ALGEBRA 2

UNIT 2

FUNCTIONS, EQUATIONS, AND GRAPHS

ANSWER KEY

Unit Essential Questions:

- Does it matter which form of a linear equation that you use?
- How do you use transformations to help graph absolute value functions?
- How can you model data with linear equations?

SECTION 2.1: RELATIONS AND FUNCTIONS

MACC.912.F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

RATING	LEARNING SCALE
4	I am able to • evaluate functions in real-world scenarios or more challenging problems that I have never previously attempted
TARGET 3	I am able to • graph relations • identify functions
2	I am able to • graph relations with help • identify functions with help
1	I am able to • understand the definition of a relation

WARM UP

Ticket prices for admission to a museum are $8 for adults, $5 for children, and $6 for seniors.

1) What algebraic expression models the total number of dollars collected in ticket sales?

$8a + 5c + 6s$

2) If 20 adult tickets, 16 children's tickets, and 10 senior tickets are sold one morning, how much money is collected in all?

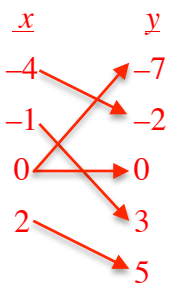
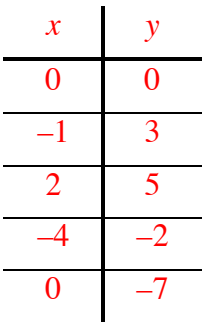
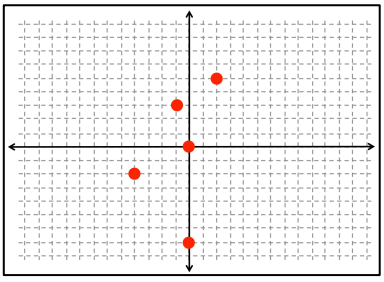
$300

KEY CONCEPTS AND VOCABULARY

Relation - a set of pairs of input and output values.

Domain - the set of all inputs (*x*-coordinates)

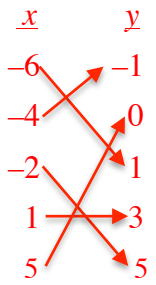
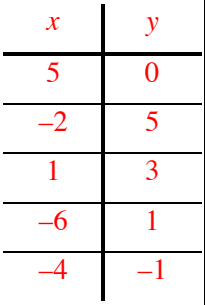
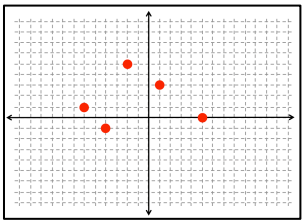
Range - the set of all outputs (*y*-coordinates)

Ordered Pairs	Mapping Diagram	Table	Graph
(0, 0) (-1, 3) (2, 5) (-4, -2) (0, -7)			

EXAMPLES

EXAMPLE 1: REPRESENTING A RELATION

Express the relation as a table, a graph, and a mapping.

Ordered Pairs	Mapping Diagram	Table	Graph
(5, 0) (-2, 5) (1, 3) (-6, 1) (-4, -1)			

EXAMPLE 2: DETERMINING DOMAIN AND RANGE

Determine the domain and range for each relation.

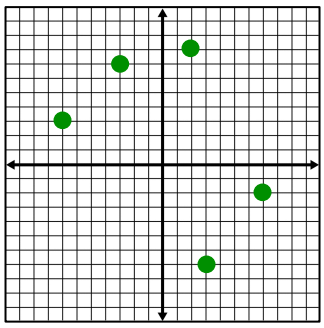
a) {(2, 3), (-1,5), (-5, 5), (0, -7)}

Domain: {-5, -1, 0, 2}
Range: {-7, 3, 5}

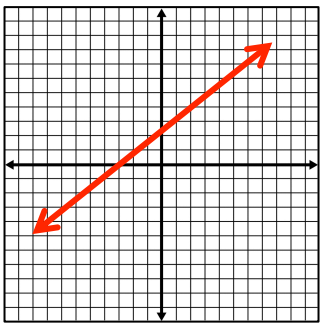
b)

x	y
1	0
2	3
3	-4
4	12

Domain: {1, 2, 3, 4}
Range: {-4, 0, 3, 12}

c) 

Domain: {-7, -3, 2, 3, 7}
Range: {-7, -2, 3, 7, 8}

d) 

Domain: All real numbers
Range: All real numbers

KEY CONCEPTS AND VOCABULARY

A function is a relationship that pairs each input value with exactly one output value.

In a relationship between variables, the dependent variable changes in response to the independent variable.

Vertical Line Test - is a test to see if the graph represents a function. If a vertical line intersects the graph more than once, it fails the test and is not a function.

Equations that are functions can be written in a form called function notation. It is used to find the element in the range that will correspond the element in the domain.

A properly working vending machine is an example of a function. You put in a code (input B15) and it gives you exactly one item (output Mountain Dew).

EQUATION	FUNCTION NOTATION
$y = 4x - 10$	$f(x) = 4x - 10$
Read: y equals four x minus 10	Read: f of x equals four x minus 10

EXAMPLES

EXAMPLE 3: IDENTIFYING A FUNCTION

Determine whether each relation is a function.

a) $\{(0, 1), (1, 0), (2, 1), (3, 1), (4, 2)\}$

Yes

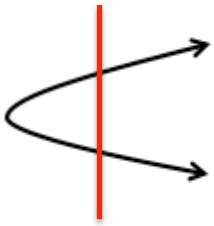
b) $\{(4, 9), (4, 3), (4, 0), (4, 4), (4, 1)\}$

No

EXAMPLE 4: USING THE VERTICAL LINE TEST

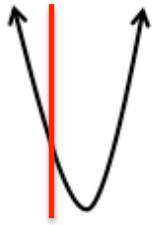
Use the vertical line test. Which graphs represent a function?

a)



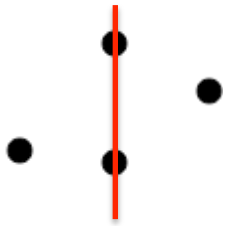
Not a Function

b)



Function

c)



Not a Function

EXAMPLE 5: EVALUATING FUNCTION VALUES

Evaluate each function for the given value.

a) $f(x) = -2x + 11$ for f(5), f(-3), and [3 – f(0)]

f(5) = 1
f(–3) = 17
[3 – f(0)] = –8

b) $f(x) = x^2 + 3x - 1$ for f(2), f(-1), and [f(0) + f(1)]

f(2) = 9
f(–1) = –3
[f(0) – f(1)] = –4

EXAMPLE 6: EVALUATING FUNCTION VALUES FOR REAL WORLD SITUATIONS

Write a function rule to model the cost per month of a cell phone data plan. Then evaluate the function for given number of data.

Monthly service fee: $24.99
Rate per GB of data uses: $5
GB of data used: 13

$C(x) = \$24.99 + 5x$
$C(13) = \$89.99$

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.2: DIRECT VARIATION

MACC.912.A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

RATING	LEARNING SCALE
4	I am able to • write and solve an equation of a direct variation in real-world situations or more challenging problems that I have never previously attempted
TARGET 3	I am able to • write and graph an equation of a direct variation
2	I am able to • write and graph an equation of a direct variation with help
1	I am able to • understand the definition of direct variation

WARM UP

Solve each equation for y.

1) $12y = 3x$ $\quad y = \frac{1}{4}x$

2) $-10y = 5x$ $\quad y = -\frac{1}{2}x$

3) $\frac{3}{4}y = 15x$ $\quad y = 20x$

KEY CONCEPTS AND VOCABULARY

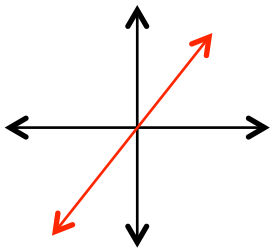
Direct Variation- a linear function defined by an equation of the form y=kx, where $k \neq 0$.

Constant of Variation - k, where $k = y/x$

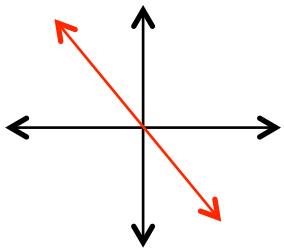
GRAPHS OF DIRECT VARIATIONS

The graph of a direct variation equation $y = kx$ is a line with the following properties:

- The line passes through (0, 0)
- The slope of the line is k.



$k > 0$



$k < 0$

EXAMPLES

EXAMPLE 1: IDENTIFYING A DIRECT VARIATION

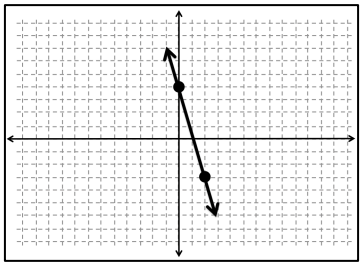
For each function, tell whether y varies directly with x. If so, find the constant of variation.

a) $3y = 7x + 7$ No

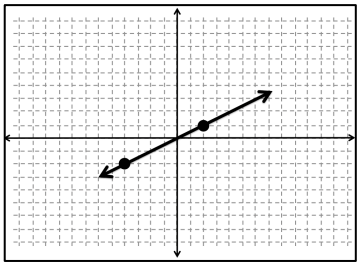
b) $5x = -2y$ Yes; $-\frac{5}{2}$

EXAMPLE 2: FINDING THE CONSTANT OF VARIATION

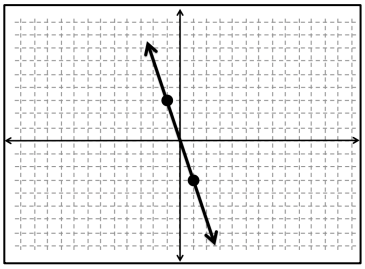
Determine if each graph has direct variation. If does, identify the constant of variation.

a) 

No

b) 

Yes; $\frac{1}{2}$

c) 

Yes; -3

EXAMPLE 3: WRITING A DIRECT VARIATION EQUATION

Suppose y varies directly with x, and $y = 15$ when $x = 27$. Write the function that models the variation. Find y when $x = 18$.

$y = \frac{5}{9}x$; 10

EXAMPLE 4: WRITING A DIRECT VARIATION FROM DATA

For each function, determine whether y varies directly with x. If so, find the constant of variation and write the equation.

a)

x	y
-1	3
2	-6
5	15

No

b)

x	y
7	14
9	18
– 4	– 8

Yes; $k = 2$, $y = 2x$

EXAMPLE 5: USING DIRECT VARIATION IN REAL-WORLD SITUATIONS

Weight on the moon y varies directly with weight on Earth x. A person who weighs 100lbs on Earth weighs 16.6lbs on the moon. What is an equation that relates weight on Earth x and weight on the moon y? How much will a 150lb person weigh on the moon?

$y = 0.166x$; 24.9lbs

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.3: LINEAR FUNCTIONS AND SLOPE-INTERCEPT FORM

MACC.912.F-IF.B.6: Calculate and interpretthe average rate of changeof a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MACC.912.F-IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.
MACC.912.A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

RATING	LEARNING SCALE
4	I am able to • write a linear equation in real world situations in slope-intercept and use the model to make predictions
TARGET 3	I am able to • find the slope • write and graph linear equations using slope-intercept form
2	I am able to • find the slope with help • write and graph linear equations using slope-intercept form with help
1	I am able to • understand the components of slope-intercept form

WARM UP

Tell whether the given ordered pair is a solution of the equation.

1) $4y + 2x = 3$; (0, 0.75) Yes

2) $y = 6x - 2$; (0, 2) No

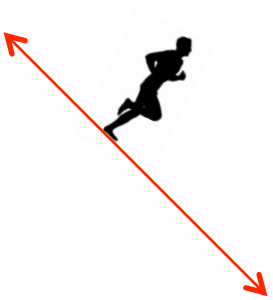
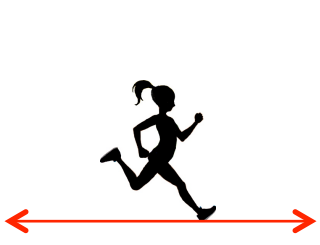
KEY CONCEPTS AND VOCABULARY

Rate of Change – a ratio that shows the relationship, on average, between two changing quantities

Slope is used to describe a rate of change. Because a linear function has a constant rate of change, any two points can be used to find the slope.

RATE OF CHANGE

$$\text{Slope} = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{y_2 - y_1}{x_2 - x_1}$$

POSITIVE	NEGATIVE	ZERO	UNDEFINED
			

EXAMPLES

EXAMPLE 1: DETERMINING A CONSTANT RATE OF CHANGE

Determine the rate of change. Determine if the function is linear. Justify your answer.

a)

x	y
1	4
5	6
9	8
13	10
17	12

Linear;
Rate of Change between all points is 1/2

b)

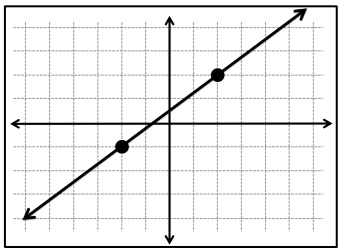
x	y
1	0
2	2
3	6
4	8
5	12

Non-Linear;
Rate of Change varies between 2 and 4

EXAMPLE 2: FINDING THE SLOPE USING A GRAPH

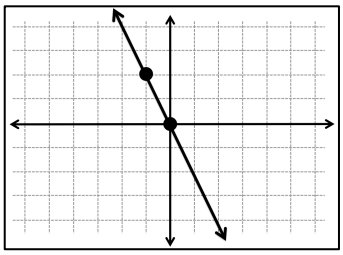
Find the slope of each line.

a)



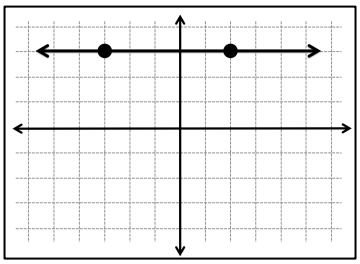
3/4

b)



–2

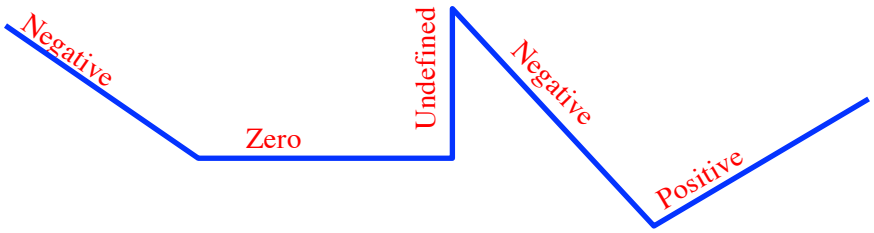
c)



0

EXAMPLE 3: IDENTIFYING SLOPES

Label the slopes of the lines below (positive, negative, etc.).



EXAMPLE 4: FINDING SLOPES USING POINTS

Find the slope of the line through the given points.

a) (3, 2) and (4, 8)

6

b) (2, 7) and (8, –6)

$-\frac{13}{6}$

c) $\left(\frac{1}{3},\frac{1}{2}\right)$ and $\left(\frac{4}{3},\frac{7}{2}\right)$

3

KEY CONCEPTS AND VOCABULARY

SLOPE-INTERCEPT FORM

$y = mx + b$

m = slope; $(0, b)$ = y-intercept

Steps for Graphing a Linear Function (Slope-Intercept Form)

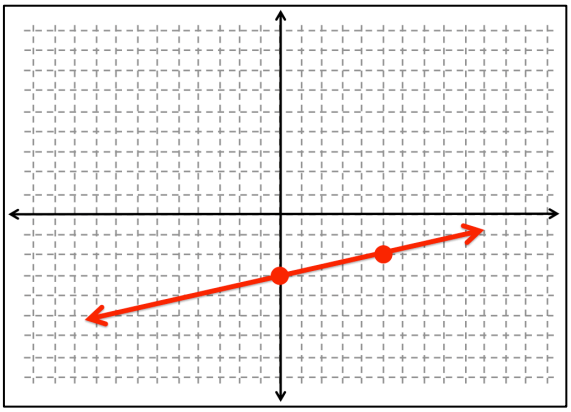
- Identify and plot the y-intercept
- Use the slope to plot an additional point (Rise/Run)
- Draw a line through the two points

EXAMPLES

EXAMPLE 5: WRITING AND GRAPHING LINEAR EQUATIONS GIVEN A Y-INTERCEPT AND A SLOPE

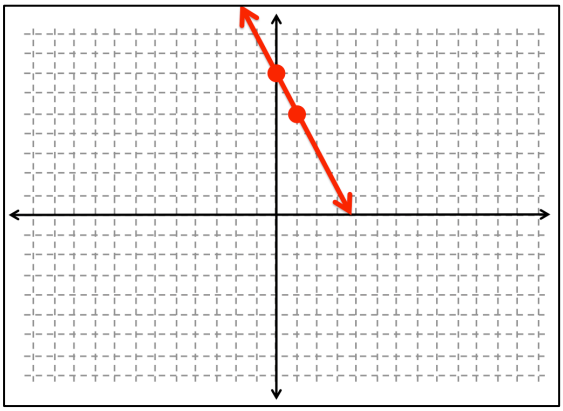
Write an equation of a line with the given slope and y-intercept. Then graph the equation.

a) slope of 1/5 and y-intercept is $(0, -3)$



$y = \frac{1}{5}x - 3$

b) slope of -2 and y-intercept is $(0, 7)$

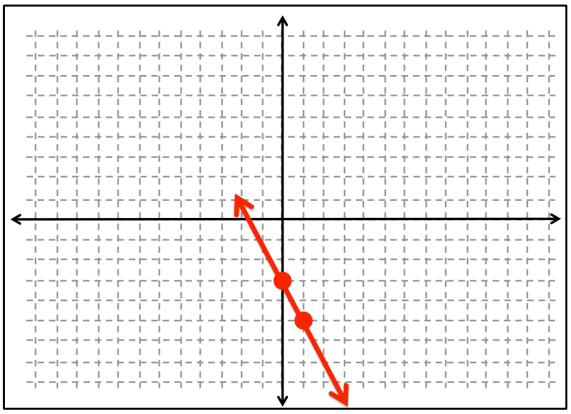


$y = -2x + 7$

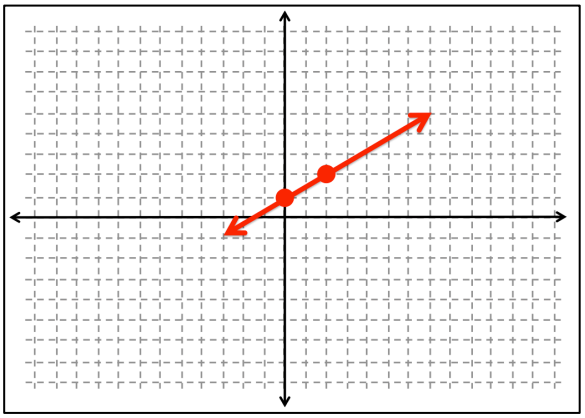
EXAMPLE 6: GRAPHING LINEAR EQUATIONS

Graph the linear equation.

a) $4x + 2y = -6$



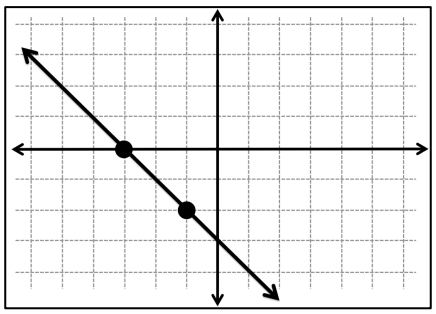
b) $-3x + 6y = 6$



EXAMPLE 7: WRITING A LINEAR EQUATION IN SLOPE-INTERCEPT FORM

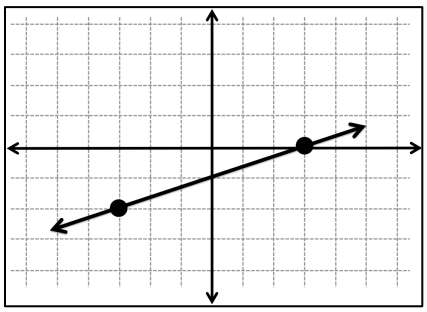
What is the equation of the line in slope-intercept form?

a)



$y = -x - 3$

b)



$y = \frac{1}{3}x - 1$

EXAMPLE 8: FINDING THE Y-INTERCEPT GIVEN TWO POINTS

In slope-intercept form, write an equation of the line through the given points.

a) (4, –3) and (5, –1)

$y = 2x - 11$

b) (3, 0) and (–3, 2)

$y = -\frac{1}{3}x + 1$

EXAMPLE 9: USING LINEAR EQUATIONS IN A REAL WORLD SITUATION

To buy a $1200 stereo, you pay a $200 deposit and then make weekly payments according to the equation: ***a = 1000 – 40t***, where ***a*** is the amount you owe and ***t*** is the number of weeks.

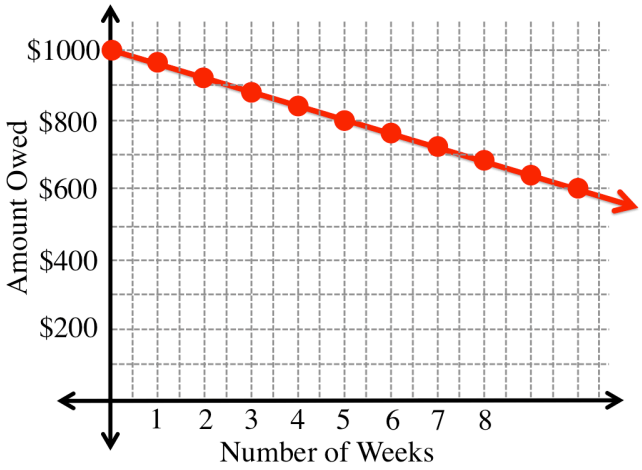
a) How much do you owe originally on layaway?

$1000

b) What is your weekly payment?

$40

c) Graph the model.



RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.4: MORE ABOUT LINEAR EQUATIONS

MACC.912.F-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.
MACC.912.F-IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.
MACC.912.A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MACC.912.F-LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

RATING	LEARNING SCALE
4	I am able to • write and graph linear equations in real-world situations or more challenging problems that I have never previously attempted
TARGET 3	I am able to • write and graph linear equations using point-slope form and standard form • write equations of parallel and perpendicular lines
2	I am able to • write and graph linear equations using point-slope form and standard form with help • write equations of parallel and perpendicular lines with help
1	I am able to • understand the components of point-slope form and standard form

WARM UP

A line passes through the points (–1, 5) and (3, k) and has a y-intercept of 7. Find the value of k.

$k = 13$

KEY CONCEPTS AND VOCABULARY

POINT-SLOPE FORM

$$(y - y_1) = m(x - x_1)$$

Use this form when you are given a point (x_1, y_1) and the slope (m).

Steps for Graphing a Linear Function (Point-Slope Form)

- Identify and plot the given point on the line
- Use the slope to plot an additional point (Rise/Run)
- Draw a line through the two points

EXAMPLES

EXAMPLE 1: WRITING LINEAR EQUATIONS GIVEN A POINT AND A SLOPE

Write an equation of a line with the given slope and point.

a) passes through (–4, 1) with slope 2/5

$(y-1)=\frac{2}{5}(x+4)$

b) passes through (3, 5) with slope 2

$(y-5)=2(x-3)$

EXAMPLE 2: WRITING LINEAR EQUATIONS GIVEN TWO POINTS

Write the equation of a line in point-slope form given two points.

a) through (4, –3) and (5, –1)

$(y+3)=2(x-4)$ or $(y+1)=2(x-5)$

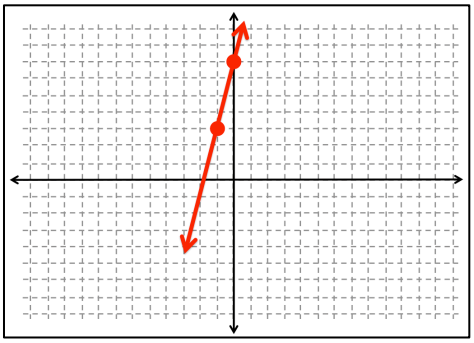
b) through (2, 0) and (–2, 6)

$(y-0)=-\frac{3}{2}(x-2)$ or $(y+2)=-\frac{3}{2}(x-6)$

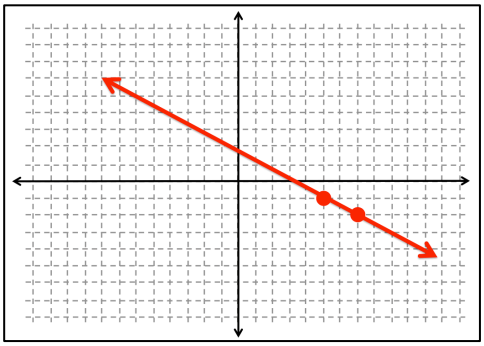
EXAMPLE 3: GRAPHING USING POINT-SLOPE FORM

Graph each equation.

a) $y-3=4(x+1)$



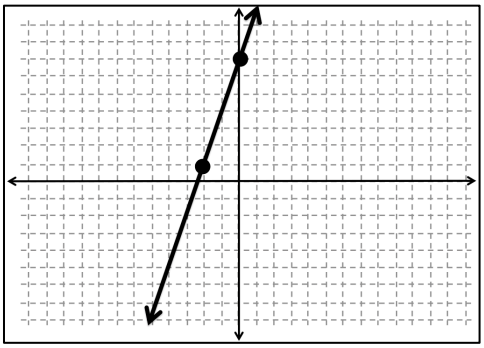
b) $y+1=-\frac{1}{2}(x-5)$



EXAMPLE 4: WRITING LINEAR EQUATIONS IN POINT-SLOPE FORM

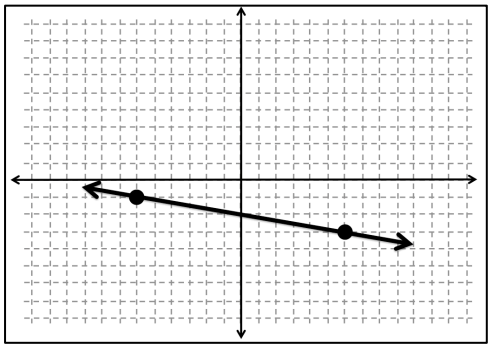
What is the equation of the line in point-slope form?

a)



$(y-1)=3(x+2)$ or $(y-7)=3(x-0)$

b)



$(y+1)=-\frac{1}{6}(x+6)$ or $(y+3)=-\frac{1}{6}(x-6)$

EXAMPLE 5: USING POINT-SLOPE FORM IN REAL-WORLD SITUATIONS

In 1996, there were 57 million cats as pets in the U.S. By 2003, this number was 61 million. Write a linear model for the number of cats as pets. Then use the model to predict the number of cats as pets in 2015?

$(y-61)=\frac{4}{7}(x-2003)$ or $(y-57)=\frac{4}{7}(x-1996)$; 68 million cats

KEY CONCEPTS AND VOCABULARY

STANDARD FORM OF A LINEAR EQUATION

$$Ax + By = C$$

where A, B, and C are integers, and A and B are not both zero.

Steps for Graphing a Linear Function (Standard Form)

- Identify and plot the *y*-intercept
- Identify and plot the *x*-intercept
- Draw a line through the two points

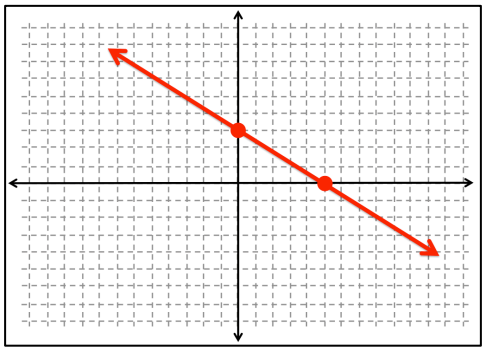
EXAMPLES

EXAMPLE 6: FINDING INTERCEPTS IN STANDARD FORM

Identify the intercepts and graph each equation.

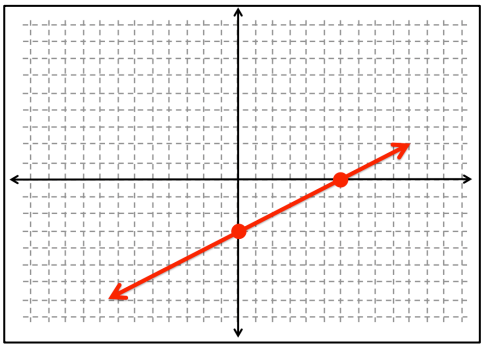
a) $3x + 5y = 15$

(5, 0)
(0, 3)



b) $2x - 4y = 12$

(6, 0)
(0, –3)



EXAMPLE 7: WRITING EQUATIONS IN STANDARD FORM

Write each equation in standard form. Use integer coefficients.

a) $y = -2x + 5$

$2x + y = 5$

b) $y + 1 = 3(x - 2)$

$-3x + y = -7$

c) $y = \frac{3}{4}x - 5$

$-3x + 4y = -20$

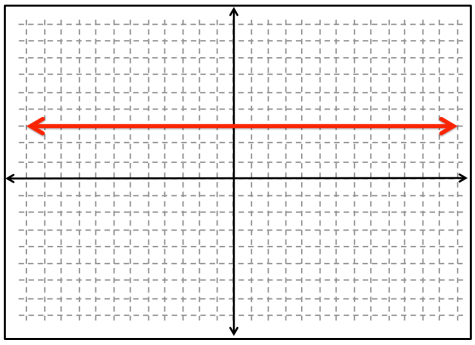
d) $y = -4.2x - 5.5$

$42x + 10y = -55$

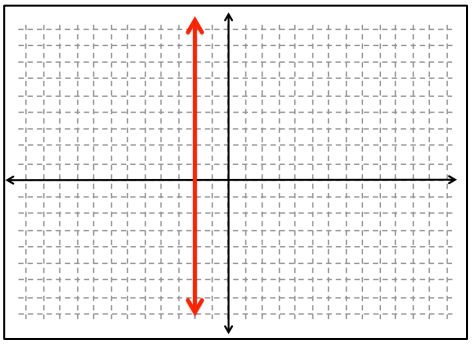
EXAMPLE 8: GRAPHING VERTICAL AND HORIZONTAL LINES

What is the graph of each equation?

a) $y = 3$



b) $x = -2$



EXAMPLE 9: USING STANDARD FORM IN REAL-WORLD SITUATIONS

You received a gift card for $100 to download songs and movies. Each songs costs $1.30 and each movie costs $20.00. Write and graph an equation that describes the items you can purchase. Give 2 examples of what you could purchase with your gift card.

$\$1.30x + \$20.00y = \$100$

Answers Vary; 0 songs and 5 movies, 30 songs and 3 movies

KEY CONCEPTS AND VOCABULARY

The slopes of Parallel Lines are equal. $m_1 = m_2$

The slopes of Perpendicular Lines are opposite reciprocals of each other. $m_1 = -\frac{1}{m_2}$

EXAMPLES

EXAMPLE 10: FINDING AN EQUATION OF A PARALLEL LINE

Write in slope-intercept form an equation of the line through (1, –3) and parallel to $y = 6x - 2$.

$y = 6x - 9$

EXAMPLE 11: FINDING AN EQUATION OF A PERPENDICULAR LINE

Write in slope-intercept form an equation of the line through (8, 5) and perpendicular to $y = -4x + 6$.

$y = \frac{1}{4}x + 3$

EXAMPLE 12: CLASSIFYING LINES

Determine if the lines are parallel, perpendicular, or neither.

a) $y = 2x - 5$
$2y = 4x - 8$

Parallel

b) $3x + 4y = 12$
$8x - 6y = -60$

Perpendicular

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.4 CONCEPT BYTE: PIECEWISE FUNCTIONS

MACC.912.F-IF.C.7b: Graph square root, cube root, and piecewise-defined functions, includingstep functions and absolute value functions.

RATING	LEARNING SCALE
4	I am able to • evaluate and graph piecewise functions in real-world applications or in more challenging problems that I have never previously attempted
TARGET 3	I am able to • evaluate and graph piecewise functions
2	I am able to • evaluate and graph piecewise functions with help
1	I am able to • understand that piecewise functions have different rules for different part of the domain

WARM UP

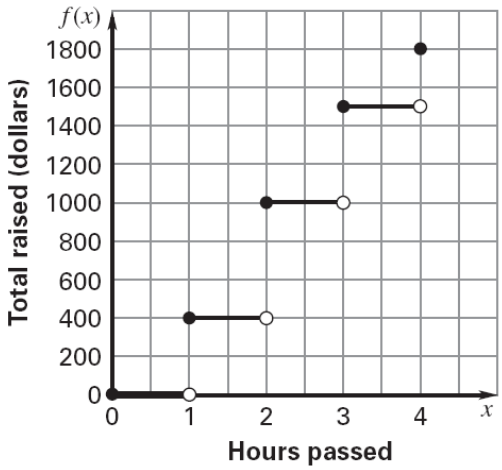
Center High School held a four-hour fundraising pledge drive. The students organizing the drive counted the total money raised at the end of each hour. The results are shown in the graph.

1) How much money had the students raised after 2 hours?
   $1000

2) How much money did they raise in all?
   $1800

3) If x is the number of hours of the drive that have passed, then the function f(x) shown by the graph gives the number of dollars raised. What is f(3.5)?
   $1500

4) On what interval does $f(x) = 400$?
   [1, 2)



KEY CONCEPTS AND VOCABULARY

Piecewise Function- A function that is represented by a combination of functions, each representing a different part of the domain.

- The graph of a piecewise function shows the different behaviors of a function over the different portions of the domain.

EXAMPLES

EXAMPLE 1: EVALUATING A PIECEWISE FUNCTION

Evaluate $f(x)$ for each of the following

$$f(x) = \begin{cases} 3x+5, & \text{if } x < 5 \\ -x+3, & \text{if } x \geq 5 \end{cases}$$

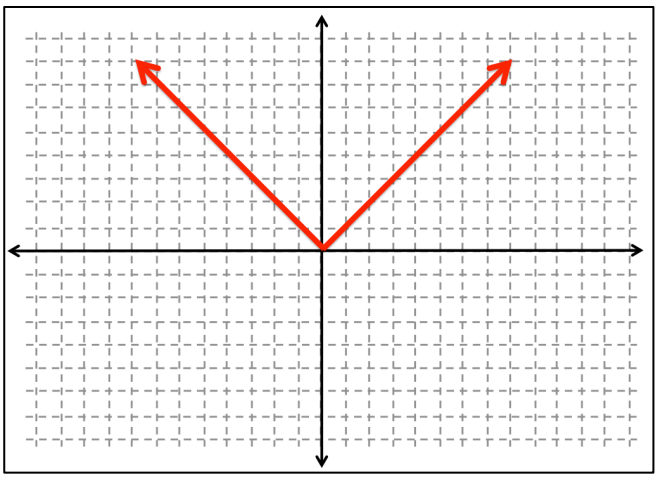
a) $f(5)$ b) $f(-4)$ c) $f(3)$ d) $f(10)$

EXAMPLE 2: GRAPHING A PIECEWISE FUNCTION

Graph the following. Identify the domain and range in interval notation.

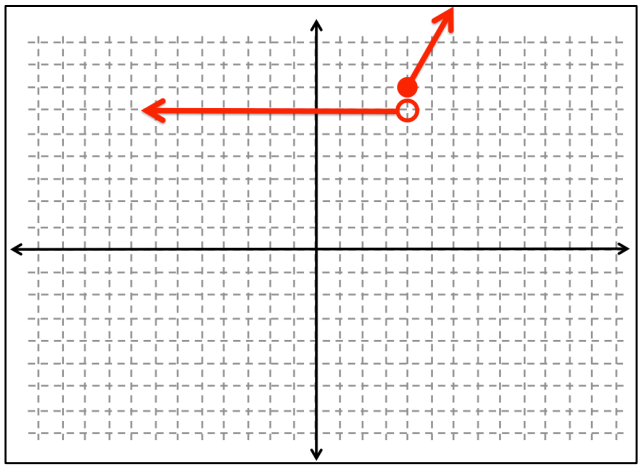
a) $f(x)=\begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

b) $h(x)=\begin{cases} 6, & x < 4 \\ 2x-1, & x \geq 4 \end{cases}$



Domain: $(-\infty,\infty)$

Range: $[0,\infty)$



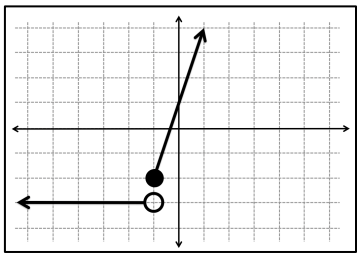
Domain: $(-\infty,\infty)$

Range: $[7,\infty)$

EXAMPLE 3: WRITING A PIECEWISE FUNCTION GIVEN A GRAPH

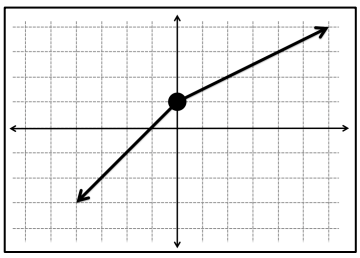
Write equations for each of the piecewise functions.

a)



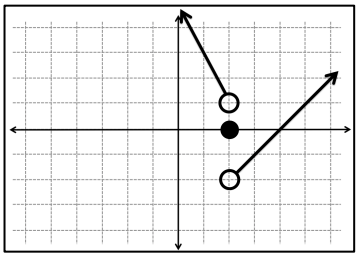
$f(x)=\begin{cases} -3, & x < -1 \\ 3x+1, & x \geq -1 \end{cases}$

b)



$f(x)=\begin{cases} x+1, & x < 0 \\ \frac{1}{2}x+1, & x \geq 0 \end{cases}$

c)



$f(x)=\begin{cases} -2x+5, & x < 2 \\ 0, & x = 2 \\ x-4, & x > 2 \end{cases}$

EXAMPLE 4: WRITING AND EVALUATING PIECEWISE FUNCTIONS IN REAL WORLD SITUATIONS

A plane descends from 5000 ft at 250 ft/min for 6 minutes. Over the next 8 minutes, it descends at 150 ft/min. Write a piecewise function for the altitude A in terms of the time t. What is the plane's altitude after 12 min?

$f(x)=\begin{cases} 5000-250x, & \text{for } 0 \leq x \leq 6 \\ 3500-150(x-6), & \text{for } 6 < x < 14 \end{cases}$

2600 ft

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.5: LINEAR MODELS

MACC.912. S-ID.B.6c: Fit a linear function for a scatter plot that suggests a linear association.
MACC.912.S-ID.B.6a: Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
MACC.912.S-ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of data.

RATING	LEARNING SCALE
4	I am able to • interpret a line of best fit by understanding the meaning of key components like intercepts and slope.
TARGET 3	I am able to • write a line of best fit and use it to make predictions
2	I am able to • write a line of best fit and use it to make predictions with help
1	I am able to • estimate the correlation for a data set

WARM UP

Write an equation for a line that goes through the points (1, 2) and (-4, 2).

$y = 2$

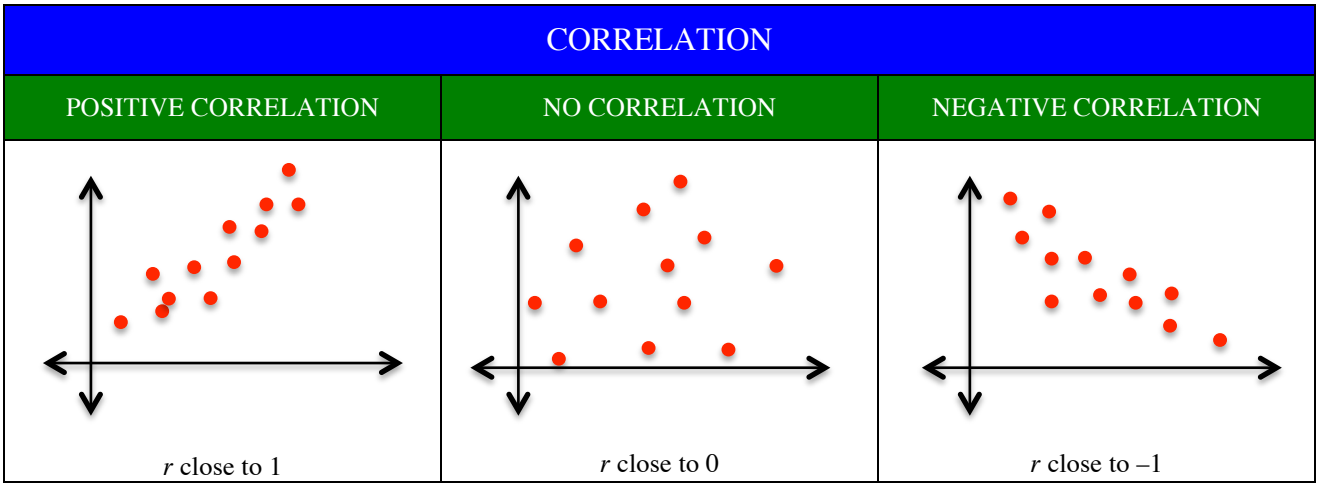
KEY CONCEPTS AND VOCABULARY

One method of visualizing two-variable data is called a scatter plot. A scatter plot is a graph of points with one variable plotted along each axis.

Correlation is a measure of the strength and direction of the relationship between two variables.

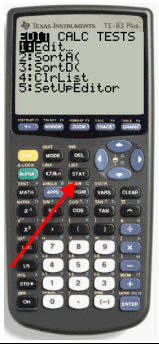
One way to quantify the correlation of a data set is with the correlation coefficient (denoted by *r)*. The correlation coefficient varies from -1 to 1. The sign of *r* corresponds to the type of correlation (positive or negative).

A line of best fit is a line through a set of two-variable data that illustrates the correlation. You can use a line of fit as the basis to construct a linear model for the data.

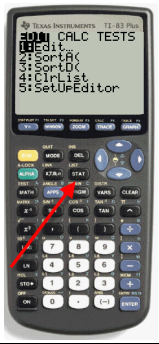


Finding the Line of Best Fit Using LinReg on a TI-83/84

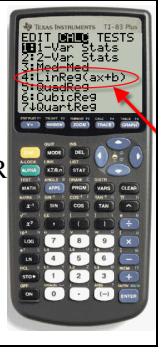
Press the STAT key.



EDIT will be highlighted, so just press ENTER. Now you need to enter your data. Usually we put the *x*-values in L_1 (list one) and the *y*-values in L_2 (list two).



Press the STAT key but this time use the right arrow key to move to the middle menu CALC and press ENTER. We want the fourth item: LinR Press ENTER

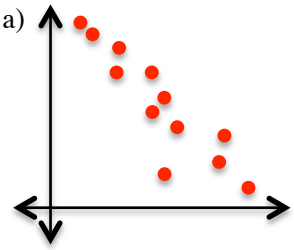


EXAMPLES

EXAMPLE 1: IDENTIFYING CORRELATION AND ESTIMATING THE CORRELATION COEFFICIENT

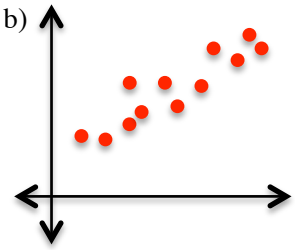
Describe the type of correlation the scatterplot shows. Estimate the value of r for each graph.

a)



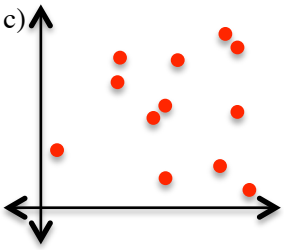
Negative

b)



Positive

c)



No Correlation

EXAMPLE 2: WRITING AN EQUATION OF A LINE OF BEST FIT

Use the data to make a scatter plot for the data below.

Heights and Arm Spans									
Height (in)	63	70	60	62	64	65	72	59	61
Arm Span (in)	62	67	60	61	63	65	70	59	60

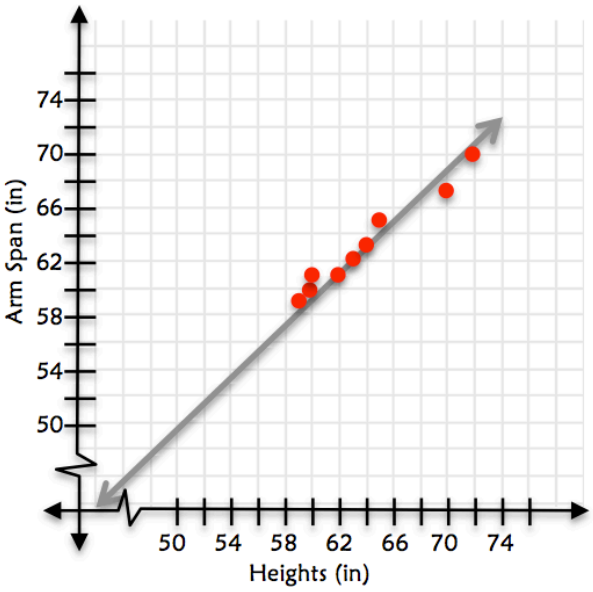
a) Draw a line of best fit.

b) Estimate the correlation coefficient.
Close to 1

c) Find the equation for the line of best fit.
$y = 0.82x + 10.5$

d) Estimate the arm span of a person who is 67 inches tall.
65.44 inches

e) Estimate the height of a person who has an arm span of 48 inches
45.7 inches

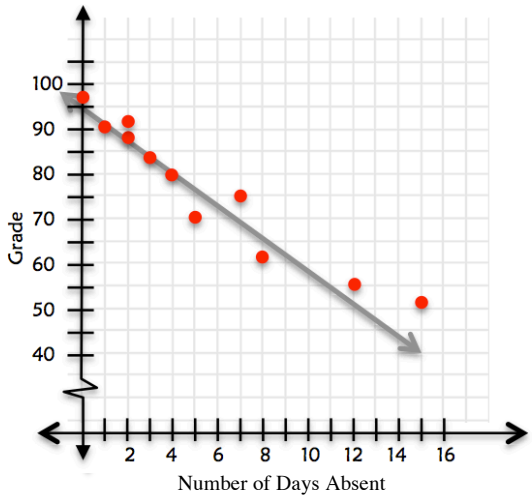


EXAMPLE 3: INTERPRETING A LINE OF BEST FIT

Use the data to make a scatter plot for the data below.

Grades and Number of Absences											
Number of Absences (days)	2	1	12	8	0	4	5	7	15	2	3
Grade	88	90	55	61	96	80	70	75	52	93	83

a) Draw a line of best fit.



b) Estimate the correlation coefficient.

Close to –1

c) Find the equation for the line of best fit.

$y = -3.1x + 93.3$

d) Estimate the grade for a student who has missed 10 days of school.

62.3%

e) Using the line of best fit from part c, what is the *x*-intercept? What does it mean in context of the problem.

(30.1, 0) The x-intercept means that if a student misses more than 30 days, they will get a 0%

f) Using the line of best fit from part c, what is the slope? What does it mean in context of the problem.

–3.1; The slope means that for every day of school that is missed, the student's grade will drop 3.1%.

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.6: FAMILIES OF FUNCTIONS

MACC.912.F-BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

RATING	LEARNING SCALE
4	I am able to • write transformed functions from parent functions in more challenging problems that I have never previously attempted
TARGET 3	I am able to • analyze and graph transformations of functions
2	I am able to • analyze and graph transformations of functions with help
1	I am able to • understand that functions can be horizontally and vertically shifted from a parent function

WARM UP

Evaluate each expression for $x = -2$, and 0.

1) $f(x) = 2x + 7$

$f(-2) = 3$

$f(0) = 7$

2) $f(x) = 3x - 2$

$f(-2) = -8$

$f(0) = -2$

KEY CONCEPTS AND VOCABULARY

TRANSFORMATIONS OF FUNCTIONS

TRANSLATIONS

A translation is a horizontal and/or a vertical shift to a graph.
The graph will have the same size and shape, but will be in a different location.

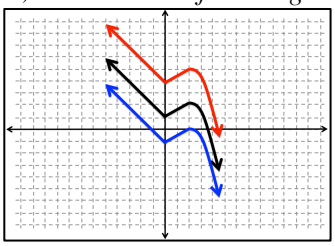
VERTICAL TRANSLATIONS

k units up if k is positive, k units down if k is negative

$y = f(x)$

$y = f(x) + 3$

$y = f(x) - 2$



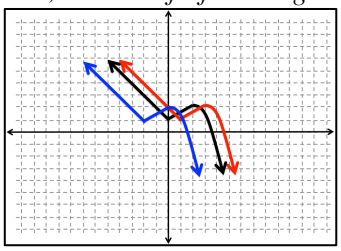
HORIZONTAL TRANSLATIONS

h units right if h is positive, h units left if h is negative

$y = f(x)$

$y = f(x - 1)$

$y = f(x + 2)$



REFLECTIONS

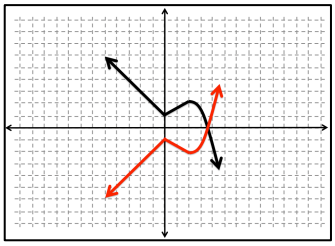
A reflection flips a graph across a line

The graph opens up if a > 0,

the graph opens down if a < 0

$y = f(x)$

$y = -f(x)$



DILATIONS

A dilation makes the graph narrower or wider than the parent function.

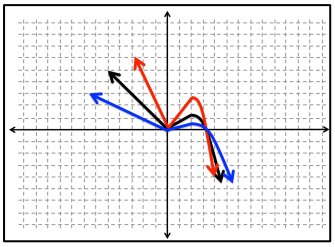
The graph is stretched if |a| > 1,

the graph is compressed if 0 < |a| < 1

$y = f(x)$

$y = \frac{1}{2} f(x)$

$y = 2f(x)$



EXAMPLES

EXAMPLE 1: IDENTIFYING TRANSFORMATIONS

Describe how the functions are related.

a) $y = 2x$ and $y = 2x + 3$

Shifted up 3 units

b) $y = x^2$ and $y = 3(x + 1)^2 - 5$

Shifted down 5 units, left 1 unit, stretched 3

EXAMPLE 2: CREATING A TABLE FOR SHIFTING FUNCTIONS

Below is a table of values for $f(x)$. Make a table for $f(x)$ after shifting the function 4 unit up.

x	$f(x)$	$f(x) + 4$
–2	4	8
–1	6	10
0	8	12
1	10	14
2	12	16
3	14	18
4	16	20
5	18	22

EXAMPLE 3: TRANSLATING FUNCTIONS

Write an equation to translate the graph.

a) $y = 4x$, 5 units down

$y = 4x - 5$

b) $y = 6x$, 3 units to the right

$y = 6(x - 3)$

EXAMPLE 4: WRITING EQUATIONS OF TRANSFORMATIONS

Write an equation for each translation of $y = x^2$.

a) 3 units up, 7 units right, reflect over x - axis

$y = -(x - 7)^2 + 3$

b) 5 units down, 1 unit left, stretch 2 units

$y = 2(x + 1)^2 - 5$

EXAMPLE 5: TRANSFORMING A FUNCTION

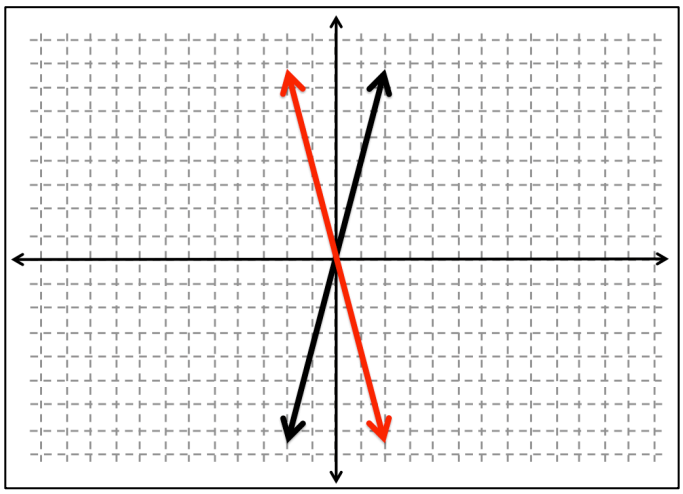
The graph of $g(x)$ is the graph of $f(x) = 6x$ compressed vertically by the factor 1/2 and then reflected in the x-axis. What is the function $g(x)$?

$g(x) = -3x$

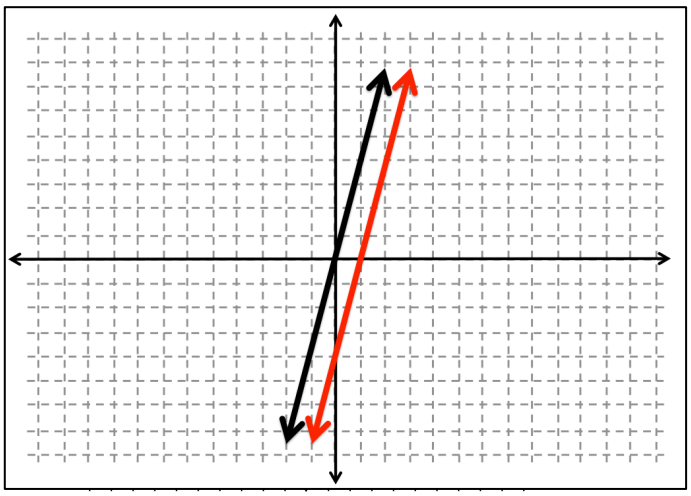
EXAMPLE 6: GRAPHING TRANSFORMATIONS

Graph $f(x) = 4x$. Graph each transformation.

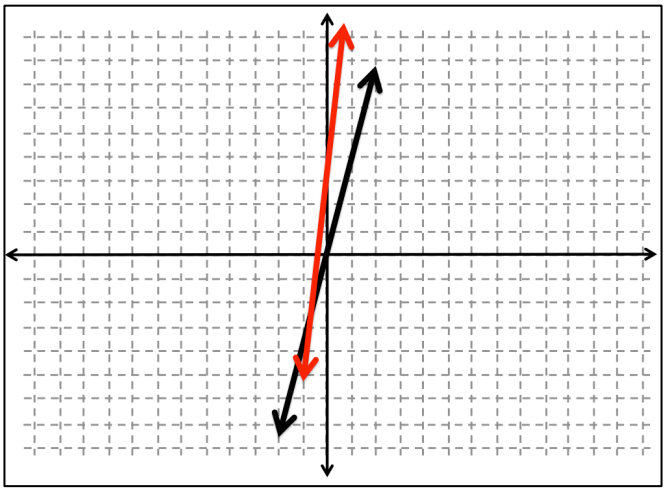
a) $g(x) = -f(x)$



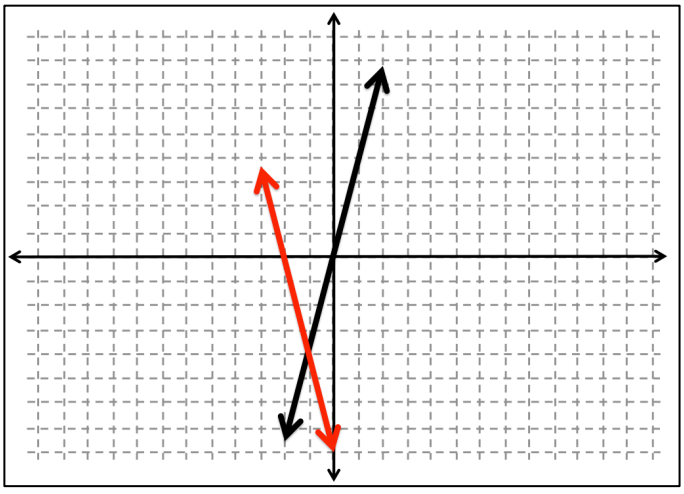
b) $g(x) = f(x - 1)$



c) $g(x) = 2f(x) + 3$



d) $g(x) = -f(x + 1) - 4$



RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.7: GRAPHING ABSOLUTE VALUE FUNCTIONS

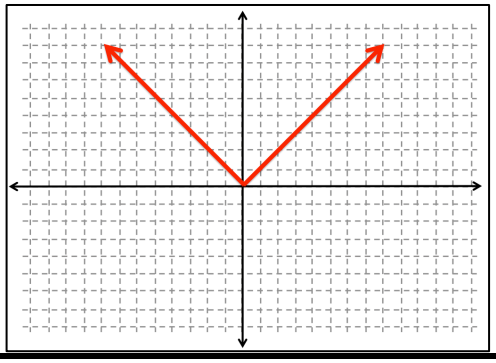
MACC.912.F-BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

MACC.912.F-IF.C.7b: Graph square root, cube root, and piecewise-defined functions, includingstep functions and absolute value functions.

RATING	LEARNING SCALE
4	I am able to • graph an absolute value function in real-world situations or more challenging problems that I have never previously attempted
TARGET 3	I am able to • graph an absolute value function
2	I am able to • graph an absolute value function with help
1	I am able to • understand the shape of the graph of an absolute value function

WARM UP

Graph the piecewise function. $f(x)=\begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$



KEY CONCEPTS AND VOCABULARY

GRAPH OF AN ABSOLUTE VALUE FUNCTION

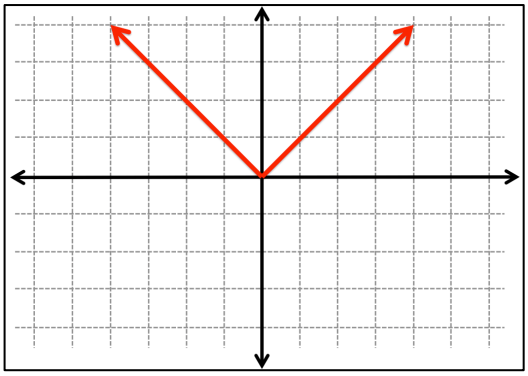
Parent Function: $f(x)=|x|$

Vertex Form: $f(x)=a|x-h|+k$

Type of Graph: V-shaped

Axis of Symmetry: $x=h$

Vertex: (h,k)



EXAMPLES

EXAMPLE 1: IDENTIFYING FEATURES OF AN ABSOLUTE VALUE FUNCTION

For each function, find the vertex and axis of symmetry.

a) $y=5|x-2|+1$

Vertex: = (2, 1)
Axis of Symmetry: $x = 2$

b) $y=|x+7|-9$

Vertex = (–7, –9)
Axis of Symmetry: $x = -7$

KEY CONCEPTS AND VOCABULARY

TRANSFORMATIONS OF ABSOLUTE VALUE FUNCTIONS

TRANSLATIONS

A translation is a horizontal and/or a vertical shift to a graph.
The graph will have the same size and shape, but will be in a different location.

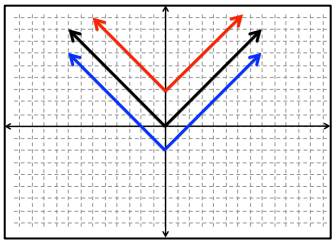
VERTICAL TRANSLATIONS

k units up if k is positive, k units down if k is negative

$y = |x|$

$y = |x| + 3$

$y = |x| - 2$



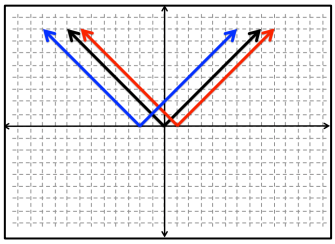
HORIZONTAL TRANSLATIONS

h units right if h is positive, h units left if h is negative

$y = |x|$

$y = |x - 1|$

$y = |x + 2|$



REFLECTIONS

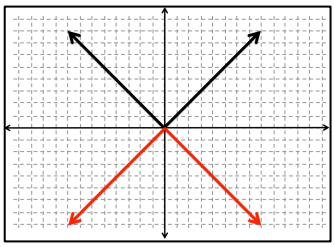
A reflection flips a graph across a line

The graph opens up if a > 0,

the graph opens down if a < 0

$y = |x|$

$y = -|x|$



DILATIONS

A dilation makes the graph narrower or wider than the parent function.

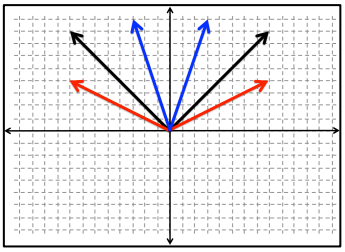
The graph is stretched if |a| > 1,

the graph is compressed if 0 < |a| < 1

$y = |x|$

$y = \frac{1}{2}|x|$

$y = 3|x|$

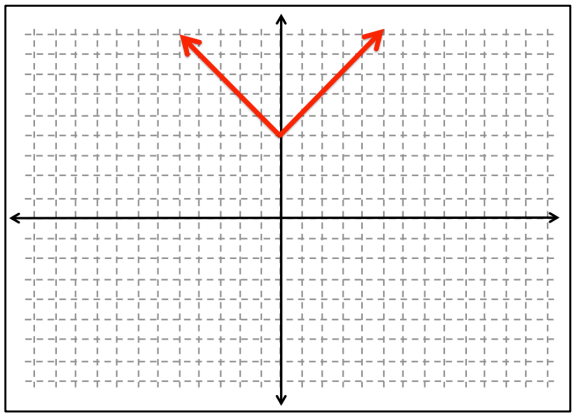


EXAMPLES

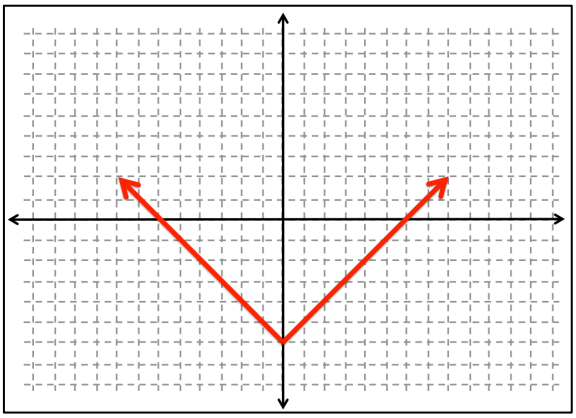
EXAMPLE 2: GRAPHING A VERTICAL TRANSLATION

Graph each absolute value function.

a) $y = |x| + 4$



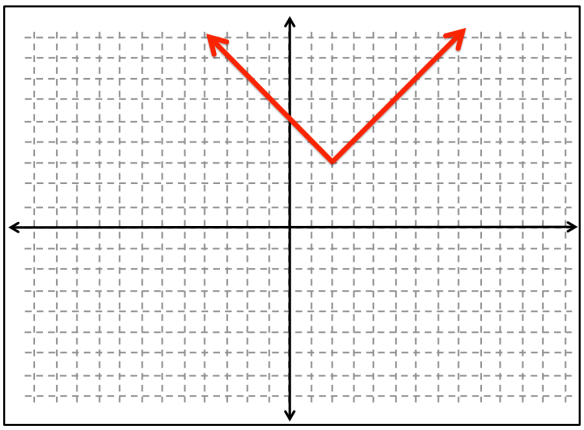
b) $y = |x| - 6$



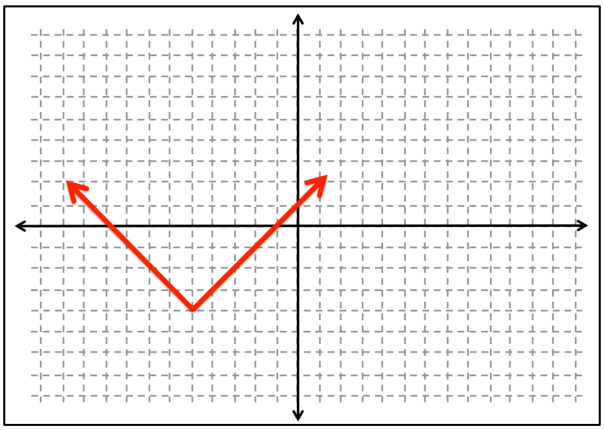
EXAMPLE 3: GRAPHING A HORIZONTAL TRANSLATION

Graph each absolute value function.

a) $y = |x - 2| + 3$



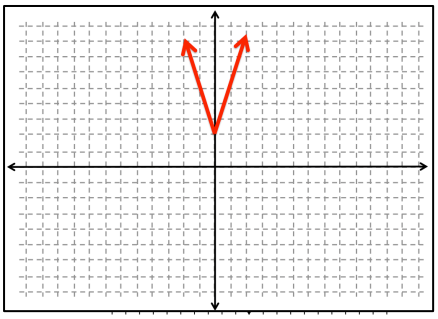
b) $y = |x + 5| - 4$



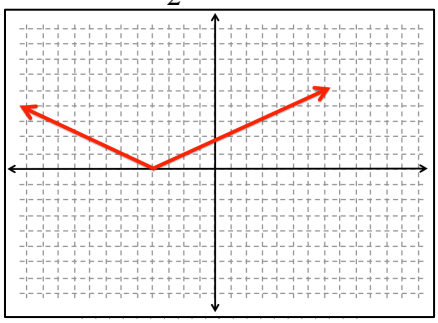
EXAMPLE 4: GRAPHING REFLECTIONS AND DILATIONS

Graph each absolute value function.

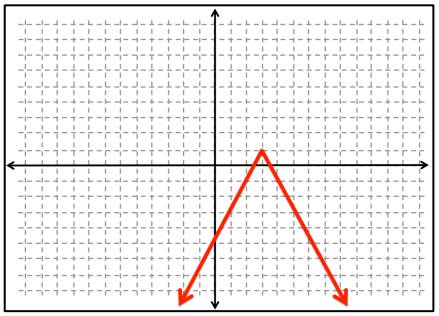
a) $y = 3|x| + 2$



b) $y = \frac{1}{2}|x + 3|$



c) $y = -2|x - 3| + 1$



EXAMPLE 5: WRITING ABSOLUTE VALUE EQUATIONS

Write the equation for each translation of the absolute value function $f(x) = |x|$.

a) left 4 units

$y = |x + 4|$

b) right 16 units

$y = |x - 16|$

c) down 12 units

$y = |x| - 12$

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1

SECTION 2.8: TWO-VARIABLE INEQUALITIES

MACC.912.A-REI.D.12: Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

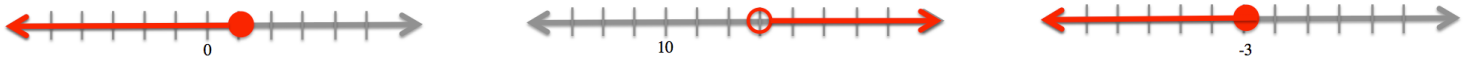
RATING	LEARNING SCALE
4	I am able to • graph linear inequalities in two variables in real-world situations or more challenging problems that I have never previously attempted
TARGET 3	I am able to • graph linear inequalities in two variables
2	I am able to • graph linear inequalities in two variables with help
1	I am able to • understand how to graph a boundary line of a linear inequality

WARM UP

Solve each inequality. Graph the solution on a number line.

1) $12p \le 15$ 2) $4 + t > 17$ 3) $5 - 2t \ge 11$

$p \le \frac{5}{4}$ $t > 13$ $t \le -3$



KEY CONCEPTS AND VOCABULARY

Linear Inequality - an inequality in two variables whose graph is a region of the coordinate plane that is bounded by a line.

Steps to Graphing a Linear Inequality

- Graph the boundary line
  - Dashed if the inequality is > or <.
  - Solid if the inequality is ≥ or ≤.
- Shade the solutions
  - Shade above the y-intercept if the inequality is ≥ or >.
  - Shade below the y-intercept if the inequality is ≤ or <.

EXAMPLES

EXAMPLE 1: IDENTIFYING SOLUTIONS OF A LINEAR INEQUALITY

Determine if the ordered pair is a solution to the linear inequality.

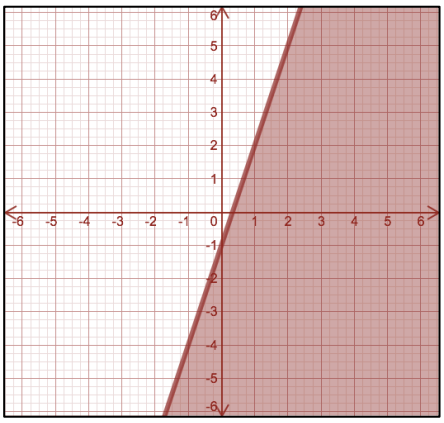
a) $y > -3x + 7; (6,1)$ b) $y \le 6x - 1; (0,3)$ c) $x \ge -4; (2,0)$

Yes No Yes

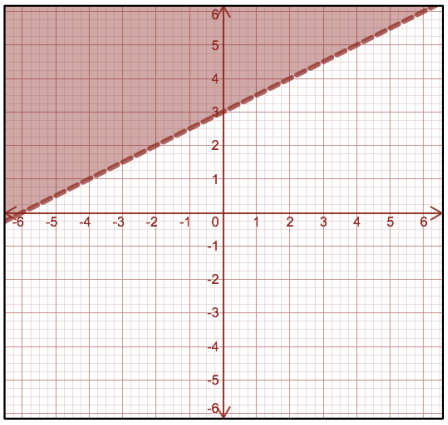
EXAMPLE 2: GRAPHING A LINEAR INEQUALITY IN TWO-VARIABLES

Graph.

a) $y \leq 3x - 1$



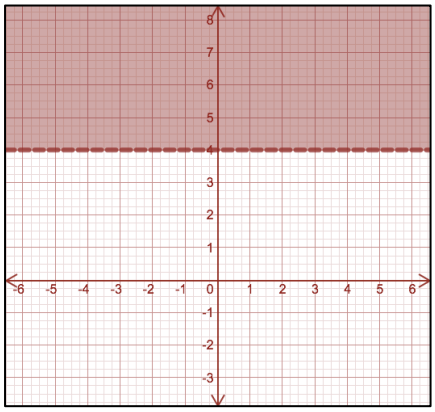
b) $y - 3 > \frac{1}{2}x$



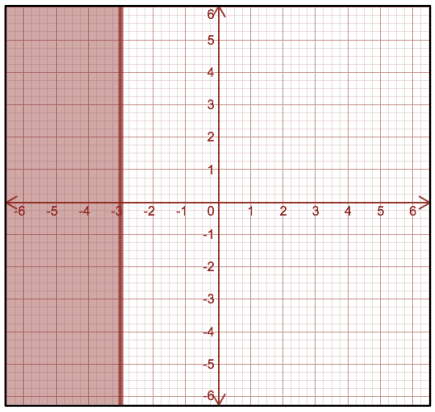
EXAMPLE 3: GRAPHING A LINEAR INEQUALITY IN ONE-VARIABLE

Graph.

a) $y > 4$

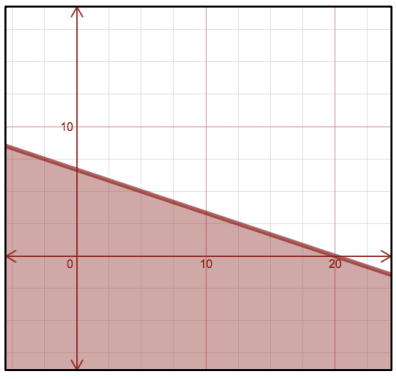


b) $x \leq -3$



EXAMPLE 4: WRITING AND SOLVING LINEAR INEQUALITIES FOR REAL WORLD SITUATIONS

A flooring company is putting 100 square feet of ceramic tile in a kitchen and 300 square feet of carpet in a bedroom. The owners can spend $2000 or less. What are two possible prices for the tile and carpet?



Samples:
$5 for tile per square foot and $5 for carpet per square foot
$10 for tile per square foot and $3.33 for carpet per square foot

RATE YOUR UNDERSTANDING *(Using the learning scale from the beginning of the lesson)*

Circle one: 4 3 2 1