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The Effect of Gearbox Architecture on Wind Turbine Enclosure Size

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Abstract

Gearbox architecture – the type of gearing used, the overall gear ratio, the number of increaser stages, the number of meshes, the ratio combinations, and the gear proportions- can have a profound effect on the "package" size of a wind turbine. In this paper the author applies a common set of requirements to a variety of potential gearbox designs for a 2.0 mW wind turbine and compares the resulting "geared component" weights, gearbox envelope sizes, generator sizes, and generator weights. Each design option is also evaluated for manufacturing difficulty via a relative cost estimate.

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The importance of macro geometry

Much has been written in recent years on optimizing the "micro" geometry of gears, *i.e.*, determining the best profile or lead modifications. With this paper we propose to take a step back and consider the "macro" geometry instead. By "macro geometry" we mean the number of stages in the gear train, the type of gears used, and the amount of gear ratio used in each stage. This basic architecture of a gearbox, its "macro geometry", is a fundamental factor in meeting the overall design objectives. Enhanced micro geometry can improve performance in the field but cannot make up for poor decision making on the basic design. Through the design exercise described in this paper we will also illustrate the interaction of "architecture" with the overall size of the drive package. One of the issues we have with the recent emphasis on micro geometry is that the modifications can only be optimized for a specific load condition. For many applications, such as wind turbines, the gearbox will be subjected to a very wide range of conditions, for most of which it will not be "optimized." If the basic gear train design is well thought out it will be less dependent upon "optimization" for its success.

Design conditions

The design conditions selected represent a simplified specification for a 2.0 mW wind turbine gearbox, see Table 1. They do not reflect any actual design project and the results presented in this paper are not intended to be applied to any future project. The typical wind turbine design specification will include a much more detailed load spectrum, for example, along with requirements for intensive gear rating analysis. The conditions used for this paper provide a "level playing field" by which preliminary designs could be rapidly developed. The objective is to compare preliminary designs in such a way as to identify those which merit further consideration on actual projects.

Table 1. Design conditions

Design inputs	Transmitted power: 2.0 mW x 1.5 application factor = 3.0 mW [4,023 HP] Required life = 85,000 hours at full load Input speed: 15 rpm Output speeds: 150, 300, 600, 900, 1200, 1500, 1800 rpm Corresponding increaser ratios: 10, 20, 40, 60, 80, 100, and 120:1
Design constraints	Minimum number of pinion teeth: 18 Maximum face width/pinion pitch diameter ratio: 1.25 [per helix] Minimum face contact ratio [m_f] =1.00 per helix Number of planets - 5 for ratios up to 4:1 - 4 for ratios between 4.05:1 & 6:1 - 3 for ratios between 6.05:1 & 13:1 Maximum individual mesh ratio: 6.5:1 [exception made for 10:1 single reduction] No divided power path arrangements which require radially timed sub-assemblies Compliance with AGMA rating standards for load sharing between planets Compliance with AGMA rating standards for load distribution factor Gear quality set at AGMA Q-11 per AGMA 2000 All external gears carburized and hardened
Gear arrangements considered	Single, double, triple and quadruple reduction external helical One planetary stage with zero, one, or two external helical stages Two planetary stages with zero, one, or two external helical stages
Design evaluation criteria	Number of components Estimated weight of gears and non-housing components Approximate envelope dimensions Relative manufacturing costs

Design constraints

An experienced gearbox designer has usually developed a set of guiding principles to speed his or her work. The author has spent much of his career designing special, one-off gearboxes where a conservative design philosophy is required out of respect for a lack of qualification testing and development time. The constraints adopted for this paper are reflective of that experience and the author recognizes that other designers may disagree with the limits he has established. The reasons for each of these constraints is discussed in the following paragraphs.

Minimum number of pinion teeth

The choice of 18 for a minimum number of pinion teeth was made based upon maximizing the tooth strength, achieving a minimum profile contact ratio of 1.30, and reducing the grind cycle time. [Form grinding cycle times are a function of the number of teeth, stock allowance, and face width.] Having designed parallel axis gear sets with as few as 3 pinion teeth and as many as 42 pinion teeth, 18 is a good minimum to avoid hobbing issues [undercutting, problems with start of active profile overlapping the top of the fillet] while still providing an acceptable profile contact ratio.

Maximum face width/pinion pitch diameter ratio

As gear capacity and cost tend to follow a volume function, pay careful attention to the "FD squared" principle [where F is the face width and D is the pinion pitch diameter]. It was not unheard of, back in the 1960s and 1970s, to have a face diameter ratio of up to 2.00 in through hardened industrial gearboxes. As the service hours accumulated on these long thin pinions it became apparent that torsional deflection adversely effected the life of these drives. In later design work we have had the opportunity to see the beneficial effects of reducing the F/D ratio to the 1.00/1.25 range and have avoided using a higher value ever since.

Minimum face contact ratio, M_f

If helical geometry is to be fully effective, a minimum face contact ratio of 1.00 per helix is needed. The adjustments in the gear rating formulas to account for M_f values of less than 1.00 have limited testing behind them so they should be avoided. Once the complications of thrust and overturning moment are introduced to the bearing evaluation process, it seems prudent to insure that the gears will enjoy the full benefits of helical load sharing.

Number of planets

Figure 1 shows the geometry behind my limits on the number of planets. We recognize that non-standard geometries can allow some adjustment to these ratio limits but find them to be good guidelines for general design. As ratings are all about "power per mesh" we have chosen to use the maximum number of planets wherever possible.

Maximum individual mesh ratio

The "FD squared" principle referenced earlier plays a big part in the decision to limit individual mesh ratios to less than 6.5:1 except in the case of a single stage 10:1 double helical gear set. That exception serves as an excellent illustration of how rotating mass increases very rapidly as set ratio goes up, see Figure 2, case A.

Radial timing

As mentioned above, rating calculations are based upon power per mesh. When multiple meshes are used to share the load it becomes incumbent upon the designer to insure that load sharing is uniform or that the drive train can accommodate the anticipated degree of inequality. Our experience with industrial divided power path drives makes us very skeptical that uniform distribution ever occurs and the highly variable nature of the loads in wind turbines further increases my discomfort. For this reason we have limited the designs in this paper to those which do not require radial timing or load sharing adjustment outside the planetary stages.

Planet load sharing

Load sharing within planetary stages is widely understood within the gear design community. We are aware of the creative approaches used to reduce the variation in load between planets but decided it was best to comply with AGMA standard adjustment factors for this exercise.

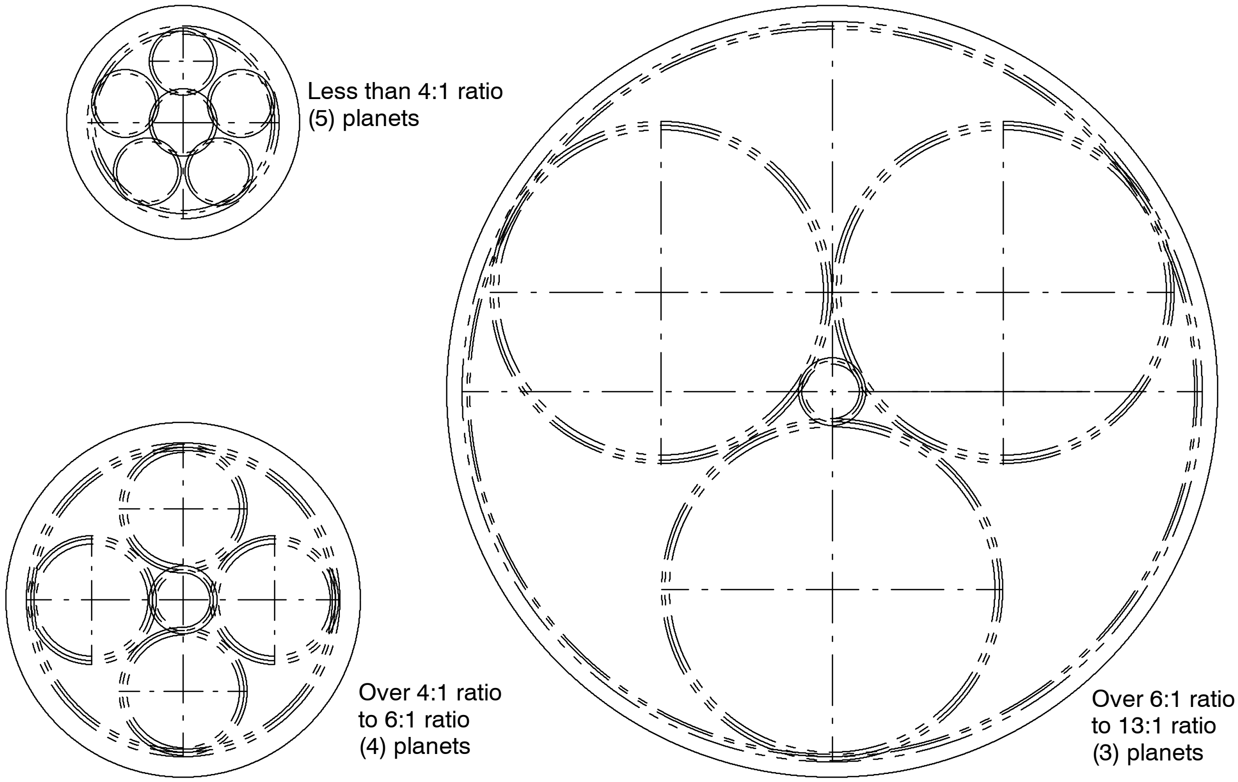

Figure 1. Number of planets vs. stage ratio

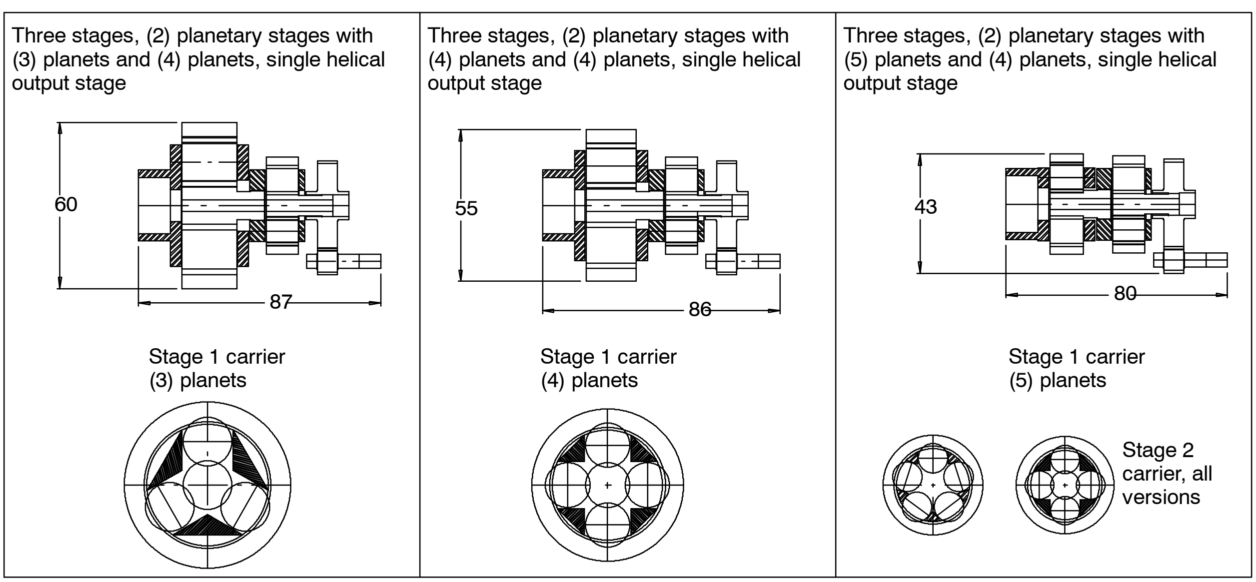

Figure 2 The effect of increasing the number of planets

Load distribution factor, C_m

While recognizing the advanced methodology being widely used to modify tooth geometry to improve operating load distribution, we have elected to comply with the C_m calculations in AGMA 2001. The purpose of this exercise is to demonstrate the effect of macro geometry on overall drive size and the potential improvement available through additional effort on C_m was not significant.

Gear set quality

Modern computer controlled gear grinding equipment is capable of consistently producing AGMA Q13 (AGMA 200-A88) parts. Considering the accuracy and loaded deflections of the mountings, however, we have reduced the gear quality to AGMA Q11 levels for this exercise. The highly variable nature of the wind turbine duty cycle along with the complexity of the assemblies contributed to our decision. The effect of improved mounted quality would not change the relative size of one design solution compared to another.

Heat treat

All external gearing in this study is Grade 2 carburized and hardened. As the durability rating of the internal gears was not a limiting factor they are calculated as through hardened [285 BHN minimum]. The alloy selection on the carburized parts and the addition of surface hardening to internal gears does not effect the final envelope size.

Evaluation of gear arrangements

As with most widely studied applications, current wind turbine gearboxes have coalesced around a narrow range of designs, typically one or two planetary stages with one or two helical stages at the high speed end. Many other arrangements are possible and the purpose of this paper is to evaluate competing designs for this demanding service. Comparison of the overall size, weight, and relative cost of each arrangement will determine whether alternate designs are worthy of further study. The size of a drive system and its weight are major factors in the design of a tower. The number and size of the geared components have a major influence on the cost of a gearbox. If only the geared components are considered, planetary arrangements have an obvious advantage in terms of physical size and weight. When the planet carriers enter the discussion, however, the weight advantage begins to diminish.

Methodology

Using the guidelines described above, the first step in this exercise was to design the anticipated gear sets in 1 NDP. As gear ratings are parametric in nature, the approximate tooth size needed to carry a specific load can be found by taking the cube root of the ratio between the 1 NDP rating and the target rating. All other dimensions for the set can be found by dividing the 1 NDP dimensions by the final NDP selected. As the dynamic factor decreases as size decreases, the rating summary charts show ratings slightly higher (<10%) than the minimum acceptable values.

Once the required gear sets were designed they were arranged into typical gear trains in a CAD program. Bearing journals, shaft extensions, planet carriers, and output hubs were sized using conservative stress levels. ***No attempt has been made to execute detailed design on the (28) gear trains studied. The preliminary layouts could be developed further but met the purposes of this exercise in the condition presented.*** Each design was then evaluated for approximate enclosed volume, estimated weight, and relative cost to manufacture. Figures 3 through 9 show the CAD layouts for each increaser ratio in the same scale. Table 2 shows the relative cost, estimated weight, and approximate volume comparison for the (28) designs. Tables 3 through 10 provides the gear geometry for the gear sets used in the designs.

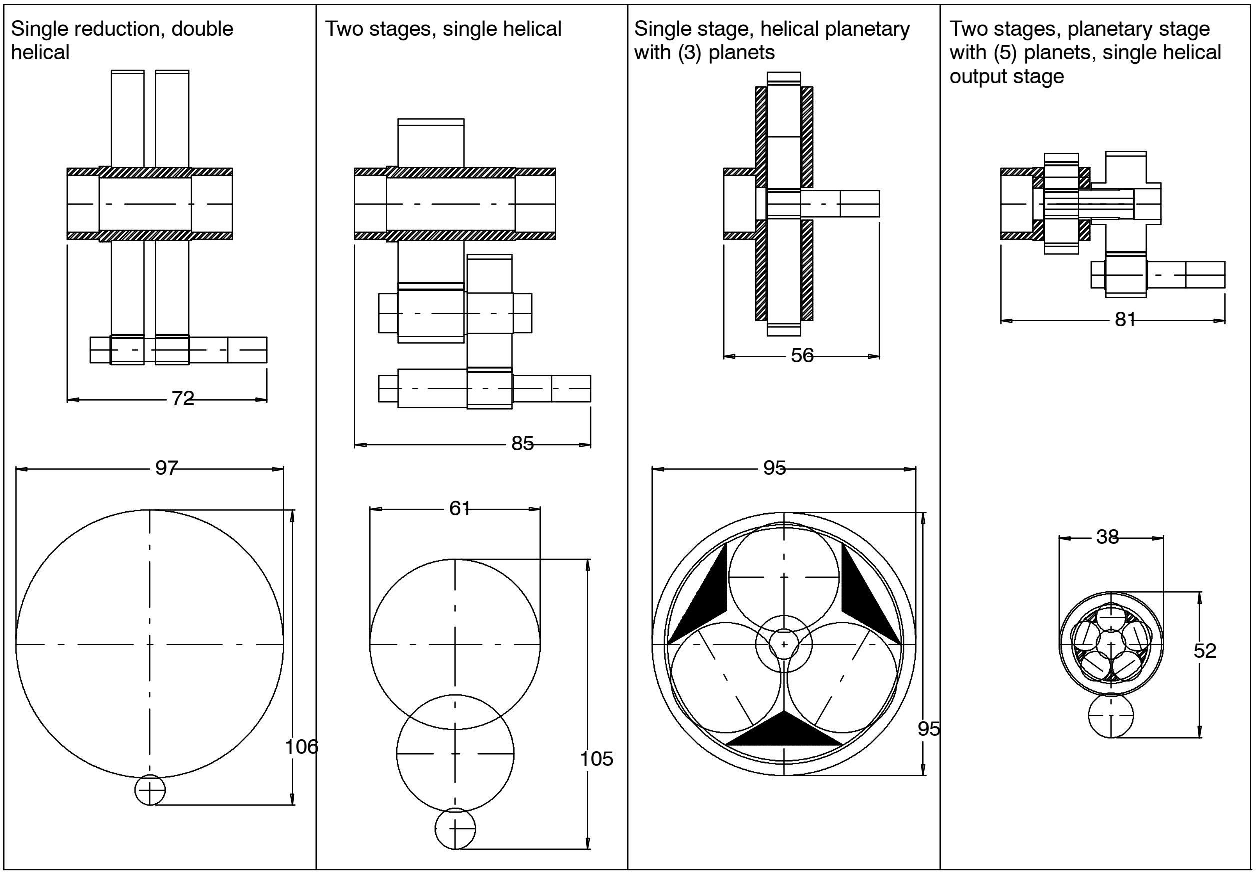

Figure 3. 10:1 gear train options

Conclusions

The popularity of planetary gear trains is very logical based upon this design exercise. For each output speed condition, a planetary design was "best" for minimum enclosed volume, lowest weight, and lowest relative cost. Once the overall ratio exceeds 40:1 the two planetary stage and one helical stage design was preferred over the one planetary stage and two helical stage design. Relative gearbox cost trends point to little influence by the overall gear ratio within a particular gearbox architecture over the 60:1 to 120:1 range. This makes sense as high volume gearbox costs are very dependent upon material cost and the weights of the planetary drives over the 60:1 to 120:1 ratio range are very similar.

Non-planetary designs may be of some interest in the future if a link between gearbox inertia and long service is found; i.e., the rotational inertia of the gearbox acts as a flywheel to smooth out load fluctuations. They might also offer a better opportunity to repair or rebuild the gearbox without removing it from the tower.

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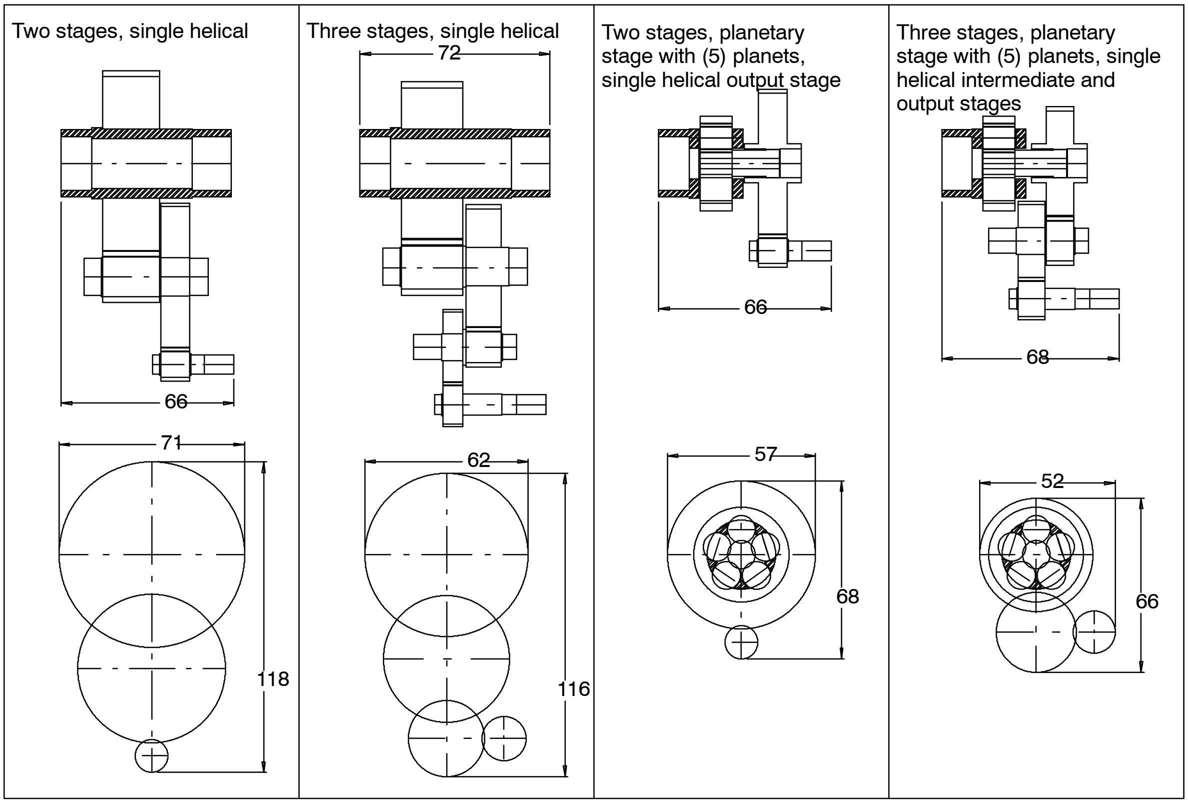

Figure 4. 20:1 gear train options

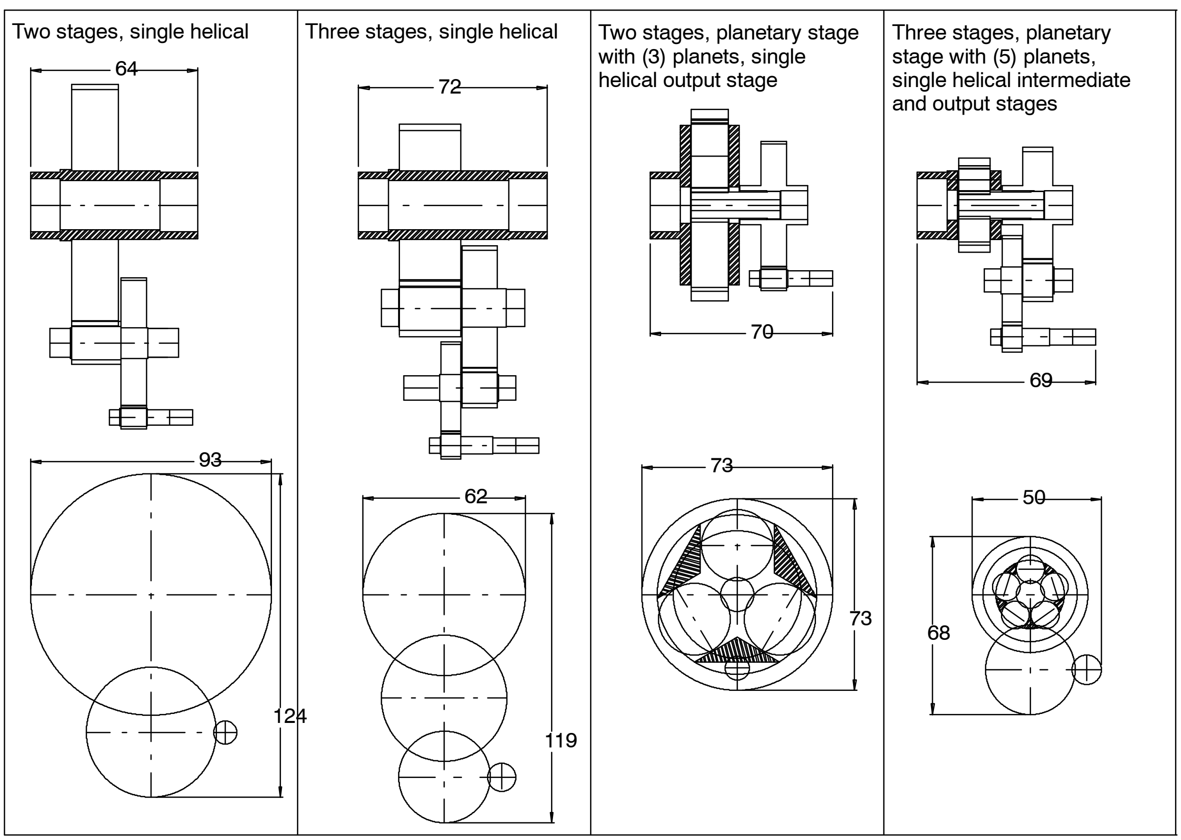

Figure 5. 40:1 gear train options

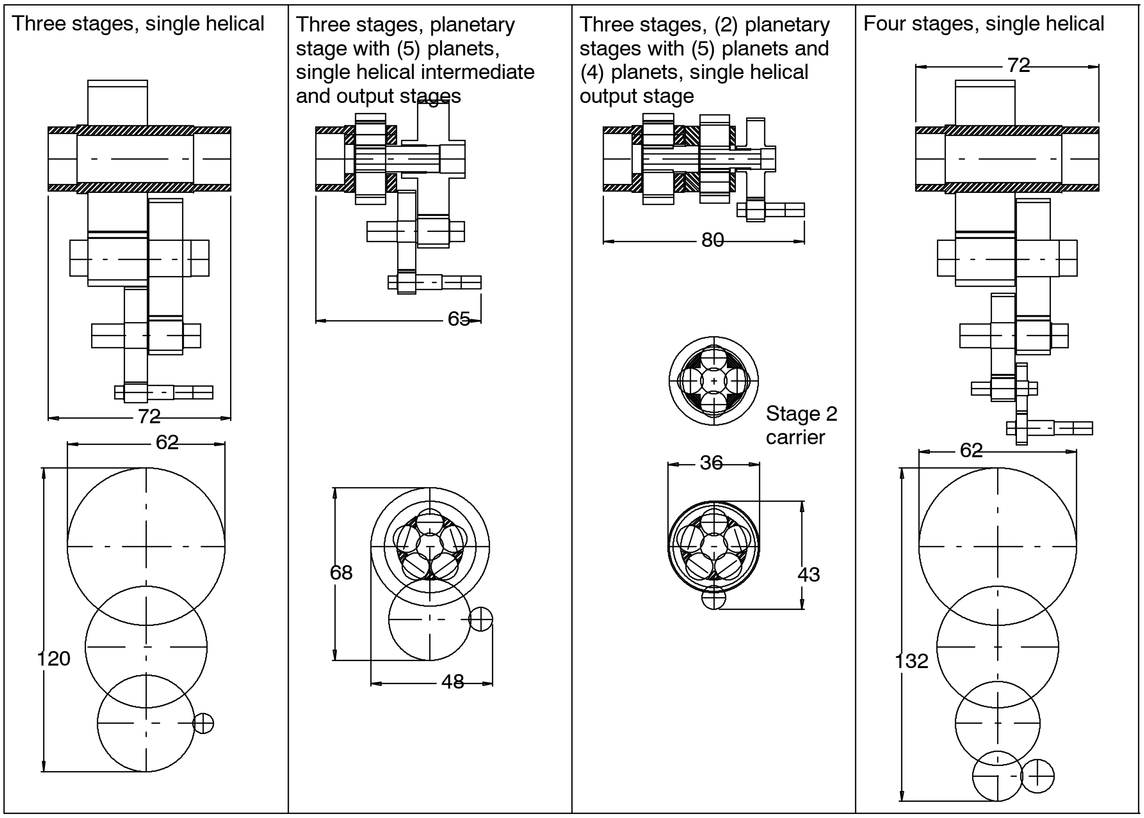

Figure 6. 60:1 gear train options

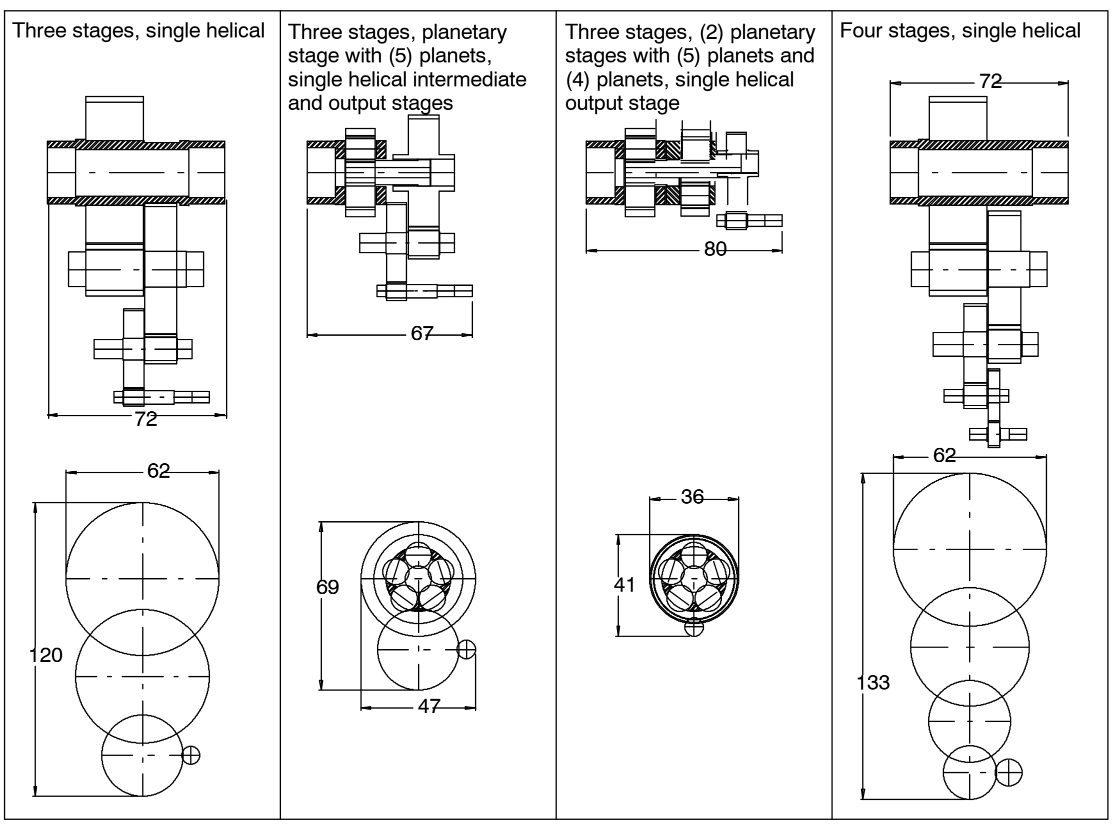

Figure 7. 80:1 gear train options

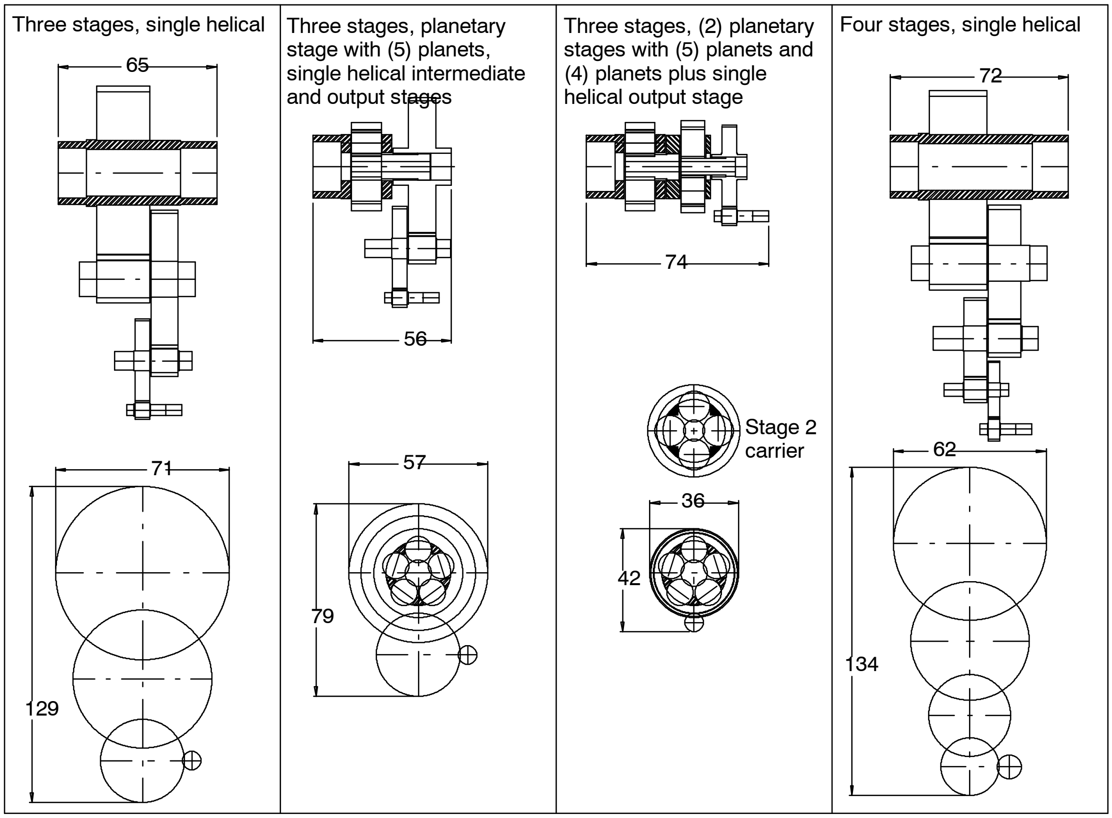

Figure 8. 100:1 gear train options

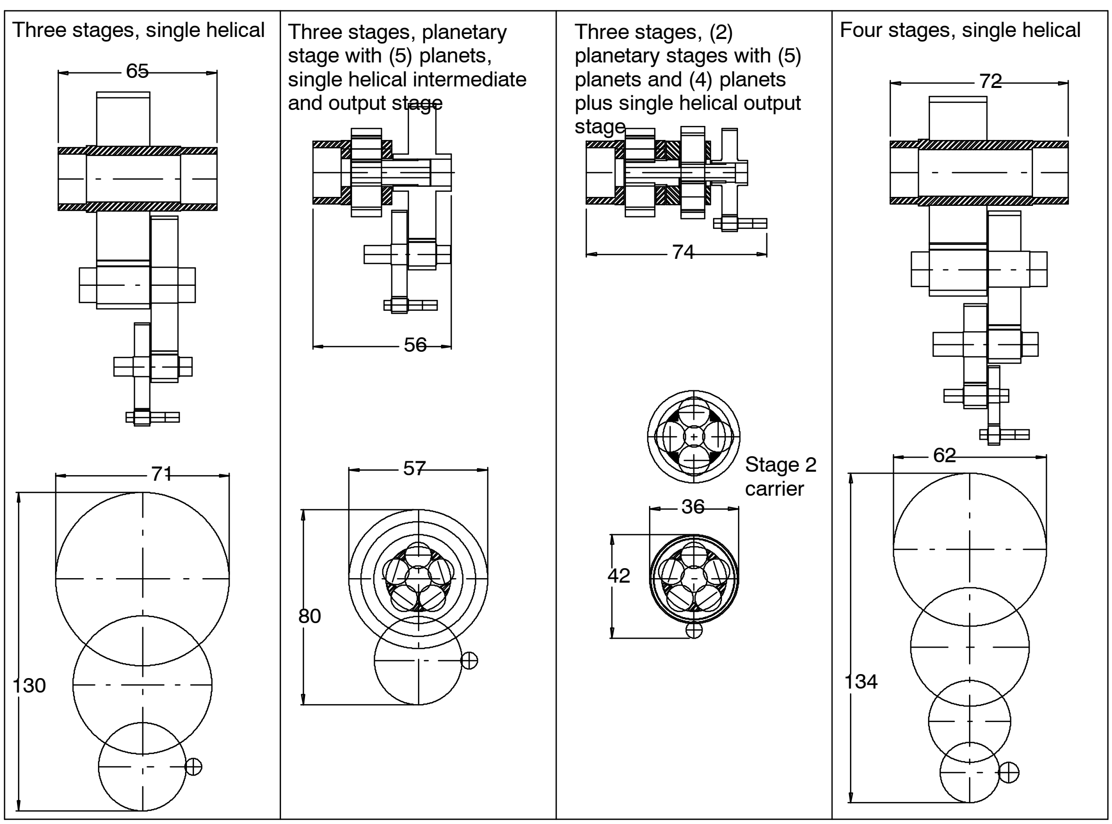

Figure 9. 120:1 gear train options

Table 2. Evaluation of design cases

Case ID	Gearbox type	Cost comparison	Volume comparison		Weight comparison	
		Relative cost	Relative volume	Approximate volume, ft^3	Relative weight	Estimated total weight, lb
10:ratios						
A	1DH	3.44	6.62	428	5.84	50,041
B	2HH	2.38	4.87	315	3.66	31,329
C	1P	2.04	4.52	292	4.12	35,289
D	2PH	1.12	1.43	93	1.25	10,724
20:ratios						
A	2HH	2.46	4.94	320	4.01	34,340
B	3HHH	2.74	4.63	300	3.85	32,964
C	2PH	1.21	2.29	148	1.52	13,054
D	3PHH	1.53	2.09	135	1.53	13,141
40:ratios						
A	2HH	2.82	6.60	427	4.91	42,050
B	3HHH	2.71	4.75	307	4.38	37,498
C	2PH	1.82	3.33	216	3.08	26,363
D	3PHH	1.45	2.10	136	1.52	13,039
60:ratios						
A	3HHH	2.74	4.79	310	3.95	33,813
B	3PHH	1.46	1.90	123	1.58	13,519
C	3PPH	1.25	1.11	72	1.28	10,989
D	4HHHH	2.97	5.27	341	3.95	33,795
80:ratios						
A	3HHH	2.75	4.79	310	4.00	34,251
B	3PHH	1.56	1.94	126	1.70	14,577
C	3PPH	1.26	1.06	68	1.32	11,345
D	4HHHH	2.94	5.31	344	3.94	33,781
100:ratios						
A	3HHH	2.77	5.32	345	4.19	35,894
B	3PHH	1.64	2.25	146	1.77	15,170
C	3PPH	1.20	1.00	65	1.28	10,985
D	4HHHH	2.94	5.35	346	3.95	33,862
120:ratios						
A	3HHH	2.79	5.36	347	4.23	36,268
B	3PHH	1.54	2.28	148	1.75	14,982
C	3PPH	1.00	1.00	65	1.00	8,565
D	4HHHH	2.64	5.35	346	4.13	35,357
Number of stages; DH = double helical, P = planetary, H = helical						

Table 3. 10:1 ratio – 150 RPM output speed design cases

	Case A	Case B		Case C	Case D	
	Stage 1	**Stage 1**	**Stage 2**	**Stage 1**	**Stage 1**	**Stage 2**
Number of stages	1	2		1	2	
Overall ratio	10	10		10	10	
Gear data summary						
Stage	1	1	2	1	1	2
Type	DH	External Helical		Planetary	Planetary	External helical
CD (inches)	52.6678	39.4254	27.1214	24.2133	9.7102	25.5917
CD (mm)	1338	1001	689	615	247	650
cd1/cd2	NA	NA	0.69	NA	NA	2.64
FW (total)	23.94	23.706	16.307	12.107	12.1378	14.624
FW/CD	0.45	0.60	0.60	0.50	1.25	0.57
F/D [per helix]	1.25	1.25	1.25	1.25	1.25	1.00
Np	18	19	19	18	20	22
Planet teeth	NA	NA	NA	72	20	NA
Number of planets	NA	NA	NA	3	5	NA
Ng	180	60	60	162	60	55
Ratio	10	3	3	10	4	3
NDP	2	1	1	2	2	2
Normal module	13	25	17	13	12	17
NPA	23	25	25	25	25	25
Helix	12	12	12	12	12	12
Pinion PD	10	19	13	10	10	15
Gear PD	96	60	41	39	10	37
Ring PD	NA	NA	NA	87	29.1306	NA
Pinion OD	11	21	15	11	11	16
Gear OD	97	61	42	40	11	38
Ring OD	NA	NA	NA	95	36	NA
Ring ID	NA	NA	NA	86	28	
X1	0.20	0.22	0.22	0.00	0.00	0.20
Mp	1.50	1.41	1.41	1.44	1.37	1.42
Mf (per helix)	1.52	1.61	1.61	1.52	1.69	1.49
Rating summary						
RDC HP	4,023	4,023	4,023	4,023	4,023	4,023
RDC kW	3,000	3,000	3,000	3,000	3,000	3,000
Pinion rpm	150	47.37	150	150	60.00	150
Cm	1.3	1.3	1.25	1.3	1.3	1.22
Number of meshes	1	1	1	3	5	1
Mesh factor	1	1	1	2.7	4.5	1
PacP	4,224	4,024	4,025	4,094	4,100	4,073
PacG	4,696	4,243	4,243	4,590	4,415	4,248
PatP	4,089	4,810	4,969	4,922	7,380	4,755
PatG	4,674	4,837	4,997	4,299	5,316	4,702
SF(dur)	1.05	1.00	1.00	1.02	1.02	1.01
SF(str)	1.02	1.20	1.24	1.22	1.32	1.18
Number of geared parts	2	4		5	9	

Table 4. 20:1 ratio – 300 RPM output speed design cases

	Case A		Case B			Case C		Case D		
	Stage 1	Stage 2	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 1	Stage 2	Stage 3
Number of stages	2		3			2		2		
Overall ratio	20		20			20		20		
Gear data summary										
Stage	1	2	1	2	3	1	2	1	2	3
Type	External helical		External helical			Planetary	External helical	Planetary	External helical	
CD (inches)	43.3237	33.3353	39.7666	30.2992	21.9501	9.7102	33.3353	9.7102	29.5268	22.0967
CD (mm)	1100	847	1010	770	558	247	847	247	750	561
cd1/cd2	NA	0.77	NA	NA	0.55	NA	3.43	NA	NA	2.28
FW	21.6805	11.112	23.546	13.466	7.683	12.1378	11.112	12.1378	10.334	10.414
FW/CD	0.50	0.33	0.59	0.44	0.35	1.25	0.33	1.25	0.35	0.47
F/D	1.25	1.00	1.25	1.00	0.49	1.25	1.00	1.25	0.61	0.71
Np	18	18	18	18	31	20	18	20	22	25
Planet teeth	NA	NA	NA	NA	NA	20	NA	20	NA	NA
Number of planets	NA	NA	NA	NA	NA	5	NA	5	NA	NA
Ng	72	90	58	63	55	60	90	60	55	50
Ratio	4	5	3	4	2	4	5	4	3	2
NDP	1	2	1	1	2	2	2	2	1	2
Normal module	24	15	26	18	12	12	15	12	19	14
NPA	25	25	25	25	23	25	25	25	25	25
Helix	12	15	12	15	18	12	15	12	18	20
Pinion PD	17	11	19	13	16	10	11	10	17	15
Gear PD	69	56	61	47	28	10	56	10	42	29
Ring PD	NA	NA	NA	NA	NA	29.1306	NA	29.1306	NA	NA
Pinion OD	20	13	21	15	17	11	13	11	19	16
Gear OD	71	57	62	48	29	11	57	11	42	30
Ring OD	NA	NA	NA	NA	NA	36	NA	36	NA	NA
Ring ID	NA	NA	NA	NA	NA	28	NA	28	NA	NA
X1	0.20	0.20	0.20	0.20	0.13	0.00	0.20	0.00	0.20	0.17
Mp	1.41	1.39	1.40	1.38	1.48	1.37	1.39	1.37	1.37	1.36
Mf	1.52	1.54	1.52	1.54	1.56	1.69	1.54	1.69	1.39	1.52
Rating summary										
RDC HP	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023
RDC kW	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Pinion rpm	60.00	300	48.33	52.50	86	60.00	300	60.00	150.00	300.00
Cm	1.3	1.25	1.3	1.25	1.22	1.3	1.25	1.3	1.25	1.22
Number of meshes	1	1	1	1	1	5	1	5	1	1
Mesh factor	1	1	1	1	1	4.5	1	4.5	1	1
PacP	4,087	4,256	4,044	4,060	4,415	4,100	4,256	4,100	4,060	4,114
PacG	4,356	4,583	4,268	4,301	4,533	4,415	4,583	4,415	4,234	4,248
PatP	4,852	5,083	5,003	5,158	4,162	7,380	5,083	7,380	4,806	4,748
PatG	5,154	5,444	5,126	5,328	4,140	5,316	5,444	5,316	4,747	4,655
SF(dur)	1.02	1.06	1.01	1.01	1.10	1.02	1.06	1.02	1.01	1.02
SF(str)	1.21	1.26	1.24	1.28	1.03	1.32	1.26	1.32	1.19	1.18
Number of geared parts	4		6			9		11		

Table 5. 40:1 ratio – 600 RPM output speed design cases

	Case A		Case B			Case C		Case D		
	Stage 1	**Stage 2**	**Stage 1**	**Stage 2**	**Stage 3**	**Stage 1**	**Stage 2**	**Stage 1**	**Stage 2**	**Stage 3**
Number of stages	2		3			2		3		
Overall ratio	40		40			40		40		
Gear data summary										
Stage	1	2	1	2	3	1	2	1	2	3
Type	External helical		External helical			Planetary	External helical	Planetary	External helical	
CD (inches)	52.8838	28.507	39.7666	30.2992	22.084	18.9089	28.2145	9.7102	28.8384	21.752
CD (mm)	1343	724	1010	770	561	480	717	247	732	553
cd1/cd2	NA	0.54	NA	0.76	0.56	NA	NA	NA	NA	0.75
FW	18.029	10	23.546	13.466	7.73	14.182	9.875	12.1378	11.535	7.613
FW/CD	0.34	0.34	0.59	0.44	0.35	0.75	0.35	1.25	0.40	0.35
F/D	1.25	1.25	1.25	1.00	0.80	1.25	1.22	1.25	0.80	0.76
Np	18	19	18	18	22	18	18	20	19	18
Planet teeth	NA	NA	NA	NA	NA	42	NA	20	NA	NA
Number of planets	NA	NA	NA	NA	NA	3	NA	5	NA	NA
Ng	114	120	58	63	78	102	108	60	57	60
Ratio	6	6	3	4	4	7	6	4	3	3
NDP	1	3	1	1	2	2	2	2	1	2
Normal module	20	10	26	18	11	16	11	12	18	13
NPA	25	25	25	25	23	25	25	25	25	20
Helix	12	15	12	15	18	12	20	12	18	20
Pinion PD	14	8	19	13	10	11	8	10	14	10
Gear PD	91	49	61	47	34	26	8	10	43	33
Ring PD	NA	NA	NA	NA	NA	64.2902	NA	29.1306	NA	NA
Pinion OD	16	9	21	15	11	13	9	11	16	11
Gear OD	93	50	62	48	35	27	49	11	44	34
Ring OD	NA	NA	NA	NA	NA	73	NA	36	NA	NA
Ring ID	NA	NA	NA	NA	NA	63		28		
X1	0.25	0.25	0.20	0.20	0.20	0.00	0.25	0.00	0.20	0.22
Mp	1.41	1.39	1.40	1.38	1.55	1.39	1.33	1.37	1.36	1.48
Mf	1.52	2.03	1.52	1.54	1.81	1.52	2.55	1.69	1.57	1.58
Rating summary										
RDC HP	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023
RDC kW	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Pinion rpm	95.00	600	48.33	52.50	171	100.00	600	60.00	180.00	600.00
Cm	1.3	1.25	1.3	1.25	1.22	1.3	1.25	1.3	1.25	1.22
Number of meshes	1	1	1	1	1	3	1	5	1	1
Mesh factor	1	1	1	1	1	2.7	1	4.5	1	1
PacP	4,045	4,109	4,044	4,060	4,295	4,066	4,168	4,100	4,062	4,135
PacG	4,404	4,472	4,268	4,301	4,552	4,446	4,526	4,415	4,273	4,370
PatP	4,459	4,633	5,003	5,158	4,176	5,852	4,965	7,380	5,178	4,778
PatG	4,735	4,891	5,126	5,328	4,319	4,078	5,202	5,316	5,231	4,856
SF(dur)	1.01	1.02	1.01	1.01	1.07	1.01	1.04	1.02	1.01	1.03
SF(str)	1.11	1.15	1.24	1.28	1.07	1.01	1.23	1.32	1.29	1.19
Number of geared parts	4		6			8		10		

Table 6. 60:1 ratio – 900 RPM output speed design cases

	Case A			Case B			Case C			Case D			
	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 4
Number of stages	3			3			3			4			
Overall ratio	60			60			60			60			
Gear data summary													
Stage	1	2	3	1	2	3	1	2	3	1	2	3	4
Type	External helical			Planetary	External helical		Planetary		External helical	External helical			
CD (inches)	39.7666	30.2992	22.5313	9.7102	28.7577	20.1552	9.7102	9.2473	20.1552	39.7666	30.2992	20.964	15.7452
CD (mm)	1010	770	572	247	730	512	247	235	512	1010	770	532	400
cd1/cd2	NA	0.76	0.57	NA	NA	0.70	NA	NA	NA	NA	0.76	0.69	0.75
FW	23.546	13.466	8.919	12.1378	12.478	7.054	12.1378	11.559	7.054	23.546	13.466	9.317	4.328
FW/CD	0.59	0.44	0.40	1.25	0.43	0.35	1.25	1.25	0.35	0.59	0.44	0.44	0.27
F/D	1.25	1.00	1.25	1.25	1.08	0.83	1.25	1.25	0.83	1.25	1.00	1.00	0.35
Np	18	18	19	20	18	20	20	18	20	18	18	18	25
Planet teeth	NA	NA	NA	20	NA	NA	20	18	NA	NA	NA	NA	NA
Number of planets	NA	NA	NA	5	NA	NA	5	4	NA	NA	NA	NA	NA
Ng	58	63	101	60	72	75	60	54	75	58	63	63	38
Ratio	3	4	5	4	4	4	4	4	4	3	4	4	2
NDP	1	1	3	2	2	2	2	2	2	1	1	2	2
Normal module	26	18	9	12	16	10	12	13	10	26	18	13	12
NPA	25	25	20	25	25	23	25	20	23	25	25	25	20
Helix	12	15	18	12	15	18	12	12	18	12	15	15	20
Pinion PD	19	13	7	10	12	8	10	9	8	19	13	9	12
Gear PD	61	47	38	10	46	32	10	9	32	61	47	33	19
Ring PD	NA	NA	NA	29.1306	NA	NA	29.1306	27.741	NA	NA	NA	NA	NA
Pinion OD	21	15	8	11	13	9	11	10	9	21	15	11	13
Gear OD	62	48	38	11	47	32	11	10	32	62	48	33	20
Ring OD	NA	NA	NA	36	NA	NA	36	35	NA	NA	NA	NA	NA
Ring ID	NA	NA	NA	28	NA	NA	28	27	NA	NA	NA	NA	NA
X1	0.20	0.20	0.25	0.00	0.25	0.25	0.00	0.00	0.25	0.20	0.20	0.20	0.00
Mp	1.40	1.38	1.52	1.37	1.38	1.43	1.37	1.49	1.43	1.40	1.38	1.38	1.52
Mf	1.52	1.54	2.46	1.69	1.67	1.72	1.69	1.52	1.72	1.52	1.54	1.54	1.00
Rating summary													
RDC HP	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023
RDC kW	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Pinion rpm	48.33	169.17	899	60.00	240.00	900.00	60.00	240.00	900.00	48.33	169.17	592.08	900
Cm	1.3	1.25	1.22	1.3	1.25	1.22	1.3	1.25	1.22	1.3	1.25	1.22	1.22
Number of meshes	1	1	1	5	1	1	5	4	1	1	1	1	1
Mesh factor	1	1	1	4.5	1	1	4.5	3.6	1	1	1	1	1
PacP	4,044	4,060	4,220	4,100	4,032	4,350	4,100	4,109	4,350	4,044	4,060	4,452	4,185
PacG	4,268	4,301	4,557	4,415	4,297	4,623	4,415	4,380	4,623	4,268	4,301	4,716	4,266
PatP	5,003	5,158	4,272	7,380	5,057	4,897	7,380	7,187	4,897	5,003	5,158	5,862	4,701
PatG	5,126	5,328	4,478	5,316	5,118	4,926	5,316	5,157	4,926	5,126	5,328	6,053	5,020
SF(dur)	1.01	1.01	1.05	1.02	1.00	1.08	1.02	1.02	1.08	1.01	1.01	1.11	1.04
SF(str)	1.24	1.28	1.11	1.32	1.26	1.22	1.32	1.79	1.22	1.24	1.28	1.46	1.25
Number of geared parts	6			10			10			8			

Table 7. 80:1 ratio - 1200 RPM output speed design cases

	Case A			Case B			Case C			Case D			
	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 4
Number of stages	3			3			3			4			
Overall ratio	80.5555555			80			80			80.2177419			
Gear data summary													
Stage	1	2	3	1	2	3	1	2	3	1	2	3	4
Type	External helical			Planetary	External helical		Planetary		External helical	External helical			
CD (inches)	39.7666	32.2628	19.5925	9.7102	28.7577	19.5925	9.7102	9.2473	19.5925	39.7666	30.2992	20.964	15.7558
CD (mm)	1010.07164	819.47512	497.6495	246.63908	730.44558	497.6495	246.63908	234.88142	497.6495	1010.07164	769.59968	532.4856	400.19732
cd1/cd2	NA	0.81130395	0.49268733	NA	NA	2.01772363	NA	NA	2.01772363	NA	0.76192583	0.69189945	0.75156458
FW	23.546	12.905	8.164	12.1378	12.478	8.164	12.1378	11.559	8.164	23.546	13.466	9.317	4.727
FW/CD	0.59210493	0.39999628	0.41669005	1.25000514	0.43390118	0.41669005	1.25000514	1.24998648	0.41669005	0.59210493	0.44443417	0.44442854	0.3000165
F/D	1.24996682	1.19998512	1.25007656	1.25000514	1.08475106	1.25007656	1.25000514	1.24998648	1.25007656	1.24996682	0.99996287	0.99994633	0.45479473
Np	18	18	18	20	18	18	20	18	18	18	18	18	31
Planet teeth	NA	NA	NA	20	NA	NA	20	18	NA	NA	NA	NA	NA
Number of planets	NA	NA	NA	5	NA	NA	5	4	NA	NA	NA	NA	NA
Ng	58	90	90	60	72	90	60	54	90	58	63	63	63
Ratio	3.22222222	5	5	4	4	5	4	4	5	3.22222222	3.5	3.5	2.03225806
NDP	0.9769	1.7328	2.898	2.1057	1.62	2.898	2.1057	1.99	2.898	0.9769	1.3838	2	3.174
Normal module	26.0006141	14.6583564	8.76466528	12.062497	15.6790123	8.76466528	12.062497	12.763819	8.76466528	26.0006141	18.3552536	12.7	8.00252047
NPA	25	25	20	25	25	20	25	20	20	25	25	25	22.5
Helix	12	15	18	12	15	18	12	12	18	12	15	15	20
Pinion PD	18.8373	10.7543	6.5308	9.7102	11.5031	6.5308	9.7102	9.2473	6.5308	18.8373	13.4665	9.3175	10.3937
Gear PD	60.6979	53.7713	32.6541	9.7102	46.0123	32.6541	9.7102	9.2473	32.6541	60.6979	47.1328	32.6112	21.1226
Ring PD	NA	NA	NA	29.1306	NA	NA	29.1306	27.741	NA	NA	NA	NA	NA
Pinion OD	21.294	12.197	7.3945	10.66	13.046	7.3945	10.66	10.2523	7.3945	21.294	15.201	10.517	11.0222
Gear OD	62.335	54.637	33.1717	10.66	46.938	33.1717	10.66	10.2523	33.1717	62.335	48.288	33.411	21.7494
Ring OD	NA	NA	NA	36.1591415	NA	NA	36.1591415	35.1781859	NA	NA	NA	NA	NA
Ring ID	NA	NA	NA	28.2757773	NA	NA	28.2757773	26.8364773	NA	NA	NA	NA	NA
X1	0.2	0.25	0.25	0	0.25	0.25	0	0	0.25	0.2	0.2	0.2	0
Mp	1.4023	1.3804	1.5135	1.3716	1.3759	1.5135	1.3716	1.4888	1.5135	1.4023	1.3809	1.381	1.4751
Mf	1.5223	1.8404	2.3272	1.6915	1.6654	2.3272	1.6915	1.5223	2.3272	1.5223	1.5352	1.5352	1.6334
Rating summary													
RDC HP	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023
RDC kW	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511
Pinion rpm	48.3333333	241.666666	1208.33333	60	240	1200	60	240	1200	48.3333333	169.166666	592.083333	1203.26612
Cm	1.3	1.25	1.2	1.3	1.25	1.2	1.3	1.25	1.2	1.3	1.25	1.22	1.22
Number of meshes	1	1	1	5	1	1	5	4	1	1	1	1	1
Mesh factor	1	1	1	4.5	1	1	4.5	3.6	1	1	1	1	1
PacP	4044.46	4091.8	4240	4100.13	4031.84	4240	4100.13	4109.04	4240	4044.46	4060.16	4451.97	4485.73
PacG	4268.11	4406.23	4565.82	4415.22	4297.32	4565.82	4415.22	4379.616	4565.82	4268.11	4301.01	4716.06	4634.47
PatP	5002.61	4893.6	4564.05	7379.73	5057.24	4564.05	7379.73	7187.256	4564.05	5002.61	5157.85	5862.14	4068.58
PatG	5126.1	5071.02	4767.33	5315.94	5118.26	4767.33	5315.94	5156.784	4767.33	5126.1	5328.07	6053.07	4413.64
SF(dur)	1.00533432	1.01710166	1.05393984	1.01917225	1.00219736	1.05393984	1.01917225	1.02138702	1.05393984	1.00533432	1.00923688	1.10662938	1.11502112
SF(str)	1.24350236	1.21640566	1.13448918	1.32138702	1.25708177	1.13448918	1.32138702	1.78654138	1.13448918	1.24350236	1.28209047	1.45715635	1.09710166
Number of geared parts	6			10			14			8			

Table 8. 100:1 ratio – 1500 RPM output speed design cases

	Case A			Case B			Case C			Case D			
	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 4
Number of stages	3			3			3			4			
Overall ratio	100			100			100			100			
Gear data summary													
Stage	1	2	3	1	2	3	1	2	3	1	2	3	4
Type	External helical			Planetary	External helical		Planetary		External helical	External helical			
CD (inches)	43.3237	33.3353	20.1634	9.7102	33.3353	21.555	9.7102	9.7821	21.555	39.7666	30.2992	20.964	16.0727
CD (mm)	1100	847	512	247	847	547	247	248	547	1010	770	532	408
cd1/cd2	NA	0.77	0.47	NA	NA	0.65	NA	NA	2.22	NA	0.76	0.69	0.77
FW	21.6805	11.112	6.05	12.1378	11.112	6.467	12.1378	9.782	6.467	23.546	13.466	9.317	4.8218
FW/CD	0.50	0.33	0.30	1.25	0.33	0.30	1.25	1.00	0.30	0.59	0.44	0.44	0.30
F/D	1.25	1.00	0.90	1.25	1.00	0.90	1.25	1.25	0.90	1.25	1.00	1.00	0.53
Np	18	18	18	20	18	18	20	18	18	18	18	18	28
Planet teeth	NA	NA	NA	20	NA	NA	20	27	NA	NA	NA	NA	NA
Number of planets	NA	NA	NA	5	NA	NA	5	4	NA	NA	NA	NA	NA
Ng	72	90	90	60	90	90	60	72	90	58	63	63	71
Ratio	4	5	5	4	5	5	4	5	5	3	4	4	3
NDP	1	2	3	2	2	3	2	2	3	1	1	2	3
Normal module	24	15	9	12	15	10	12	11	10	26	18	13	8
NPA	25	25	25	25	25	23	25	25	23	25	25	25	23
Helix	12	15	20	12	15	20	12	12	20	12	15	15	20
Pinion PD	17	11	7	10	11	7	10	8	7	19	13	9	9
Gear PD	69	56	34	10	56	36	10	12	36	61	47	33	23
Ring PD	NA	NA	NA	29.1306	NA	NA	29.1306	31.3054	NA	NA	NA	NA	NA
Pinion OD	20	13	8	11	13	8	11	9	8	21	15	11	10
Gear OD	71	57	34	11	57	36	11	13	36	62	48	33	24
Ring OD	NA	NA	NA	36.1591415	NA	NA	36.1591415	37.5992549	NA	NA	NA	NA	NA
Ring ID	NA	NA	NA	28	NA	NA	28	31	NA	NA	NA	NA	NA
X1	0.20	0.20	0.25	0.00	0.20	0.25	0.00	0.00	0.25	0.20	0.20	0.20	0.00
Mp	1.41	1.39	1.33	1.37	1.39	1.40	1.37	1.39	1.40	1.40	1.38	1.38	1.47
Mf	1.52	1.54	1.88	1.69	1.54	1.88	1.69	1.52	1.88	1.52	1.54	1.54	1.72
Rating summary													
RDC HP	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023
RDC kW	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Pinion rpm	60.00	300	1500	60.00	300	1500.00	60.00	300.00	1500.00	48.33	169.17	592.08	1501
Cm	1.3	1.25	1.2	1.3	1.25	1.2	1.3	1.25	1.2	1.3	1.25	1.22	1.22
Number of meshes	1	1	1	5	1	1	5	4	1	1	1	1	1
Mesh factor	1	1	1	4.5	1	1	4.5	3.6	1	1	1	1	1
PacP	4,087	4,256	4,252	4,100	4,256	4,226	4,100	4,133	4,226	4,044	4,060	4,452	4,710
PacG	4,356	4,583	4,579	4,415	4,583	4,551	4,415	4,488	4,551	4,268	4,301	4,716	4,916
PatP	4,852	5,083	5,370	7,380	5,083	4,878	7,380	6,815	4,878	5,003	5,158	5,862	4,326
PatG	5,154	5,444	5,539	5,316	5,444	5,059	5,316	5,338	5,059	5,126	5,328	6,053	4,792
SF(dur)	1.02	1.06	1.06	1.02	1.06	1.05	1.02	1.03	1.05	1.01	1.01	1.11	1.17
SF(str)	1.21	1.26	1.38	1.32	1.26	1.21	1.32	1.69	1.21	1.24	1.28	1.46	1.19
Number of geared parts	6			10			14			8			

Table 9. 120:1 ratio – 1800 RPM output speed design cases

	Case A			Case B			Case C			Case D			
	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 1	Study 2	Study 3	Study 4
Number of stages	3			3			3			4			
Overall ratio	20			120			120			119.995555			
Gear data summary													
Stage	1	2	3	1	2	3	1	2	3	1	2	3	4
Type	External helical	External helical	External helical	Planetary	External helical	External helical	Planetary	Planetary	External helical	External helical	External helical	External helical	External helical
CD (inches)	43.3237	33.3353	20.6923	9.7102	33.3353	20.6923	9.7102	9.7821	20.6923	39.7666	30.2992	20.964	15.8682
CD (mm)	1100.42198	846.71662	525.58442	246.63908	846.71662	525.58442	246.63908	248.46534	525.58442	1010.07164	769.59968	532.4856	403.05228
cd1/cd2	NA	0.7694472	0.47762079	NA	NA	2.13098597	NA	NA	2.13098597	NA	0.76192583	0.69189945	0.75692615
FW	21.6805	11.112	6.208	12.1378	11.112	6.208	12.1378	9.782	6.208	23.546	13.466	9.317	4.76
FW/CD	0.50043048	0.33334033	0.30001498	1.25000514	0.33334033	0.30001498	1.25000514	0.99998977	0.30001498	0.59210493	0.44443417	0.44442854	0.29997101
F/D	1.25002162	1.00001799	1.05004989	1.25000514	1.00001799	1.05004989	1.25000514	1.24987222	1.05004989	1.24996682	0.99996287	0.99994633	0.60593716
Np	18	18	18	20	18	18	20	18	18	18	18	18	25
Planet teeth	NA	NA	NA	20	NA	NA	20	27	NA	NA	NA	NA	NA
Number of planets	NA	NA	NA	5	NA	NA	5	4	NA	NA	NA	NA	NA
Ng	72	90	108	60	90	108	60	72	108	58	63	63	76
Ratio	4	5	6	4	5	6	4	5	6	3.22222222	3.5	3.5	3.04
NDP	1.061	1.6771	3.24	2.1057	1.6771	3.24	2.1057	2.3515	3.24	0.9769	1.3838	2	3.3867
Normal module	23.9396795	15.1451911	7.83950617	12.062497	15.1451911	7.83950617	12.062497	10.8016159	7.83950617	26.0006141	18.3552536	12.7	7.49992618
NPA	25	25	25	25	25	25	25	25	25	25	25	25	22.5
Helix	12	15	20	12	15	20	12	12	20	12	15	15	20
Pinion PD	17.3441	11.1118	5.9121	9.7102	11.1118	5.9121	9.7102	7.8264	5.9121	18.8373	13.4665	9.3175	7.8556
Gear PD	69.3766	55.5588	35.4726	9.7102	55.5588	35.4726	9.7102	11.7395	35.4726	60.6979	47.1328	32.6112	23.8809
Ring PD	NA	NA	NA	29.1306	NA	NA	29.1306	31.3054	NA	NA	NA	NA	NA
Pinion OD	19.5894	12.542	6.7454	10.66	12.542	6.7454	10.66	8.6762	6.7454	21.294	15.201	10.517	8.4461
Gear OD	70.8244	56.513	35.8738	10.66	56.513	35.8738	10.66	12.5891	35.8738	62.335	48.288	33.411	24.471
Ring OD	NA	NA	NA	36.1591415	NA	NA	36.1591415	37.5992549	NA	NA	NA	NA	NA
Ring ID	NA	NA	NA	28.2757773	NA	NA	28.2757773	30.5399311	NA	NA	NA	NA	NA
X1	0.2	0.2	0.35	0	0.2	0.35	0	0	0.35	0.2	0.2	0.2	0
Mp	1.4102	1.3887	1.3161	1.3716	1.3887	1.3161	1.3716	1.3857	1.3161	1.4023	1.3809	1.381	1.4659
Mf	1.5223	1.5353	2.1898	1.6915	1.5353	2.1898	1.6915	1.5222	2.1898	1.5223	1.5352	1.5352	1.755
Rating summary													
RDC HP	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023	4023
RDC kW	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511	2999.9511
Pinion rpm	60	300	1800	60	300	1800	60	300	1800	48.3333333	169.166666	592.083333	1799.93333
Cm	1.3	1.25	1.2	1.3	1.25	1.2	1.3	1.25	1.2	1.3	1.25	1.22	1.22
Number of meshes	1	1	1	5	1	1	5	4	1	1	1	1	1
Mesh factor	1	1	1	4.5	1	1	4.5	3.6	1	1	1	1	1
PacP	4086.89	4256.32	4285.27	4100.13	4256.32	4285.27	4100.13	4132.84	4285.27	4044.46	4060.16	4451.97	4349.89
PacG	4356	4583.39	4653.43	4415.22	4583.39	4653.43	4415.22	4487.9	4653.43	4268.11	4301.01	4716.06	4578.15
PatP	4851.6	5082.81	5334.57	7379.73	5082.81	5334.57	7379.73	6815.2	5334.57	5002.61	5157.85	5862.14	4212.49
PatG	5154.18	5444.37	5271.35	5315.94	5444.37	5271.35	5315.94	5337.94	5271.35	5126.1	5328.07	6053.07	4777.99
SF(dur)	1.01588118	1.05799652	1.06519264	1.01917225	1.05799652	1.06519264	1.01917225	1.027303	1.06519264	1.00533432	1.00923688	1.10662938	1.08125528
SF(str)	1.20596569	1.26343773	1.31030325	1.32138702	1.26343773	1.31030325	1.32138702	1.69405915	1.31030325	1.24350236	1.28209047	1.45715635	1.1876684
Number of geared parts	6			10			14			8			

Table 10. 80:1 ratio – 1200 RPM output speed design cases, effect of number of planets on stage 1 size

	Case A, 3 planets			Case B, 4 planets			Case C, 5 planets		
	Study 1	**Study 2**	**Study 3**	**Study 1**	**Study 2**	**Study 3**	**Study 1**	**Study 2**	**Study 3**
Number of stages	3			3			3		
Overall ratio	80			80			80		
Gear data summary									
Stage	1	2	3	1	2	3	1	2	3
Type	Planetary		External helical	Planetary		External helical	Planetary		External helical
CD (inches)	15.7593	9.2473	19.5925	14.3823	9.2473	19.5925	9.7102	9.2473	19.5925
CD (mm)	400	235	498	365	235	498	247	235	498
cd1/cd2	NA	NA	1.24	NA	NA	1.36	NA	NA	2.02
FW	19.699	11.559	8.164	17.978	11.559	8.164	12.1378	11.559	8.164
FW/CD	1.25	1.25	0.42	1.25	1.25	0.42	1.25	1.25	0.42
F/D	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
Np	18	18	18	18	18	18	20	18	18
Planet teeth	18	18	NA	18	18	NA	20	18	NA
Number of planets	3	4	NA	4	4	NA	5	4	NA
Ng	54	54	90	54	54	90	60	54	90
Ratio	4	4	5	4	4	5	4	4	5
NDP	1	2	3	1	2	3	2	2	3
Normal module	22	13	9	20	13	9	12	13	9
NPA	25	20	20	25	20	20	25	20	20
Helix	12	12	18	12	12	18	12	12	18
Pinion PD	16	9	7	14	9	7	10	9	7
Gear PD	16	9	33	14	9	33	10	9	33
Ring PD	47.2778899	27.741	NA	43.146848	27.741	NA	29.1306623	27.741	NA
Pinion OD	17	10	7	16	10	7	11	10	7
Gear OD	17	10	33	16	10	33	11	10	33
Ring OD	60	35	NA	55	35	NA	36	35	NA
Ring ID	46	27	NA	42	27	NA	28	27	NA
X1	0.00	0.00	0.25	0.00	0.00	0.25	0.00	0.00	0.25
Mp	1.49	1.49	1.51	1.49	1.49	1.51	1.37	1.49	1.51
Mf	1.52	1.52	2.33	1.52	1.52	2.33	1.69	1.52	2.33
Rating summary									
RDC HP	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023	4,023
RDC kW	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
Pinion rpm	60.00	240.00	1200.00	60.00	240.00	1200.00	60.00	240.00	1200.00
Cm	1.3	1.25	1.2	1.3	1.25	1.2	1.3	1.25	1.2
Number of meshes	3	4	1	4	4	1	5	4	1
Mesh factor	2.7	3.6	1	3.6	3.6	1	4.5	3.6	1
PacP	4,035	4,109	4,240	4,042	4,109	4,240	4,100	4,109	4,240
PacG	4,244	4,380	4,566	4,309	4,380	4,566	4,415	4,380	4,566
PatP	6,732	7,187	4,564	6,799	7,187	4,564	7,380	7,187	4,564
PatG	4,806	5,157	4,767	4,879	5,157	4,767	5,316	5,157	4,767
SF(dur)	1.00	1.02	1.05	1.00	1.02	1.05	1.02	1.02	1.05
SF(str)	1.19	1.79	1.13	1.21	1.79	1.13	1.32	1.79	1.13
Number of geared parts	5	6	2	6	6	2	7	6	2
Number of geared parts	13			14			15		