Ain Shams University
Faculty of Engineering
Design & Production Engineering Department

MECHANICS OF MACHINES (1)

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Syllabus

1. Mechanisms
2. Velocity and Acceleration
3. Equilibrium of Machines & Turning Moment Diagram(Flywheel)
4. Cams
5. Gear(Geometry and Train)
6. Balancing

References

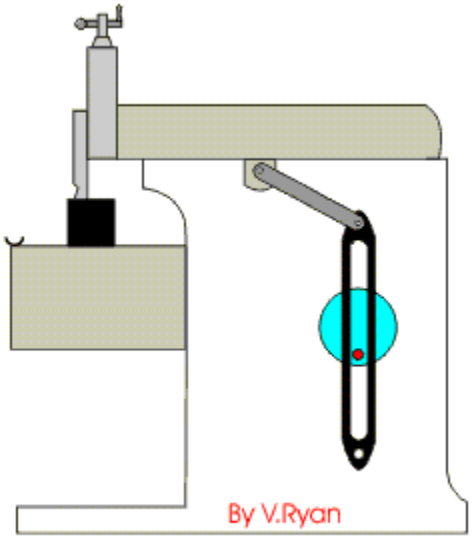
1. The theory of machines, T. Bevan
2. The theory of machines, P. L. Ballaney
3. The theory of machines, R. S. khurmi & J. K. Gupta
4. The theory of machines(worked example), Ryder
5. The theory of machines(solved example), Onvoner
6. The theory of machines, W. Grean
7. Mechanics of machine, Ham & Crane
8. Mechanics for engineering, Duncan & Macmillan
9. Mechanics of machine, Hannah & Stephens

CHAPTER 1
MECHANISMS

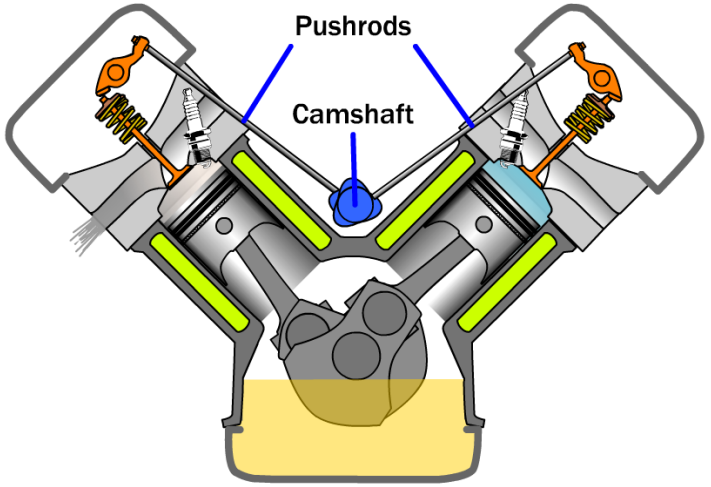
Definitions

Theory of machines

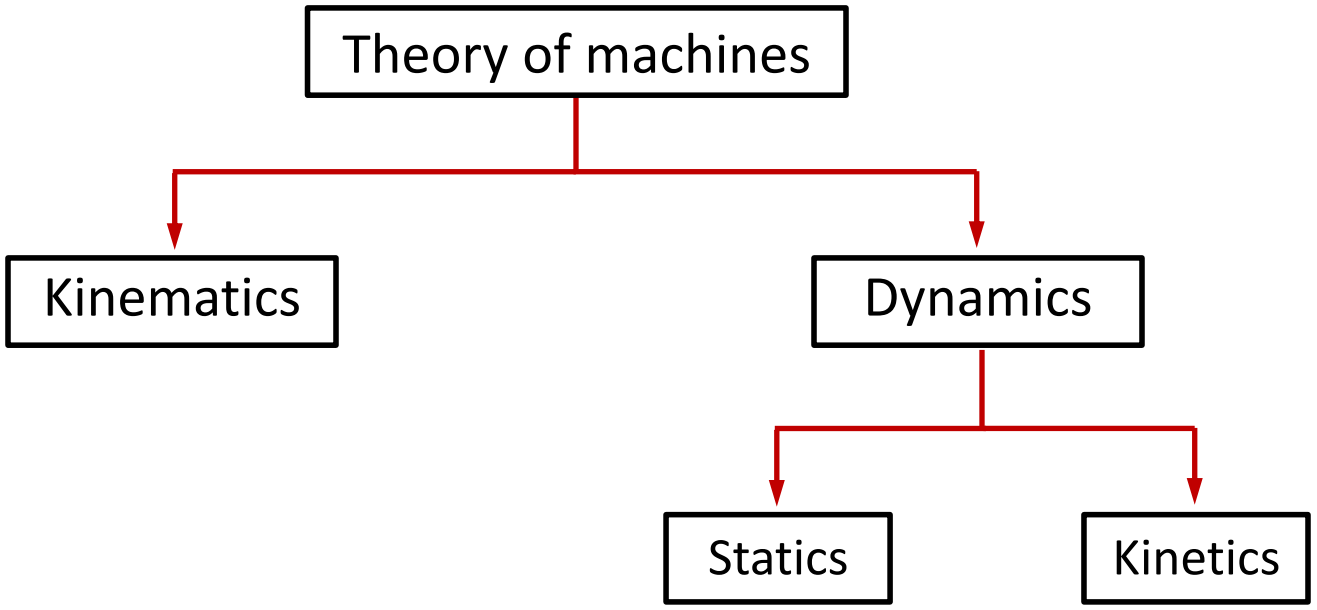
This branch of engineering- science is very essential for an engineer in designing various parts of a machine .



Shaping Machine



V-Engine



1. Kinematics

Study of the relative motion between the various parts of a machine

2. Dynamics

Study of the forces which acts on the machine parts

2.1. Statics

Deals with the forces assuming the machine parts to be massless

2.2. Kinetics

Deals with the inertia forces arising from the combined effect of the mass and the motion of the parts

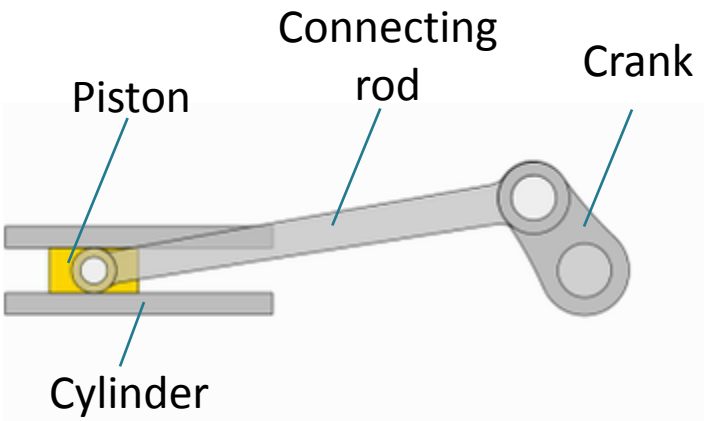
Definitions

Example: Reciprocating engine

Rotary speed of the crank shaft relative to the reciprocating speed of the piston form a **kinematic problem**

The thrust exerted by the steam or gas on the piston and force produced on the connecting rod form a **static problem**

Skeleton outline

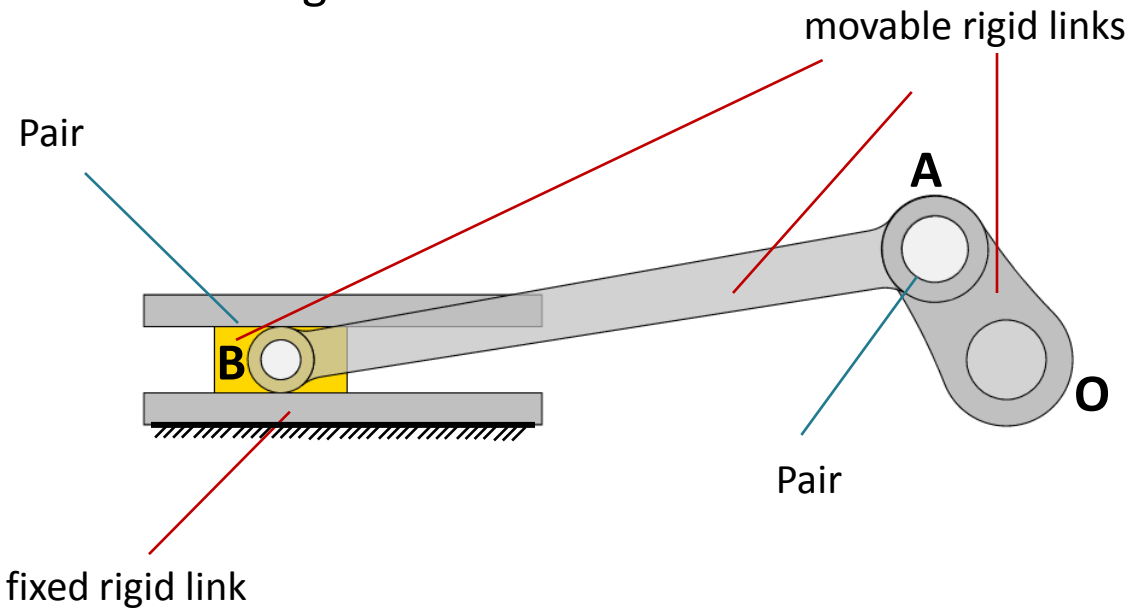


Definitions

Link or element

A link may be defined as a resistant (rigid or non rigid) body fixed or in motion which transmits force with negligible deformation

It has 2 or more pairing elements by which it may be connected to other bodies for transmitting force or motion



Definitions

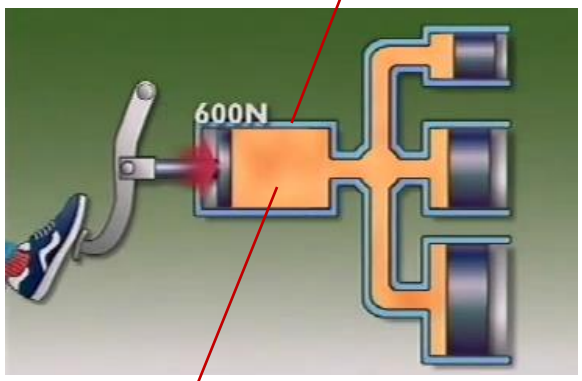
Examples of links which are resistant but not rigid:

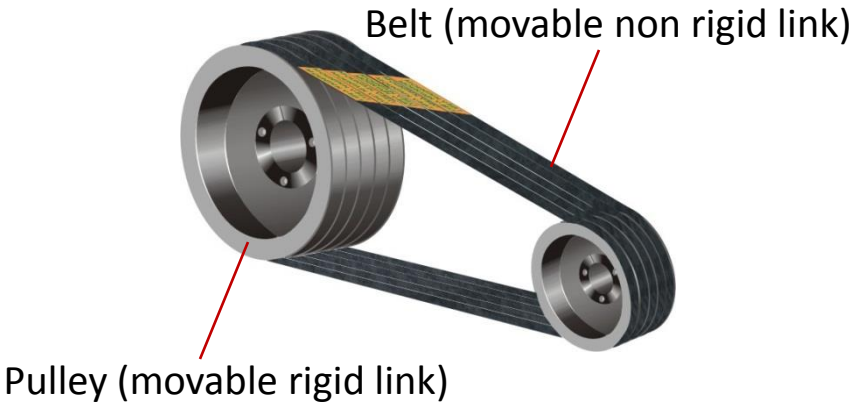
A) Liquids

Resistant to compressive forces and used as links in hydraulic presses

B) Chains & Belts

which are resistant to tensile forces and transmitting motion and forces





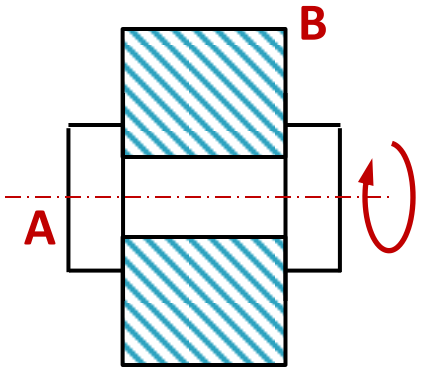
Definitions

Kinematic pair

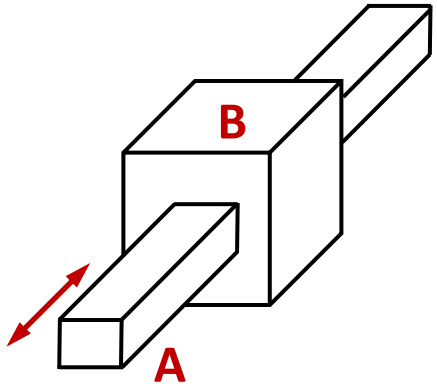
Two links which are connected together in such a way that their relative motion is completely constrained

Complete constrain pair

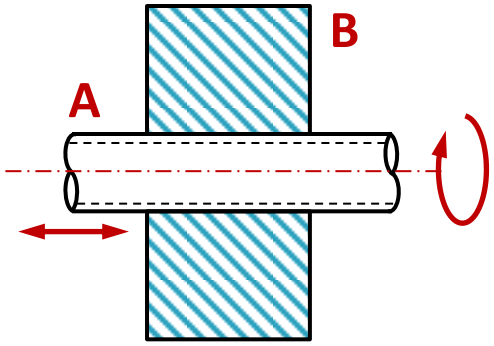
The relative motion is limited to a definite direction



Turning Pair



Sliding Pair



Screw Pair

There is a relation between the rotation of **A** and the axial displacement of **A** relative to **B**

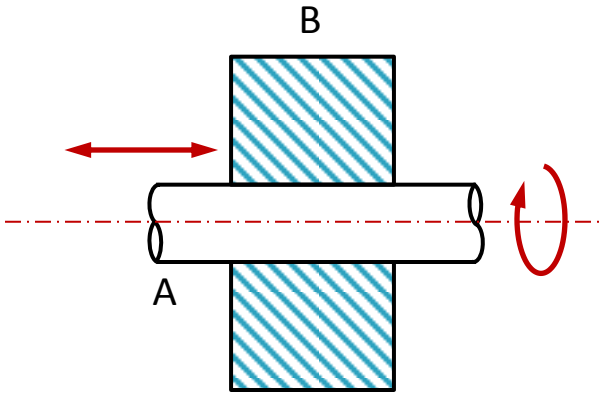
Definitions

Incomplete pair

As an example of this pair

The relative motion may be slide- rotate- sliding and rotation

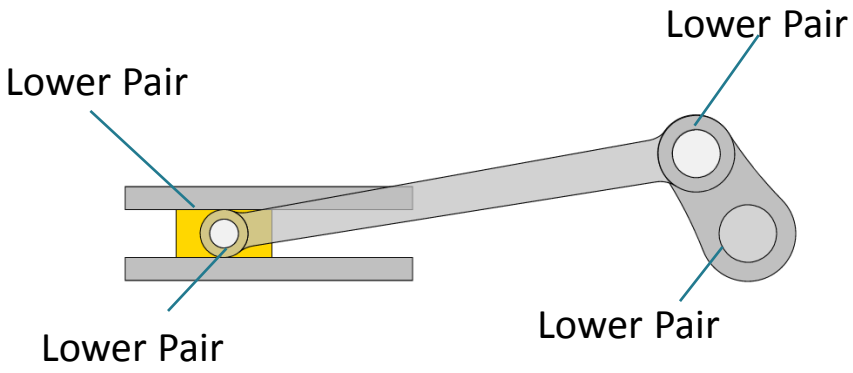
So there is nothing in connection A & B to determine which of the motions take place



Definitions

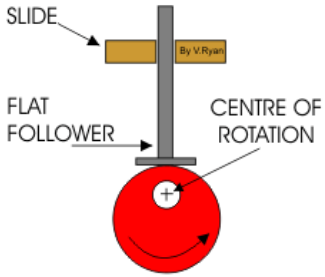
- Pairs
  - Lower
  - Higher

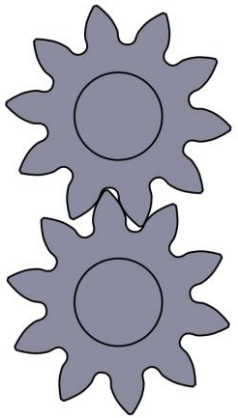
When relative motion takes place, there is a **contact surface** between the two links (turning pair- sliding pair- screw pair)

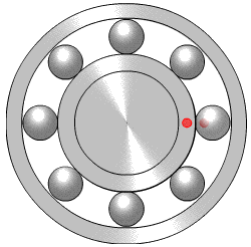


The two links have **line or point contact** while they are in motion(Cams- Gears- Bearings)

The pair must be force-closed in order to provide completely constrained motion



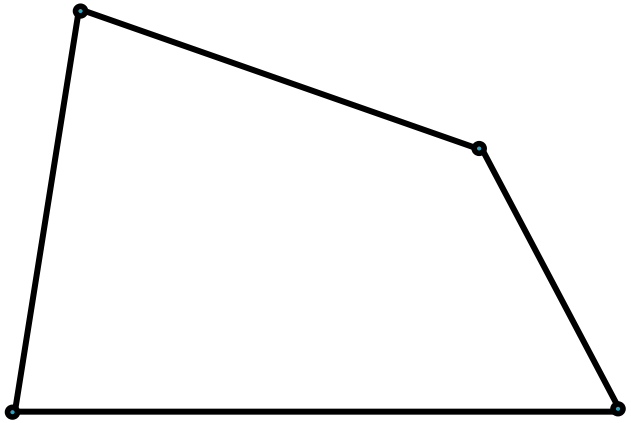




Definitions

Kinematic chain

when a number of links are connected by means of pairs the resulting assemblage is called kinematic chain

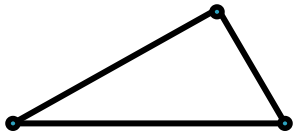


Definitions

Kinematic chain

Locked

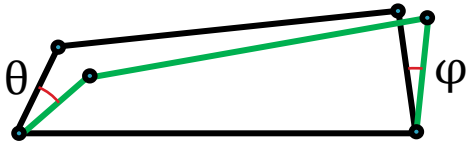
n = 0



No relative motion is possible(Structure)

Constrained

n = 1



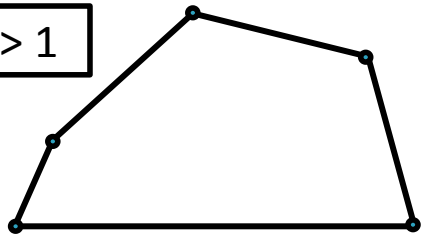
Definite relative motion is possible

The basis of all machine

Single input –single output

Unconstrained

n > 1

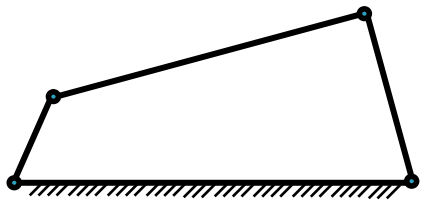


the relative motion is possible but not definite

Definitions

Mechanism

If one of the of the links of the kinematic chain is fixed, the chain became mechanism(inversions different fixed links)



Definitions

Machine

Is a mechanism which receive energy in some available form and uses it to do some particular kind of work

Definitions

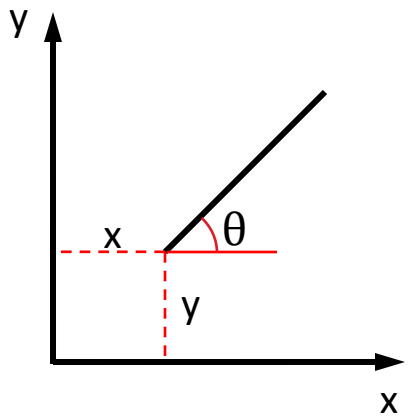
Degrees of freedom n

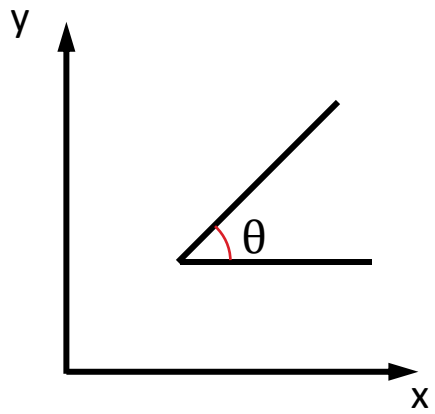
The link have 3 degrees of freedom

Two links have 6 degrees of freedom

If two links jointed together by turning pair the degree of freedom become 4

i.e. one lower pair removes 2 degree of freedom from the system





Definitions

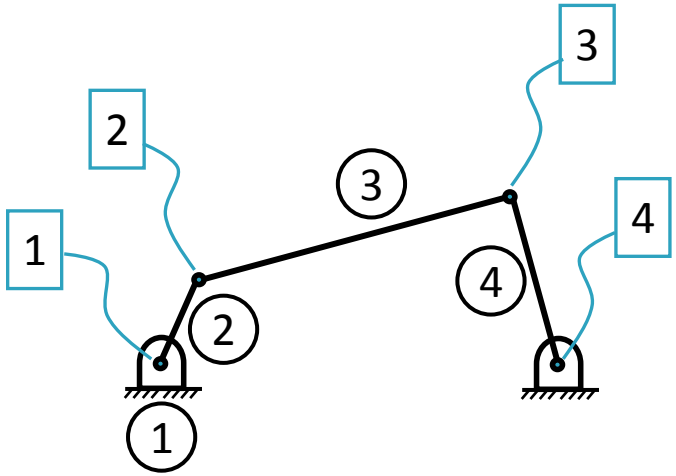
$\therefore n = 3L - 2P_l - 3$

$= 12 - 8 - 3 = 1$

Where

n: is the degrees of freedom

P_l **:** number of lower pairs



L = 4

$P_l = 4$

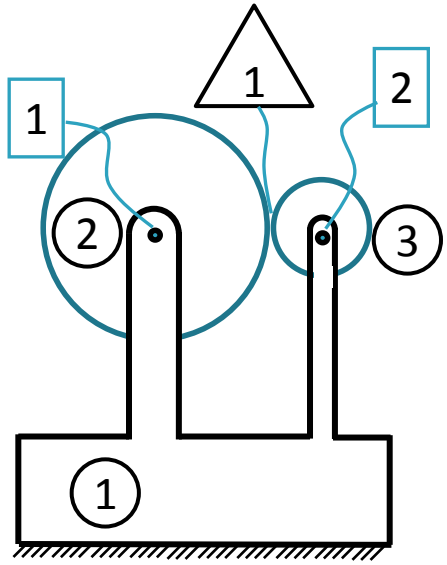
$n = 3L - 2P_l + cP_h - 3$

$1 = 9 - 4 + cP_h - 3$

$c = -1$

$n = 3L - 2P_l - P_h - 3$

P_h **:** number of higher pairs



L = 3

$P_l = 2$

$P_h = 1$

Inversions

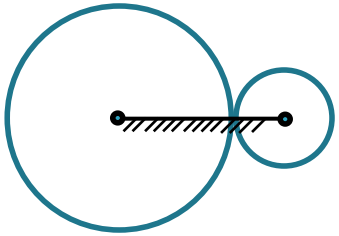
Different mechanisms can be obtained by fixing in turn different links in a kinematic chain

It is important to note that inverting a mechanism doesn't change the motions of its links relative to each other, but does change their absolute motions

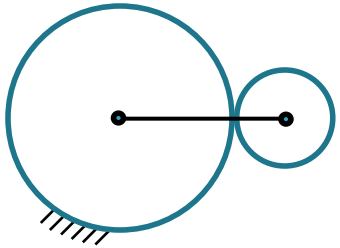
Inversions

Example 1: original gear train, epicyclic gear train

1st inversion: Original Train



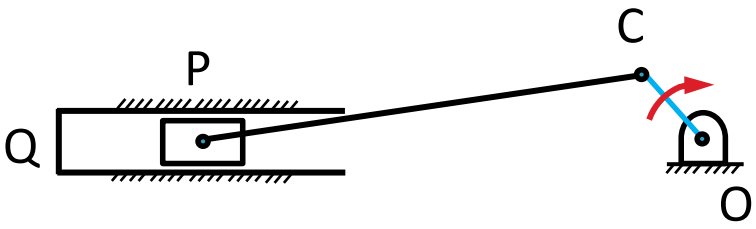
2nd inversion: Epicyclic Gear Train

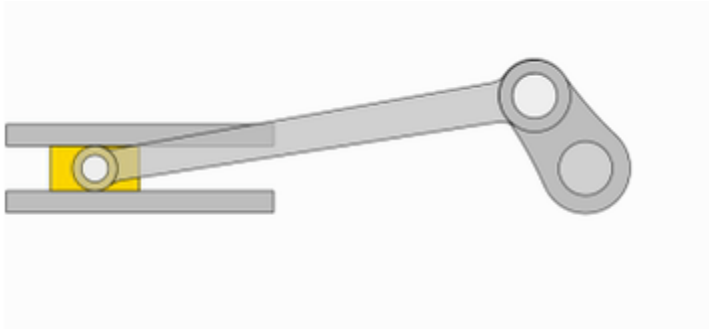


Inversions

Example 2: Inversions of slider crank chain 3T, 1S

1ST Inversion: the cylinder is fixed: reciprocating engine mechanism

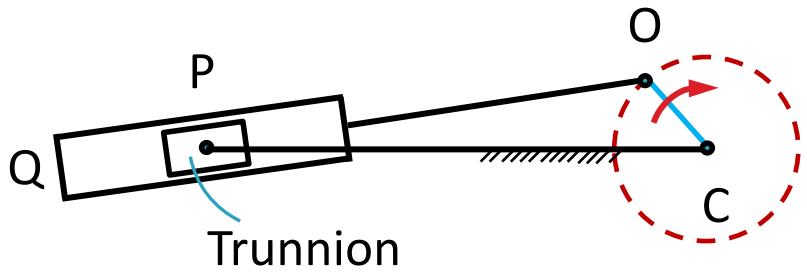




Inversions

Example 2: Inversions of slider crank chain

2nd inversion: PC becomes fixed: oscillating cylinder engine



Inversions

Example 2: Inversions of slider crank chain

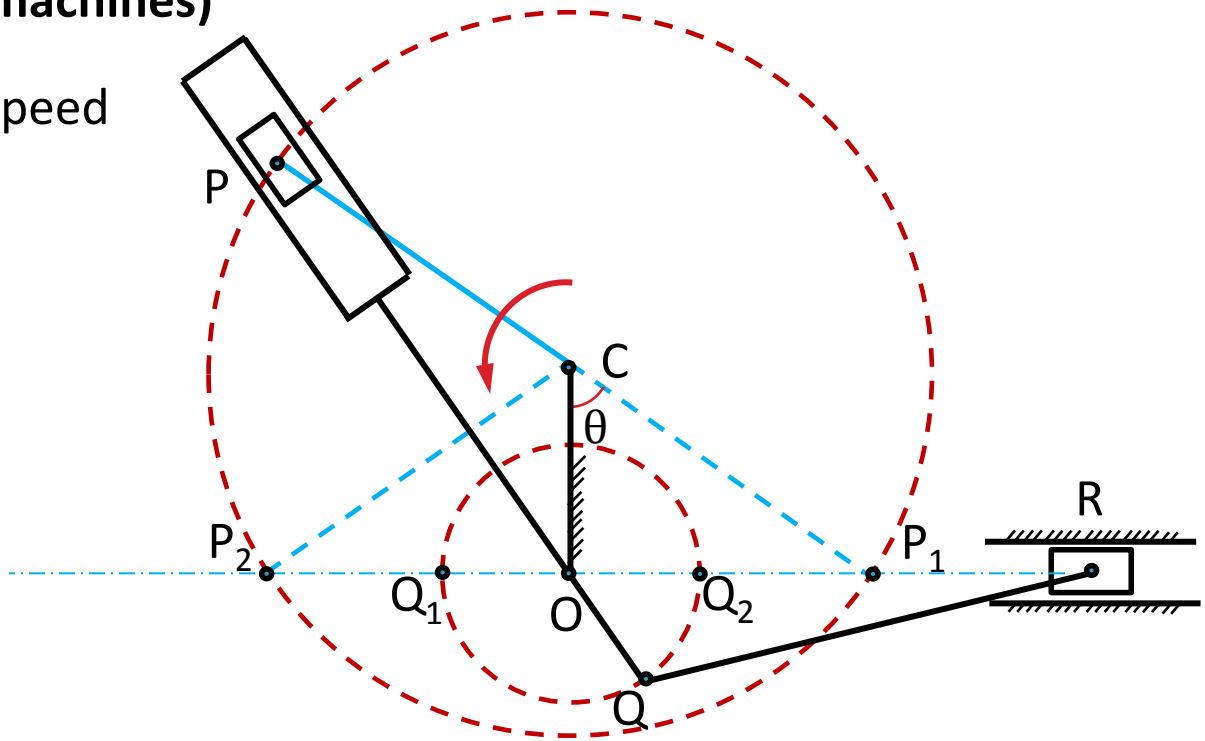
3rd Inversion: fixing the link OC: Whitworth or quick return motion mechanism (slotting and shaping machines)

CP rotates at uniform speed

$$\frac{t_C}{t_R}=\frac{180-\theta}{\theta}$$

$$\omega = \frac{d\theta}{dt}=k$$

$$\theta \propto t$$

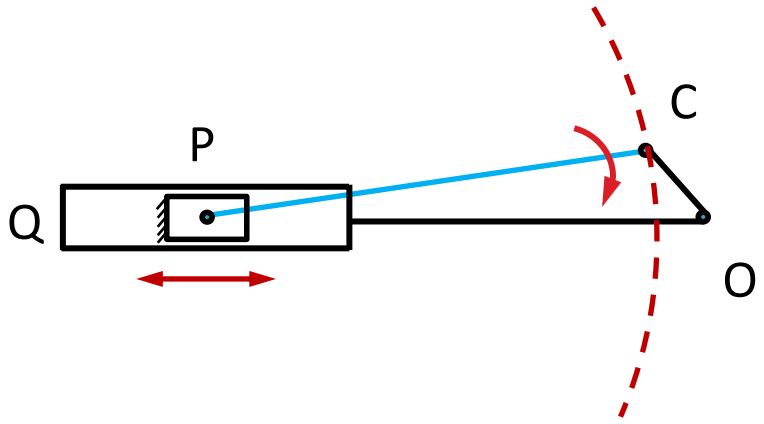


Inversions

Example 2: Inversions of slider crank chain

4th inversion: fixing the piston: pendulum pump

CP will oscillate, QO will reciprocate



Inversions

Example 3: Inversions of double slider crank chain

2T, 2 S

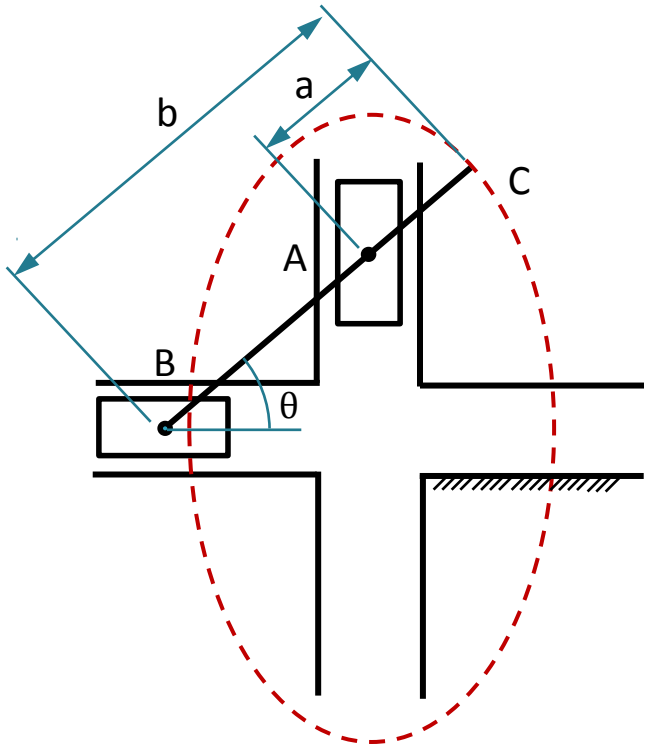
1st inversion: If the slotted frame is fixed: ellipse trammels

$x = a\,cos\theta$ $\qquad$ $y = b\,sin\theta$

$(\frac{x}{a})^2+(\frac{y}{b})^2= cos^2\theta + sin^2\theta = 1$

i.e. $\frac{x^2}{a^2} + \frac{y^2}{b^2}=1$

a=semi-minor axis, b= semi- major axis



Inversions

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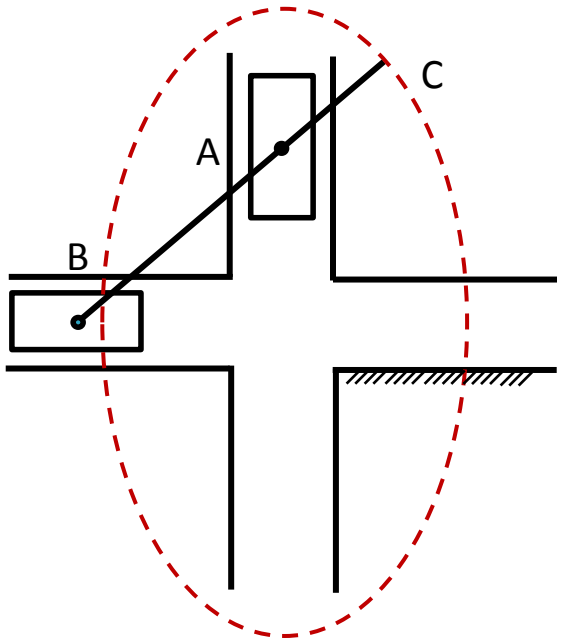
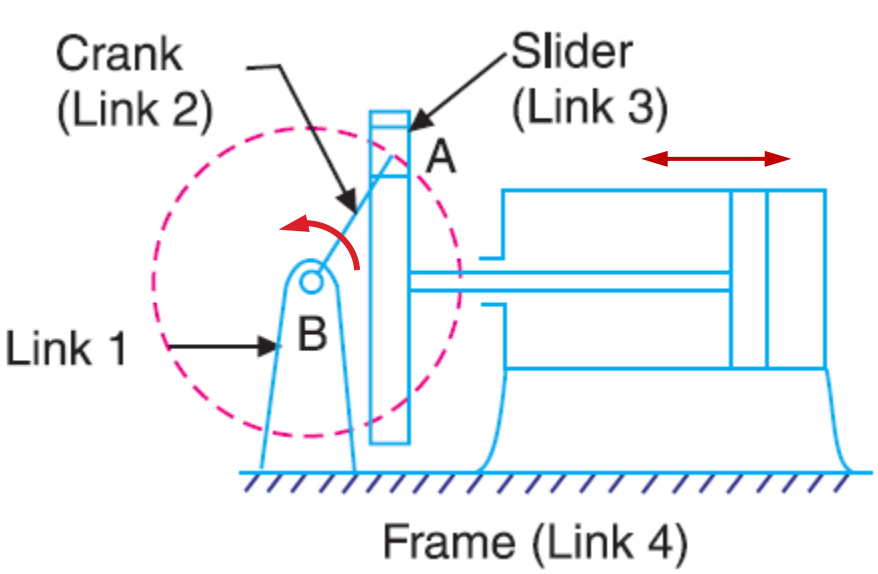


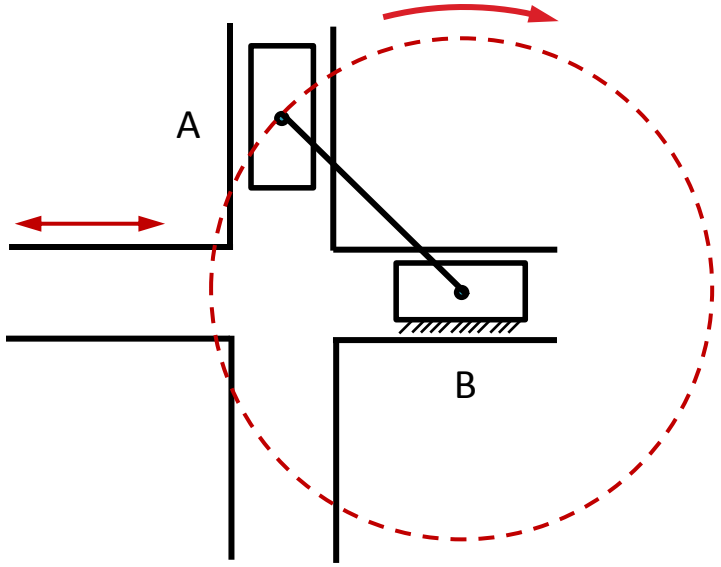
Inversions

Example 3: Inversions of double slider crank chain

2nd inversion: If one of the two blocks is fixed: scotch yoke

it is used for converting rotary into reciprocating motion





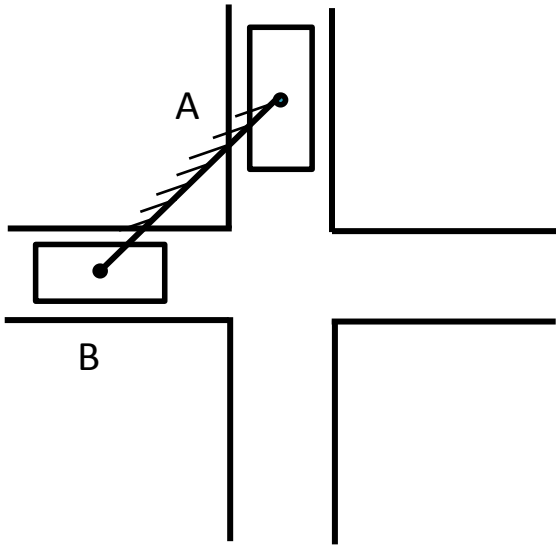
Scotch yoke mechanism

Inversions

Example 3: Inversions of double slider crank chain

3rd inversion: Coupling link AB is fixed: Oldham's coupling

If one block is turned through a definite angle, the frame and the other block must turn through the same angle



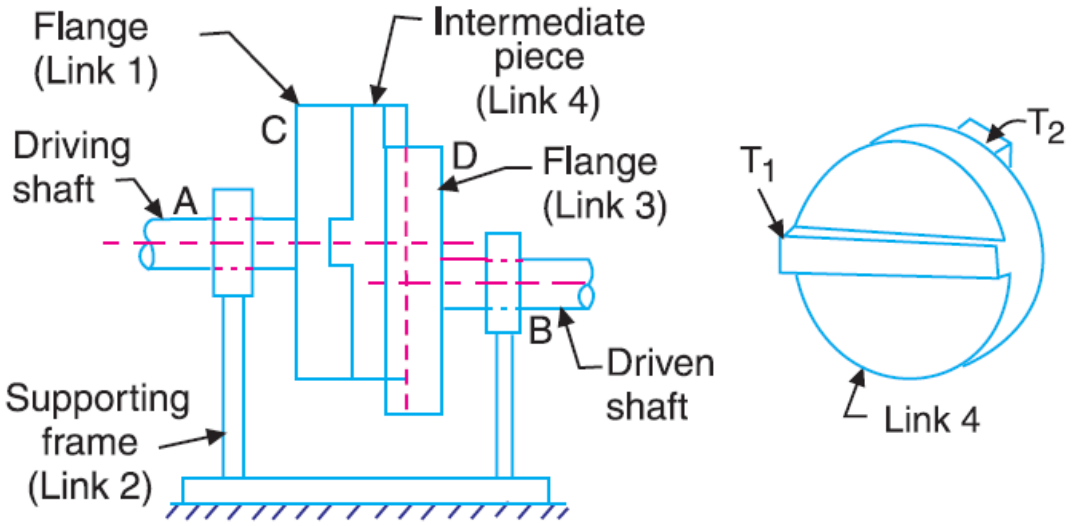
Inversions

Example 3: Inversions of double slider crank chain

3rd inversion: Coupling link AB is fixed: Oldham's coupling

If the two shafts remain parallel the distance h may vary while the shafts are in motion without affecting the transmission of uniform motion from one shaft to the other

The centre of the disc will describe a circular path with h as a diameter

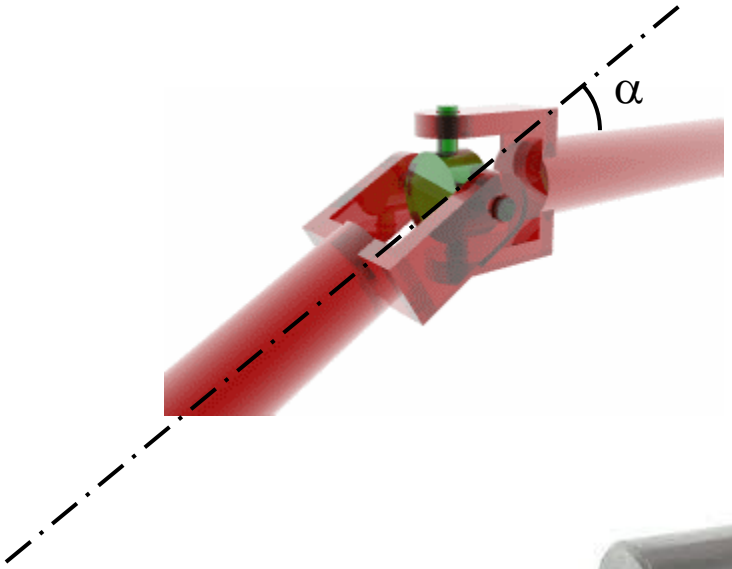


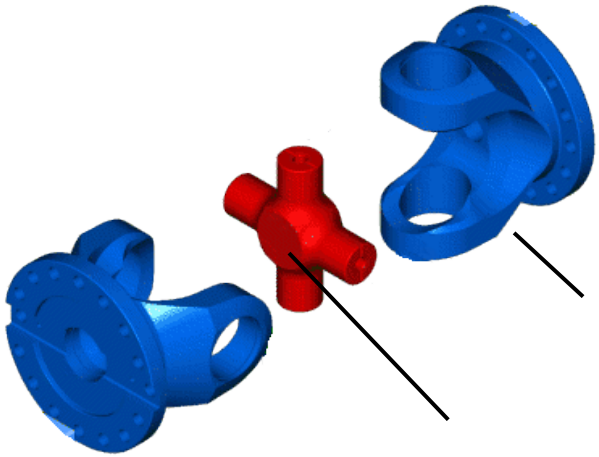
Hooke's joint (Universal Joint)

To transmit the motion between two intersecting shafts

Where a shaft drive has to be fitted to a flexible frame (tractors)

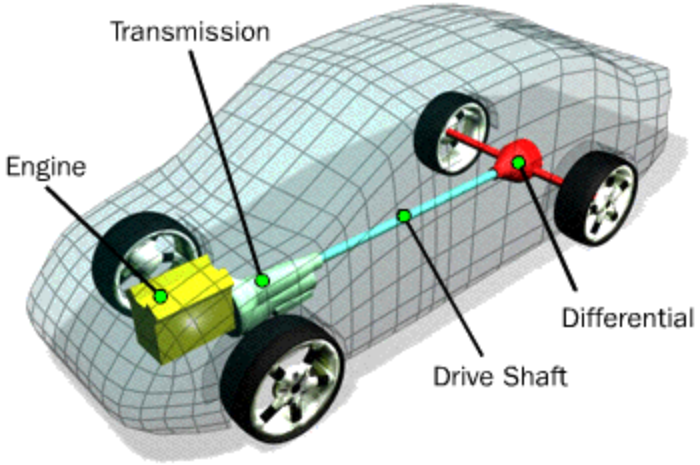
The centre of the cross must lies on the axis of each shaft

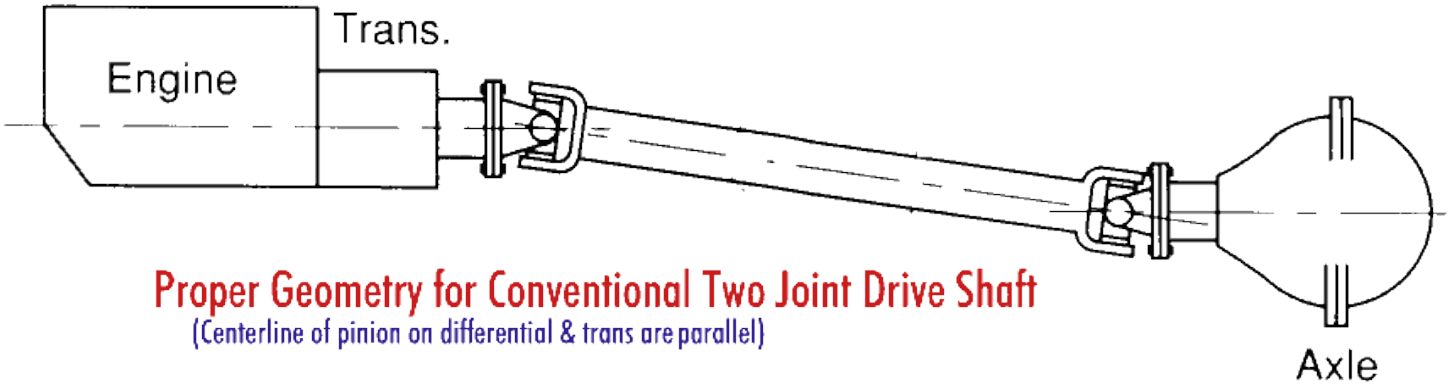




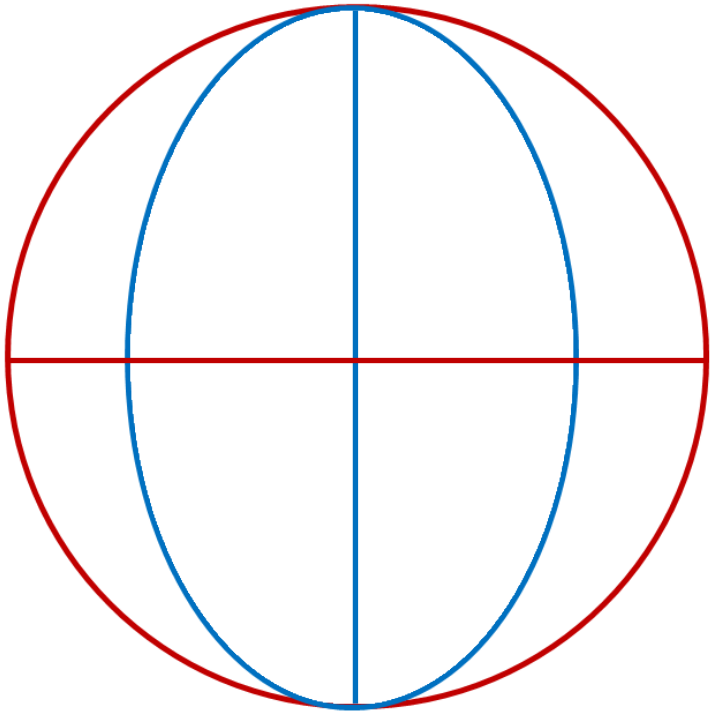
Hooke's joint (Universal Joint)

Gear box to back axel





Proper Geometry for Conventional Two Joint Drive Shaft
(Centerline of pinion on differential & trans are parallel)



Hooke's joint

Relation between the angular velocities

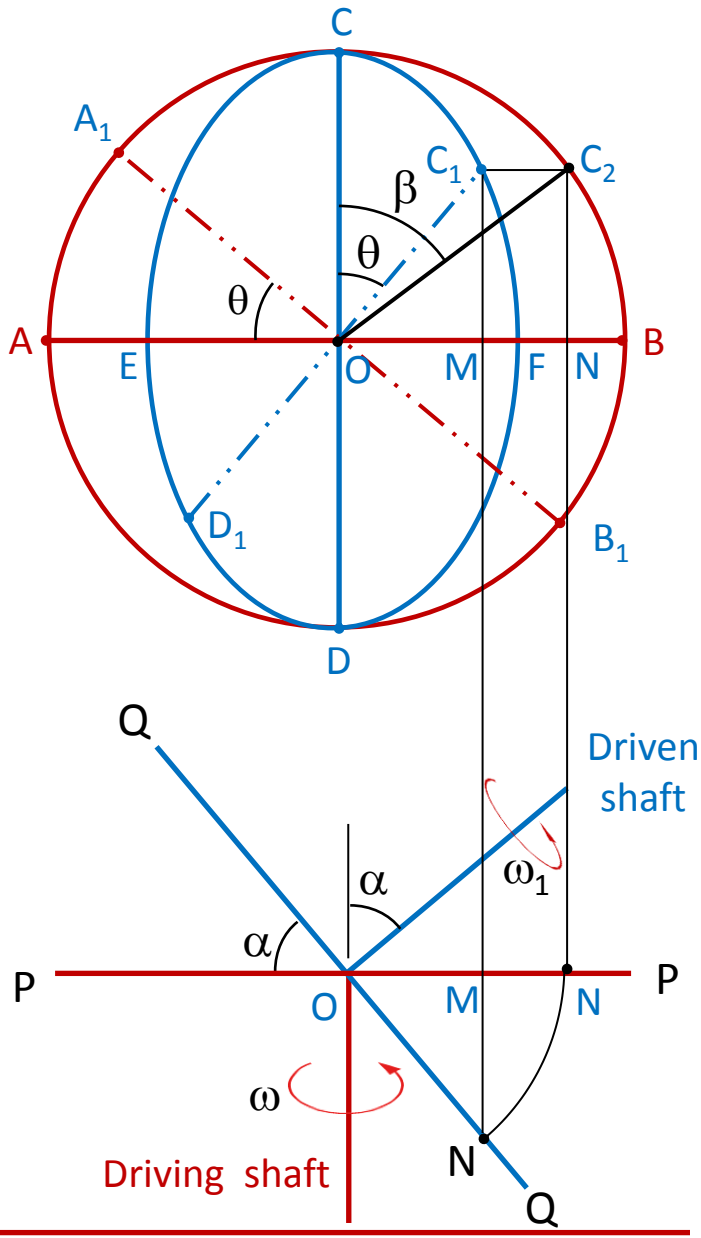
θ: Angular displacement of the driver $\quad \omega = \frac{d\theta}{dt}$

β: Angular displacement of the driven $\quad \omega_1 = \frac{d\beta}{dt}$

$$\tan\beta = \frac{ON}{NC_2} \qquad \tan\theta = \frac{OM}{MC_1} \qquad = \frac{OM}{NC_2}$$

$$\frac{\tan\beta}{\tan\theta} = \frac{ON}{OM} = \frac{1}{cos\alpha}$$

$$\tan\theta = \tan\beta \cdot \cos\ \alpha$$



Hooke's joint (Universal Joint)

$\tan\theta = \tan\beta \cdot \cos\alpha$

Differentiating this equation

$$sec^2\theta \frac{d\theta}{dt} = \cos\alpha \cdot sec^2\beta \cdot \frac{d\beta}{dt}$$

($\frac{d\theta}{dt} = \omega$, $\frac{d\beta}{dt} = \omega_1$)

$$\frac{\omega}{\omega_1} = \cos\alpha \cdot cos^2\theta \, sec^2\beta$$

$$sec^2\beta = 1 + tan^2\beta = 1 + \frac{tan^2\theta}{\cos^2\alpha} = 1 + \frac{sin^2\theta}{cos^2\theta \cdot \cos^2\alpha}$$

$$= \frac{cos^2\theta \cdot \cos^2\alpha + sin^2\theta}{cos^2\theta \cdot \cos^2\alpha} = \frac{cos^2\theta \cdot \cos^2\alpha + 1 - cos^2\theta}{cos^2\theta \cdot \cos^2\alpha} = \frac{1 - cos^2\theta(1 - \cos^2\alpha)}{cos^2\theta \cdot \cos^2\alpha}$$

$$= \frac{1 - cos^2\theta \cdot sin^2\alpha}{cos^2\theta \cdot \cos^2\alpha}$$

Hooke's joint (Universal Joint)

Hence

$$\boxed{\frac{\omega}{\omega_1}=\frac{1-cos^2\theta.sin^2\alpha}{\cos\alpha}}$$

$\frac{\omega}{\omega_{1max}}$ at $\cos\theta = \pm 1$ i.e. at θ=0, π, 2π.. etc.

$\frac{\omega}{\omega_{1max}} = cos\alpha$, $\omega_{1max}=\frac{\omega}{cos\alpha}$

$\frac{\omega}{\omega_{1min}}$ at $\cos\theta = 0$ i.e. at $\theta=\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$.. etc.

$\frac{\omega}{\omega_{1min}}=\frac{1}{cos\alpha}$, $\omega_{1min}= \omega\ cos\alpha$

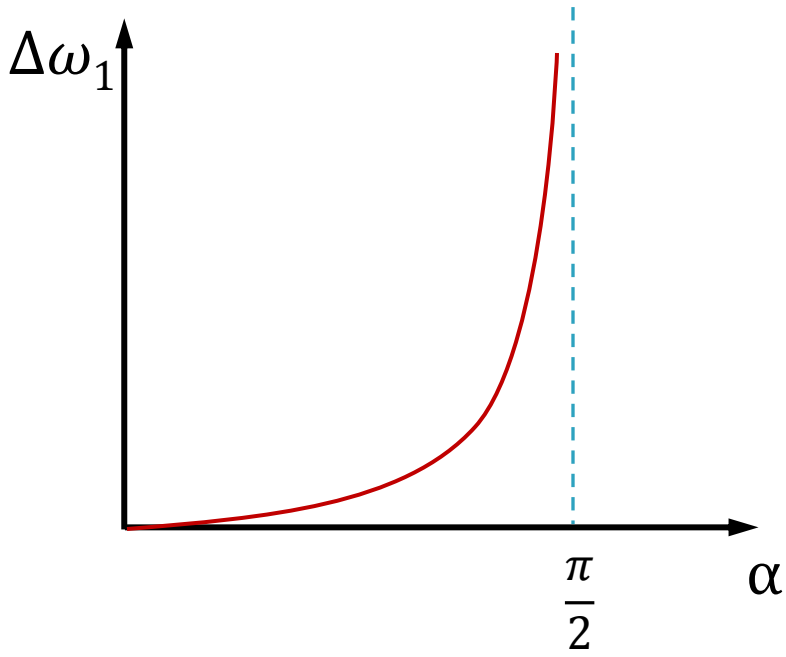
Hooke's joint (Universal Joint)

$$\omega_{1max} = \frac{\omega}{cos\alpha} \qquad \omega_{1min} = \omega\ cos\alpha$$

$$\frac{\Delta\omega_1}{\omega} = \frac{1}{cos\alpha} - cos\alpha = \frac{1 - cos^2\alpha}{cos\alpha}$$

$$= \frac{sin^2\alpha}{cos\alpha} = \frac{sin\alpha\ sin\alpha}{cos\alpha} = sin\alpha\ tan\alpha$$

$$\Delta\omega_1 \propto \alpha^2$$



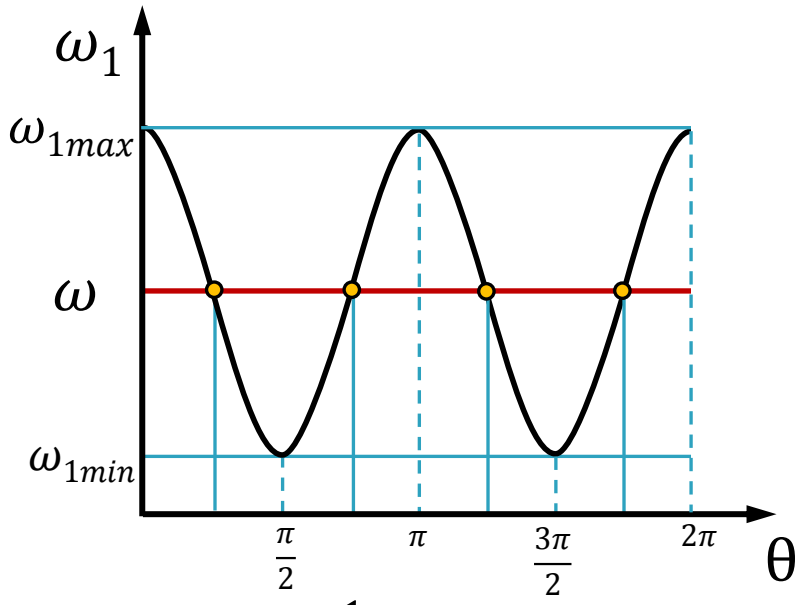
Hooke's joint (Universal Joint)

Conditions of equal speeds

Put $\frac{\omega}{\omega_1}=1$, $\frac{1-cos^2\theta.sin^2\alpha}{\cos\alpha}=1$

$$1-cos^2\theta.sin^2\alpha=\cos\alpha$$

$$1-\cos\alpha=cos^2\theta.sin^2\alpha$$



$$cos^2\theta=\frac{1-\cos\alpha}{sin^2\alpha}=\frac{1-\cos\alpha}{1-\cos^2\alpha}=\frac{1-\cos\alpha}{(1-\cos\alpha)(1+\cos\alpha)}=\frac{1}{(1+\cos\alpha)}$$

$$sin^2\theta=1-cos^2\theta=1-\frac{1}{(1+\cos\alpha)}=\frac{\cos\alpha}{(1+\cos\alpha)}$$

$$\frac{sin^2\theta}{cos^2\theta}=\frac{\cos\alpha}{(1+\cos\alpha)}.(1+\cos\alpha)=\cos\alpha$$

$$\boxed{\tan\theta=\pm\sqrt{\cos\alpha}}$$

Hooke's joint (Universal Joint)

Angular acceleration of the driven shaft

$$\omega_1 = \omega \frac{\cos\alpha}{1-cos^2\theta.sin^2\alpha} = \omega \cos\alpha(1 - cos^2\theta.sin^2\alpha)^{-1}$$

$$\frac{d\omega_1}{dt} = \omega \cos\alpha \left[-(1 - cos^2\theta.sin^2\alpha)^{-2}.(sin^2\alpha . \frac{2\, cos\theta.sin\theta}{sin2\theta}\right] \frac{d\theta}{dt}$$

$$\boxed{\alpha_1 = \frac{-\omega^2 \cos\alpha\ sin^2\alpha\ sin2\theta}{(1 - cos^2\theta.\ sin^2\alpha)^2}}$$

α_1 will increase by increasing α, in normal practice such α don't exceed 10°

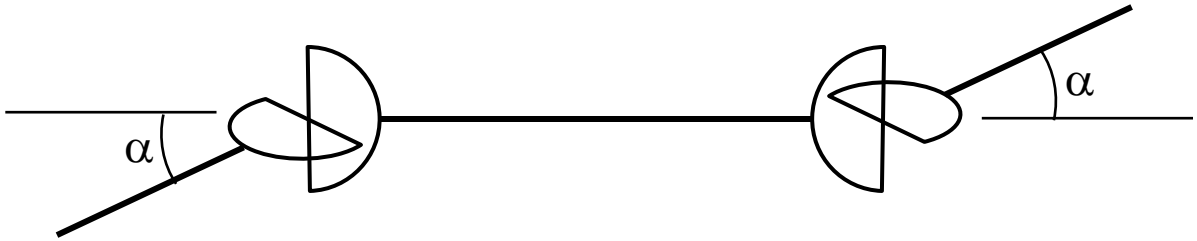
Maximum acceleration occur when

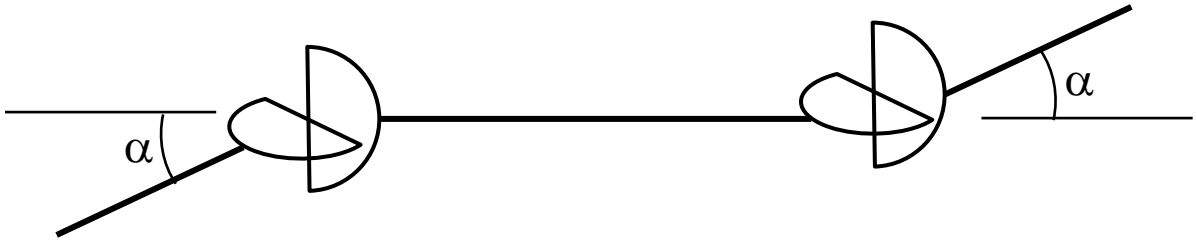
$$cos2\theta \cong \frac{2\, sin^2\alpha}{2 - sin^2\alpha}$$

This relation is valid if $\alpha < 30°$

Hooke's joint

If the driving and the driven shafts are equally inclined to the intermediate shaft and the 2 forks on the intermediate shaft lie in the same plane, it is evident that speeds of driving and driven shafts are identical and the fluctuation of speed are confined to intermediate shaft, which may be made short and light



Hooke's joint

If the forks of the intermediate shaft lie in planes perpendicular to each other, the fluctuation of the driven shaft shall vary between $cos^2\alpha$ and $\frac{1}{cos^2\alpha}$