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Research Article

A General MINLP Model for the Multiway Valve Channel-Limited Location-Allocation Problem

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As an important aspect of oil and gas field construction, oil and gas field surface engineering often affects the efficiency and safety of oil and gas production, and it requires a large investment. In the past, much work has been done on layout optimization for gathering pipeline networks. However, few studies have considered multiway valves in the optimization of pipeline networks. Compared to traditional metering processes, a process using multiway valves can reduce construction and operation costs and enable automation of the well-selection operation. In this paper, an MINLP model is established in which the number of multiway valves and their numbers of channels are considered special constraints and the number of multiway valves and the associations between wells and multiway valves are treated as optimization variables. A specific heuristic algorithm for solving this problem is also proposed in this paper. We consider the coordinates of real-world wells and of randomly generated well locations as different examples to analyze the performance of this algorithm. These examples demonstrate that the algorithm can initially adjust the network through single-step iteration, while double-step iteration is efficient when all channels of a multiway valve have been associated with pipelines, and multistep iteration can help the objective function escape from local optima. Finally, numerical analysis results prove that the proposed algorithm can be used to efficiently solve the problem of interest and exhibits stable convergence.

1. Introduction

In recent years, as the production scale of oil and gas fields has expanded, the construction and labor costs incurred for the measurement of oil and gas production per well have also increased. In addition, the practical implementation of conventional metering methods is subject to certain limitations, especially in areas such as deserts and deep-sea regions. There are three main metering methods for oil and gas fields: single-well metering, metering by manual rotation, and automatic rotation metering. The single-well metering process requires the installation of metering equipment for each well, as shown in Figure 1(a). The initial investment is relatively expensive, especially when the well production rate is below the initial estimate. In the manual rotation measurement process, as shown in Figure 1(b), each valve that is to be opened or closed needs to be operated manually. Although it is not necessary to install metering devices at each well site, metering a well will require the manual operation of more than a dozen valves, and after metering, these valves will need to be operated again to discharge the liquid in the metering separator, thus requiring more operation time and incurring a higher cost. In addition, the metering device occupies a large area, and such devices are not suitable for offshore platforms. Regarding the third method, retrofitting an actuator onto the original manual valve of an existing manifold will greatly increase the construction cost. Therefore, it is necessary to find a device that can enable automatic well-selection metering while incurring lower construction and operation costs.

At present, many oil and gas field production enterprises use multiway valves (MVs) to achieve automatic rotation metering. The MV-based metering process can reduce the amount of manual operation required in remote areas, which is greatly beneficial for reducing operation costs. The metering process with an MV is shown

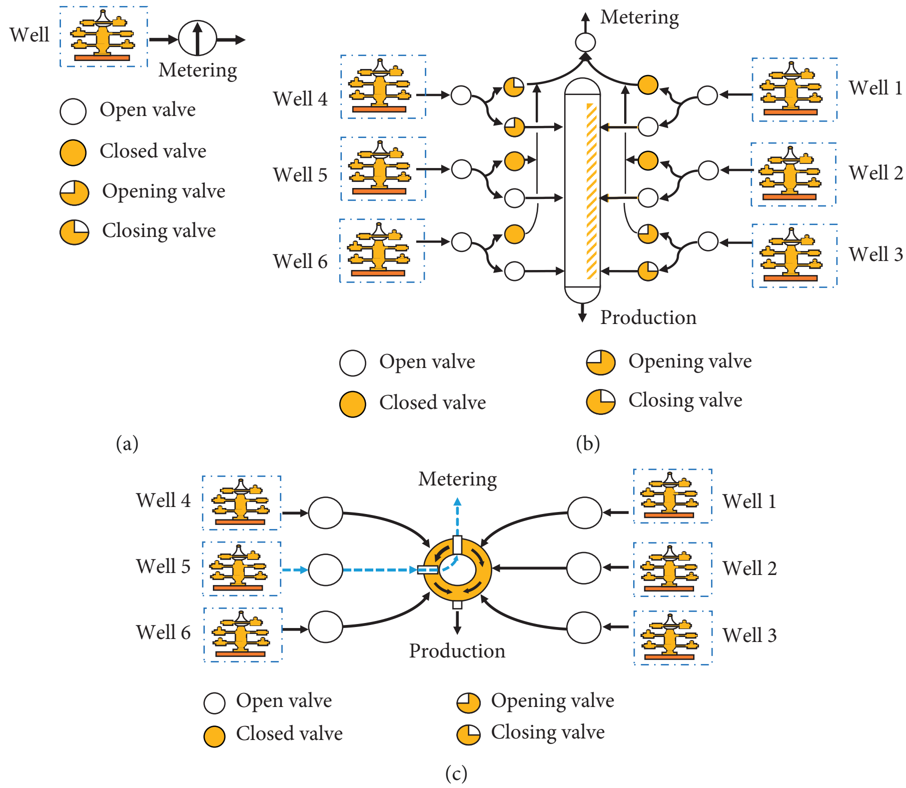

FIGURE 1: Comparison of metering processes: (a) single-well metering; (b) metering by manual rotation; (c) automatic rotation metering.

in Figure 1(c). The blue dashed arrow indicates the flow that needs to be metered at present, and the black arrows indicate the flows from production wells. When switching to metering another well, the control system requires only remote operation, thus leading to considerable savings in operation cost. Compared to the traditional methods, the MV-based metering process reduces the numbers of valves, gauges, and actuators that need to be installed on the pipelines. Each device also occupies a smaller space, making these devices more suitable for a wider variety of oil and gas fields, especially for coalbed methane mining, shale gas, and other unconventional oil and gas resources. From here on, when the term "valve" is used in this paper, it refers to an MV.

The number of channels of an MV is a characteristic that is defined by the manufacturer and determines how many pipelines can access the MV. If an MV has 16 channels, this means that at most 16 wells can be connected to the MV. Manufacturers make MVs with various channel counts, such as 12, 14, and 16. Depending on the conditions in the field or other requirements such as consistency principle, the designer needs to select MVs with an appropriate number of channels to meet the requirements of the project. Therefore, during the design process, the number of wells connected to an MV is constrained by the number of channels of the MV, which depends on the designer's selection. Depending on the constraint relaxation degree, there are three possible ways to specify MV-related constraints:

(1) The number of MVs is unrestricted

(2) Allowable ranges are given for the number of MVs and their channel counts

(3) The channel counts of the MVs and the number of MVs are fixed in advance

For a given target oil and gas reservoir, the traditional solution is to invite experts to design feasible solutions based on real scenarios, but this approach does not necessarily guarantee the most economical cost. By contrast, optimization methods can help a designer to obtain the best solution. However, in the past, researchers have rarely considered constraints related to the channel counts of the MVs and the number of MVs, although such constraints are important to ensure that the optimal solution satisfies the required service conditions. In this paper, the problem of interest is formulated as a special P-median problem, and the constraints mentioned above are integrated into the mathematical model. By treating the associations between wells and MVs and the number of MVs as variables and the minimum cost as the objective function, a general mixed integer nonlinear programming (MINLP) model is established. Based on the connection and foresight matrices of the production wells and MVs, a foresight matrix search algorithm (FMSA) is proposed in this paper to solve the MINLP problem. In this algorithm, the nonlinear part of the problem is treated as a subproblem, and the optimization results are obtained through iteration. By examples, the convergence of the algorithm can be ensured, as can the quality and efficiency of the solution. In addition, it is proven that, with this algorithm, the solution process can be completed in a reasonable time even if the problem is large in scale.

This paper is divided into six sections: In Section 1, we have introduced the problem motivating this paper and what we have done to address it. In Section 2, we review the related research contributions that have been reported in the past. In Section 3, the detailed mathematical model is presented, and the special constraints are formulated. Subsequently, the details of solving the mathematical model using the proposed algorithm are introduced in Section 4. The results are analyzed in Section 5. Finally, conclusions and discussions are presented in Section 6. The data used in this paper can be found in Tables 1–5.

2. Literature Review

The complexity and large scale of the corresponding mathematical models make MINLP problems a popular and difficult topic to address for scholars around the world. The methods for solving MINLP problems can be generally divided into deterministic methods and heuristic methods. Methods of the former type can yield globally optimal solutions for small-scale problems but are often inefficient for large-scale problems, whereas the quality of the solutions obtained with the heuristic methods that have been proposed to date usually cannot be guaranteed, and these methods cannot be used to solve large-scale problems within a limited time.

The problem of interest here is different from the well group partitioning (WGP) problem, which usually considers factors such as the gathering radius and pipeline diameter. The purpose of the conventional WGP model is to determine the optimal mapping between wells and stations. In other words, subject to well constraints and other constraints, the wells are divided among different gathering stations to save construction costs. This problem is a kind of location-allocation problem. In a discrete location-allocation problem, P facilities are selected from a set of available candidate sites such that a given objective function is minimized. This problem is specifically referred to as the P-median problem. However, there are some differences between the model established in this paper and the general P-median model. In the problem considered here, the alternative facility (MV) locations are the same as the well locations, meaning that $m = n$, whereas in the conventional P-median problem, m is often less than n. Moreover, the optimization target of the conventional P-median problem is usually to minimize the total distance from facilities to customers, while in this paper, the target is to minimize the cost of all equipment, including pipelines and devices, and the model is constrained by the special constraints of MVs. In addition, special constraints on the numbers of channels and facilities (MVs) are imposed in the model considered here. All of these factors increase the scale of the optimization problem, making the problem more complicated and difficult to solve. In [1, 2], the authors have proven that the P-median problem is an NP-hard problem when the number of facilities is uncertain. That is, the MINLP problem solved in this paper is a special case of the location-allocation problem. In this problem, the number of MVs m and the associations between wells and valves are treated as optimization variables. The associations between pipelines and various facilities are precisely the elements of interest in oil and gas field surface engineering, and reasonable optimization of the pipeline network structures between facilities can effectively reduce the engineering and construction costs. There are many possible pipeline network structures, including star, tree, and ring structures. With the development of methods for exploiting unconventional oil and gas resources, tree-tree network structures [3, 4] have been used in marginal gas reservoirs for resources such as coalbed gas and shale gas. Such a structure can considerably reduce costs by virtue of the connection configuration between the wells and co-construction with existing wells and stations. Similarly, research on star-tree [5] and star-star [6] pipeline networks has become quite mature.

A pipeline network also includes facilities such as manifolds, platforms, and compressors. Many studies have investigated oil and gas network models that contain various types of facilities. Rodrigues et al. [7] proposed a 0-1 programming model to determine the connections between FPSO units, manifolds, and facilities as well as the sizes and locations of platforms. In [8, 9], the authors proposed a mixed integer linear programming (MILP) model to optimize the connections between wells and platforms while considering not only the installation time but also the linear drop of the reservoir pressure. Furthermore, Ramos Rosa et al. [10] considered the reservoir dynamics and secondary development to establish an MILP model seeking the maximum net present value (NPV) while simultaneously optimizing the manifold layout, routing, and pipeline diameter. Most of the oil and gas field layout optimization problems mentioned above can be transformed into mixed integer programming (MIP) problems, i.e., programming problems in which some of the independent variables take integer values while others take continuous values. MIP first became an independent branch of mathematical problem-solving when Gomory [11] proposed the cutting-plane method. Generally, MINLP problems can be solved using two types of methods: deterministic algorithms and heuristic algorithms. A deterministic method of solving an MINLP problem is to simplify it to an MILP problem, which can be effectively solved by means of dual simplex or sequential quadratic programming. However, the main difficulty is that appropriate linear approximations must be adopted for nonlinear functions. To this end, Möller [12], Martin et al. [13], Tomasgard [14], and Nørstebø [15] et al. used piecewise approximation to deal with the nonlinear functions in the model. Mikolajková et al. [16] proposed a linearization method for solving MINLP problems. Deterministic methods can guarantee a globally optimal solution when solving an MINLP problem, but they usually require considerable resources to solve large-scale problems.

Because of the complexity of MINLP problems, meta-heuristic algorithms, including genetic algorithms (GAs) [17] and particle swarm optimization (PSO) [18], are also used to solve problems related to oil and gas pipeline networks. However, the computational burden of heuristic

Table 1: Coordinates of 192 wells from a gas field.

x (m)	y (m)	Number	x (m)	y (m)	Number	x (m)	y (m)	Number
7430	2770	W1	4176	5039	W65	5570	3473	W129
7177	2698	W2	3975	4957	W66	5801	3634	W130
7193	3084	W3	4068	5358	W67	4985	3832	W131
7112	3341	W4	4429	5300	W68	7389	5655	W132
7572	2515	W5	4722	5370	W69	8522	5800	W133
8827	4759	W6	4904	5150	W70	8103	5715	W134
9009	5366	W7	4273	5543	W71	7542	5441	W135
9587	5795	W8	4558	5660	W72	7186	5593	W136
9458	5509	W9	5138	5270	W73	6566	5406	W137
9135	5732	W10	4969	5575	W74	6475	4116	W138
8935	5562	W11	5231	5060	W75	6024	4415	W139
8746	5209	W12	6030	5549	W76	6221	4302	W140
8774	5516	W13	5468	4806	W77	6465	4443	W141
8565	5454	W14	6472	5940	W78	6295	4625	W142
8261	4764	W15	7846	5656	W79	6120	4893	W143
8253	4440	W16	6208	5351	W80	6374	5033	W144
8396	4255	W17	5788	5334	W81	6650	5172	W145
8479	4649	W18	5557	5251	W82	6523	4829	W146
8713	4407	W19	5749	4979	W83	6711	4567	W147
9132	4226	W20	5994	5116	W84	5997	3457	W148
8857	4207	W21	6756	4798	W85	5990	3836	W149
8479	4998	W22	5723	4398	W86	6292	3902	W150
722	1355	W23	5573	4520	W87	6206	3526	W151
1779	1152	W24	5965	2209	W88	6492	3690	W152
1497	1374	W25	6276	2157	W89	6760	3709	W153
1971	1398	W26	6395	2580	W90	6593	3934	W154
1952	952	W27	6095	2413	W91	6250	2881	W155
1617	989	W28	5827	2630	W92	5532	2840	W156
1671	763	W29	5791	2352	W93	5819	2923	W157
1352	1125	W30	5384	2478	W94	5774	3200	W158
890	1183	W31	5587	2195	W95	6151	3119	W159
535	738	W32	5421	1986	W96	6405	3284	W160
264	29	W33	5718	2003	W97	6638	3422	W161
140	460	W34	6953	1171	W98	6812	3137	W162
459	285	W35	6758	1383	W99	6962	2906	W163
284	2160	W36	6489	1199	W100	6693	2722	W164
1805	2128	W37	6641	978	W101	6525	2993	W165
1466	1868	W38	6971	904	W102	7089	4415	W166
4185	4644	W39	7321	1026	W103	7254	4146	W167
4266	4411	W40	7397	1444	W104	7067	4733	W168
3996	4274	W41	7178	1236	W105	7036	5093	W169
4167	4079	W42	7074	1509	W106	7340	5187	W170
4047	3589	W43	6615	1674	W107	7832	5149	W171
4213	3355	W44	6959	2201	W108	8062	5110	W172
4280	3785	W45	6836	2432	W109	7609	4981	W173
4439	4188	W46	6529	2291	W110	7273	4862	W174
4642	3905	W47	6646	2086	W111	7835	4858	W175
4506	3500	W48	6450	1842	W112	7626	4730	W176
4745	3665	W49	6228	1732	W113	7819	4479	W177
4902	3436	W50	7117	1848	W114	7883	4211	W178
4624	3289	W51	7220	2425	W115	8024	4616	W179
4849	4060	W52	6837	1771	W116	7521	4356	W180
4417	4898	W53	6076	1312	W117	7333	4573	W181
4553	4667	W54	6304	1463	W118	7031	3908	W182
4614	4356	W55	5601	1025	W119	7409	3886	W183
4767	4755	W56	5263	808	W120	7728	4028	W184
4910	4514	W57	5429	1272	W121	7150	3733	W185
4671	5018	W58	5809	1246	W122	7370	3515	W186
5016	4291	W59	5721	1462	W123	7599	3580	W187
5021	4926	W60	5238	1591	W124	7878	3778	W188

Table 1: Continued.

x (m)	y (m)	Number	x (m)	y (m)	Number	x (m)	y (m)	Number
5243	4666	W61	5533	1698	W125	7735	3369	W189
5220	4048	W62	5887	1725	W126	7497	3253	W190
5352	4367	W63	5219	3276	W127	7883	3225	W191
5465	4197	W64	5441	3674	W128	7696	2928	W192

Table 2: Coordinates of 35 wells from a gas field.

x (m)	y (m)	Number	x (m)	y (m)	Number	x (m)	y (m)	Number
264	29	W1	4213	3355	W13	4767	4755	W25
140	460	W2	4280	3785	W14	4910	4514	W26
459	285	W3	4439	4188	W15	4671	5018	W27
284	2160	W4	4642	3905	W16	5016	4291	W28
1805	2128	W5	4506	3500	W17	5021	4926	W29
1466	1868	W6	4745	3665	W18	5243	4666	W30
1793	2404	W7	4902	3436	W19	5220	4048	W31
4185	4644	W8	4624	3289	W20	5352	4367	W32
4266	4411	W9	4849	4060	W21	5465	4197	W33
3996	4274	W10	4417	4898	W22	4176	5039	W34
4167	4079	W11	4553	4667	W23	3975	4957	W35
4047	3589	W12	4614	4356	W24			

Table 3: Coordinates of 40 wells randomly generated by MATLAB.

x (m)	y (m)	Number	x (m)	y (m)	Number	x (m)	y (m)	Number
855.158	2624.822	W1	1365.531	7212.275	W15	9786.806	7126.945	W29
8010.146	292.2028	W2	1067.619	6537.573	W16	5004.716	4710.884	W30
9288.541	7303.309	W3	4941.739	7790.517	W17	596.1887	6819.719	W31
4886.09	5785.251	W4	7150.371	9037.206	W18	424.3114	714.4546	W32
2372.836	4588.488	W5	8909.225	3341.631	W19	5216.498	967.3003	W33
9630.885	5468.057	W6	6987.458	1978.098	W20	8181.486	8175.471	W34
5211.358	2315.944	W7	305.4095	7440.743	W21	7224.396	1498.654	W35
4888.977	6240.601	W8	5000.224	4799.221	W22	6596.053	5185.949	W36
6791.355	3955.152	W9	9047.222	6098.666	W23	9729.746	6489.915	W37
3674.366	9879.82	W10	6176.664	8594.423	W24	8003.306	4537.977	W38
377.3887	8851.68	W11	8054.894	5767.215	W25	4323.915	8253.138	W39
9132.868	7961.839	W12	1829.225	2399.32	W26	834.6981	1331.71	W40
987.1228	2618.712	W13	8865.119	286.7415	W27			
3353.568	6797.28	W14	4899.014	1679.271	W28			

algorithms is usually heavy, making them an inefficient means of solving large-scale problems. In addition, either heuristic methods converge slowly or the quality of their results cannot be guaranteed. Because of the special characteristics of MVs, no related research on optimization problems involving MVs has been reported in recent years; therefore, to address this lack, a corresponding MINLP model and an algorithm for solving it are proposed in this paper.

3. Problem Definition and Formulation

3.1. Connection Matrix and Assumptions. In general, optimal solutions to various problems can be found through various optimization methods, but the solution found may not necessarily be suitable for the actual conditions and engineering design requirements of a particular situation. For a given oil and gas field, especially an unconventional reservoir with scattered wells, it is necessary to reserve some MV channels for future connections to additional production wells; therefore, either the channel counts of the MVs and the number of MVs need to be fixed in advance or ranges should be defined for these parameters to satisfy the need to reserve valves or channels for future use. If experts define several possible schemes in accordance with the given requirements, the most appropriate solution can be obtained by comparing the results of these schemes, but the quality of a solution chosen in this way cannot be guaranteed; thus, it is necessary to formulate and solve the corresponding optimization problem with constraints on the number of MVs and their channel counts.

Suppose that there are n wells that need to be divided into m sets and connected to the corresponding MVs. The MVs will be placed at well sites because no additional land acquisition is allowed; therefore, all well sites are possible alternative locations for MVs, and the number of MVs m is unknown. Let $L = (l_{ij})_{m\times n}$ be the connection matrix, which reflects the connection relationships between the wells and

Table 4: Coordinates of 80 wells randomly generated by MATLAB.

x (m)	y (m)	Number	x (m)	y (m)	Number	x (m)	y (m)	Number
1269.868	9133.759	W1	6554.779	1711.867	W28	5383.424	9961.347	W55
1576.131	9705.928	W2	7060.461	318.328	W29	8173.032	8686.947	W56
4217.613	9157.355	W3	6550.980	1626.117	W30	8692.922	5797.046	W57
971.318	8234.578	W4	5852.678	2238.119	W31	8530.311	6220.551	W58
3516.595	8308.286	W5	5472.155	1386.244	W32	9027.161	9447.872	W59
1621.823	7942.845	W6	4505.416	838.214	W33	2784.982	5468.815	W60
2289.770	9133.374	W7	5498.602	1449.548	W34	1868.726	4897.644	W61
1523.780	8258.170	W8	4387.444	3815.585	W35	2760.251	6797.027	W62
1066.528	9618.981	W9	4455.862	6463.130	W36	1189.977	4983.641	W63
46.342	7749.105	W10	5059.571	6990.767	W37	119.021	3371.226	W64
2598.704	8000.685	W11	6160.447	4732.888	W38	3112.150	5285.331	W65
4314.138	9106.476	W12	5852.641	5497.236	W39	1656.487	6019.819	W66
3377.194	9000.538	W13	3804.458	5678.216	W40	2629.713	6540.791	W67
9571.669	4853.756	W14	5688.237	4693.906	W41	781.755	4426.783	W68
8002.805	1418.863	W15	3509.524	5132.495	W42	844.358	3997.826	W69
7431.325	3922.270	W16	4908.641	4892.526	W43	2769.230	461.714	W70
6948.286	3170.995	W17	8147.237	9057.919	W44	1492.940	2575.083	W71
9502.220	344.461	W18	9575.068	9648.885	W45	1965.953	2510.839	W72
9597.440	3403.857	W19	7922.073	9594.924	W46	758.543	539.501	W73
7512.671	2550.951	W20	8491.293	9339.932	W47	1818.470	2638.029	W74
8407.173	2542.822	W21	6787.352	7577.401	W48	1455.390	1360.686	W75
8142.848	2435.250	W22	7655.168	7951.999	W49	4018.080	759.667	W76
9292.636	3499.838	W23	7093.648	7546.867	W50	2399.162	1233.189	W77
9171.937	2858.390	W24	8909.033	9592.914	W51	1839.078	2399.525	W78
9340.107	1299.062	W25	7572.002	7537.291	W52	4172.671	496.544	W79
6323.592	975.404	W26	5307.976	7791.672	W53	3692.468	1112.028	W80
6557.407	357.117	W27	6892.145	7481.516	W54			

the MVs. If $l_{ij} = 1$, then well i is in well set j, which means that well i is connected to MV j. Thus, the remaining elements in the ith column of the matrix are all equal to 0. Here, a matrix $L^{in}(l_1^{in}, l_2^{in}, \ldots, l_n^{in}), l_i^{in} \in (0, 1), i \in 1, 2, \ldots, n$, with dimensions of $1 \times n$ is introduced to indicate whether an MV is placed at the ith well site. If $l_i^{in} = 1$, this means that an MV is placed at the ith well site. If $l_i^{in} = 0$, this indicates that there is no MV placed at the ith well site. By incorporating L^{in} into the matrix L, we obtain the matrix $\overline{L}$, as shown in Figure 2. If $l_i^{in} = 1$ in the matrix $\overline{L}$, then the corresponding MV can be identified by finding the row j that contains the element in the ith column of $\overline{L}$ (below the first row) that has a value of 1; that is, MV j is placed at the ith well site. Thus, $u_j(x_j^g, y_j^g)$ and the other wells connected to valve j can be found at the same time. Now, considering the characteristics of MVs, the following assumptions are adopted before solving the mathematical model in this paper:

(1) Generally, the number of channels of an MV ranges from 6 to 16. According to the designer's requirements, the corresponding MV specification will be given as either a constant or a range before optimization. Thus, the special constraints on the MVs will be specified beforehand.

(2) All wells need to connect directly to an MV because this is the only way in which wells can be selected for metering; there are no pipeline connections between wells.

(3) The possible locations of the MVs are preselected by the engineering company. One of the strategies that is usually adopted is to place the MVs at well sites to reduce land acquisition, which means that all well sites are potential alternative locations for MVs.

(4) In this paper, we use the Euclidean distance to quantify the length of the pipeline between a well and an MV. Additionally, terrain and obstacles are not considered.

(5) The costs of valves and instruments other than the MVs are not considered. These costs are neglected in the calculation since the use of MVs simplifies the metering process.

(6) We assume that the same material and diameter are used for all pipelines. Therefore, factors such as pressure and flow rate are not considered.

3.2. MINLP Model

3.2.1. Objective Function. Given the above assumptions, the objective function can be simply expressed as follows:

$$\min C = C_{\text{pipe}} + C_v, \tag{1}$$

where C_{pipe} represents the cost of all pipelines, which is equal to the sum of the costs of the pipelines connected to each MV:

Table 5: Coordinates of 191 wells randomly generated by MATLAB.

x (m)	y (m)	Number	x (m)	y (m)	Number	x (m)	y (m)	Number
5438.859	7210.466	W1	6951.63	4991.16	W65	2547.902	2240.4	W129
3658.162	7635.046	W2	377.3887	8851.68	W66	1886.62	2874.982	W130
3934.564	6714.311	W3	1365.531	7212.275	W67	2702.704	1970.538	W131
5522.913	6298.834	W4	1067.619	6537.573	W68	2702.943	2084.614	W132
4670.682	6481.984	W5	305.4095	7440.743	W69	2955.338	3329.363	W133
4241.668	5078.583	W6	596.1887	6819.719	W70	8443.088	1947.643	W134
4886.09	5785.251	W7	154.8713	9840.637	W71	8010.146	292.2028	W135
5000.224	4799.221	W8	526.77	7378.581	W72	6987.458	1978.098	W136
5004.716	4710.884	W9	326.0082	5611.998	W73	8865.119	286.7415	W137
5268.758	4167.995	W10	1564.05	8555.228	W74	7224.396	1498.654	W138
4896.876	3394.934	W11	911.1346	5762.094	W75	9990.804	1711.211	W139
5985.237	4709.243	W12	444.5409	7549.333	W76	8177.606	2607.28	W140
4400.851	5271.427	W13	5.223754	8654.386	W77	6951.405	679.9277	W141
4624.492	4243.49	W14	834.8281	6259.598	W78	9159.912	11.51057	W142
5822.492	5407.393	W15	319.9102	6147.134	W79	8699.41	2647.79	W143
4045.8	4483.729	W16	426.5241	6351.979	W80	6962.663	938.2003	W144
1671.684	1062.163	W17	252.2818	8422.066	W81	7688.543	1672.535	W145
1781.325	1280.144	W18	3507.271	9390.016	W82	8013.476	2278.429	W146
1614.847	1787.662	W19	3111.023	9233.796	W83	8448.557	2094.051	W147
2362.306	1193.962	W20	3674.366	9879.82	W84	9479.331	820.7121	W148
2186.766	1057.983	W21	4323.915	8253.138	W85	7378.417	634.045	W149
1096.975	635.9137	W22	5478.709	9427.37	W86	4709.233	2304.882	W150
1920.283	1388.742	W23	4177.441	9830.525	W87	4302.074	1848.163	W151
1117.057	1362.925	W24	4607.259	9816.38	W88	4388.7	1111.192	W152
1476.082	549.7415	W25	4574.244	8753.716	W89	5211.358	2315.944	W153
169.8294	1208.596	W26	5180.521	9436.226	W90	4899.014	1679.271	W154
1057.094	1420.411	W27	5224.953	9937.046	W91	3724.097	1981.184	W155
1339.313	308.8955	W28	4386.45	8335.006	W92	4252.593	3127.189	W156
2276.643	4356.987	W29	5144.235	8842.81	W93	4228.857	942.2934	W157
2580.647	4087.198	W30	5827.91	8153.972	W94	4734.86	1527.212	W158
2372.836	4588.488	W31	6125.665	9899.502	W95	3180.741	1192.145	W159
1733.886	3909.378	W32	4980.943	9008.525	W96	3477.127	1499.973	W160
2919.841	4316.512	W33	5746.612	8451.782	W97	3773.955	2160.189	W161
2691.194	4228.356	W34	5216.498	967.3003	W98	3624.115	495.3258	W162
1904.333	3689.165	W35	4820.221	1206.116	W99	4895.7	1925.104	W163
1909.237	4282.53	W36	5895.075	2261.877	W100	4170.29	2059.755	W164
2427.854	4424.023	W37	5840.693	1077.69	W101	4888.977	6240.601	W165
3308.579	4243.095	W38	5943.563	225.1259	W102	4941.739	7790.517	W166
1998.628	4069.548	W39	6385.308	336.0384	W103	3846.191	5829.864	W167
1897.104	4950.058	W40	6753.321	67.15314	W104	5308.643	6544.457	W168
2818.669	5385.967	W41	4713.572	357.6273	W105	4076.192	8199.812	W169
1239.323	4903.573	W42	4423.054	195.7762	W106	4609.164	7701.597	W170
1776.025	3985.895	W43	5880.261	1547.523	W107	3411.246	6073.892	W171
855.158	2624.822	W44	5340.641	899.5068	W108	4257.288	6444.428	W172
987.1228	2618.712	W45	5737.098	520.7789	W109	4587.255	6619.448	W173
604.7118	3992.578	W46	6224.751	5870.447	W110	4794.632	6393.17	W174
688.061	3195.997	W47	6028.431	7112.158	W111	5447.161	6473.115	W175
908.2329	2664.715	W48	6596.053	5185.949	W112	9132.868	7961.839	W176
1536.567	2810.053	W49	6568.599	6279.734	W113	7150.371	9037.206	W177
1230.837	2054.942	W50	6663.389	5391.265	W114	6176.664	8594.423	W178
1465.149	1890.722	W51	6981.055	6665.279	W115	8181.486	8175.471	W179
6443.181	3786.094	W52	6959.493	6998.878	W116	8313.797	8033.644	W180
5948.961	2622.117	W53	6833.632	5465.931	W117	7183.589	9686.493	W181
6791.355	3955.152	W54	6476.176	6790.168	W118	6377.091	9576.939	W182
6447.645	3762.722	W55	6278.964	7719.804	W119	6678.327	8443.922	W183
6170.909	2652.809	W56	6834.159	7040.474	W120	6357.867	9451.741	W184
7302.488	3438.77	W57	7386.403	5859.87	W121	8419.292	8329.168	W185
5313.339	3251.457	W58	6609.446	7297.519	W122	8085.141	7550.771	W186
6021.705	3867.712	W59	7690.291	5814.465	W123	7487.057	8255.838	W187
6073.039	4501.377	W60	5649.796	6403.118	W124	2259.218	1707.08	W188

Table 5: Continued.

x (m)	y (m)	Number	x (m)	y (m)	Number	x (m)	y (m)	Number
6620.096	4161.586	W61	2077.423	3012.463	W125	2217.467	1174.177	W189
5860.921	2621.453	W62	2966.759	3187.783	W126	424.3114	714.4546	W190
6877.961	3592.282	W63	1829.225	2399.32	W127	834.6981	1331.71	W191
6786.523	4951.77	W64	2518.061	2904.407	W128			

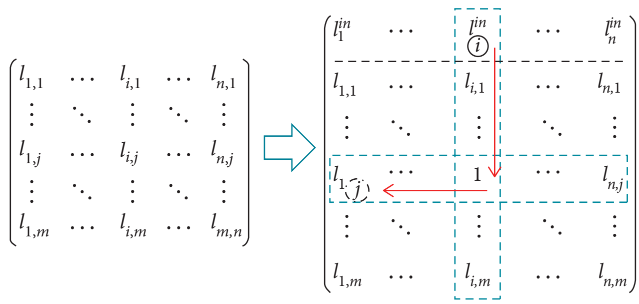

Figure 2: Matrix of MVs and wells.

$$C_{\text{pipe}} = \sum_{j=1}^{m} C_j^{\text{pipe}}, \tag{2}$$

where C_j^{pipe} represents the total cost of all pipelines connected to MV j, which can be expressed as follows:

$$C_j^{\text{pipe}} = W^p \times L_j \times D^{\text{pipe}}, \quad j = 1, 2, \ldots, m, \tag{3}$$

where "×" denotes the matrix dot product operation.

The total cost of the pipelines connected to valve j is the dot product of D^{pipe}, L_j, and $W^p\,[w_1^p, w_2^p, \ldots, w_n^p]$, where W^p is the pipeline cost coefficient matrix, L_j is the jth row of the connection matrix, and $D^{\text{pipe}} = [d_1^{\text{pipe}}, d_2^{\text{pipe}}, \ldots, d_n^{\text{pipe}}]$ is a matrix consisting of the pipeline lengths from the wells to the valve. o_i represents the coordinates of the ith well, $i = 1, 2, \ldots, n$. Specifically, $o_i\,(x_{i,j}, y_{i,j}), i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$, means that the ith well is connected to the jth MV. Then, d_i^{pipe} represents the pipeline length from well i to the corresponding MV under the assumption that this well is connected to valve j; this pipeline length can be expressed as

$$d_i^{\text{pipe}} = \sqrt{\left(x_{i,j} - x_j^g\right)^2 + \left(y_{i,j} - y_j^g\right)^2}. \tag{4}$$

$u_j\,(x_j^g, y_j^g) \in O_j$ represents the coordinates of the jth MV, which are unknown and are determined by solving a subproblem. The detailed solution method will be described in Section 4. Then, the cost of all pipelines can be written as follows:

$$C_{\text{pipe}} = \sum_{j=1}^{m}\sum_{i=1}^{n} w_i^p l_{i,j} \sqrt{\left(x_{i,j} - x_j^g\right)^2 + \left(y_{i,j} - y_j^g\right)^2}. \tag{5}$$

Considering the costs of the MVs themselves, the total cost of all MVs can be expressed as

$$C_v = W^v \times B, \quad j = 1, 2, \ldots, m, \tag{6}$$

where $B[\beta_1, \beta_2, \ldots, \beta_m]$ is a matrix with dimensions of $1 \times m$ that is used as a compact representation of the connection matrix, in which β_j is a binary variable. If all elements in the jth row of the connection matrix are 0, then $\beta_j = 0$; otherwise, $\beta_j = 1$. W^v is the matrix of the MV cost coefficients. C_v is the dot product of W^v and B. By summing the costs of the pipelines and valves, the objective function is obtained as follows:

$$C = W^p \times L_j \times D^{\text{pipe}} + W^v \times B$$
$$= \sum_{j=1}^{m}\sum_{i=1}^{n} w_i^p l_{i,j} \sqrt{\left(x_{i,j} - x_j^g\right)^2 + \left(y_{i,j} - y_j^g\right)^2} + \sum_{j=1}^{m} w_j^v \beta_j. \tag{7}$$

3.2.2. Constraints. The objective function has been established, as shown in equation (7). Now, we will describe the constraints of the model. In this paper, the variable m is constrained by $\underline{\omega}$ and $\overline{\omega}$, as follows:

$$\underline{\omega} \le m \le \overline{\omega}, \tag{8}$$

where $\overline{\omega}$ is the upper bound on the number of MVs, while $\underline{\omega}$ is the minimum value that m can take; both $\underline{\omega}$ and $\overline{\omega}$ are integers. As the range of m shrinks, the number of MVs will finally be specified; that is, $\underline{\omega} = m = \overline{\omega}$. Only one nonzero element is allowed to exist in each column of the connection matrix because each well can be connected to only one MV. This constraint can be expressed as follows:

$$\sum_{j=1}^{m} l_{i,j} = l_{i,1} + l_{i,2} \cdots l_{i,m} = 1, \quad i = 1, 2, \ldots, n, \tag{9}$$

$$l_{i,j} \in (0, 1). \tag{10}$$

Given that row in the connection matrix represents one MV, the sum of the elements in each row of this matrix cannot be greater than the channel count of the corresponding MV. Since the channel counts of the MVs selected by the designer might differ, N^{up} is used to represent the maximum MV channel count provided by the manufacturer, and $N_{r,j}^{\text{up}}$ is used to represent the specified channel count of the jth MV:

$$\sum_{i=1}^{n} l_{i,j} = l_{1,j} + l_{2,j} \cdots l_{n,j} \le N_j^{\text{up}}, \quad j = 1, 2, \ldots, m. \tag{11}$$

The following constraints should be satisfied:

$$N_{r,j}^{\text{up}} \le N^{\text{up}}, \quad j = 1, 2, \ldots, m, \tag{12}$$

$$N_j^{\text{up}} \le N^{\text{up}}, \quad j = 1, 2, \ldots, m. \tag{13}$$

If $N_{r,j}^{\text{up}}$ is the channel count of MV j as chosen or required by the designer and a_j^{up} is a binary variable that represents whether a specified channel count for the jth valve is actually used to constrain the mathematical model, according to the constraint relaxation degree, then the maximum number of wells that can actually be connected to MV j is defined as

$$N_j^{\mathrm{up}} = N^{\mathrm{up}}\left(1 - a_j^{\mathrm{up}}\right) + a_j^{\mathrm{up}} N_{r,j}^{\mathrm{up}}, \quad j = 1, 2, \ldots, m. \tag{14}$$

By combining this equation with equation (11), we obtain

$$\sum_{i=1}^{n} l_{i,j} \le N^{\mathrm{up}}\left(1 - a_j^{\mathrm{up}}\right) + a_j^{\mathrm{up}} N_{r,j}^{\mathrm{up}}, \quad j = 1, 2, \ldots, m. \tag{15}$$

Usually, it is not permitted to connect zero wells to a given MV. Therefore, taking 1 as the default lower bound N^{lb} for each row of the matrix, we have

$$\sum_{i=1}^{n} l_{i,j} = l_{1,j} + l_{2,j} \cdots l_{n,j} \ge N^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m. \tag{16}$$

Sometimes, it is unreasonable to connect only a few wells to an MV. In this case, we use $N_{r,j}^{\mathrm{lb}}$ to indicate a specified lower bound on the number of wells to which an MV is allowed to be connected. The corresponding constraint can be expressed as

$$\sum_{i=1}^{n} l_{i,j} = l_{1,j} + l_{2,j} \cdots l_{n,j} \ge N_{r,j}^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m. \tag{17}$$

The minimum number of wells that can be connected to MV j is determined by a_j^{lb}. a_j^{lb} is a binary variable that controls the value of N_j^{lb}. If $a_j^{\mathrm{lb}} = 1$, a specific lower bound is not required and the default is used, whereas if the value of this variable is 0, a specific lower bound is applied. Thus, using the same formulation used for the upper bound, we obtain the following constraint:

$$N_j^{\mathrm{lb}} = N^{\mathrm{lb}}\left(1 - a_j^{\mathrm{lb}}\right) + a_j^{\mathrm{lb}} N_{r,j}^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m. \tag{18}$$

By combining equations (16)~(18), we can write the constraint related to the lower bound as follows:

$$\sum_{i=1}^{n} l_{i,j} \ge N^{\mathrm{lb}}\left(1 - a_j^{\mathrm{lb}}\right) + a_j^{\mathrm{lb}} N_{r,j}^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m. \tag{19}$$

The following relationships should be satisfied:

$$N_{r,j}^{\mathrm{lb}} \le N_j^{\mathrm{up}}, \quad j = 1, 2, \ldots, m, \tag{20}$$

$$N_{r,j}^{\mathrm{lb}} \le N_{r,j}^{\mathrm{up}}, \quad j = 1, 2, \ldots, m, \tag{21}$$

$$N^{\mathrm{lb}} \le N_{r,j}^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m, \tag{22}$$

$$N^{\mathrm{lb}} \le N_j^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m, \tag{23}$$

$$N_j^{\mathrm{lb}} \le N_{r,j}^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m. \tag{24}$$

Equations (20) and (21) are used to ensure that the lower bound will not exceed the upper bound. Equations (22) and (23) ensure that both the required lower bound and the bound that is actually applied are at least as large as the default lower bound. Finally, equation (24) stipulates that the real lower bound should not exceed the required lower bound. Since the matrix L^{in} represents the MVs, the sum of the elements of the matrix L^{in} should satisfy

$$\begin{gathered} \sum_{i=1}^{n} l_i^{in} = m, \\ l_i^{in} = (0, 1). \end{gathered} \tag{25}$$

Because only one MV is allowed in each well group, when we subtract L^{in} from any row of the connection matrix L and then add the absolute value of the results together, we will have $\mathrm{sum}_j + m - 1$ for each row of L. Then, the constraint that only one MV is allowed in each well group can be written as

$$\sum_{i=1}^{n} \left| l_i^{\mathrm{in}} - l_{i,j} \right| = \mathrm{sum}_j + m - 1, \quad j = 1, 2, \ldots, m. \tag{26}$$

According to the above equations, the mathematical model can be expressed as

$$\begin{aligned}
C &= \sum_{j=1}^{m} \sum_{i=1}^{n} w_i^p l_{i,j} \sqrt{\left(x_{i,j} - x_j^g\right)^2 + \left(y_{i,j} - y_j^g\right)^2} + \sum_{j=1}^{m} w_j^v \beta_j, \\
\text{s.t} \quad & \underline{\omega} \le m, \\
& m \le \overline{\omega}, \\
& \sum_{i=1}^{n} l_{i,j} \le N^{\mathrm{up}}\left(1 - a_j^{\mathrm{up}}\right) + a_j^{\mathrm{up}} N_{r,j}^{\mathrm{up}}, \quad j = 1, 2, \ldots, m, \\
& N^{\mathrm{lb}}\left(1 - a_j^{\mathrm{lb}}\right) + a_j^{\mathrm{lb}} N_{r,j}^{\mathrm{lb}} \le \sum_{i=1}^{n} l_{i,j}, \quad j = 1, 2, \ldots, m, \\
& N_j^{\mathrm{up}} \le N^{\mathrm{up}}, \quad j = 1, 2, \ldots, m, \\
& N_{r,j}^{\mathrm{up}} \le N^{\mathrm{up}}, \quad j = 1, 2, \ldots, m, \\
& N_{r,j}^{\mathrm{lb}} \le N_j^{\mathrm{up}}, \quad j = 1, 2, \ldots, m, \\
& N_{r,j}^{\mathrm{lb}} \le N_{r,j}^{\mathrm{up}}, \quad j = 1, 2, \ldots, m, \\
& N^{\mathrm{lb}} \le N_{r,j}^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m, \\
& N^{\mathrm{lb}} \le N_j^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m, \\
& N_j^{\mathrm{lb}} \le N_{r,j}^{\mathrm{lb}}, \quad j = 1, 2, \ldots, m, \\
& \sum_{j=1}^{m} l_{i,j} = 1, \quad i = 1, 2, \ldots, n, \\
& \sum_{i=1}^{n} l_i^{in} = m, \\
& \sum_{i=1}^{n} \left| l_i^{in} - l_{i,j} \right| = \mathrm{sum}_j + m - 1, \quad j = 1, 2, \ldots, m, \\
& a_j^{\mathrm{up}}, a_j^{\mathrm{lb}}, \beta_j, l_{i,j}, l_i^{in} \in (0, 1).
\end{aligned} \tag{27}$$

The decision variables used in the mathematical model represent the associations between the wells and the MVs, and the number of valves and their channel counts are treated as constraints. The binary variables a_j^{up} and a_j^{lb} determine the range of the channel counts of the MVs, while $\overline{\omega}$ and $\underline{\omega}$ determine the range of the number of MVs.

4. The Proposed Algorithm for Solving the MINLP Problem

The underlying principle of methods such as steepest gradient descent [19] is to find the direction in which the objective function decreases the fastest at the present point, take a step in that direction, and then repeat this process, thus iteratively reducing the value of the objective function. We use the same idea to search for the optimal solution in a discrete space. In this paper, the mathematical model is described in the matrix form; thus, the step size and the number of steps must be expressed in terms of a measure of the distance between two matrices. If we treat the whole matrix as a point, then we can move one nonzero element in the matrix to obtain a new matrix, and this new matrix can be treated as a point adjacent to the original point; that is, the new matrix is one step away from the original matrix. All the points obtained by changing one individual element are the points closest to the current point. As an example, let us consider a connection matrix that is equivalent to the 5×5 unit matrix; then, we can obtain a foresight matrix corresponding to this connection matrix, as shown in Figure 3. The square columns represent the objective function values, where each row of the matrix represents a well and each column represents an MV. Then, the values in the jth column of the matrix represent the values of the objective function after the corresponding nonzero elements are moved to the jth valve; that is, the square column in the ith row and jth column represents the objective function value after well i is connected to MV j.

If the initial connection matrix is a unit matrix, then initially, the values on the diagonal of the foresight matrix are equal to the objective function value of the original matrix because the values on the diagonal of the foresight matrix, represented by square columns with stripes, are obtained only when no element is moved. For a case in which the constraints are not satisfied after a single nonzero element is moved to another position, the corresponding objective function value is represented by a dark gray square column; all such objective function values are replaced by a sufficiently large penalty value M to ensure that the algorithm will not select such a point. White columns are used to represent the feasible domain, meaning that these positions in the matrix represent points that satisfy the constraints of the mathematical model. The minimum value in the feasible domain, denoted by v_{n+1}, is found as the result of the current iteration, as shown in Figure 3. As shown in the figure, the minimum objective function value is found in the 4th row and 1st column; therefore, we set the element in the 4th row and 1st column to 1 and set the remaining elements in the 4th row to 0. In this way, we can find the path that causes the objective function value to drop the fastest from the current point, thus completing one single-step iteration. When all values in the feasible domain are greater than or equal to the current value, the current point is considered to be a locally optimal solution.

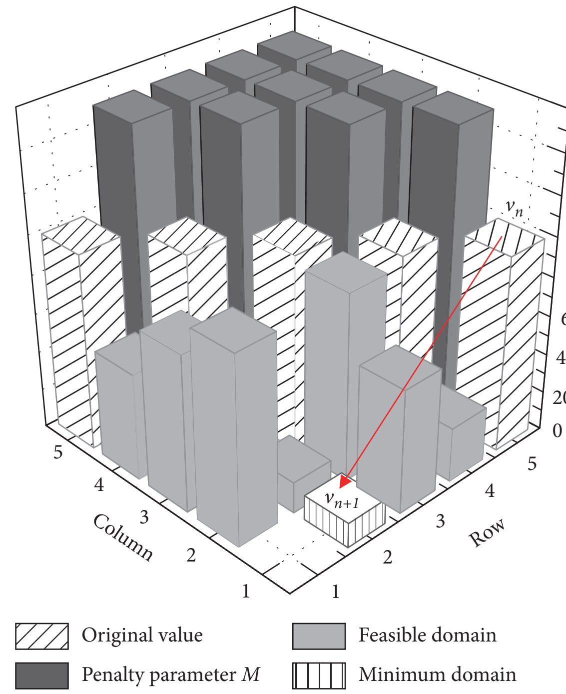

Figure 3: Foresight matrix of the unit matrix.

We can always construct a foresight matrix such as that shown in Figure 3 after every step and then find the point that minimizes the objective function for the next step until no value less than the current value of the objective function is found. To obtain a better solution once the current point has fallen into a local optimum, we can consider increasing the step size, thus enabling the algorithm to escape from the local optimum and find a better feasible solution after further iterations.

4.1. Solving the Subproblem. All wells connected to MV j are assigned to the same well set. The MV will be placed at one of the wells in this set such that the total weighted length from the wells to the MV is as short as possible, as expressed in equation (23). Thus, a subproblem must be solved to find the placement of valve j that results in the shortest pipeline length in well group j:

$$\min_{S_j} \sum_{i=1}^{n} l_{i,j} d_i^{\mathrm{pipe}}, \quad j = 1, 2, \ldots, m. \tag{28}$$

This subproblem can be solved using a simple algorithm. The coordinates of the wells connected to valve j are represented by $(x_{i,j}, y_{i,j})$, $i = 1, \ldots, \mathrm{num}_j$, $j = 1, \ldots, m$. Because the location of MV j is unknown, $u_j(x_j^g, y_j^g) \in O_j$ is assumed to represent the valve coordinates that minimize the sum of the pipeline lengths connected to valve j. The valve will be placed at each well site in turn; then, we can compare the results to find the optimal placement position for the valve.

4.2. Details of the Algorithm. The algorithm for solving the MINLP problem obtains the solution through iteration. The

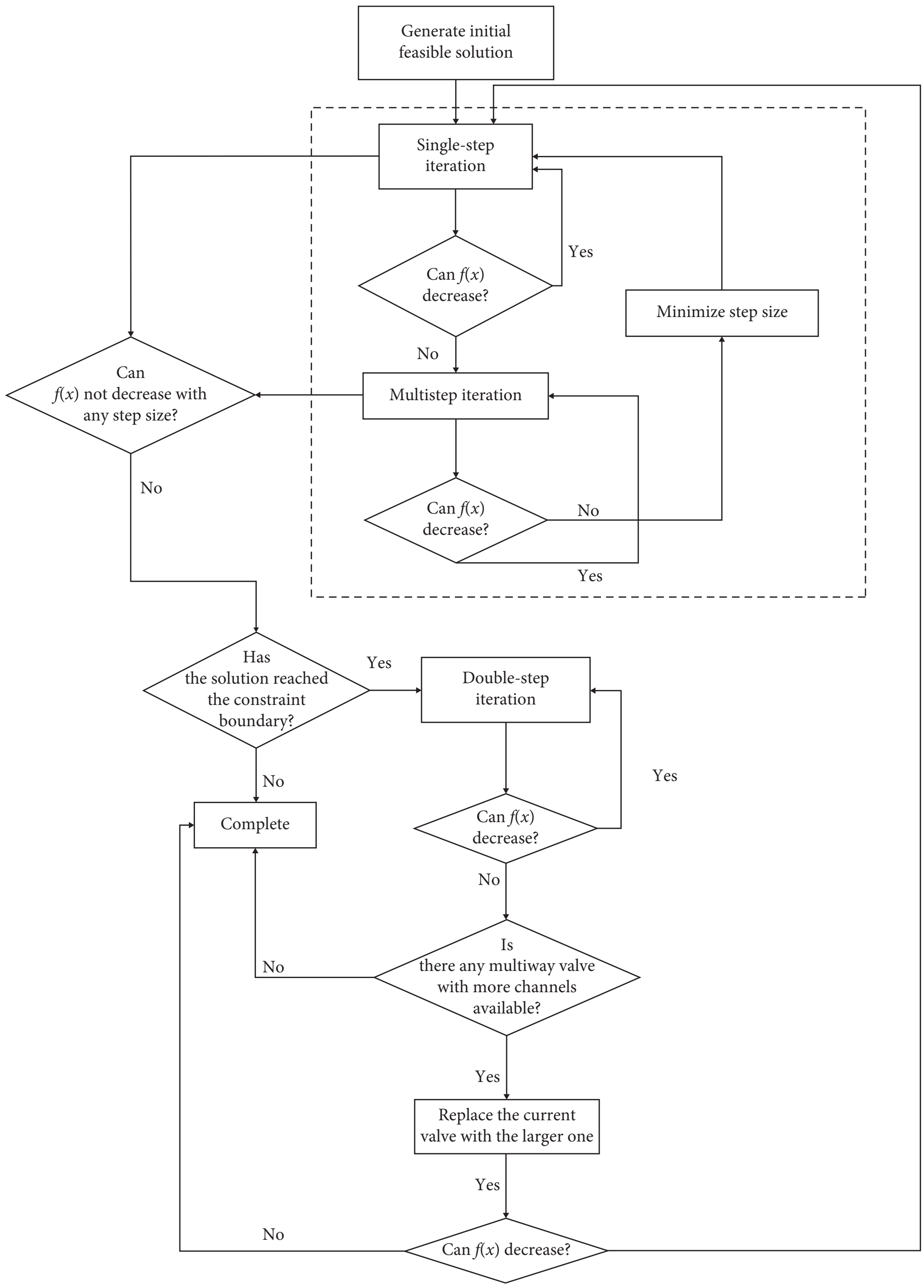

Figure 4: Algorithm flow diagram.

iterative process mainly comprises 4 steps; this process is depicted in Figure 4 and described in detail below.

4.2.1. Step 1: Initialization. If the constraint on the number of MVs m is relaxed, m can range from 1 to n. In the extreme case, a unit matrix is taken as the initial feasible solution, meaning that each MV is initially assumed to be connected to only one well. Generally, if $1 \leq \underline{\omega} \leq m \leq \overline{\omega} \leq n$, we can initially set $m = \overline{\omega}$, and the wells will be sequentially assigned to the n MVs in accordance with the constraint conditions.

4.2.2. Step 2: Iteration. The initial costs can be obtained by substituting the initial distribution matrix into the objective function. Then, a blank matrix with dimensions of $m \times n$ is initialized as the foresight matrix.

Starting from the first column, each nonzero element 1 in the connection matrix is moved up or down by one row; each movement will change the distribution, and the objective function value will change as the connection matrix changes. The corresponding objective function value is calculated for every movement, and the new objective function value is filled into the corresponding position of the foresight matrix.

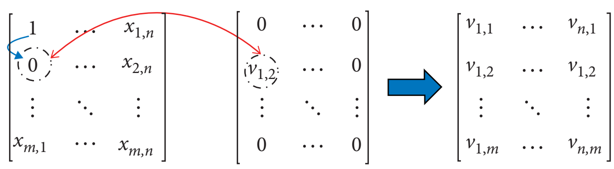

Figure 5: Process of filling in the foresight matrix.

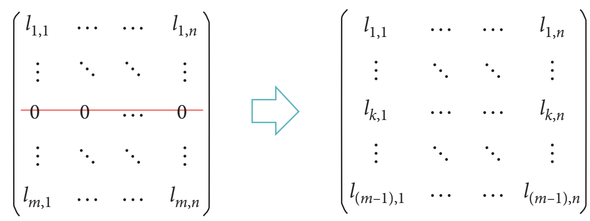

Figure 6: Removal of a row that does not contain any wells.

In the example shown in Figure 5, element 1 in the first row and first column is moved to the second row and first column to obtain a new connection relationship. After substituting this new connection relationship into the objective function, the value $v_{1,2}$ corresponding to the new connection relationship is obtained. $v_{1,2}$ is then filled into the second row and first column of the foresight matrix, corresponding to the new position of the nonzero element in the connection matrix. If the constraints are not satisfied by this new connection relationship, a sufficiently large penalty value M is placed in the corresponding position instead. Once the foresight matrix has been completely filled, the path of fastest descent can be determined. This process is illustrated in Figure 5.

According to the constraints, the nonzero elements in the connection matrix can move only up and down; they cannot move left or right. Since each column of the matrix represents a well, when a nonzero element is moved from the jth row to the kth row in column i, this represents that this well is disconnected from MV j and connected to MV k, and the value of the element in the ith row and jth column returns to 0.

In this way, each 1 value in the connection matrix is moved from the top row to the bottom. Each time a 1 value is moved to a new position, a new objective function value is generated, filling in the corresponding position in the foresight matrix. When the new matrix represents a connection relationship that does not satisfy the constraints, a sufficiently large penalty value M is filled into the foresight matrix instead. Once the $m \times n$ foresight matrix has been completed, we then search for the minimum value in this matrix that is smaller than the current value. For example, if $v_{i,j}$ is the smallest value in the foresight matrix, then for the next iteration, 1 in the ith column of the connection matrix is moved to the jth row, and the value at the original position is set to 0; thus, one single-step iteration is completed. By repeating this process, the current value of the objective function can be iteratively reduced, and new connection relationships can be formed. Figure 5 illustrates this iterative process for a 5×5 foresight matrix.

If all nonzero elements in a given row of the connection matrix leave that row by moving up or down, this means that the corresponding MV does not exist; in this case, the empty row should be removed from the connection matrix, decreasing the feasible domain. In this way, the number of MVs can be found. Specifically, if the elements of the jth row are all 0, then the jth row should be removed from the connection matrix, as shown in Figure 6.

4.2.3. Step 3: Increasing the Step Size. If a lower cost cannot be obtained by moving a single element in the connection matrix up or down, this means that the solution has become stuck in a local optimum. Therefore, the step size must be gradually increased to allow the algorithm to escape from this local optimum. For example, suppose that the matrix L is initially set as the unit matrix and that the initial step size is 1. When the objective function value cannot continue to decrease after a certain number of iterations, then increasing the step size can help to find a new connection relationship that will allow the objective function to continue to decrease. More specifically, suppose that the ith row of the matrix L contains two nonzero elements, meaning that MV i is connected to two wells, while the jth row contains three nonzero elements. In this case, both nonzero elements in the ith row are added to the jth row, setting all elements in the ith row to zero, and the new total cost $v_{i,j}$ under the new connection relationship is obtained. This cost value is filled into the ith row and jth column of the foresight matrix. Then, the matrix is restored to its previous state, and the algorithm proceeds to fill in the other elements in the foresight matrix in the same way. Each element in the foresight matrix is an objective function value, and these values are used as the basis for iteration.

In the foresight matrix for multistep iteration, the values on the diagonal should be equivalent to the objective function value when no wells are moved because for each element on the diagonal, the row and column correspond to the same well; therefore, the penalty value M is filled into the positions on the diagonal of the matrix. Moreover, the matrix elements below the diagonal to the left are symmetric to the matrix elements above the diagonal to the right; therefore, all matrix elements in either the lower left or the upper right can be directly set equal to M to avoid redundant calculations. As before, if the constraints are not satisfied after the elements of the connection matrix are moved, the penalty value M is also directly filled into the corresponding position. After the foresight matrix has been fully filled in, the smallest value in this matrix can be found. If this minimum value is smaller than the cost obtained in the previous iteration, then we can find the row and column corresponding to this minimum value, add the elements of the ith row to the jth row, and remove the ith row, whose elements are now all 0. At this time, the number of MVs m is set to m minus 1.

If the remaining channels of an MV do not allow connection to any further wells, the wells will be sequentially assigned to other MVs. This iterative process will repeat until the objective function value can no longer be reduced by such multistep iterations. Then, the algorithm will either return to Step 1 or proceed to Step 4.

4.2.4. Step 4: Adjustment. If there is no MV whose channels are fully occupied or for which the number of wells connected to that valve has reached the lower bound N_j^{lb}, then the optimal solution can be obtained through single-step or multistep iteration. However, according to the constraints, if an MV is fully occupied, then it cannot be connected to any new wells, whereas if a valve has reached its lower bound N_j^{lb}, then no more wells can be moved out of the corresponding row. In this case, the iterative process can be continued as follows:

(1) The step size can be increased to 2 to test whether the objective function value will continue to decrease. First, we look for the fully occupied MV. Suppose that num_j wells are connected to this MV; then, the remaining $n - \mathrm{num}_j$ wells will each be moved to this MV in turn, replacing one of the existing wells, which is moved to another MV that meets the constraints. By repeating this process, the new values of the objective function are obtained and written into the foresight matrix based on the corresponding well numbers i and j. Finally, the path of steepest descent can be determined in the same way as before.

(2) For an MV j that has been fully connected, we look for an MV t with more channels than MV j, where b_j denotes the number of channels of MV j. Then, $b_j < b_t$, and the total number of wells connected to each valve satisfies $\mathrm{sum}_t \le \mathrm{sum}_j$. Since $\mathrm{sum}_j = b_j$, $b_j \ge \mathrm{sum}_t$ must be satisfied to allow the jth and tth valves to be exchanged; that is, the following conditions should be satisfied simultaneously:

$$\begin{cases} b_j < b_t, \\ b_j \ge \mathrm{sum}_t. \end{cases} \quad (29)$$

If these conditions are satisfied, then the channel counts of the jth and tth MVs are exchanged, and we continue iterating steps 2 and 3 mentioned above. However, if i and t do not satisfy $b_i^{\mathrm{sum}} < b_t$, $b_i < b_t$, and $\mathrm{sum}_{b,t} \le \mathrm{sum}_{b,t}$, then the penalty value M is written into the corresponding position of the foresight matrix. The process above is repeated; if, after the Nth iteration, the objective function value cannot continue to decrease, meaning that $v_{N+1}^{\min} - v_N^{\min} = 0$, then the current objective function value corresponds to the optimal solution.

5. Analysis of Examples

5.1. Verification of the Validity of the Algorithm. First, a simple example is presented to demonstrate that the proposed algorithm can effectively solve the problem of interest. The associations between the wells and the MVs and the number of MVs are taken as variables, and their values are found by the algorithm via iteration; in particular, during the iterative process, multistep iteration can help the objective function value to escape from a local optimum. Let us consider a simple example for which we can make a reasonable prediction of the expected solution. Then, we can apply the proposed algorithm to solve the example and compare its solution with our expected result to prove that the algorithm can effectively solve the presented MINLP model.

For all examples in this paper, we consider steel pipes with an inner diameter of 53 mm and a wall thickness of 3.5 mm as the pipes linking the wells and valves; thus, the cost coefficient w_i^p is a constant. Moreover, the cost of an MV w_j^v is a known constant once the vender is decided. In this paper, the cost of an MV is treated as an average value that does not vary with the number of channels. Here, we consider an example of a field with 26 wells (Table 1; W10~W35), for which the channel counts of the MVs and the number of MVs are not limited; therefore, the number of channels of each MV is uniformly set to 16, which is the maximum number of channels per MV that can be provided by general manufacturers.

We first perform single-step iteration, during which the number of MVs will usually stay at a relatively large value, as shown in Figure 7. When the identity matrix is used as the initial matrix, the number of MVs cannot be decreased below 8 via this process. However, increasing the step size can lead to further reduction in the value of the objective function, demonstrating that the reason that the objective function cannot be further reduced via single-step iteration is that the step size of each advance is too small. Thus, the objective function becomes stuck in a local optimum, and it is necessary to increase the step size to escape from that local optimum to allow the objective function to continue to decrease. The objective function decreases almost linearly during the initial 18 steps because the main factor affecting the decrease in the objective function value during the initial iterations is the number of MVs rather than the pipeline cost. However, the value of the objective function no longer decreases once it drops into a local optimum. After the 18th iteration, the step size must be increased to continue to reduce the objective function value. The single-step iteration process converges prematurely after the 18th iteration; thus, the findings demonstrate that the strategy of increasing the step size can efficiently help the objective function to escape from a local optimum.

The result of optimization with an increase in step size is shown in Figure 8. The hollow square symbols represent the positions of the MVs, the triangular symbols represent the wells connected to MV 1, and the circular symbols represent the wells connected to MV 2. For the 26 wells considered in this calculation, the algorithm completely divides them into two groups, and the solution satisfies all the constraints, which is obviously consistent with our expectations. This example proves that the proposed algorithm can be applied to solve the problem of interest and obtain reasonable optimization results.

5.2. Convergence of the Algorithm. Usually, we can judge whether an algorithm is valid according to whether the value of the objective function converges during the iterative process. If the algorithm is divergent, no matter how many

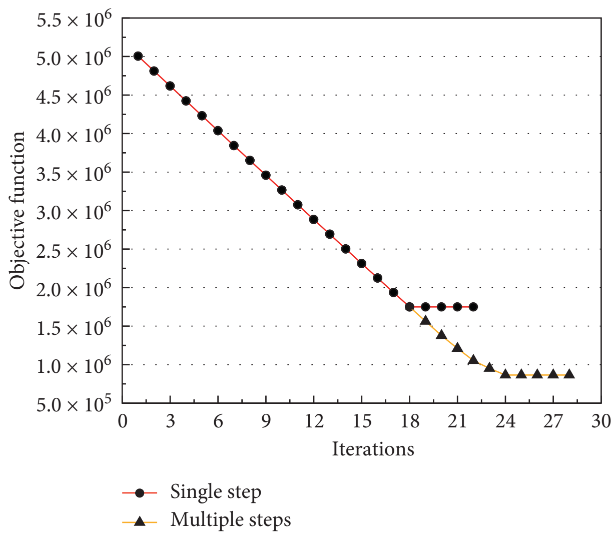

Figure 7: Comparison of two different step sizes.

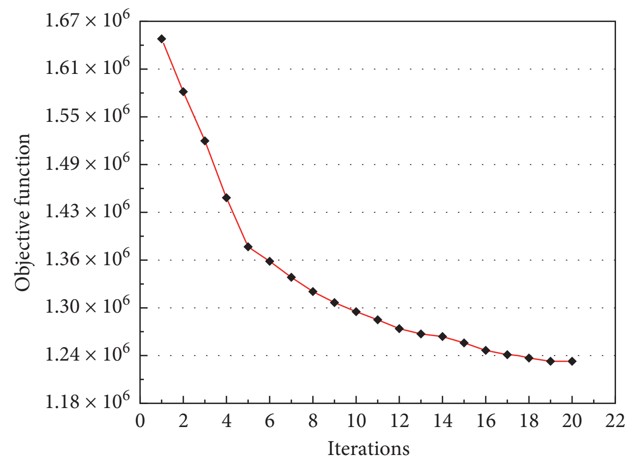

Figure 9: Iterative process for 35 wells under a fixed number of MVs with fixed channel counts.

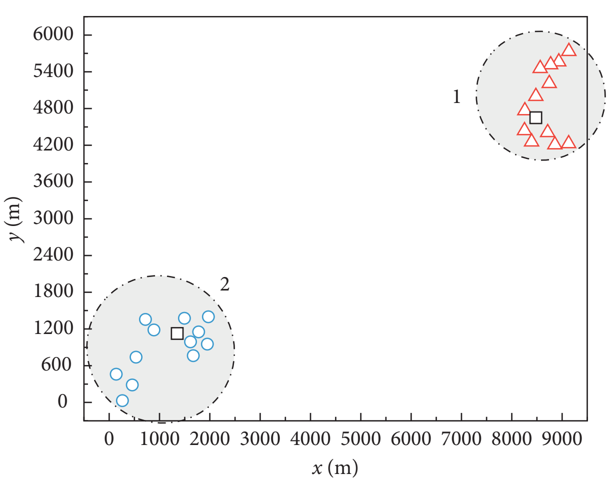

Figure 8: Results of optimization with an increase in step size.

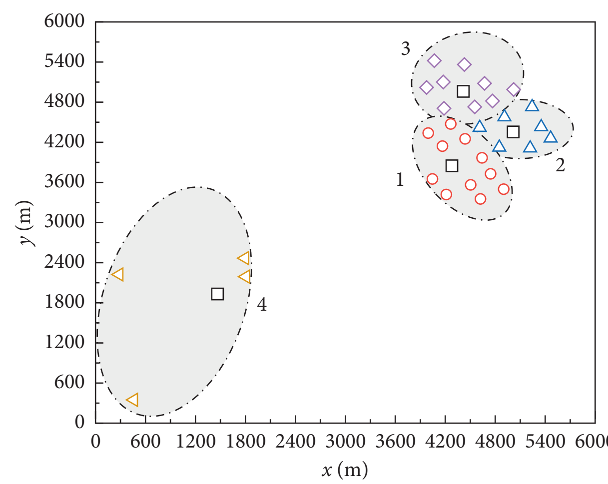

Figure 10: Optimization results for 35 wells.

times it is iterated, the value of the objective function will always fluctuate and will not approach a constant. In contrast, convergence of the algorithm means that the objective function value will tend toward stability after some numbers of iterations. In summary, convergence is a necessary condition for the algorithm to be valid.

Next, various examples are presented to analyze the convergence of the algorithm. First, we consider a field with 35 wells (Table 2; W1~W35) as an example. MVs with four different channel counts are specified: $N_1^{up} = 14$, $N_2^{up} = 12$, $N_3^{up} = 10$, and $N_4^{up} = 8$. Thus, the number of channels of each MV is fixed in advance, the limitations on the numbers of pipelines that are allowed to be connected to different valves are different, and the number of MVs is also given, i.e., $m = 4$.

Figure 9 shows the iterative process for these 35 wells, which includes single-step iteration, double-step iteration, and redistribution of different kinds of valves. The iterative process still converges, and the optimal solution is obtained after the 21st iteration. After 14 iterations, the redistribution of different valve types is necessary to allow the objective function value to continue to decrease. Finally, a difference of 0 between two consecutive steps indicates that the final solution has been obtained.

As seen from Figure 9, the objective function value gradually tends toward stability and converges as iteration continues. The result is shown in Figure 10. Red circular symbols, blue triangular symbols, purple diamond symbols, and yellow triangular symbols are used to represent the individual wells connected to MVs 1, 2, 3, and 4, respectively, and square symbols are used to represent the MVs. The initial channel counts of the MVs corresponding to these different well groups are 14, 12, 10, and 8, respectively, while the final channel counts are 12, 10, 14, and 8, in sequence. In the case of tight constraints, the minimum value of the objective function can be obtained by using the proposed optimization algorithm.

In addition to the example above, an example with 40 wells (Table 1; W69~108) and various channel counts is considered to prove the convergence of the algorithm; the results are shown in Figures 11(a) and 11(b). It can be seen from the figures that, during the initial iterations, the

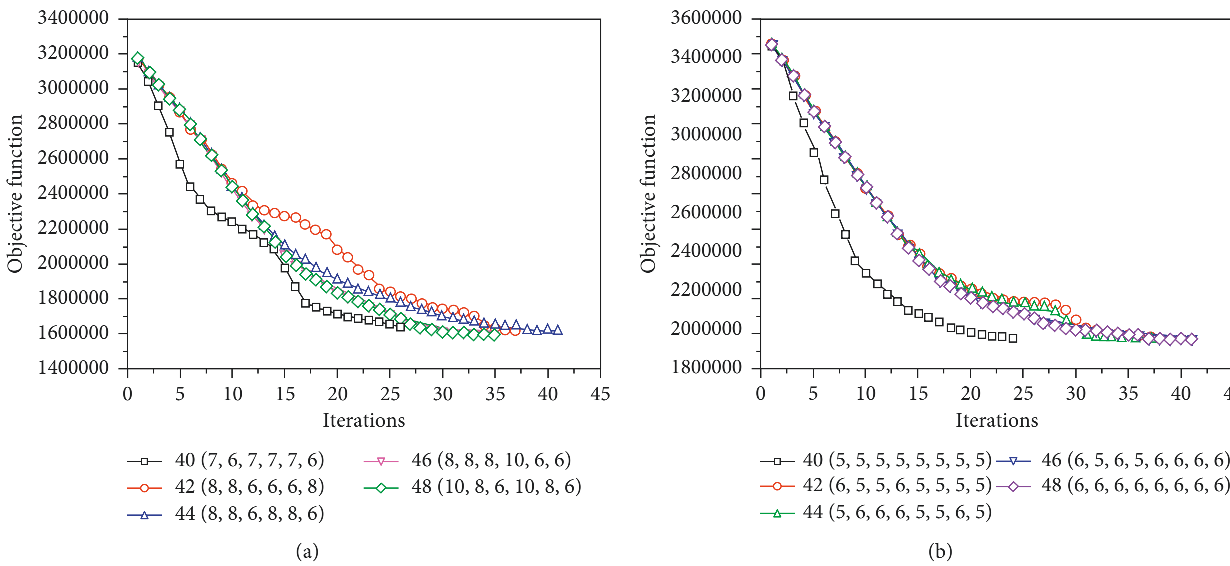

Figure 11: Convergence examples under different constraints: (a) 6 valves, 40~48 channels; (b) 8 valves, 40~48 channels.

objective function value decreases at a relatively fast rate. Then, the rate of decrease slowly declines as iteration proceeds. The algorithm eventually converges when $v_{N-1}^{\min} - v_N^{\min} = 0$.

Again, the value of the objective function gradually converges and tends toward stability. In Figure 11(a), the iterative processes with a total of 46 and 48 channels are identical, indicating that once the number of channels exceeds a certain threshold, a further increase in the number of channels has no effect on the iterative process or the result. Additionally, although the iterative process for an initial feasible solution with 40 channels is different from the other 4 cases, the final result is not significantly different from the others, indicating that the initial solution does not have a great influence on the result.

The iterative processes for total channel counts of 46 and 48 are also fundamentally similar. Once the total number of channels exceeds 46, the iterative processes are basically the same. The results show that the algorithm presented above can stably converge. Therefore, these examples demonstrate that even under different constraints and in different scenarios, the algorithm can drive the objective function value toward stable convergence.

5.3. Parameter Analysis. In general, the designer of a pipeline system should consider the possibility of reserving channels for new wells in the future along with other factors when specifying the channel counts of the MVs and the number of MVs. However, different combinations of MVs with different numbers of channels might affect the results; therefore, we will analyze the effects of different channel counts and different numbers of MVs on the results of our algorithm. We consider a field with 40 wells (Table 1; W69~108) as our example here, varying the constraints on the numbers of channels and valves to obtain different results. The results are shown in Table 6, from which we can see the influence of these different constraint parameters on the cost of the pipeline system.

Six different numbers of MVs are specified: 3, 4, 5, 6, 7, and 8. The table shows that the requirement of the use of too few MVs will lead to additional costs when the total number of channels is the same. The cost also rises if more than 4 MVs are used, and the smaller the total number of channels is, the faster the increase of cost is. When the total number of channels is fewer than or equal to 42, the cost rises at a significantly higher rate than in the other cases. Therefore, the total number of channels should be set to greater than 42 to make it easier to achieve lower costs. However, increasing the number of channels does not significantly reduce the overall cost once the total number of channels exceeds 48. The number of MVs has a greater impact on the cost. These findings suggest that the optimization of the number of MVs is the dominant factor affecting the value of the objective function, while the number of channels is the secondary influencing factor. These observations can provide a reasonable basis for practical design.

Next, a set of well distribution data from the field and a set of data randomly generated by MATLAB are used as examples to analyze how the well distribution density affects the optimization result. In the first of these examples, the coordinates of 40 wells (Table 1; W69~108) that are densely distributed in an actual production field are considered for optimization. The number of MVs m is treated as an optimization variable, and each MV is specified to have 16 channels; in the optimization result obtained, four MVs are used. It can be seen from Figure 12(a) that the optimization result is reasonable.

In the second example, the positions of the 40 wells are randomly generated using the rand function (Table 3; W1~W40), and the number of MVs is constrained to a range. The upper limit on the number of channels is uniformly set to 16 for each MV, and the problem is solved using the proposed algorithm. The well connections of the six MVs are shown in Figure 12(b). These optimization results are also reasonable.

5.4. Stability of the Algorithm. In this section, scatter plots are presented in Figure 13 to show the well and MV

Table 6: Costs with various numbers of channels and MVs.

Total channels	Cost (¥)	Number of MVs	Individual channel counts
40	1.424×10^6	3	16, 16, 8
42	1.396×10^6	3	16, 16, 10
44	1.396×10^6	3	16, 14, 14
46	1.396×10^6	3	16, 16, 14
48	1.396×10^6	3	16, 16, 16
50	1.304×10^6	3	16, 16, 18
40	1.306×10^6	4	10, 10, 10, 10
42	1.287×10^6	4	12, 10, 10, 10
44	1.287×10^6	4	12, 12, 10, 10
46	1.287×10^6	4	12, 12, 12, 10
48	1.287×10^6	4	12, 12, 12, 12
50	1.287×10^6	4	14, 12, 12, 12
40	1.647×10^6	5	8, 8, 8, 8, 8
42	1.520×10^6	5	8, 8, 8, 8, 10
44	1.426×10^6	5	8, 8, 8, 10, 10
46	1.426×10^6	5	8, 8, 10, 10, 10
48	1.426×10^6	5	8, 8, 10, 10, 12
50	1.426×10^6	5	8, 8, 10, 12, 12
40	1.641×10^6	6	7, 6, 7, 7, 7, 6
42	1.619×10^6	6	8, 8, 6, 6, 6, 8
44	1.615×10^6	6	8, 8, 8, 8, 6, 6
46	1.592×10^6	6	8, 8, 8, 10, 6, 6
48	1.592×10^6	6	10, 8, 8, 10, 6, 6
50	1.592×10^6	6	10, 8, 8, 8, 8, 8
40	1.893×10^6	7	6, 6, 6, 6, 6, 4, 6
42	1.870×10^6	7	6, 6, 6, 6, 6, 6, 6
44	1.775×10^6	7	6, 8, 6, 6, 6, 6, 6
46	1.766×10^6	7	8, 8, 6, 6, 6, 6, 6
48	1.766×10^6	7	6, 8, 6, 8, 8, 6, 6
50	1.766×10^6	7	8, 8, 6, 8, 6, 6, 6
40	1.957×10^6	8	5, 5, 5, 5, 5, 5, 5, 5
42	1.950×10^6	8	6, 6, 5, 5, 5, 5, 5, 5
44	1.941×10^6	8	6, 6, 5, 5, 6, 6, 5, 5
46	1.941×10^6	8	6, 6, 5, 5, 6, 6, 6, 6
48	1.941×10^6	8	6, 6, 5, 5, 6, 6, 8, 6
50	1.919×10^6	8	8, 6, 6, 6, 6, 6, 6, 6

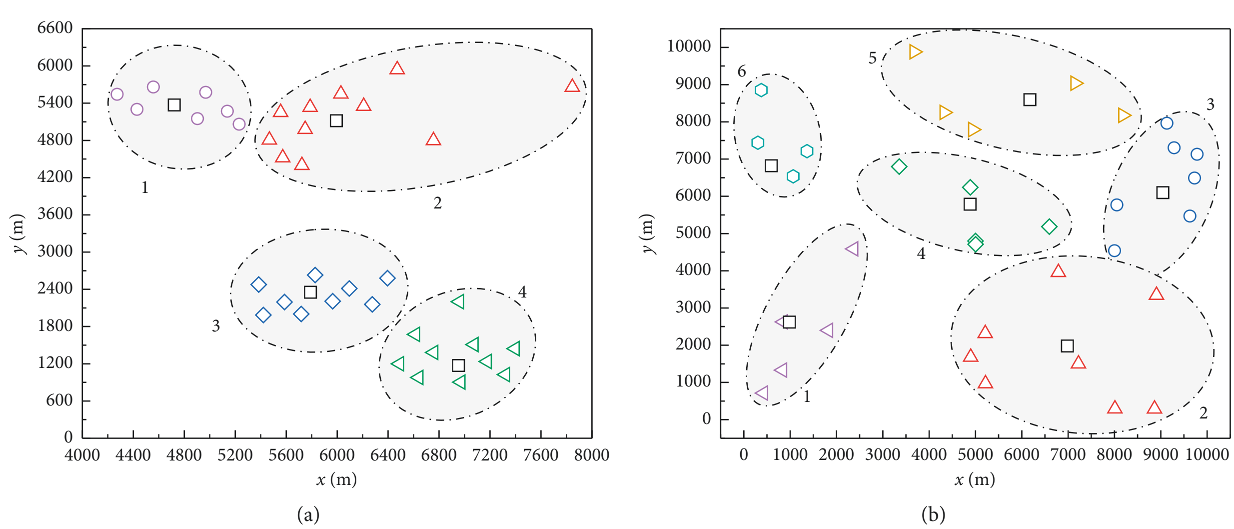

Figure 12: Optimization results for 40 wells: (a) 40 wells in a real production field; (b) 40 wells generated randomly by MATLAB.

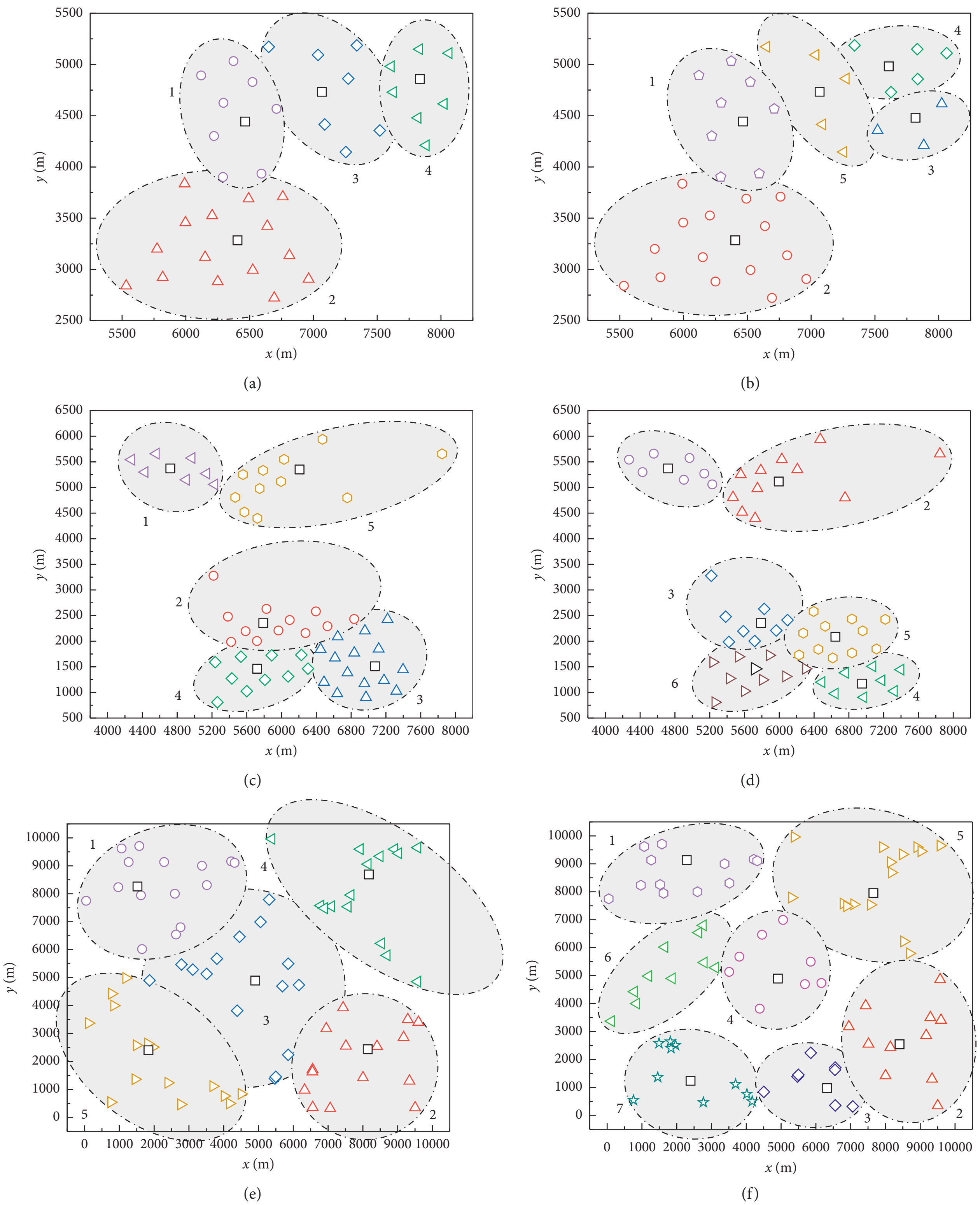

FIGURE 13: Results for various situations: (a) 41 wells, 4~8 MVs, 16 channels per MV; (b) 41 wells, 5 MVs, 16 channels per MV; (c) 60 wells, 5 MVs, 16 channels per MV; (d) 60 wells, 6~10 MVs, 16 channels per MV; (e) 80 wells, 5 MVs, 16 channels per MV; (f) 80 wells, 6~10 MVs, 16 channels per MV.

distribution results calculated using the proposed algorithm under the condition that the maximum number of channels per MV is constrained to 16 and the number of MVs is constrained to a certain range. The results prove that the algorithm can stably solve the MINLP problem and obtain reasonable results. Figures 13(a) and 13(b) show the result

Table 7: Performance of the algorithm.

Wells	Type	Channels per MV	m (range)	$f(x)$ (¥)	m (outcome)	Time
26	Field	All 16	2~6	0.866×10^6	2	0.255 s
26	Field	16, 16	2	0.866×10^6	2	0.335 s
35	Field	All 16	3~6	1.118×10^6	3	0.836 s
35	Field	8, 10, 8, 10	4	1.394×10^6	4	0.854 s
41	Field	All 16	3~8	1.316×10^6	4	0.877 s
41	Field	All 16	4	1.316×10^6	4	1.277 s
60	Field	All 16	6~10	1.91×10^6	6	2.554 s
60	Field	16, 8, 12, 12, 8, 10	6	1.945×10^6	6	3.418 s
80	Random	All 16	6~10	4.330×10^6	7	5.553 s
80	Random	10, 10, 16, 16, 8, 8, 12, 12	8	4.381×10^6	8	8.336 s
191	Field	All 16	12	5.350×10^6	12	42.817 s
191	Random	All 16	12	7.382×10^6	12	78.645 s

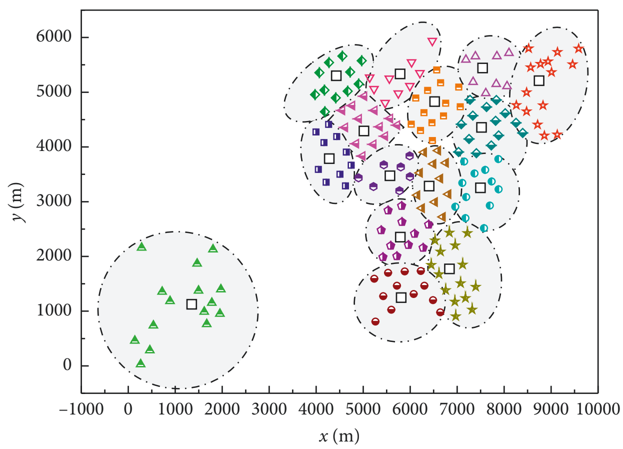

Figure 14: Optimization results for a large-scale example.

generated using the data based on W140~W180 (Table 1), Figures 13(c) and 13(d) show the result generated using the data based on W67~W128 (Table 1), Figure 13(e) shows the result generated using the data based on W1~W80 (Table 1), and Figure 13(f) shows the result generated using the data based on W1~W80 (Table 4).

5.5. Algorithm Performance. The scale of the computations performed in the proposed algorithm mainly depends on the size of the foresight matrix and the number of iterations. Under the assumption that all solutions are feasible, the maximum time complexity of the algorithm is $o(mnN)$, where m and n denote the numbers of rows and columns, respectively, of the matrix and N is the number of iterations. If the coordinate data are given in order, the time consumption is smaller, and if they are random, the time consumption is larger, as seen by comparing the examples with 191 wells in Table 7. The random data for 191 wells are shown in Table 5. For the examples with 26 and 41 wells, for the first example of each type listed in Table 7, the identity matrix is considered the initial feasible solution, and the computational scale is then reduced through multistep iteration. By comparison with the second example in each case, it is confirmed that the multistep iteration strategy can efficiently reduce the scale of the computations performed, thus saving time. However, regardless of the type of iteration used, the same optimal solutions can be found. In the second listed examples with 35, 60, and 80 wells, there is a specified upper bound on the number of channels for each MV, and the number of MVs is constant. The computation times are increased in these cases mainly because the single-step iteration process requires more iterations to approach the optimal solution. The solution times shown in Table 7 essentially meet our expectations with regard to the algorithm complexity. Thus, it is proven that the proposed algorithm can be used to solve various problem instances within a short time.

Finally, a large-scale problem with 192 wells (Table 1; W1~W192) is considered, and the result is shown in Figure 14. Here, the channel count for each valve is constrained to 16, and the number of valves is allowed to vary from 15 to 39. It can be seen that the algorithm can still successfully separate different wells into different blocks in the face of a large number of wells with multiple valves. The results show that the algorithm can still effectively solve such a large-scale problem and yield reliable results. All of the above solution processes were implemented in MATLAB on an ASUS GX501VSK laptop (Intel Core i7-7700HQ Processor, 16 GB of DDR4 RAM).

6. Conclusion

This paper proposes the replacement of traditional metering processes with a process based on MVs, thus simplifying and automating the necessary operations and reducing the investment in redundant valve instrumentation and other equipment, the amount of space occupied by this equipment, and the cost of the station. A generalized MINLP model is presented, and a corresponding heuristic algorithm for MV optimization design is proposed for solving the presented MINLP problem. The numbers of channels of the MVs and the number of MVs are treated as special constraints in the MINLP problem.

The coordinates of actual production well sites and randomly generated data are used as examples for calculation and analysis. From the analysis of these examples, it can be seen that the behavior of the objective function value in the proposed algorithm is strictly convergent. A large collection of optimization results demonstrates that the algorithm shows stable convergence and good reliability. The analysis results show that the proposed heuristic algorithm can quickly and effectively solve the presented MINLP model for the design of a pipeline system with MVs. In this paper, we do not consider the optimization of the MV process itself in an integrated gathering system; this problem will be addressed in the future.

Nomenclature

a_j^{up}: Binary variable. If $a_j^{\mathrm{up}} = 1$, then a specific number of channels are given. By default, if $a_j^{\mathrm{up}} = 0$, the number of channels of the jth MV is given by the manufacturer

a_j^{lb}: Binary variable. If $a_j^{\mathrm{lb}} = 1$, then a specific lower bound on the number of wells to be connected to the jth MV is given, whereas a value of 0 indicates that the default lower bound is used

b_t: Number of channels of a fully connected MV

b_j: Number of channels of MV j

B: Matrix used as a compact representation of the connection matrix

β_j: Binary variable used to indicate MVs without any connected channels. If all elements in the jth row of L are 0, then $\beta_j = 1$; otherwise, $\beta_j = 0$

C: Total cost

C_{pipe}: Total cost of pipelines

C_v: Total cost of MVs

C_j^{pipe}: Total cost of the pipelines connected to the jth MV

D^{pipe}: Total length of pipelines

D_j^{pipe}: Length of the pipelines connected to MV j

$D_{\mathrm{sum}}^{\mathrm{pipe}}(d_{1,j}^{\mathrm{pipe}}, d_{2,j}^{\mathrm{pipe}}, \ldots, d_{\mathrm{num}_j,j}^{\mathrm{pipe}})$: Set of the total pipeline lengths obtained by placing MV j at all possible different well sites

L: Matrix representing the connection relationship between the wells and the valves

$l_{i,j}$: Element in the ith row and jth column of L. $l_{i,j} = 1$ indicates that the ith well is connected to the jth valve, and $l_{i,j} = 0$ indicates that there is no connection between the ith well and the jth valve

$L^{in}(l_1^{in}, l_2^{in}, \ldots, l_n^{in})$: Matrix with dimensions of $1 \times n$ used to express where the valves are placed. $l_i^{in} = 1$ indicates that an MV is placed at the ith well site, and a value of 0 indicates no MV is placed at the corresponding well site

$\overline{L}$: Connection matrix incorporating the information about the placement of the MVs

M: Penalty value

m: Number of MVs

n: Number of wells

N_j^{up}: Number of channels ultimately selected for MV j

N_j^{lb}: Lower bound actually applied on the number of wells connected to MV j

$N_{r,j}^{\mathrm{up}}$: Specified number of channels for MV j

$N_{r,j}^{\mathrm{lb}}$: Specified lower bound on the number of wells to be connected to valve j

N^{lb}: Default lower bound on the number of wells to be connected to valve j

N^{up}: Maximum number of channels per MV that a manufacturer can provide and also the maximum value that $N_{r,j}^{up}$ can take

$O(o_1, o_2, \ldots, o_n)$: Set of well coordinates

$o_i(X_{i,j}, Y_{i,j})$: Indicator that the ith well is connected to the jth MV

$U(u_1, u_2, \ldots, u_m)$: Set of MV coordinates

$v_{i,j}$: Objective function value obtained by moving the nonzero element in the jth column to the ith row

$V_{m\times n}$: Foresight matrix

$v_N^{\min}$: Minimum objective function value after N iterations

$\underline{\omega}$: Lower bound on the number of MVs

$\overline{\omega}$: Upper bound on the number of MVs

$W^p[w_1^p, w_2^p, \ldots, w_n^p]$: Cost coefficient matrix for pipelines

$W^v[w_1^v, w_2^v, \ldots, w_n^v]$: Cost coefficient matrix for MVs.

Data Availability

The coordinate data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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