	Linear and Exponential Functions *Topic A Linear and Exponential Sequences* *Topic B Functions and Their Graphs* *Topic C Transformations of Functions* *Topic D Using Functions and Graphs to Solve Problems*		**Topic A – November 14 to November 29 (10 days)** **Topic B – November 30 to December 13 (10 days)** **Review/Reteach/Assessment** **December 14 to December 14 (2 days)** **Topic C – January 4 to January 20 (12 days)** **Topic D – OPTIONAL (IF TIME)** **Review/Reteach/Assessment** **January 26 to January 27 (2 days)**
Content Standards	**Write expressions in equivalent forms to solve problems.** **A-SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12t})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. **Create equations that describe numbers or relationships** **A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **Represent and solve equations and inequalities graphically** **A-REI.D.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **Understand the concept of a function and use function notation** **F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.* **Interpret functions that arise in applications in terms of the context** **F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where*		

the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Build a function that models a relationship between two quantities

F-BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Build new functions from existing functions

F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Construct and compare linear and exponential models and solve problems

F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

	F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **Interpret expressions for functions in terms of the situation they model** **F-LE.B.5** Interpret the parameters in a linear or exponential function in terms of a context.

Technology	Graphing Calculator (***Highly Recommended for Topic C***) Lesson 5 Video How folding paper can get you to the moon (http://www.youtube.com/watch?v=AmFMJC45f1Q) Lesson 13 Video Curiosity Rover (http://mars.jpl.nasa.gov/msl/mission/timeline/edl/) Lesson 13 Video Landing Sequence (http://www.jpl.nasa.gov/video/index.php) Lesson 23 Video Newton's Law of Cooling (http://demonstrations.wolfram.com/NewtonsLawOfCooling/) Lesson 23 Video Coffee Cooling (http://demonstrations.wolfram.com/TheCoffeeCoolingProblem/)
Assessment	Possible Formative Assessments ● Mid-Module Assessment and Rubric ● End-of-Module Assessment and Rubric ● Teacher Created ● Exit Tickets ● SCA's using Kuta software Summative Assessment ● District Assessment: 2016-17 D6 AlgI Eureka Module 3AB Common Assessment and 2016-17 D6 AlgI Eureka Module 3CD Common Assessment ● Teacher Created Assessment to address other standards
Instructional Notes	***<u>Topic A Linear and Exponential Sequences</u>*** ***Problems from Module 5: Lesson 3 Exit Ticket, Lesson 5 Exercise 1-3, Exit Ticket and Problem Set 1-4*** **Lesson 1 Integer Sequences (Problem Set)** Student Outcomes: ● Students examine sequences and are introduced to the notation used to describe them. NOTE States: ● Function notation is first covered in this lesson. ● In 8th grade students define, evaluate and compare functions NOT using function notation. ● Multiple representations of writing function notation. ● Introduction to function notation in this lesson without calling attention to it.

Lesson 2 Recursive Formulas for Sequences(Problem Set)

Student Outcomes:

- Students write recursive and explicit formulas for sequences

  NOTE States:

  - The recursive definition is synthesized with exponential and linear sequences.
  - Students need to write the definition for the following sequences: 1, 2, 4, 8, 16, … as 2^{n-1}
  - The focus is to obtain a high level understanding of recursive notation.

Lesson 3 Arithmetic and Geometric Sequences (Problem Set)

Student Outcomes:

- Students learn the structure of arithmetic and geometric sequences

  NOTE States:

  - Students differentiate between the two sequences.
  - Write both explicit and recursive definitions of Arithmetic and Geometric sequences. "Lesson 4 Why Do Banks Pay YOU to Provide Their Services?"
  - Generate the Simple Interest formula $I(t)=Prt$
  - Generate the Compound Interest $FV = PV(1 + r)^t$

Lesson 4 Why do Banks Pay YOU to Provide Their Services? (Problem Set)

Student Outcomes:

- Students compare the rate of change for simple and compound interest and recognize situations in which a quantity grows by a constant percent rate per unit interval.

Lesson 5 The Power of Exponential Growth (Socratic Lesson)

Student Outcomes:

- Students are able to model with and solve problems involving exponential formulas.

  NOTE States:

  - Video “How folding paper can get you to the moon” (http://www.youtube.com/watch?v=AmFMJC45f1Q)

Lesson 6 Exponential Growth – US Population and World Population (Modeling)

Student Outcomes:

- Students compare linear and exponential models of population growth.

Lesson 7 Exponential Decay (Problem Set)

Student Outcomes:

- Students describe and analyze exponential decay models; they recognize that in a formula that models exponential decay, the growth factor bb is less than 1; or equivalently, when bb is greater than 1, exponential formulas with negative exponents could also be used to model decay

Topic B Functions and Their Graphs

Problems from Module 5: Lesson 2 Exercise 1-2 and Exit Ticket, Lesson 3 Problem Set 2

Lesson 8 Why Stay with Whole Numbers? (Problem Set)

Student Outcomes:

- Students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- Students create functions that represent a geometric situation and relate the domain of a function to its graph and to the relationship it describes.

Lesson 9-10 Representing, Naming, and Evaluating Functions (Socratic and Problem Set)

Student Outcomes:

- Students understand that a function from one set (called the domain) to another set (called the range) assigns each element of the domain to exactly one element of the range.
- Students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- Students understand that a function from one set (called the domain) to another set (called the range) assigns each element of the domain to exactly one element of the range and understand that if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.
- Students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

    NOTE States:

    - Read through lengthy explanation in lesson 9. This will aid in assessing what students know and need to know about function.
    - Lesson 10 addresses domain and range in interval notation using brackets instead of inequalities.

Lesson 11 The Graph of a Function (Exploration)

Student Outcomes:

- Students understand set builder notation for the graph of a real-valued function.
- Students learn techniques for graphing functions and relate the domain of a function to its graph.

    NOTE States:

    - The pseudocode is not a necessary component of the lesson.

Lesson 12 The Graph of the Equation *y = f(x)* (Exploration)

Student Outcomes:

- Students understand the meaning of the graph of $y = f(x)$, namely $\{(x, y) \mid x \in D \text{ and } y = f(x)\}$. Students understand the definitions of when a function is increasing or decreasing.

    NOTE States

    - The pseudocode is not a necessary component of the lesson.

Lesson 13 Interpreting the Graph of a Function (Modeling Cycle)

Student Outcomes:

- Students create tables and graphs of functions and interpret key features including intercepts, increasing and decreasing intervals, and positive and negative intervals.

  NOTE States

  - Students create tables and graphs of functions and interpret key features including intercepts, increasing and decreasing intervals, and positive and negative intervals.

Lesson 14 Linear and Exponential Models – Comparing Growth Rates (Problem Set)

Student Outcomes:

- Students compare linear and exponential models by focusing on how the models change over intervals of equal length. Students observe from tables that a function that grows exponentially eventually exceeds a function that grows linearly.

  NOTE States:

  - The lesson stresses end behavior of a linear and exponential. An exponential function overtakes the linear as x goes to infinity.

****Alg 1 Eureka Module 3AB Common Assessment 2016-17**

**** Semester 1 Final Exam**

-Topic C Transformations of Functions ****Graphing calculator is highly recommended for Topic C****

Problems from Module 5: Lesson 1 Exercises 2-6

Lesson 15 Piecewise Functions (Exploration)

Student Outcomes:

- Students examine the features of piecewise functions, including the absolute value function and step functions. Students understand that the graph of a function f is the graph of the equation $y = f(x)$.

  NOTE States:

  - *Introduction and solving Absolute Value equation needs to added in this lesson*. Making the connection to the graph of the absolute value function and absolute value equations will help the students with their reasoning of the function. Absolute value equations are not solved in Algebra II.
  - Graphing by hand.
  - Students need to do the opening exercise in order to do the explore in the lesson.
  - Introduction to the Greatest Integer function (Step Function).

Lesson 16 Graphs Can Solve Equations Too (Problem Set)

Student Outcomes:

- Students discover that the multi-step and exact way of solving $|2x - 5| = |3x + 1|$ using algebra can sometimes be avoided by recognizing that an equation of the form $f(x) = g(x)$ can be solved visually by looking for the intersection points of the graphs of $y = f(x)$ and $y = g(x)$.

  NOTE States:
  - Solving by graphing calculator with appropriate viewing window.
  - Discuss the limitations of solving by graphing; refer to closing of lesson.

Lesson 17-20 Four Interesting Transformations of Functions (Exploration, Problem Set, Exploration, Exploration)

Student Outcomes:

- Students examine that a vertical translation of the graph of $y = f(x)$ corresponds to changing the equation from $y = f(x)$ to $y = f(x) + k$.
- Students examine that a vertical scaling of the graph of $y = f(x)$ corresponds to changing the equation from $y = f(x)$ to $y = kf(x)$.
- Students examine that a horizontal translation of the graph of $y = f(x)$ corresponds to changing the equation from $y = f(x)$ to $y = f(x - k)$.
- Students examine that a horizontal scaling with scale factor k of the graph of $y = f(x)$ corresponds to changing the equation from $y = f(x)$ to $y = f(x)$ to $y = f(x-k)$.
- Students apply their understanding of transformations of functions and their graphs to piecewise functions

  NOTE States
  - Lesson 17 - Vertical translating and vertical scaling
  - Lesson 18 - Horizontal translation
  - Lesson 19 - Horizontal scaling
  - Lesson 20 - Applying transformations
  - **Eureka** uses "*k*" exclusively to describe transformations instead of *a*, *h* and *k*.
  - Students may struggle with the generalized notation *f(x) + 3 & f(x - 6).*
  - Students will write piecewise functions - introduction in lesson 15.
  - Transformations are revisited again in Module 4 Lessons 19 & 20.

Topic D Using Functions and Graphs to Solve Problems (Optional, if time permits)

Problems from Module 5: Lesson 2 Problem Set 1-4

Lesson 21 Comparing Linear and Exponential Models Again (Problem Set)

Student Outcomes:

- Students create models and understand the differences between linear and exponential models that are represented in different ways.

  NOTE States:
  - A suggestion with the opening exercise is to make cards of each square in the table and have students match in the correct columns.

Lesson 22 Modeling an Invasive Species Population(Modeling)

Student Outcomes:

- Students apply knowledge of exponential functions and transformations of functions to a contextual situation.

	Lesson 23 Newton's Law of Cooling (Modeling) Student Outcomes: ● Students apply knowledge of exponential functions and transformations of functions to a contextual situation. NOTE States: ● http://demonstrations.wolfram.com/NewtonsLawOfCooling/ ● At first , transformations do not seem relevant but then students need to describe Newton's Law with transformations. **Lesson 24 Piecewise and Step Functions in Context (Modeling)** Student Outcomes: ● Students create piecewise and step functions that relate to real-life situations and use those functions to solve problems. ● Students interpret graphs of piecewise and step functions in a real-life situation. NOTE States ● (http://www.albanyairport.com/parking_rates.php). ● Big exploratory project will take more than one day. ● Be sure to work each problem out to anticipate what students will struggle with and how long the lesson will take and how to group students. ****Alg 1 Eureka Module 3CD Common Assessment 2016-17**