does note its association with reproduction: *wari* in the Ghanaian language Twi means "he/she marries." Herskovits (1930), noting that the "awari"

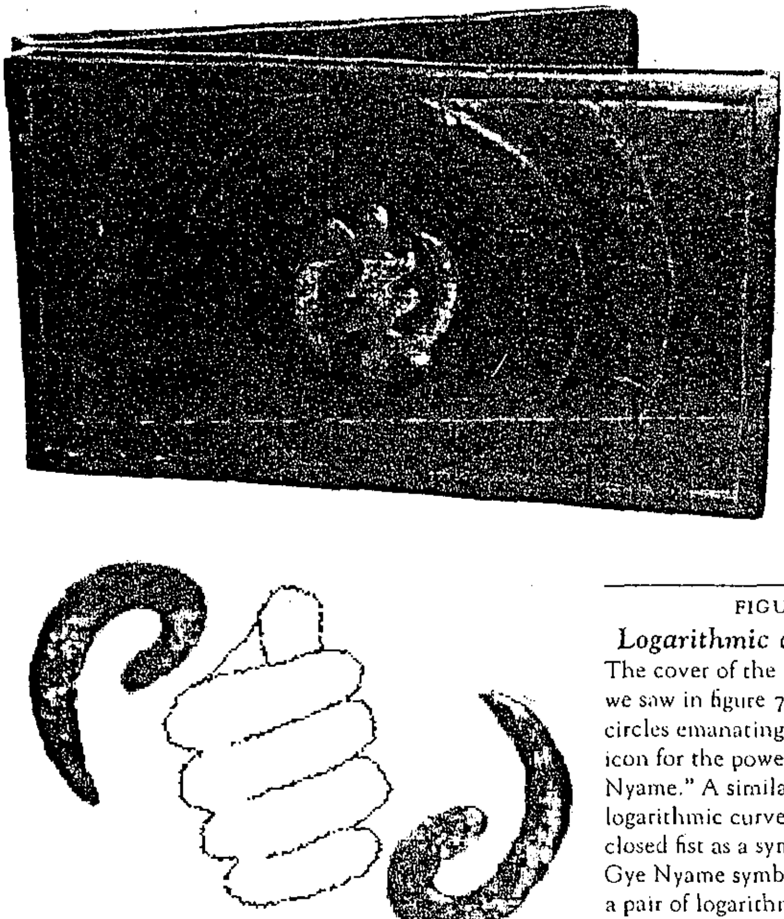


FIGURE 7.14

Logarithmic curves and owari

The cover of the hinged owari board we saw in figure 7.10 shows concentric circles emanating from the Adinkra icon for the power of god, "Gye Nyame." A similar icon, without the logarithmic curves, is attributed to a closed fist as a symbol of power. The Gye Nyame symbol thus appears to be a pair of logarithmic curves held in a fist: God holding the power of life.

game played by the descendants of African slaves in the New World had retained some of the precolonial cultural associations from Africa, reports that awari had a distinct "sacred character" to it, particularly involving the carving of the board. Owari boards with carvings of logarithmic spirals (fig. 7.14) can be commonly found in Ghana today, suggesting that Western scientists may not be the only ones who developed an association between discrete self-organizing patterns and biological reproduction. It is a bit vindictive, but I can't help but enjoy the thought of von Neumann, apostle of a mechanistic New World Order that would wipe out the irrational cacophony of living systems, spinning in his grave every time we watch a cellular automaton—whether in pixels or owari cups—bring forth chaos in the games of life.

Conclusion

Both tarumbeta and owari's marching-group dynamics are governed by the triangular numbers. There is nothing special about the triangular number series—similar nonlinear growth properties can be found in the numbers that form successively larger rectangles, pentagons, or other shapes. Nor is there anything special about the powers of two we found in divination—similar aperiodic properties can be produced by applications of mod 3, mod 4, etc. What is special is recursion the underlying concept of recursion—the ways in which a kind of mathematical feedback loop can generate new structures in space and new dynamics in time. In the next chapter, we will see how this underlying process is found in both practical applications and abstract symbolics of African cultures.

CHAPTER 8

Recursion

Recursion is the motor of fractal geometry; it is here that the basic transformations—whether numeric or spatial—are spun into whole cloth, and the patterns that emerge often tell the story of their whirling birth. We will begin by defining three types of recursion.[1] While it is possible to categorize the examples in this chapter solely on the basis of these three types, it is more illuminating to combine the analysis with cultural categories. It is in examining the interaction between the two that the use of fractal geometry as a knowledge system, and not just unconscious social dynamics, becomes evident. The cultural categories begin with the concrete instances of recursive construction techniques and gradually move toward the abstractions of recursion as symbolized in African iconography.

Three types of recursion

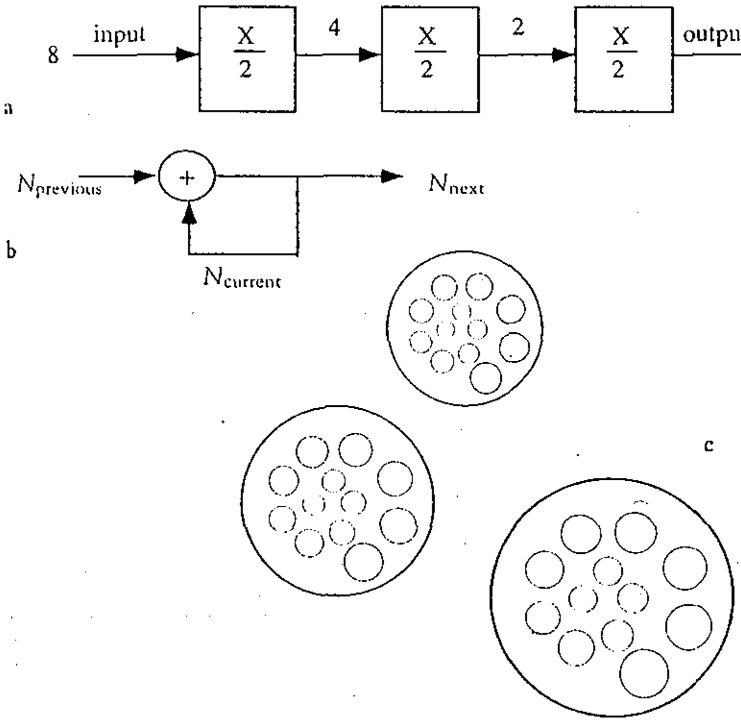
The least powerful of the three is cascade recursion, in which there is a predetermined sequence of similar processes. For example, there is a children's story in which a man buys a Christmas tree, but discovers it is too tall for his ceiling and cuts off the top. His dogs find the discarded top, and put it in their doghouse, but they too discover it is too tall, and cut off the top. Finally the 1 mice drag this tiny top into their hole, where it fits just fine—the recursion "bottoms out." Note that these were all independent transformations; it is only by coincidence, so to speak, that they happened to be the same. Figure 8.1a shows the numeric version of cascade recursion, in which we divide a number by two in each part of the sequence. This is not a very powerful type of recursion, for two reasons. First, it requires that we know how many transformations we want ahead of time—and that is not always possible. If the mouse was in charge, he would have said "just keep dividing until it's small enough to fit in my hole." Second, we have to know what transformation to make ahead of time, and that is not always possible, either. Recall, for example, the generation of the Fibonacci series we saw in chapter 7 (fig. 8.1b). Although the generation is just using addition, it cannot be created by a recursive cascade, because the amount to be added in each transformation changes in relation to previous results. Generating the Fibonacci series requires a feedback loop or, as mathematicians call it, iteration.

2 In iteration, there is only one transformation process, but each time the process creates an output, it uses this result as the input for the next iteration, as we've seen in generating fractals. A particularly important variety of iteration is "nesting," which makes use of loops within loops. Hofstadter (1980, 103–129) nicely illustrates nesting with a story in which one of the characters starts to tell a story, and within *that* story a character starts to read a passage from a book. But at that point the recursion "bottoms out": the book passage gets finished and we start to ascend back up the stories. Nested loops are very common in computer programming, and we can illustrate this with a program for drawing the architecture of Mokoulek (fig. 8.1c) we examined in chapter 2. The Ba-ila architecture we saw in chapter 2 can also be simulated this way, using one loop for the rings-within-rings, and another for the front-back scaling gradient that makes up each of those rings. In chapter 6 the first corn-row hairstyle (*ipako elede*) showed braiding as an iterative loop; the second corn-row example added another iterative loop of successive perimeters of braids.[2] It is common for computer programs to do such nesting several layers deep, and keeping track of all those loops within loops can be quite a chore.

3 The third type of recursion is "self-reference." We are all familiar with the way that symbols or icons can refer to something: the stars and stripes flag refers to America, the skull-and-cross-bones label refers to poison, the group of letters c-a-t refers to an animal. But it's also possible for a symbol to refer to itself. Kellogg's cornflakes, for example, once came in a box that featured a picture of a family sitting down to breakfast. In this picture you could see that the family had a box of Kellogg's cornflakes on their breakfast table, and you could see that

this box showed the same picture of the family, with the same box on their table, and so on to infinity (or at least to as small as the Kellogg company's artisans could draw).

Self-reference is best known for its role in logical paradox. If, for example, you were to accuse someone of lying, it would be an ordinary statement. But suppose you accuse yourself of lying? This is the paradox of Epimenides of Crete, who declared that "all Cretans are liars." If he's telling the truth, he must be lying, but if he's lying, then he's telling the truth. The role of self-reference in logical




```
WHILE e-count < 4 do:
  • draw enclosure
  WHILE g-count < 12 do:
    • draw granery
    • rotate toward center
    • shrink granery size
    • increase g-count by 1
  END of g-count's loop
  • reset g-count to 0
  • rotate toward center
  • shrink enclosure size
  • increase e-count by 1
END of e-count's loop.
```

FIGURE 8.1

Recursive cascade versus iteration

(a) A recursive cascade, in which the same transformation (division by two) happens to be used in each part of a sequence. This requires knowing how many times the transformatin should happen ahead of time. It also requires that the transformation is independent of previous results. (b) The Fibonacci sequence is produced by adding the previous number to the current number to get the next number, starting with $1 + 1 = 2$. In the Fibonacci sequence we add a different amount in each iteration—we could not know how much each transformation should add ahead of time, so a recursive cascade would not do the job. (c) In some cases it is necessary to put an iterative loop inside another iterative loop ("nesting"). Here is an example of nesting in a computer program for drawing the architecture of Mokoulek we examined in chapter 2. It is written in what programmers call "pseudocode," a mixture of a programming language and ordinary English. The first loop draws three large enclosures, and the inner loop draws 12 graneries inside each enclosure. Variable "e-count" tracks the number of enclosures, and g-count tracks the number of graneries.

paradox has been important for mathematical theory, but it has also been put to practical use in computer programming. Most programming has little routines called "procedures," and often a procedure will need to call other procedures. In self-referential programming the procedure calls itself.

Practical fractals: recursion in construction techniques

In his discussion of the metal-working techniques of Africa, Denis Williams gives a poetic description of recursive cascade in the *edan* brass sculptures of the Yoruba: "The image proliferates like lights in a bubble: one edan bears in its lap another, smaller version of itself, which bears in turn a smaller in its lap, and this bears another in its lap, etc.—a sort of sculptural relay race" (1974, 245). While the edan sculptures are unique to the Yoruba, recursive construction techniques are quite common in Africa. For example, Williams goes on to note that much African metalwork, unlike European investment casting, uses a "spiral technique" to build up structures from single strands (whether before casting, as in the lost wax technique,[3] or afterwards as wire), resulting in "helical coils formed from smaller helical coils." A wig made from metal wires (fig. 8.2a) shows a similar iterative construction using coils made of coils. In chapter 6 we saw some examples of African hair styles in which either adaptation to contours or abstract spatial transformation resulted in a scaling pattern. The fractal braids shown in figure 8.2b have nothing to do with the shape of the head; they are rather the result of successive iterations that combine strands of hair into braids, braids into braids of braids, and so on. Figure 8.2c shows another wig, this one for a sculpture, that features braids of many scales.

This collection of sculpture, metalwork, and hairstyling sounds like a motley assortment, but once we start looking for recursion we see a close relation: all examples used a single transformation—stacking, braiding, coiling—that was applied several times. Looking at the relation between the basic transformation and its final outcome can help us distinguish among different types of recursion. The braiding pattern of figure 8.2b, for example, is based on iteration, because the way each stage is braided depends on the braids produced in previous stages; they are braids of braids. The braids in figure 8.2c, on the other hand, are of different scales simply because each stage uses different amounts of single-hair strands—a cascade of predetermined transformations. Similarly, the coils of coils indicate iteration, because the output of one stage becomes the input for the next.

Recursive construction techniques are also used for the decorative designs of African artisans. In our discussion of the fractal esthetic in chapter 4, we examined decorative patterns which did not provide evidence for a formal geometric method. That doesn't mean no formal method could possibly exist; it's just that none could be readily discerned from the design itself, and the artisans did not report anything beyond intuition or esthetic taste. But there are some designs that do indicate an explicit recursive technique from the pattern itself. Figure 8.2e shows a Mauritanian textile with two such scaling patterns. Intentional application of iteration as a construction technique is indicated by the way the X fractal's seed shape is shown on either side, and by having iteration carried out on two completely different seed shapes in the same piece. The triangle fractal (close to what mathematicians call the "Sierpinski gasket") is also found in Mauritanian stonework (fig. 8.2f). A three-dimensional version from Ghana (fig. 8.2h) may have been inspired by these designs.

Both of the above are examples of additive construction, as we saw in the Koch curve of chapter 1, but subtractive iterations, as we saw for the Cantor set, are also found in African decorative fractals (fig. 8.2i). Carving designs include applications of iterative construction, particularly for calabash decorations (fig. 8.2l). A geometric algorithm for producing nonlinear scaling through folding was invented by the Yoruba artisans who produced the adire cloth of figure 8.2n. It is not merely a metaphor to refer to a specified series of folds as algorithmic; in fact, one of the classic fractals, the "dragon curve," was discovered in 1960 when physicist John Heighway experimented with iterative paper folding (Gardner 1967). The adire cloth also shows the application of reflection symmetry at every scale from single-stitch rows, which are reflected on either side of the fold edges, to the entire fabric, which is created by the joining of two mirror image cloths.

So far we have only discussed the technical method employed, but of course cultural meaning is often attached to these techniques as well. Recursive hairstyles, for example, embed layers of social labor with each iteration, a way to invest physical adornment with social meaning (such as friendship between stylist and stylee). Figure 8.3a shows a Fulani wedding blanket, in which spiritual energy is embedded in the pattern through its iterative construction. Prestige can also be associated with increasing iterations, as we find for brass casting and beadwork in the grassland areas of Cameroon (fig. 8.3b,c). The scaling iterations in one of the brass sculptures (fig. 8.3d) was reported to be symbolic as well: it showed three generations of royalty. But kinship groups are not just static entities; they change across time, and in the following two sections we will see that African representations of such temporal processes often involve recursion.

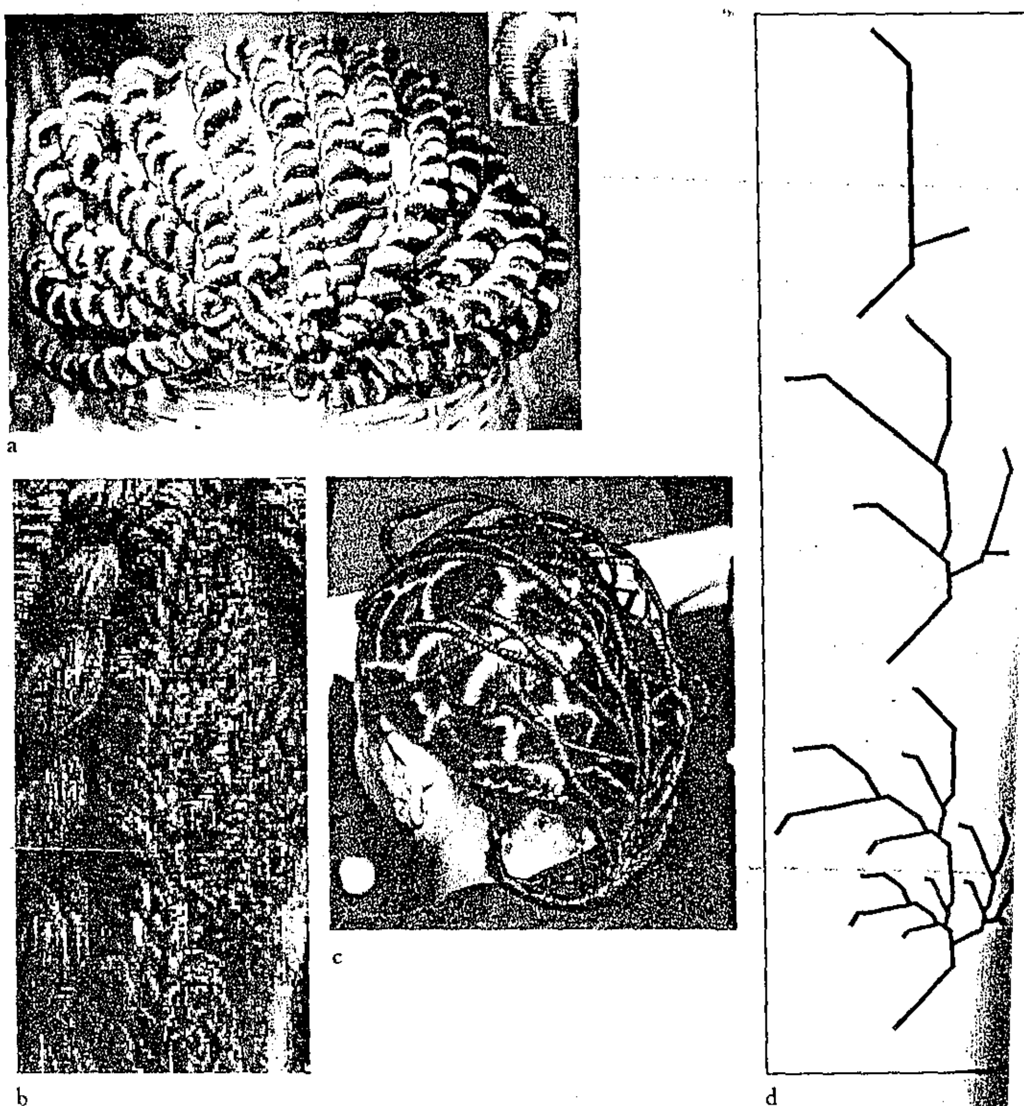


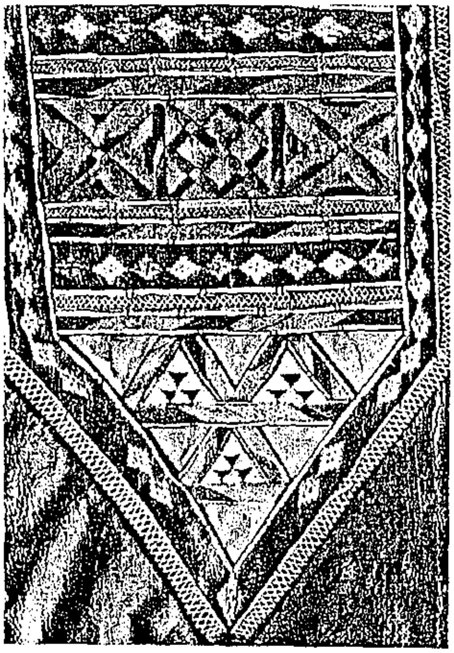
FIGURE 8.2

Recursive construction techniques

(a) Coils of coils are used to create this metal wig from Senegal. (b) A scaling cascade of brai a mask from the Dan societies of Liberia and Côte d'Ivoire. (c) Iterative braiding in this hair from Yaounde, Cameroon, *la tresse de fil*, can be simulated by fractal graphics. (d) Three itera of the *tresse de fil* simulation.

(b, *from Barbier-Mueller 1988.*)

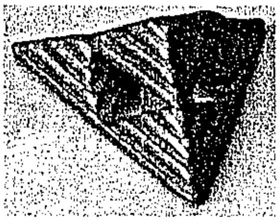
(*figure co*



e

f

g



h

FIGURE 8.2 *(continued)*
Iterative construction in Mauritanian decoration
(e) Recursive construction with triangles and X-shapes in Tuareg leatherwork. The X-shape is related to the quincunx discussed in chapter 4. (f) Designs using several iterations of triangles can also be found in Mauritanian stonework. (g) The use of triangles in this nomadic architecture from Mauritania may be one reason for the popularity of the design. Unlike rectangles, triangles can create a rigid frame using *flexible joints—an important feature in* a landscape where long poles are scarce and lashing is the most common joinery. (h) A single iteration of a three-dimensional version of the recursive triangle construction, created by Akan artists in Ghana.
(e, *from Jefferson 1973;* f *and* g, *photos courtesy IFAN, Dakar;* h, *from Phillips 1995, fig. 5.103.)*
(figure continues)

i

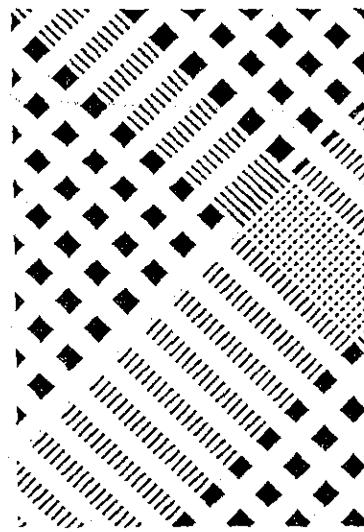


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FIGURE 8.2 *(continued)*

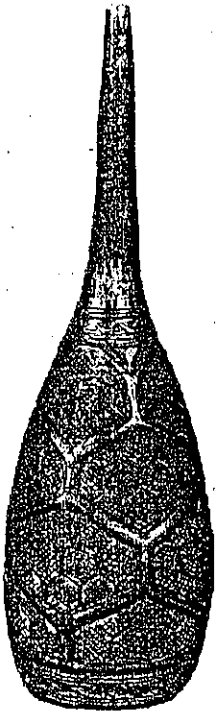
Scaling pattern from subtractive iteration

(i) A Fante woman posing in front of a painted studio backdrop, Cape Coast, Ghana, 1860.
(j) The Fante pattern can be thought of as two iterations of scaling subtraction (that is, erasing). Strips are erased from an all-black background. Where the thick strips intersect, we get large squares, and where the thin strips intersect we get small squares.
(i, photo from the National Museum of African Art, Smithsonian Institution.) *(figure continu*

Representing recursion as a process in time: part I, luck and age

A simple example of African representation for recursion as a time-varying process is shown in figure 8.4, where we see three designs that depict wishes for catches of everlarger fish. Since the experience of bad luck or good luck in fishing can occur on a daily basis, it is easy to see how a big fish could become an icon for good luck. But in these designs the artisans take the concept a step further. Good fortune is not in terms of a singular chance event, as one sees in the myths of the Native American trickster.[4] The wish is for an iterative process—that each fish is to be successively larger than the last one.

While these good luck icons are often a more informal part of cultural practice, other recursive processes are taken much more seriously. Anthropologists



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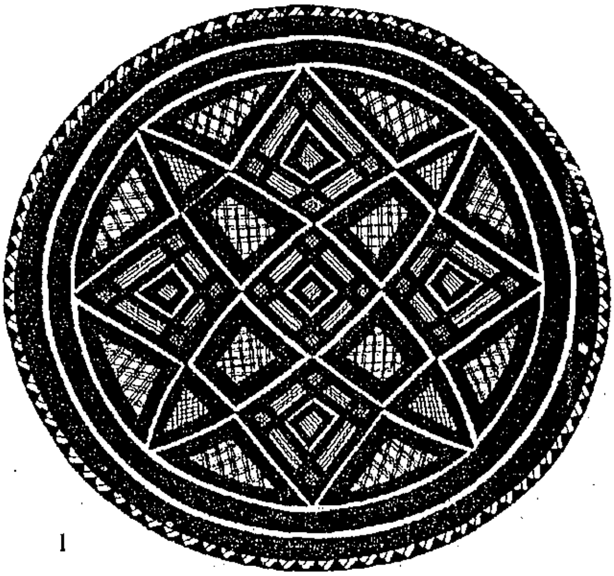

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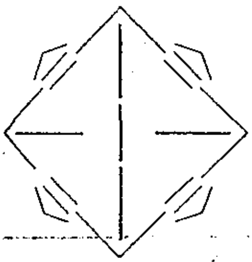
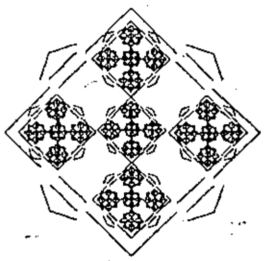
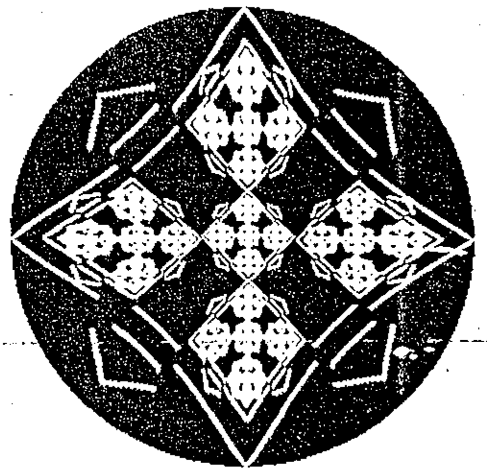


Seed shape, with active lines in gray.



Fourth iteration.



Fourth iteration enlarged, with adaptive scaling (mapping from a sphere to a plane) applied to match the adaptive scaling of the calabash design.

m

FIGURE 8.2 *(continued)*

Iteration in carvings

(k) The Bakuba of Zaire created several carvings that feature a self-similar design. This Bakuba wooden bottle makes use of hexagons of hexagons as well as adaptive scaling as it narrows into the neck. **(l)** Chappel (1977) records a wide variety of calabash designs, many with scaling attributes. This is probably the best example of iterative construction in these carvings. The design simulation not only requires recursion but adaptive scaling as well. **(m)** Seed shape and fourth iteration; fourth iteration enlarged, with adaptive scaling applied.

(k, *courtesy Musée Royal de l'Afrique Central, Belgium.)*

(figure continues)

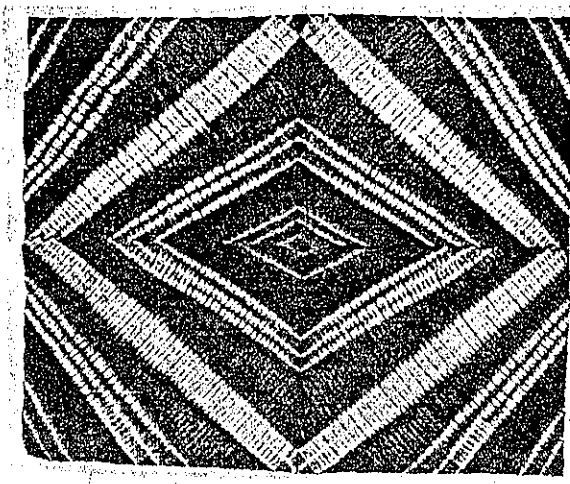
FIGURE 8.2 (*continued*)

Adire cloth: scaling from iterative folding

(n) This Yoruba adire cloth is actually two separate pieces attached along the horizontal midline. The dye pattern is created by sewing along folds before dye is applied and then removing the threads so that the white lines are left where the dye did not penetrate. **(o)** The folding method is based on reflection symmetry across a diagonal. It is easiest to understand by making a paper model.

The adire artisans have not only developed an algorithm for generating this nonlinear scaling series, but have done so in a way that maximizes efficient production: all folds fall along the same two edges, so only two edges need be sewn. Your paper model can imitate this effect by running a heavy felt marker along the two edges, so that the ink bleeds through all the layers (you can cheat by inking each fold as you unfold it). Note that the white lines in the adire are triple—this, too, is created by a reflection symmetry, sewing next to the fold to create the two outer lines (one on each side of the fold), and sewing right on the edge of the fold to create the center line.

(**n**, *photo from Picton and Mack 1979.*)



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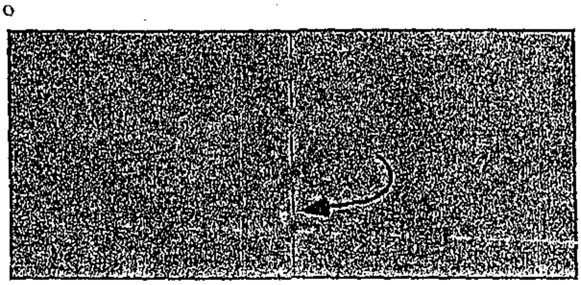
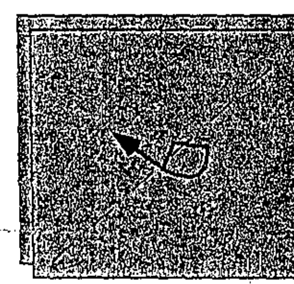
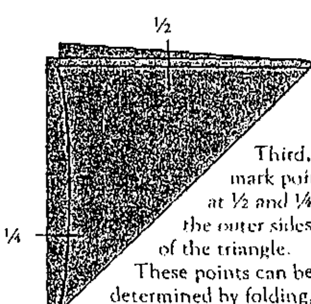


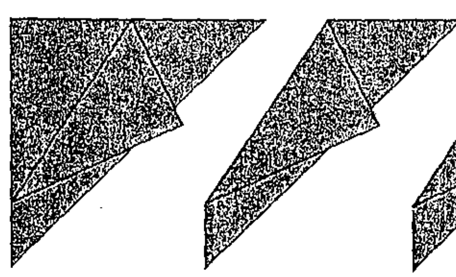
First, cut out a paper rectangle with width twice the height, and fold it in half, making a square.



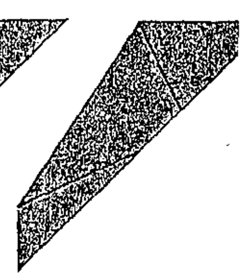
Second, fold the square along i diagonal, making a triangle.



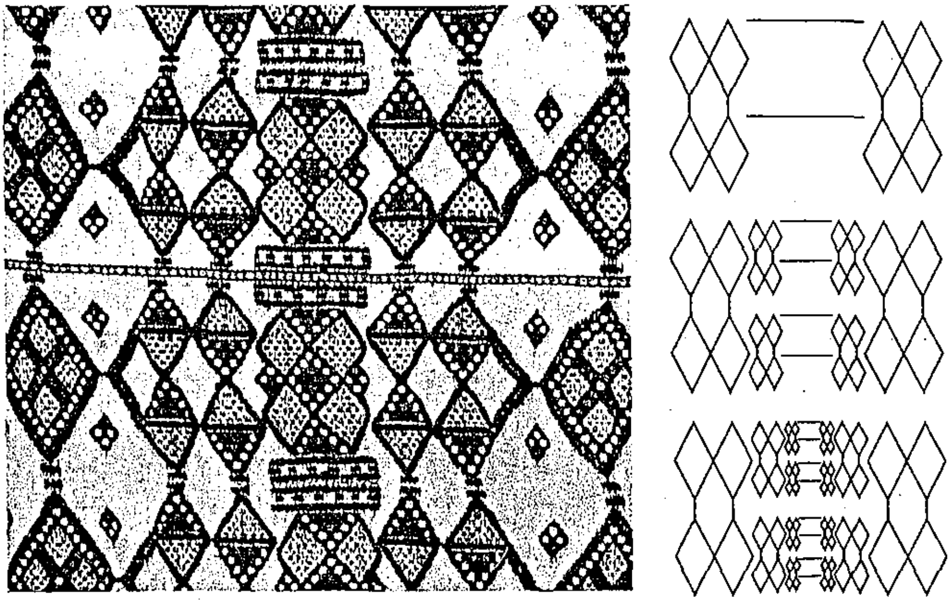
Third, mark points at ½ and ¼ of the outer sides of the triangle. These points can be determined by folding, if one wishes to maintain the origami equivalent of compass and straight-edge construction, but doing it by eyeball works just fine.



Fourth, fold from the corners on opposite sides along the line between the ½ and ¼ marks.



Finally, fold in the sma overlapping corner on side.



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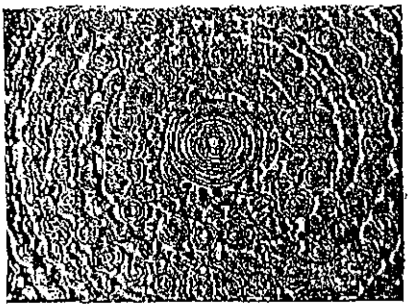

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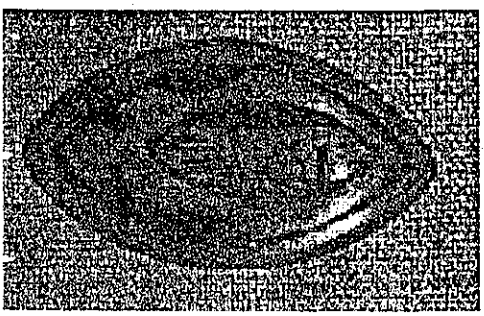
c

d

FIGURE 8.3
Making meaning through iterative construction
(a) This Fulani wedding blanket from Mali is based on diamonds that scale from either side as we move toward the center; a pattern that is easily simulated using a fractal (see diagram). The weavers who created it report that spiritual energy is woven into the pattern, and that each successive iteration shows an increase in this energy. Releasing this spiritual energy is dangerous, and if the weavers were to stop in the middle they would risk death. The engaged couple must bring the weaver food and kola nuts to keep him awake until it is finished. (b) The prestige bronze of Foumban, Cameroon, often makes use of self-similar iterations. (c) Prestige is also symbolized by the labor and artistry required to produce the many iterations of bead pattens for this elephant mask. (d) According to Salefou Mbetukom, the leading castor of Foumban, this sculpture shows the succession of kings in the royal family.
(c, from agence Hoa-Qui.)

have always been interested in the contrast between the elaborate political and economic hierarchy of European societies and the relatively "classless" (sometimes even rulerless) structure of many precolonial African societies. If it is not political and economic structure that governs their society, then what does? One part of the answer is age. All human cultures differentiate between chil-

a



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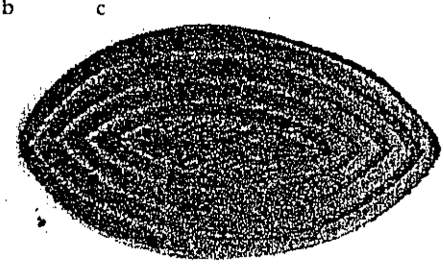


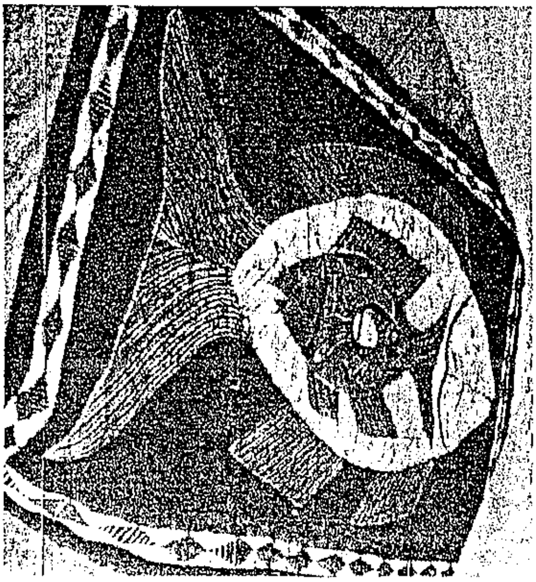


FIGURE 8.4

If wishes were fishes

(a) Scaling scales: this Bamana tattoo, created with henna, is said to represent the scales of fish. It is good luck, signifying ever-larger fish catches. (b) This is an "abbia," a carved gambling chip from Cameroon. Given the high stakes of the game, it could be a more aggressive symbolism than just luck, e.g., "just as you have swallowed others, I will swallow you." Other chips appear to carry the iteration out several more levels, although they are less recognizable as fish (c). (d) This print with four iterations of fish is from northern Ghana. It was reported to be a fertility symbol.

(d, *photo courtesy of Traci Roberts and Ann Campbell.*)

d

dren, adults, and elders, but in many African societies the divisions are much more elaborate and structured. In these age-grade systems, all community members born within a given number of years will move together through a series of ritual initiations. In chapter 5 we saw one example in which these initiation stages appeared to be accompanied by an iterative scaling geometry, the lusona. Figure 8.5a shows another geometric visualization of age-grade initiation: a hexagonal mask created by the Bassari of the Senegambian and Guinea-Bissau region.

Although the mask is only a linear-concentric scaling of hexagons, and thus not a fractal, it does suggest an iterative process, and we might well suspect a link between stages in age-grade and stages in iteration. The initiation process is a closely guarded secret, so it is not simply a matter of asking Bassari experts, but during my visit with the Bassari in 1994 I found that the meaning of other mathematical patterns in Bassari culture can be used to make some educated guesses about the meaning of the mask. Despite the extensive migrations from the villages to cities (Nolan 1986), there is still strong participation in the age groups and transition rituals. The "forest spirit" Annakudi, for example, seems to be undaunted by the city of Tambacounda, where a local age group hosted him at a well-attended dance during my stay. Indeed, I found the stereotype of traditional elders and irreverent youths to be somewhat reversed (which was explained to

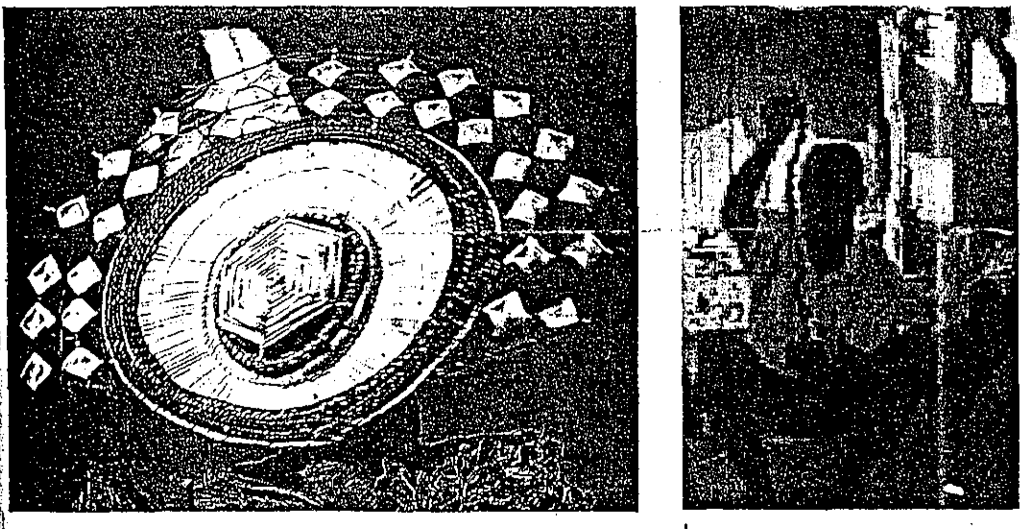


FIGURE 8.5

Scaling hexagons in a Bassari mask

(a) The Bassari initiation masks frequently feature scaling hexagons in the center. This appears to be a linear scaling. (b) One of the Bassari elders demonstrates the traditional string talleys, with knots in groups of six.

(a, *photo from agence Hoa-Qui/Michel Renaudeau*.)

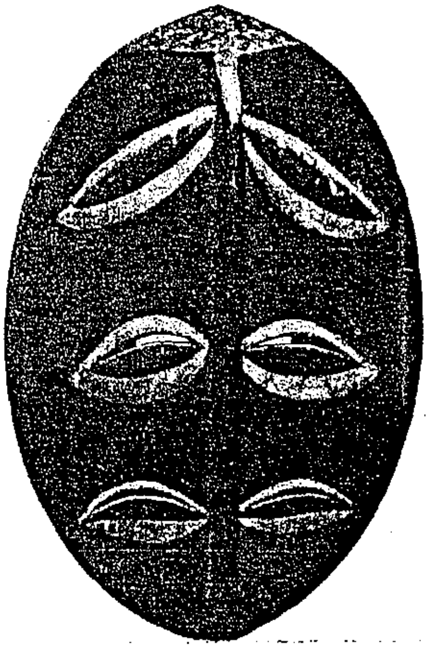
me as an effect of the strong hierarchy of secret knowledge: the youth are often more wary about breaking taboos because they are less certain about boundaries and consequences). This is not to say that there is any overt presence of fear. In fact, it is the positive aspects of the secrets that are stressed, as became obvious when elders gleefully refused my questions while emphasizing the wonderful nature of the information they could not divulge.

The number six is a prominant feature of Bassari mathematics in many areas of their life. They have a popular game, for example, played with pebbles on a sand pattern, which makes use of two axes with six holes in each line. In their traditional calendar there are six months per year, each of 30 (6 × 5) days, with an initiation about every 12 (6 × 2) years (to a total of nine initiations). Each of these rites of passage involves a lengthy education in a new level of traditional knowledge. The most important is the passage to adulthood, which lasts for six days. In addition to these time measures, the number six also appears in the Bassari counting system. String tallies, traditionally used for recording various counts, often used knots grouped by six. The Bassari elder who demonstrated these tallies to me (fig. 8.5b) told me that he did not know much about traditional forms of calculation, but he did know that in precolonial times it was performed by specialists who were trained in the memorization of sums. This practice may explain the origins of the famous African American calculating prodigy, Thomas Fuller. In 1724, at the age of 14, he was captured—quite possibly from the geographic areas that included the Bassari[5]—and sold into slavery in Virginia, where he astonished both popular and professional audiences with his extraordinary calculating feats (Fauvel and Gerdes 1990).

Finally, there is the Bassari divination system. Although the cast shells are interpreted by images rather than any numeric reading, they are cast six times. Each cast provides the answer to a specific question (or verification of a previous question) relevant to the client's problem; the final sixth cast shows the problem as a whole. If we compare this divination to the initiation system, the number six can be seen as a marker for information clusters, a punctuation point which, like the tally system, allows the distinctions that maintain a comprehensive structure. And like the initiation, each cycle of six provides an expanding view of the whole. Thus it seems likely that the scaling hexagons of the initiation mask represent this six-stage iteration of knowledge.

Nonlinear scaling iterations can also be found in African initiation masks. Figure 8.6a shows a Bakwele mask in which both size and curvature have a nonlinear increase with each stage. My guess—I have not found any cultural descriptions that can confirm this—is that it suggests "to open your eyes" as a metaphor of knowledge, and thus maps the scaling iterations of the mask to iterations of knowledge gained in initiation stages. Figure 8.6b shows a Bembe mask used in the first of a three-stage initiation for a voluntary association, the *bwami* (Biebuyck 1973). Before the ceremony, the mask is hidden behind a screen, and during the ritual the screen is gradually lifted by a high-ranking senior member. Both the relation between the number of eyes in the mask and the number of stages in initiation, as well as this method of visually exposing the pattern as a sequence, again suggest intentional use of a scaling geometric design to represent scaling iterations of knowledge.

a



b

FIGURE 8.6

Nonlinear scaling in initiation masks

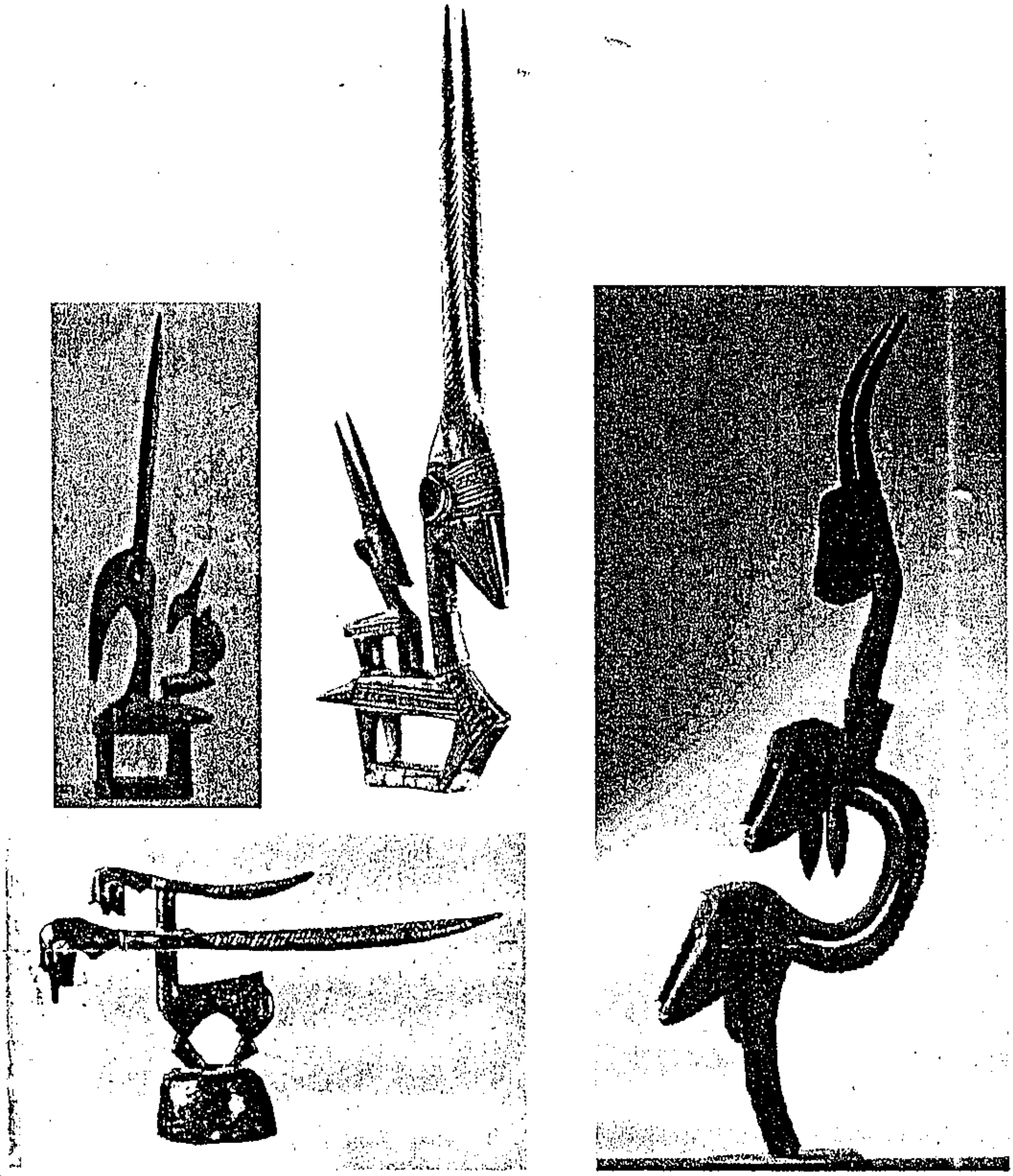
(a) Bembe mask, from western Congo, used in the first of a three-stage initiation for a voluntary association, the *bwami*. (b) Mask used in initiation by the Bakwele of Congo. (a, *photo courtesy Gene Isaacson*; b, *courtesy Musée de l'Homme*.)

Representing recursion as a process in time: part II, kinship and descent

If age-grade systems are one part of the standard anthropological explanation for how "classless" societies are structured, kinship is the other.[6] Kinship systems are primarily based on genetic ties ("blood relations") and marriage, although most societies also have "fictive" kin (e.g., adoption) which are just as real—kinship is a cultural phenomenon. Descent is also culturally based. Most Western European and American societies think of descent as biological, but that is because most of them have bilateral descent, in which both parents are used to establish kinship. Unilineal descent, where a kin group traces their lineage through one sex only, is actually more common (in about 60 percent of the world's cultures). A "clan" is a unilineal kinship group whose members report that they are descended from a common distant ancestor, often a mythological figure. Clans often have important religious and political functions, although they are typically spread out across many villages and usually prohibit marriage between clan members.

We have already seen how the Bamana use recursion to generate a binary code in their divination; here we will look at their representation of descent as recursion. The antelope figure in Bamana iconography is associated with both human and agricultural fertility. In the *chi wara* association, which is open to both men and women, the antelope appears in a striking headdress (fig. 8.7a), which represents the recursion of reproduction: mother and child. When seeing one headdress individually, the scaling seems trivial, but with several examples together the extraordinary insistence on self-similarity becomes apparent. This icon acts as the seed transformation in an iterative loop: the child becomes a mother, who has a child, who becomes a mother, and so on. Figure 8.7b shows the descent carried to a third iteration.

In chapter 2 we saw several examples in which descent was tied to scaling architecture. The Batammaliba, who live in the northern parts of Ghana, Benin, and Togo, have developed an elaborate system for this relationship (Blier 1987). Figure 8.8 shows a diagram of their two-story house, based on the circle of circles found in much of the West African interior. In front of the house lies the first of two scaling transformations. It is the "soul mound," a circle of cylinders representing the spirits of those currently living in the house and physically structured like a scaled-down version of the house architecture. As the current family gives way to a new generation, the soul mound undergoes a second transformation in which it is divided into a single cylinder and is moved inside. A scaling sequence of these single cylinders—one for each generation—can be seen wrapped around the central tower inside the house.

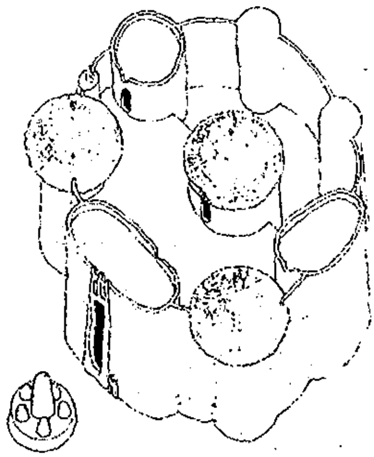


a b

FIGURE 8.7

Recursion and reproduction in Bamana sculpture

(a) The chi wara figure, used in ritual dances for agricultural fertility, shows a striking self-similarity: although the figures vary widely, each one is similar to itself. This can be attributed to the Bamana view of reproduction as cyclic iterations. (b) Here the cycle is carried out to three iterations.
(a: *upper left, from the de Hevenon Collection, Museum of African Art, Smithsonian Institution; upper right, courtesy Musée de l'Homme; lower, from Carnegie Institute* 1970. b, *courtesy Musée de l'Homme.)*



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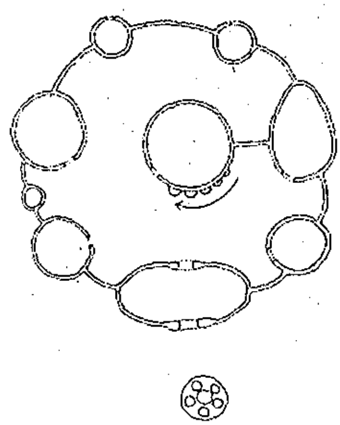


b

FIGURE 8.8

Recursion in Batammaliba architecture

(a) Diagram of the Batammaliba two-story house. In front of the house lies the "soul mound," representing the spirits of those currently living in the house.
(b) Inside the house, single mounds representing ancestors are found in the scaling arrays, with the size of the ancestral mounds increasing from youngest to oldest. Here only one such array is shown, but typically there are several in the same household.
(a, *from Blier 1987*.)

Blier's diagram indicates that the size of the ancestral mounds increases from youngest to oldest, and she notes that this reflects the Batammaliba's idea of a spiritual power in proportion to age. So far it would appear that there are only two scaling cascades—one to shrink houses to soul mounds, and another to divide soul mounds into cylinder rows—and no iterative loop. But if the largest mound represents the oldest, then recent mounds would be increasingly threatened by vanishing scale. How would the first descendant have known how large to make the first mound? Blier notes that many of the symbolic features of the architecture are replastered with additional layers of wet clay on ritual occasions, and we can surmise that this applies to the ancestral mounds as well. Thus an iterative loop, in which each new ancestor adds power to the older ones by increasing their mound's size, would be at work in the scaling sequence we see accumulating around the central tower.

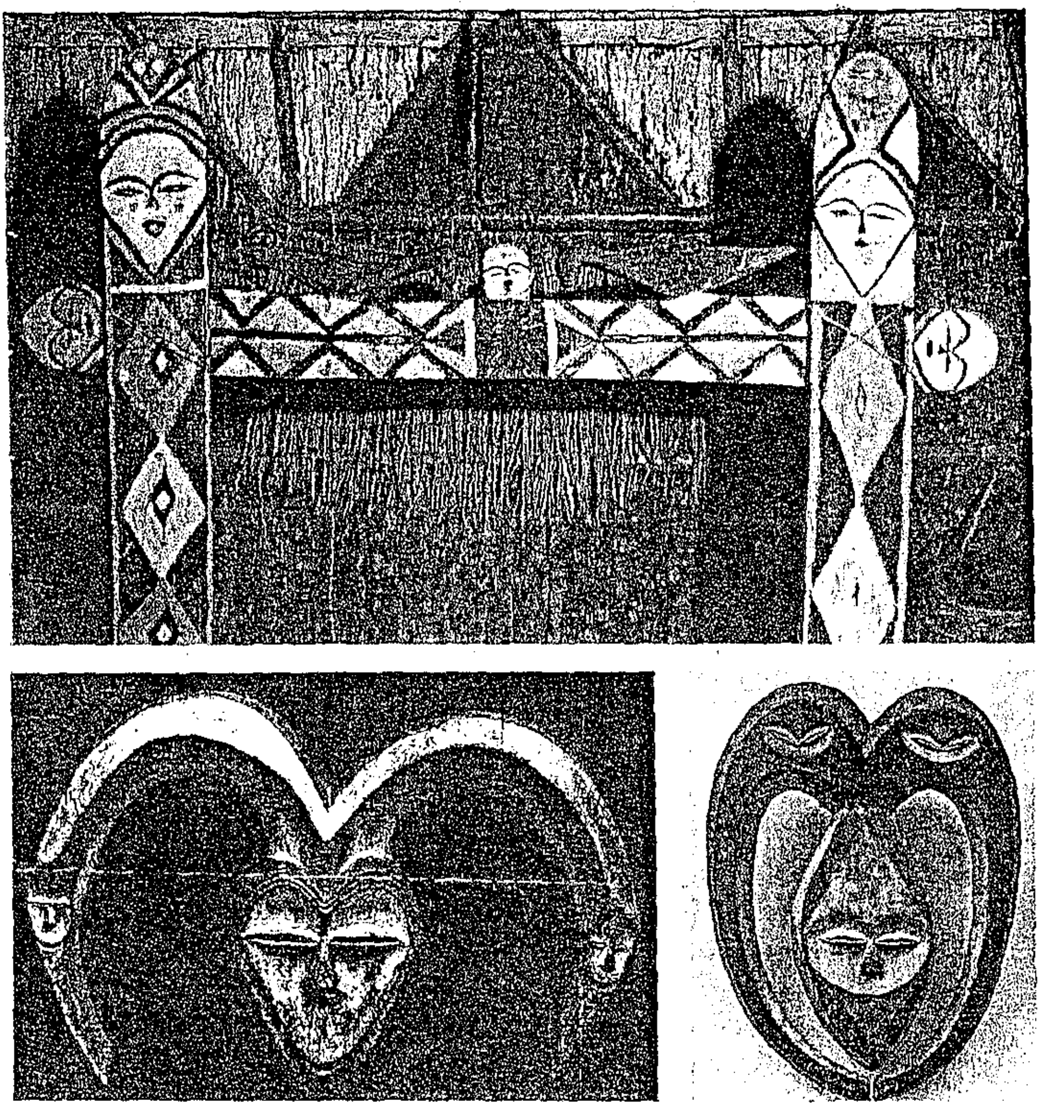
The Mitsogho society of Gabon includes several religious associations that are housed in the same temple (*ebandza*). Figure 8.9a shows the central post of an ebandza featuring scaling pairs of human figures. As in the chi wara figure, there is only one iteration; the significance lies in this figure as the seed transformation for a recursive process. The use of a cross shape may be due to Christian influence, but the bilateral scaling is quite indigenous, as we see in the classic Bakwele sculpture (fig. 8.9b) elsewhere in Gabon. Most important, the ebandza post provides a visualization for the iterative concept of descent that is widely used in this culture area. This is beautifully described by Fernandez (1982) in a detailed ethnography of the Mitsogho's neighbors and cultural relatives, the Fang.

Although the Fang are patrilineal, they believe that the active principle of birth—a tiny human (what was called a "homunculus" in early European medical theory)—is contained in the female blood. The idea of the new existing within the old, and vice versa, is a strong cultural theme. For example, in one ritual the mother places a newborn child on the back of her oldest sibling to symbolize continuity of the lineage. Fernandez (1982, 254) notes that the rebirth concept is so strong that "Fang fathers often called their infant sons *ata*, the familiar form of father." In many of the Fang and Mitsogo religious practices, the spirit is explicitly described as traveling a vertical cyclic path. Ancestors rise from the earth to become born again, and by proper living they can rise higher with each rebirth.

These cyclic iterations are visualized in the Nganga dance of the Bwiti religion (fig. 8.9c). Even in Christian-animist syncretism, biblical characters are reinterpreted as cyclic rebirths: the African gods Zame and Nyingwan become Adam and Eve, who become Cain and Abel (understood as male and female), who become Christ and the Virgin Mary. Fernandez notes that these cycles are not mere repetition, but rather iterative transformations: "The spiritual-fraternal relation of Zame and his sister is converted into the carnal relation of Adam and Eve which degenerates into the materialistic and divisive relation of Cain and Abel which then is regenerated as the immaculate and filial relationship of Mary and Jesus" (p. 339). According to Fernandez, these degeneration/regeneration differences are visualized as horizontal versus vertical,[7] which could explain the alternation in the ebandza posts. In applying this cyclic conception to the ebandza structure (fig. 8.9d), we can see the descent model in its full fractal expansion.

The Tabwa, who occupy the eastern section of the Democratic Republic of Congo (Zaire), have also developed several geometric figures to serve as models for their conceptions of kinship and descent. Maurer and Roberts (1987, 25) explain that in the Tabwa origin story, an aardvark's winding tunnel results in

a



b

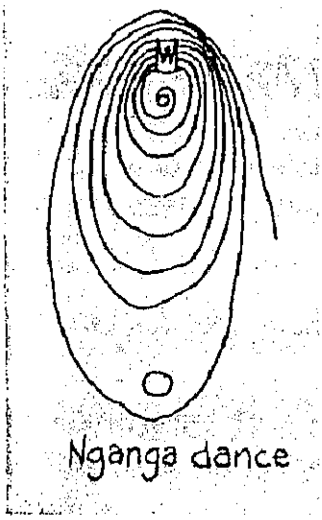
FIGURE 8.9

Recursive kinship in Gabon

(a) The central post of the ebandza temple in western Gabon suggests an iterative descent concept. This is actually a museum reproduction. (b) Bakwele masks from eastern Gabon show similar bilateral scaling.

(a, *from Perrois 1986*; b: *left, from Perrois 1986; right, Metropolitan Museum of Art; from Zaslavsky 1973*.)

(figure continues)



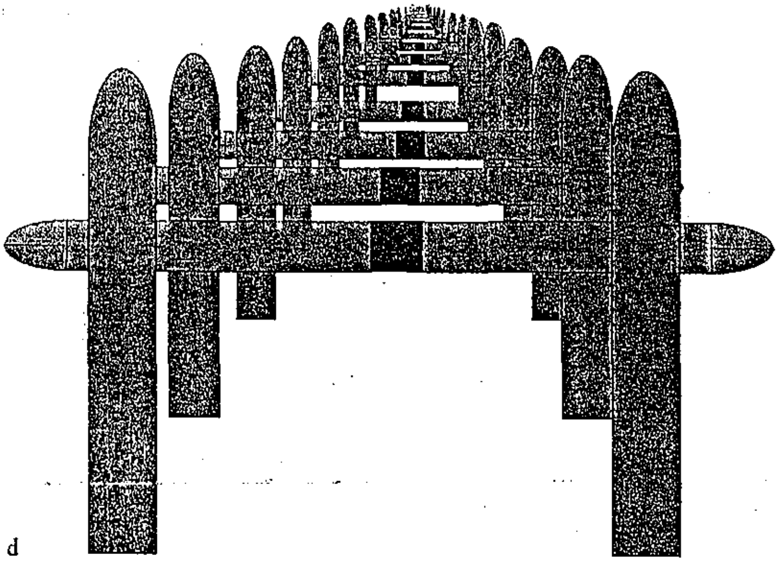
c

FIGURE 8.9 (*continued*)

Recursive descent in Gabon

(c) In many of the Fang and Mitsogo religious practices, the spirit is explicitly described as traveling a vertical cyclic path. Ancestors rise from the earth to be born again, and by proper living they can rise higher with each rebirth. These cyclic iterations are visualized in the Nganga dance of the Bwiti religion. (d) We can apply the explicit mapping of cyclic generations given by the Nganga dance to the iterative posts of the ebandza temple and see the descent model in its full fractal expansion. The implication of infinite regress is discussed in chapter 9.

(c, *from Fernandez 1982.*)



d

a "bottomless spring" from which emerges the first human, Kyomba, whose descendants spread in all directions from this central point. This spread is visualized by the *mpande*, a disk cut from the end of a cone snail, which is worn as a chest pendant (fig. 8.10a). The central point is drilled out, representing the emergence of Kyomba from the deep spring, and the logarithmic spiral of the shell end symbolizes the expansion of kin groups from this origin.[8]

One way to represent these expanding iterations through time is to take a series of portraits as the structure changes: projections at different points along the time axis. Figure 8.10b shows the first step toward this design: a more linear version of the mpande disk, in which an Archimedean spiral fits between a series of triangles (which represent the wives of the guardian of the ancestors). In figure 8.10c we see that the linear spiral has become concentric squares, but they are now portrayed in a scaling sequence, suggesting a series of portraits of the kinship spiral as it expands through time. Similar scaling square sequences, carried out to a great number of iterations, can be seen in the staffs of their northern neighbors, the Baluba (fig. 8.10d).

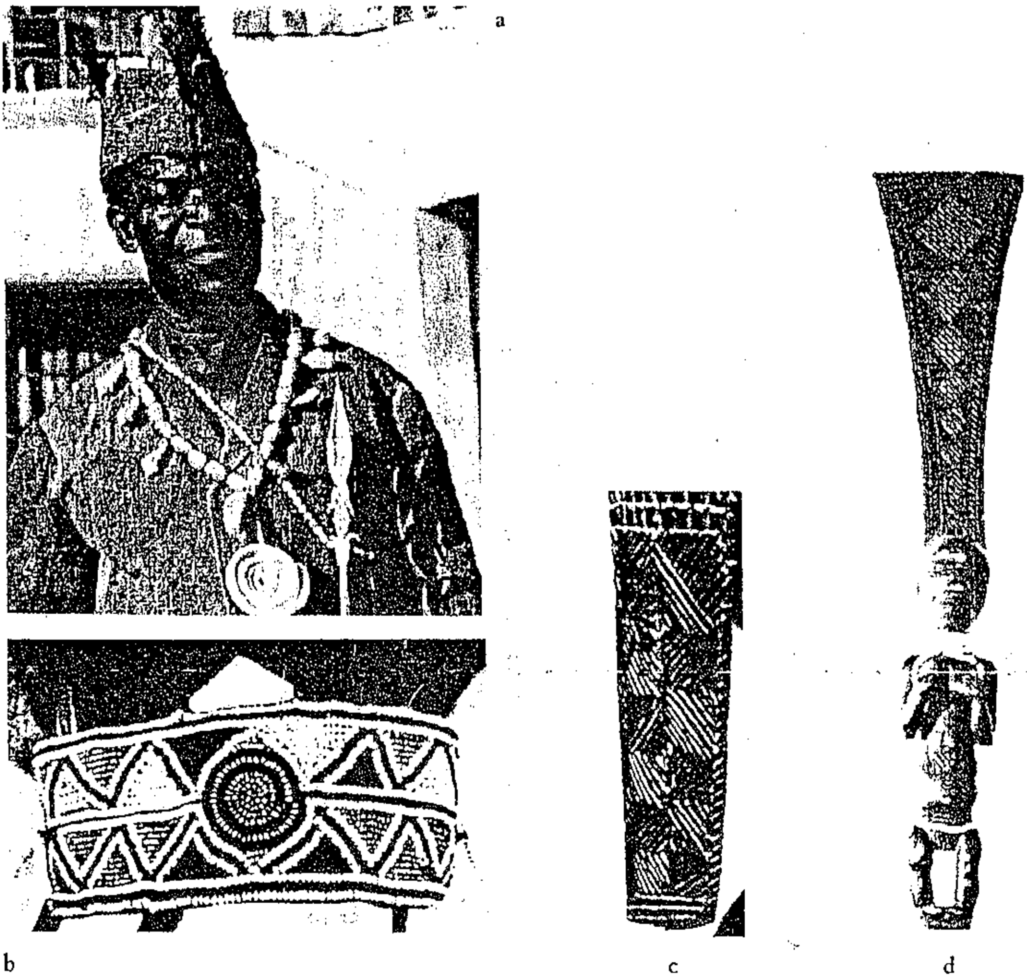


FIGURE 8.10

Tabwa kinship representations

(a) The mpande shell worn by Chief Manda Kaseke Joseph. (b) A more linear version of the mpande disk, in which an Archimedean spiral fits between a series of triangles (which represent the wives of the guardian ancestors). (c) The linear spiral has become concentric squares, but they are now portrayed in a scaling sequence, suggesting a series of portraits of the kinship spiral as it expands through time. (d) Similar scaling of square sequences can be seen in the staffs of their northern neighbors, the Baluba.

(a–c, *from Roberts and Maurer 1985*; d, *Museum für Volkerkunde, Frankfurt.*)

Recursive cosmology

In all the descent representations we have examined, kinship groups trace themselves to a mythological ancestor at the beginning of the world, and thus we move from the origins of humanity to the origins of the cosmos. African creation concepts are often based on a recursive nesting. The best-known example is that of the Dogon, as described by French ethnographer Marcel Griaule (1965). His work began during the 1930 Dakar-Djibouti expedition, where he first made contact with the Dogon of Sanga in what is now Mali. In 1947 his studies took a dramatic turn of events when one of the Dogon elders, Ogotemmêli, agreed to introduce Griaule to their elaborate knowledge system. Clifford (1983) provides a detailed review of the strong reactions to Griaule's resulting ethnography. While many of the critiques were really about the failings of modernist anthropology in general—the tendency to prefer a static past over the present, or a singular "tradition" over individual invention—there were also those who simply did not believe that such elaborate abstractions could be indigenous.

For the Dogon the human shape is not only a biological form, but maps meaning at all levels: "The fact that the universe is projected in the same manner on a series of different scales—the cosmos, the village, the house, the individual—provides a profoundly unifying element in Dogon life" (Duly 1979). The Dogon house is physically structured on a model of the human form, with a large rectangle for the body, smaller rectangles on each side for arms, a door for the mouth, and so on. The Dogon village, however, represents the human form with a symbolic structure rather than a geometric structure: it is not physically arranged as a human shape, but various buildings are assigned meaning according to their social function (the smithy stands for the head, the menstrual lodges as hands, and so on). The use of two different systems of representation prevents self-similarity in the physical structure of the architecture, but some of the Dogon's religious icons do show human forms made out of human forms (fig. 8.11a).

A threefold scaling appears in several aspects of the Dogon religion, and it is here that we find an indication that the Dogon are using more than just a cascade. Griaule (1965, 138) summarizes Ogotemmêli's creation story: "God . . . had three times reorganized the world by means of three successive Words, each more explicit and more widespread in its range than the one before it." But these reorganizations are not merely layering one on top of the other; rather the output of each reorganization becomes the input for the next. The earth gives birth to the first spirits; these "Nummo" regenerate ancestral beings into humanlike reptiles; the reptile-ancestors are again reborn as the first true humans. Within rebirth, the threefold iteration is again enacted. In the first

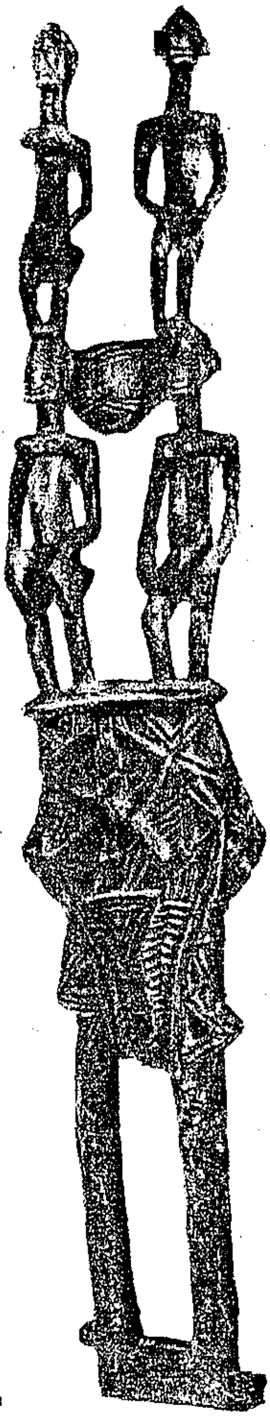
(a) In the Dogon cosmology, the structure of the human form is created from human form.

(b) The symbolism of the stacked pots, representing the breath of life, within the feteus, within the womb. We can use an iterative drawing procedure to better understand how this kind of scaling can result from a recursive loop. Suppose we have a routine that can draw the circle of the pot given a diameter, and one that can draw a lid.

```
While diameter ≥ minimum do:
    Draw a circle of size diameter
    If size = minimum, draw a lid
    Shrink diameter by 2/3
End of "while" loop.
```

This procedure first checks to see if we are past the smallest diameter possible. If not, it draws a pot, shrinks the diameter value by ⅔s, and then goes back to the start of the while loop. In other words, the output of one iteration—a given diameter—becomes the input for the next iteration.

(c) Dogon recursive image of mother and child.



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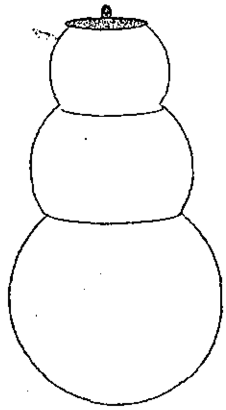

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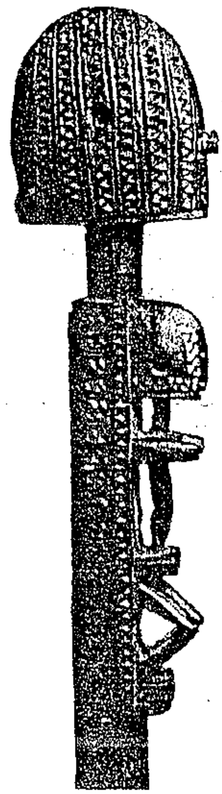


c

FIGURE 8.11

Scaling in Dogon religious icons

(a, *from Laude 1973*; courtesy Lester Wunderman; c, *from Carnegie Institute 1970*; courtesy of Jay C. Leff.)

regeneration, for example, each ancestral being enters the earth's womb, which turns each of them into a fetus, which allows the breath of life (*nummo*) to enter. The cosmological narrative suggests that in the Dogon view the birthing processes at all scales are, in some sense, iterations through the same transformation, and that these iterations are actually nested loops.

Why should the Dogon require such deep iterative nesting? I suspect that there are two motivations. First, there is an insight into modeling the world: recursion is an important feature in biological morphogenesis, as well as in environmental and social change. The second is the cultural context of this knowledge: elders need to ensure that the younger generation respects their authority, which can only be done by giving them gradual access to the source of this power, which is knowledge. A knowledge system in which endless exegesis is possible makes the initiation process a lifetime activity. But having so much explanatory elbow room also presents a problem with translating such narratives into mathematics.[9] We had to be careful with translations for more formal practices, such as interpreting the Bamana divination system as a binary code, or *adire* cloth as a geometric algorithm. A narrative is not a quantitative or geometric pattern, and its ambiguity requires all the more care in producing a mathematical translation that does not embellish indigenous concepts. First, we have to distinguish between modeling the narrative—something a structural anthropologist like Claude Lévi-Strauss would do—and the narrative as an indigenous model, such as the Dogon's system for representing their own abstract ideas. The best way to limit our translation to ideas that the Dogon themselves are trying to convey is to compare these abstractions of the narrative with other, more formal Dogon systems. This means missing some ideas that do not have such formal counterparts, but it is better to err on the safe side in this context.

The material designs of the Dogon are more restricted than the narrative in terms of their iterative depth. The best case is probably in the iconography of the granary, where Ogotemmêli explains a stack of three pots: the largest represents the womb; the one on top of it, creating its lid, represents the fetus; and the lid of that pot is the smallest pot, containing a perfume that represents the breath of life (Griaule 1965, 39). The smallest pot is capped by a normal lid; at this point the recursion "bottoms out." This is not merely a stack of different sizes; in the Dogon view the womb creates the preconditions that give rise to the fetus, which is the precondition for the entry of the breath of life. The recursion is emphasized in the way that each new pot begins before the previous pot ends (fig. 8.11b), that is, one pot's lid is the next pot's body (Griaule 1965, 199). In the sculpture in figure 8.11c the mother's breasts become the child's head—again, a new one begins before the previous one ends. As we saw in the chi wara sculpture of the Dogon's Bamana neighbors, reproduction is modeled as recursion.

The Dogon view of a cosmos structured as nested human-form is quite similar to certain ancient Egyptian representations. Figure 8.12 shows a relief from a tomb in which the cosmos encloses the sky, which encloses the earth. It is interesting to note that there are again three iterations of scale. A three-iteration numeric loop is indicated for the Egyptian god of wisdom, Thoth. He is referred to as Hermes Trismegestus, which means "thrice great Hermes," but he is also referred to as "eight times great Hermes." Why both three and eight? It makes sense if we think in terms of those common elements of African numeric systems, recursion and base-two arithmetic. Thrice great because while an ordinary human may rise as high as the master of masters, Hermes Trismegestus is the master of masters of masters (three iterations); thus we can surmise "eight times great" refers to $2^3 = 8$.

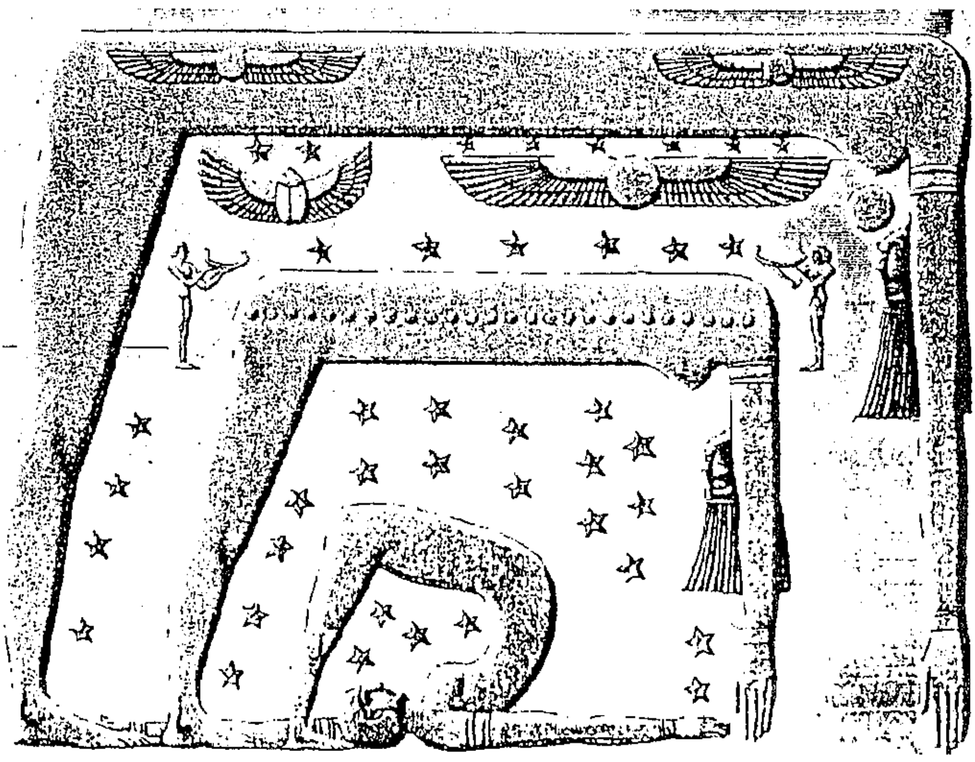


FIGURE 8.12

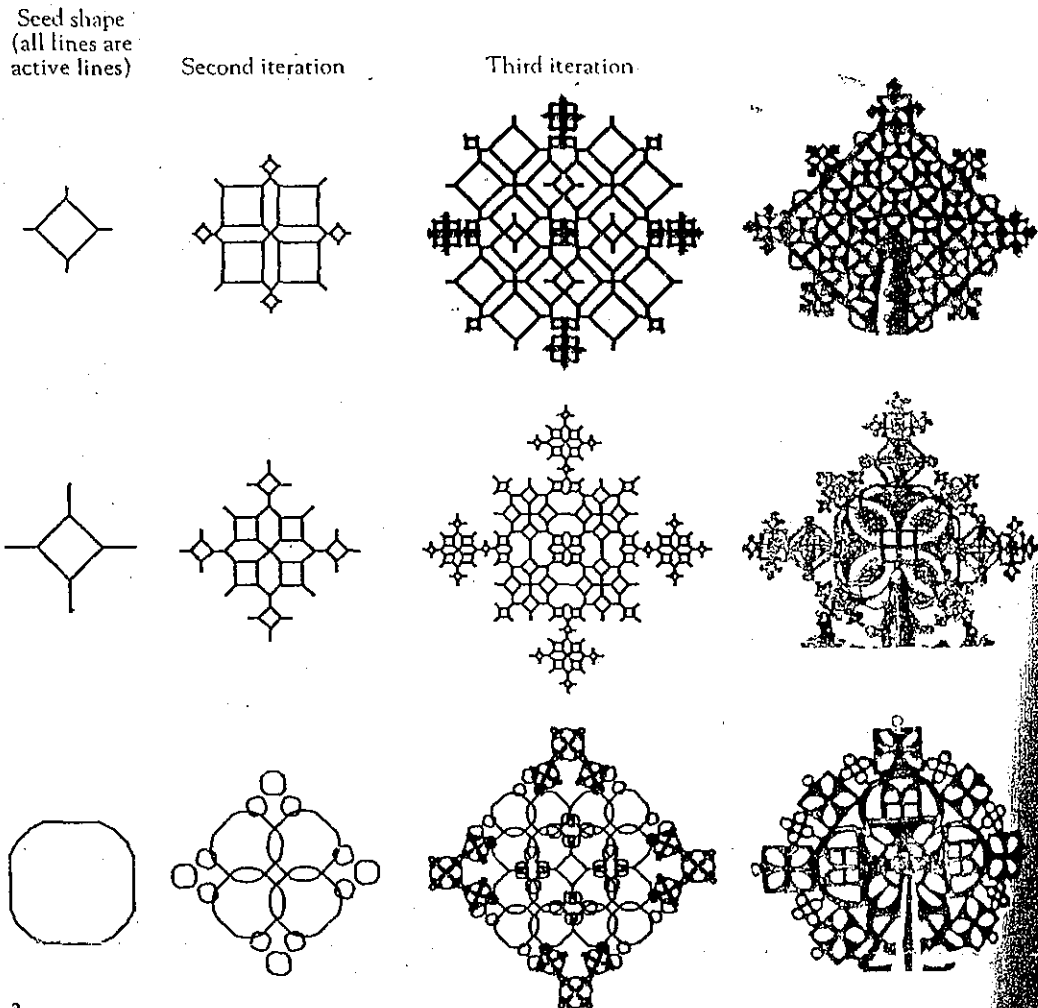
Recursion in the cosmology of ancient Egypt

Geb, the Earth, enclosed by Shu, space, enclosed by Nut, the stellar canopy. (*From Fourier 1821.*)

Many of the processional crosses of Ethopia also indicate a threefold iteration (fig. 8.13). Although the crosses are now used in Christian church proceedings, Perczel (1981) reports that related designs can be found on ornaments excavated from the city of Axum in northern Ethopia in the second half of the first millennium B.C.E., so we should not assume that the threefold iteration was originally related to the Christian trinity, although a connection may have occurred later (fig. 8.13b). Could there be a common history behind all these occurrences of triple iterations in the religious icons of the Sudan and North Africa? I think the common use of recursion itself is due to a mutual influence, but the occurrence of triple iteration may be only due to the similarity of circumstances rather than diffusion. For one thing, given the materials the artisans are working with, minute scales are difficult, so that the tendency to be limited to three iterations may simply be a practical consequence of the craft methods. It may also be that if one wishes to get the concept of iteration across, two is too few, while more than three is unnecessary (which is why modern mathematicians often represent an infinite series by the first three elements, e.g., "1,2,3 . . ."). On the other hand, there are cases where many such "unnecessary" iterations are made in the most difficult of craft materials. Figure 8.14 shows an ancient Egyptian design, carved in stone, representing the origin myth in which the lotus flower (its petals-within-petals illustrated by a multitude of scaling lines) begins the self-generating creation of the material world.

Self-reference

Self-reference is the most powerful type of recursion. The ability of a system to reflect on itself is at the heart of both the limits of mathematical computation as well as our subjective experience of consciousness. But there are relatively trivial applications of self-reference as well (one can always use a blowtorch to light a candle). Self-reference first came to the attention of mathematicians in simple examples of logical paradox; for example, the "liar's paradox" we examined earlier. To see how self-reference can be more than just a logician's joke, let's examine how it works in programming. Recall that a simple cascade could not be used if we did not know how many transformations were needed ahead of time. The same problem occurred for the Batammaliba ancestral mounds; since the first descendant did not know how many would be needed, the system has to allow for iterative resizing. We also saw the possibility of nested iterative loops, illustrated by the two-loop drawing program for Mokoulek architecture. But suppose we didn't know how many nested loops we were going to need? In the same way that the recursive cascade could not deal with an unknown number of iter-

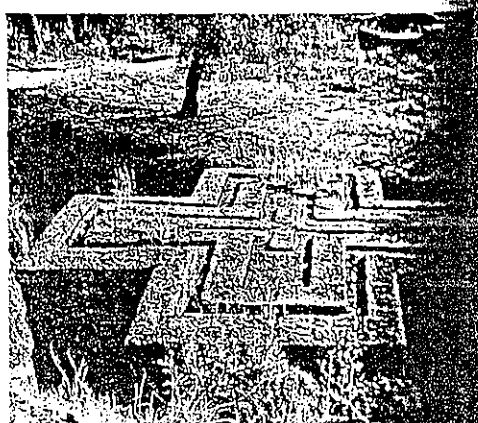


a

FIGURE 8.13
Fractals in Ethiopian processional crosses

(a) Fractal simulations for Ethiopian processional crosses through three iterations. (b) Ethiopia converted to Christianity in 333 C.E., and in the thirteenth century King Lalibela directed the construction of churches to be cut from massive rocks in one of the mountain regions. The church of St. George (at right) shows a triple iteration of nested crosses.

(a, *all Ethiopian processional crosses from Portland Museum in Oregon; photos courtesy of Csilla Perczel*, b, *photo by Georg Gerster.*)



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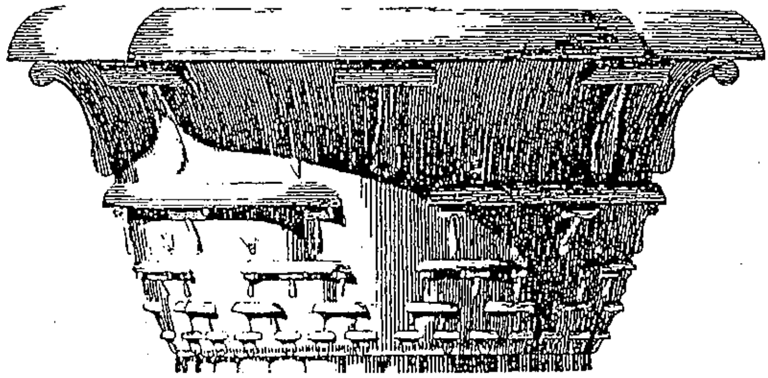


FIGURE 8.14
The lotus icon in ancient Egyptian cosmology
In the origin story of ancient Egypt the lotus flower was often used as an image of the unfolding of the universe, its petals-within-petals signifying the expansion of scales. This is a very stylized representation used in the capitals of columns in temples.
(From Fourier 1821.)

ations, nested iteration has trouble with an unknown number of loops.[10] Here is where self-reference can help out. An example of self-reference in programming is illustrated for the Dogon pot stack in figure 8.15.

We know that the Dogon pot stack can be drawn with a single iterative loop—it does not *require* self-reference. But the task can be accomplished by self-reference, and we might similarly ask if there are cases of scaling in African designs in which self-reference plays a role, regardless of whether it is required. In European history, self-reference begins with the story of Epimenides of Crete, the "liar's paradox." Similar utilizations of narrative self-reference to create uncertainty can be found in certain African trickster stories. For example, in an Ashanti story of Ananse (who became "Aunt Nancy" in African American folklore), a man named "Hates-to-be-contradicted" is tricked into contradicting himself. Pelton (1980, 51) notes that the application of such self-referential paradox is a theme in many Ananse stories: "Thus Ananse rejects truth in favor of lying, but only for the sake of speech; temperance in favor of gluttony for the sake of eating; chastity in favor of lasciviousness for the sake of sex." The following tale is not nearly as sparse but carries the flavor of self-referential paradox quite well:

> One of the most common of all stories in Africa describes the encounter of a man and a human skull in the bush. Among the Nupe of Nigeria, for instance, they tell of the hunter who trips over a skull while in pursuit of game and exclaims in wonderment, "What is this? How did it get here?" "Talking

FIGURE 8.15

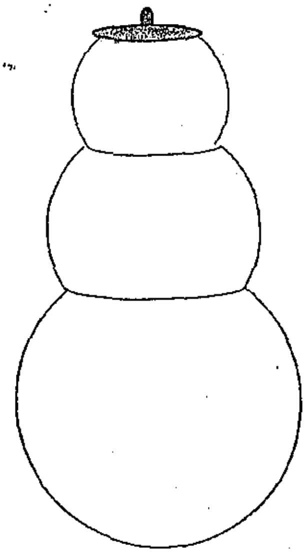
Drawing the Dogon pot stack by self-reference

The symbolism of the stacked pots represents the breath of life, within the fetus, within the womb. We have aready seen how this can be drawn using an iterative loop; now let's see how it can be drawn using self-reference.

Suppose we have a routine that can draw the semicircle of the pot given a diameter.

```
Procedure DRAW-POT
   If size = minimum, draw a lid.
   Else
      Draw a circle of size diameter
      Shrink diameter by ⅔
      DRAW-POT
   End of "else" clause
End of procedure
```

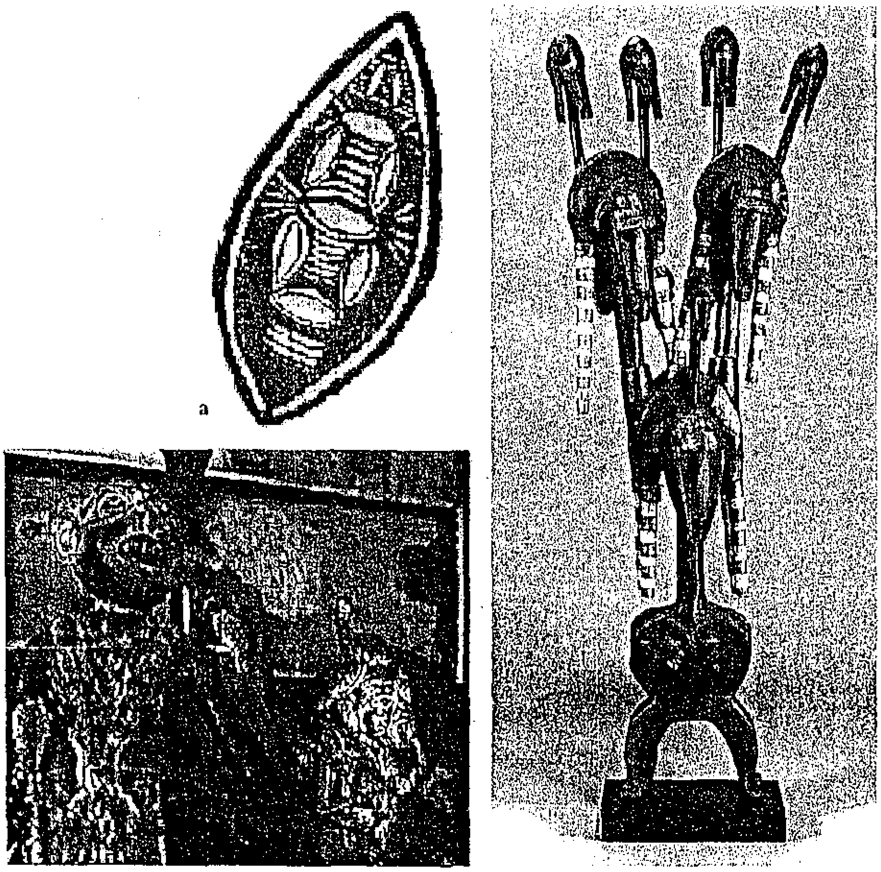
Notice that this procedure first checks to see if we are at the smallest diameter possible. If not, it draws a pot, shrinks the diameter value it by ⅔s, and then calls itself—an application of *self-reference*. Now the program has to execute a DRAW-POT procedure again. The recursion will "bottom-out" when it finally draws a lid. The program then skips to the "End of procedure" line and can finally pop back up to the place it left off after executing the previous DRAW-POT call.



brought me here," the skull replies. Naturally the hunter is amazed and quickly runs back to his village, exclaiming about what he has found. Eventually the king hears about this wonder and demands that the hunter take him to see it. They return to the place in the bush where the skull is sitting, and the hunter points it out to his king, who naturally wants to hear the skull's message. The hunter repeats the question: "How did you get here?" but the skull says nothing. The king, angry now, accuses the hunter of deception, and orders his head cut off on the spot. When the royal party departs, the skull speaks out, asking the hunter "What is this? How did you get here?" The head replies, "Talking brought me here!" (Abrahams 1983, 1)

Self-reference is also visually portrayed in some African designs. Figure 8.16a shows another abbia carving from Cameroon, seen also in the nested fish earlier in this chapter. But this abbia carving is an icon for itself—it is an abbia of abbia. According to the *Cameroon Cultural Review* (inside cover, June 1979), its meaning is "reproduction." Another example of self-reference from Cameroon is shown in figure 8.16b, a life-size bronze statue of the king of Foumban. Here we see the king smoking his pipe, the bowl of which is a figure of the king smoking his pipe, the bowl of which is a figure of the king smoking his pipe. Like the Kellogg's cornflakes box described earlier, the visual self-reference instantly leads to infinite regress. But it could be more than just humor in the bronze sculpture. Since the pipe is a well-known symbol of royal prestige in Foumban, it may be that the artisans were making purposeful use of the infinite regress: "The king's power is never-ending."

Figure 8.16c shows a Bamana headdress, that is, a sculpture worn on the head during ceremonies. Fagg (1967) suggests that this enacts self-reference: a headdress of a person wearing a headdress of a person wearing a headdress. Others (cf. Arnoldi 1977) have described this as a symbol of fertility spirits, but the two interpretations may not be mutually exclusive. Returning to the



b c

FIGURE 8.16

Self-reference in African icons

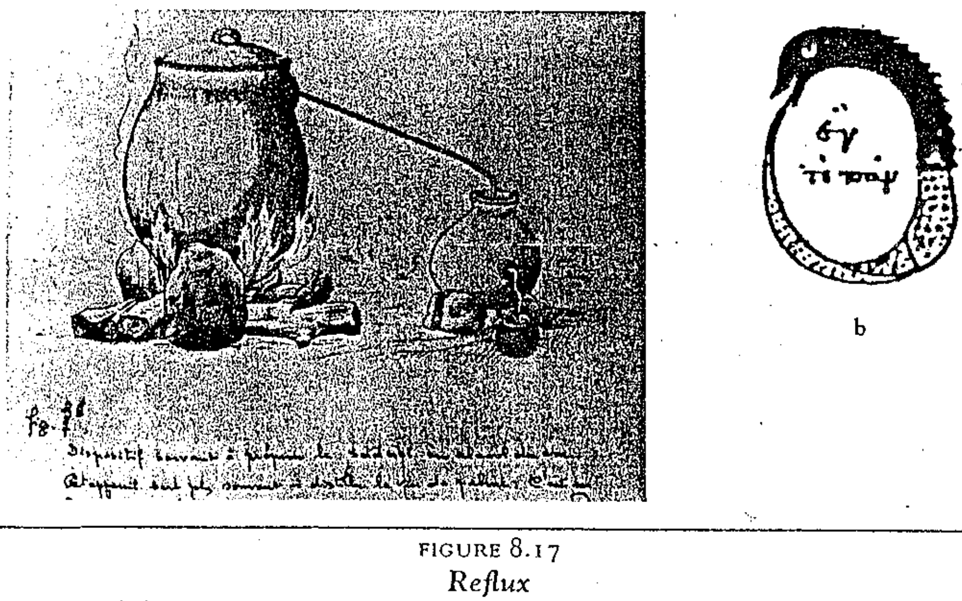
(a) The abbia carvings from Cameroon show a wide variety of images, but this abbia carving is an icon for itself—it is an abbia of abbia. (b) A life-size bronze statue of the king of Foumban. Here we see the king smoking his pipe, the bowl of which is a figure of the king smoking his pipe. (c) Bamana headdress.

(*a, drawing based on abbia pictured on the cover of* Cameroon Cultural Review, *1979; c, photo courtesy* [illegible]*ana University Museum of African Art.*)

Bamana's close cultural relatives the Dogon, we see self-reference suggested by Ogotemmêlli's description of how the eighth ancestor, "who was Word itself," was able to use Word (that is, the breath of life) to self-generate into the next iteration of humanity. In examining the self-similar iterations of the Dogon mother and child in figure 8.11c, we noted a structural characteristic that can be expressed in the phrase "a new one begins before the old one ends." This would also describe the structure of the pipe in the statue of the king of Foumban, which we know to be explicitly self-referential. Perhaps the self-referential version of the Dogon pot stack was the correct one after all.

Iconic representations of recursion

The abbia of abbia, as a symbol of "reproduction," is more than just an application of self-reference; it represents the concept itself. If recursion is really a conscious (that is, self-conscious!) aspect of African knowledge systems, then we should expect such representations, rather than just instances in which the concept is applied. Figure 8.17a shows the application of recursion in the tra-



a b

FIGURE 8.17
Reflux

(a) This sketch from the notebook of a nineteenth-century ethnographer in southern Senegal shows an indigenous apparatus for the distillation of liquor from palm wine using a scaling cascade.
(b) Ancient Egyptian alchemists drew this snake symbol to represent their reflux technique.
A tube comes out of a heated pot and reenters after cooling. This cyclic refinement was used in the creation of dyes and perfumes, but it also symbolized the alchemists' goal of refinement of the human soul.
(a, *photo courtesy IFAN, Dakar;* b, *drawing based on Taylor 1930.*)

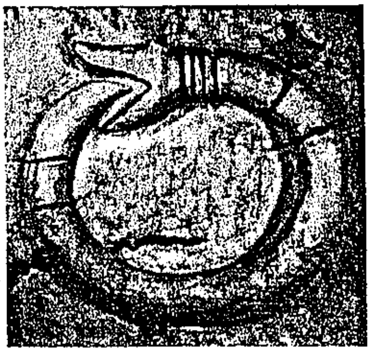
ditional distillation of palm wine into liquor in the Casamance region of Senegal. Such distillation techniques were developed to sophisticated levels in ancient Egypt, where the process became an iterative loop which modern chemists call a "reflux" apparatus. Figure 8.17b shows the iconic representation of the reflux system in the oldest known alchemical writings (first century C.E.), which are attributed to Maria (who wrote under the name of Miriam, sister of Moses), Cleopatra (not the famous queen), Comarius, and the mythic figure of Hermes Trismegestus (Thoth). Taylor (1930) notes that although these were written in Greek, "the religious element . . . links them to Egypt rather than to Greece," and he suggests that the most likely origin is from the traditions of the ancient Egyptian priesthood.[11] In these writings we find the reflux icon associated with the aphorism "as above, so below," recalling the self-similar scaling cosmology we have seen in sub-Saharan Africa, as well as its links to the recursion of self-fertilization.[12]

Of course, one can go too far in attributing links between ancient Egypt and sub-Saharan Africa (see Oritz de Montellano 1993; Martel 1994; Lefkowitz 1996). There is good evidence for the origins of the Egyptian base-two arithmetic system from sub-Saharan Africa, and for the persistent use of recursion in knowledge systems across the African continent. But it would be unwise to assume that one can attribute more specific features to diffusion. In particular, it is highly unlikely that the same figure of a serpent biting its tail, appearing as an icon for the god Dan in the vodun religion of Benin (fig. 8.18a) could have derived from the Egyptian image, or vice versa. As we shall see, the meaning of the vodun icon has nothing to do with the Egyptian reflux concept.

In August 1994, thanks to the aid of Martine de Sousa (one of the African descendants of the famed Francisco de Souza), I was granted an interview with the chief of the Dan temple in Ouidah, Benin. Both the chief and his wife were quite responsive to my interest in the geometric features of Dan's representations and identified the sinusoidal icon in iron (fig. 8.18b) as "Dan at work in the world," pointing out that he creates order in wind and water. The cyclic Dan was more abstract, existing in a domain where he was in communication with other gods of vodun. Maupoil (1981, 79) also found that Dan (Dangbe) was there "to assure the regularization of the forces," and Blier (1995) summarizes his role as "powers of movement through life, and nature's blessings." Regular phenomena in nature—the periodic aspects of weather, water waves, biological cycles, etc.—are attributed to the action of Dan.

The relation between the undulatory Dan "at work in the world" and the circular form of Dan as a more abstract spiritual force maps neatly on to the difference between the sinusoidal waves we see in space and time—waves in

a



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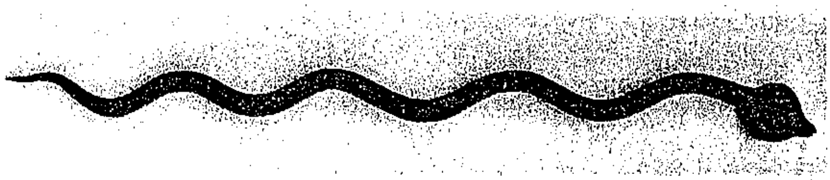
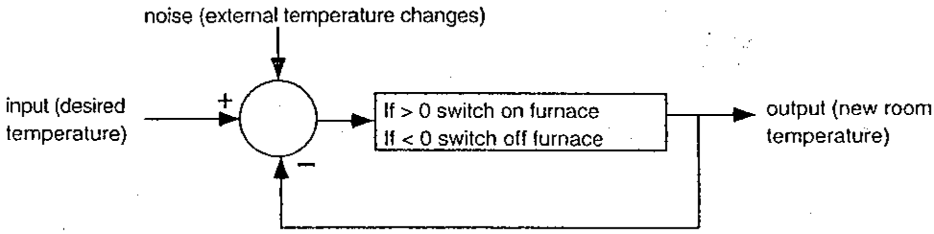
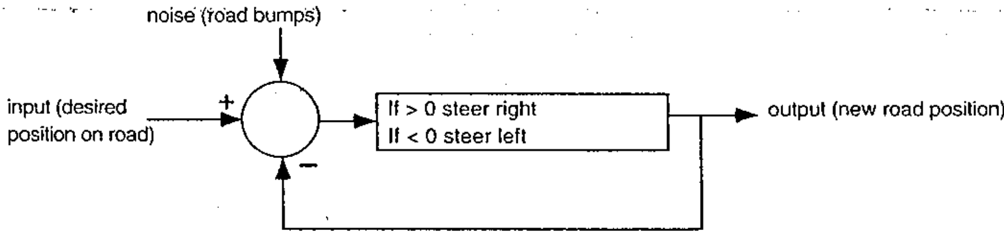




The thermostat that regulates temperature in a house is a negative feedback loop. The word "negative" is used because we subtract the current room temperature from the desired temperature set by the thermostat control. Over time this will tend to produce cycles of heat and cold.



Driving a car can also be modeled by a negative feedback loop. The driver attempts to stay in the center of the lane, and will correct to adjust for bumps. Again, given enough bumps, we will tend to see cycles of swerving to get back to the center.

c

FIGURE 8.18

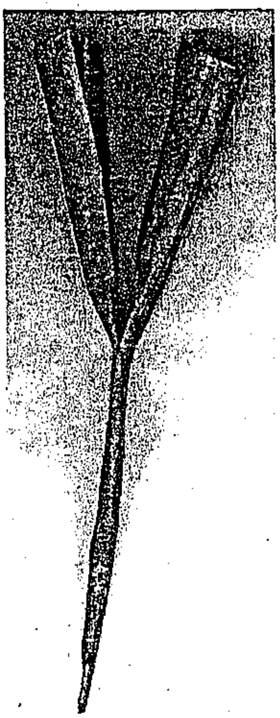
The vodun god Dan

In the vodun religion of Benin, the snake god Dan represents the cyclic order of nature. Dan's shape reflects this idea in two ways. As an abstract force, he is represented as a feedback loop (a). As a concrete manifestation, his body is always oscillating in a periodic wave (b). This same idea of a periodic time series from cyclic feedback is also used in Western models of nature (c).
(a, *photo courtesy IFAN, Dakar.*)

water and cirrus clouds, daily fluctuations in heat and light, the biannual rainy seasons, etc.—and the abstract idea of an iterative loop that generates these waveforms. The association can be derived from the kind of empirical observation one gets in everyday occurrences. A lopsided wheel will produce undulatory tracks in sand; friends who periodically give gifts are in a "cycle of exchange," and so forth. What did take great insight and intellectual labor, however, was the religious practitioners' generalization of such observations into specific, abstract, universally applicable categories, represented by icons with the appropriate geometric structure.

The mathematical equivalents in nonlinear dynamics are limit cycles and point attractors—the results of what engineers call a "negative feedback loop." We have already seen such characterizations in cellular automata and owari, where spatial patterns remain bounded within a cycle or frozen in a static pattern. Figure 8.18c shows some commonplace examples of negative feedback loops, and how they act to keep the behavior of systems bounded or stabilized, even in the presence of noise. But the vodun system would not be complete if it could only account for regularity—what causes deviation in the first place? Hence the role of Legba, god of chaos. Figure 8.19a shows another iron icon, the forked path of Legba, "god of the crossroads." As explained to me by Kake S. Alfred, a divination priest of vodun in Cotonou, Benin, Legba is represented by the fork because "the answer could be yes or no; you don't know which path he will take." For divination, in which a "path" (question) is often pursued for further questions, the image becomes one of endless bifurcations. At the Palais Royal in Porto Novo, Benin, I was told that the shrine to Legba was placed at the threshold because his force was so disruptive that it would undo both good and evil, creating a purification at the entrance. Kake also explained that while the music of Dan was slow and regular, the music of Legba was both fast and slow—signifying his unpredictable nature—an observation I was able to confirm by recording the drumming that was used to call each god at the temple of Dan in Ouidah.[13] As the converse to Dan, the bifurcating uncertainties of Legba are like a positive feedback loop, amplifying deviation and noise (fig. 8.19b).

Contrasts between a negative feedback loop, creating stability, and the positive feedback of uncontrolled disorder are also featured in the iconic carvings of the Baule. Vogel (1977, 53) notes that the Baule chief is chosen by consensus, and that in all important decisions he serves as mediator in public meetings rather than as an autocrat. The Baule carving in figure 8.20a shows two caimans (relatives of the alligator) biting each other's tails. It is said to represent the chief and the people in balance—if one bites, the other will bite back. It nicely recalls the kinds of negative feedback loop models that are often proposed in West-

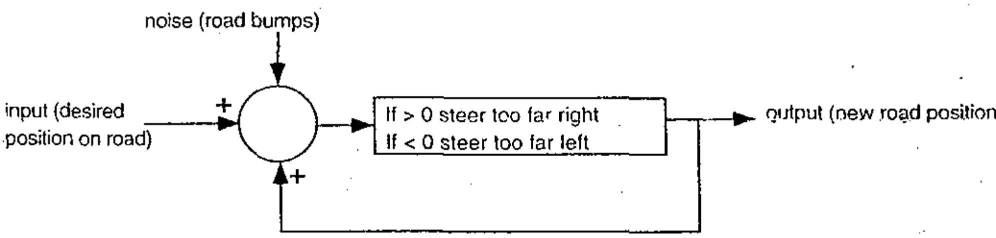


a

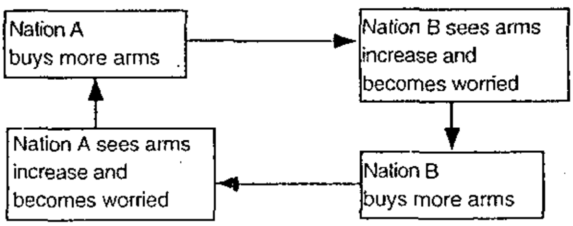
FIGURE 8.19
Legba

(a) The vodun god Legba represents the forces of disorder. Vodun divination priests explain this icon as the path to the future: with Legba there is no way to know which path will be taken. *Since one crossroad leads to another, the resulting image* is one of bifurcating unknowns, the uncertainty multiplying with each crossroad.

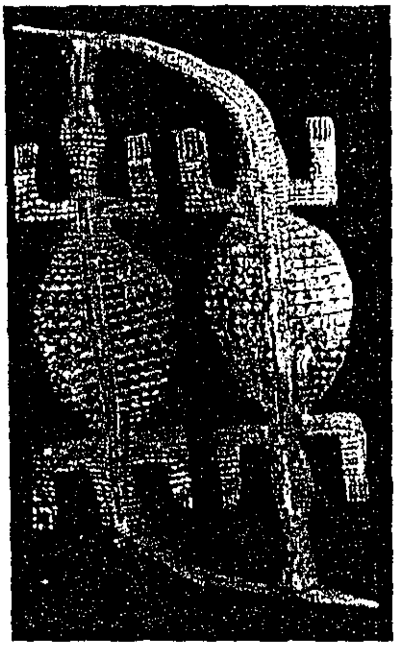
b



In contrast to negative feedback, which will help stabilize a system, positive feedback will destabilize it. A drunken driver, for example, can overshoot the center line and create increasingly large oscillations, eventually running off the road.



Here we see positive feedback in an arms race.



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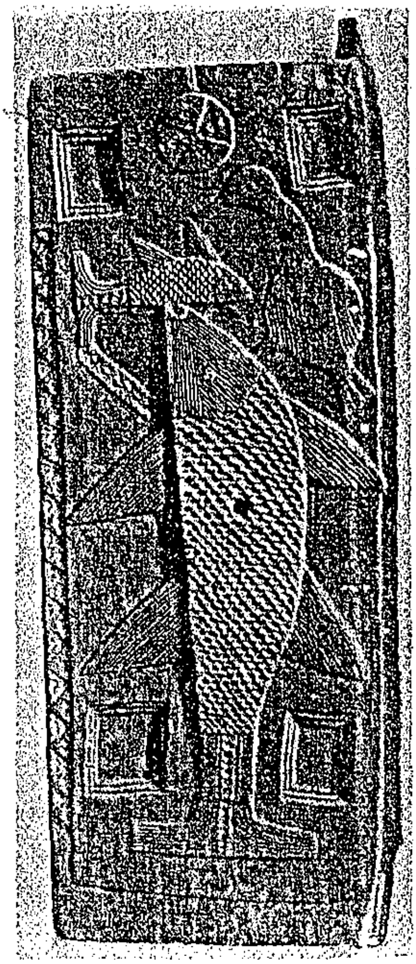


b

FIGURE 8.20
Feedback loops in Baule iconography
(a) This Baule carving shows two crocodiles biting each other's tails. It is a symbol showing the chief and the people in equal power, the idea of social forces in a cycle of balance. (b) Baule door. Holas (1952, 49–50) describes this as a *circuit fermé* of *fécondité* (closed circuit of fecundity); Soppelasa (1974) and Odita (1971) identify these animals as symbols of "increase." (a and b, photo courtesy of IFAN, Dakar.)

ern political theory, but this flowchart is a purely indigenous invention. So, too, is the Baule positive feedback loop of figure 8.20b, showing that "power creates the appetite for more power"—little fish are eaten by bigger fish, who then become even bigger fish. The fish-within-fish abbia from Cameroon we saw earlier may have had similar connotations.

Conclusion

Recursion can be found in almost every corner of African material culture and design, from construction techniques to esthetic design, and in cultural representations from kinship to cosmology. Most of these are specific enough to allow us to distinguish between the first two types of recursion—cascade versus iteration—and in some cases the third type, self-reference, is also made explicit by the indigenous knowledge system. We have seen several cases in which the iterative loops are nested, but these are rarely more than two loops deep, so it would not appear that the application of self-reference is motivated by the complexity of the computation. The only potential exception is the cosmological narrative of the Dogon, and this narrative is too vague to serve as a mathematical foundation. There is, however, another route to the limits of computation. As we will find in chapter 10, the combination of negative and positive feedback indicated by certain recursion icons provides another path to the heights of computational complexity, one we will explore in detail. But first, we need to take a short detour through infinity.

CHAPTER
9
Infinity

The first time I submitted a journal article on African fractals, one reviewer replied that Africans could not have "true" fractal geometry because they lacked the advanced mathematical concept of infinity. On the one hand, that reviewer was wrong about fractals at a pragmatic level. If he or she saw a fractal on a computer screen it would be taken as a "true" example, and in fact no physically existing fractal is infinite in its scales; at best it will have to bottom out into subatomic particles. On the other hand, it raises an interesting question. Infinity has been an important part of fractal mathematics in Europe; so how does that compare to the use of infinity in Africa?

To the ancient Greeks, infinity was associated with what they thought of as the horrors of infinite regress. Aristotle tamed this problem by redefining infinity: it was a limit that one could tend toward, but it was not considered to be a legitimate object of mathematical inquiry in itself. Most European mathematicians kept to this definition until the Cantor set, Europe's first fractal, created the proper definition of an infinite set, thus allowing infinity itself to be considered. We will discuss this in more detail in chapter 13, but for now it is sufficient to note that this distinction does not shape African concepts of infinity. Many African knowledge systems using infinity in the sense of a progression without limit do not hesitate to represent it with iconic symbols suggesting "the infinite" in its Cantorian sense as a completed whole. This is by no means a more sophisticated or elaborated definition than that of pre-Cantorian European mathematics; it is rarely linked to much more than either a narrative or a geometric visualization. But far from being nonexistent, these culturally specific representations show a strong engagement with the same concepts that coupled infinity and fractals in contemporary Western mathematics.

The most common African visualizations for infinity are snail shells. The Baluba, for example, use spiral land snails (fig. 9.1), and the Jola use the spiral end of a sea snail, which forms a drinking cup that can only be used by the chief. Unlike the ancient Greek associations with troubling paradox and pathology, the African infinite is typically a positive association, in this case to invoke prosperity without end. If these infinity icons were only meant to communicate this desire they would fit Aristotle's definition: a process without end. But the spiritual element of these icons adds another requirement: the icons need to convey the sense that they are drawing on the power of infinity itself. Snail shells are used because of the scaling properties of their logarithmic spirals; one can clearly see the potential for the spiral to continue without end despite its containment in a finite space—indeed, it is only because of its containment in a finite space that there is a sense of having gained access to or grasped at the infinite.

We have already seen another example of an infinity icon in the Nankani architecture discussed in chapter 2. There the coils of a serpent of infinite

FIGURE 9.1
Baluba use of snail shells to symbolize infinity
Davidson (1971, 120) describes this as a fertility figure, and notes that the snail shells represent endless growth.
(Collection Tristan Tzara, Paris; photo by Eliot Elisofon.)

length, sculpted into the house walls, made use of the same association between prosperity without end, and a geometric length without end. The conscious creation of this infinity concept is more clear than in the case of the snail shells, because one cannot actually see the infinite coils of the snake. And unlike the naturally occurring shells, the packing of this infinite length into a finite space (the Nankani describe it as "coiling back on itself indefinitely") cannot be mistaken for mere mimicry of nature; it is rather the artifice of fractals. This snake icon does not exist in isolation; we saw that the Nankani map out a scaling progression that passes through their architecture, the *zalanga* and the *kumpio*, which provides a recursive pathway to this concept of infinity.

In chapter 8 we discussed the Mitsogho and Fang iterative model of descent. Fernandez (1982, 338) notes the contrast to Christian theology: "The question as to whether God was one or many may have bothered the missionaries in their contacts with Fang more than the Fang themselves. Holding Christian beliefs in the 'Uncreated Creator' and 'Unmoved Mover,' missionaries were challenged by the 'infinite regress' of the genealogical model employed by the Fang—their belief that the God of this world is one of a long line of gods and like man has his own genealogy."

The Fang theory of infinite regress is a complete, coherent view; it does not need further amendment, for the Christian theory of uncreated creator is no more free of contradiction—and perhaps less so. Of course, as Fernandez himself warns, one cannot simply proclaim that a particular African narrative is just another work of theology or philosophy—or, for that matter, mathematics. Recent works such as Mudimbe's *Invention of Africa* (1988) have shown that such translations to specific European disciplines are always partial, highly interpretive, and in danger of misrepresenting the indigenous view. Yet Mudimbe is also respectful of the work that has been done. Of particular relevance here are his citations of African theologian Engelbert Mveng.

Mveng included several notes on infinity in his studies of the relation between the African and Christian views. His beautiful text, *L'Art d'Afrique Noire* (1964), contains diagrams (pp. 100–103) showing what he termed "radiation amplificative," scaling patterns in African art and music that he interpreted as representations of a transcendental path to infinity. "Une fois de plus, nous découvrons que le mouvement rythmique, dans notre art, n'est autre chose qu'une course vers l'infini" (Once again, we discover that the rhythmic movement in our art is none other than the path toward infinity) (p. 102). Father Mveng was a wonderful inspiration during my research in Cameroon, both for his deep cultural knowledge as well as for his courageous work as a cross-cultural mediator. During our last meeting we discussed Mudimbe's book, and I promised to send him a copy. Shortly after doing so a reply came from the American Cultural Center in Yaoundé: Mveng had been murdered "under suspicious ?! circumstances"—apparently the result of opposition to his cross-cultural activism. He has finally taken the *course vers l'infini.*

CHAPTER

10

Complexity

In ordinary speech, "complex" just means that there is a lot going on. But for mathematicians the term is precisely defined, and it gives us a new way to approach mathematics in African material culture. In chapter 7 we saw how certain African symbolic systems, like the Bamana divination code, could be generated by a recursive loop. Such numeric systems clearly translate into the Western definitions of what it means to "compute." But the translation was less clear for some of the physically recursive structures in African material culture. Can a system of physical dynamics be said to "compute"? Mathematical complexity theory, which is based on fractal geometry, provides a way to measure the computation embedded in physical structures, rather than just symbol systems. By looking at African material culture in the framework of complexity theory, we can better understand the presence of fractal geometry as an African knowledge system.

Analog computing

By the mid-1960s it was clear to many researchers that digital computers would be the wave of the future. But before then, analog computers held their own, and they may yet make a comeback. In digital systems, information is represented by physically arbitrary symbols. As Bateson (1972) said, "There is nothing sevenish about the numeral seven." The geometric structure of a digital symbol has little or nothing to do with its meaning, which is simply assigned to it by social convention. In analog systems, the physical structure of the signal changes in proportion to changes in the information it represents.[1] Rather than being arbitrary, the physical structure is a direct reflection of its information. Loudness in human speech is a good example of analog representation. As I get more excited, I speak louder: the physical parameter changes in proportion to the semantic parameter. This is not true for the digital parts of speech, such as the average pitch ("fomat frequency") of each word. In English the word "cat" has a higher pitch than the word "dog," but that does not infer a relation in meaning—in fact, the difference is reversed in Spanish, since "gato" has a lower average pitch than "perro." This same analog/digital distinction occurs in neural signals. In the frog retina, for example, some neurons have a firing rate in proportion to the speed of small moving images (Grusser and Grusser-Cornehls 1976). That is, the faster a fly moves across the eye, the faster the pulses of the neuron: an analog system. A digital example can be found in the motor neurons that fling open the crayfish claw. Here a specific firing pattern (off-on-on-off) switches the claw to this defense reflex (Wilson and Davis 1965).

So far we have only examined how analog systems can represent information; figure 10.1 shows a simple example of how analog computing works. Although most computer scientists eventually settled on digital systems, analog computers were quite popular up until the 1960s. Even when they began to die out as practical machines, there was an increasing awareness that much of our own brain operates by analog computing, and this led some scientists toward the development of what are now called "neural nets"—computing devices that mimic the analog operations of natural neurons (fig. 10.2). By the mid-1980s neural nets and related analog devices had achieved enough success (and digital computers had run into enough barriers) to begin to compare the two. There was an odd moment of analog optimism, when a few brash claims were made about the potential superiority of analog computing (see Dewdney 1985; Vergis et al. 1985), but these assertions were eventually proved incorrect (Blum, Shub, and Smale 1989; Rubel 1989). As it turns out, analog systems have the same theoretical limits to computing as digital systems.[2] Although the studies did not result in releasing the known limitations, they did produce a new framework for thinking about computing in physical dynamics: complexity theory.

Before this time, mathematicians had defined complexity in terms of randomness, primarily based on the work of Soviet mathematician A. N.

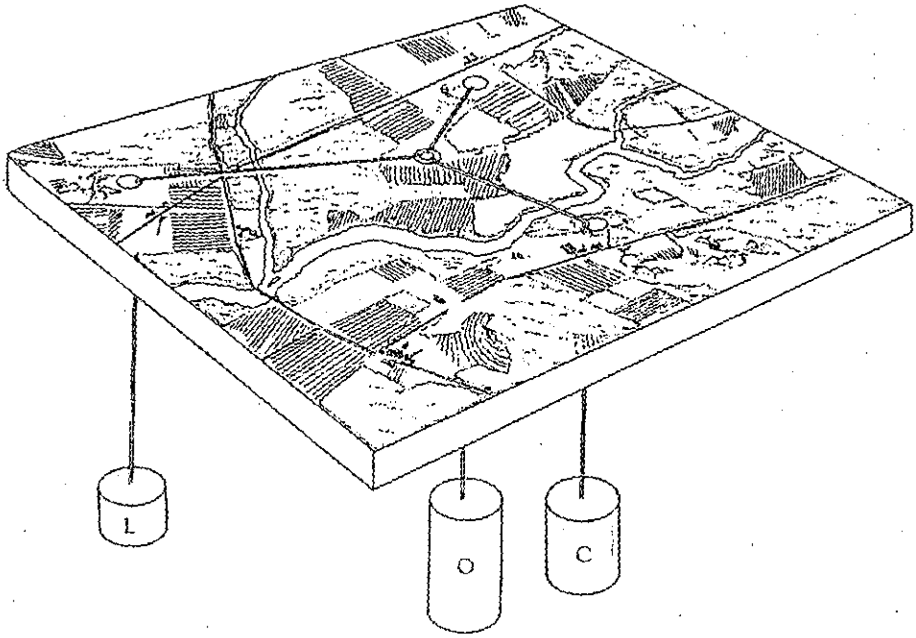


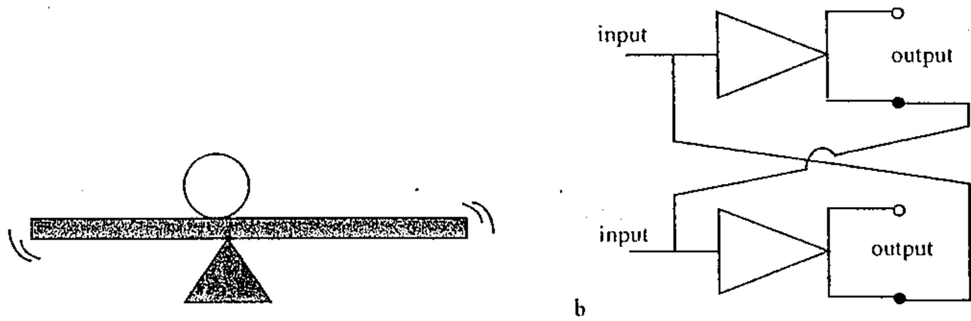
FIGURE 10.1
Analog computation

Dewdney (1985) shows a great variety of simple physical devices that demonstrate analog computing. This device, created by J. H. Lueth of the U.S. Metals Refining Company, solves the following optimization problem: a refinery must be located to minimize its costs. If transportation in dollars per mile of ore, coal, and limestone are values of O, C, and L, and distances of these sources are o, c, and l, then the refinery should be located at the point where oO + cC + lL is at a minimum. The holes through which the strings pass are at the source locations, and the weights on the ends of the strings are proportionate to O, C, and L. The brass ring attached to the strings quickly moves to the optimal location on the map.
(Courtesy A. K. Dewdney.)

Kolmogorov and Americans Gregory Chaitin and Ray Solomonoff. In this definition, the complexity of a signal (either analog or digital) is measured by the length of the shortest algorithm required to produce it (fig. 10.3). This means that periodic numbers (such as .2727272 . . .) will have a low algorithmic complexity. Even if the number is infinitely long, the algorithm can simply say, "Write a decimal point followed by endless repetitions of '27,'" or even shorter: "3/11." Truly random numbers (e.g., a string of numbers produced by rolling dice) will have the highest algorithmic complexity possible, since their only algorithm is the number itself—for an infinite length, you get infinite complexity. In analog systems a periodic signal such as the vibration from a single guitar string or the repetitive swings of a pendulum would have the lowest algorithmic complexity, and random noise such as static from a radio that has lost its station (what is often called "white noise") would have the highest algorithmic complexity.

One problem with defining complexity in terms of randomness is that it does not match our intuition. While it's true that the periodic signal of a ticking metronome is so simple that it becomes hypnotically boring, the same could be said for white noise—in fact, I sometimes tune my radio between stations if I want to fall asleep. But if I want to stay awake I listen to music. Music somehow satisfies our intuitive concept of complexity: it is predictable enough to follow along, but surprising enough to keep us pleasantly attentive. Mathematicians eventually caught up with their intuition and developed a new measure in which the most complex signals are neither completely ordered nor completely disordered, but rather are halfway in between. These patterns (which include almost every type of instrumental music) also happen to be fractals—in fact, as we will see, the new complexity measure exactly coincides with the measure of fractal dimension.

The first step in this direction was through studies of cellular automata. Recall from chapter 7 that computer scientists in the early 1980s had started to think

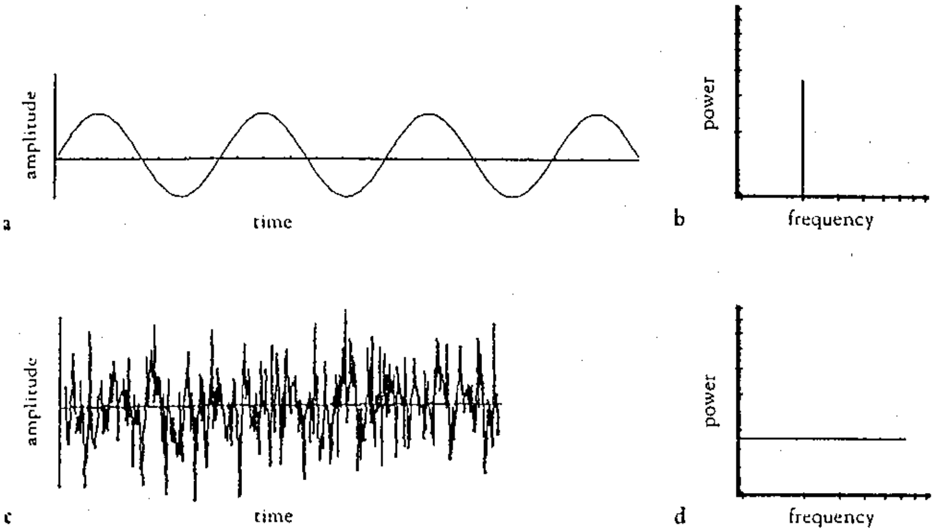


a b

FIGURE 10.2
Neural nets

(a) Suppose we balance a ball on a teeter-totter. Unless the ball is at the precise center, the *teeter-totter will start to slope toward one side, which will cause the ball to roll even farther* toward that side. In other words, there are two stable states, and anything in between (except fo tiny neutral point) will get caught up in the positive-feedback loop leading rapidly to a stable st (b) This is an electrical circuit that works much like the teeter-totter. Each triangle is an amplit with two outputs, one normal and the other (black circle) an inverted output. Since the inverte output is connected to the input of the other amplifier in each, they will balance out like the ba at the exact center of the teeter-totter, but rapidly flip to one of the two stable states in which t amplifier is at its maximum ("saturated"). That means that this circuit can solve a simple task: which of two numbers is larger? By putting an initial charge proportionate to one of the two numbers at each input, the system rapidly flips to the saturated stable state favored by the large number. Linking thousands of these simple amplifiers together allows computer scientists to ma sophisticated machines for pattern recognition and other artificial intelligence tasks.

about cellular automata as the simulation of complicated physical dynamics, such as that seen in living organisms. Physicist Stephen Wolfram began to wonder: just how complicated is it? Clearly, living systems are more complex than random noise, so he knew that the old complexity measure of Kolmogorov would not do. But Wolfram had studied a good deal of computer science, and he realized that the way in which different types of recursions are used to measure computing power could also be applied to physical dynamics. Recall from chapter 8 that we divided recursion into three types: cascades, iterations, and self-reference.



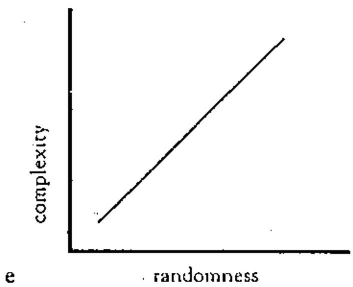


FIGURE 10.3

Kolmogorov-Chaitin complexity measure

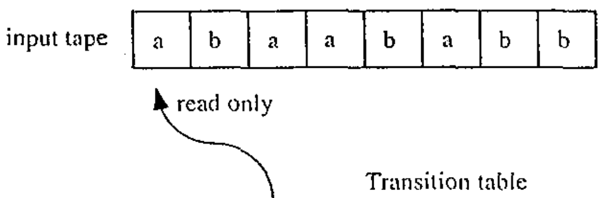
(a) Whether it is in digital or analog signals, complexity can be measured in terms of the information content. The first such measure was that of Kolmogorov and Chaitin, who thought of complexity in terms of randomness. The sine wave is about as nonrandom as we can get. Here it is given as a time-varying signal, although the same would apply to a spatial pattern, such as waves in water or sand (in which case we could measure it as wavelength, which is simply the reciprocal of frequency). (b) The same signal in a spectral density plot. This tells you how much power is at each frequency. In the case of the sine wave, all the signal power is at one frequency. (c) White noise is a completely random signal, such as that produced by the sound of bacon frying. By the Kolmogorov-Chaitin definition, white noise is the most complex signal. Again, this would also apply to a spatial pattern, such as dust sprinkled on a table. (d) Spectral density plot for white noise. Because it is completely random, there is an equal likelihood of any wavelength occurring at any time, so the signal's power is equally distributed across the spectrum. (e) In summary, the Kolmogorov-Chaitin complexity measure is simply a measure of randomness. (c, *courtesy R. F. Voss.*)

These correspond approximately to the three formal categories of recursion used in computer science, which we will now examine in detail.

Three types of recursion: the Chomsky hierarchy

In a recursive system, present behavior depends on past behavior. It is the capability of this access to memory that defines the relative difference in recursive power. The scaling cascade, for example, could not produce the Fibonacci sequence, because it could not recall previous members of the sequence. Similar distinctions are used in computer science to rank computational power into three types of abstract machines, referred to as "Chomsky's hierarchy." These abstract machines are compared by their ability to recognize certain categories of character strings. A machine that can recognize periodic character strings such as "ababa . . ." occurs at the lowest level of the hierarchy: the Finite State Automaton (FSA). An example of the FSA is shown in figure 10.4.

What would it be like to *be* an FSA? Since the FSA has no memory storage, the experience would be somewhat analogous to neurosurgery patients who have had bilateral hippocampal lesions (Milner 1966). These patients are fully aware and intelligent but have lost the capacity to transfer knowledge to long-term memory. The hippocampal surgery patient who finds herself at the end of a book can deduce that she has read its contents, although she does not know what the previous chapters were about. An FSA has only an *implicit* memory, FSA because its present state cannot reveal anything about its past, other than the fact that it must have passed through one of the sequences of states that terminate in the present state.



Transition table

Current state	Current symbol on input tape	New state
S_1	a	S_1
S_1	b	S_2
S_2	a	S_2
S_2	b	S_1

FIGURE 10.4
The finite state automaton
The finite state automaton (FSA) has a list of transition rules that tell it how to change from one state to the next, depending on its current state and the symbol it is reading on the input tape. It has no memory, other than that implied by its curent state. This FSA will end up in the "accept" state S_1 if the tape ends after an even number of b's.

The set of palindromic strings (e.g., aabbaa) is a good example of the limitation of the FSA: it lacks the ability to memorize the first half of the string and therefore cannot compare it with the second. The least powerful machine capable of this memory storage is the Push-Down Automaton (PDA), illustrated in figure 10.5. The stack memory of the PDA is usually compared to the spring-loaded tray stack often used in cafeterias; once a symbol is read from memory it is gone. As a knowledge analogy, we might think of a reader who accumulates stacks of books but gets rid of each book after it is read. This is a temporary explicit memory, since the PDA can make two different transitions given the same state and input, depending on its past. It is important to understand that greater recursive capability does not necessarily imply *larger* memory storage; it means an improved ability to *interact* with memory. Size only matters insofar as it restricts the interaction. PDA

Although the PDA can recognize all sets of strings recognized by an FSA, as well as many others, there are still (infinitely) many sets of strings that it cannot recognize. For example, it cannot recognize the set of all strings of the form $a^N b^N c^N$ (where we have N repetitions of a, followed by the same for b and c), because it has to wipe out its memory in the process of comparing the number of a's and b's, leaving no information for checking the number of c's.

At the top of the hierarchy (fig. 10.6), the Turing Machine (TM) can recognize all computable functions. It is simply a PDA with unrestricted memory, but because of this capability it can achieve full self-reference: the ability to analyze its own program. Again, it is not a difference in memory size, but in memory access—unlike the PDA stack, the TM memory interactions can occur over any past sequences of any length, and it does not lose memory TM

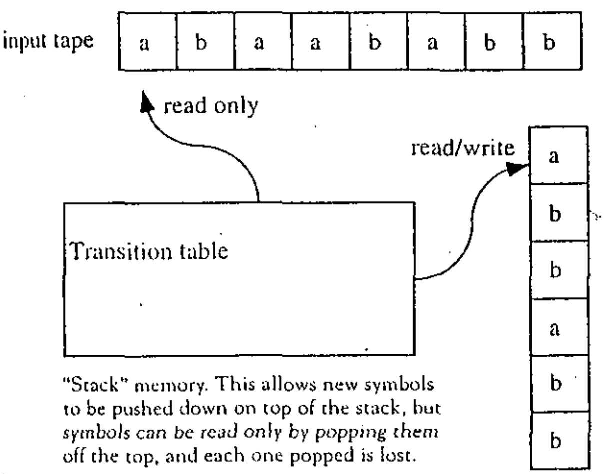


FIGURE 10.5
The push-down automaton
The push-down automaton (PDA) has a list of transition rules, but these make use of an explicit memory storage as well as internal states.

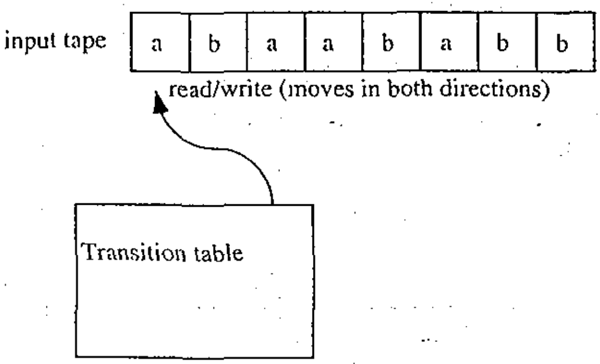


FIGURE 10.6
The Turing machine
The Turing machine has an unconstrained memory; it can implement any algorithm that can possibly exist.

after it is read. To continue the text analogy, if the FSA is a person who accomplishes tasks with no books, and the PDA is a person whose simple tasks are limited to books that are removed after they are read, then the TM would be able to collect and recall all books, in any order. Unfortunately this does not solve all of our problems, because the unbounded nature of the TM means that it foolishly accepts some tasks that require an infinite library. This is called the "halting problem," and Turing himself proved that it is unavoidable. Mathematician Rózsa Péter showed that one can define a restricted set of programs that are haltable (which she called the set of "primitive recursive functions"), but in doing so we would always sacrifice some of the TM's computing power.

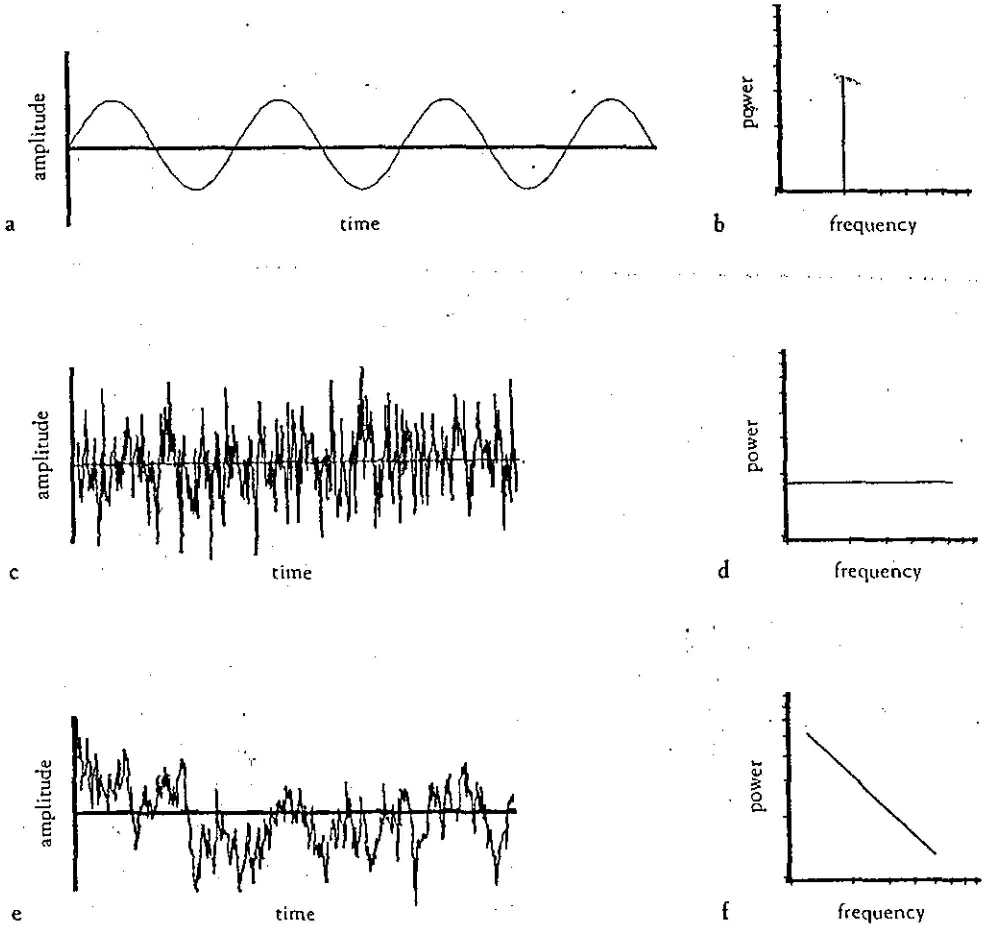
These three machines, FSA, PDA, and TM, illustrate the ascent up the Chomsky hierarchy. They differ in having implicit memory, temporary explicit memory, and permanent explicit memory. By looking at memory as the basis for the recursive loop in these systems—that is, as the element that governs the ability of the system to perform interactions between its present input and past behavior—we can see that the differences in computational power for these machines depends on the differences in recursive power.

Measuring analog complexity with digital computation

Now let's return to Wolfram and his cellular automata. After running thousands of trials, Wolfram found that all cellular automata generally divided into four specific classes. Classes 1 and 2 were those that either died out, or went into a periodic cycle. Class 3 was just the opposite: it was uncontrolled growth that led to apparently random behavior, like white noise. But class 4, which included the "game of life" cellular automaton, had something that Wolfram described as "complex" behavior: not as random as white noise, but not as boring as a periodic cycle. Wolfram found that this highest complexity also demanded the highest computability: while pure order and pure disorder could be recognized by an FSA, the patterns of the complex behavior required a Turing machine.

Mathematical physicist James Crutchfield (1989) found an even simpler example of recursive computation in a physical system. Crutchfield used the population equation made famous by biologist Robert May (1976): $P_{n+1} = P_n R(1 - P_n)$ (where P is a population number, scaled so that it is between 0 and 1, and R is the birth rate). May found that when R is low, the population is simply a periodic cycle, switching back and forth between the same sequence of levels. As you increase R, the length of the cycle (that is, the number of different population levels you pass through before returning to the first one) increases extremely fast. At $R = 3.1$, the population is in a two-level cycle, at $R = 3.4$ in a four-level cycle, and at $R = 4.0$ the cycle length is at infinity: deterministic chaos. Crutchfield was able to measure the computability of these chaotic fluctuations and found results similar to those of Wolfram: both completely periodic waves and completely disordered waves were computationally quite simple, but those in between, with a mix of order and disorder, had a high degree of computational complexity. The simple equation examined by Crutchfield required only a PDA, but other researchers (Blum, Shub, and Smale 1989) demonstrated that more complex analog feedback systems would be capable of signal complexity equivalent to TM computability.

Figure 10.7 shows how these complex waveforms, called "1/F noise," compare to periodic and white noise waveforms. This is easiest to see in the spectral density plots. A periodic signal has all its power at one wavelength, while a white-noise signal has the same power at all wavelengths. 1/F noise is a compromise between the two—biased so that it has the greatest amount of power at the longest wavelength, and the least at the shortest. For this reason, 1/F noise is fractal; it has fluctuations within fluctuations within fluctuations. When we think of the length of these waveforms in terms of memory, we can begin to see a connection to computational power. If a system had the same behavior over and over again, it would be too fixed on memory. If it randomly picked a new behavior every time, then it would be too free from memory. But useful behavior is generally a mixture between the two. For example, think of something unusual you did today—moving socks to a new side of the drawer, or eating pretzels instead of crackers. Whatever it was, chances are it was pretty trivial. If we took the same whimsical approach to major life-events each day—"today I think I'll move to Spain, or get pregnant, or become a podiatrist"—we would be in trouble. Our life is typically arranged as 1/F noise: high-power events should be long-term changes, and low-power events should be short-term changes.[3] In fact,



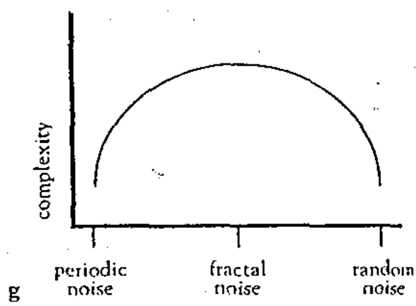


FIGURE 10.7

Crutchfield-Smale complexity measure

(a–b) *Periodic noise:* A simple signal. (c–d) *White noise:* From the viewpoint of the Crutchfield-Smale measure, this is also of low complexity. An FSA, for example, could define this noise by making all state transitions equally probable. (e–f) *Fractal noise:* The most complex signals in the Crutchfield-Smale measure are "scaling noises" in which there are fluctuations within fluctuations. These signals have the greatest amount of their power in the lowest frequencies (longest wavelength). Since power is the reciprocal of frequency, it is often referred to as 1/F noise. (g) In summary, the Crutchfield-Smale complexity measure is a reflection of the fractal dimension. The "most fractal" (e.g., dimension of 1.5) will be the most complex, and the function decreases with both higher and lower dimensions.

(c and e, courtesy R. F. Voss.)

many of the analog waveforms produced by intelligent human behavior appear to be 1/F signals (Voss 1988; Eglash 1993).

As more scientists began to think of complexity in terms of computation and 1/F noise, they began to accumulate examples that suggested that this was what it meant to have a "self-organizing" system. In the evolution of life, for instance, most of the genetic information stores long-term events, such as the physiology that underwent change in life's evolution from water to land. More short-term adaptations, such as skin color, take up very little of the genetic material. Here again, we have something like 1/F noise, with long-term events taking up the bulk of the system, and short-term events taking up proportionately less. Physicists Per Bak and Chao Tang (Bak and Chen 1991) found several examples of simple physical self-organizing systems that produced 1/F noise. In forest fires, for example, very dry woods would spread fire in an orderly circle, while fires in wet wood would be too sporadic or random, and thus die out. But in-between fires spread in a fractal pattern, with most of the fire in long-length patches, less of the fire in medium patches, even less in smaller patches, and so on. In water we have orderly crystals and disorderly liquids, but in between we can get the fractal patterns of snowflakes.

Since we are familiar with our own recursive interactions with memory, we have a good intuitive sense for why 1/F noise should accompany complex behavior, and clearly it can characterize many varieties of self-organizing systems—perhaps all of them if we use the proper definition. But how does this happen? What is the mechanism that makes it work? Complexity theorists have not hesitated to suggest implications of their work for culture; here I would like to suggest the reverse: that certain aspects of African culture can provide important implications for complexity theory. More so than any of the previous ethnomathematics models we have seen, this part of my research was much more of a collaboration, much closer to my sense of the "participant simulation" method—although if truth be known I had to be dragged kicking and screaming much of the way.

Christian Sina Diatta: an African physicist looks at culture

"Rhab." "Phantom." "Rhab!" "Phantom!!" A strange dialog flew across the computer lab at the Institut de Technologie Nucleaire Appliquée at Senegal's University of Dakar. I was seated with Professor Christian Sina Diatta, director of the lab, watching the pulsating forms of cellular automata flow about the screen. Dr. Diatta was the local sponsor for research under the United States' Fulbright Fellowship program, and was eager to discuss his own ideas. His physics lab was an inspiring place to be. I had already been able to sit in on a graduate student's presentation; after having witnessed the same ritual in the physics department at the University of California at Santa Cruz, it made for a fascinating bit of cross-cultural comparison. I tried to make myself useful by setting up a demo of an electrical circuit that produced deterministic chaos ("Chua's circuit") and installing various types of software for simulations of nonlinear dynamics. It was one of these software demos, Rudy Rucker's CALAB, that caused our multilingual exchange.

As noted in chapter 7, some of Rucker's most interesting programs are those he calls "Zhabotinsky CAs," which can produce paired log spirals. In addition to the two states of live cell and dead cell, these cellular automata require at least one "ghost state." Since someone had previously mentioned the indigenous term for ghost, *rhab*, it seemed like an opportunity for creative translation. I explained (in French, the official language of Senegal) that after *l'état mort* (the dead state) the cell went to *l'état rhab*. To my surprise, Diatta corrected *rhab* back to the French: "phantom." We went back and forth a couple of times before I realized that it was not just my poor pronunciation. Only later did I discover my blunder: Diatta was not from the Islamic Wolof majority (in whose language *rhab* occurs) but from one of the animist minority groups, the Jola. Using Wolof was no more of a cultural translation for him than it would have been to use English.

This was only the start of my mistranslations. Although Dr. Diatta was greatly enthusiastic about my work on fractals in African architecture, he seemed disinterested in the fractal generation software. But he persistently brought up African architecture during the cellular automata demos. I found this entirely too frustrating: the whole point of my research on African fractals was to explore the intentional side of these designs. Cellular automata create patterns not by preplanned design, but rather by the interactions of its aggregate cells. From my point of view, having fractal architecture as the result of aggregate self-organization destroyed the possibility of intentionality. By focusing on cellular automata as an architectural model, Diatta seemed to be undoing all my carefully prepared research. His enthusiasm was unbeatable, however, and I began to study aerial photos of his place of origin, the Jola settlements south of the Casamance River. Figure 10.8 shows the settlement of Mlomp, not far from Diatta's hometown; its paired log spiral structure could have come right out of Rucker's Zhabotinsky CAs.

A trip to the Casamance was clearly called for. I was fortunate in finding Nfally Badiane, a Jola graduate student who had done his master's thesis on indigenous architecture of the southern Casamance, as a guide. Nfally's background is ideal for an anthropologist: raised among the Islamic majority in Dakar, he is both

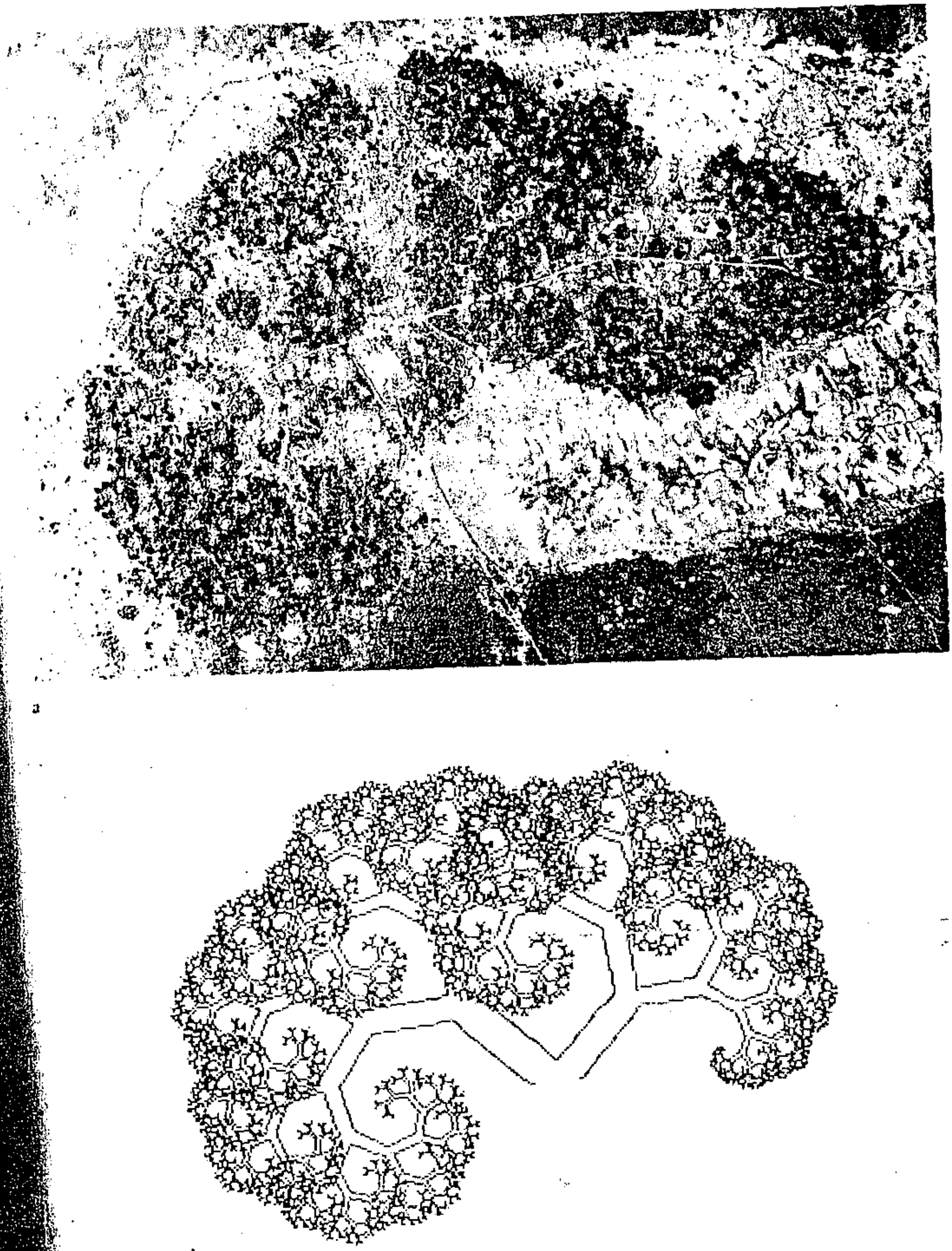


FIGURE 10.8

The Jola settlement of Mlomp, Senegal

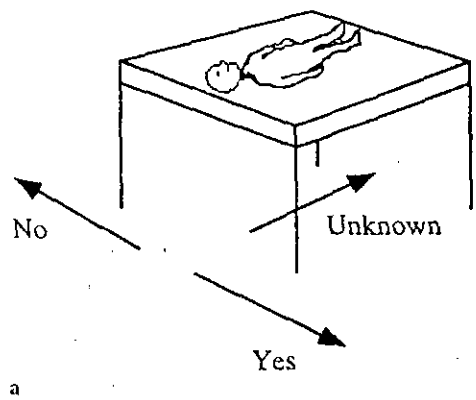
(a) Mlomp. (b) Mlomp model generated by combination of stochastic and recursive process.
(a, courtesy Institut Geographique de Senegal; b, courtesy of Egondu Onyejekwe.)

stranger to and member of the Jola society. As we traveled the delta area of the Casamance River, using cars, trucks, canoes, and anything else that moved, his warnings about the secrecy of Jola religious knowledge were repeatedly confirmed. Secular information about technical methods of house construction, precolonial and postcolonial social changes, kinship groups, and many other aspects of Jola society were readily forthcoming (Eglash et al. 1994). We were told that the circular building complexes were not preplanned, nor were the broad curves of these complexes in each neighborhood, but that they could not tell us anything about the sequence of construction because, unlike the Wolof, "we do not have a *griot* [oral historian] in Jola society." The spiral structure visible in the photo was mainly due to the carefully maintained sacred forest surrounding each local neighborhood. But the mechanisms for creating such coherent structures over such an enormous range of scales remained hidden. A tantalizing glimpse of the Jola's sacred geometry, however, led us to suspect that there was a conscious element to the CA-like settlement structure. First, there was the symbolism of the chief's drinking vessel: a spiral shell. Second, Nfally had seen the interior of one of the settlement altars, and said that it consisted of a spiral passage.

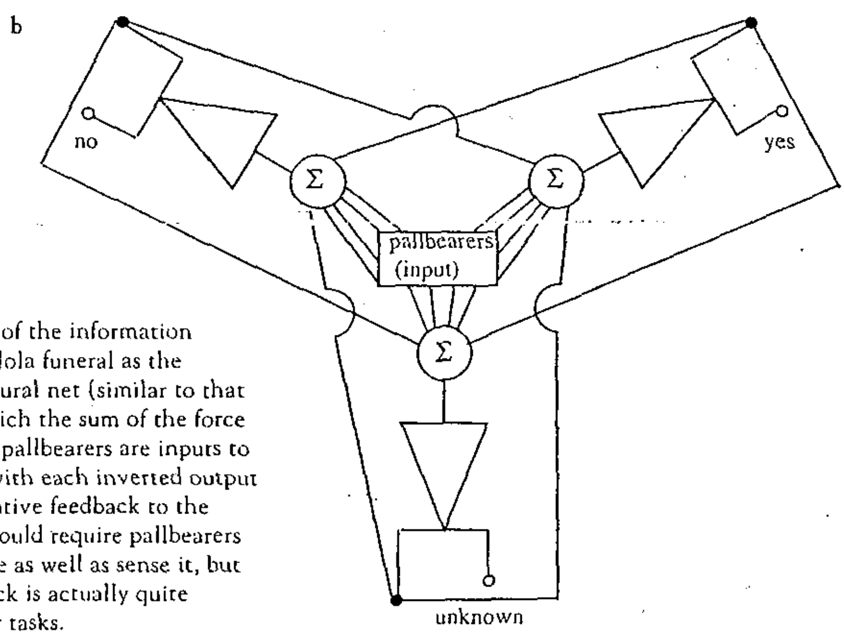
The best clue we found was from Diatta himself, who described a log spiral path in certain rituals that took place in the sacred forest. But how to reconcile this self-conscious modeling with what appeared to be the emergence of the settlement structure through aggregate self-organization? I finally confessed my disturbance to Diatta, and asked him how I might understand the apparent contradiction. He suggested yet another simulation: the Jola funeral ritual (fig. 10.9a). We had been alerted to this ceremony as a result of a suspicious death during our visit, but were not allowed to attend. Diatta described the ritual in detail. The body of the deceased was placed on a platform, and posts at each of the four corners are held aloft by pallbearers. If critical knowledge is thought to have been held by the deceased (e.g., as in the case of a murder), a priest asks questions. The pallbearers, reacting to the force of the deceased, move the platform to the right for yes, left for no, and forward for "unknown."

The simulation for this ritual (fig. 10.9b) is based on an analog feedback network. We don't need to make any assumptions about whether the pallbearers are exerting force due to conscious opinions or subconscious beliefs; it is only necessary to assume that they exert force in proportion to this motivation. Since they can both exert force and sense it from others, this would theoretically allow the summation of knowledge among the participants to be expressed in the most effective way possible. In fact, the technique is more effective than a vote, since voting can lead to the paradox of a minority opinion win if there are more than two options.[4] The information emerged from the bottom-up interaction of the parts, yet it was also intentional in the sense that this mechanism for aggregate self-organization of knowledge had been consciously designed. This was not intentionality as I knew it; it sounded more like the description of a neural network in computer science:

> If a programmer has a neural network model of vision, for example, he or she can simulate the pattern of light and dark falling on the retina by activating certain input nodes, and then letting the activation spread through the



(a) In the Jola funeral ritual four pallbearers hold a platform aloft and move it in response to questions. Since the information (whether one believes it to be of spiritual or mundane origin) is held by the pallbearers, we can model the force of each corner as having direction and magnitude (a vector) determined by the pallbearer's conviction. Decision making based on a continuous range rather than on yes/no is called "fuzzy logic" in mathematics.



(b) We can think of the information processing in the Jola funeral as the equivalent of a neural net (similar to that in fig. 10.2) in which the sum of the force vectors of all four pallbearers are inputs to three amplifiers, with each inverted output connected as negative feedback to the other two. This would require pallbearers to both exert force as well as sense it, but such force-feedback is actually quite common in motor tasks.

FIGURE 10.9

Neural net model for the Jola funeral ritual

> connections into the rest of the network. The effect is a bit like sending shiploads of goods into a few port cities along the seacoast, and then letting a zillion trucks cart the stuff along the highways among the inland cities. But if the connections have been properly arranged, the network will soon settle into a self-consistent pattern of activation that corresponds to a classification of the scene. "That's a cat!" (Waldrop 1992, 289–90)

The tricky part is "if the connections have been properly arranged." Clearly it could be arranged for four people, but could it for this city of Mlomp, with dozens of local neighborhoods and hundreds of people in each? And Mlomp is not an anomaly. While we saw a more explicit formal system in the construction of several fractal settlement architectures in chapter 2, there are also many African settlements that have a large, diffuse fractal structure (see Denyer 1978, 144). Self-organizing mechanisms that arrange such vast aggregations into coherent patterns would have to be more global and less explicit.

One key mechanism in complexity theory is memory; the theory predicts that self-organizing systems will utilize 1/F distributions in memory length. The lukasa, a visual "memory board" developed by the Baluba of Congo (Zaire), shows just such fractal scaling (fig. 10.10). The memory system of the lukasa is partly based on digital (that is, physically arbitrary) coding, such as color, but Roberts (1996) notes that much of the lukasa is a "geometry of ideas," mapping the beaded spatial structure to analogous historical events. Although there is considerable interpretive and coding variation, there is a tendency to have single beads representing individuals, groups of beads representing royal courts, and larger bead arrangements showing the sacred forests that have been growing over many generations. This visualization of a 1/F-like distribution of memory suggests at least the possibility of indigenous awareness of scaling properties in maintaining self-organized complexity.

The strongest candidate for a mechanism underlying self-organization is the complementary pair of indigenous feedback concepts we examined in chapter 8. In the vodun religion of Benin, we found Dan representing the stabilizing force of negative feedback, and Legba representing the disruptive force of positive feedback. Similar feedback pairs were found in the Baule door carvings; the caimans biting each other's tails are a symbol of negative feedback, and the fish eating ever larger fish represent positive feedback. This combination of opposing feedback loops also appears to be at the heart of the conditions that sustain self-organizing structures. Of course, most self-organizing systems will have more than two loops; but in every case I have examined, at least one of each is present, and it is through this interaction that sustained complexity can arise.

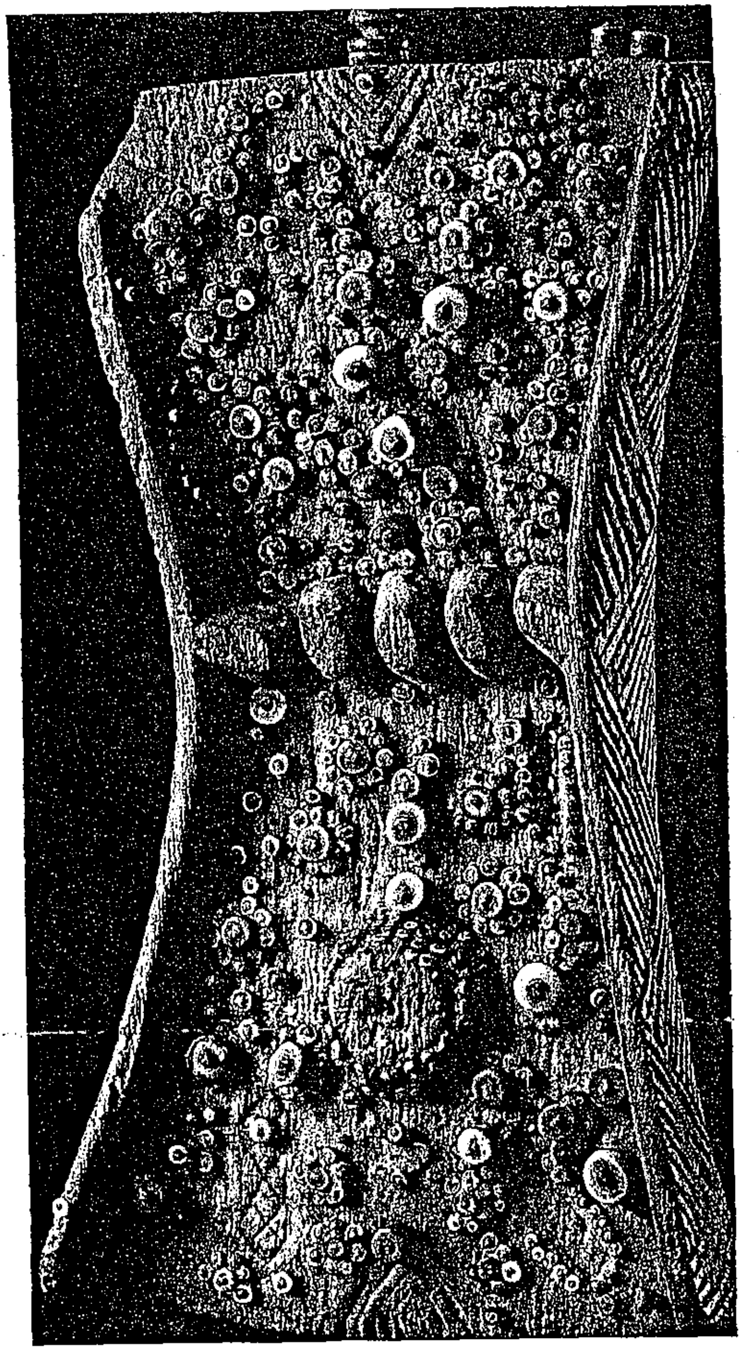


FIGURE 10.10
Lukasa
(From Roberts and Roberts 1996; photo by Dick Beaulieux.)

Returning to the most basic example of complex behavior, May's population equation, we have two components. On the one hand, there is population growth: $P_{n+1} = P_n R$. Next year's population will be this year's population times the growth rate. As long as R is a positive number, this will be a positive feedback loop. But the other part of the equation, multiplying by $(1 - P_n)$, was a negative feedback loop, acting like an epidemic that kills more people with larger population size.[5] Together they create deterministic chaos: the positive feedback keeps expanding the population, and the negative feedback keeps it within bounds. This works for other chaos equations as well. Figure 10.11 shows a chaos equation called the "Rossler attractor" modeling a car with two drivers. One is drunk and overcompensates by steering too far with each correction; the other is sober and pulls it back on the road when the drunken oscillations get too large. Because it always steers back to a slightly different position, the oscillations never repeat—deterministic chaos.[6]

We can see the same combination of negative and positive feedback creating self-organization in aggregate systems. The "game of life" cellular automaton offers a particularly clear illustration of this phenomenon. If we give a rule set that makes birth too easy (e.g., the cell goes to the "live" state if there is one or more nearest neighbors alive), then there is too much positive feedback and we get a rapidly spreading disk. If we make death too easy (e.g., the cell goes to the "dead" state if there is one or more nearest neighbors alive), the screen goes

FIGURE 10.11

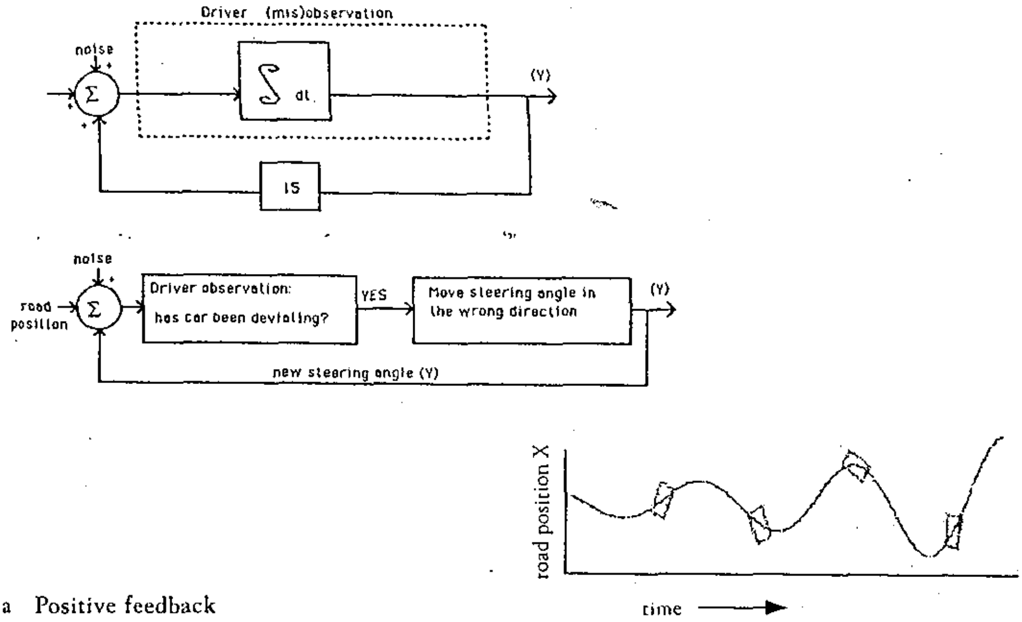
Rössler attractor as feedback in automobile driving

The Rössler attractor is a set of three simple equations whose output is deterministic chaos, that is, a signal with variable oscillations which remain bounded but never repeat the exact same pattern. How can such a simple system produce infinite variation? An automobile driving model can help us see what these equations are doing.

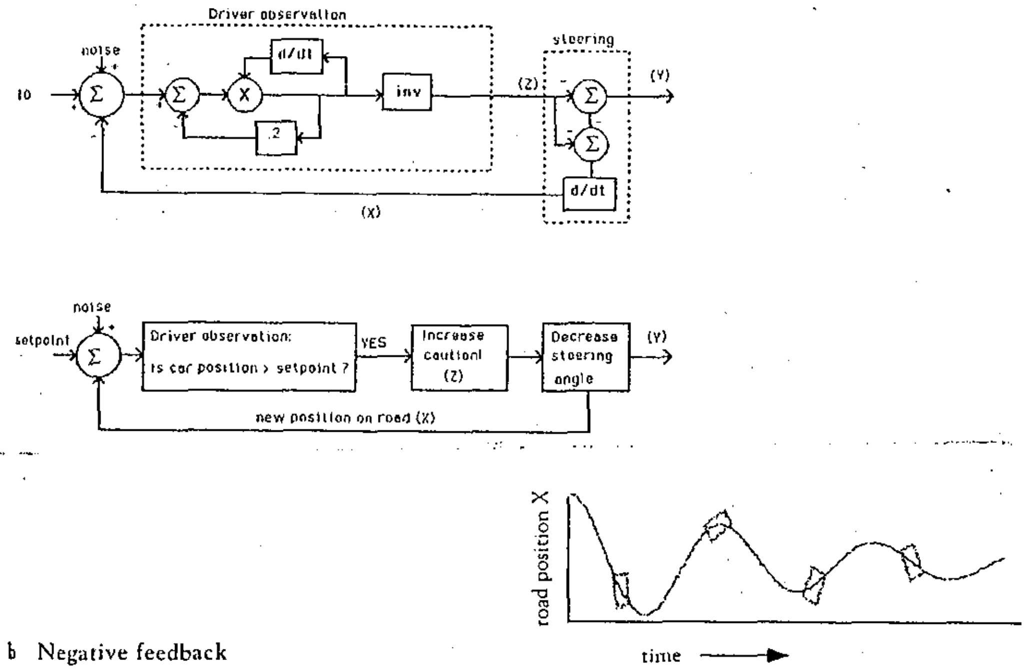
(a) *Positive feedback.* First, there is a part of the system that provides a positive feedback loop; this acts like a drunken driver who swerves farther and farther off the road. Note that the car is not properly aligned with the direction of travel; this skidding is the nonlinear relationship between road position X and steering angle Y.

(b) *Negative feedback.* The other part of the system is a negative feedback loop; given a swerving input, this cautious driver steers back toward the center of the road. "Caution" is represented by the third variable, Z.

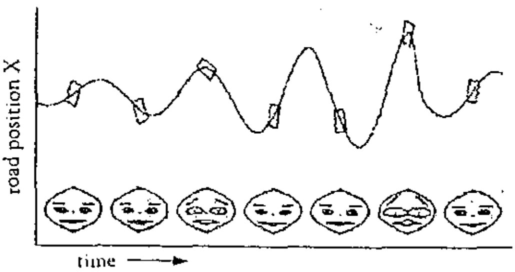
(c) *Combination of negative and positive feedback.* Here we see the complete Rössler system at work. The "caution" variable Z controls the facial expression (diameter of eyes and mouth, angle of eyebrows). Note that after the oscillation gets large enough, the negative feedback kicks in, and we go back toward the center of the road. Because the car never steers back to exactly the same position on the road, the behavior never repeats. If, for example, you looked at the number of increasing oscillations that occur before the negative feedback dampens it back toward the center, it would appear to be completely random, with no predictable pattern. Yet the pattern is entirely deterministic (that is, determined only by this set of equations); it could be predicted if you knew the initial conditions with infinite precision.



a Positive feedback



b Negative feedback



c Combination of negative and positive feedback

Facial expression = Z

Total system: $x' = -(y + z)$
$y' = x + 0.15y$
$z' = 0.2 + z(x - 10)$

blank in a few generations. The "classic" life rule set (found by John Horton Conway in 1970) is often referred to as "3-4" life because it takes 3 nearest neighbors to give birth, but 4 results in death. Conway discovered that this combination of negative and positive feedback maximized the complexity of behavior. Similarly, when Per Bak found empirical data for self-organization in physical systems—forest fires, earthquakes, avalanches, etc.—he noted that it occurred only at a "critical state" in which there was a balance between noise-suppressing mechanisms—which would correspond to negative feedback—and the positive feedback of noise-amplifying loops.

It is unfortunate that so much of the classic research on African social mechanisms came from functionalist anthropology, since they made an almost exclusive emphasis on the role of negative feedback in achieving equilibrium. When it comes to conscious knowledge systems, African societies do not exclusively focus on balance, harmony, and stasis. The complimentary roles of Dan and Legba, of order and disorder, are much more common, as we see in this passage: "In the mind of the Bambaras the air, wind and fire . . . are indispensable elements of the wörld's onward movement. But as these principles may be active in an uncontrolled, that is, unruly and often excessive manner, Nyalé is considered to be a profuse and extravagant being. . . . So by her very nature Nyalé is, to a certain extent, a factor of disorder. That is why it is said that Bemba . . . took away her 'double' to entrust it to Faro . . . whose essential attribute is equilibrium" (Zahan 1974, 3).

A similar pairing occurs in the Dogon religion, where Amma, the high god, creates the Nummo to enact order, and accidentally creates the disorderly Ogo; together the two generate life as we know it. In the repertoire of dynamical concepts occurring in several African knowledge systems, there is recognition of the useful tension between equilibrium and disequilibrium, the dance between order and chance that results in self-organized complexity. And just as Stuart Kauffman has shown a bias toward order in evolution's "edge of chaos," the high god ensures that the trickster can act only sporadically, thus creating more power toward long-term order in these African cosmologies.

Although fractals resulting from geometric algorithms are usually seen as static structures, they too can be viewed as the combination of feedback loops. A seed shape with a huge number of tiny line segments (fig. 10.12a) will tend to be shape-preserving under self-replacement iterations; here deviations due to replacement are damped—the difference between a line segment and the seed shape is usually not important (and the resulting graph will have a low fractal dimension, i.e., tending toward 1.0). But for seed shapes made up of only a few large lines (fig. 10.12b), the difference between a line segment and its

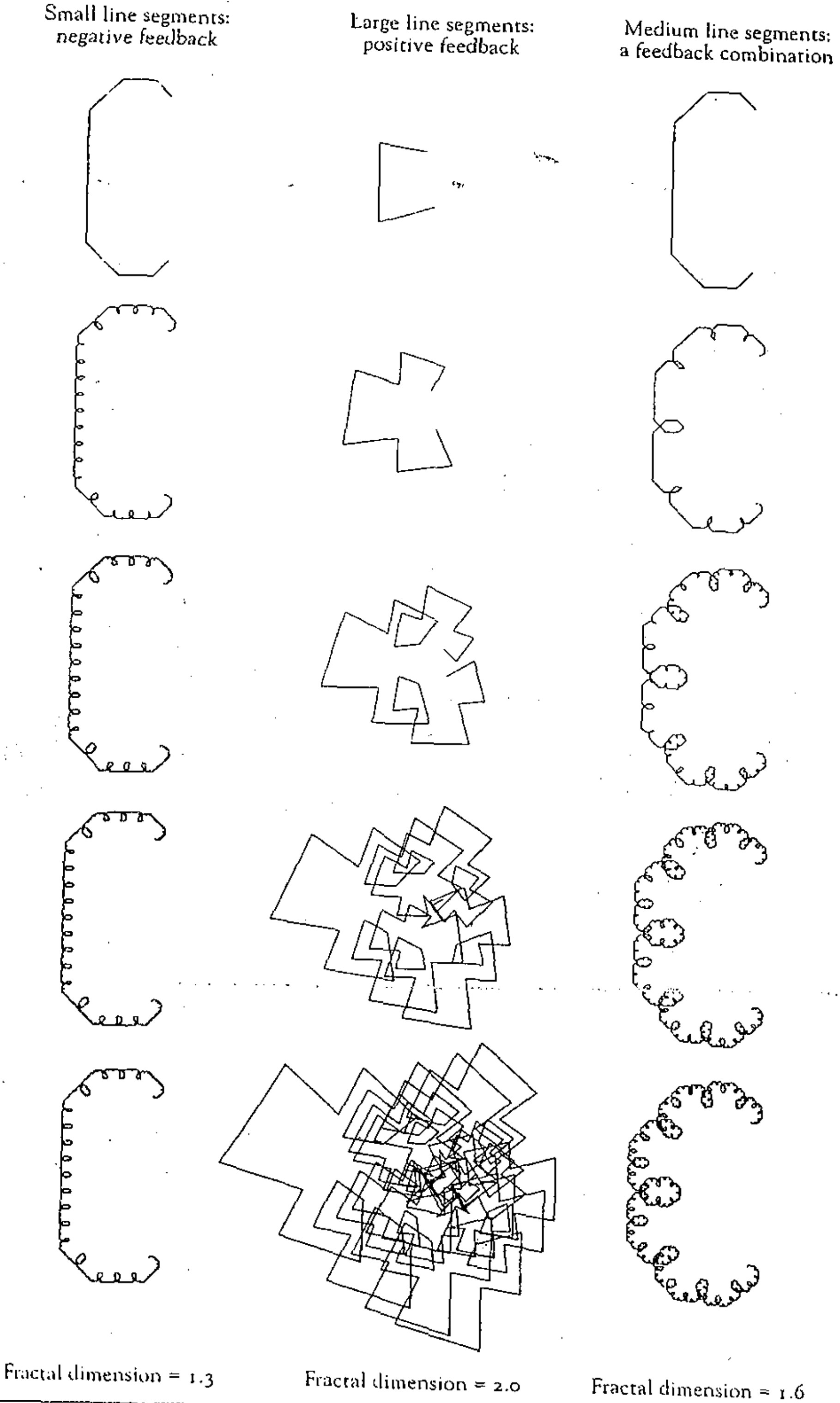


FIGURE 10.12
Fractal graphics as feedback

replacement shape will be very important. Large deviations tend to be amplified in a quick positive feedback, sometimes explosively growing out of bounds in only a few iterations. Figure 10.12b has been scaled down to fit on the page, but the actual fractal graph will quickly grow out of bounds and blacken the screen entirely (i.e., a fractal dimension close to 2.0). Figure 10.12c shows a fractal dimension close to 1.5, the "most fractal" measure, which results from a balance between the negative feedback of small segment shape preservation and the positive feedback of large segment replacement deviation.

There is no quantitative measure of fractal dimension in precolonial African knowledge systems. But the idea of a spectrum progressing from more orderly to less orderly is vividly portrayed in certain material designs. The best examples are in the raffia palm textiles of the Bakuba (fig. 10.13a). These tend to show periodic tiling along one axis, and aperiodic tiling—often moving from order to disorder—along the other. Similar geometric visualizations of the spectrum

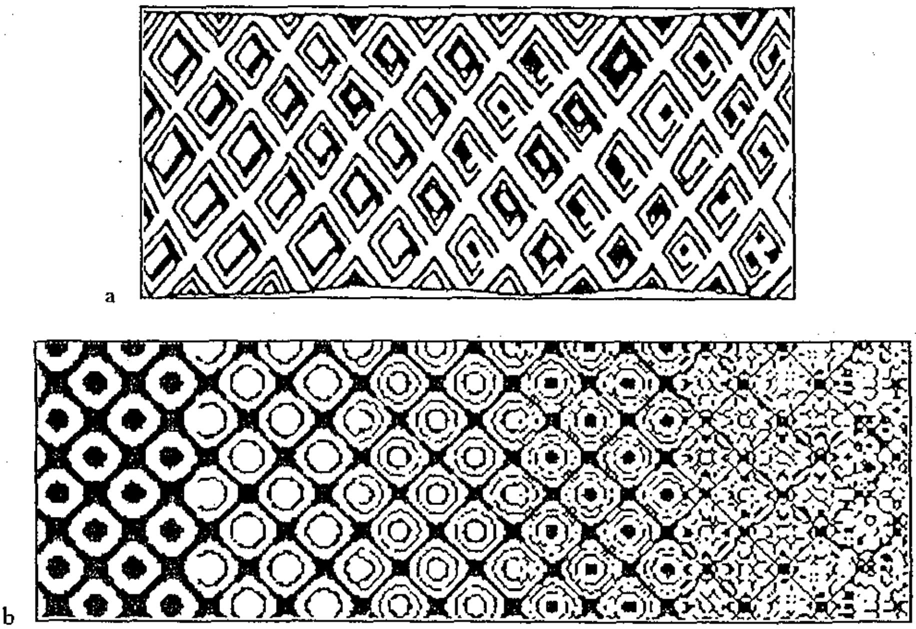


FIGURE 10.13

From order to disorder in a Bakuba cloth

(a) The Bakuba often create cloth designs that stay fairly constant along the vertical axis, but gradually change along the horizontal axis. In many cases, the horizontal transformation suggests an order-disorder range. (b) Computer scientist Clifford Pickover created this pattern to show how a spectrum from order to disorder could be visualized by allowing a random variable to have increasing influence on the graph's equation. Thus it, too, makes use of periodic tiling along the vertical axis and aperiodic along the horizontal.

(**a**, *from Meurant 1986, by permission of the author;* **b**, *from Pickover 1990, by permission of the author.)*

from order to disorder have been used in computer science (fig. 10.13b). As far as I can tell, the Bakuba weavings never reach more than halfway across the spectrum—they are typically moving between 1 and 1.5, that is, from periodic to fractal, rather than stretching all the way to pure disorder.[7]

I know of only one African textile that takes this last step, and that is the block print shown in figure 10.14. This pattern is reminiscent of the title of Nigerian author Chinua Achebe's famous novel, *Things Fall Apart*. Given the anticolonial context of Achebe's writing, it might be tempting to read it as an indication that white noise only comes with white people, but at least in terms

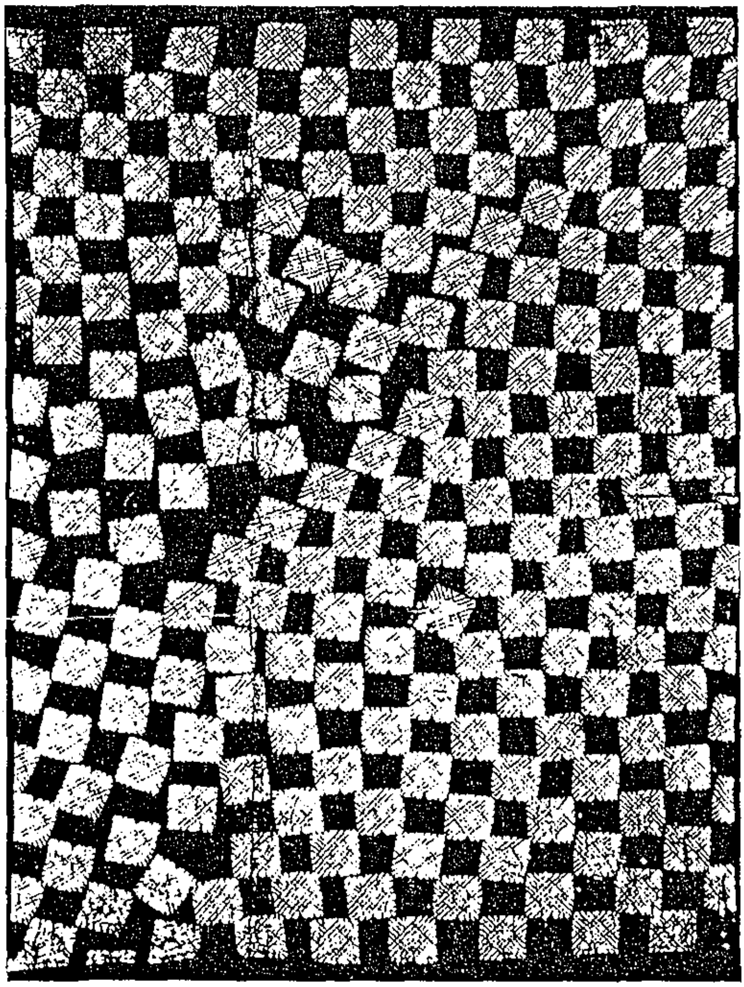


FIGURE 10.14
Block print textile
This print from West Africa suggests the full spectrum from order to disorder.
(*From Sieber 1972.*)

of the indigenous knowledge system such assumptions are unfounded.[8] There is, for example, a form of music indigenous to Nigeria that has something like a white noise distribution of sounds. Akpabot (1975) describes "the random music of the Birom," a flute ensemble designed to allow each musician to express individual feelings through whatever idiosyncratic noise (or even silence) he or she chooses, resulting in "an indeterminate process [in which] the sounds produced by the players are not obstructed by a conscious attempt to organize the rhythms and harmonies" (p. 46). Pelton (1980) refers to the Nigerian (Yoruba) trickster Eshu as the "lord of random," and notes that there is a coupling between the orderly work of Olirun and this unpredictable spirit, similar to the negative feedback/positive feedback combinations we noted earlier. The characterization of extreme disorder might well be applied to the experience of colonial rule, but we should not assume that the concept was unknown before then. A summary of selected African complexity concepts is shown in figure 10.15; note that the central peak of spiritual power is analogous to the central peak of computational power in the Crutchfield-Smale complexity measure.

Conclusion

This chapter is only the bare outline of what I hope will be future areas of research, examining the relations between technical, cultural, and political systems through the new frameworks offered by complexity theory. For the moment, we will have to limit ourselves to the few fragments that my Senegalese colleagues pointed out so diligently. First, this does not negate the previous examples of explicit algorithmic design in African fractals,[9] but it does suggest that at least in the case of settlement architecture they can arise from another source as well. The creation of fractal settlement patterns through aggregate self-organization, while unlike the planned structures we saw in chapter 2, do not seem to be the result of unconscious social dynamics (as we saw for the urban sprawl of European cities in chapter 4). This may be due to a difference between African concepts of intention, which can apply to a group project created over several generations, versus the Western focus on an individual performing immediate action in defining intentionality. Most important, there are indications that this pattern creation through group activity is supported by conscious mechanisms specific to self-organization as defined in complexity theory. Both the scaling distribution of interactions with memory and the spectrum from order to disorder have at least some graphic counterparts in African designs. The best candidate for a conscious mechanism is the combination of negative and positive feedback. We did not examine every possible case of deterministic chaos and

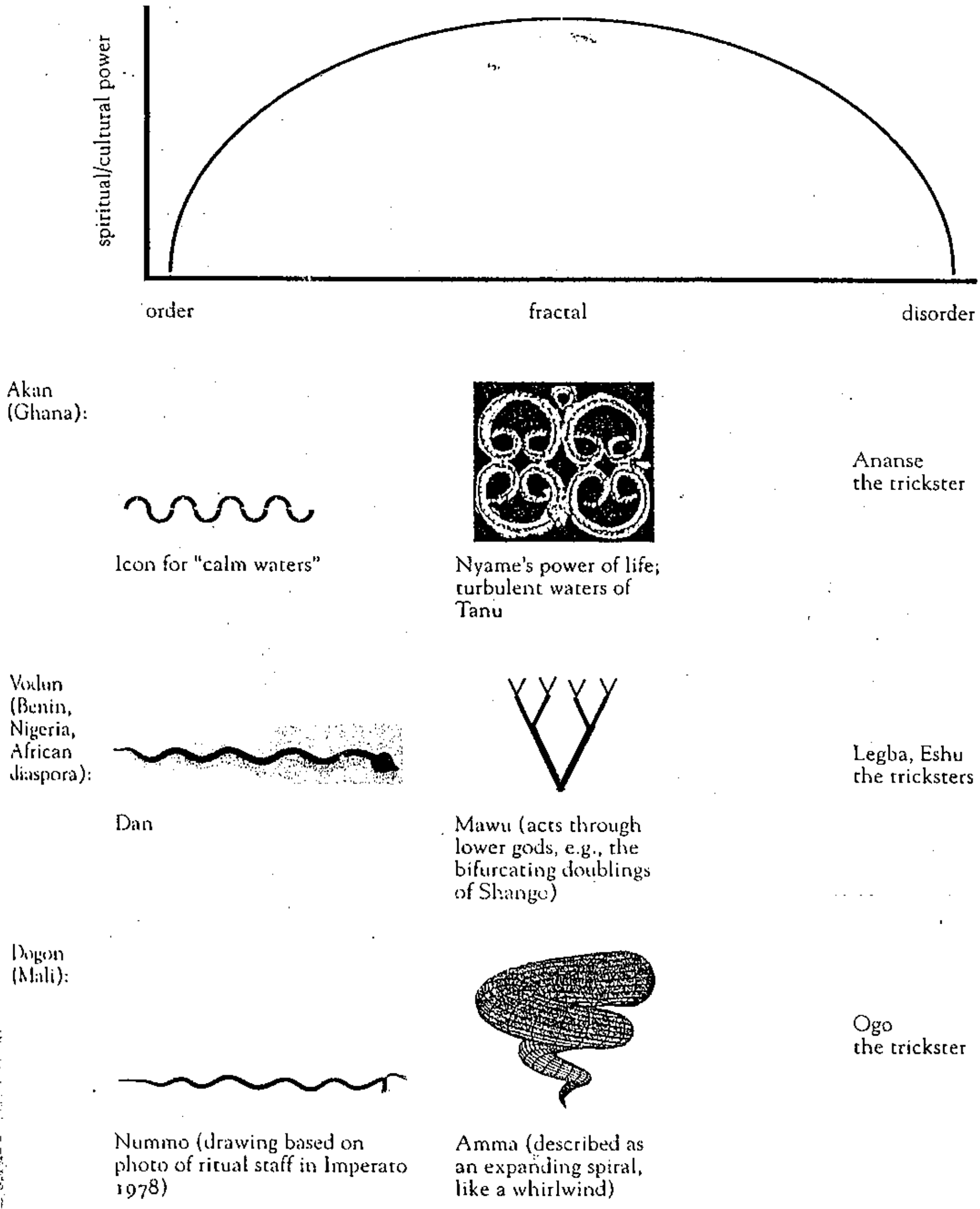


FIGURE 10.15

African complexity concepts in religion aggregate self-organization, but it would appear that the combination of negative and positive feedback loops, which form the basis of several African knowledge systems, also form a key mechanism of general self-organizing systems.

As noted in the first chapter, it is just as important to find what is missing as it is to find what is present. While four of the five basic concepts of fractal geometry—scaling, self-similarity, recursion, and infinity—are all potent aspects of African mathematics, *a quantitative measure of dimension (the Hausdorf-Besicovitch measure) is completely absent*. There is a weak sense of a complexity spectrum of order-disorder, which would covary with the Hausdorf-Besicovitch measure, but that spectrum is neither quantitative nor (to my knowledge) ever compared to a concept of dimension in any indigenous African system. This is an enormous gap in the African knowledge of fractal geometry, especially since the dimensional measure is often considered the most valuable component by contemporary researchers in the field.

On the other hand, we also need to appreciate all knowledge systems in their own right, and African fractals have a surprisingly strong utilization of recursion. Indeed, in Mandelbrot's seminal text, *The Fractal Geometry of Nature* (1977), the index lists "recursion" only twice, and the terms iteration, self-reference, self-organization, and feedback are entirely absent. As we will see, this absence is no accident; it reflects a European historical trend. But why have Europeans traditionally placed such little importance on recursion, and why was it so strongly emphasized in African fractals? In part III of this book we will take up such cross-cultural comparisons in detail.

PART III

Implications

CHAPTER
11
Theoretical frameworks in cultural studies of knowledge

Parts I and II of this book emphasized the geometric, symbolic, and quantitative aspects of African fractals. Some cases were more speculative than others—a difference that I hope was clearly indicated—but even in the use of mythic narrative, I generally restrained conclusions to those that had geometric or quantitative counterparts. In other words, the claims made in parts I and II should be falsifiable in the sense of Karl Popper; the data either supports the hypothesis or refutes it.[1] But the chapters in this last section will switch to topics in cultural politics and other humanities. These issues are too complex and multidimensional to be reduced to formal representations; they can only be approached by exploring their interpretative depths. Poetry can reveal as much truth about the world as any science; we only need to keep in mind that it is a different way of going about it. While the philosophy, politics, and poetics of culture are not strictly falsifiable, they can often approach the areas of life that Popperian positivism cannot—areas we cannot live without.

Given that one can make a good case for at least four of the five basic elements of fractal geometry in African mathematics, what should we make of it in terms of culture? To ask this question effectively we need to avoid two pitfalls. The first is the possibility of "overdetermined" explanations for African fractals, explanations that seem to be waiting for us before we've even begun to examine the evidence. The second is the difficulty of sustaining skepticism in a racially charged environment, the possibility that we might shy away from critique over fears that expressing a negative view could be taken as having an ethnocentric or racist motivation. Both failings are equally damaging. Recently, researchers have drawn attention to the ways that theories of knowledge (epistemology) can sneak unexamined into cultural portraits. If we are to avoid the trap of seeing African fractals as an indication that Africans are "closer to nature," or concrete rather than abstract thinkers, or unified in a single homogeneous culture, then we need to know a bit about the origin of these misconceptions. The first step in that process is to examine the epistemological frameworks that are applied to the study of culture.

The unity/diversity debate and thin description

According to Mudimbe (1988), the concept of a unitary, traditional "African culture" is an invention created first by colonialists, who sought to rationalize their conquest with the myth of the primitive, and subsequently by anticolonialists seeking to consolidate their opposition. A similar critique is provided by Appiah (1992), who suggests that the differences among various African societies were much too broad to allow any generalizations (p. 25): "Surely differences in religious ontology and ritual, in the organization of politics and the family, in relations between the sexes and in art, in styles of warfare and cuisine, in language—surely all these are fundamental kinds of differences?"

Appiah and Mudimbe promote various kinds of solidarity in contemporary Africa (as well as internationally in the diaspora); they only caution that this cultural unity is of relatively recent origin, and that attempts to see an African *"essence" or a unified African culture preceding major* European *intervention* (i.e., previous to the First World War) will eventually have to fall back on racially defined categories, which is certainly a self-defeating basis for antiracist movements. From Appiah's antiessentialist point of view one cannot discuss precolonial "African culture;" only "African cultures."

On the other extreme of the unity versus diversity debate lies the Afrocentric position. While its proponents also agree that there was no single, homogeneous African culture, they emphasize the shared elements. Asante and Asante's *African Culture: Rhythms of Unity* (1985), for example, begins by stating that while black unity cannot be based on genetic grounds, broadly shared cultural undercurrents were found throughout the diverse societies of precolonial Africa:

> Although the precise actions and ideas may differ within the acceptable range and still remain squarely in the category of African culture, there are some behaviors among some African ethnic groups which may have the opposite meaning among others. Twinness is commonly considered a positive characteristic in African societies, yet there are some ethnic groups which accept twinness as a negative characteristic. . . .Yet this particularistic emphasis would not make the ethnic group unrelated to the others. Patterned behaviors by African ethnic groups are cultural, not rigid or fixed, but related to history and experience. Culture can vary over time, but in the case of African culture, it will always be articulated in the same way.

There is a lot going on in this paragraph, but the crucial point for my analysis is Asante and Asante's distinction between the *surface* particularities of various ethnic groups, which may differ, and *deeper* cultural sensibilities or patterns of articulation (which they later illustrate with "the three traditional values: harmony with nature,[2] humaneness, and rhythm" [p. 7]). In this Afrocentrism, it is only at the deep level in which we find important cultural attributes held in common.

Appiah also makes this distinction between trivial surface and the "fundamental" depths. The only disagreement between him and the Asantes is whether or not the depths reveal differences. One way around this question is in the "thick description" proposed by anthropologist Clifford Geertz (1973). Geertz was motivated in part by his dissatisfaction with the ways that Claude Lévi-Strauss's structuralism seemed to reduce symbolic culture to a flat, mechanistic syntax. For Geertz, cultural symbols should be in a kind of dynamic play, and the ethnographer should show their turbulent expansion through layers of meaning, not their reduction to a single fixed structure. Geertz defined these deep elements, which tend to be more subjective and literary, as specific to a particular community. For him, it would be extremely difficult to compare deep elements from one location to the next, because the deep elements are the result of local interpretations. Taken to the extreme, Geertz's thick description would simply reply that the question Appiah and the Asantes are asking cannot be answered.

The framework I have used in parts I and II of this book, which is that of ethnomathematics in general, might be referred to as thin description: a study of the surface particularities, such as material designs and symbolic formulas. As the Asantes point out, a mathematical element like doubling ("twinness" in their quotation) is just a surface feature. Whether or not it has deeper meanings—and thus the entire Afrocentrism/antiessentialism debate—is a question outside of thin description. For this reason, the thin description use of African icons to represent specific mathematical concepts or structures (e.g., the trickster = disorder) is not necessarily in conflict with the thick description of these symbols in their deep semiotic dynamics. Pelton (1980) sets up just such a conflict, and perhaps rightly so—there has indeed been a tendency for structuralists to claim that they had reduced culture to its true essence. Their error was to insist that these bare-bones structures were the truly deep mechanisms of culture, and that the discursive play of meaning should be disregarded as shallow distraction. As long as we keep the thick stuff as the deep, and the pared-down structures as the surface, there is no conflict.

While the lack of African unity in "twinness" is not a problem for those concerned only with deeper meanings, wouldn't it present a problem for thin description? That is, if doubling is supposed to be an important feature of African mathematics, then how does one explain the African societies that do not use it? Indeed, how is fractal geometry supposed to be an African knowledge system if the examples of its use are so disparately scattered across the continent? To answer this question, we need to consider what Wittgenstein called a "family resemblance." When we look at the photograph of a large family we can see that everyone is related, even if there is no single characteristic that they all share (some have big noses and some small, some light hair and some dark, etc.). In the same way, it is not uncommon for a group of mathematical ideas to share many commonalities without a singular essence. In James Gleick's (1987) history of chaos theory, for example, he shows that the emergence of nonlinear dynamics as a discipline was due to a slow gathering of many different strands of mathematics—strange attractors, fractal geometry, cellular automata, and so on. In order for scientists to collaborate on this development, there was a long period in which several researchers worked hard to point out the family resemblance of these disparate mathematical tools, and many aspects of their relationships are still uncertain today. Similarly, African fractal geometry is not a singular body of knowledge, but rather a pattern of resemblance that can be seen when we describe a wide variety of African mathematical ideas and practices. And as we saw in the case of Banneker's quincunx, it is not the only pattern possible.

Participant simulation

Whether one believes in Geertz's thick description or in some other method for researching the deeper meanings of a local culture, anthropologists generally agree that it requires long-term local ethnographic study. My thin description fieldwork lasted only a year and moved through Senegal, Mali, Burkina Faso, The Gambia, Cameroon, Benin, and Ghana. This dispersed investigation is quite unlike what is undertaken by most anthropologists, who often spend a couple of years in one village alone, using "participant observation" to traverse the depths of the local culture by actively living it. There is, however, an important difference: I was not trying to understand how the Yoruba experience grief, or to determine the inner meaning of communal spirit among the Baka. My interest was primarily in the formal properties of design, in methods of construction, and in other technical questions that could often be answered in a direct and simple fashion. Many of the Africans I spoke with were clearly relieved to hear that I was a mathematician.[3] Of course I was still faced with several of the same problems involving ethnographic accuracy and authority (see Clifford 1983). But even these were sometimes differently posed. In particular, I began to think of my methodology not as participant observation, but rather as *participant simulation*, seeking to collaborate in mathematical analysis and virtual reconstruction with my African colleagues.

Participant simulation was carried out to conclusion only in the research with Christian Sina Diatta, but I tried to maintain the practice at some level with everyone I had the opportunity to work with. That meant hauling diagrams of fractal graphics with me into the equatorial rain forest and across the savannah, and disrupting research time with math lectures, but in the end it was well worth it.[4] There was the potential problem that someone who knew what I was after might fabricate what I wanted to hear (as in St. Louis, Senegal, when one of the local children heard me talking about Benjamin Banneker and claimed to know him personally). A more pressing problem was my resistance to their suggestions, as occurred in my initial disappointment with the lack of place value notation in the Bamana divination code, or hearing the description of the oscillatory snake as "Dan at work" (all I could think of at the time was a road construction sign). Of course, there are always the aftereffects—Senegalese sociologist Fatou Sow said "if there are not fractals in Africa now, there surely will be by the time you leave"—but then that is a feature of all ethnography; and participant simulation is about turning that into an advantage.

The reason collaborative approaches like participant simulation were not traditionally used in ethnography comes from concerns over accuracy—the desire to obtain an objective account—and concerns over authority, a suspicious motive in the colonial context of most traditional anthropology. Clifford (1983) describes the move toward collaborative techniques as both the anthropologists' own self-critique of authority and as a growing recognition that since the ethnographer has as much motivation as the informant does, accuracy and objectivity can be better approached by sharing authority with indigenous voices than by using them in a kind of ventriloquist act. Simply proclaiming a collaborative approach is of course no guarantee that you will have one, and participant simulation is perhaps even more susceptible to manipulation due to the role of technological expertise.

On the other hand, since the creation of virtual worlds—simulations—is in some ways the production of something fake, participant simulation does have the advantage of avoiding some old-fashioned concepts of authenticity. It was, after all, the creation of an "authentic native" (see Appadurai 1995) that helped colonists to jail rebels among black South Africans and Native Americans; and one could even hear the occasional guilt-ridden lament among the colonial rulers that they themselves were to blame for having accidentally polluted the natural purity of these "children of the forest" with their own troubling artifice (see the apartheid culture comedy, *The Gods Must Be Crazy*). Locating indigenous activity in virtual worlds can, if done properly, counter this habitual tendency to place artificial on the Western side and natural on the indigenous side.

Doing it properly relies on the other root, which comes from the old-fashioned—and, I think, still crucial—method of participant observation. Participant observation recruits a kind of responsibility that can be sadly lacking in virtual ethnographies. Take, for example, the growing field of cyber-ethnography, in which anthropologists study the virtual communities of the Internet. Since "lurking" (observing the electronic exchanges *without participating*) is so easy, there have been a number of studies in which the ethnographer is reduced to eavesdropper or spy, with no attempt to work with the community in either off-line or on-line lives. On the other hand, recruits can include both draftees, who have little real interest in working collaboratively, and fanatics, who are all too interested in what Gayatri Spivak (1987) calls the "benevolence of the western gaze."

Thus participant simulation is an attempt to take the best of both approaches, and to use them in a kind of checks-and-balances system. By insisting on participation we can help avoid glib irresponsibility; and by using simulation we can strive to avert the policing of boundaries around constructions of authenticity and realism. From this point of view we do not need to emphasize tradition over invention; the mathematical creations of a single individual are still examples of indigenous mathematics, even if she is the only one who knows they exist.

Intentionality and ethnomathematics

There are clear advantages to a methodology that can credit the inventions of a single individual, but what about those creations that do not have a single inventor? As we saw in the case of complexity in chapter 10, it is possible to err on the other side by insisting that conscious creations can only come from singular inventors. A better understanding of this problem can be gained through the contrast between ethnomathematics and mathematical anthropology. Mathematical anthropology is generally focused on revealing patterns that are not consciously detected by its subjects of study. In part this is due to a conviction that many of the underpinnings of society are forces unnoticed by its members—not only because such forces operated at levels beyond individual awareness, but also because regulatory mechanisms would have to be covert, obscured, or otherwise protected from manipulation and conscious reflection. For these reasons, mathematical anthropology makes good sense, and it has indeed produced wonderful insights. But its emphasis on unconscious process also arose from imitation of the researcher-object relation in the natural sciences: if anthropologists were simply reporting indigenous discourse, then they would not count as scientists. This problem of mere reporting is indeed the case for "non-Western mathematics," which is mainly focused on direct translations for Chinese, Hindu, and Muslim mathematics and thus considered a subject for historians. Hence mathematical anthropology's tendency to avoid intentionality can be problematic.

The intentionality problem in mathematical anthropology can be seen in Koloseike's (1974) model for mud terrace construction in the low hills of Ecuador. Koloseike began with two hypotheses: either the Indians learned from the Inca stone terraces in the high mountains above, or they were unintentional by-products of cultivation on hillsides. He then made a list of nine observations that were relevant to deciding between the two. Of particular interest are the following:

> 3. The same hillside soil is used in rammed-dirt houses and fence walls, and these stand for years.
> 4. But I never saw a terrace being constructed, nor did people talk about such a project.
> 5. Small caves are often dug into the terrace face for shelter during rainstorms. That this potentially weakens the terrace face does not seem to concern people. (1974, 29–30)

Koloseike concludes that these terraces are the unintentional result of an accretion process from the combination of cultivation and erosion, and then proceeds to develop a mathematical model for the rate of terrace growth. My point is not in questioning the accuracy of the model, but rather the way that indigenous intentionality is positioned as an obstacle that must be overcome before mathematics can be applied. Even a small degree of awareness—being aware that a cave dug into a terrace face might weaken it—must be eliminated.

In addition, it reveals a particular cultural construction of the supposed universal attribute of "intention." As a Westerner, Koloseike is used to a society in a hurry. Projects to be done must get done, and always with someone in charge. The idea of a long-term intentional project, perhaps extending over several generations, or the constitution of collective intentionality rather than individual intent, is not brought under consideration. It may well be that the mathematical model Koloseike offered was not only accurate, but also had an indigenous counterpart.

Ethnomathematics, in contrast, has emphasized the possibilities for indigenous intentionality in mathematical patterns. For example, Gerdes (1991) used the Lusona sand drawings of the Chokwe people of northeastern Angola to demonstrate indigenous mathematical knowledge. While it would have been possible to attribute this practice to an unconscious social process, such as the regulation of authority, Gerdes chose to focus on their properties as conscious indigenous inventions. Ascher (1991) notes the same type of Eulerian path drawings in the South Pacific, and shows them to be primarily motivated by symbolic narratives, in particular their use by the Malekula islanders as an abstract mapping of kinship relations. Again, this is in strong contrast to the tradition of mathematical anthropology, where kinship algebra was considered a triumph of Western analysis (and even a source of mathematical self-critique; Kay [1971] harshly notes the anthropologists' tendency to invent a new "pseudo-algebra" for various kinship systems rather than apply one universal standard).

Ascher's description of the Native American game of Dish shows this contrast in a more subtle form. In the Cayuga version of the game, six peach stones, blackened on one side, are tossed, and the numbers landing black side or brown side up were recorded. The traditional Cayuga point scores for each outcome are (to the nearest integer value) inversely proportional to the probability. Ascher does not posit an individual Cayuga genius who discovered probability theory, nor does she explain the pattern as merely an unintentional epiphenomenon of repeated activity. Rather, her description (p. 93) is focused on how the game is embedded in community ceremonials, spiritual beliefs, and healing rituals, specifically through the concept of "communal playing" in which winnings are attributed to the group rather than to the individual player. Juxtaposing this context with detailed attention to abstract concepts of randomness and predictability in association with the game—in particular the idea of "expected values" associated with successive tosses—has the effect of attributing the invention of probability assignments to collective intent.

At the skeptical extreme in ethnomathematics, Donald Crowe has refrained from making any inferences about intentionality and insists that his studies of symmetry in indigenous pattern creations (see Washburn and Crowe 1988) are simply examples of applied mathematics. But since Crowe has restricted his work to only those patterns which *could* be attributed to conscious design (painting, carving and weaving), it creates the opposite effect of mathematical anthropology's attempt to eliminate indigenous intent. This is evidenced by Crowe's dedication to the use of these patterns in mathematics education, particularly his teaching experience in Nigeria during the late 1960s, which greatly contributed to Zaslavsky's (1973) seminal text, *Africa Counts*.

While non-Western mathematics is exclusively focused on direct translations (such as Hindu algebra or Muslim geometry), ethnomathematics can be open to any systematic pattern discernable to the researcher. In fact, even that description is too restrictive: before Gerdes's study there was no Western category of "recursively generated Eulerian paths"; it was only in the act of their participant simulation that Gerdes—and the Chokwe—created that hybrid. And unlike mathematical anthropology, ethnomathematics puts an emphasis on the attribution of conscious intent to these patterns. At the same time, it demands quantitative or geometric confirmation that is lacking in the purely interpretive approach of New Age mysticism, such as that of Fritjof Capra's *Tao of Physics* (see critiques in Restivo 1985). Claims that ancient knowledge systems reveal the structure of the atom or the equivalence of matter and energy do more harm than good—first because they are wrong, and second because there is no means by which such knowledge could be obtained. Such mystification damages credible research in indigenous knowledge systems, and removes the attribution of intentionality and intellectual labor from the putative knowers.

Evolution is a bush and not a ladder: the cultural location of African fractals

We are increasingly surrounded by explanations based on biological determinism, and there is none more virulent than racism. Even in the supposed liberal climate of U.S. academia, my lectures on fractals in Africa are frequently followed by a question about neuroscience. Typically this is an innocent remark concerning Noam Chomsky's ideas on universal cognitive structure, but even so, it is quite telling that a lecture on European fractals invokes questions about the genius of individuals, while African fractals are compulsively attached to biology.

The mythology of race is too complex to recount here (see note 6), but it is useful to distinguish between two categories of racism. Primitivist racism operates by making a group of people too concrete, and thus "closer to nature"—not really a culture at all, but rather beings of uncontrolled emotion and direct bodily sensation, rooted in an edenic ecology. Orientialist racism operates by making a group of people too abstract, and thus "arabesque"—not really a natural human, but one who is devoid of emotion, caring only for money or an inscrutable spiritual transcendence.

The alternative to biogenetic explanation is sociocultural, and here the categories of primitive and oriental can be much more complex. Historically, many researchers who strongly opposed both racism and ethnocentracism have been located in institutions with titles like "Museum of Primitive Arts" or "Department of Orientalist Studies," and it would be unwise to simply sneer at their work, particularly considering the antiracist contributions by black anthropologists such as Zora Neale Hurston or Jomo Kenyetta. There is value to be found in even the weakest of these oppositional theories, and problems in even the strongest.

In general these theories can be grouped into two strategies: sameness and difference. Sameness can usually avoid orientalism and primitivism, since it argues that what is important about a non-Western culture are those things held in common with the Euro-Americans, and what is different is (in this context) trivial. Claude Lévi-Strauss, for example, argued that the "savage mind" is based on systems of symbolic structures, just like the European mind, so that an African working with a system of mythological symbols is performing the same cognitive operations as a European working with a system of computer code symbols. One drawback of sameness is that we become players in a game created by someone else: "I am worthwhile only insofar as I am the same as you." Difference can avoid this trap, although it has more trouble avoiding primitivism and orientalism. For example, Aimé Césaire's neologism "negritude" began as a way of speaking about the difference of African culture in open-ended, dynamic, creative terms, but later (in the hands of others) the comparison was frozen into a set of binary oppositions (intuitive vs. analytic, concrete vs. abstract, etc.).[5] In other words, both sameness and difference have moments of failure as well as moments of success.

The recent focus on ancient Egypt in certain circles of African studies has certainly seen both moments. Motivated by considerable scholarly work (e.g., Drake 1984), it has also become attached to some disreputable and questionable claims (see critiques in Oritz de Montellano 1993; Martel 1994; Lefkowitz 1996). It is worth noting, however, that some of the critiques have been equally lacking in their restraint. In his review of the Portland Baseline Essays, for example, Rowe (1995)—while rightly pointing to a number of unsupported assertions—implied that claims for an ancient Egyptian glider should be dismissed because the author was merely an aerodynamics technician rather than a Ph.D. Rowe was quite right in objecting to the wild leap from empirical tests of a small wooden carving to the authoritative claims for ancient Egyptians flying from pyramids; but to imply that simple experiments are automatically suspect because they were made by a technician is nothing but classist prejudice. On the other hand, the fact that this researcher was a technician rather than a PhD speaks to the underlying cause for these problems: the lack of institutional resources and precarious economics among many black educational communities.

Appeals to ancient Egypt can also encounter problems as a strategy of sameness. On the one hand, ancient Egypt's status as a state empire directly opposes primitivist assumptions that Africa consists of nothing but tribal villages. On the other hand, it reinforces the view that the knowledge systems of nonstate indigenous societies are not comparable to those of state societies. This view comes from the old idea of cultural evolution as a ladder, a unilineal progression from "primitive" to "advanced." In the ladder model the small-scale decentralized ("band") societies would be on the bottom rung, the more hierarchical ("tribal") societies would be on the next rung, and the most hierarchical ("state") societies would be on the top rung. Of course, simply positing that the societies with complex social organization (e.g., labor specialization and political hierarchy) have greater technological complexity is not inherently demeaning; but it is not entirely accurate. Anthropological research has persistently shown that neither social structures nor their knowledge systems can be consistently ranked in a unilineal sequence; for example, monotheistic religions tend to occur in band and state societies more than in tribal. Just as biological evolution has been revised from Lovejoy's "great chain of being" to Gould's "copiously branching bush,"[6] so too cultural evolution is now typically portrayed as a branching diversity of forms. There is no reason to focus on state societies over nonstate societies in the pursuit of antiprimitivist portraits.

The difficulties of theoretical frameworks in the epistemology of nonstate societies have been much more mixed. Appiah (1992) provides an extensive discussion of this intersection, starting with ethnophilosophy. His analysis weaves between the positions of Wiredu (1979), who critiques the focus on comparison to Western science rather than religion (noting that it leaves the superstitions and folk philosophies of the West unexamined), and Hountondji (1983), who argues against any mimetic comparison, suggesting that ethnophilosophy and its allies are dressing European motivations in autochthonous garb. Both critiques could certainly be applied to African fractals. But like Mudimbe's (1988) Foucaultian analysis of African epistemology, and Gilroy's (1993) fractal history (which we will examine in the following chapter), Appiah's dialectical contour maps African epistemology as an historical process rather than an object of strictly pre- or post-Western presence. The cautions of Wiredu, Hountondji, and others are serious reminders that African fractals can only succeed as participant simulation, not as Indiana Jones discovering another lost temple.

Given those precautions, it makes sense to see African fractals as just another moment in a historical sequence. One could, for example, place them in Mudimbe's history of ethnophilosophy, or Zaslavsky's (1973) history of research on African math. But there are other researchers who have pointed out some of the fractal characteristics of African designs and practices, and it is useful to examine them as a group, even if they lack the clear historical trajectory of other categories. We have already mentioned the observation of nonlinear scaling by British art historian William Fagg (chapter 6), and the interpretation of scaling designs as signifiers of infinity in the work of Cameroonian theologian Engelbert Mveng (chapter 9). Léopold Senghor, the distinguished man of letters who became Senegal's first president, also had an eye for African fractals. His term was "dynamic symmetry," which he took from art historians. But Senghor's motivation was primarily ideology; defining a "negritude" that would encompass the kind of cultural politics he saw as necessary to independence. Most recently, Henry Louis Gates (1988) explored the doubling practices of vodun divination in terms of a literary version of deterministic chaos; here the recursion generates a cultural uncertainty that frees gender identity from static boundaries: "The Fon and Yoruba escape the Western version of discursive sexism through the action of doubling the double; the number 4 and its multiples are sacred in Yoruba metaphysics. Esu's two sides 'disclose a hidden wholeness,' rather than closing off unity, through the opposition, they signify the passage from one to the other as sections of a subsumed whole."

While all four have hit upon mathematical aspects of African fractals, none of these authors have focused on representations of mathematical knowledge. Mveng, the theologian, provides a theological interpretation. Fagg, the artist, concludes with a comparison to D'Arcy Thompson's famous nature drawings. Senghor, the statesman, sees his dynamic symmetry as a sign of cultural—and thus national—identity. And Gates, as a literary critic, sees it as discursive technique. Surely my insistence on indigenous mathematics is no less an imposition of seeing the world though my own lenses, but since that is no different from the other explanations, why does ethnomathematics appear to be so much more controversial? It is because a portrait of mathematical sophistication in nonstate societies creates a strong conflict with the old ladder model of cultural evolution, a model that is itself overdue for extinction.

Conclusion

So far we have outlined several theoretical frameworks that could raise problems for African fractals. On the one hand, there are theories in which the designs could be dismissed as unconscious biological or social process. On the other hand, great care must be taken to avoid either inflated claims or a mathematical version of negritude. With the exception of biological determinism, none of the frameworks reviewed here are necessarily good or bad. There are cases in which mathematical anthropology is more appropriate than the ethnomathematical approach, or when sameness is a better strategy than difference, or when attention to ancient Egypt needs to supersede attention to sub-Saharan Africa—just as there are cases in which the opposite is true. Our goal is not to find the one true final framework—it does not exist—but to keep a well-stocked toolbox and know how to pick the right tool for the right job. Now that we are well prepared for constructive tasks, it is time to move to politics.

CHAPTER 12

The politics of African fractals

Given the possible dangers in misinterpreting African fractals, how can we put them to good use? Social theorists from many different disciplines have used two mathematical concepts we have discussed, recursion and the analog-digital dichotomy, in constructing their ideologies. Many theories of communication have assumed that there is some kind of universal ethical or social difference between using analog signals and using digital symbols. Other theories have maintained that recursion has some kind of universal ethical or social value. Both are ultimately failures in the sense that ethics and values do not lie within mathematical distinctions. Yet they are also on the right track in that such associations can be *locally* formed—it is just that different locations will result in different social meanings. Such locally specified social attachments to fractals can be useful for understanding cultural politics in Africa and beyond.

The politics of the analog-digital distinction

Jean Jacques Rousseau is often credited as a founder of "organic romanticism," the theory that the Natural is inherently better than the Artificial. Whether or not this is deserved, Jacques Derrida (1974) takes him to task for proposing that a natural/artificial difference can be found between different languages. Jean Jacques

Rousseau proposed that the "natural crys of animals," music, and "accentuation" (that is, pitch intonation in the human voice) are all a similar type of communication. In this I would tend to agree, since instrumental music and human pitch intonation are for the most part analog representations and since he was probably thinking of analog examples of animal communication (although many animals, for example vervet monkeys, use digital communication as well). Rousseau contrasted this to "articulation" in the human voice, by which he meant the linguistic (and hence digital) parts of speech. But instead of seeing the distinction as two different types of representation, one analog and the other digital, Rousseau claimed that analog signals were not a form of representation at all. In his view, digital versus analog was representation versus The Real. Music, animal cries, and emotional intonation were somehow more natural and authentic. Worse yet, he inflated this into a cultural difference, maintaining that while European languages were largely based on (digital) articulation, the language of the nobel savage was closer to nature.

One might hope that Derrida would correct the matter and point out that analog signals are just as much a representation—just as much fakes, just as easy to lie or tell truth with, and just as artificial—as digital symbols are. But he too failed to produce a balanced portrait. Derrida did insist that all human linguistics is fundamentally digital (quite true), but he did not bother to say a word about other modes of vocal representation. This error is due to Derrida's concern over the authoritarian ideology that organic romanticism can produce. For example, history is full of dictators who claimed that their ethnic group was the real or natural one, and that others were artificial pollutants in their Eden. Rousseau himself did not have such fascistic tendencies, but Derrida is right in pointing out that organic romanticism can always be used in that way, no matter who it is coming from.[1] One need not panic so much, however, and banish analog signals from existence; it is enough to give them the same epistemological status as digital symbols—no more and no less.

I have found this egalitarian view of the analog/digital distinction very difficult to promote; it seems that everyone has their own favorite view. When I spoke to chaos theorist Ralph Abraham, for example, he explained that analog systems were in his view the realm of spirit, the vibrations of Atman. Postmodern theory maven James Clifford, to the contrary, insisted that only digital representation is capable of the flexible rearrangements that constitute human thought. This same battle has been played out in the history of African cultural studies. During the 1960s, realism was in vogue, and what could have been a wonderful exploration of the analog representation techniques in African culture was often reduced to romantic portraits of the "real" and "natural," while African symbol systems suffered from neglect. During the late 1970s, this began to reverse itself—with the advent of postmodernism, African cultural portraits became increasingly focused on discourse and symbol systems, even at the expense of ignoring analog representations.

It is important, however, to see how these restrictions have been contested, particularly in black intellectual communities. Hooks (1991, 29) summarizes her own reaction to romantic organicism: "This discourse created the idea of 'primitive' and promoted the notion of an 'authentic' experience, seeing as 'natural' those expressions of black life which conformed to a pre-existing pattern or stereotype." Rose (1993) describes the history of rap music, also arising in the mid-1970s, as not just a resistance to organic romanticism, but as a technocultural rebellion that makes Derrida look like Gutenberg. Cornel West, Houston Baker, Hortense Spillers, and Hazel Carby have made interventions in African American intellectual discourse in similar ways, as have works of black science fiction such as George Schyler's *Black No More*, Ralph Ellison's *Invisible Man*,[2] Toni Cade Bambara's *The Salt Eaters*, Samuel R. Delany's *Dhalgren*, and Octavia Butler's *Xenogenesis* trilogy. An egalitarian view of the natural/artificial dichotomy can be seen in black intellectual history running from George Washington Carver's concept of "God's Kingdom of the Synthetic" to Mudimbe's "Invention of Africa."[3] Indeed, Carver and Mudimbe's concepts are quite similar; it is not Mudimbe's contention that African unity lacks a spiritual bond, but rather a celebration of the spirit of invention, which requires resistance to the European claim that spirit can exist only in categories of the natural. African animism is marked by an extraordinary acceptance of the religious significance of artifice,[4] from gris-gris to the mojo hand, and its techniques for passing information through the physical dynamics of sound and movement show that this faith in the power of analog representation is not misplaced.

The politics of recursion

While Derrida was trashing organic romanticism, Michel Foucault was attempting to do the same for humanism. His historical studies demonstrate that humanist goals of recursion—to be self-governed, self-controlling individuals—are not innocent; but rather develop historically in combination with various techniques of social control. In an era where "self-management" usually means that the corporation you work for has developed improved techniques for self-exploitation, it is not hard to see what Foucault is getting at. As in the case of Derrida's warnings against claims that analog representation will automatically lead to more ethical living, Foucault warns against seeing recursion as a moral formula.[5] While African analog systems raise the problem of someone making claims about what is more real or more natural, African recursion—especially the recursive architecture of African settlements—raises the problem of humanist claims.

To see how this can be a problem, consider the following two case studies of African architecture. Caplan (1981) studied the relation between housing and women's autonomy in Tanzania. She described how the flexibility of housing allowed women to create new homes if they wanted a divorce, or to extend old homes if they wanted to shift the family structure. As in many African settlements, this self-organized housing created a self-similar structure—fractals—which allowed greater social self-control for women. When socialism brought modernization programs, this autonomy was threatened by the "improved" housing design, which sometimes resembled concrete army barracks.[6] Here one would conclude that fractal is better.

Stoller (1984) described a Songhai town in which a caste system ensured that the best land was voluntarily given over to the highest caste members. It was not a matter of forcing people against their will, but simply unquestioned common sense that one should want to be located in their proper place. This fractal, self-organized architecture was a form of self-exploitation. Eventually several members of the community decided to break out of this oppressive structure by building houses along the new highway. Thus liberation in this case meant leaving the fractal geometry, and lining up in straight Euclidean formation—exactly the opposite of the Tanzanian village studied by Caplan. Stoller's work nicely illustrates Michel Foucault's warning against simplistic humanist formulas: self-determination is not necessarily liberating; it can serve to support social control rather than resist it. Neither fractal nor Euclidean geometries have any inherent ethical content; such meanings arise from the people who use them.

Colonialism and architectural fractals

René Descartes was not much of a humanist; in his view self-organized architecture is junk. He makes this clear in his famous *Discourse on Methodology*:

> [T]here is less perfection in works made of several pieces and in works made by the hands of several masters than in those works on which but one master has worked. Thus one sees that buildings undertaken and completed by a single architect are commonly more beautiful and better ordered than those that several architects have tried to patch up. . . . Thus I imagined that people who, having once been half savages and having been civilized only gradually, have made their laws only to the extent that the inconvenience caused by crimes

> and quarrels forced them to do so, would not be as well ordered as those who, from the very beginning of their coming together, have followed the fundamental precepts of some prudent legislator. (1673, 12)

For Descartes, "self-organized" is synonymous with savages, the imperfection of both material and social structure. Lack of complete Euclidean regularity means randomness: for "streets crooked and uneven, one will say that it is chance more than the will of some men using their reason that has arranged them thus" (p. 12). The lack of Cartesian coordinates in many African settlements would thus evidence their need for the guidance of colonial reason. As Hull (1976) notes, huge centers of urban life in Africa were indeed disregarded by Europeans as "unstructured bush communities" on just these principles. While Timbuktu was granted cityhood due to its grid pattern of streets, the Yoruba cities of equal population size and economic, technical, and labor specialization have been disregarded as merely giant villages due to their lack of Cartesian regularity.[7] Thus fractal architecture was used as colonial proof of primitivism. This debate over the urban status of non-Euclidean settlements continues in the postcolonial era (see Schwab 1965; Lloyd 1973).

The occasional Cartesian linearity in African architecture threw a hitch into this colonial justification. In 1871 the German geologist Carl Mauch "discovered" the ruins of Great Zimbabwe. Stunned by the evidence of precise stone cutting on a massive scale, he proposed that the buildings were not of African design, but were instead due to the Queen of Sheba's visit to Solomon. The Rhodesian government used this explanation as a part of its propaganda against Black rule (MacIntosh and MacIntosh, 1989). Actually, they had much less to fear in the truth: the stone was not cut, but it naturally broke into linear sheets (after heating) due to its geologic properties. Moreover, most of the outside walls were originally covered with smooth clay, creating a nonlinear set of scaling shapes (which Connah [1987] refers to as "random curved forms"). This is not to diminish the remarkable technological skill of the construction, but to point out that one culture's sign for "artificial" can be another's sign for "natural." Euclidean versus fractal does not necessarily mean artificial versus natural; that, too, is culturally influenced.

During the development of colonial cities, the chaos of African architecture was used as both symbol and symptom of European fears over social chaos. Pennant (1983) provides an example of this concern about proper settlement geometry in his examination of colonial development in Malawi: "The language of this 1930s policy discourse is significant. Medical experts wrote of 'investigations' showing 'unquestionably' and of 'abundant proof.' . . . Lay Europeans showed 'concern,' 'alarm,' and 'horror.' Africans, with their 'primitive habits,' of 'promiscuous defecation' formed a 'floating' or 'scattered' population in need of 'control' and 'supervision' in a 'properly laid-out village or location.'"

In the above case where "primitive" mixes with modern, the fractal tradition was a threat. But kept in what colonialists thought of as its natural role, it could make fractal settlements appear to benefit the colonial enterprise. The novelist Karen Blixen (Isak Dinesen), in *Out of Africa* (1988), described her attempts to lay out grids for African workers' houses on her ranch. They refused to follow these linear instructions and fit their houses in patterns matching the irregular configuration of the land. That such ecological fit could be quite efficient was not, however, lost on the colonists. "The squatters' land was more intensely alive than the rest of the farm, and was changing with the seasons" (p. 9). Architectural fractals could be part of colonial romanticism as long as they ensured a supply of self-supporting workers.

Even in the case of social control, indigenous fractals could be utilized. British colonial policy, for example, at first failed in cases where there was a decentralized network rather than a large hierarchy. This was approached in the case of the Ibo with a system of "indirect rule" based on "warrent chiefs" (Isichei 1976). The Ibo autonomy of self-organization was turned against them; in a sense it was grass-roots colonialism. The architectural equivalent of this system can be seen in a manual for colonial-era housing designs from the Agency for International Development (Hinchcliff 1946, 31). Here the Ibos' fractal settlement pattern (radial houses around a center in each village, radial villages around the settlement center) is tidied up to suit European conceptions of symmetry while retaining the overall indigenous fractal structure.

Fractals and racial redistricting

In the introduction to his seminal *Fractal Geometry of Nature*, Benoit Mandelbrot examines some of the disparaging comments that were made about the early fractal forms of Georg Cantor, Helge von Koch, and others. Rejected as "bizarre" and "torturous," these "dragons" were consigned to the oddities section at the end of the few math texts that would even consider them. Strikingly similar language has been used to reject the outlines of voting districts that were altered to include larger African American populations, and these do indeed appear to be fractals (fig. 12.1).[8] Were the courts as mistakenly hasty to disregard fractals as mathematicians were?

The Euclidean shape of voting districts is not an arbitrary sampling—this could only be done by randomly selecting voters from everywhere in the state. According to the 1993 Supreme Court ruling in *Shaw v. Reno*, it is meant to

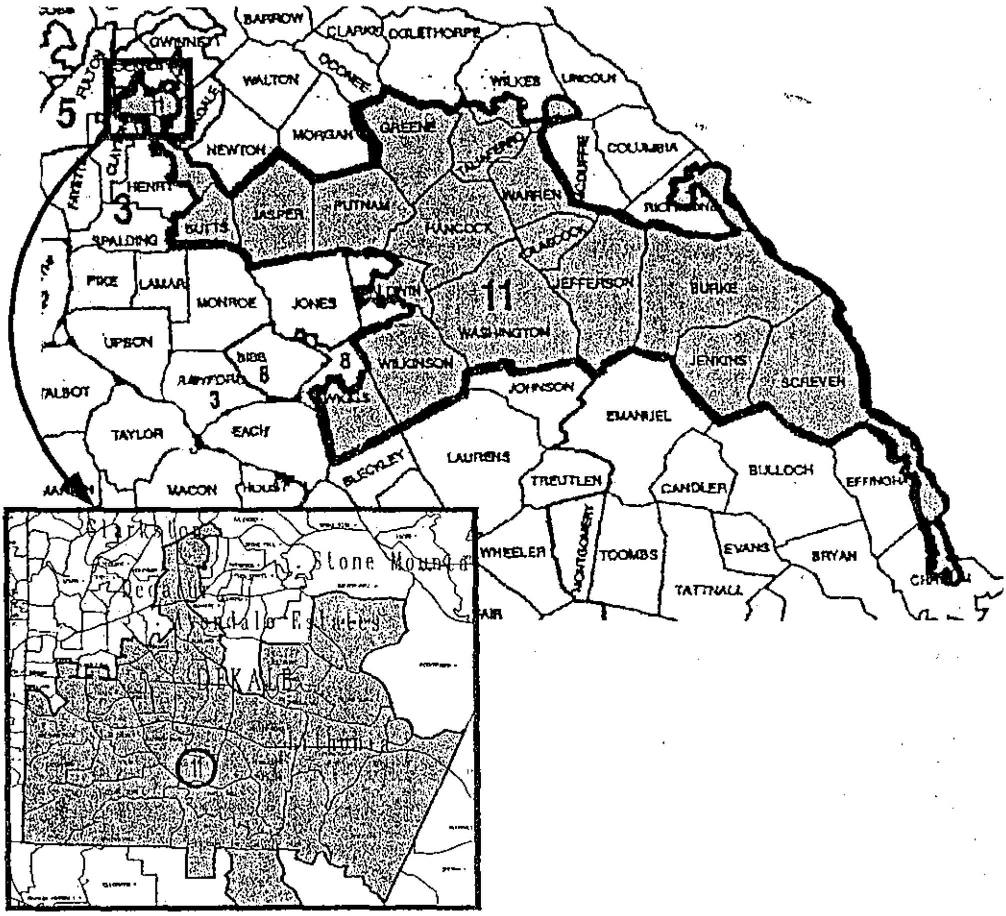


FIGURE 12.1.

Georgia congressional district 11 in 1992

Similarity of irregular redistricting pattern can be seen at multiple scales.

(Original maps courtesy of the Carl Vinson Institute, graphic highlights by the author.)

designate a geographic locale in which "shared interests" inform the vote. The objection to creating a district in which contours are "predominantly motivated" by race is that it creates a bias in the sampling of the geographic location. This would certainly be the case if we were to take a random sampling, separated voters by ethnicity, and then designated those ethnic groups as the voting districts. However, if some ethnic groups are distributed in Euclidean settlement patterns, and others in fractal settlement patterns, then why consider Euclidean district shapes to be unbiased, and fractal district shapes to be biased? I do not know if African American housing distributions are more fractal than others—and even if they are, I would not necessarily assume a cultural connection to African fractals—but the fact that we now know of societies in which fractal settlement patterns are beautiful fusions of form and function suggests that we might reconsider their potential role in American politics.

African fractals from cultural visionaries

Fractals and chaos theory have been increasingly mentioned in the humanities as either a tool or an object of cultural analysis, but too often the approach of these studies has left the impression of mathematical ink blots allowing writers to see whatever they please. Lyotard (1984) saw fractal geometry as contributing to a "postmodern condition" whose contradictory nature would disrupt authoritarian certainty; a more cautious version of this thesis is floated in Deleuze and Guattari (1987). At least two authors (Steenburg 1991; Argyros 1991) have argued that fractals and other branches of chaos theory have created a direct challenge to postmodernism, integrating the disruptions it created. Porush (1991) and others insist that "deterministic chaos" is attempting to substitute a feeling of free will for fatality.[9] Sobchack (1990) suggests that it implies "an embrace of irresponsibility in a world already beyond control." When Sobchack cites Peitgen and Freeman in her condemnation of chaos theory as a denial of "the specificity of human embodiment and historical situation," I can't help but think of Peitgen's fractal geometry course at the University of California at Santa Cruz, where he commented on German mathematicians who altered their careers to oppose Nazi anti-Semitism or support peace efforts; or of Freeman's (1981) use of Martin Luther King in his discussion of chaos in neurophysiology. How can we critique the work of chaos theorists as lacking historical specificity and embodiment if we ignore their own histories and bodies?

Hayles's *Chaos Bound* (1990) took a more subtle approach. Like Porush and others, Hayles's literary method allows her to glide far too easily between unrelated ideas; by the time she has tossed together quantum theory, entropy, and Gödel's theorem with deconstruction and "holism," one can only conclude that any complicated idea can be a metaphor for any other complicated idea. But her detailed analysis of literary works, showing deep parallels between self-reflexive writing and self-referential mathematics, suggests that when grounded in specific locations the fusion of fractal geometry and cultural interpretation can be profoundly rewarding.

Paul Gilroy makes explicit use of fractals in his portrait of the diversity and dynamicism with which both traditional Africa and the African diaspora have organized their cross-cultural flows. The recursive construction of his Black Atlantic can be seen, for example, in this quote from James Brown on a visit to hear Fela Kuti in Nigeria: "[B]y this time he was developing Afro-beat out of African music and funk. His band had a strong rhythm; I think Clyde picked up on it in his drumming, and Bootsy dug it too. Some of the ideas my band was getting from that band had come from me in the first place, but that was okay with me. It made the music that much stronger" (1993, 199).

Gilroy cites the impact of the Virginia Jubilee Singers on tour in South Africa in 1890, the return of slaves from Brazil to Nigeria, the Rastafari culture in Zimbabwe, and other examples of "mutations produced during its contingent loops and fractal trajectories." Perhaps his most radical move is a claim for diasporic mixing with Jewish culture—W.E.B. Du Bois passing for a Jew to maintain safety in Eastern Europe, the use of the Exodus theme in Martin Luther King, Jr., and Marcus Garvey, and E. W. Blyden's childhood in a Jewish community.[10] The fractal imagery works in many different ways for Gilroy—from the turbulent metaphor of hybridity to the concrete description of ships' paths and travelers' routes (or "roots/routes" as he puts it).[11] While music is, without doubt, Gilroy's strongest example, he does slip into the problematic labels of representational versus "nonrepresentational" rather than digital versus analog,[12] but he makes it clear that the music reverberating across his Black Atlantic is neither pure nor natural.

While Gilroy is primarily focused on fractals as spatial representations of blurred boundaries, he also briefly mentions their potential for "a striking image of the scope of agency within restricted conditions" (1993, 237n28); that is, the ability for geometric expansion within bounded space becomes an analogy for oppositional political expansion in human bondage. The metaphor is carried to a more exacting relationship in Gary Van Wyk's study of Sotho-Tswana murals under the apartheid system of South Africa. Van Wyk (1993) found that the *litema*, or the house painting patterns of the Sotho-Tswana women, utilize alternations of irregularity and regularity at several scales, sometimes resulting in a resemblance to fractal patterns. Noting that the scaling is associated with the geometric structure of flowers, and flowers with the regenerative power of women (both spiritually and in social struggles), Van Wyk's ethnography concludes that the murals expressed political opposition to apartheid by providing a visual analog in which "a woman can be secretive while at the same time hiding nothing" (Deleuze and Guattari 1987, 289–290).

Although the word "fractal" is nowhere mentioned in his text, Anton Shammas's novel *Arabesques* (1988) is an exemplar of nonlinear insight and recursive cultural commentary. Heaver (1987) analyzes the novel through the North African artistic form of the "arabesque," and shows how Shammas has used this fractal to sustain the cyclic time and multiple identities required to articulate a

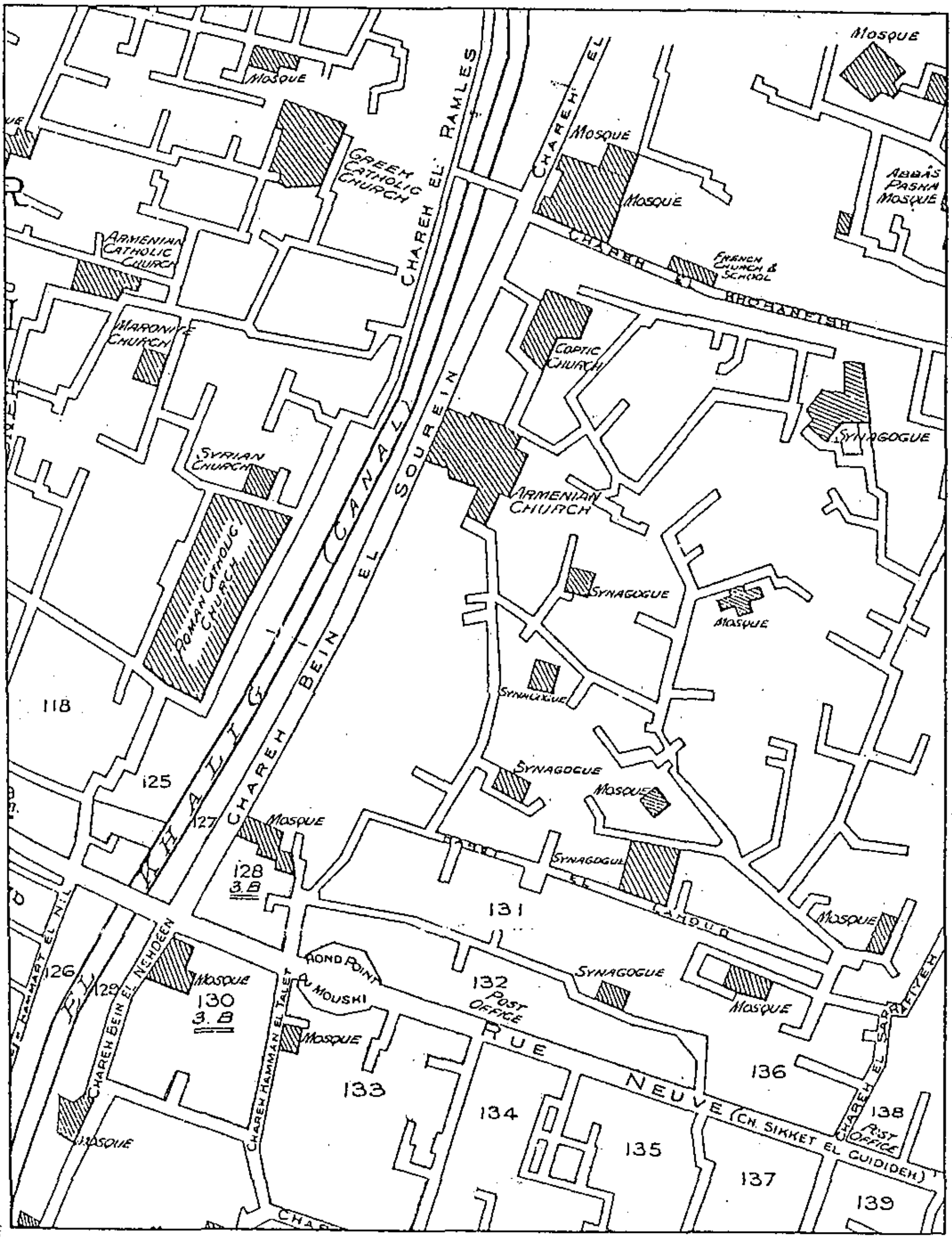


FIGURE 12.2

Religious institutions in the map of Cairo, 1898 political flexibility crucial to the survival of cultural diversity. "As an 'Israeli Arab,' Shammas is a member of a minority group—but as a Christian, he falls outside the Islamic mainstream of the minority. . . . On the other hand, he writes in Hebrew, the language of the dominantly Jewish culture, which is itself a minority within the predominantly Arab Middle East" (p. 49).

Such recursive nesting is emphasized throughout the temporal flow, narrative structure, and conceptual dynamics of the novel. Heaver suggests that the "nonmimetic geometrical abstractions of the arabesque" are a spatial model for Shammas. He notes that in part these cyclic reentries act to negate one another; undermining, for example, the fruitless argument of "I was here first." But negation is not the only meaning behind the arabesque, as Heaver points out in a passage that ties Islamic social structure to analog representation, recursion, and the scaling properties of fractals.

> The arabesque does not serve only a negative, critical function; it also bears a positive, utopian message. It acts as an analogue, in the area of visual arts, to the position of Islamic "contractualism" in the social sphere. . . . In contrast to western corporativism, with its preference for hierarchical structures in which a limited number of conclusions are drawn from a limited number of premises (on the model of geometry), the cyclical rhythms of the arabesque could well be said to characterize an "indefinitely expandable" structure. The arabesque provides a framework within which it becomes possible to reduce the apparently "chaotic variety of life's reality" to manageable proportions, yet without "arbitrarily setting bounds to it." (Heaver 1987, 61)

Clearly, when Heaver refers to the limiting dangers of a "model of geometry" he is thinking of Euclidean structures; it is the fractal geometry of the arabesque which conveys the hopeful message of Shammas. In chapter 2 we examined the arabesque branches of streets that appear in a map of Cairo, Egypt. In another section of this map (fig. 12.2), a wide diversity of religious institutions flower at the ends of these branches, attesting to the positive potential of fractals in cultural politics.

CHAPTER 13

Fractals in the European history of mathematics

Anthropologists have recently taken an increasing interest in the cultural analysis of Euro-American societies. In part this is a reaction to the many decades of focus on indigenous societies, as if their behavior required explanation while that of Europeans was self-evident. At first this "reflexive ethnography" sounded like an ingenious way to turn tables on some very troubling aspects of anthropological authority, but it too has drawbacks. Occasionally one suspects a hidden sigh of relief from anthropologists who decide they can place themselves on the cutting edge by "studying their own tribe" (just as cyberethnography sometimes seems suspiciously convenient). Nevertheless, there is an important place for anthropological studies of Euro-Americans. It would be an unbalanced portrait if we were to see African fractals in need of cultural analysis, and Western fractals as merely self-evident mathematics.

A cultural history of European fractals

Ancient Greek philosophy is often remembered for Plato's rational realm of unchanging, static forms. But in the history of mathematics, it is important to consider other intellectual currents in that society, in particular the paradoxes of the philosopher Zeno of Elea and the discovery of irrational numbers by the Pythagoreans.

According to ancient historians, Pythagoras of Samos gathered knowledge in Egypt and Babylon in the sixth century B.C.E. and established a secret society in Magna Graecia (what is now southeastern Italy). His disciples, including one of the first recorded women mathematicians, Theana, swore an oath to maintain strict dietary regulations, secrecy, and a religious faith in numbers. The Pythagorean cosmology was a harmonious unity based on whole numbers (1, 2, 3 . . .) and their ratios (fractions such as 2/3, 5/2, etc.). From the motion of heavenly bodies to the laws of music, they found increasing evidence for their arithmetic religion. But at some point—and much ink has been spilled in the date debate—came the discovery of what they termed *alogos*, the "irrational" numbers (a name that we have kept to this day). Unlike whole number ratios, which either terminate (5/2 = 2.50000 . . .) or repeat (13/11 = 1.181818 . . .), irrational numbers, such as the square root of two (1.41421356 . . .), continue to change forever. They cannot be expressed as the ratio of two finite integers; as geometric magnitudes they are "incommensurable lines." The most plausible origin for the Pythagorean knowledge of irrationals is in an attempt to determine the diagonal of a pentagon. If you wish to determine the ratio of diagonal to sides for a regular hexagon, it is quite easy, because all diagonals intersect in the center. But the diagonals of a pentagon just form a smaller pentagon. Since the same operation can be repeated again and again, an irrational number is exposed.[1] This "irrationality" in the heart of their spiritual practice was too much, and members of the group agreed not to reveal this secret on pain of death.

Zeno of Elea (fl. ca. 450 B.C.E.), a disciple of Parmenides, provided a series of paradoxes that also conflicted with the numerical faith of the day. His most famous example is a race between Achilles, the fleetest of runners, and a tortoise. Allowing the tortoise a sporting chance, Achilles gives it a considerable lead (let's say 100 feet). But by the time he caught up to the place where the tortoise began, it had already advanced 10 feet. By the time he gained that distance, the tortoise has crept forward one foot. Zeno concluded that although experience proves otherwise, logically the tortoise should win the race. Back in 450 B.C.E., these paradoxes of infinity (and infinity's flip side, the infinitesimal) were unnerving, even shocking to philosophers who depended on rationality as the gateway to religious perfection.

In Plato's philosophic cosmology, spiritual perfection was seen as the higher level of transcendent stasis, and illusion and ignorance were the result of life in our lower realm of changing dynamics ("flux," which in ancient Greek also means "diarrhea"). Several of Plato's students attempted to improve the match between the characteristics of mathematics and the requirements of the static realm. Eudoxus proposed to eliminate irrationals by redefining "ratio," and

Xenocrates introduced a doctrine of indivisibles to oppose Zeno's paradoxes. Aristotle, noting that infinity + infinity = infinity, suggested that this "self-annihilating" characteristic could be eliminated by restricting reference to infinity as a limit to be approached, rather than as a thing itself, a proper object of mathematical inquiry.[2]

The Platonic reform was quite successful, and as a result mathematicians in the following centuries paid little attention to the kinds of recursion that led to Zeno's troubling infinite regress. One early exception was that of Leonardo Fibonacci in the twelfth century. He introduced the first recursive series shown to be of use in modeling the natural world. In chapter 7 we saw that the Fibonacci series appears to have been utilized in the temple architecture and weight balances of ancient Egypt. There may actually be a connection between the two. While little biographical material is available, Gies and Gies (1969) and other sources have put together a good account of what life was probably like for young Leonardo of Pisa. Following schooling in Pisa, in which arithmetic was largely based on the Latin writings of Boethius (circa 500 C.E.), Leonardo's father sent for him from the North African city of Bugia (Bougie). There he learned the Indian place-value notation (probably through Arabic sources). He was inspired by this innovation and traveled along the Mediterranean to Constantinople, Egypt, Syria, Sicily, and Provence, collecting mathematical knowledge from both scholars and ordinary merchants.

The resulting text, *Liber Abaci* (Book of the Abacus), has a strong Islamic influence. Levey (1966), for example, shows that many of abu Kamil's sixty-nine problems can be found in Leonardo's text. But the Fibonacci series, introduced unobtrusively as the solution to a problem in rabbit population growth, does not have a known Islamic counterpart. Perhaps it is simply an independent invention, but if the weight balance system was in use at that time, Leonardo could have easily picked it up from a merchant during his travels in Egypt. And it is possible that through its religious use in ancient Egypt the series had retained some significance as an item of sacred or mystical knowledge and was thus transmitted through scholarly contact.

Gies and Gies (1969, 61) note that Leonardo's practice of reducing all fractions to 1 in the numerator "went back to ancient Egypt, and perhaps derived from the fact that fractions were regarded less as numbers in their own right than as signs of division." Boyer (1968, 281) suggests that the *Liber Abaci* problem with recursive nesting of sevens ("Seven old women went to Rome, each woman had seven mules; each mule carried seven sacks . . .") originated in its ancient Egyptian counterpart (Rhind Mathematical Papyrus problem #79). And Fibonacci does provide a narrative statement of the recursive construction,[3] highlighting the same self-generating aspect of the series that would be emphasized by the ancient Egyptian belief system.

If this influence (whether merely contextual or direct) does in fact exist, it should not detract from the genius of Leonardo's work. His general solution for finding "congruent numbers" for squares has been hailed as "the finest piece of reasoning in number theory of which we have any record before the time of Fermat." But when it comes to the use of the Fibonacci series in the contemporary history of mathematics (cf. Brooke 1964), there is actually no evidence of a direct contribution from Fibonacci himself. By all accounts, German astronomer Johannes Kepler rediscovered the series independently in 1611, and it was only in the mid-1800s, with the formal publication of *Liber Abaci*, that French mathematician Edouard Lucas found the Pisan historical predecessor and named it accordingly. This fact has received little attention, and most texts present Fibonacci's discovery as if it were in a direct intellectual line of descent rather than an honorary title given to a well-deserving but disconnected antecedent. Fibonacci himself seemed unhesitant about the multicultural contributions to his work; the first sentence of *Liber Abaci* states, "The nine Indian figures are . . . ." No doubt he would have been quite content attributing the series to originators of any heritage.

Fibonacci's series was simply unbounded growth; there was no introduction of the infinite except in ways that Aristotle would have approved. The seventeenth century brought attention to the concept of the "infinitesimal" (revived from its Greek banishment in Kepler's *Stereometria* [1615]), and convergence to a limit as infinity is approached (e.g., the algorithms for generating *pi*); but infinity would still exist only as a never-reached orientation rather than a legitimate object of study. The Aristotelian voice could still be heard in 1831, when mathematician Carl Friedrich Gauss (1777–1855) cautioned his friend Schumacher against infinity: "I must protest most vehemently against your use of the infinite as something consummated, as this is never permitted in mathematics. The infinite is but a façon de parler, meaning a limit to which certain ratios may approach as closely as desired when others are permitted to increase indefinitely." But Gauss's distinction was short-lived. As we saw in chapter 1, the work of Georg Cantor, which had produced the first fractal, the Cantor set, ended the Aristotelian view on infinity. Like Fibonacci, Cantor too may have had some non-European influence in his work.

The Cantor set (fig. 13.1a) was his visualization of transfinite number theory. It shows the interval of zero to one on the real number line, and indicates that the number of points is not denumerable—that is, greater than infinity. But at the time, pure mathematics was only one of Cantor's concerns. His real fascination was in the theological implications; the increasing classes of infinity he discovered seemed to point toward a religious transcendental. Cantor's biographers differ greatly on the cultural significance of this point. E. T. Bell felt that Cantor's Jewish ethnic origin ruled his life, and he made several remarks about the inheritance of personality traits—particularly disturbing in light of his

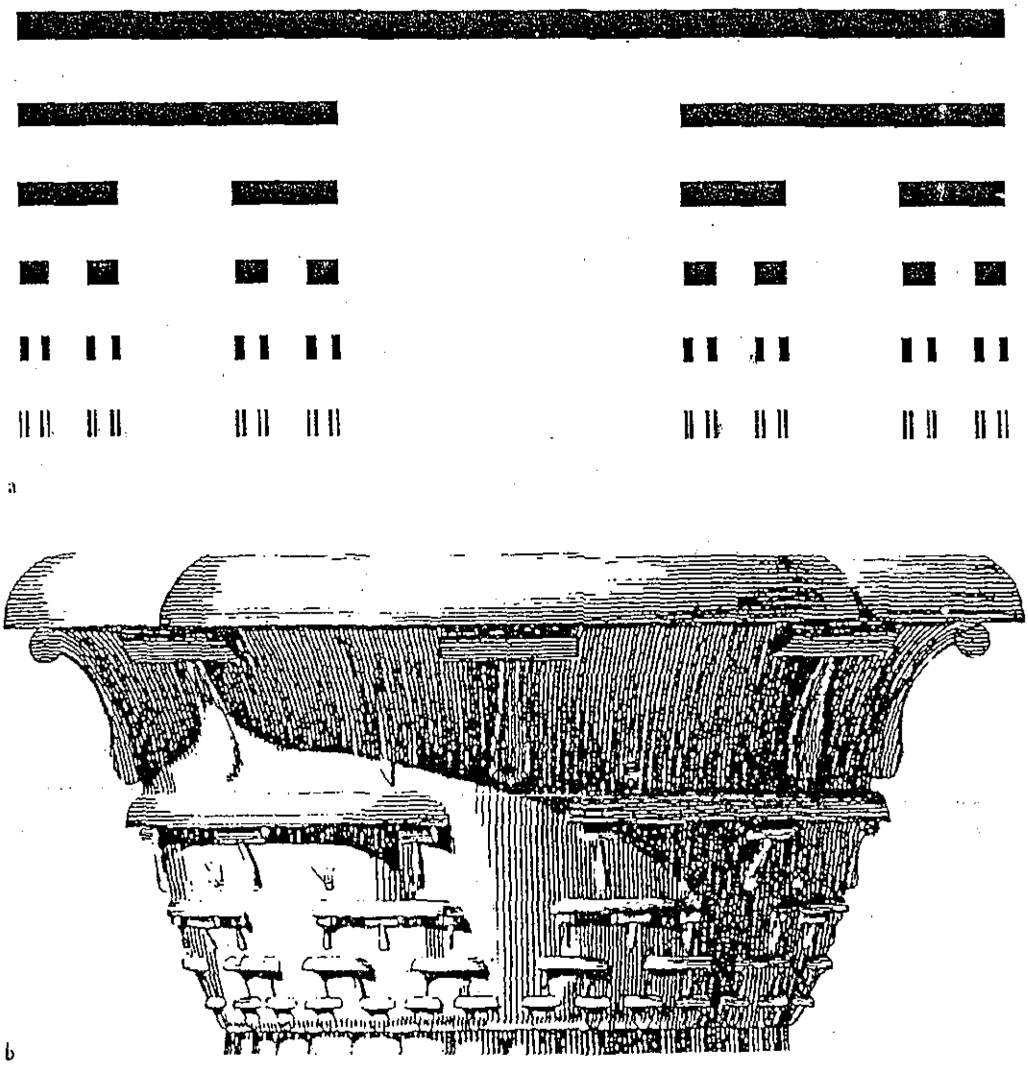


FIGURE 13.1

The Cantor set

(a) The first fractal, created by Georg Cantor in 1877. (b) This design is found on the top of columns in the temples of ancient Egypt. Georg Cantor's Rosicrucian beliefs and his cousin Moritz Cantor, an expert on the geometry of Egyptian art, may have put him in contact with this Egyptian design.

(b, *from Fourier 1821.*)

remarks on Cantor's arch rival, the Jewish mathematician Leopold Kronecker: "There is no more vicious academic hatred than that of one Jew for another when they disagree on purely scientific matters. When two intellectual Jews fall out they disagree all over, throw reserve to the dogs, and do everything in their power to cut one another's throat or stab one another in the back" (Bell 1939, 562–563).

Another Cantor biographer, J. W. Dauben, says that since Cantor's mother was Roman-Catholic "she was by definition non-Jewish, thus it follows that Georg Cantor was not Jewish, contrary to the view which has prevailed in print for many years" (Dauben 1979). Nazi scholars solved their worries by spreading a story that Cantor was found abandoned on a ship bound for St. Petersberg (Grattan-Guiness 1971, 352).

Actually Cantor's Jewish identity was quite complex. His family had indeed converted to Christianity, but he was well aware of his heritage. He referred to his grandmother as "the Israelite" and wrote a religious tract that attempted to show that there was no virgin birth, and that the real father of Jesus Christ was Joseph of Arimathea. Cantor eventually joined the Rosicrucians, whose mystical/scientific approach to a supposed Egyptian origin for all religions probably appealed not only to his intellectual interests, but also to his syncretic ethnicity. Cantor chose a Hebrew letter as his new symbol: the aleph, beginning of the alphabet, was used to represent the beginning of the nondenumerable sets. While his biographers argued Jew or not-Jew, off or on, zero or one, Cantor himself proved that the continuum from zero to one cannot be delimited by any subdivision process, no matter how long its arguments.

Given Cantor's Rosicrucian theology and the proximity of his cousin Moritz Cantor—at that time a leading expert in the geometry of Egyptian art (Cantor 1880)—it may be that Georg Cantor saw the ancient Egyptian representation of the lotus creation myth (fig. 13.1b), and derived inspiration from this African fractal for the Cantor set. We may never know for certain, but the geometric resemblance is quite strong.

As noted in chapter 1, Cantor's mathematics was considered utterly impractical; it was not until Benoit Mandelbrot that fractal geometry became useful to science. Mandelbrot reports that his inspiration came from a study of long-term river fluctuations by British civil servant H. E. Hurst. Hurst examined the flood variations over several centuries and concluded that it could be characterized in terms of a scaling exponent. Later, Mandelbrot realized that this was the same scaling mathematics that the "remarkable curves" of Cantor and others described. But where did Hurst find a reliable source for several centuries of flood data? Hurst lived in Egypt for 62 years and was able to demonstrate long-term scaling in Nile flood records because of the accurate "nilometer" readings going back fifteen centuries. Attempts to find patterns in the floods are quite ancient in the Nile valley; in some ways, Hurst and Mandelbrot were simply the latest—and most successful—participants in that search.

Recursion and sex: a cross-cultural comparison

Throughout the exploration of African fractals, we failed to find any one cultural feature that was persistently associated with these forms. They ranged from practical construction techniques to abstract theological icons, from windscreens to kinship structures, from esthetic patterns to divination techniques. There is no singular "reason" why Africans use fractals, any more than a singular reason why Americans like rock music. Such enormous cultural practices just cover too much social terrain. At best we can make a lower-dimensional projection of such high-dimensional dynamics, the silhouette that appears given one particular axis of illumination. This section will focus on the relation between recursion in mathematics and sexuality in culture. Sex is convenient in that other researchers have developed African-European comparisons, and that sexual reproduction is often connected to recursive concepts.

Taylor (1990) describes sexuality in Rwanda as based on the concept of a "fractal person" in which society is perceived "not in terms of monadic individuals but in terms of . . . structures of meaning whose patterns repeat themselves in slightly varying forms like the contours of a fractal topography" (p. 1025). His analysis on expressions of this sociality in terms of the circulation of fluids is used to examine the failure of programs to encourage condom use. Carolyn Martin Shaw (1989, 1995) analyzes Kikuyu sexuality in related ways and provides an illuminating contrast to European sexuality. Using Foucault's critique of humanism, Shaw challenges the usual portrait of European sexual repression and African sexual license. She demonstrates that in both cases, the social system controls sexual behavior, but while the European locus of control is in the privatization of pleasure, the Kikuyus's sexual regulation occurs through public expressions of pleasure and "sociality of individual conscience." For example, she highlights the practice of *ngweko*, in which teenagers wrap themselves with a few leather strips, oil their bodies, and engage in a public display of sexual behavior. From a European point of view this sounds like an unregulated orgy, but Shaw found that the practice was a method of preventing teenage pregnancies and channeling the teens' sexual desire into socially approved forms.[4]

When we look at many African fractals we can see an emphasis on sexuality in terms of reproduction. The self-similarity of the Bamana chi wara antelope headdress and merunkun fertility puppet, the self-generating Dogon cosmology, the cyclic kinship iconography of the Mitsogho, Fang, and Baluba, the iterations of birthing in the Nankani architecture, and many other cases of recursion are closely tied to sexual reproduction. Thus one contributing factor to the African mathematical emphasis on recursion could be this African construction of sexuality through positive public domain expressions.

The European counterpart of Shaw's theory would predict the opposite, and indeed we find that the banishing of infinite regress in the Platonic reform was closely tied to a kind of sexual prohibition. In Plato's *Symposium*, Socrates explains that there is a hierarchy of reproduction. Love between a man and a woman will only result in a flesh child, a creature of flux who will eventually die, at best producing more flux. Love between a man and a boy (lover and beloved) is higher, because it can result in raising the boy to a higher plane—that of a philosopher. And a philosopher can have a "brain child," a perfect idea that never changes or dies. The Platonic ideal of static, eternal perfection conflicts with the ever-changing dynamic of sexual reproduction. The Greek preference for the static shape of the Archimedean spiral suggests this Platonic ideal, just as the growing shape of the logarithmic spiral suggests the African emphasis on fertility and reproduction. Of course, this is a gross generalization; there are, for example, plenty of Archimedean spirals in African designs. Conversely, European mathematician Jacobo Bernoulli was utterly dedicated to the logarithmic spiral and specified that one would be engraved on his tombstone. But the stone cutter did not go against the grain of his culture; Bernoulli's grave is still marked with an Archmedean spiral (fig. 13.2).

It would be dangerous to suggest that there is an ethical difference at stake here, as so many organic romanticists have maintained. Again, there is no historical evidence for a consistent relationship between mathematical distinc-

FIGURE 13.2
Bernoulli's tombstone
Although Bernoulli asked for a logarithmic spiral to be inscribed on his tombstone, the engraver was apparently only familiar with the linear spiral.
(From Maor 1987, courtesy Birkhäuser Verlag AG, Basel, Switzerland.)

tions and the ethics of their users. Some strictly linear, logical thinkers like Bertrand Russell and Noam Chomsky have been famous for their progressive ethical standpoints, just as some holistic organicists have been prone to fascism. And of course vice versa. What does count for ethics is how people are able to use mathematics in the particular events and ideas that surrounded their life. With that in mind, let's look at three of the innovators who brought recursion into European computational mathematics.

The story of Ada Lovelace is well known in computing science history. Her fame stems from her writings in 1843 on the mathematical possibilities of Charles Babbage's proposed "analytical engine"—a plan for a mechanical digital computer. Lovelace is often promoted as a recovered feminist ancestor, a position that tends to overestimate her achievements and obscure her own thinking. Against these reductive portraits, Stein (1985) has written a detailed, critical examination of Lovelace that reveals a much more interesting and complex story than the popularizations have allowed.

Lovelace's mother was always worried that she might have inherited the notorious sexual proclivities of her father, Lord Byron. Her childhood revolved around strictly prescribed educational activities, and at times she was forced to lie perfectly still, with bags over her hands to ward off any "wildness." This repressed upbringing eventually inspired rebellion in the form of an attempted elopement, but the failed affair left her humiliated and repentant. She wrote to a family friend, William King, requesting mathematical instruction as a cure for her sinful impulses. King agreed, sending her both mathematical and religious texts. But despite her declarations to apply her mathematical imagination "to the greater glory of God," she turned away from the moralizing of King to the more glamorous social company of Babbage and his famous "thinking machines."

Babbage's motivations were far removed from King's religious intellectualism. He was primarily concerned with economic and scientific progress. This switch from King to Babbage was an act of independence, and Lovelace began to turn her imagination loose. While pursuing a much more intense area of mathematical study, her religious thinking also took an expanded turn. She began to describe herself and her work in terms of magical imagery: the mechanisms of symbol manipulation were "mathematical sprites," and she advised Babbage to allow himself to be "unresistingly bewitched" by "the High Priestess of Baggage's Engine."

Stein also notes that it was actually Babbage who first drew up the "table of steps" constituting the first computer programs. Babbage was having difficulty obtaining funding for his work, however, and realized that Lovelace's social position and notoriety—both as the daughter of Byron as well as a "Lady of

Mathematics"—could be put to his advantage. The reputation of Lovelace as the originator of programming stems from this public relations ploy of Babbage. There was, however, one table for which Ada was wholly responsible: the recursive generation of a sequence known as the Bernoulli numbers. Moore (1977) states that this table used recursive programming. Huskey and Huskey (1984), apparently referring to this claim, suggest that this is a confusion with Lovelace's description of mathematical "recurrence groups" and note that the term "recursive programming" generally refers to a procedure that calls itself (i.e., self-reference)—impossible for Lovelace since her code had no procedures. But they also note that Lovelace introduced a new code notation to describe what she referred to as "a cycle of a cycle," which would be equivalent to the recursive structure of nested iteration in use today.

Significantly, this iterative recursion was the one program for which Babbage claimed credit: "We discussed various illustrations that might be introduced: I suggested several, but the selection was entirely her own. So also was the algebraic working out of the different problems, except, indeed, that relating to the numbers of Bernouilli [*sic*], which I had offered to do to save Lady Lovelace the trouble" (quoted in Stein 1985, 89).

The appropriation may have been anticipated by Lovelace: Stein notes that in the letters concerning this program, Lovelace is atypically vague—she had always been overdependent on Babbage for mathematical specifics—and speculates that the vagueness was a deliberate move to protect her iterative innovation. Many feminists have written about male envy of women's reproductive capacity,[5] and there might well be a parallel in Babbage's appropriation of Lovelace's recursive achievement. But the organicist versions of such analyses portray the conflict in terms of women being more natural or embodied, and men being more artificial or abstract. In this story of male womb envy and the protective mother, it is the digital abstraction of recursion, not concrete embodiment, over which the struggle is fought. The birthing metaphor was mentioned by Lovelace herself; the finished programming study was "her first child." Contrary to Plato, sexual reproduction is not in opposition to the abstract realm of mathematics; Lovelace used her mathematics to rebel against attempts to limit her to a repressive femininity and used this artificial sexuality—a bewitching high priestess, jealously guarding her programming progeny—to develop the first computational recursion.

In the discussion of the mathematical theory of computability in chapter 10, we noted that the set of "primitive recursive functions," developed by Rózsa Pèter, had the greatest computing power short of a Turing machine. Unlike Lovelace, Pèter's capability as a mathematician is uncontested; in fact, she is widely regarded as "the mother of recursive function theory" (Morris and Harkleroad 1990). But she, too, implied that parallels existed between her gender identity and mathematics; maintaining that women could provide a special insight that men could not (Andréka 1974, 173). Since we know that, as a mathematician, she would not be thinking of this special insight as being more concrete or less logical, it may be that Pèter also made connections between sexual reproduction and recursion.

Following Pèter's class of primitive recursive functions, one reaches the upper limit of recursive power in the Turing machine. Alan Turing's contributions were not only in the mathematical abstractions of computing, but in its application to artificial intelligence as well. In his classic paper titled "Computing Machinery and Intelligence," he proposed what is now called the Turing test. At first, most definitions of machine intelligence were based on a particular task or behavior (e.g., chess playing). But as the field of artificial intelligence (AI) has developed, these have shown to be increasingly inadequate, and the Turing test is widely regarded as the most reliable definition for AI (in fact, yearly Turing tests are now held, with no machine winners thus far).

Turing begins by describing a game in which a man and a woman are behind a door and answer questions from an interrogator by written replies. The interrogator must determine who is the man and who is the woman; both must try to deceive him in their answers. Turing then suggested replacing one person with an AI machine; the Turing test holds that if the interrogator cannot distinguish person from machine, then one has created true machine intelligence. Turing's biographer, Andrew Hodges, suggests that this "imitation game" was inspired by Turing's own life: struggling to define his identity as a homosexual in a homophobic society. Both the Turing machine's ability to imitate other machines and this game of cognitive imitation echo the social experience of gays who live in a community where they must pretend to be someone they are not. And to some extent, the endless self-reference of metamathematics was Turing's hiding place from the antigay world surrounding him. But the sexual guessing game on which the Turing test was based worked against such normative gender restrictions: it suggested gender as something more fluid, less fixed—a feature which the virtual communities on the Internet have started to demonstrate (cf. Stone 1995; Turkle 1995). Douglas Hofstadter (1985, 136–167), a modern master of recursion, has also written about the potential for a more fluid gender identity in digital dynamics.

Mathematics had a double meaning for Turing. It was both an emotional shield, a closed world of endless interior self-reference, as well as an opening into consciousness and community. In the end, this desire for opening killed Turing:

during a robbery investigation he admitted his homosexuality to police detectives and was arrested and forced to submit to hormone treatments. This eventually drove him to suicide. It was a tragic fairy-tale ending: he killed himself by eating an apple dipped in poison. Hodges writes about this death in terms of the double meaning that mathematics had in his life. "Lonely consciousness of self-consciousness was at the center of his ideas. But that self-consciousness went beyond Gödelian self-reference, abstract mind turning upon its abstract self. There was in his life a mathematical serpent, biting its own tail forever, but there was another one that had bid him eat from the tree of knowledge."

In Africa these two serpents are one; sexual reproduction exists in the same public realm as social intercourse. That is one possible reason why we see recursion—the snake that bites its own tail—so prominently emphasized in African fractals, and a possible explanation for why these pioneers of recursion in Europe happened to be people who took issue with sexual repression. That's not to say there is a deterministic link between the two. In analog feedback theory, for example, we see both anti-authoritarian feminists, like Norbert Wiener (Heims 1984), as well as authoritarian prudes like Howard Odum (Taylor 1988). Mathematics is not a mere reflection of personal interests, nor is it an abstraction that is entirely divorced from our lives. We make meaning for ourselves out of whatever metaphors—technical or otherwise—we find useful; conversely, personal meanings can often inspire new technical ideas.

While recursion is prominent in African fractals, it has been less so in European fractal geometry.[6] In the historical appendix to *The Fractal Geometry of Nature*, Mandelbrot provides an erudite history of mathematical developments that led to his work; recursion is never mentioned. Even when recursion does come up in the fractal geometry literature, the treatment is typically informal or cursory. For example, Saupe (1988, 72) merely notes that "in some cases the procedure can be formulated as a recursion."[7] Similarly, the fractal time series produced by deterministic chaos is rarely regarded as the product of feedback loops, and in one of the few studies that is focused on this relationship, Mees (1984, 101) merely states that "chaos is certainly possible in feedback systems." On the contrary, it is not that chaos is possible with feedback, but that chaos is impossible without it.

It would be inaccurate to say that European mathematics has disregarded recursion in general, and perhaps the observation I am making is simply due to disciplinary specialization; there is no reason why someone studying applications of graphics to analysis and mensural theory should necessarily be thinking about Turing machines or recursive functions. But it is precisely this lack of necessity in mathematics that is so easily forgotten in a discipline where certainty goes beyond that of any empirical science imaginable. Mathematics is both an invention and a discovery. We discover the constraints inherent in the fabric of space and time, constraints that are the stuff of which our universe is composed. But mathematics does not stop there. The constraints are not just negations, but rather the building blocks with which further mathematics is constructed. And like any construction, there are choices to be made, decisions about how these building blocks are to be connected, interrogated, and deployed in further discovery. This is where the human side of mathematics enters the picture, especially that most human of endeavors, culture. Conversely, culture is not mere whim, a purely subjective matter of choosing favored social practices. This is where the mathematical side of humanity enters the picture, for we are only free to construct culture within the constraints of the universe in which we live. Neither mathematics nor culture should be viewed as firmly fixed on the universal/local divide; there are divisions within divisions never ending.

CHAPTER

14

Futures for African fractals

Most anthropologists have long abandoned the tendency to create a frozen "ancient tradition" in defining indigenous society; change and synthesis are now integral parts of the cultural portrait. So, too, with African fractals; they are necessarily as much of the future as they are of the past.

Fractals in African contemporary arts

There are many works of modern African professional art which incorporate aspects of fractals, spanning a wide range of cultural viewpoints. At the National Museum in Yaoundé, Cameroon, one can see organic romanticism in Nyame's paintings of logarithmic spirals morphing into people. The double-sided post-modern metal sculptures of Legba in Benin, by artists such as Kouass, show a cyborg[1] trickster whose traditional bifurcating abilities are ready for the binary codes of new technologies. In East Africa, painter Gebre Kristos Desta produces nonlinear scaling circles he describes as pure abstractionism (Mount 1973, 118). African fractals continue to evolve. Besides being present in professional studio art, fractals have also appeared in large-scale public art works, such as on the facade of the University of Dakar library (fig. 14.1). This scaling design, in which the alternation of painted rectangles at the small scale

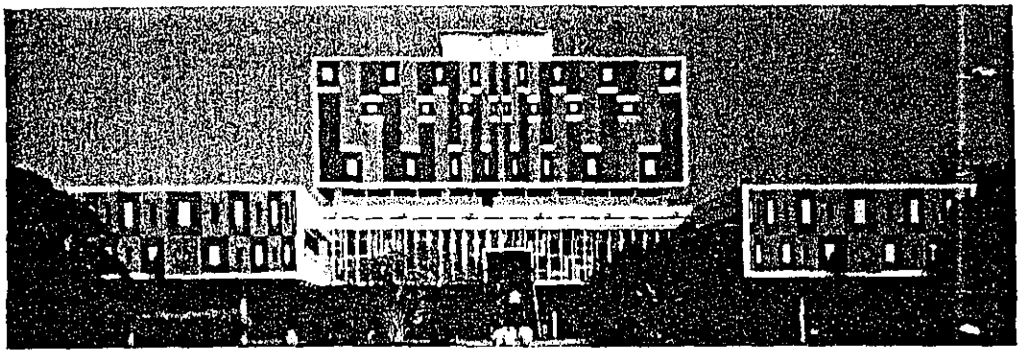


FIGURE 14.1

The library of the University of Dakar

This design makes use of both self-similarity (the vertical alternation of painted rectangles looks like the alternation of buildings) and nonlinear scaling (the rectangle width decreases rapidly as you go toward the center).

matches the alternation of the building walls at the large scale, is reminiscent of certain African fabrics.

One of the most active areas of today's African art comes not from professional studios, but rather from the undistinguished sellers of tourist art. Tourist art was formally disregarded in the professional art world, but cultural studies have increasingly shown that this is a dubious position. First, neither the "traditional artist" creating royal works for a king, nor art students trying to please their instructors, nor even professional studio artists who must also be concerned with sales are completely free to create whatever they wish, so there is no reason to single out the creators of tourist art for being constrained. Second, opportunities for professional studio artists are few, and the tourist market creates a large number of economic opportunities; it seems suspicious to disregard this vibrant activity in favor of a tiny elite. And finally, as Cullers (1981) notes, tourism is not the opposite of authentic culture, rather tourism *creates* authenticity.

Cullers's observation was repeated to me by Max (he did not want his last name to be used), a Senegalese artist in Dakar who sells to the tourist trade. Max complained that his most creative work—the designs which came to him in dreams—was difficult to sell because of the tourist conception of tradition and authenticity. Like many creators of tourist art in Dakar, he produces imitations of the kora, the Senegalese stringed instrument that features a single fret running down the center and a hand grip on both sides. Figure 14.2 shows the usual kora model, along with Max's innovation, the recursive kora. The recursive kora makes use of each hand grip as the fret of two smaller koras. I asked Max

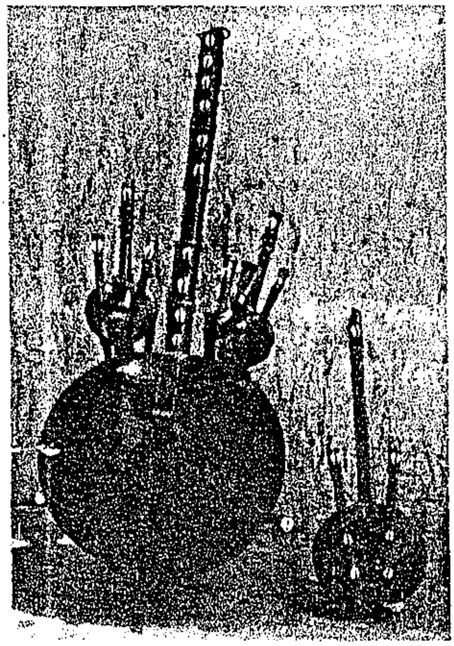


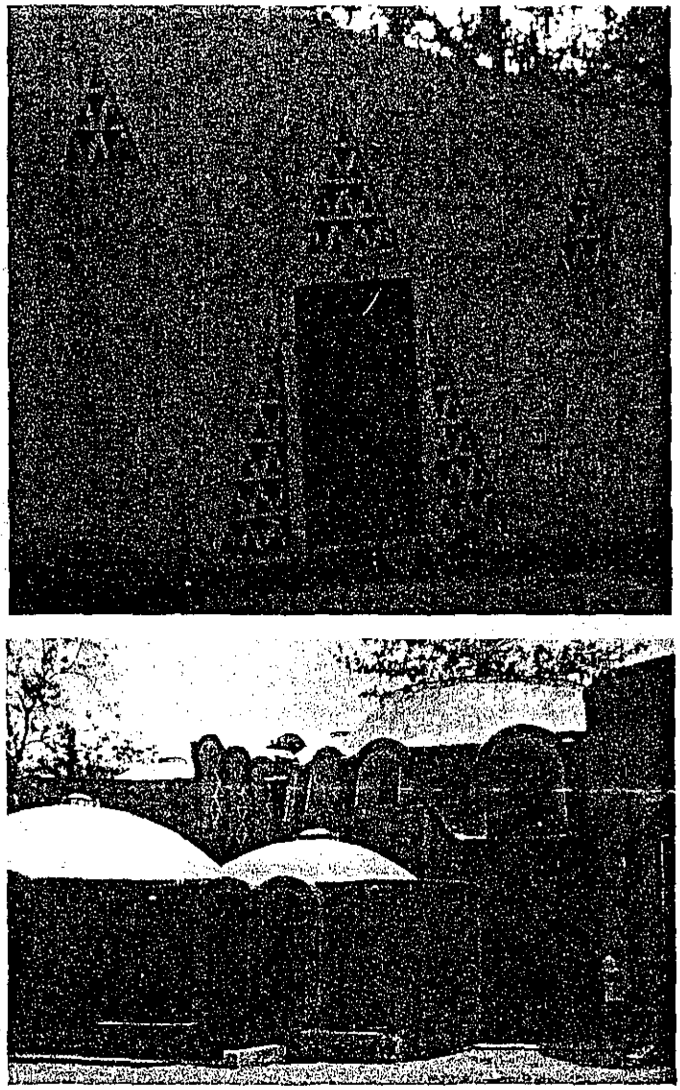
FIGURE 14.2
The recursive kora
At *right*, a typical kora; at *left*, the innovative recursive kora created by Senegalese artist Max.

if he had ever considered continuing to smaller scales, and he said that he had once done so, but that it was impossible to sell such innovative work; tourists did not want anything that smacked of originality.

Fractals in African contemporary architecture

Many indigenous African designs have been incorporated into modern architectural projects in Africa, and some of these have been fractals. For example, the Sierpinski-like iterative triangles from Mauritania were used in an institutional building in Senegal, and the circles of circles in the architecture of West African villages became the basis of a design for a building complex in downtown Bamako, the capital of Mali (fig. 14.3).

One of the most potent visions of an African fractal future has come from the architectural studies of Dr. David Hughes at Kent State University in Ohio. Working as a Fulbright scholar in several African countries, Hughes (1994) put together a portrait of what he termed "Afrocentric architecture," which embodies several aspects of the fractal model. First, Hughes combined a characterization of the self-organizing properties of African building design (an "organic architecture" which "grows from its site") with its self-similar properties (what he termed "the outside/inside relationship," a mutual shaping of units, clusters of units, and communal spaces formed by the surrounding clusters). Second, he explicitly rejected primitivist or naturalizing portraits. While noting its environmental harmony, Hughes also emphasized that African architecture is always an intentional act of design and semiotics, not merely an unconscious adaptation to the ecosystem. In his framework, "tradition" includes the tradition of innovation, or as Gates (1988) puts it, the African theme of "repetition with revision."



a

b

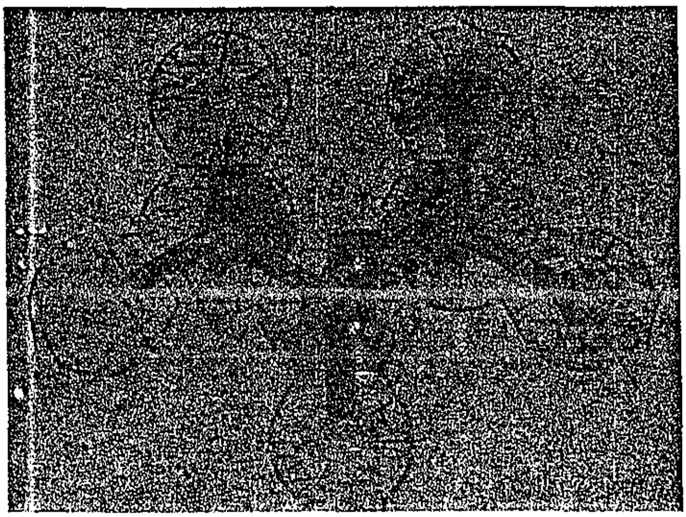
FIGURE 14.3

Indigenous fractals in modern architecture

(a) Here a traditional Mauritanian fractal design is used in a modern building in the Casamance, Senegal. (b) *The DPC building in Burkina Faso, using traditional scaling cylinders with contemporary construction techniques.* Architects such as Issiaka Isaac Drabo have made many large-scale buildings based on this syncretic approach.

Given this combination of self-organized structure and intentional design, it is not surprising that Hughes's work led him to a beautiful example of the potential fractal future. Figure 14.4 shows a design by Alex Nyangula, one of Hughes's students at the Copperbelt University in Zambia (Hughes 1994, 165–166). This architecture provides a powerful syncretic fusion of indigenous and modern forms. The figure traced by the walkway shown in the ground plan is a classic

a



b

FIGURE 14.4
Design for Kitwe Community Clinic
(a) Kitwe Community Clinic in Zambia; design by David Hughes and Alex Nyangula.
(b) Kitwe Community Clinic ground plan.
(Photos courtesy David Hughes.)

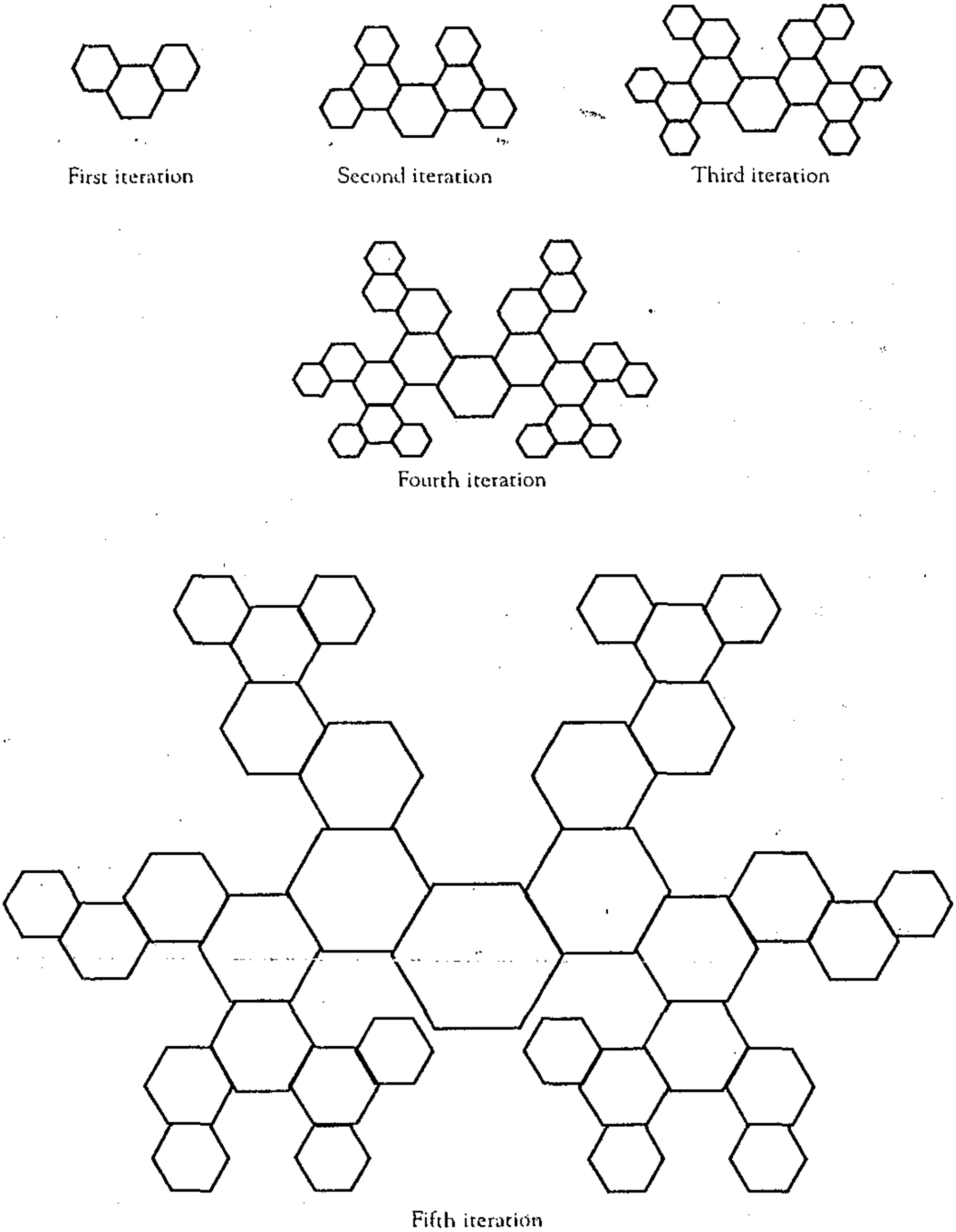


FIGURE 14.5

Fractal iterations of Nyangula's community clinic design

Fractal based on Nyangula's architectural design. The "active lines" of the generation process have been removed, as have any self-intersecting hexagons.

example of the fractal branching pattern referred to as a Cayley tree (see Schroeder 1991, 87–88; Peitgen et al. 1991, 19–20), and can be extended from the two iterations given by Nyangula to infinity. Adding the hexagons (a syncretism between the cylinder of Zambian indigenous architecture and the rectilinear forms of modern materials) violates the Cayley requirement that the graph is self-avoiding (that is, that the branches do not intersect). Since I was interested in exploring the fractal structure by taking Nyangula's design to higher iterations, I made two adjustments for this problem. One is suggested by the approach elevation sketch (Hughes 1994, 167), where it is clear that the central unit is slightly larger than the others. This means that self-intersection will be forestalled to higher iterations.[2] The other is simply the elimination of units whenever they overlap. With these two qualifications, Nyangula's design makes for an infinitely expandable (yet bounded) architecture, as shown in figure 14.5. Such flexibility could contribute to the efforts to encourage a more participatory approach to African architectural design (Fathy 1973; Özkan 1997).

If we take an aerial view of the modern European settlement of Paris, France, we would see linear concentric circles surrounding its center of social power. The difference between this linear, radially symmetric series of circles and Africa's nonlinear, decentralized architecture is perhaps subtle, but important. The term "Afrocentric" is misleading in that "centric" is much more the geometry of Paris than of Logone-Birni, Mokoulek, Labbezanga, and the other African architectures we have explored. Hughes's call for a "multidimensional Afrocentrism" is both an affirmation of "Afro" and a challenge to "centrism"; it is a call for cultural portraits that do not reduce to a single one-dimensional center but rather combine the boundaries of tradition with the infinite expansion of innovation.

African fractals in math education

Several researchers have independently explored fractal aspects of African mathematics. Chonat Getz of the University of the Witwatersrand has created Iterated Function System simulations of Zulu basket weaving. John Sims, mathematician and artist at the Ringling School of Design in Florida, has developed fractal patterns based on Bakuba rafia cloth (and inspired by his African heritage). In chapter 5 we encountered the lusona analysis of Paulus Gerds, a professor at Universidade Pedagogica of Maputo, Mozambique, whose prolific writings have recently ranged from the ethnomathematics of women's art in southern Africa (Gerdes 1998a) to the use of Mozambique basket weaving geometry in modeling fullerene molecules (Gerdes 1998b).

While there are clearly benefits to utilizing indigenous knowledge for development and education in Africa, African fractals might also be of use in the United States. Despite the low mathematics participation of African American students as an ethnic group, it has been demonstrated that changes in the learning environment can improve their mathematics proficiency to levels equal to the majority population. Evidence suggests that although direct institutional barriers in economically disadvantaged schools, such as the emphasis of vocational over academic subjects (Davis 1986) and lack of computer access (Anderson, Welch, and Harris, 1984) can account for some of this difference, more subtle curricular changes can play a key role in retention and achievement. For example, Baratz et al. (1985) found that African American students are more likely to use computers for routine drill; hence, the problem is not simply the availability of computers, but also their style of utilization. The National Assessment of Educational Progress (1983) study of math performance in seventeen-year-old African Americans reported the greatest deficiencies at the applications level, and several researchers (Usiskin 1985; Davis 1989; Malcom 1983) have recommended revision of courses to emphasize more interdisciplinary and "real-world" mathematics instruction as well as "action-oriented" pedagogy. Computer-based learning has demonstrated the capability for both interactive and interdisciplinary mathematics instruction (Keitel and Ruthven 1993), and Stiff et al. (1993) specifically point to computer-based learning as a promising forum for bringing these changes to African American students. These needs could be directly addressed by applying African fractals to the classroom.

In addition to changes in structural aspects of mathematics teaching, several researchers and instructors have initiated culturally enriched curricula. The rationale for this approach comes from a variety of perspectives (e.g., Vygotskian learning theory). Powell (1990) notes that pervasive mainstream stereotypes of scientists and mathematicians conflict with African American cultural orientation. Similar conflicts between African American identity and mathematics education in terms of self-perception, course selection, and career guidance have been noted (cf. Hall and Postman-Kammer 1987; Boyer 1983). But we should not assume that this constitutes a problem of "self-esteem." The relation between cultural identity and learning is quite complex; it would be naive to suggest that today's African American students have the same relation to ideas about their ancestry as did students in previous decades, and in no case has there ever been a simple "mimicry" of African culture. Rather, ethnographic research (Hebdige 1987; Mercer 1988; Rose 1994) shows that African American youth actively construct identity using a wide variety of cultural signifiers.

For this reason, applications of African fractals will have to stress design tools and guided discovery, and avoid passive presentation. While "interactive" has become a catchword in multimedia, many of these systems merely use the computer like a slide projector, with students pressing different buttons to see various images. Multimedia in this form has a distinctly "canned" feel to it. The design approach, in contrast, offers students tools for constructing patterns of their own creation. Thanks to many participants—in particular, programmers TQ Berg and Jaron Sampson, and minority math education specialist Gloria Gilmer—we have started development of an African fractals software math lab. The lab begins with simulations of traditional African patterns and shows students how the mathematical structure behind these designs offers them tools to create their own.

Again, it is important to stress that African American students are not expected to be interested in the material out of a simple identity reflection, anymore than they would necessarily be interested in wearing Dashiki shirts and Afros. Rather, it is the opportunity to create new configurations and syntheses that combine tradition and innovation that are significant. At the June 1996 meeting of the Columbus Urban Youth Conference, we explored these connections with a class of eighteen 12-year-old African American students. The first class meeting introducing traditional architecture was a near disaster; despite multimedia and manipulatives, it appeared that the primitivist associations with "mud huts" were a strong deterrent. The following session, using the Ghanaian log spiral—cellular automata—owari relations, was quite successful, probably because the combination of traditional religious knowledge and mathematical graphics sent a more clear antiprimitivist message.[3] But in a design exercise where the students began with computer graphics simulations of the Ghanaian logarithmic spiral patterns, they showed little interest in producing further *imitations* of the African designs. Rather, the students quickly caught on to visual correlates of the equation parameters and began a free-for-all competition to see who could make the most bizarre patterns. Their interest appeared to be sparked by the African connections, but quickly went beyond them.

Perhaps more important than mitigating a direct conflict between ethnic identity and mathematics, using African fractals in the classroom might help guard against an overemphasis on biological determinism, which has been found adversely to affect mathematics learning. Geary (1994) reviews cross-cultural studies that indicate that while children, teachers and parents in China and Japan tend to view difficulty with mathematics as a problem of time and effort, their American counterparts attribute differences in mathematics performance to innate ability (which can then become a self-fulfilling prophecy). For African Americans, biological determinism has been closely associated with mythic stereotypes about "primitive people" (e.g., the fable that Africans count "one, two, three, many"). By showing the presence of complex mathematical concepts in African culture, we can mend some of that damage. Since reductive myths of biological determinism are detrimental to mathematics learning for students of all ethnic backgrounds, all students could potentially benefit from this material.

Finally, we should note that the increasing use of multicultural curriculum materials in the arts and humanities have not been matched in the sciences. This could send a message to minority students that their heritage is only pertinent to the arts and humanities, and that the sciences are really for people from other ethnic groups. In addition, some texts such as *Multicultural Mathematics* (Nelson 1993) have emphasized only Chinese, Hindu, and Muslim examples, so that even in cases where multiculturalism is used, African math may be left out (see Katz 1992 for a similar critique). And of the few texts that do use African math, almost all examples are restricted to primary school level. Again, this restriction might unintentionally imply primitivism (i.e., that mathematical concepts from African culture are only childlike). For this reason, our lab's inclusion of advanced topics such as fractal geometry, cellular automata, and complexity are worth the extra effort to tie into a secondary school curriculum (without overlooking the use of standard topics such as logarithmic scaling, geometric construction, and trigonometry).

While the multimedia lab's most significant potential for improving education is in mathematics, we should not ignore African Studies. African art, for example, is increasingly used in secondary schools across the nation, and use of our lab could greatly enhance such courses. First, as noted above, it provides an alternative to detrimental misrepresentations of Africans as "primitive" people. In art history lessons, for instance, students often learn about the geometric basis for Greek architecture or Renaissance painting, while commentary on African works is often restricted to discussion of "naturalness" or "emotional expression." Second, the lab aids in integrative curricula development (see Roth 1994 on difficulties in this area). It would allow math teachers who would like to include ethnomathematics components in their teaching to refer to examples in which students are already engaged, and would provide art teachers with new tools for design and analysis. Similar advantages could be obtained in other African Studies areas.

Information technologies and sustainable development

The use of indigenous knowledge systems in development goes back to colonial appropriations, but in the postcolonial context these systems have taken on new meaning as a sign of either epistemological independence, or at least a more egalitarian view of knowledge systems. In chapter 10, for example, we saw the scaling spirals of Jola settlement architecture that arose from their circular buildings; the French research organization ENDA has built an impluvium created by the combination of modern materials and this traditional Jola design. Another of ENDA's rural development projects that incorporate both traditional fractal architecture and modern techniques is shown in figure 14.6.

In chapter 6 we saw how the scaling patterns of kente cloth were created to match the scaling of saccadic eye movements as they scan from the face to the body. The Ghanaian Broadcasting Corporation, Ghana's national television channel, has continued this practice in the context of modern information technologies, utilizing the scaling pattern of kente cloth in their test pattern (fig. 14.7). Whereas the traditional scaling was applied to the human visual scan, this technologized version makes use of the same pattern for testing the video scan. A simple application, but it shows that African fractals are not just restricted to low-tech adaptations; they can also provide some useful bridges between traditional and high-tech worlds.

In chapter 10 we saw that there were ties between the traditional knowledge systems supported by African fractals and the productive maintenance of these societies in what Per Bak would call a state of self-organized criticality. This suggests that most of the indigenous African societies were neither utterly anarchic, nor frozen in static order; rather, they utilized an adaptive flexibility that could be applied to modern development. But decades of research have shown

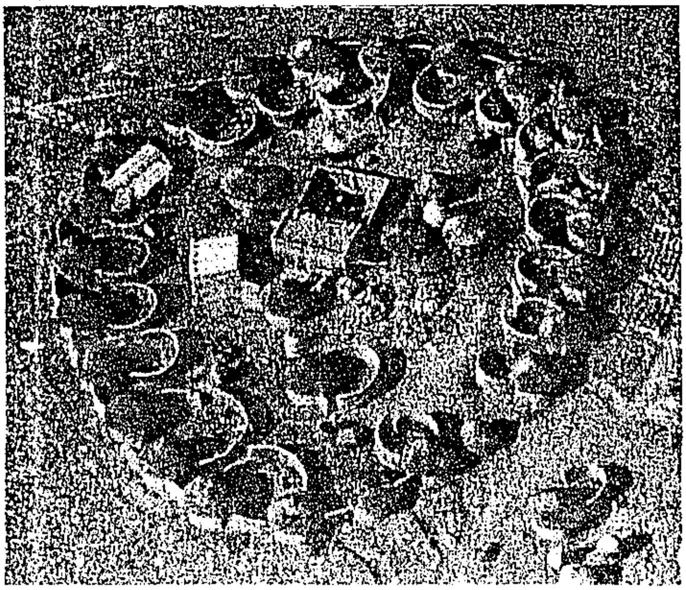


FIGURE 14.6
Modernized fractal village
This ENDA project in Burkina Faso combined the traditional fractal structure with modern construction techniques.

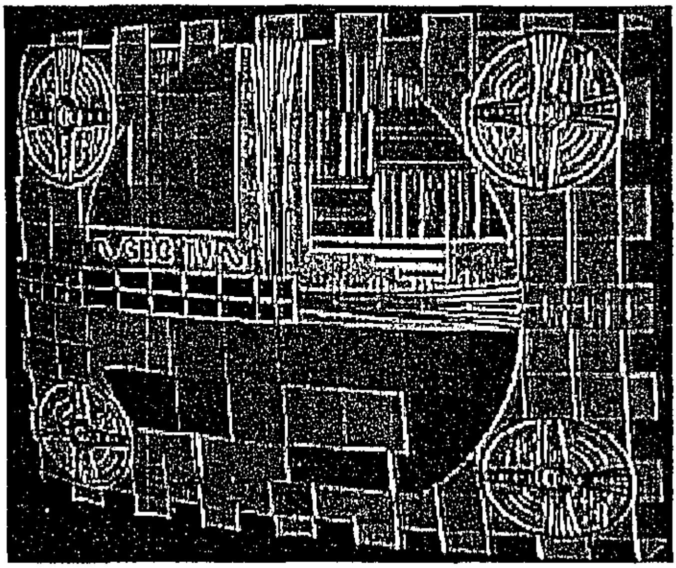


FIGURE 14.7
Kente cloth in the Ghanaian Broadcasting Corporation test pattern
Kente cloth pattern is used in the upper right-hand quadrant of the large circle.

that a top-down approach to development, even that making use of indigenous knowledge, is often less effective than a bottom-up, "grass roots" approach. Adopting information technology to rural areas could provide the opportunity for putting African fractals to work in sustainable development.

In addition to the need for bottom-up authority, researchers have demonstrated the critical role of women in African development (e.g., Boserup 1970; Nelson 1981; Adepoju and Oppong 1994); particularly in terms of the gendered division of labor in rural societies (Beneria 1982). While much of this analysis has focused on the vulnerability of women in bearing the brunt of economic change, it has also started a new appreciation for the extensive knowledge systems that existed in precolonial women's activities. Since many of these practices continue today (albeit in modified form), women's indigenous knowledge systems have become an important resource in new approaches to development.

Some obvious challenges include environmental damage (increasing salinization, deforestation, and desertification), external economic pressures (the move to cash-cropping, tourism, and migration to cities; abuse of power by private corporations), increased disease (AIDS and other viruses), political unrest (ethnic conflict, uncontrolled military force, abuse of authority), and damage to the sociocultural system (disruptions of women's traditional authority, loss of traditional knowledge systems). While all of these are far too large to be addressed by any one approach, none of them can be viewed in isolation from the others. In Nigeria, for example, the Shell Petroleum Development Company began operations in Ogoniland that eventually led to widespread environmental damage; attempts to protest through the press and other communication eventually led to the execution of Ogoni writer Ken Saro-Wiwa (Soynika 1994). Freedom of the press is not a separate issue from protection of the environment.

It is right to decry abuse of authority, but replacing one authority with another is not necessarily going to provide a long-term solution. African fractals suggest two alternative approaches. First, what is needed is not E. F. Schumaker's call for "small is beautiful," but rather a self-organized approach to changes in the relations between scale and the socioenvironmental systems—not just appropriate technology, but appropriate scaling. Second, more critical attention needs to be paid to the artificial/natural dichotomy, which tends to be trapped in either the organicists' desire for untouched nature (e.g., Hughes 1996), or the techno-optimist's desire for resource extraction.

An alternative to these damaging extremes can be found in Calestous Juma's 1989 classic, *The Gene Hunters*. Rather than a preservationist perspective, in which indigenous society would be portrayed as natural elements of an unchanging ecosystem, or a technocratic profiteering perspective, in which agricultural development is merely a question of maximizing yields with imported strains, Juma provides evidence for indigenous agricultural activity as sustainable biotechnology. His studies show a long-standing African tradition of new seed variety development that combined ecological sustainablilty with innovation and experimentation. These practices have been threatened by corporate monocropping, which can cause soil depletion, over-dependance on insecticides, loss of genetic variation, and other social and ecological crises, as well as the appropriation of these genetic resources by a biotechnology industry with little interest in indigenous legal rights. Juma notes that the challenge now facing African agriculturalists is not just preservation of biodiversity, but also access to the legal, technical, and financial apparatus that would allow them to reap the profit that could sustain such ecologically sound efforts.

From the viewpoint of complexity theory, Juma's critique suggests that we are trapped between the periodic stasis of the preservationists' limit cycle, and the white noise of the profiteering positive feedback loop. As we saw in these mathematical models, both are lacking in flexible interactions with memory; the limit cycle being too tied to it, and the white noise being too free from it. Information technologies have the potential to provide this memory, documenting indigenous knowledge from seed varieties and soil types to gene sequences to ecotopes. By providing informed rural access to information technologies, African agriculturalists can take a step toward protecting their genetic resources from appropriation and move toward Juma's approach, which we might call "biotech-diversity" (cf. Haraway 1997; Shiva 1997).

To view indigenous knowledge as a self-organizing system is one thing, but creating the same bottom-up approach for a synthesis of ecological sustainability and technological development is a much greater challenge. For example, Russel Barsh notes: "There is an implicit assumption in the research methodology used to elicit traditional pharmacological knowledge that this information is recorded and transmitted digitally (numbers and/or words) . . . [rather than] internalizing an analog model" (1997, 33–34).

Native Seeds, a botanical organization dedicated to the continuation of indigenous plant stock, has been creating a "cultural memory bank" that will record both analog and digital information on Native American agriculture. The concept, originating from Philippine ethnobotanist Virginia Nazarea-Sandoval (1996), documents the combination of cultural and biological information about the crops, seeds, farming, and utilization methods. The information, including video interviews, is stored on CD-ROM, with access controlled entirely by the indigenous farmers. In the U.S. context, which is overloaded with electronic technology and ethnocide, this approach makes sense, but the African context, with its enormous indigenous population and sparse electronic technology, will call for techniques that can have a wider impact, one that includes development of a technological infrastructure as well.

If there is to be social transformation through grass-roots technological innovation, it will require much more participation than agricultural systems alone. Other kinds of information technology development could include flexible economic networks, which allow small-scale businesses to collaborate in the manufacture of products and services they could not produce independently. These networks have created strong revitalization in certain rural areas of Europe (Sabel and Piore 1990), and have shown promise in pilot studies in the rural United States as well (e.g., ACEnet in southern Ohio). The use of computers to organize production and vending and provide dynamic searches for the appropriate market niche—one which would be environmentally and socially sustainable as well as profitable—could spread the benefits of new information technologies to the microbusiness level without having to put a laptop in every pushcart, and microfinancing programs have already proved successful in many Third World countries (Serageldin 1997).

African traditions of decentralized decision making could also be combined with new information technologies, creating new forms that combine democratic rule with collective information sharing. The idea of "electronic democracy" has slowly been developing over the Internet; but the efforts have been hampered by the tendency to assume that virtual voting must be the same as ordinary voting. Perhaps the neural net style of African decision making could be utilized in the West as well, with voters indicating proportional strengths for various options. Conversely, perhaps there are ways to apply computer media to enhance African decision making. One approach would be the development of community networks through public-access terminals (Schuler 1995). And the enormous development in electronic security measures, creating systems that stymie even the most sophisticated hackers (encryption codes, fingerprint scanners, etc.), might find uses in preventing voter fraud that is so common in unstable political regimes.

Nigerian American computer engineer Egondu Onyejekwe has started efforts to apply information technology networking in African developmental projects using complexity theory as a guiding principle. One area she cites is the problem of land ownership (for example, see Charnley 1996). She notes that the continual division of land promoted by the colonial legacy often results in unproductive economies of scale, but that government ownership tends to make conditions worse by adding more hierarchy. "Resolving the land problem requires a non-hierarchical method of organization, a system in which cooperative behavior is rewarded at the same time that individual innovation can flourish; a combination of cooperation and competition like we see in cellular automata and other computational models of self-organizing systems. What better way to encourage this than through computing and information networks?"[4]

Neither the African fractals framework nor dissemination of information technologies offers panaceas. My point is, rather, that the shift in perspective often called for in development need not be either conservative return to the past, nor the epistemological equivalent of an alien invasion. African fractals offer a framework that is both rooted in indigenous cultures and cross-pollinates with new hybrids.

APPENDIX

Measuring the fractal dimension of African settlement architecture

There are several different ways to estimate the fractal dimension of a spatial pattern. In the case of Mokoulek (fig. 2.4 of chapter 2) we have a black-and-white architectural diagram, which allows us to do a two-dimensional version of the ruler size versus length plots we saw in chapter 1. By placing the architectural diagram of Mokoulek under grids of increasing resolution, and counting the number of grid cells that contain some part of the diagram, we can plot the increase of area with decreasing cell size (just as we obtained a plot of the increasing length with decreasing ruler size). Figure A.1 shows the results, indicating a fractal dimension of 1.67—not too far from the 1.53 fractal dimension that is obtained analytically from the computer simulation.

For the aerial photo of Labbazanga (fig. 2.5 of chapter 2) we have an image in shades of gray, and the simple grid-counting method cannot be applied. It is possible to reduce the gray scale to black and white, but an alternative method allows us to make a more direct measure of the scaling properties. Figure A.2a shows the method for finding the scaling slope of 1/F noise in a one-dimensional time series by applying a Fourier transform. In figure A.2b we see how this can be applied to a two-dimensional spatial distribution by sweeping the same spectral density measure around in polar coordinates. Rather than the line of one-dimensional 1/F noise, a two-dimensional distribution is characterized by a cone. It is difficult to show the entire cone, but we can take horizontal slices (fig. A.2b), which show similar characteristics for both Labbazanga and its fractal simulation (fig. A.3).

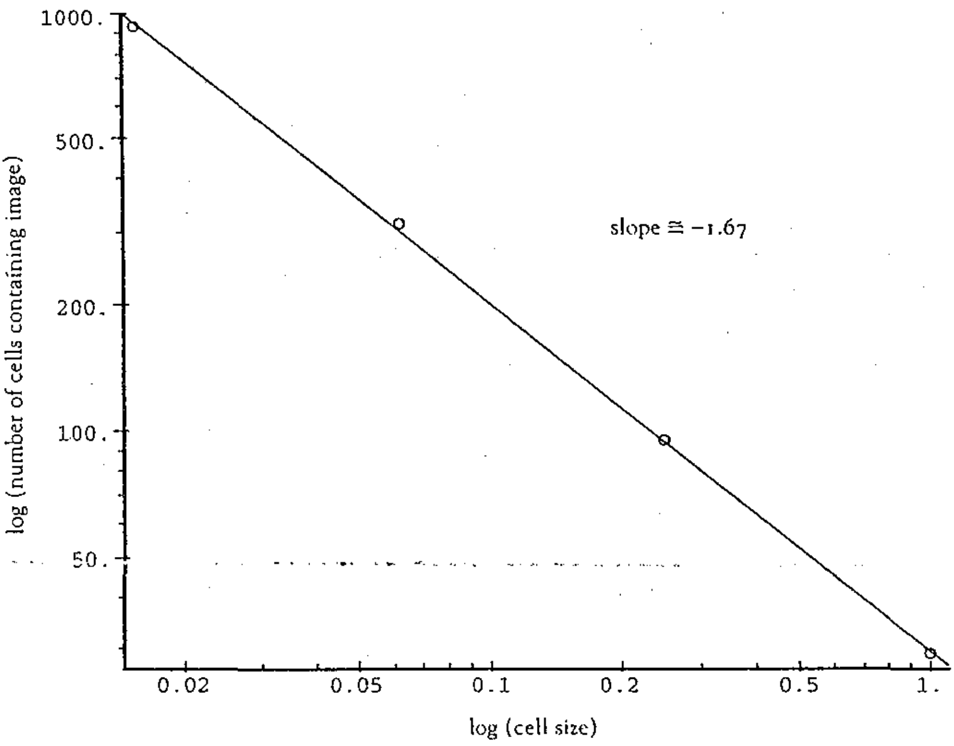


FIGURE A.1
Measuring the fractal dimension of Mokoulek

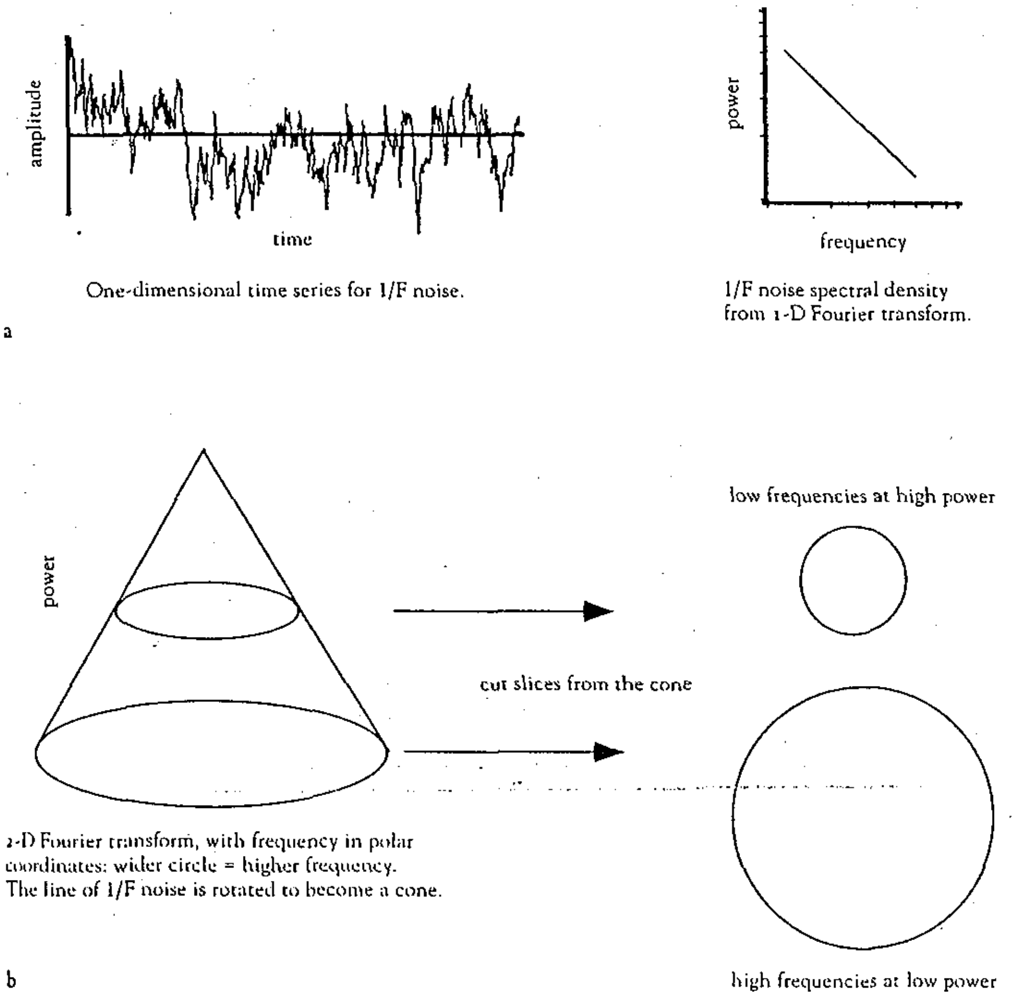


FIGURE A.2

Using a 2-D Fourier transform to detect fractal spatial distributions

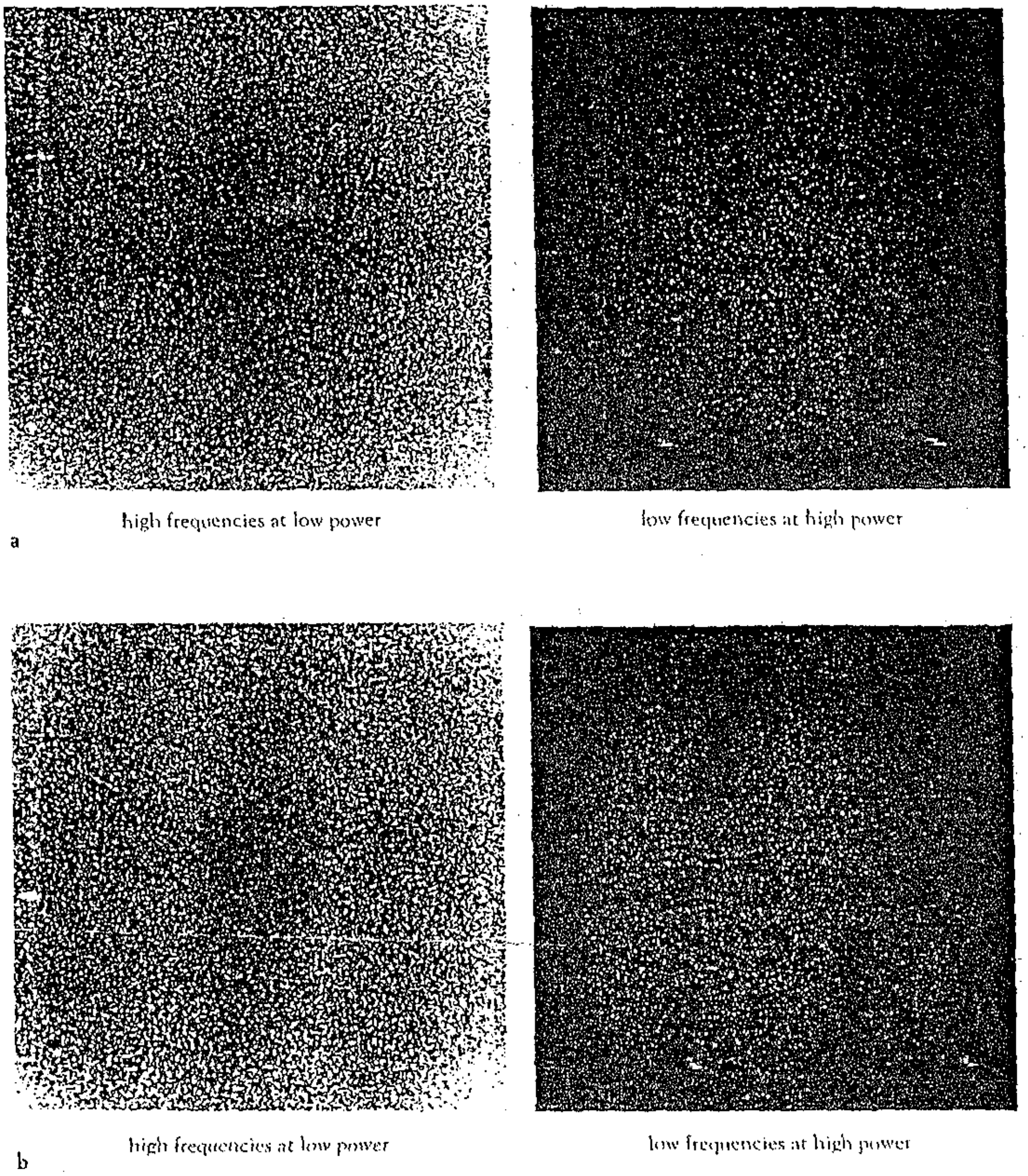


FIGURE A.3

Results of a 2-D Fourier transform applied to aerial photo of Labbazanga

(a) Spectra for aerial photo of Labbazanga (fig. 2.5a from chapter 2). (b) Spectra for fractal image (fig. 2.5b from chapter 2). Note that the axes of symmetry in the fractal can be seen in this spectral density distribution, while none exist for that of Labbazanga.

Notes

CHAPTER 1 *Introduction to fractal geometry*

1. For a hexagon example, see Washburn and Crowe (1988, 237). Numerical examples can be found in Crump (1990, 39–40, 50–54, 105–106, 128–133).
2. The number 10 was not only a basis for counting, but it also appeared in Chinese natural philosophy. In acupuncture, for example, the number 10 is created by the combination of the "five elements" (*wu-yün*) and the binary yin/yang.
3. Michael Polanyi (1966) referred to this as "tacit knowledge."

CHAPTER 2 *Fractals in African settlement architecture*

1. On triangular churches, see Norberg-Schulz (1965, 172); for the Pantheon, see ibid., 124.
2. Another passage, "path of the serpent," is used only by royalty. It alternates left and right as it approaches the center of the palace, and thus creates a scaling zigzag pattern. The implication seems to be that even royalty must negotiate the fractal ranking, but they can traverse it in a more direct route.
3. American readers are probably most familiar with nuclear families, but in Africa the family structure typically extends to much larger networks. The English term "cousins," for example, emphasizes the nuclear family by lumping all these relatives together, while many African kinship systems have distinct terms for paternal parallel cousins, maternal parallel cousins, paternal cross cousins, etc.
4. The status difference between front and back is also expressed in the Ba-ila term for slave: "one who grows up at the doorway" (Smith and Dale 1968 [1920] vol. 1, 304).
5. This is another meaning for the term "participant simulation." In the first meaning, briefly mentioned in the introduction, I defined it as an effort in cooperative modeling and analysis, a technologized version of recent attempts in collaborative ethnography by some anthropologists and their informants. In that sense it supports the humanist goals

of self-governing autonomy. But in the Mokoulek case I am also using it in the postmodernist sense, a participant in a virtual world. The contrasting meanings and their consequences are discussed in detail in chapter 10, where the two are brought together.

6. The results were published in Eglash and Broadwell (1989), and are reproduced in the appendix.

CHAPTER 3 *Fractals in cross-cultural comparison*

1. In general, anthropologists divide nonstate societies between "band" organization, which is entirely decentralized and based mainly on consensus, and "tribal" organization, in which there is an official leader but otherwise little political hierarchy. The term "tribe" is controversial, however, since colonialists often used it to deny the existence of indigenous state societies, so it is important to separate the technical designation from its colloquial use.
2. This is a complex designation in cultural studies, since the label of "traditional"—or worse yet, "authentic"—was used by colonial authorities to exercise control over indigenous populations, and still occurs in the neocolonial context to valorize the "vanishing native" while appropriating their cultural resources. See Minh-ha (1986), Anzaldúa (1987), Clifford (1988), and Bhabha (1990) for discussion of some of these issues.
3. Crowe and Nagy (1992), for example, have done extensive analysis of Fiji decoration, and found 12 of the 17 mathematically possible two-color strip symmetries, but none of the designs they show are fractal.
4. Of course, nothing is absolutely certain when it comes to ancient history. Several researchers have suggested that the Coptic designs from Egypt were an important influence on the Celtic interlace patterns, and some Italian floor tiles were created by North African artisans (Argiro 1968, 22). But one could just as easily argue the influence in reverse. Given the history of trade routes and travel, we should not attempt to reduce designs to a singular origin; the goal is to see how any one society has built up its particular repertoire of designs—from whatever sources—as part of a dynamic yet culturally specific practice.

CHAPTER 4 *Intention and invention in design*

1. This spatial metaphor of "underlying"—truth beneath the surface—can be a delusion if we assume that there is never more than one true "essence" to be found. On the other hand, claiming that no model is more accurate a generalization than any other is equally misguided.
2. The postwar era marked a significant change in the role of nature as a potential model for scientific discovery, as seen in the emerging disciplines of cybernetics and bionics (Gray 1995).

CHAPTER 7 *Numeric systems*

1. It is unfortunate that an otherwise excellent paper comparing African and Australian ethnomathematics (Watson-Verran and Turnbull 1994) fails to make this distinction between the iterative generation of linear and nonlinear number series.
2. Readers who recall the definition of nonlinear functions as involving, at minimum, something like x^2 may be puzzled by the idea of a nonlinear additive series. That is because most of us were first exposed to the definition of "nonlinear" in the context of continuous functions (e.g., differential equations). But discrete iteration (what is often called a "difference equation") can produce nonlinear steps with simple addition.
3. After giving a lecture on Bamana divination in the United States, I was approached by a mathematics faculty member who was quite taken by this phrase. "That's just like us," he exclaimed. "We get the power of mathematics only at the cost of our social deformity as nerds."

4. The series was first introduced as an example of a recursively computable aperiodic string by Axel Thue (1863–1922), using the replacement rules 0 → 01, 1 → 10, with an initial 0. Morse discovered its application to deterministic chaos, in which it models the fractal time series produced by certain nonlinear equations. See Schroeder (1991, 264–268) on these aspects of the sequence.
5. One-dimensional versions can show all the dynamics of two dimensions, and can even be used as a kind of parallel computer. Consider, for example, a rule that in each iteration the number of counters in a cup is replaced by the sum of itself and its left neighbor. Starting with one: 0100000 → 0110000 → 0121000 → 0133100 → 0146410. This fourth iteration gives us the binomial coefficients for expansion of $(a + b)^4$, which equals $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.

CHAPTER 8 *Recursion*

1. The standard terminology is somewhat ambiguous, since "recursion" is sometimes used to refer specifically to what we will call "self-reference," and at other times it is used in the more general sense applied here. "Iteration" is used in its normal definition, and for the least powerful we will use the term "cascade." Technically, these three types of recursion roughly correspond to Turing machines, push-down automata, and finite-state automata, but these models are a little too abstract to be directly useful in helping readers develop a sense of the distinctions that are of interest here.
2. Sagay (1983) explicitly mentions starting with the small shape in the center, whereas the Ipako Elede rows look like they might be better described as a preestablished linear sequence (although Sagay does not give details here).
3. Actually, it is not wax that is used in much of Africa, but rather a latex created by boiling the sap of the Euphorbia plant. Williams notes that it can produce long, delicate threads that are impossible for wax.
4. Pelton (1980, 230) contrasts the singular random events of the Native American trickster myths with "the less episodic, more narrative myths of Legba and Ogo-Yuruga [in Africa]." The reason for the difference is partly mathematical. The Native American concept of unpredictability is based more on chance (see Ascher 1991, 87–94), while the African concept tends to be closer to deterministic chaos, as we saw in Bamana sand divination.
5. Curtin (1971) shows that the slave trade from what is now northern Senegal diminished after 1700, and that the Nigerian area did not begin major activity until after 1730. This still leaves the possibility that Fuller came from the area of present-day Benin and Ghana, which would be too far south to have directly shared influences with the Bassari, but Holloway (1990, 10) notes that Virginians showed some preference for Africans from the Senegambian region.
6. I qualified this as "standard" because there has been a growing concern that anthropologists may have overemphasized the importance of age-grade and kinship by projecting their own desires as well as the interests of their informants. Shaw (1995), for example, shows how Louis Leaky's description of the extreme obedience of the Kikuyu to their age-grade system was colored both by Leaky's desire for the order of a "small English village" that he never experienced (having grown up with missionary parents) and the Kikuyu elders' own interests in receiving the initiation payments that were overdue to them.
7. In addition to the association of the vertical with the spiritual, Fernandez suggests that the spatial distinction derives from the Fang's periodic clan fission/relocation. The fragmentation of a social group comes with horizontal movement and is seen as the result of stagnation or strife, while the establishment of the group in a new location is seen as positive regeneration, building from the ground up.
8. Maurer and Roberts (1985, 25) describe the Tabwa belt, a leather strip with bands of beads or wire as representations of a single descent line. Since the Tabwa use the mpande

disk to represent the expansion of all kinship groups from a singular origin, it is not unreasonable to think of the mukaba belt as a lower-dimensional projection of the mpande disk. If one is willing to speculate so wildly that even I would hesitate to do so, the aardvark's winding tunnel could be viewed as a three-dimensional spiral projected onto the two-dimensional mpande disk, just as the belt is a one-dimensional projection of the mpande spiral. A similar practice, the "Poincaré slice," is used in nonlinear dynamics (see Abraham and Shaw 1982).

9. It is important to understand that the problem is *not* one of "authenticity." I agree with the critiques of modernist anthropology's tendency to make one individual representative of an entire society and to focus on a false homogeneous past. In ethnomathematics we are interested in the invention of mathematical concepts; so it doesn't matter whether the source is an entire society or a single creative individual. What does matter is the precision and accuracy of the math, and it is here that the interpretive flexibility offered by narratives presents problems.
10. Note that I wrote "has trouble with" rather than "cannot do"—in fact, a programmer could write a kind of "metaloop" of iteration that would figure out how many nestings are needed. But in doing so, the program has to be able to refer to a part of itself (its loops), so this is already a partial or limited self-reference. Of course we could then play the same trick, demanding that we can't tell ahead of time how many metaloops will be needed, and our smarty-pants programmer could again make a meta-metaloop, and so on. It is only when we generalize the trick itself that full self-reference will be required. And even then, it too will meet up with undoable tasks—because that very property of not bounding the process ahead of time leaves it vulnerable to other problems. As Alan Turing proved for computing, and as Kurt Gödel showed for all mathematics in general, any system that is sufficiently powerful to fully utilize self-reference will have to be incomplete in its ability to resolve all the theorems it can ask (see Hofstadter 1980).
11. The most specific connection made by Taylor is the possibility that the material attributed to Hermes-Thoth was derived from some of the Egyptian priesthood writings mentioned by Clement of Alexandria.
12. Stéphanidès (1922, 192) suggests a more direct sub-Saharan origin of alchemy, entering Egypt around 718 B.C.E., following the invasions of Ethiopia.
13. That's not to say that the Legba drum beats were random; but the drumming did indeed have an unexpected change of pace.

CHAPTER 10 *Complexity*

1. The analog/digital dichotomy in computing is often confused with other dualisms. The same terms are used by engineers to describe the continuous/discrete dichotomy, and by cognitive scientists to discuss "reasoning by analogy" versus inductive analysis, but these distinctions are irrelevant to the sense in which it is used here. Musical notes, for example, are excellent examples of analog communication, but they are entirely discrete. See Eglash (1993) for details.
2. Blum et al. show that an analog Turing machine would be susceptible to the halting problem. See Eglash (1992, 1998c) for more details on this recent history of cybernetics. We can think of the wave/particle duality in physics as another indication that the analog/digital distinction is fundamentally egalitarian.
3. We can also look at this in terms of psychopathology. A neurotic will often repeat the same phrase over and over, while a psychotic tends to be talking "word salad," a jumble of nonsense. In both cases, their mental relation to memory is pathologically simplified: the neurotic slavishly follows memory, while the psychotic completely ignores it. Complex information processing requires a *dynamic* interaction with memory, a nontrivial recursive loop.
4. For example, say there are choices A, B, and C. A wins, but B and C voters say, "If only I had known A was going to win, I would have been willing to vote the other way."

Tank and Hopfield (1987, 106) contrast this one-shot majority rule voting with the collective-decision-making process in neural nets: "In a collective-decision committee the members vote together and can *express a range of opinions; the members* know all about the other votes and can change their opinions. The committee generates . . . what might be called a sense of the meeting."

5. Recall that we scaled down P to a number between 0 and 1. That means that $(1 - P_n)$ will always be a fraction, which reduces P_n—in fact, the larger P_n, the smaller the fraction.
6. The reason it never lands back on exactly the same spot is not because of external noise; it is rather for the same reason that the number P never repeats. Gottfried Mayer-Kress suggested that a good way to understand this is to note that the drunken driver never stops missteering, even while the sober one is overpowering him. I suspect that this combination of negative feedback and positive feedback is at the heart of every case of deterministic chaos, although I have yet to prove it. In Eglash (1992) I reported that the Lorenz attractor consisted of only positive feedback, but this turns out to be incorrect. In terms of dynamical systems theory (Abraham and Shaw 1982; Devaney 1986), positive feedback is the counterpart to spreading in phase space, and negative feedback corresponds to folding in phase space. The phase-space combination of local spreading and global folding is a common definition for chaos; the conjecture simply translates the phase-space definition into a control theory formulation.
7. I've oversimplified the relations here. For example, a finer distinction can be made about "disorder" if we consider white-noise versus brown-noise distribution on a surface (Gardner 1978; Voss 1990). In Brownian motion, a particle moves in a random, continuous trajectory; given an infinite amount of time, such "brown noise" will approach a two-dimensional curve. In white noise, single points on the surface are selected at random, so an infinite amount of time will still only leave us with disconnected points, which is a zero-dimensional curve. Between zero and one dimension, we have objects like the Cantor set, and between one and two dimensions we have objects like the Koch curve. This is slightly different when we think about noise as a single time-varying signal (as in acoustic noise) because the single points of the white distribution will also be connected into a continuous (but nondifferentiable) curve, now of dimension one, while brown noise as a time series will still be at dimension two.
8. Achebe himself prevents such a reading by highlighting a *precolonial catastrophe* that befalls his main character, Okonkwo. At the same time, the contrast between Okonkwo's misery due to indigenous accident and his suicide as a result of the colonial encounter makes it clear that these are entirely different orders of chaos.
9. There is also a good illustration of collective fractal generation in the arts: the Mbuti bark-cloth design shown in chapter 3 is actually the product of multiple artists.

CHAPTER 11 *Theoretical frameworks in cultural studies of knowledge*

1. Popper might object to the characterization of "fractal geometry minus dimensional measures," since it sounds like an ad hoc adjustment, but the important thing is that the four attributes (scaling, recursion, infinity, and dimension) were tested in a more or less falsifiable manner. Whether or not one can still call it fractal geometry if one of the four is missing is an important question; but we need to address the possibility of a weak characterization of recursion in European fractals before making that judgment.
2. This should not necessarily be assumed to mean "closer to nature," since it could also refer to an indigenous knowledge system that promotes good ecological practices; but the ambiguity is problematic.
3. In fact I'm not—my *master's degree is in systems engineering*, and although I took a few graduate seminars in mathematics for my interdisciplinary Ph.D. (thanks to the flexibility of the History of Consciousness board at the University of California at Santa Cruz), I wouldn't dare call myself a mathematician in professional company. I have always

tried to introduce myself as an ethnomathematician during field work, but sometimes translation problems took time to get that across.

4. Worth it not just in ethical and methodological terms; it often came to my aid in dire circumstances. On a hot road near the Lake Chad region, I was stopped by military police who were clearly looking for a bribe. I was released only when I began to launch into a lengthy explanation of fractal geometry. Knowing the Baka counting system saved my skin when a group of teenagers in a village in southern Cameroon took me for a disrespectful tourist; unlike the gendarmes, they were delighted to find mathematics in their midst.
5. On the role of neologisms in the work of Cesaire, see Clifford (1988). On the construction of negritude as a set of binary oppositions, see Mudimbe (1988).
6. For example, the octopus arose millions of years before vertebrates but has a nervous system more sophisticated than that of some reptiles (see Eglash 1984, 161). This is a dangerous analogy, of course, because people often confuse biological and cultural evolution. Here are two crucial differences. First, cultural evolution is Lamarckian—we can pass our acquired knowledge to the next generation—while biological evolution is Darwinian, with the rare lucky mutant having an advantage that is then passed on. Second, the timescales are of different orders of magnitude. Significant biological evolution requires on the order of a million years, while dramatic cultural evolution requires no more than a few thousand years. This is why human beings have such a tiny amount of genetic variation: the first modern humans, from their singular origin in Africa, quickly spread across the earth over a few thousand years. Our nearly identical genetic composition is a result of speedy Lamarckian cultural evolution adapting us into these new environments.

CHAPTER 12 *The politics of African fractals*

1. Derrida's promotion of arbitrary signifiers and artificiality was not the sole voice for this position. Black activists like James Boggs (1968) have also been champions of artifice. Wittig's (1973) *Lesbian Body* takes a topic that was often treated as the unassailable ground of feminist meaning, the authentic physical self, and dismantles this construction through textual erotics. Like Derrida, she shows that a system of arbitrary symbols is just as capable of carrying the kind of human essence often attributed to the Real or Natural.
2. Angela Davis has pointed out Ellison's denaturalizing tropes in lectures at UCSC; her recent work continues to tease out these threads of self-assembly in black cultural identity and community.
3. My favorite illustration of analog artifice in black intellectual works occurs in chapter 11 of Audre Lourd's *Zami*. Like Wittig (1973), she describes the self-assembly of a lesbian body, but her techniques for this artificial reconstruction come through the analog media of scent, vibration, and form. See Eglash (1995) for other examples.
4. Consider, for example, the mojo hand/dataglove comparision in Dery (1994, 210), or the following passage from Williams (1974, 40): " 'Simply anything can become a God,' a Yoruba informant once remarked. 'This button (pointing to the dashboard of the car in which we were), 'it only needs to be built up by prayer' (by invocation)."
5. Similar views can be found in several other intellectual works of the time; e.g., Joreen's (1972) critique of the women's movement, "Tyranny of Structurelessness." There are, of course, many centralist critiques of decentralization, but Joreen's text took a more complex angle of analysis. See Ehrlich (1979) for a critical view. Invocations of African royalty in black cultural representations are typically viewed as commentary on self-esteem. While that may be true, in most cases there are hints that it also serves to question the humanist control enacted in a political democracy that can support such deep economic subservience (see Queen Latifa's "Queen of Royal Badness" in Smith 1990).

6. In fact, this was how I got started on African fractals. It occurred to me that aerial photos might show the difference between these architectural designs as fractal versus Euclidean. Pat Caplan generously provided me with aerial photos of the area in which she worked, and the indigenous housing did indeed appear to be less Euclidean.
7. Recursive architectural structure is linguistically indicated by the Yoruba term for homestead: *ot ka ot*, or "house within the house."
8. The 1993 Supreme Court ruling in *Shaw v. Reno* used the terms "bizarre" and "snakelike," the latter echoing historian John Fiske's 1812 characterization of a "dragonlike" contour, a phrase changed to "salamander" and finally to "gerrymander" (after Massachusetts governor Elbridge Gerry) by political cartoonist Gilbert Stuart.
9. The insistence that stochastic variation implies free will and deterministic variation implies domination is made by several authors besides Porush (e.g., Hakim Bey). I think that individuals or groups can indeed create such associations, just as they can create the opposite (e.g., that a simple bounded system can still have the liberty of infinite variation, as we will see argued by Gilroy, Van Wyk, and Heaver). The error is in assuming universal meaning to what has to be local semiotics. A closer examination of the social meanings for statistics (Porter 1986) reveals that its political associations are often dependent on modernist concepts of humanist individualism, which is strongly critiqued in the Foucaultian and other postmodernist analyses championed by Porush, Hayles, Sobchack, and others.
10. Just as important is the reverse influence, e.g., Jewish jazz musician Mezz Mezzrow passing for black while in prison so that he could play in the band.
11. Gilroy's work in this area should be seen as part of a larger community of researchers and cultural workers (e.g., artists) who have developed a postmodern emphasis on hybridity, creolization, and other impure identities (cf. Minh-ha 1986; Anzaldúa 1987; Bhabha 1990; Sandoval 1995; Haraway 1996).
12. Digital and analog are also confusing terms because digital technology is now commonly used to generate the analog waveforms of music. But it is necessary to see how these representations are layered. The electronic "on-off" code pulses are actually noisy waveforms that must be processed with analog control circuits at the lowest level of the silicon chip; eventually they are decoded in binary form, then converted to an electrical waveform that will modulate the speaker. The resulting acoustic waveform can be analog, digital, or—especially in the case of rap music—somewhere in between. See Eglash (1993) for details.

CHAPTER 13 *Fractals in the European history of mathematics*

1. According to ancient accounts, the discovery of irrationals was in the middle of the fifth century B.C.E. Modern scholars generally agree that the proof for the incommensurability of the square of a diagonal with respect to its side, first mentioned explicitly in Plato's dialog *Theaetus*, is too abstract to have been used at this time. Von Fritz (1944) provides a resolution for this conflict in his speculative reconstruction of Hippasus' analysis of the pentagon. See Knorr (1975) and Fowler (1987) for discussion of the original texts relevant to this area.
2. Plato was not the only influence at the time, nor were irrationals only granted one perspective. Fowler (1987), for example, maintains that the significance of irrationals has been misunderstood and suggests that even Plato presented their proof as "a source of interesting and fruitful problems" rather than as a disturbing paradox. Nevertheless, it was the homogeneous representations of Platonic thought deployed centuries later, not its contemporary diversity, which would matter for the intuition and practice of modern mathematicians.
3. "We add to the first number the second one, i.e., 1 and 2, the second to the third; the third to the fourth; the fourth to the fifth . . . and it is possible to do this order for an infinite number of months" (trans. Maxey Brooke).

4. Similar analysis was provided by Henry Louis Gates (1990) and others in the censorship trial of rap group 2 Live Crew, maintaining that the explicit sexual lyrics were not acultural profanity but rather modern variations of a long-standing black tradition of public sexual commentary.
5. Tuana (1989), for example, notes that the male homunculus theory, which locates the active principle of birth in sperm only, dominated European medical thinking from Aristotle to van Leeuwenhoek (and in some senses even to the present; see Hartouni 1997). Again, the African version is in strong contrast; recall from chapter 8 that the Fang believe that the homunculus or active principle is contained in the female blood (the division is more egalitarian than the European model, however, since the male Fang are said to provide a complementary protective, skeletal principle).
6. That is, prior to complexity theory, at which point advances in the application of fractal geometry were made precisely because of the growing recognition of a relationship between computational recursion and self-organizing phenomena. Complexity theory is a marker distinguishing the transitional postmodernism of the 1970s from the stable postmodernism of the 1980s (Eglash 1998c).
7. The qualification is not inaccurate; the problem is that sometimes the authors of this text (*The Science of Fractal Images*) use the term "recursion" to mean iteration, and sometimes (as in this case) it means self-referential programming. This level of ambiguity would not be tolerated for any other mathematical terminology used in the text.

CHAPTER 14 *Futures for African fractals*

1. For more on cyborgs, see Haraway (1996) and Gray (1995).
2. In fact, if I had used a large enough size difference, self-intersection could have been avoided altogether, but I think that would not do justice to the African tradition of putting similar-sized houses together—a tradition that has its roots in egalitarian socioeconomic structure, and one to which Nyangula was no doubt sensitive.
3. But there was more to it than that. Perhaps in part because it implied a Platonic view, it made sense to the students that religious symbolism would be mathematical, while something as concrete as a mud wall was too hard to reimage. There was also the visual effect of seeing computer simulations of the African log spirals; for a generation brought up on video games and MTV, this placed it in a contemporary framework. Finally, there was something about the religious subject matter itself—the very concept of a "life force" expressed as a self-organizing system—that may have created a resonance for these students.
4. Onyejekwe's African Women Global Network is available from http://www.osu.edu/org/awognet.

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